CHAPTER P
Preparation for Calculus

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Section P.1 Graphs and Models

1. $y = -\frac{1}{2}x + 2$

x-intercept: $(4, 0)$

y-intercept: $(0, 2)$

Matches graph (b).

3. $y = 4 - x^2$

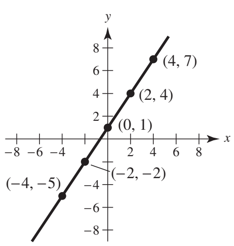
x-intercepts: $(2, 0), (-2, 0)$

y-intercept: $(0, 4)$

Matches graph (a).

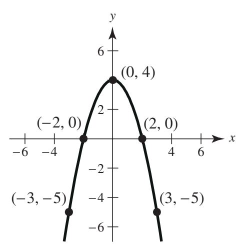
5. $y = \frac{3}{2}x + 1$

x	-4	-2	0	2	4
y	-5	-2	1	4	7



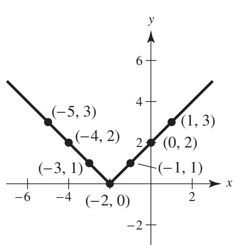
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



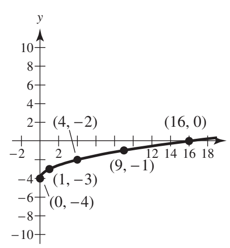
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



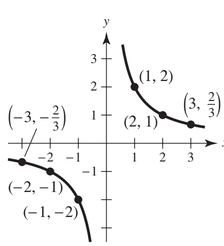
11. $y = \sqrt{x} - 4$

x	0	1	4	9	16
y	-4	-3	-2	-1	0



13. $y = \dfrac{2}{x}$

x	-3	-2	-1	0	1	2	3
y	$-\frac{2}{3}$	-1	-2	Undef.	2	1	$\frac{2}{3}$

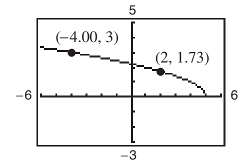


15.

Xmin = -3
Xmax = 5
Xscl = 1
Ymin = -3
Ymax = 5
Yscl = 1

Note that $y = 4$ when $x = 0$.

17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2; (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1; (-2, 0), (1, 0)$

21. $y = x^2\sqrt{25 - x^2}$

y-intercept: $y = 0^2\sqrt{25 - 0^2}$

$y = 0; (0, 0)$

x-intercepts: $0 = x^2\sqrt{25 - x^2}$

$0 = x^2\sqrt{(5 - x)(5 + x)}$

$x = 0, \pm 5; (0, 0); (\pm 5, 0)$

23. $y = \dfrac{3\left(2 - \sqrt{x}\right)}{x}$

y-intercept: None. x cannot equal 0.

x-intercept: $0 = \dfrac{3\left(2 - \sqrt{x}\right)}{x}$

$0 = 2 - \sqrt{x}$

$x = 4; (4, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept:

$0^2(y) - 0^2 + 4y = 0$

$y = 0; (0, 0)$

x-intercept:

$x^2(0) - x^2 + 4(0) = 0$

$x = 0; (0, 0)$

27. Symmetric with respect to the y-axis since

$y = (-x)^2 - 2 = x^2 - 2.$

29. Symmetric with respect to the x-axis since

$(-y)^2 = y^2 = x^3 - 4x.$

31. Symmetric with respect to the origin since

$(-x)(-y) = xy = 4.$

33. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

35. Symmetric with respect to the origin since

$$-y = \frac{-x}{(-x)^2 + 1}$$

$$y = \frac{x}{x^2 + 1}.$$

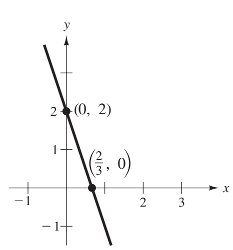
37. $y = \left|x^3 + x\right|$ is symmetric with respect to the y-axis since $y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|$.

39. $y = -3x + 2$

Intercepts:

$\left(\frac{2}{3}, 0\right), (0, 2)$

Symmetry: none

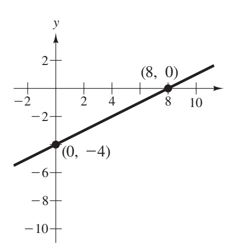


41. $y = \dfrac{1}{2}x - 4$

Intercepts:

$(8, 0), (0, -4)$

Symmetry: none

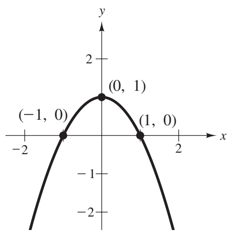


43. $y = 1 - x^2$

Intercepts:

$(1, 0), (-1, 0), (0, 1)$

Symmetry: y-axis

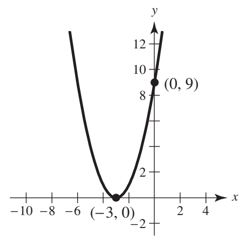


45. $y = (x + 3)^2$

Intercepts:

$(-3, 0), (0, 9)$

Symmetry: none

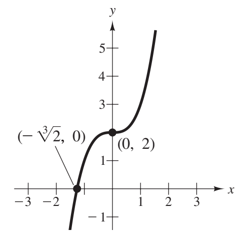


47. $y = x^3 + 2$

Intercepts:

$\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



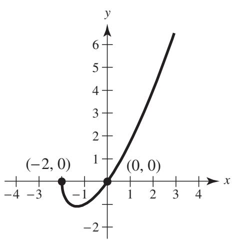
49. $y = x\sqrt{x + 2}$

Intercepts:

$(0, 0), (-2, 0)$

Symmetry: none

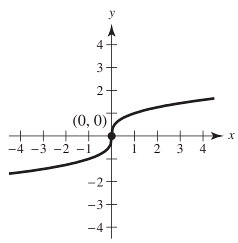
Domain: $x \geq -2$



51. $x = y^3$

Intercept: $(0, 0)$

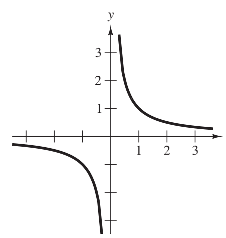
Symmetry: origin



53. $y = \dfrac{1}{x}$

Intercepts: none

Symmetry: origin

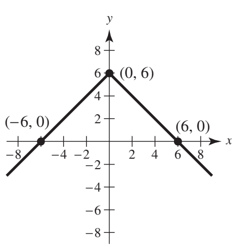


55. $y = 6 - |x|$

Intercepts:

$(0, 6), (-6, 0), (6, 0)$

Symmetry: y-axis



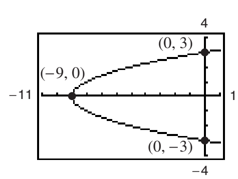
57. $y^2 - x = 9$

$$y^2 = x + 9$$
$$y = \pm\sqrt{x + 9}$$

Intercepts:

$(0, 3), (0, -3), (-9, 0)$

Symmetry: x-axis



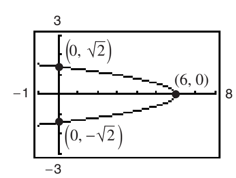
59. $x + 3y^2 = 6$

$$3y^2 = 6 - x$$
$$y = \pm\sqrt{2 - \frac{x}{3}}$$

Intercepts:

$(6, 0), \left(0, \sqrt{2}\right), \left(0, -\sqrt{2}\right)$

Symmetry: x-axis



61. $x + y = 2 \Rightarrow y = 2 - x$

$2x - y = 1 \Rightarrow y = 2x - 1$

$$2 - x = 2x - 1$$
$$3 = 3x$$
$$1 = x$$

The corresponding y-value is $y = 1$.

Point of intersection: $(1, 1)$

63. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$$6 - x^2 = 4 - x$$
$$0 = x^2 - x - 2$$
$$0 = (x - 2)(x + 1)$$
$$x = 2, -1$$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2), (-1, 5)$

65. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$$5 - x^2 = (x - 1)^2$$
$$5 - x^2 = x^2 - 2x + 1$$
$$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ and $y = 1$.

Points of intersection: $(-1, -2), (2, 1)$

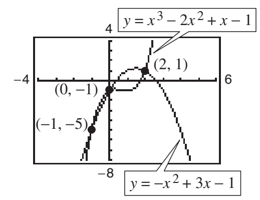
67.
$$y = x^3$$
$$y = x$$
$$x^3 = x$$
$$x^3 - x = 0$$
$$x(x + 1)(x - 1) = 0$$
$$x = 0, x = -1, \text{ or } x = 1$$

The corresponding y-values are $y = 0$, $y = -1$, and $y = 1$.

Points of intersection: $(0, 0), (-1, -1), (1, 1)$

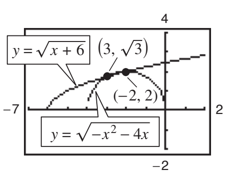
69.
$$y = x^3 - 2x^2 + x - 1$$
$$y = -x^2 + 3x - 1$$
$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$
$$x^3 - x^2 - 2x = 0$$
$$x(x - 2)(x + 1) = 0$$
$$x = -1, 0, 2$$

$(-1, -5), (0, -1), (2, 1)$



71. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



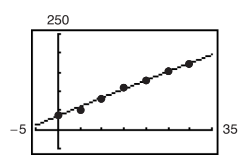
Points of intersection: $(-2, 2), \left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically,
$$\sqrt{x + 6} = \sqrt{-x^2 - 4x}$$
$$x + 6 = -x^2 - 4x$$
$$x^2 + 5x + 6 = 0$$
$$(x + 3)(x + 2) = 0$$
$$x = -3, y = \sqrt{3} \Rightarrow \left(-3, \sqrt{3}\right)$$
$$x = -2, y = 2 \Rightarrow \left(-2, 2\right).$$

73. (a) Using a graphing utility, you obtain

$y = -0.007t^2 + 4.82t + 35.4.$

(b)



(c) For 2010, $t = 40$ and $y = 217$.

75.
$$C = R$$
$$5.5\sqrt{x} + 10{,}000 = 3.29x$$
$$\left(5.5\sqrt{x}\right)^2 = (3.29x - 10{,}000)^2$$
$$30.25x = 10.8241x^2 - 65{,}800x + 100{,}000{,}000$$
$$0 = 10.8241x^2 - 65{,}830.25x + 100{,}000{,}000 \qquad \text{Use the Quadratic Formula.}$$
$$x \approx 3133 \text{ units}$$

The other root, $x \approx 2949$, does not satisfy the equation $R = C$.

This problem can also be solved by using a graphing utility and finding the intersection of the graphs of C and R.

77. $y = (x + 2)(x - 4)(x - 6)$ (other answers possible)

79. (i) $y = kx + 5$ matches (b).

Use $(1, 7)$:
$$7 = k(1) + 5 \Rightarrow k = 2, \text{ thus, } y = 2x + 5.$$

(ii) $y = x^2 + k$ matches (d).

Use $(1, -9)$:

$-9 = (1)^2 + k \Rightarrow k = -10$, thus, $y = x^2 - 10$.

(iii) $y = kx^{3/2}$ matches (a).

Use $(1, 3)$: $3 = k(1)^{3/2} \Rightarrow k = 3$, thus, $y = 3x^{3/2}$.

(iv) $xy = k$ matches (c).

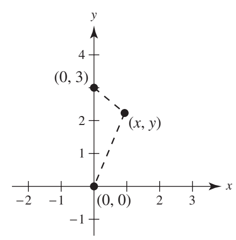
Use $(1, 36)$: $(1)(36) = k \Rightarrow k = 36$, thus, $xy = 36$.

81. False; x-axis symmetry means that if $(1, -2)$ is on the graph, then $(1, 2)$ is also on the graph.

83. True; the x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

85.
$$\begin{aligned}
2\sqrt{(x - 0)^2 + (y - 3)^2} &= \sqrt{(x - 0)^2 + (y - 0)^2}\\
4[x^2 + (y - 3)^2] &= x^2 + y^2\\
4x^2 + 4y^2 - 24y + 36 &= x^2 + y^2\\
3x^2 + 3y^2 - 24y + 36 &= 0\\
x^2 + y^2 - 8y + 12 &= 0\\
x^2 + (y - 4)^2 &= 4
\end{aligned}$$

Circle of radius 2 and center $(0, 4)$



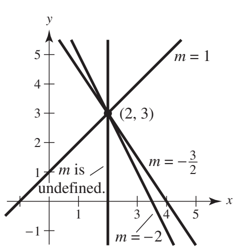
Section P.2 Linear Models and Rates of Change

1. $m = 1$

3. $m = 0$

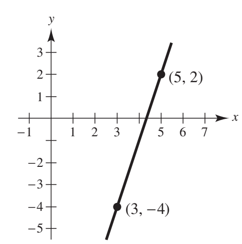
5. $m = -12$

7.



9. $m = \dfrac{2 - (-4)}{5 - 3}$

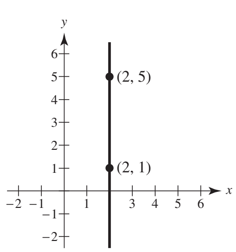
$= \dfrac{6}{2} = 3$



11. $m = \dfrac{5 - 1}{2 - 2}$

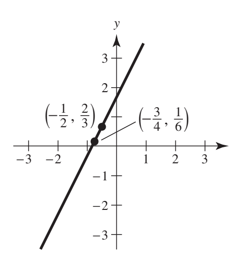
$= \dfrac{4}{0}$

undefined



13. $m = \dfrac{2/3 - 1/6}{-1/2 - (-3/4)}$

$= \dfrac{1/2}{1/4} = 2$



15. Since the slope is 0, the line is horizontal and its equation is $y = 1$.
Therefore, three additional points are $(0, 1)$, $(1, 1)$, and $(3, 1)$.

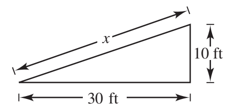
17. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are $(0, 10)$, $(2, 4)$, and $(3, 1)$.

19. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

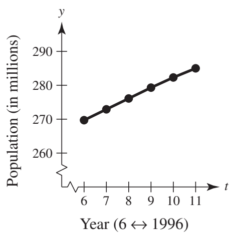
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

21. (a)



(b) The slopes of the line segments are:

$$\frac{272.9 - 269.7}{7 - 6} = 3.2$$
$$\frac{276.1 - 272.9}{8 - 7} = 3.2$$
$$\frac{279.3 - 276.1}{9 - 8} = 3.2$$
$$\frac{282.3 - 279.3}{10 - 9} = 3.0$$
$$\frac{285.0 - 282.3}{11 - 10} = 2.7$$

The population increased least rapidly between 2000 and 2001.

23. $x + 5y = 20$

$$y = -\tfrac{1}{5}x + 4$$

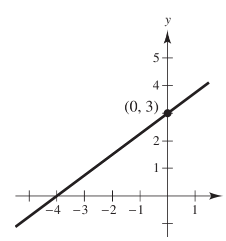
Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is $(0, 4)$.

25. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

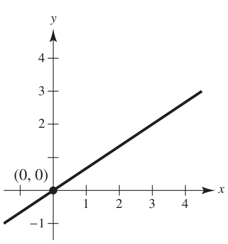
27.

$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$



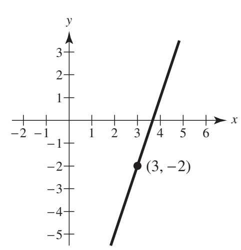
29.

$$y = \tfrac{2}{3}x$$
$$3y = 2x$$
$$2x - 3y = 0$$



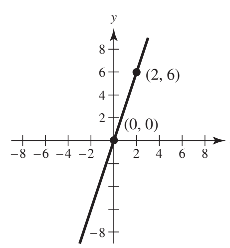
31.

$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$y - 3x + 11 = 0$$



33. $m = \dfrac{6 - 0}{2 - 0} = 3$

$$y - 0 = 3(x - 0)$$
$$y = 3x$$

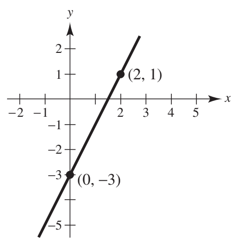


35. $m = \dfrac{1 - (-3)}{2 - 0} = 2$

$y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$0 = 2x - y - 3$

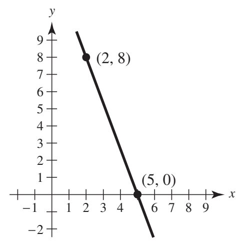


37. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$y - 0 = -\dfrac{8}{3}(x - 5)$

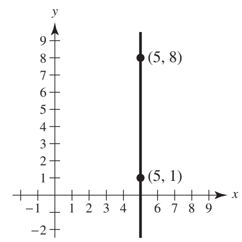
$y = -\dfrac{8}{3}x + \dfrac{40}{3}$

$3y + 8x - 40 = 0$



39. $m = \dfrac{8 - 1}{5 - 5}$ Undefined

Vertical line $x = 5$

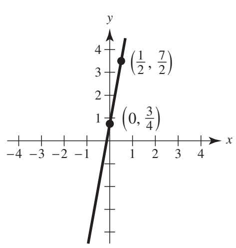


41. $m = \dfrac{7/2 - 3/4}{1/2 - 0} = \dfrac{11/4}{1/2} = \dfrac{11}{2}$

$y - \dfrac{3}{4} = \dfrac{11}{2}(x - 0)$

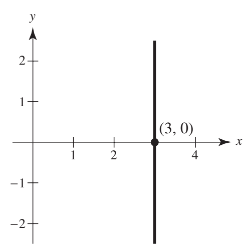
$y = \dfrac{11}{2}x + \dfrac{3}{4}$

$22x - 4y + 3 = 0$



43. $x = 3$

$x - 3 = 0$



45. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

47. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{1}{a} + \dfrac{2}{a} = 1$

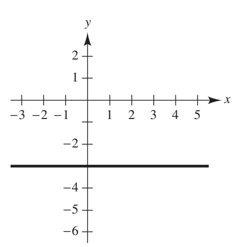
$\dfrac{3}{a} = 1$

$a = 3 \Longrightarrow x + y = 3$

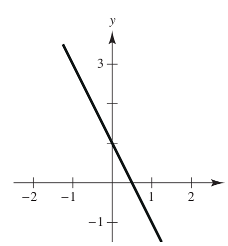
$x + y - 3 = 0$

49. $y = -3$

$y + 3 = 0$



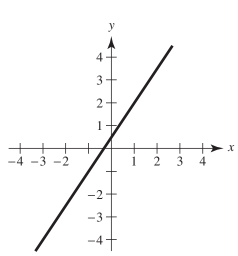
51. $y = -2x + 1$



53. $y - 2 = \frac{3}{2}(x - 1)$

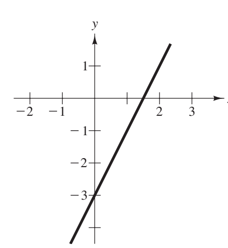
$y = \frac{3}{2}x + \frac{1}{2}$

$2y - 3x - 1 = 0$

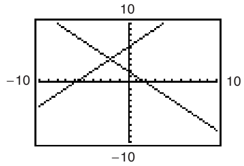


55. $2x - y - 3 = 0$

$y = 2x - 3$

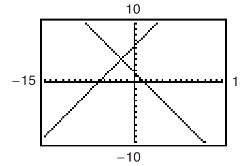


57. (a)



The lines do not appear perpendicular.

(b)



The lines appear perpendicular.

The lines are perpendicular because their slopes 1 and -1 are negative reciprocals of each other. You must use a square setting in order for perpendicular lines to appear perpendicular. Answers depend on calculator used.

59. $4x - 2y = 3$

$$y = 2x - \tfrac{3}{2}$$

$$m = 2$$

(a) $$y - 1 = 2(x - 2)$$
$$y - 1 = 2x - 4$$
$$2x - y - 3 = 0$$

(b) $$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$

61. $5x - 3y = 0$

$$y = \tfrac{5}{3}x$$

$$m = \tfrac{5}{3}$$

(a) $$y - \tfrac{7}{8} = \tfrac{5}{3}\left(x - \tfrac{3}{4}\right)$$
$$24y - 21 = 40x - 30$$
$$24y - 40x + 9 = 0$$

(b) $$y - \tfrac{7}{8} = -\tfrac{3}{5}\left(x - \tfrac{3}{4}\right)$$
$$40y - 35 = -24x + 18$$
$$40y + 24x - 53 = 0$$

63. The given line is vertical.

(a) $x = 2 \Rightarrow x - 2 = 0$

(b) $y = 5 \Rightarrow y - 5 = 0$

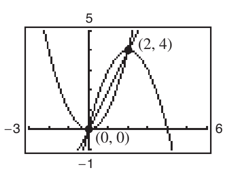
65. The slope is 125. $V = 2540$ when $t = 4$.

$V = 125(t - 4) + 2540 = 125t + 2040$

67. The slope is -2000. $V = 20{,}400$ when $t = 4$.

$V = -2000(t - 4) + 20{,}400 = -2000t + 28{,}400$

69.



You can use the graphing utility to determine that the points of intersection are (0, 0) and (2, 4). Analytically,

$$x^2 = 4x - x^2$$
$$2x^2 - 4x = 0$$
$$2x(x - 2) = 0$$
$$x = 0 \Rightarrow y = 0 \Rightarrow (0, 0)$$
$$x = 2 \Rightarrow y = 4 \Rightarrow (2, 4).$$

The slope of the line joining (0, 0) and (2, 4) is $m = (4 - 0)/(2 - 0) = 2$. Hence, an equation of the line is

$$y - 0 = 2(x - 0)$$
$$y = 2x.$$

71. $$m_1 = \frac{1 - 0}{-2 - (-1)} = -1$$

$$m_2 = \frac{-2 - 0}{2 - (-1)} = -\frac{2}{3}$$

$$m_1 \neq m_2$$

The points are not collinear.

73. Equations of perpendicular bisectors:

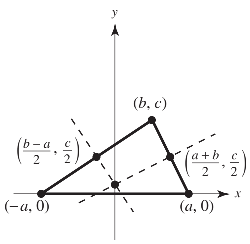
$$y - \frac{c}{2} = \frac{a-b}{c}\left(x - \frac{a+b}{2}\right)$$
$$y - \frac{c}{2} = \frac{a+b}{-c}\left(x - \frac{b-a}{2}\right)$$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$$\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right).$$

This point lies on the third perpendicular bisector, $x = 0$.

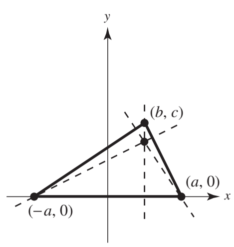


75. Equations of altitudes:

$$y = \frac{a-b}{c}(x + a)$$
$$x = b$$
$$y = -\frac{a+b}{c}(x - a)$$

Solving simultaneously, the point of intersection is

$$\left(b, \frac{a^2 - b^2}{c}\right).$$



77. Find the equation of the line through the points $(0, 32)$ and $(100, 212)$.

$$m = \tfrac{180}{100} = \tfrac{9}{5}$$
$$F - 32 = \tfrac{9}{5}(C - 0)$$
$$F = \tfrac{9}{5}C + 32$$

or

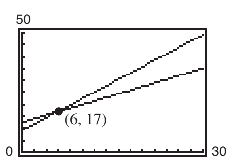
$$C = \tfrac{1}{9}(5F - 160)$$
$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

79. (a) $W_1 = 0.75x + 12.50$

$W_2 = 1.30x + 9.20$

(b)



Using a graphing utility, the point of intersection is $(6, 17)$.
Analytically,

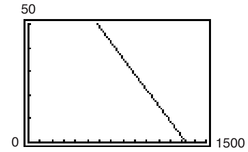
$$0.75x + 12.50 = 1.30x + 9.20$$
$$3.3 = 0.55x \Rightarrow x = 6$$
$$y = 0.75(6) + 12.50 = 17.$$

(c) Both jobs pay \$17 per hour if 6 units are produced. For someone who can produce more than 6 units per hour, the second offer would pay more. For a worker who produces less than 6 units per hour, the first offer pays more.

81. (a) Two points are $(50, 580)$ and $(47, 625)$. The slope is

$$m = \frac{625 - 580}{47 - 50} = -15.$$
$$p - 580 = -15(x - 50)$$
$$p = -15x + 750 + 580 = -15x + 1330$$
$$\text{or } x = \frac{1}{15}(1330 - p)$$

(b)



If $p = 655$, $x = \frac{1}{15}(1330 - 655) = 45$ units.

(c) If $p = 595$, $x = \frac{1}{15}(1330 - 595) = 49$ units.

83. The tangent line is perpendicular to the line joining the point $(5, 12)$ and the center $(0, 0)$.

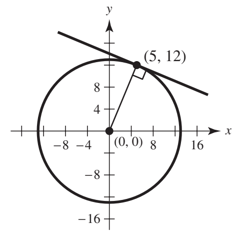
Slope of the line joining $(5, 12)$ and $(0, 0)$ is $\frac{12}{5}$.

The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$

$$y = \frac{-5}{12}x + \frac{169}{12}$$

$$12y + 5x - 169 = 0.$$



85. $4x + 3y - 10 = 0 \Longrightarrow d = \dfrac{|4(0) + 3(0) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{10}{5} = 2$

87. $x - y - 2 = 0 \Longrightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

89. A point on the line $x + y = 1$ is $(0, 1)$. The distance from the point $(0, 1)$ to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

91. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.)

The slope of the line $Ax + By + C = 0$ is $-A/B$. The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$

$$Ay - Ay_1 = Bx - Bx_1$$

$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Longrightarrow \quad A^2x + ABy = -AC \qquad (1)$$

$$Bx - Ay = Bx_1 - Ay_1 \Longrightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2)$$

$$(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$

$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Longrightarrow \quad ABx + B^2y = -BC \qquad (3)$$

$$Bx - Ay = Bx_1 - Ay_1 \Longrightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4)$$

$$(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$

$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

—CONTINUED—

91. —CONTINUED—

The distance between (x_1, y_1) and this point gives us the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$\begin{aligned} d &= \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2} \\ &= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2} \\ &= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} \\ &= \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} \\ &= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

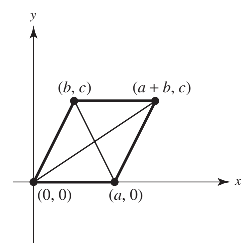
93. For simplicity, let the vertices of the rhombus be $(0, 0)$, $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure. The slopes of the diagonals are then

$$m_1 = \frac{c}{a + b} \text{ and } m_2 = \frac{c}{b - a}.$$

Since the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and we have

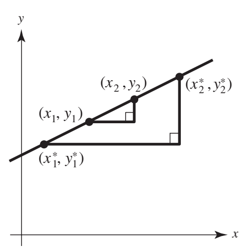
$$m_1m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



95. Consider the figure below in which the four points are collinear. Since the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



97. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

Section P.3 Functions and Their Graphs

1. (a) Domain of f: $-4 \le x \le 4$

Range of f: $-3 \le y \le 5$

Domain of g: $-3 \le x \le 3$

Range of g: $-4 \le y \le 4$

(b) $f(-2) = -1$

$g(3) = -4$

(c) $f(x) = g(x)$ for $x = -1$

(d) $f(x) = 2$ for $x = 1$

(e) $g(x) = 0$ for $x = -1$, 1 and 2

3. (a) $f(0) = 2(0) - 3 = -3$

(b) $f(-3) = 2(-3) - 3 = -9$

(c) $f(b) = 2b - 3$

(d) $f(x - 1) = 2(x - 1) - 3 = 2x - 5$

5. (a) $g(0) = 3 - 0^2 = 3$

(b) $g(\sqrt{3}) = 3 - (\sqrt{3})^2 = 3 - 3 = 0$

(c) $g(-2) = 3 - (-2)^2 = 3 - 4 = -1$

(d) $g(t-1) = 3 - (t-1)^2 = -t^2 + 2t + 2$

7. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

9. $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \dfrac{(x+\Delta x)^3 - x^3}{\Delta x} = \dfrac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2, \Delta x \neq 0$

11. $\dfrac{f(x) - f(2)}{x-2} = \dfrac{(1/\sqrt{x-1} - 1)}{x-2}$

$$= \frac{1-\sqrt{x-1}}{(x-2)\sqrt{x-1}} \cdot \frac{1+\sqrt{x-1}}{1+\sqrt{x-1}} = \frac{2-x}{(x-2)\sqrt{x-1}(1+\sqrt{x-1})} = \frac{-1}{\sqrt{x-1}(1+\sqrt{x-1})}, x \neq 2$$

13. $h(x) = -\sqrt{x+3}$

Domain: $x + 3 \geq 0 \Longrightarrow [-3, \infty)$

Range: $(-\infty, 0]$

15. $f(t) = \sec\frac{\pi t}{4}$

$\frac{\pi t}{4} \neq \frac{(2k+1)\pi}{2} \Longrightarrow t \neq 4k + 2$

Domain: all $t \neq 4k + 2$, k an integer

Range: $(-\infty, -1] \cup [1, \infty)$

17. $f(x) = \frac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

19. $f(x) = \sqrt{x} + \sqrt{1-x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1$

21. $g(x) = \dfrac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n an integer

23. $f(x) = \dfrac{1}{|x+3|}$

$|x+3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

25. $f(x) = \begin{cases} 2x+1, & x < 0 \\ 2x+2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2+1) = 2(t^2+1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

27. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2+1) = -(b^2+1) + 1 = -b^2$

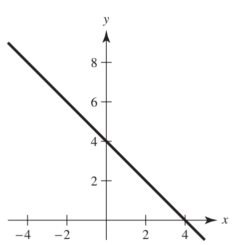
Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

29. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

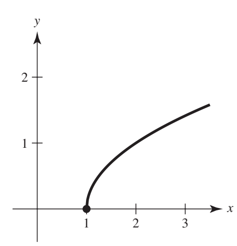
Range: $(-\infty, \infty)$



31. $h(x) = \sqrt{x-1}$

Domain: $[1, \infty)$

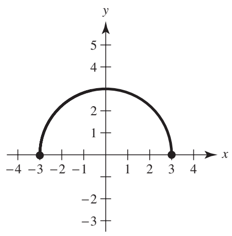
Range: $[0, \infty)$



33. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

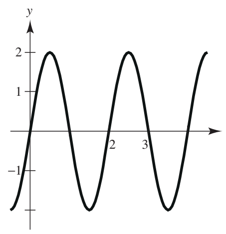
Range: $[0, 3]$



35. $g(t) = 2 \sin \pi t$

Domain: $(-\infty, \infty)$

Range: $[-2, 2]$



37. The student travels $\dfrac{2-0}{4-0} = \dfrac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the following 2 minutes. Finally, the student travels $\dfrac{6-2}{10-6} = 1$ mi/min during the final 4 minutes.

39. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

41. y is a function of x. Vertical lines intersect the graph at most once.

43. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

y is not a function of x since there are two values of y for some x.

45. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

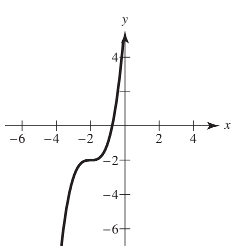
y is not a function of x since there are two values of y for some x.

47. $y = f(x + 5)$ is a horizontal shift 5 units to the left. Matches d.

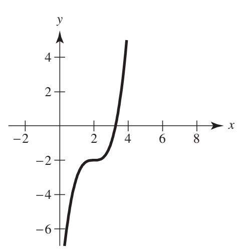
49. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

51. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

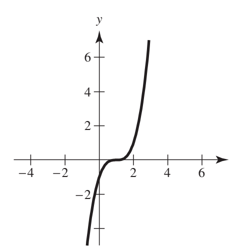
53. (a) The graph is shifted 3 units to the left.



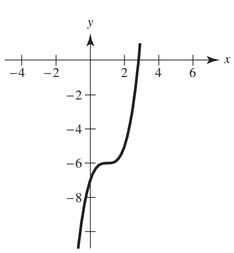
(b) The graph is shifted 1 unit to the right.



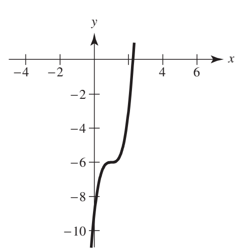
(c) The graph is shifted 2 units upward.



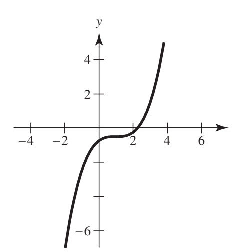
(d) The graph is shifted 4 units downward.



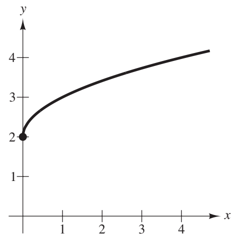
(e) The graph is stretched vertically by a factor of 3.



(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.

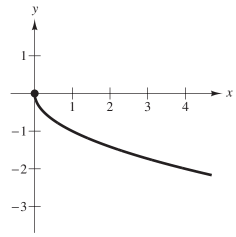


55. (a) $y = \sqrt{x} + 2$



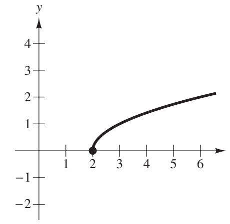
Vertical shift 2 units upward

(b) $y = -\sqrt{x}$



Reflection about the x-axis

(c) $y = \sqrt{x - 2}$



Horizontal shift 2 units to the right

57. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \quad (x \geq 0)$

59. $f(x) = x^2$, $g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x, \quad x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different.
$(f \circ g) = (g \circ f)$ for $x \geq 0$.

61. $f(x) = \dfrac{3}{x}$, $g(x) = x^2 - 1$

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \frac{3}{x^2 - 1}$$

Domain: all $x \neq \pm 1$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1 = \frac{9}{x^2} - 1 = \frac{9 - x^2}{x^2}$$

Domain: all $x \neq 0$

No, $f \circ g \neq g \circ f$.

63. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1) = f(-4)$, which is undefined

65. $F(x) = \sqrt{2x - 2}$

Let $h(x) = x - 1$, $g(x) = 2x$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(x - 1)) = f(2(x - 1)) = \sqrt{2(x - 1)} = \sqrt{2x - 2} = F(x)$.

[Other answers possible]

67. $f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

Even

69. $f(-x) = (-x)\cos(-x) = -x \cos x = -f(x)$

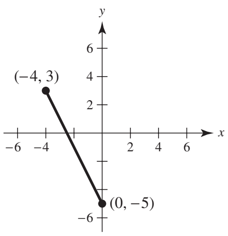
Odd

71. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

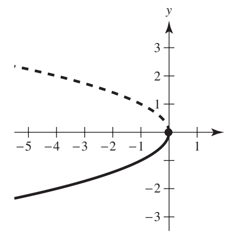
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

73. f is even because the graph is symmetric about the y-axis.
g is neither even nor odd.
h is odd because the graph is symmetric about the origin.

75.
$$\text{Slope} = \frac{3+5}{-4-0} = -2$$
$$y + 5 = -2(x - 0)$$
$$y = -2x - 5$$
$$f(x) = -2x - 5, \quad -4 \le x \le 0$$



77.
$$x + y^2 = 0$$
$$y^2 = -x$$
$$y = -\sqrt{-x}$$
$$f(x) = -\sqrt{-x}, \quad x \le 0$$



79. Matches (ii). The function is $g(x) = cx^2$. Since $(1, -2)$ satisfies the equation, $c = -2$. Thus, $g(x) = -2x^2$.

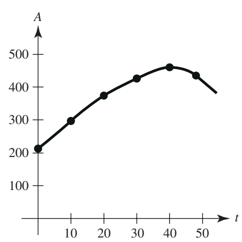
81. Matches (iv). The function is $r(x) = c/x$, since it must be undefined at $x = 0$. Since $(1, 32)$ satisfies the equation, $c = 32$. Thus, $r(x) = 32/x$.

83. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the program would turn on (and off) one hour later.

(c) If $H(t) = T(t) - 1$, then the overall temperature would be reduced 1 degree.

85. (a)



(b) $A(15) \approx 345$ acres/farm

87. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2 = 2(1 - x)$.

If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2 = 2(x - 1)$.

Thus,
$$f(x) = \begin{cases} 2(1 - x), & x < 0 \\ 2, & 0 \le x < 2 \\ 2(x - 1), & x \ge 2 \end{cases}.$$

89.
$$\begin{aligned} f(-x) &= a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x) \\ &= -[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x] \\ &= -f(x) \end{aligned}$$

Odd

91. Let $F(x) = f(x)g(x)$ where f and g are even. Then

$$F(-x) = f(-x)g(-x) = f(x)g(x) = F(x).$$

Thus, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

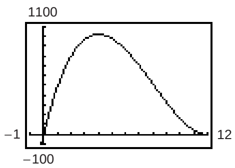
$$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x).$$

Thus, $F(x)$ is even.

93. (a) $V = x(24 - 2x)^2 = 4x(12 - x^2)$

Domain: $0 < x < 12$

(b)



The dimensions for maximum volume are $4 \times 16 \times 16$ cm.

(c)

x	*length and width*	*volume*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

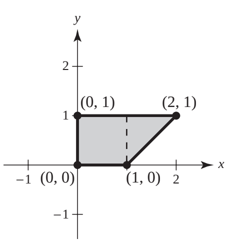
The dimensions for maximum volume appear to be $4 \times 16 \times 16$ cm.

95. False; let $f(x) = x^2$.

Then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

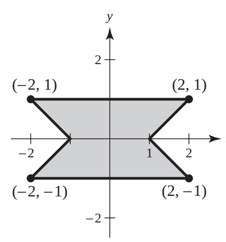
97. True, the function is even.

99. First consider the portion of R in the first quadrant: $x \geq 0$, $0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

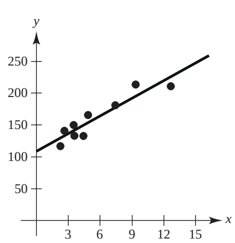
[49th competition, Problem A1, 1988]

Section P.4 Fitting Models to Data

1. Quadratic function

3. Linear function

5. (a), (b)

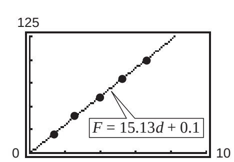


Yes. The cancer mortality increases linearly with increased exposure to the carcinogenic substance.

(c) If $x = 3$, then $y \approx 136$.

7. (a) $d = 0.066F$ or $F = 15.1d + 0.1$

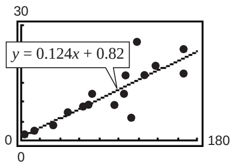
(b)



The model fits well.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ cm.

9. (a) Using a graphing utility, $y = 0.124x + 0.82$.
$r \approx 0.838$ correlation coefficient

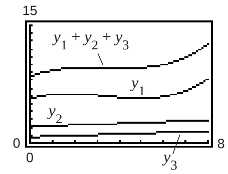
(b)


(c) The data indicates that greater per capita electricity consumption tends to correspond to greater per capita gross national product.

The data for Hong Kong, Venezuela and South Korea differ most from the linear model.

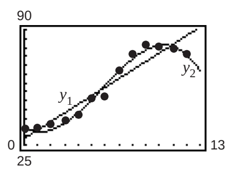
(d) Removing the data $(118, 25.59)$, $(113, 5.74)$ and $(167, 17.3)$, you obtain the model $y = 0.134x + 0.28$ with $r \approx 0.968$.

11. (a) $y_1 = 0.0343t^3 - 0.3451t^2 + 0.8837t + 5.6061$
$y_2 = 0.1095t + 2.0667$
$y_3 = 0.0917t + 0.7917$

(b)


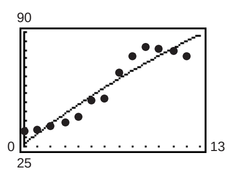
For $t = 12$, $y_1 + y_2 + y_3 \approx 31.06$ cents/mile.

13. (a) Linear: $y_1 = 4.83t + 28.6$
Cubic: $y_2 = -0.1289t^3 + 2.235t^2 - 4.86t + 35.2$

(b)


(c) The cubic model is better.

(d) $y = -0.084t^2 + 5.84t + 26.7$

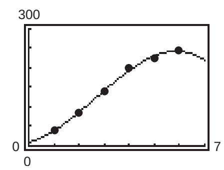


(e) For $t = 14$: Linear model $y_1 \approx 96.2$ million

Cubic model $y_2 \approx 51.5$ million

(f) Answers will vary.

15. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

(b)


(c) If $x = 4.5$, $y \approx 214$ horsepower.

17. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

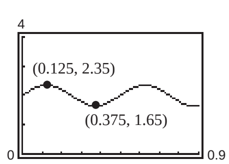
(b) The amplitude is approximately

$$(2.35 - 1.65)/2 = 0.35.$$

The period is approximately

$$2(0.375 - 0.125) = 0.5.$$

(c) One model is $y = 0.35 \sin(4\pi t) + 2$.

(d)


19. Answers will vary.

Review Exercises for Chapter P

1. $y = 2x - 3$

$x = 0 \Longrightarrow y = 2(0) - 3 = -3 \Longrightarrow (0, -3)$ y-intercept

$y = 0 \Longrightarrow 0 = 2x - 3 \Longrightarrow x = \frac{3}{2} \Longrightarrow \left(\frac{3}{2}, 0\right)$ x-intercept

3. $y = \dfrac{x-1}{x-2}$

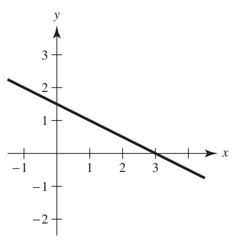
$x = 0 \Longrightarrow y = \dfrac{0-1}{0-2} = \dfrac{1}{2} \Longrightarrow \left(0, \dfrac{1}{2}\right)$ y-intercept

$y = 0 \Longrightarrow 0 = \dfrac{x-1}{x-2} \Longrightarrow x = 1 \Longrightarrow (1, 0)$ x-intercept

5. Symmetric with respect to y-axis since

$$(-x)^2y - (-x)^2 + 4y = 0$$
$$x^2y - x^2 + 4y = 0.$$

7. $y = -\frac{1}{2}x + \frac{3}{2}$



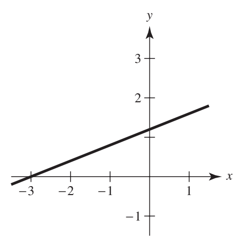
9. $-\frac{1}{3}x + \frac{5}{6}y = 1$

$-\frac{2}{5}x + y = \frac{6}{5}$

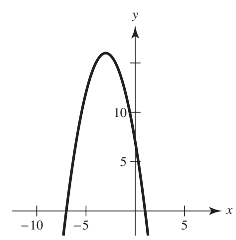
$y = \frac{2}{5}x + \frac{6}{5}$

Slope: $\frac{2}{5}$

y-intercept: $\frac{6}{5}$

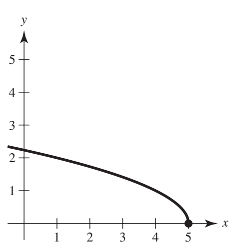


11. $y = 7 - 6x - x^2$



13. $y = \sqrt{5 - x}$

Domain: $(-\infty, 5]$



15. $y = 4x^2 - 25$

Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -30
Ymax = 10
Yscl = 5

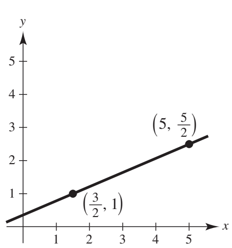
17.
$$\begin{aligned} 3x - 4y &= 8 \\ \underline{4x + 4y} &\underline{= 20} \\ 7x \quad &= 28 \\ x &= 4 \\ y &= 1 \end{aligned}$$

Point: $(4, 1)$

19. You need factors $(x + 2)$ and $(x - 2)$. Multiply by x to obtain origin symmetry.

$$\begin{aligned} y &= x(x + 2)(x - 2) \\ &= x^3 - 4x \end{aligned}$$

21.



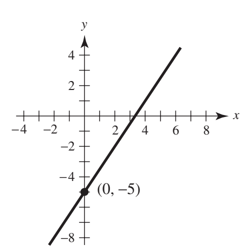
$$\text{Slope} = \frac{(5/2) - 1}{5 - (3/2)} = \frac{3/2}{7/2} = \frac{3}{7}$$

23. $\dfrac{1-t}{1-0} = \dfrac{1-5}{1-(-2)}$

$1 - t = -\dfrac{4}{3}$

$t = \dfrac{7}{3}$

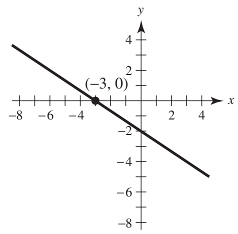
25.
$$\begin{aligned} y - (-5) &= \tfrac{3}{2}(x - 0) \\ y &= \tfrac{3}{2}x - 5 \\ 2y - 3x + 10 &= 0 \end{aligned}$$



27. $y - 0 = -\frac{2}{3}(x - (-3))$

$y = -\frac{2}{3}x - 2$

$3y + 2x + 6 = 0$



29. (a) $y - 4 = \dfrac{7}{16}(x + 2)$

$16y - 64 = 7x + 14$

$0 = 7x - 16y + 78$

(b) Slope of line is $\dfrac{5}{3}$.

$y - 4 = \dfrac{5}{3}(x + 2)$

$3y - 12 = 5x + 10$

$0 = 5x - 3y + 22$

(c) $m = \dfrac{4 - 0}{-2 - 0} = -2$

$y = -2x$

$2x + y = 0$

(d) $x = -2$

$x + 2 = 0$

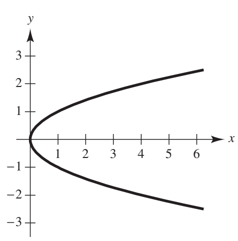
31. The slope is -850. $V = -850t + 12{,}500$.

$V(3) = -850(3) + 12{,}500 = \$9950$

33. $x - y^2 = 0$

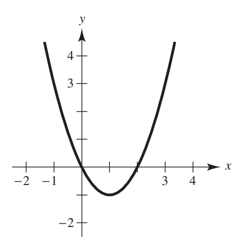
$y = \pm\sqrt{x}$

Not a function of x since there are two values of y for some x.



35. $y = x^2 - 2x$

Function of x since there is one value of y for each x.



37. $f(x) = \dfrac{1}{x}$

(a) $f(0)$ does not exist.

(b) $\dfrac{f(1 + \Delta x) - f(1)}{\Delta x} = \dfrac{\dfrac{1}{1 + \Delta x} - \dfrac{1}{1}}{\Delta x} = \dfrac{1 - 1 - \Delta x}{(1 + \Delta x)\Delta x}$

$= \dfrac{-1}{1 + \Delta x}, \Delta x \neq -1, 0$

39. (a) Domain: $36 - x^2 \geq 0 \Longrightarrow -6 \leq x \leq 6$ or $[-6, 6]$

Range: $[0, 6]$

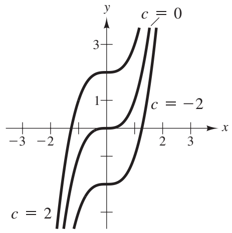
(b) Domain: all $x \neq 5$ or $(-\infty, 5) \cup (5, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

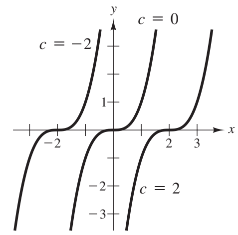
(c) Domain: all x or $(-\infty, \infty)$

Range: all y or $(-\infty, \infty)$

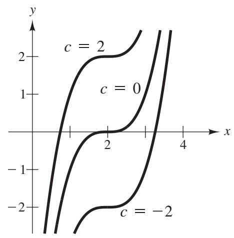
41. (a) $f(x) = x^3 + c, c = -2, 0, 2$



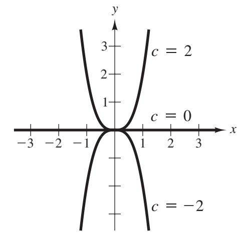
(b) $f(x) = (x - c)^3, c = -2, 0, 2$



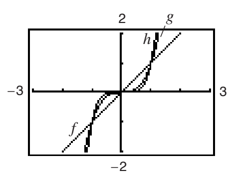
(c) $f(x) = (x - 2)^3 + c, c = -2, 0, 2$



(d) $f(x) = cx^3, c = -2, 0, 2$

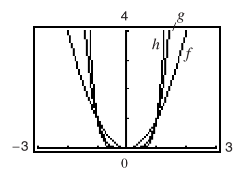


43. (a) Odd powers: $f(x) = x, g(x) = x^3, h(x) = x^5$



The graphs of f, g, and h all rise to the right and fall to the left. As the degree increases, the graph rises and falls more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, -1)$.

Even powers: $f(x) = x^2, g(x) = x^4, h(x) = x^6$

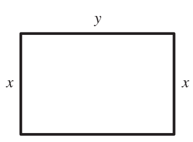


The graphs of f, g, and h all rise to the left and to the right. As the degree increases, the graph rises more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$.

(b) $y = x^7$ will look like $h(x) = x^5$, but rise and fall even more steeply.

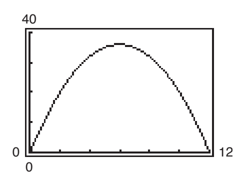
$y = x^8$ will look like $h(x) = x^6$, but rise even more steeply.

45. (a)



$$2x + 2y = 24$$
$$y = 12 - x$$
$$A = xy = x(12 - x) = 12x - x^2$$

(b) Domain: $0 < x < 12$



(c) Maximum area is $A = 36$. In general, the maximum area is attained when the rectangle is a square. In this case, $x = 6$.

47. (a) 3 (cubic), negative leading coefficient

(b) 4 (quartic), positive leading coefficient

(c) 2 (quadratic), negative leading coefficient

(d) 5, positive leading coefficient

49. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

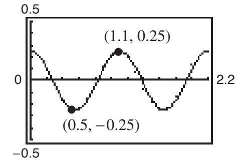
(b) The amplitude is approximately

$$(0.25 - (-0.25))/2 = 0.25.$$

The period is approximately 1.1.

(c) One model is $y = \frac{1}{4}\cos\left(\frac{2\pi}{1.1}t\right) \approx \frac{1}{4}\cos(5.7t)$

(d)



Problem Solving for Chapter P

1. (a)

$$x^2 - 6x + y^2 - 8y = 0$$

$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$

$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: $(3, 4)$ Radius: 5

(b) Slope of line from $(0, 0)$ to $(3, 4)$ is $\frac{4}{3}$. Slope of tangent line is $-\frac{3}{4}$. Hence,

$$y - 0 = -\frac{3}{4}(x - 0) \implies y = -\frac{3}{4}x \quad \text{Tangent line}$$

(c) Slope of line from $(6, 0)$ to $(3, 4)$ is $\frac{4-0}{3-6} = -\frac{4}{3}$.

Slope of tangent line is $\frac{3}{4}$. Hence,

$$y - 0 = \frac{3}{4}(x - 6) \implies y = \frac{3}{4}x - \frac{9}{2} \quad \text{Tangent line}$$

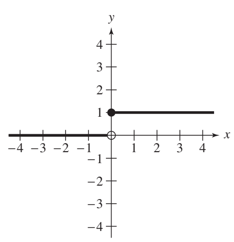
(d) $-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$

$$\frac{3}{2}x = \frac{9}{2}$$

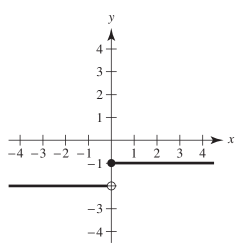
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

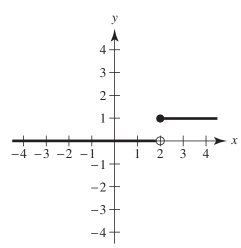
3. $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



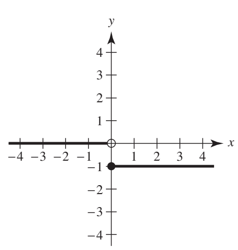
(a) $H(x) - 2$



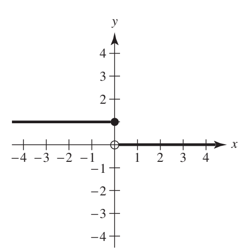
(b) $H(x - 2)$



(c) $-H(x)$



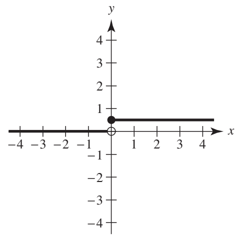
(d) $H(-x)$



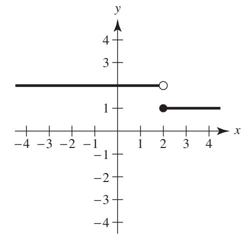
—CONTINUED—

3. —CONTINUED—

(e) $\frac{1}{2}H(x)$



(f) $-H(x - 2) + 2$

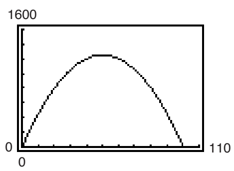


5. (a) $x + 2y = 100 \implies y = \dfrac{100 - x}{2}$

$$A(x) = xy = x\left(\frac{100 - x}{2}\right) = -\frac{x^2}{2} + 50x$$

Domain: $0 < x < 100$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c)
$$\begin{aligned} A(x) &= -\frac{1}{2}(x^2 - 100x) \\ &= -\frac{1}{2}(x^2 - 100x + 2500) + 1250 \\ &= -\frac{1}{2}(x - 50)^2 + 1250 \end{aligned}$$

$A(50) = 1250$ m^2 is the maximum.
$x = 50$ m, $y = 25$ m

7. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. Hence, the total time is

$$T = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4} \text{ hours.}$$

9. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$. Slope of tangent line is less than 5.

(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$. Slope of tangent line is greater than 3.

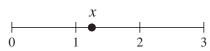
(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$. Slope of tangent line is less than 4.1.

(d)
$$\begin{aligned} \text{Slope} &= \frac{f(2 + h) - f(2)}{(2 + h) - 2} \\ &= \frac{(2 + h)^2 - 4}{h} \\ &= \frac{4h + h^2}{h} \\ &= 4 + h, h \neq 0 \end{aligned}$$

(e) Letting h get closer and closer to 0, the slope approaches 4. Hence, the slope at $(2, 4)$ is 4.

11. (a)

$$\frac{I}{x^2} = \frac{2I}{(x-3)^2}$$



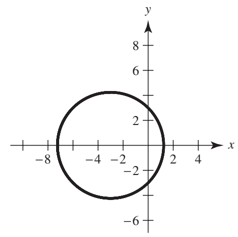
$$x^2 - 6x + 9 = 2x^2$$

$$x^2 + 6x - 9 = 0$$

$$x = \frac{-6 \pm \sqrt{36+36}}{2} = -3 \pm \sqrt{18} \approx 1.2426, -7.2426$$

(b)

$$\frac{I}{x^2+y^2} = \frac{2I}{(x-3)^2+y^2}$$



$$(x-3)^2 + y^2 = 2(x^2+y^2)$$

$$x^2 - 6x + 9 + y^2 = 2x^2 + 2y^2$$

$$x^2 + y^2 + 6x - 9 = 0$$

$$(x+3)^2 + y^2 = 18$$

Circle of radius $\sqrt{18}$ and center $(-3, 0)$.

13.

$$d_1 d_2 = 1$$

$$\left[(x+1)^2 + y^2\right]\left[(x-1)^2 + y^2\right] = 1$$

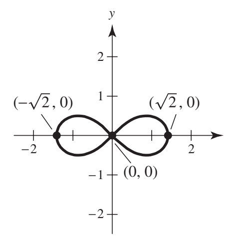
$$(x+1)^2(x-1)^2 + y^2\left[(x+1)^2 + (x-1)^2\right] + y^4 = 1$$

$$(x^2-1)^2 + y^2\left[2x^2+2\right] + y^4 = 1$$

$$x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 = 1$$

$$(x^4 + 2x^2y^2 + y^4) - 2x^2 + 2y^2 = 0$$

$$(x^2+y^2)^2 = 2(x^2-y^2)$$



Let $y = 0$. Then $x^4 = 2x^2 \implies x = 0$ or $x^2 = 2$.

Thus, $(0, 0)$, $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$ are on the curve.