Quantile Methods for Financial Risk Management

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Abstract

This thesis develops new methods to assess two types of financial risk. Market risk is defined as the risk of losing money due to drops in the values of asset portfolios. Systemic risk refers to the breakdown risk for the financial system induced by the distress of individual companies. During the financial crisis 2007–2009, both types of risk materialized, resulting in huge losses for investors, companies, and tax payers all over the world. Therefore, considering new risk management alternatives is of interest for both financial institutions and regulatory authorities.

A common feature of the models used throughout the thesis is that they adapt quantile regression techniques to the context of financial risk management in a novel way. Firstly, to predict extreme market risk, nonparametric quantile regression is combined with extreme value theory. The resulting extreme Value at Risk (VaR) forecast framework is applied to different international stock indices. In many situations, its performance is superior to parametric benchmark models.

Secondly, a systemic risk measure, the *realized systemic risk beta*, is proposed. In contrast to exististing measures it is tailored to account for tail risk interconnections within the financial sector, individual firm characteristics, and financial indicators. To determine each company's relevant risk drivers, model selection techniques for high-dimensional quantile regression are employed. The realized systemic risk beta corresponds to the total effect of each firm's VaR on the system's VaR. Using data on major financial institutions in the U.S. and in Europe, it is shown that the new measure is a valuable tool to both estimate and forecast systemic risk.

Zusammenfassung

In dieser Dissertation werden neue Methoden zur Erfassung zweier Risikoarten entwickelt. Markrisiko ist definiert als das Risiko, auf Grund von Wertrückgängen in Wertpapierportfolios Geld zu verlieren. Systemisches Risiko bezieht sich auf das Risiko des Zusammenbruchs eines Finanzsystems, das durch die Notlage eines einzelnen Finanzinstituts entsteht. Im Zuge der Finanzkrise 2007–2009 realisierten sich beide Risiken, was weltweit zu hohen Verlusten für Investoren, Unternehmen und Steuerzahler führte. Vor diesem Hintergrund besteht sowohl bei Finanzinstituten als auch bei Regulierungsbehörden Interesse an neuen Ansätzen für das Risikomanagement.

Die Gemeinsamkeit der in dieser Dissertation entwickelten Methoden besteht darin, dass unterschiedliche Quantilsregressionsansätze in neuartiger Weise für das Finanzrisikomanagement verwendet werden. Zum einen wird nichtparametrische Quantilsregression mit Extremwertmethoden kombiniert, um extreme Markpreisänderungsrisiken zu prognostizieren. Das resultierende Value at Risk (VaR) Prognose-Modell für extreme Wahrscheinlichkeiten wird auf internationale Aktienindizes angewandt. In vielen Fällen schneidet es besser ab als parametrische Vergleichsmodelle.

Zum anderen wird ein Maß für systemisches Risiko, das *realized systemic risk beta*, eingeführt. Anders als bereits existierende Messgrößen erfasst es explizit sowohl Risikoabhängigkeiten zwischen Finanzinstituten als auch deren individuelle Bilanzmerkmale und Finanzsektor-Indikatoren. Um die relevanten Risikotreiber jedes einzelnen Unternehmens zu bestimmen, werden Modellselektionsverfahren für hochdimensionale Quantilsregressionen benutzt. Das realized systemic risk beta entspricht dem totalen Effekt eines Anstiegs des VaR eines Unternehmens auf den VaR des Finanzsystems. Anhand von us-amerikanischen und europäischen Daten wird gezeigt, dass die neue Messzahl sich gut zur Erfassung und Vorhersage systemischen Risikos eignet.

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Contents

Introduction

> *If I had better foresight, maybe I could have improved things a little bit. But frankly, if I had perfect foresight, I would never have taken this job in the first place.*
>
> –Richard F. Syron, former Chief Executive of Freddie Mac

Since the financial crisis 2007–2009, risk management practice has been discussed controversially. In response to the tremendous losses of investors, firms, and governments, and therefore societies, all over the world, the Basel Committee for Banking Supervision introduced a new regulatory framework, known as Basel III, in 2010, emphasizing that "[the] failure to capture major on- and off-balance sheet risks, [...] was a key destabilising factor during the crisis" (Basel Committee on Banking Supervision (2011, p. 2)).

Two important types of financial risk are market risk and systemic risk.[1] Market risk can be described as the risk that a portfolio of assets loses value due to market movements. Since 1996, regulation requires banks to report their market risks on a regular basis, using the so-called Value at Risk (VaR), a loss forecast corresponding to certain time horizon and a certain significance level. In practice, VaR is estimated via statistical methods, e. g. regressions, simulation, or extreme value theory. The appropriateness of each of these methods depends on whether the data meet the respective assumptions. In Chapter 1 of this thesis, we investigate if relaxing some common assumptions on distributional and functional forms leads to a gain in market VaR prediction accuracy.

The financial turmoil taking place in the wake of Lehman Brothers's bankruptcy in September 2008 revealed the financial sector's fragility when confronted with

[1] Furthermore, there are credit risk, liquidity risk, and operational risk.

the default of a key player. This systemic risk, which had been neglected in regulatory frameworks before, has since become an active field of research. In Chapters 2 and 3 of this thesis, which are joint work with Nikolaus Hautsch and Melanie Schienle, we develop and apply a new procedure to quantify systemic risk. In contrast to existing measures, the so-called realized systemic risk beta explicitly takes into account network spillover effects within the financial sector, while also including individual companies's balance sheet information and financial indicators. The model is estimated using only publicly available data. As Hellwig (2009) points out, in the crisis, "risks [were] generated by [...] interdependence and by the lack of transparency about systemic risk exposure" (p. 134). With our model, we are able address these two problematic issues.

Value at Risk

The risk measure employed throughout the thesis is conditional Value at Risk (VaR). It is defined as the negative p-quantile q_p of the conditional distribution F of random returns Y_t at time point t,

$$VaR_{p,t} := -q_p(Y_t|\mathbf{X}_t) = -\inf\{y \in \mathbb{R} : F(y|\mathbf{X}_t) \geq p\},$$

given a vector of covariates $\mathbf{X}_t$. VaR may be interpreted as a loss that, conditional on $\mathbf{X}_t$, will only be exceeded with small probability p during the time span t to $t+1$. One advantage of VaR is that it summarizes potential losses in a intuitive way.[2] Besides being used for internal risk management in financial firms, it has become a building block for regulatory capital requirements, since the "Amendment to the Capital Accord to Incorporate Market Risks" (see Basel Committee on Banking Supervision (1996, 2006, 2011)). In a much-noticed paper by Artzner, Delbaen, Eber, and Heath (1999), however, VaR is criticized for not being subadditive in general, implying that the VaR of a portfolio is not necessarily less or equal to the sum of VaRs of smaller portfolios that it is split into. Therefore, Artzner et al. (1999) suggest to use Expected Shortfall (ES), the expected loss over a target horizon, given the VaR is exceeded. In this thesis, we stick to VaR for two reasons. Firstly, even if ES might be preferable from a theoretical point of view, its estimation still re-

[2] Jorion (2007) provides a comprehensive introduction to VaR.

quires the estimation of VaR in a first step. In principle, all methods descibed in the following chapters may be extended to ES frameworks. Secondly, Danielsson, Jorgensen, Samorodnitsky, Sarma, and de Vries (2011) show that for a broad class of continuous distributions with fat tails, VaR is indeed subadditive. Exceptions are distributions with infinite first moments.[3] Since we work with VaR only in the context of distributions of daily returns which are known to possess first moments, this limitation does not affect our results.

Detailed outline of the thesis

In Chapter 1, a framework is introduced which allows us to apply nonparametric quantile regression to Value at Risk (VaR) prediction at any probability level of interest. A monotonized double kernel local linear estimator is used to estimate moderate (1%) conditional quantiles of index return distributions. For extreme (0.1%) quantiles, nonparametric quantile regression is combined with extreme value theory. The abilities of the proposed estimators to capture market risk are investigated in a VaR prediction study with empirical and simulated data. Possibly due to its flexibility, the out-of-sample forecasting performance of the new model appears to be superior to competing models. A version of Chapter 1 is published in *Computational Statistics & Data Analysis*, see Schaumburg (2012).

Chapter 2 introduces the *realized systemic risk beta* as a measure for a financial company's contribution to systemic risk given network interdependence between firms' tail risk exposures. Conditional on statistically pre-identified network spillover effects and market and balance sheet information, we define the realized systemic risk beta as the total time-varying marginal effect of a firm's VaR on the system's VaR. Suitable statistical inference reveals a multitude of relevant risk spillover channels and determines companies's systemic importance in the U.S. financial system. Our approach can be used for a transparent macroprudential regulation. The chapter is based on Hautsch, Schaumburg, and Schienle (2012a).

In Chapter 3, we extend the framework from Chapter 2, providing a methodology for forecasting the systemic impact of financial institutions in interconnected systems. For five years including the financial crisis, we demonstrate how the approach can be used for transparent systemic risk monitoring of large European

[3] In this case, ES is not defined.

banks and insurance companies. As measure for systemic impact we predict systemic risk betas which account for a company's position within the network of financial interdependencies in addition to its individual balance sheet characteristics and its exposure towards market conditions. Relying only on publicly available market data, we can determine time-varying systemic risk networks, and forecast systemic importance on a daily frequency and on a quarterly basis. We illustrate that the interplay of both could serve as a valuable tool for supervisory authorities. The chapter is based on Hautsch, Schaumburg, and Schienle (2012b).

Chapter 1

Predicting extreme Value at Risk: Nonparametric quantile regression with refinements from extreme value theory

This chapter is based on Schaumburg (2012).

1.1 Introduction

In bank regulation, the effectiveness of capital requirements in preventing funding shortfalls rests upon the estimation accuracy of market risk measures, the most widely used of which is Value at Risk (VaR). According to the Market Risk Amendment to the Basel II Capital Accord, VaR is to be calculated daily, using a “99th percentile, one-tailed confidence interval” (Basel Committee on Banking Supervision (2005, p. 40)).

Not only against the background of the financial turbulences during the crisis 2007–2009, there is a practical need for VaR models that are rich enough to capture the dynamics of quickly changing market environments, while being parsimonious at the same time in order to avoid overfitting. In this context, kernel-based quantile regression is a natural choice: Being fully nonparametric, the approach does not require any structural assumptions, neither on the form of the VaR function nor on the shape of the loss distribution. Furthermore, instead of estimating structural parameters, the primary interest is to predict quantiles accurately. Therefore, we do not lose interpretability by employing nonparametric estimation methods.

Until now, however, nonparametric quantile regression has not been widely used in risk management practice. One possible reason is that, precisely because no structure is imposed on the regression function, the speed of convergence is slower than in parametric regression settings, and generally more observations are needed for accurate estimation. Therefore, when considering nonparametric quantile regression for VaR prediction, one has to address the problem of data sparseness at the tails of the loss distribution, which gets more severe when considering quantiles corresponding to extreme probabilities.

This paper proposes a framework allowing to operationalize nonparametric quantile regression as a VaR estimation tool for both moderate and extreme probabilities. We include only past returns as regressors, which is sufficient, as a maximum of information can be exploited and any kinds of nonlinearities are captured within the modelling approach. In addition, the model is simple, overfitting is avoided, and problems arising from limited observations are minimized. A double kernel-type estimator is used to smooth distortions. Additionally, monotonization by rearrangement is applied at the boundary of the estimated function. Finally, in order to predict VaR corresponding to extreme probability levels (such as 0.1%), the peaks over threshold method is incorporated and applied to the standardized nonparametric quantile residuals, resulting in an estimator that combines nonparametric quantile regression and extreme value theory (EVT).

Several studies have compared the forecast performances of different VaR models, see, among others, Kuester, Mittnik, and Paolella (2006), Manganelli and Engle (2001) and Nieto and Ruiz (2008). They take a broad variety of models into account, but nonparametric quantile regression as a tool for VaR estimation is rarely included. There are three exceptions we are aware of: Cai and Wang (2008) suggest to estimate VaR and Expected Shortfall using a new nonparametric VaR estimator, combining the Weighted Nadaraya Watson (WNW) estimator of Cai (2002) and the Double Kernel Local Linear (DKLL) estimator of Yu and Jones (1998). In the empirical application, however, only 5% quantile curves are estimated and no forecasts are computed. Chen and Tang (2005) investigate nonparametric VaR estimation, when no regressors are present. Taylor (2008) proposes to combine double kernel quantile regression with exponential smoothing of the dependent variable in the time domain. 1% and 99% VaRs are predicted from the model along with some benchmarks, but extreme quantiles are not considered.

In contrast to the studies already available in the literature, we present a method that allows to nonparametrically estimate VaR corresponding to any probability that might be of practical interest. We estimate, predict and backtest 1% and 0.1% VaRs for four sets of index returns and a simulated time series. Our focus is on gains to loosening assumptions in comparison to existing VaR models. Therefore, in the empirical application, we choose to benchmark our model against the most flexible parametric VaR models, the Conditional Autoregressive Value at Risk (CAViaR) models of Engle and Manganelli (2004). They allow to set up different linear and nonlinear specifications, including the lagged VaR estimate as a regressor, and they do not rely on distributional assumptions.

We find that although the CAViaR models obtain almost perfect in-sample fits in the case of 1% VaR, the Double Kernel Local Linear (DKLL) estimator of Yu and Jones (1998) often outperforms them in terms of out-of-sample backtesting results, especially when the estimation period is short relative to the forecasting period. Results are promising for 0.1% VaR as well. The superiority of the EVT-refined DKLL estimator over the plain DKLL estimator is shown in a small simulation study. It turns out that, especially when the estimation window is small relative to the forecasting period, the extreme value theory-refined nonparametric model predicts extreme VaRs very accurately.

Section 1.2 of this chapter outlines the basic setup of conditional quantile models, before describing CAViaR models. The DKLL estimator used in the following is presented in Section 1.3. Furthermore, the incorporation of extreme value theory into the model is explained. The investigated data sets and the backtesting method are summarized in Section 1.4. The empirical results on 1% and 0.1% VaR estimation are stated in Section 1.5. In Section 1.6, the performance of the EVT-refined nonparametric model is further assessed via a small simulation study. Section 1.7 concludes.

1.2 Quantile regression approaches to VaR estimation

1.2.1 Conditional quantiles

Let $\{Y_t\}_{t=1}^n$ be a strictly stationary time series of portfolio returns and let $\mathbf{X}_t$ be a d-dimensional vector of regressors. The pth conditional quantile of Y_t, denoted by $q_p(\mathbf{x})$, is defined as

$$q_p(\mathbf{x}) = \inf\{y \in \mathbb{R} : F(y|\mathbf{x}) \geq p\} \equiv F^{-1}(p|\mathbf{x}), \tag{1.1}$$

or, equivalently, as the argument that solves

$$\min_{q_p(\mathbf{x})} \mathbf{E}\big[\big(p - I(Y_t < q(\mathbf{X}_t))\big)\big(Y_t - q(\mathbf{X}_t)\big)|\mathbf{X}_t = \mathbf{x}\big], \tag{1.2}$$

where $I(A)$ denotes the indicator function on some set A. Both formulations are widely used in the literature. In the seminal paper by Koenker and Bassett (1978) a sample equivalent of (1.2) where $q(\mathbf{X}_t) = \mathbf{X}_t'\boldsymbol{\beta}$, also including the special case $\mathbf{X}_t = 1$, is established. $\boldsymbol{\beta}$ is a d-dimensional vector of unknown parameters. The linear quantile model is extended to conditionally heteroskedastic processes in Koenker and Zhao (1996). In Engle and Manganelli (2004) conditional autoregressive quantile functions are estimated using (1.2) with $q(\mathbf{X}_t)$ possibly being nonlinear in parameters, see Section 1.2.2 for some examples. In a number of papers, localized kernel versions of (1.2) are estimated, leading to a nonparametric fit: Yu and Jones (1997) compare the goodness of fit of local constant and local linear models. Cai (2002), Yu and Jones (1998), Cai and Wang (2008) propose nonparametric methods to estimate the distribution function in (1.1), which, in a second step, is inverted. Section 1.3.1 contains more details on these approaches. Wu, Yu, and Mitra (2007) model (1.1) without regressors, and Chernozhukov and Umantsev (2001) operationalize a linear version of (1.1).

Following the convention of expressing VaR as a positive number, it is defined as

$$VaR_p^t(\cdot) = -q_p^t(\cdot),$$

where q_p^t is the quantile of the return distribution corresponding to probability p, at time t. VaR_p^t denotes a generic VaR measure which may depend on $\mathbf{x}$ and/or a vector of parameters $\boldsymbol{\beta}$. To simplify notation, index t is suppressed in contexts where it does not cause confusion.[1]

1.2.2 Conditional Autoregressive VaR (CAViaR) Models

The class of Conditional Autoregressive Value at Risk (CAViaR) models, first introduced by Engle and Manganelli (2004), is used to benchmark the forecast performance of the nonparametric VaR estimators considered in this chapter. Several comparison studies have done so, for example Kuester, Mittnik, and Paolella (2006) or Taylor (2008). CAViaR models are dynamic VaR models describing the quantile of a random variable at time t, e.g., the return on a financial portfolio, as possibly nonlinear function of its own lags and, in addition, of a vector of observable variables, $\mathbf{X}_t$,

$$VaR_p^t(\boldsymbol{\beta}) = \beta_0 + \sum_{i=1}^{r_1} \beta_i VaR_p^{t-i}(\boldsymbol{\beta}) + \sum_{j=1}^{r_2} \beta_j f(\mathbf{X}_{t-j}),$$

[1] Although the Basel Committee on Banking Supervision asks banks to report VaR for a 10-day holding period, we focus on one-day-ahead forecasts, which is in line with the literature.

where $d = r_1 + r_2 + 1$ is the dimension of $\boldsymbol{\beta}$, the parameter vector that may be estimated by solving

$$\min_{\beta} \frac{1}{n} \sum_{t=1}^{n} \left[p - I\left(Y_t < -VaR_p^t(\boldsymbol{\beta})\right) \right] \left(Y_t + VaR_p^t(\boldsymbol{\beta})\right). \tag{1.3}$$

A straightforward choice for $\mathbf{X}_t$ is lagged returns. Following the original article, the specifications used here include the first lagged value of $VaR_p(\cdot)$ and the first lagged value of Y_t, therefore $\mathbf{X}_t = Y_{t-1}$.

Well-known stylized facts on asset returns are, firstly, that they exhibit volatility clustering. It carries over to VaR: if high variation is observed in returns of the recent past, it is likely to continue, and risk is therefore high as well. Secondly, quantiles (or volatility) might react differently according to the sign of past returns. This possibility is captured by the Asymmetric Slope specification

$$VaR_p^t(\boldsymbol{\beta}) = \beta_1 + \beta_2 VaR_p^{t-1}(\boldsymbol{\beta}) + \beta_3 (Y_{t-1})^+ + \beta_4 (Y_{t-1})^-, \tag{1.4}$$

where $(x)^+ = \max(x, 0)$ and $(x)^- = -\min(x, 0)$, but not by the Indirect GARCH(1,1) specification

$$VaR_p^t(\boldsymbol{\beta}) = \sqrt{\beta_1 + \beta_2 (VaR_p^{t-1})^2(\boldsymbol{\beta}) + \beta_3 Y_{t-1}^2}. \tag{1.5}$$

The Asymmetric Slope CAViaR imposes a piecewise linear structure on VaR, although the true functional form might be nonlinear. As pointed out in Kuester, Mittnik, and Paolella (2006), financial returns may also have an autoregressive (AR) mean, which is neglected by the CAViaR specifications. For these reasons we combine the features of the above, by allowing for nonlinearity and asymmetric effects of past returns, and additionally incorporate an AR mean, by introducing an alternative specification, called Indirect Autoregressive Threshold GARCH (AR-TGARCH(1,1)) CAViaR:

$$VaR_p^t(\boldsymbol{\beta}) = \beta_1 Y_{t-1} + \left(\beta_2 + \beta_3 (VaR_p^{t-1})^2(\boldsymbol{\beta}) + \beta_4 Y_{t-1}^2 + \beta_5 (Y_{t-1})^2 I(Y_{t-1} < 0)\right)^{\frac{1}{2}}. \tag{1.6}$$

Including the AR term introduces the possibility for a nonzero autoregressive mean, asymmetry is present if $\beta_5 \neq 0$ and the square root allows for a nonlinear functional form.

For estimating the parameters of the CAViaR models, an algorithm similar to the one proposed in the original paper by Engle and Manganelli (2004) is applied. A grid search is conducted by generating a large number of random vectors, the dimension of which corresponds to the number of model parameters. The five vectors which lead to the lowest values of the objective function (1.3) are selected and fed into a simplex optimization algorithm. The final parameter vector is chosen to be the one minimizing (1.3). Our new AR-TGARCH specification fits into this procedure.

1.3 Nonparametric quantile regression with refinements from extreme value theory

1.3.1 Double Kernel Local Linear VaR regression

In general, estimating nonparametric models requires large amounts of data. Since VaR corresponds to a quantile at the tail of the return distribution, suitable nonparametric quantile estimators should be able to deal with areas where data are sparse. Therefore, from the variety of nonparametric quantile estimators, the Double Kernel Local Linear (DKLL) estimator of Yu and Jones (1998) is chosen for the VaR application, because it localizes the data in both x- and y-direction, which leads to smoother estimates. For more details on regularity assumptions and asymptotic properties, see the original article by Yu and Jones (1998). The Weighted Double Kernel Local Linear estimator of Cai and Wang (2008), a Nadaraya Watson type estimator, forms an alternative to the DKLL estimator. In a small simulation study, which is not reported here, the DKLL estimator performed slightly better at design points at the boundary of the support of the data. Therefore, we chose it for our application.

For notational convenience, observations $\{(X_t, Y_t)\}_{t=1}^n$ are assumed to be drawn from an underlying bivariate distribution $F(x,y)$ with density $f(x,y)$. The extension to the multivariate case is straightforward, but requires more tedious notation. The estimator is defined as inverse of a conditional distribution function as in (1.1). Throughout this section, quantiles of return distributions are discussed, so that VaR corresponds to the negative quantile.

A generic nonparametric method of estimating a conditional distribution $F(y|x)$ is

$$\tilde{F}(y|x) = \sum_{t=1}^{n} w_t(x) I(Y_t \leq y), \tag{1.7}$$

where $I(\cdot)$ is again an indicator function and the weights $w_t(x)$ are positive and sum up to one. Choosing equal weights $w = 1/n$ yields the empirical distribution function. Using instead a kernel function with bandwidth parameter h, in the following sometimes abbreviated by $K_h(\cdot) = \frac{1}{h}K(\cdot/h)$, which is often chosen to be a symmetric probability density function, results in the Nadaraya Watson estimator of a conditional distribution

$$\tilde{F}_{NW}(y|x) = \sum_{t=1}^{n} \underbrace{\frac{K_h(x - X_t)}{\sum_{t=1}^{n} K_h(x - X_t)}}_{w_t(x)} I(Y_t \leq y), \tag{1.8}$$

see for example Li and Racine (2007). It attaches a smooth set of weights to the data, and is known to be monotone increasing and bounded between zero and one. However, it suffers from boundary distortion, as shown by Fan and Gijbels (1996). They advocate the use of local polynomial estimators, the simplest of which is the local linear estimator.
One way to reduce distortions that arise due to a limited number of observations is to smooth not only the observations of the regressor variable X_t, but also the observations of the dependent variable Y_t. This requires the introduction of a second symmetric kernel $W_{h_2}(\cdot)$. Its kernel distribution is defined by

$$\int_{-\infty}^{y} W_{h_2}(Y_t - u)du = \Omega\left(\frac{y - Y_t}{h_2}\right), \tag{1.9}$$

with h_2 being the bandwidth parameter. It can be viewed as a smooth, differentiable version of the indicator function.

As a next step, the conditional distribution value of y is approximated by a linear Taylor expansion around x. The estimate $\hat{F}(y|x) = \hat{\beta}_0$ is obtained from

$$(\hat{\beta}_0, \hat{\beta}_1) = \arg\min_{\beta_0, \beta_1} \sum_{t=1}^{n} \left(\Omega\left(\frac{y - Y_t}{h_2}\right) - \beta_0 - \beta_1(X_t - x)\right)^2 K_{h_1}(x - X_t), \tag{1.10}$$

where $h_1 > h_2$. Solving for $\hat{\beta}_0$ yields the explicit expression for the conditional distribution function estimator,

$$\hat{F}(y|x) = \sum_{t=1}^{n} \underbrace{\frac{K_{h_1}(x - X_t)\left[S_2 - (x - X_t)S_1\right]}{\sum_{t=1}^{n} K_{h_1}(x - X_t)\left[S_2 - (x - X_t)S_1\right]}}_{w_t(x)} \Omega\left(\frac{y - Y_t}{h_2}\right), \tag{1.11}$$

where

$$S_l = \sum_{i=1}^{n} K\left(\frac{x - X_t}{h_1}\right)(x - X_i)^l, \qquad l = 1, 2.$$

(1.11) is a version of (1.7) where the kernel distribution function $\Omega(\cdot)$ in (1.9) replaces the indicator. The DKLL quantile estimator $\hat{q}_p(x)$, the sample analogue to (1.1), is then defined by

$$\hat{q}_p(x) = \inf\left\{y \in \Re : \hat{F}(y|x) \geq p\right\} \equiv \hat{F}^{-1}(p|x) \tag{1.12}$$

with $\hat{F}$ from (1.11). In finite samples, $\hat{F}(y|x)$ might not always be monotonically increasing. In such cases, however, the inverse is not defined. Yu and Jones (1998) suggest the following implementation scheme: For $\hat{q}_{1/2}(x)$, any value satisfying (1.12) is chosen; for $p > 1/2$, the largest, and for $p < 1/2$, the smallest solutions to (1.12) are taken as quantile estimates.

In this chapter, a stronger procedure is applied, avoiding to delete estimated values. Chernozhukov, Fernandez-Val, and Galichon (2009) show that any nonmonotone estimate of a monotone function can be improved in terms of common metrics, such as the L_p-norm, by simple rearranging. In an earlier work, Dette, Neumeyer, and Pilz (2006) propose a similar method of smoothed rearrangements. For the case of a monotone increasing (decreasing) function, the point estimates are sorted in ascending (descending) order. Making use of these theoretical results, nonmonotone distribution estimates are rearranged before inverting. In the present context of monotonizing the estimated distribution function, a further effect is that quantile crossing is circumvented. Estimated values greater than one are discarded.

1.3.2 Refining nonparametric quantile regression with extreme value theory

For extreme quantiles, usually very few data points are available, so that fully nonparametric regression does not yield reliable estimates. Extreme value theory (EVT) is an alternative to model extreme quantiles. In the following, a method of incorporating extreme value theory into CAViaR models, which was introduced by Manganelli and Engle (2001), is adapted to obtain 0.1% VaR estimates from a nonparametric model.

The strategy is to first calculate the standardized quantile residuals,

$$\frac{\hat{\epsilon}^t_{p_2}}{\hat{q}^t_{p_2}} = \frac{Y_t - \hat{q}^t_{p_2}}{\hat{q}^t_{p_2}} = \frac{Y_t}{\hat{q}^t_{p_2}} - 1. \tag{1.13}$$

While p denotes here the (very low) probability of interest, p_2 corresponds to a moderately low probability for which the quantile can be estimated nonparametrically, for example $p_2 = 0.01$ or $p_2 = 0.05$. McNeil and Frey (2000) employ a similar technique to estimate 1% VaR from a GARCH residual series. An EVT-augmented nonparametric kernel distribution estimator is also considered by MacDonald, Scarrott, Lee, Darlow, Reale, and Russell (2011), who show consistency of their method via Bayesian inference.

Reformulating the definition of the pth quantile of portfolio returns in terms of the p_2th quantile yields

$$
\begin{aligned}
P[Y_t < q_p^t] &= P\left[Y_t < q_{p_2}^t - q_{p_2}^t + q_p^t\right] \\
&= P\left[\frac{Y_t}{q_{p_2}^t} - 1 > \frac{q_p^t}{q_{p_2}^t} - 1\right] = p.
\end{aligned}
$$

The inequality sign is switched assuming that $q_{p_2}^t$ is a negative number. Let

$$
z_p \equiv \frac{q_p^t}{q_{p_2}^t} - 1
$$

denote the $(1-p)$th quantile of the standardized residuals. It is estimated by the peaks over threshold (POT) method. Alternatively, it could be estimated by other methods, such as the Hill estimator, as well. A number of applications employ the POT method to forecast extreme quantiles; for a selection of applications and an investigation of its finite sample properties see El-Arouia and Diebolt (2002). An estimate for the pth return quantile can be expressed by means of $\hat{z}_p$ and $\hat{q}_{p_2}^t$,

$$
\frac{\hat{q}_p^t}{\hat{q}_{p_2}^t} - 1 = \hat{z}_p \quad \Leftrightarrow \quad \hat{q}_p^t = \hat{q}_{p_2}^t(\hat{z}_p + 1). \tag{1.14}
$$

Again, $\widehat{VaR}_p^t = -\hat{q}_p^t$. In the remainder of this section, the underlying extreme value arguments, which are used to obtain $\hat{z}_p$ in (1.14), are discussed very briefly, following Embrechts, Klüppelberg, and Mikosch (1997).

Large observations which exceed a high threshold can be approximated reasonably well by the Generalized Pareto Distribution (GPD) with distribution function

$$
G_{\xi,\beta}(x) = \begin{cases} -(1+\xi x/\beta)^{1/\xi} & \text{for } \xi \neq 0 \\ 1 - e^{x/\beta} & \text{for } \xi = 0, \end{cases} \tag{1.15}
$$

with shape parameter ξ and scale parameter $\beta > 0$. The support is $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\frac{\beta}{\xi}$ if $\xi < 0$. The parameters can be consistently estimated if the threshold exceedances are independent, regardless of the true underlying distribution, see Smith (1987). In general, given a high threshold u and a random variable Y, the probability of Y exceeding u at most by x is given by

$$
F_u(x) = P[Y - u \leq x | Y > u] = \frac{F(x+u) - F(u)}{1 - F(u)}. \tag{1.16}
$$

Balkema and de Haan (1974) and Pickands (1975) show that for a large class of distribution functions F it is possible to find a positive function $\beta(u)$ such that

$$\lim_{u \to y_0} \sup_{0 \le x < y_0 - u} \left| F_u(x) - G_{\xi,\beta(u)}(x) \right| = 0, \tag{1.17}$$

with y_0 corresponding to the right endpoint of F. Rearranging (1.16) and using $F_u(\cdot) \approx G_{\xi,\beta}(\cdot)$, it holds that

$$1 - F(u + x) \approx [1 - F(u)][1 - G_{\xi,\beta}(x)].$$

Then, $1 - G_{\xi,\beta}(x)$ can be obtained by estimating the GPD parameters by maximum likelihood. Let N_u denote the number of exceedances over threshold u. A common way of estimating $S(u) := 1 - F(u)$ is to use the empirical distribution function $\frac{N_u}{n}$. Substituting the estimates yields

$$\widehat{S(u+x)} = \frac{N_u}{n} \left(1 + \hat{\xi} \left(\frac{x}{\hat{\beta}}\right)\right)^{-\frac{1}{\hat{\xi}}}. \tag{1.18}$$

The quantile can be estimated by inverting (1.18), employing a change of variables $y = u + x$ and fixing the distribution value at the probability of interest: $F(y) = p$. Therefore, the quantile estimator $\hat{q}_p$ is obtained from

$$\begin{aligned} 1 - p &= \frac{N_u}{n} \left(1 + \hat{\xi} \left(\frac{y - u}{\hat{\beta}}\right)\right)^{-\frac{1}{\hat{\xi}}} \\ \Leftrightarrow \quad \hat{q}_p &= u + \left[\left((1 - p) \frac{n}{N_u} \right)^{-\hat{\xi}} - 1 \right] \cdot \frac{\hat{\beta}}{\hat{\xi}}. \end{aligned} \tag{1.19}$$

In general, extreme value methods require the underlying data to be i.i.d. Although computing standardized residuals as in (1.13) should remove most of the dynamics, one cannot eliminate the possiblitity of remaining autocorrelation. However, under some conditions on the dependence structure (see e.g. Drees (2003) for details), the relationship between the limiting distributions of the maxima of a dependent but strictly stationary sequence, $(Y_t)_{t \in \mathbb{N}}$ say, and a white noise sequence $(\widetilde{Y}_t)_{t \in \mathbb{N}}$ with the same distribution function F may be described by the so-called extremal index $\theta \in (0, 1]$. If the distribution of normalized threshold exceedances in the sequence $(\widetilde{Y}_t)$ converges to an extreme value distribution $G(x)$, as in (1.17), then it can be shown that the equally normalized exceedances of $(Y_t)_{t \in \mathbb{N}}$ converge to $G^{\theta}(x)$, see Embrechts, Klüppelberg, and Mikosch (1997). Thus, the same limiting extreme value distribution may be used, while changing only the normalization parameters.

Intuitively, if our sequence of standardized residuals possesses an extremal index which is < 1, then its extremal behavior is asymptotically the same as that of a shorter white noise sequence with the same distribution. However, it might still be interesting to find out about the extent of deviation from white noise for a given data set. The extremal index may be estimated by the so-called Runs Method where θ is computed as the conditional probability that a threshold exceedance is followed by a run of r non-exceedances. The idea is that periods in between clusters of exceedances are longer than periods between independent exceedances (for details see Embrechts, Klüppelberg, and Mikosch (1997), Chapter 8). The higher the clustering tendency, the fewer runs will be present. The estimator is

$$\widehat{\theta} = \frac{\sum_{t=1}^{n-r} I(A_t)}{\sum_{t=1}^{n} I(Y_t > u)}, \tag{1.20}$$

where $I(\cdot)$ is again the indicator function and

$$A_t = \{Y_t > u, Y_{t+1} \leq u, ..., Y_{t+r} \leq u\}. \tag{1.21}$$

Drees (2003) shows that a wide variety of financial time series, including ARMA and ARCH processes, may be estimated by the POT maximum likelihood estimator which we use here. He finds that the only difference to i.i.d. data is an increased variance of the quantile estimator, but this drawback does not affect our results, as our goal is forecasting accuracy, which is checked via backtesting.

1.4 Data and backtesting method

We analyze four data sets of daily index returns, DAX, FTSE 100 (FTSE), EuroSTOXX 50 (EuroSTOXX) and S&P 500 (S&P). The longest available time series of each are used to compute in-sample fits. The common end date of the in-sample period is 28/02/2003. We predict VaR for the subsequent 1000 days, until the end of 2006 (29/12/2006). As a second step, we take the same forecast period, but additionally include 300 days to check whether model performances worsen when the data contains the beginning of the financial crisis. The 1300-day forecast period ends on 22/02/2008. Table 1.1 summarizes some features of the data.

The reason for using a rather long estimation period relative to the forecast horizon, is that our aim is to assess and compare the quality of the nonparametric model in capturing market risk, using as much information as possible. We are aware that in real life risk management, available time series are typically much shorter. Therefore, in Section 1.5.2, we additionally estimate our models using only the last 1000 data points, from 30/04/1999 to 28/02/2003 and forecast 1% VaRs for the subsequent 200 and 1000 days. Realizations of

	DAX	FTSE	EuroSTOXX	S&P
start date	03/10/1966	03/01/1984	02/01/1987	31/07/1970
end in-sample period	28/02/2003	28/02/2003	28/02/2003	28/02/2003
no. of observations	9500	4998	4215	8500
mean	0.020	0.026	0.021	0.028
median	0.000	0.020	0.057	0.007
0.5% quantile	-4.107	-3.470	-4.924	-3.009
99.5% quantile	3.686	3.258	4.157	3.239
skewness	-0.422	-0.794	-0.326	-1.468
kurtosis	11.158	13.571	8.414	39.075

Table 1.1: Data summary for the four index time series used in the long estimation period.

quantiles cannot be observed. Therefore, backtesting of the models is carried out using the dynamic quantile (DQ) out-of-sample test developed in Engle and Manganelli (2004) to test and compare the performance of VaR models. From the variety of alternatives, we choose this particular test, because it is the standard test to compare CAViaR and other models. Define the binary variable

$$Hit_t \equiv I(Y_t < -VaR_p^t) - p.$$

If the chosen model is correct,

$$\mathbf{E}[Hit_t|\Omega_t] = 0, \tag{1.22}$$

where Ω_t is any information known up to time t. Thus, VaR is estimated correctly, if independently for each day of the forecasting period, the probability of exceeding it equals p. Note that this also implies that Hit_t is uncorrelated with its own lagged values. Let Z_t denote a K-vector of variables potentially related to Hit_t, and let $\mathbf{Z}$ denote the $(N \times K)$-matrix stacking the values of Z_t, where N is the number of observations in the forecast period. Then, the moment condition in (1.22) can be checked using the test statistic

$$DQ = \frac{\mathbf{Hit}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{Hit}}{p(1-p)}, \tag{1.23}$$

where $\mathbf{Hit}$ is a vector containing all values of Hit_t. Under the null hypothesis that (1.22) holds, DQ is asymptotically χ^2-distributed with K degrees of freedom. In analogy to Engle and Manganelli (2004), Kuester, Mittnik, and Paolella (2006) and Taylor (2008), we include a constant, four lagged values of Hit_t and the current VaR estimate in the information set.

It can be seen immediately that for our information set, the DQ test statistic requires at least one out-of-sample VaR exceedance in order to be defined. Otherwise, the lagged values of Hit_t would cause multicollinearity in the matrix $\mathbf{Z}$. However, when considering extreme VaR, as we do in Section 1.5.3, it might well be that there are no exceedances within the forecasting period (note that the correct number of exceedances for 1000 forecasts of 0.1% VaR is 1). Therefore, in order to be able to compare the different models while taking the possibility of no exceedances into account, we employ the test proposed by Kupiec (1995) which is an unconditional test of the correctness of the achieved share of VaR exceedances. Define the indicator variable

$$I_t \equiv I(Y_t < -VaR_p^t).$$

The idea of the Kupiec test is to check whether $\mathbf{E}[I_t] = p$, in which case the number of exceedances

$$ex_N = \sum_{t=n+1}^{n+N} I_t$$

has a binomial distribution with parameters N and p. Under the null hypothesis of correct coverage, the corresponding likelihood ratio statistic

$$LR_{Kup} = 2\log\left[\left(1 - \frac{ex_N}{N}\right)^{N-ex_N}\left(\frac{ex_N}{N}\right)^{ex_N}\right] - 2\log\left[(1-p)^{N-ex_N}p^{ex_N}\right],$$

is asymptotically χ^2-distributed with one degree of freedom.

1.5 Application to stock index returns

1.5.1 Monotonicity of quantile curves

When forecasting from a nonparametric model, one has to balance two effects occurring at the boundary areas: The support from which predictions of the dependent variable can be computed is limited to the range in which the estimated function is located. This means that for outlying lagged returns, which are not in the support of the estimated curve, no forecasts for VaR exist. On the other hand, often only few data points are available at boundary areas, so that outliers have more influence and the resulting curve may show distortions. Therefore, one has to decide carefully about the range of the grid at which the function is evaluated, balancing possible distortions against a limited range of regressor values to compute forecasts from.

We estimate the time-varying conditional 1% VaRs of DAX, EuroSTOXX, S&P and FTSE using the DKLL estimator. Due to its double smoothing property, distortions are eased and quantile curves are smoother. Additionally, we make use of the monotonization method proposed by Chernozhukov, Fernandez-Val, and Galichon (2009). Whenever curves are not monotonically decreasing on the left of the minimum and monotonically increasing on the right, estimated values are rearranged in descending and ascending order, respectively. To illustrate possible changes in the in-sample fit, Figure 1.1 shows the original as well as the rearranged 1% VaR curves of DAX and EuroSTOXX. Both curves cover 99% of the data.

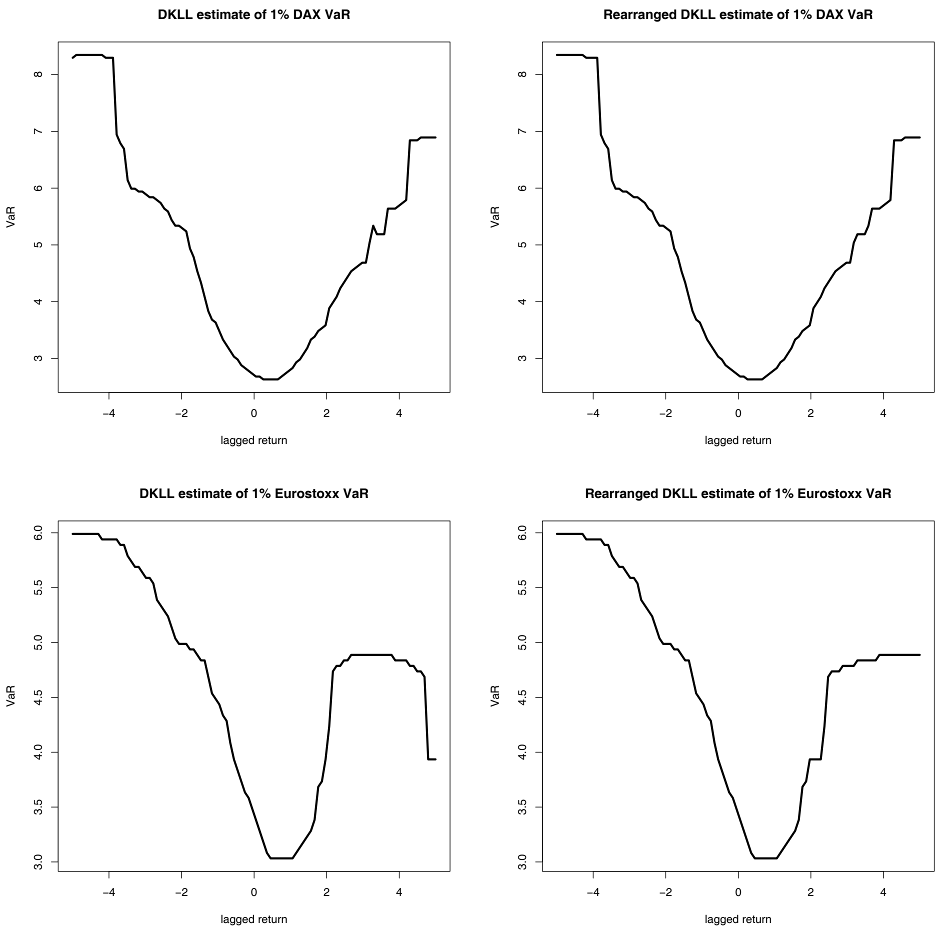

Figure 1.1: Original and rearranged DKLL estimates of 1% conditional DAX VaR curve (upper panel) EuroSTOXX VaR curve (lower panel).

Table 1.2 compares backtesting results on original and rearranged DKLL fits, on a forecast horizon of 1000 days. In-sample and out-of-sample coverages are aimed to be as close as possible to the underlying probability, in this case 1%. The p-value of the DQ test described in Section 1.4 expresses the highest significance level at which the variables in the information set are jointly significant. Therefore, a larger p-value indicates that the null hypothesis of independent VaR exceedances is more likely not to be rejected, suggesting that a model is more adequate. The theoretical results from Chernozhukov, Fernandez-Val,

	DAX		FTSE	
	DKLL orig.	DKLL rearr.	DKLL orig.	DKLL rearr.
in-sample (%)	0.78	0.78	0.94	0.95
out-of-sample (%)	1.00	1.00	0.40	0.50
DQ p-value	0.182	0.182	0.614	0.830

	EuroSTOXX		S&P500	
	DKLL orig.	DKLL rearr.	DKLL orig.	DKLL rearr.
in-sample (%)	0.81	0.81	0.97	0.97
out-of-sample (%)	0.50	0.50	0.30	0.30
DQ p-value	0.859	0.859	0.544	0.544

Table 1.2: Backtest result comparison for original and rearranged DKLL models: DQ test results for original and rearranged DKLL models as well as in-sample and out-of sample share of VaR exceedances (in percent). The forecast period is 1000 observations.

and Galichon (2009), that rearranging weakly improves estimation, are confirmed by our empirical results: Whenever values in the columns are different, they are superior for the rearranged estimates. In-sample and out-of-sample coverages are closer to 1% in case of the FTSE return series. Furthermore, the DQ test p-value substantially increases, indicating that the null hypothesis of the DQ test is “further away” from rejection than in the case of the original DKLL model. Whenever we mention results for the DKLL estimator in the following, it refers to the rearranged version.

1.5.2 Comparing 1% VaR predictions

Long estimation period

Table 1.3 lists backtest results of the CAViaR and the rearranged DKLL estimates which are obtained from the data set data set described in Table 1.1. Generally, the in-sample exceedance shares achieved by all three CAViaR specifications are very close to the underlying probability 1%. In contrast, the DKLL estimator has a slight tendency to overestimate VaR, leading to in-sample coverages below 1%.

In terms of out-of-sample forecasting, results differ among the four indices. In predicting DAX VaR, the CAViaR models perform quiete poorly on both forecast horizons. Out-of-sample coverages are too high, and the DQ test p-values raise doubt that the models are able to generate conditionally independent VaR exceedances and correct coverage. In contrast, the DKLL estimate achieves more accurate out-of-sample exceedance rates in predicting VaR over both 1000 and 1300 days, and the test results suggest that the model is appropriate at least for the shorter forecast horizon.

In the case of FTSE VaR, results are mixed: The DKLL estimator overestimates VaR in the short prediction period (coverage is 0.5%), where, however, it yields a good fit according to the backtest. For 1300 forecasts, it achieves the correct coverage, but the DQ p-value drops sharply. The CAViaR models, are strongly rejected in the short horizon, but show a better performance in the longer one.

The picture for EuroSTOXX is somewhat similar to that of FTSE, except that the results from the CAViaR models now differ more strongly among each other, and the Asymmetric Slope CAViaR beats all the other models considered. The p-value obtained by the DKLL estimator again drops when moving from the short to the longer forecast horizon, but it is still above the p-values of GARCH and AR-TGARCH CAViaR, which perform better in terms of out-of-sample coverages.

The results for S&P VaR show a different structure. Although all coverages within the short prediction period are low, the DQ test indicates adequate out-of-sample fits. For the extended horizon, all p-values drop, such that the GARCH CAViaR and the DKLL models are even rejected at a 1% significance level. One possible reason is that the additionally included observations exhibit some dynamics which are not well captured by these models, leading to a clustering of VaR exceedances. Interestingly, the AR-TGARCH is least affected by this effect.

The AR-TGARCH CAViaR model does not outperform the other two CAViaR models and the DKLL model systematically, but its results are less varying: For both in-sample and out-of-sample forecast horizons, its coverage and backtest results are often better than

	Asymm. Slope	GARCH	AR-TGARCH	DKLL
		DAX		
in-sample	1.01	1.01	1.04	0.78
out-of-sample (1000)	1.50	1.50	1.50	1.00
out-of-sample (1300)	1.54	1.54	1.61	1.07
DQ p-value (1000)	0.054*	0.011**	0.054*	0.182
DQ p-value (1300)	0.043**	0.009***	0.039**	0.040**
		FTSE		
in-sample	1.02	1.00	1.00	0.94
out-of-sample (1000)	0.60	0.60	0.60	0.50
out-of-sample (1300)	1.08	1.16	1.08	1.00
DQ p-value (1000)	0.005***	0.007***	0.007***	0.830
DQ p-value (1300)	0.040**	0.104	0.059*	0.011**
		EuroSTOXX		
in-sample	1.04	0.99	0.99	0.81
out-of-sample (1000)	0.70	0.80	0.80	0.50
out-of-sample (1300)	0.76	0.92	0.92	0.62
DQ p-value (1000)	0.970	0.058*	0.057*	0.859
DQ p-value (1300)	0.980	0.015**	0.015**	0.031**
		S&P		
in-sample	1.01	1.00	0.97	0.97
out-of-sample (1000)	0.30	0.30	0.30	0.30
out-of-sample (1300)	0.92	1.00	0.69	1.15
DQ p-value (1000)	0.547	0.285	0.547	0.544
DQ p-value (1300)	0.042**	0.000***	0.079*	0.008***

Table 1.3: Backtesting results for 1% VaR models: Long estimation period: In-sample and out-of-sample exceedance probabilities in %. Considered forecast horizons are 1000 and 1300 observations. Models which are rejected by the DQ test are marked with * for rejection on 10%, ** on 5% and *** on 1% significance level.

the results of one of the two others. We attribute this finding to the fact that the model combines the features of Asymmetric Slope and Indirect GARCH specification, and it is therefore more universally applicable.

Summing up, the out-of-sample VaR prediction results produced by the fully nonparametric DKLL estimator are satisfactory except for the extended forecast horizon in the case of the S&P. The CAViaR models are strong competitors, however, they have the drawback that it is not possible to detect one parameterization that systematically dominates others. As it is often the case with parametric models, the question remains which one to choose in practical applications. The DKLL model, on the other hand, outperforms at least one of the CAViaR models in most cases, and is therefore the most robust alternative.

Short estimation period

In real life risk management, time series available for the estimation of VaR models are rarely as long as the ones we investigate in the previous section. For this reason, we repeat the estimation using only 1000 observations, i.e. roughly the last four years up to 28/02/2003, and forecast VaRs for both the subsequent 200 and 1000 days. Table 1.4 lists the backtesting results. The good performance of the DKLL estimator carries over to the

	Asymm. Slope	GARCH	AR-TGARCH	DKLL
		DAX		
in-sample	1.10	1.10	0.90	0.80
out-of-sample (200)	2.50	1.50	2.50	0.50
out-of-sample (1000)	0.70	0.80	0.80	0.10
DQ p-value (200)	0.409	0.839	0.410	0.967
DQ p-value (1000)	0.603	0.061*	0.671	0.195
		FTSE		
in-sample	1.00	1.00	1.10	0.70
out-of-sample (200)	1.00	1.00	1.00	0.50
out-of-sample (1000)	0.70	0.60	0.60	0.10
DQ p-value (200)	0.810	1.000	0.975	0.997
DQ p-value (1000)	0.020**	0.008***	0.007***	0.225
		EuroSTOXX		
in-sample	1.10	1.00	1.10	1.10
out-of-sample (200)	1.50	1.00	1.50	1.00
out-of-sample (1000)	0.40	0.40	0.50	0.20
DQ p-value (200)	0.985	1.000	0.992	1.000
DQ p-value (1000)	0.674	0.652	0.713	0.356
		S&P		
in-sample	1.00	1.00	1.10	0.90
out-of-sample (200)	1.00	0.50	1.00	0.50
out-of-sample (1000)	0.20	0.10	0.20	0.10
DQ p-value (200)	0.536	0.742	0.390	0.975
DQ p-value (1000)	0.206	0.110	0.210	0.212

Table 1.4: Backtesting results for 1% VaR models: Short estimation period, using only 1000 observations. In-sample and out-of-sample exceedance probabilities in %. Considered forecast horizons are 200 and 1000 observations. Models which are rejected by the DQ test are marked with * for rejection on 10%, ** on 5% and *** on 1% significance level.

short estimation period. Although VaR estimates are again too conservative in particular for the longer forecasting period, the null hypothesis of valid moment conditions tested by the DQ test is never rejected even on a 10% significance level. The CAViaR models also overestimate VaR for the subsequent 1000 observations, and additionally, they are rejected by the DQ test in the case of FTSE. Based on these results, it can be said that the DKLL estimator is also applicable when the estimation period is rather short, and keeps yielding reliable VaR forecasts even for the distant future.

1.5.3 Comparing extreme VaR predictions

Following the procedure described in Section 1.3.2, standardized residuals are computed from the rearranged DKLL estimate and the time-varying 0.1% quantile of time series Y_t is calculated according to (1.14). The underlying 'moderate' probability is chosen to be 1%. Similarly, VaR estimates obtained from the EVT-augmented CAViaR models are computed, following Manganelli and Engle (2001). As mentioned at the end of Section 1.3.2,

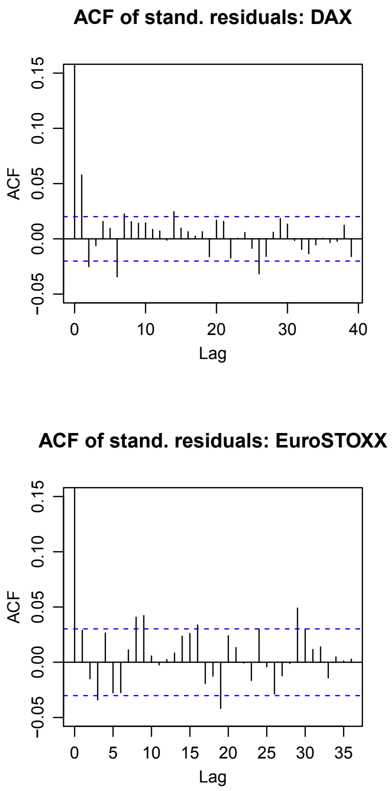

Figure 1.2: Autocorrelation functions of the standardized nonparametric residuals for the four indices. The dashed lines are 95% confidence intervals.

the data should be checked for dependence before applying extreme value methods. Figure

1.2 shows autocorrelation functions (ACFs) for the standardized residuals from the nonparametric model, together with 95% confidence intervals. Although the magnitude of the autocorrelation is not very high, for some lags, the confidence intervals are exceeded. Therefore, we carry out Ljung-Box tests on independence based on 20 lags, the results of which imply significant autocorrelation on a 1% confidence level for all four considered models and all model specifications. Fitting simple AR models to the standardized residuals, however, removes the autocorrelation entirely, see Table 1.5. The corresponding results for the three CAViaR specifications are very similar, and thus, not shown here. They are available upon request. Since McNeil, Frey, and Embrechts (2005) state that ARMA processes with innovations drawn from fat-tailed distributions exhibit values of the extremal index $\theta < 1$, we also estimate the extremal indices using the Runs Method (1.20) described in Section 1.3.2 and report them in the last column of Table 1.5. The parameter r, corresponding to the length of runs, was set to 30 for all three indices, after finding that the estimated $\widehat{\theta}$ was very robust with respect to plausible choices of r. As McNeil, Frey, and Embrechts (2005) point out, the distribution of the maximum of n dependent data points with extremal index θ can be approximated by the associated i.i.d. series with $n\theta$ observations. Given the large number of data points in all our samples, we conclude that the loss in accuracy due to dependence in the standardized residuals is not too severe. Therefore, we can apply the proposed method to estimate the 0.1%-VaRs.

	LB p-value	LB p-value for AR residuals	AR lag order	$\widehat{\theta}$
DAX	0.000	0.529	7	0.80
FTSE	0.000	0.203	9	0.91
EuroSTOXX	0.001	0.357	9	0.85
S&P	0.000	0.224	4	0.88

Table 1.5: Remaining autocorrelation and extremal index estimates: The first column reports the outcomes (p-values) of the Ljung-Box (LB) test on independence of the standardized residuals from the DKLL model. The null hypothesis of independence is always rejected on a 1% confidence level. The second column contains the LB test results after fitting an autoregressive (AR) model to the standardized residuals. In all cases, the null hypothesis cannot be rejected. The selected lag order is reported in the third column. The forth column contains estimates of the extremal index θ.

Table 1.6 contains both in-sample and out-of-sample shares of 0.1% VaR exceedances for the four models. Only the long estimation period is considered. However, the simulation study in Section 1.6 contains a discussion of results from the nonparametric model for extreme quantiles, based on a shorter space of time. Due to the occurrence of no VaR exceedances within the prediction period, we use the Kupiec test instead of the DQ test for backtesting. It checks the correctness of the achieved unconditional coverage via a likelihood ratio approach, which is based on the Bernoulli likelihood, see Section 1.4.

	Asymm. Slope	GARCH	AR-TGARCH	DKLL
		DAX		
in-sample	0.11	0.17	0.09	0.13
out-of-sample(1000)	0.00	0.00	0.00	0.10
out-of-sample(1300)	0.15	0.15	0.15	0.23
Kupiec p-value (1000)	0.157	0.157	0.157	1.000
Kupiec p-value (1300)	0.570	0.570	0.570	0.203
		FTSE		
in-sample	0.14	0.16	0.14	0.16
out-of-sample(1000)	0.10	0.20	0.10	0.10
out-of-sample(1300)	0.15	0.38	0.15	0.15
Kupiec p-value (1000)	1.000	0.379	1.000	1.000
Kupiec p-value (1300)	0.570	0.014^{**}	0.570	0.570
		EuroSTOXX		
in-sample	0.19	0.26	0.31	0.24
out-of-sample(1000)	0.10	0.20	0.20	0.00
out-of-sample(1300)	0.23	0.31	0.31	0.15
Kupiec p-value (1000)	1.000	0.379	0.379	0.157
Kupiec p-value (1300)	0.203	0.058^{*}	0.058^{*}	0.570
		S&P		
in-sample	0.20	0.22	0.19	0.12
out-of-sample(1000)	0.10	0.00	0.00	0.00
out-of-sample(1300)	0.15	0.08	0.08	0.00
Kupiec p-value (1000)	1.000	0.157	0.157	0.157
Kupiec p-value (1300)	0.570	0.784	0.784	0.107

Table 1.6: Backtesting results for 0.1% VaR models: In-sample and out-of-sample exceedance probabilities in %. Considered forecast horizons are 1000 and 1300 observations. Models which are rejected by the DQ test are marked with * for rejection on 10%, ** on 5% and *** on 1% significance level.

In contrast to the outcomes of the 1% VaR analysis, in-sample VaR exceedance shares achieved by the DKLL estimator are now less conservative, but instead always slightly higher than the target probability 0.1%. On the other hand, out-of sample coverage and backtest results are remarkably good especially for FTSE and EuroSTOXX, where the DKLL estimator shows best results on one of the two considered forecast horizon, but also for DAX, where its coverages are very close to 0.1%. Concerning S&P, the Asymmetric Slope CAViaR model yields the most accurate fit, except for the in-sample exceedance rate, which is closer to 0.1% in the case of the DKLL model. According to the Kupiec test, the differences to the nominal coverage are rarely significant, the only exception being the GARCH and AR-TGARCH CAViaRs when predicting 1300 days of EuroSTOXX VaR, and GARCH CAViaR for the extended forecast period of FTSE VaR.

It was pointed out in Kuester, Mittnik, and Paolella (2006), that for comparison of VaR prediction strategies, the focus should not be limited to one or two probability levels, but one should take a range of quantiles into account when deciding which model is the best.

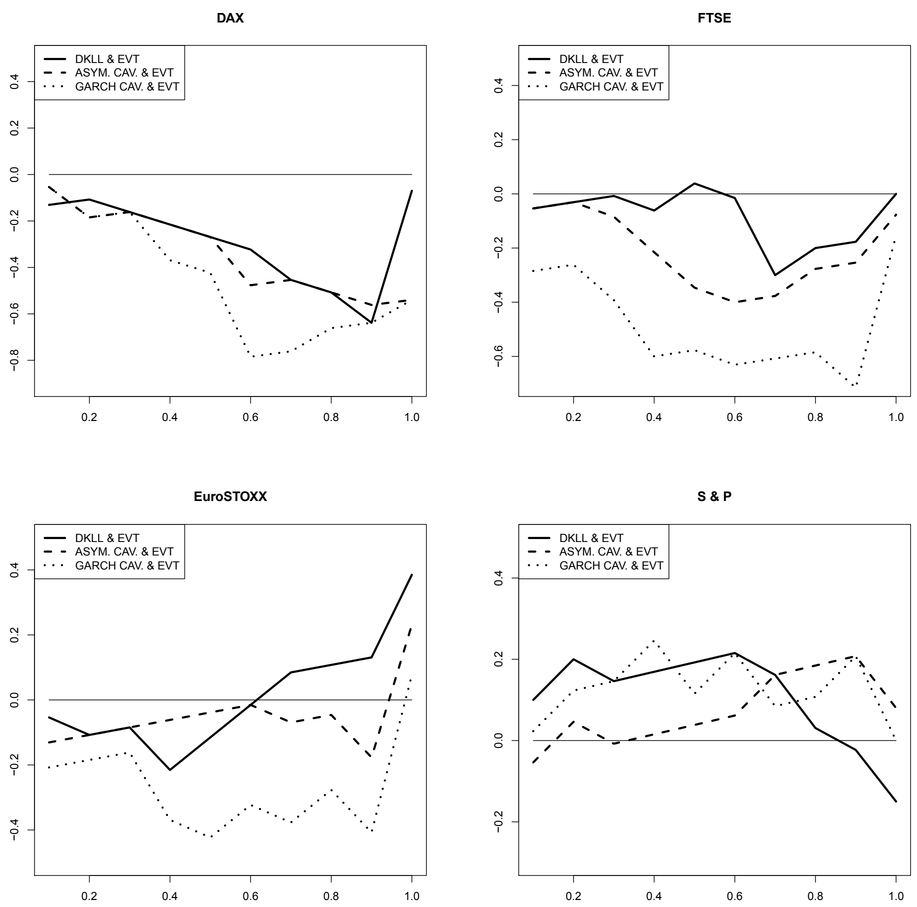

Figure 1.3: Graphical comparison of coverage results
Coverage results in % for 0.1%-1% ranges of estimated EVT-augmented index VaRs. The evaluated forecast horizon is 1300 days. Nominal coverages are on the horizontal axis, and the lines correspond to the differences of nominal and estimated VaR exceedance shares (in %). The closer they are to zero, the better the (unconditional) model fit.

We adapt their graphical representation of coverage accuracy in Figure 1.3 for VaR levels between 0.1% and 1%. For the sake of clarity, the AR-TGARCH CAViaR, which usually showed results that were similar to one of the other two CAViaR models, is not included in the graph. It turns out that the Asymmetric Slope CAViaR is the stronger competitor for the DKLL model, as in three out of four cases, the lines corresponding to its differences to the nominal coverages are closer to zero than those corresponding to the GARCH CAViaR. In some ranges, e.g. 0.1%-0.5% FTSE VaR, the DKLL model clearly yields a very good fit. In other cases, such as DAX, all three models do not hit the correct coverages. The DKLL estimator, however, goes head to head with the CAViaR models, while sometimes even beating them.

1.6 Simulation: Comparing DKLL and EVT-refined VaR estimation

This section is devoted to the question of whether it is sensible to refine the nonparametric VaR estimator with extreme value methods, instead of using the plain version even for extreme VaR estimation. One would expect that especially for small data sets, the EVT extrapolation into the far tails of return distributions yields more stable results than estimating the tail quantiles directly. In order to check this, we carry out a small simulation study to complement the previous empirical results. As the goal is to assess the relative accuracies of DKLL and EVT-refined DKLL estimators, we do not additionally include the CAViaR models.

To the FTSE time series, we fit a GARCH(1,1) model with t-distributed error terms, i.e. we estimate the unknown parameters μ, ω, α, β, and ν in

$$Y_t = \mu + \sigma_t \epsilon_t, \;\; \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \;\; \epsilon_t \sim t_\nu. \tag{1.24}$$

The estimates are listed in Table 1.7.

$\widehat{\mu}$	$\widehat{\omega}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\nu}$
0.054	0.015	0.083	0.904	10

Table 1.7: Estimated GARCH parameters from the FTSE return series with 4998 observations. This estimated model is used in the simulation.

From the estimated model, a time series of 13000 observations is simulated. To obtain a setup which is realistic with respect to usual data availability, only 2000 observations are used to estimate the models, and 0.1% VaR predictions are computed over two forecast horizons, N=5000 and N=10000. The advantages of simulated data are that they allow for much longer horizons, and that the return quantile functions

$$q_p^t = \sigma_t F_p^{-1}(\epsilon_t),$$

where $F_p^{-1}(\epsilon_t)$ denotes the p-quantile of the error term distribution, can be computed because the input parameters are known. This allows us to compare the estimated VaRs to their true counterparts. Table 1.8 shows coverages, mean squared errors, mean absolute errors and median absolute errors in-sample and out-of-sample for both models.

	in-sample: n=2000			
	cov.	$\widehat{MSE}$	$\widehat{MAE}$	$\widehat{Med.AE}$
DKLL	0.001	1.049	0.753	0.52
EVT-DKLL	0.001	0.605	0.579	0.429

	out-of-sample: N=5000				out-of-sample: N=10000			
	cov.	$\widehat{MSE}$	$\widehat{MAE}$	$\widehat{Med.AE}$	cov.	$\widehat{MSE}$	$\widehat{MAE}$	$\widehat{Med.AE}$
DKLL	0.009	3.673	1.359	0.9	0.008	3.379	1.253	0.787
EVT-DKLL	0.003	2.455	1.067	0.649	0.004	2.281	0.981	0.578

Table 1.8: Coverages and different loss functions from comparing the estimated 0.1% VaRs with the true quantile function. Cov. stands for coverage, MSE for mean squared error, MAE for mean absolute error and Med. AE for median absolute error.

Throughout, the EVT-augmented DKLL model yields lower losses and better coverages than the plain DKLL model. In order to robustify this result, we repeated the simulation for GARCH parameters estimated from EuroSTOXX data, and using different numbers of in-sample observations. All these results, which are available on request, lead to the conclusion that that the combination of standardized nonparametric residuals and extreme value theory is a valuable complement to the rearranged DKLL estimator, which we suggest to use for estimating moderately low quantiles.

1.7 Conclusion

In this paper, we propose a way to estimate and predict conditional Value at Risk using a nonparametric model. We consider probabilities that are of practical interest for financial institutions. For external market risk reporting, 1% portfolio VaRs have to be estimated on a daily basis. Internal risk management sometimes requires to take even more extreme probabilities, such as 0.1%, into account. Although typically very few observations are available in the extreme tails, models to be used should be flexible and rest upon as few structural assumptions as possible. We suggest to use nonparametric quantile regression, more specifically, a rearranged Double Kernel Local Linear VaR estimator as well as a version of the latter augmented by extreme value theory. Both are applied to different index return time series. Forecast performances are benchmarked against the widely used CAViaR models. Although these also perform well in many occasions, none of the considered specifications systematically dominates the others. In constrast to them, nonparametric regression circumvents the issue of choosing the appropriate parametrization.

Backtesting results from the evaluation of real as well as simulated data examples lead to the conclusion that the fully nonparametric and the EVT-refined nonparametric models do not only outperform the parametric alternatives in a considerable number of situations, but that they can be used to predict VaR of any probability level of interest, even when the estimation period is of moderate size. In recent years, computing power has increased to such an extent that fully nonparametric models come at little more computation cost than other models that rely on more restrictive assumptions. From the results in this paper, however, we conclude that the gains on the additional flexibility are substantial and nonparametric quantile regression with EVT refinements should be considered as a practical alternative for estimating and forecasting VaR.

Chapter 2

Financial Network Systemic Risk Contributions

This chapter is based on Hautsch, Schaumburg, and Schienle (2012a).

2.1 Introduction

The financial crisis 2007–2009 has shown that cross-sectional dependencies between assets and credit exposures can cause even small risks of individual banks to cascade and build up to a substantial threat for the stability of an entire financial system.[1] Under certain economic conditions, company-specific risk cannot be appropriately assessed in isolation without accounting for potential risk spillover effects from other firms. In fact, it is not just its size and idiosyncratic risk but also its interconnectedness with other firms which determines a company's systemic relevance i.e., its potential to significantly increase the risk of failure of the entire system – which we denote as systemic risk.[2] While there is a broad consensus that any prudential regulatory policy should account for the consequences of network interdependencies in the financial system, in practice, however, any attempt of a transparent implementation must fail, as long as suitable empirical measures for firms' individual risk, risk spillovers and systemic relevance are not available. In particular, it is unclear how to quantify individual risk exposures and systemic risk contributions in an appropriate but still parsimonious and empirically tractable way for a prevailing underlying

[1] For a thorough description of the financial crisis, see, e.g., Brunnermeier (2009).

[2] Bernanke (2009) and Rajan (2009) stress the danger induced by institutions which are "too interconnected to fail" or "too systemic to fail" in contrast to the insufficient focus on firms which are simply "too big too fail".

network structure. And there is an apparent need for empirically feasible and forward-looking measures which only rely on available data of publicly disclosed balance sheet and market information but still account for the complexity of the financial system.

A general empirical assessment of systemic relevance cannot build on the vast theoretical literature of financial network models and financial contagion, since these results typically require detailed information on intra-bank asset and liability exposures (see, e.g., Allen and Gale (2000), Freixas, Parigi, and Rochet (2000), and Leitner (2005)). Such data is generally not publicly disclosed and even regulators can only collect partial information on some sources of inter-bank linkages. Available empirical studies linked to this literature can therefore only partially contribute to a full picture of companies' systemic relevance as they focus on particular parts of specific markets at a particular time under particular financial conditions (see, e.g., Upper and Worms (2004), and Furfine (2003), for Germany and the U.S., respectively).[3] Furthermore, assessing risk interconnections on the basis of multivariate failure probability distributions has proven to be statistically complicated without using restrictive assumptions driving the results (see, e.g., Boss, Elsinger, Summer, and Thurner (2004), or Zhou (2010), and references therein). Finally, for regulators it is often unclear, how complex structures ultimately translate into dynamic and predictable measures of systemic relevance.

The objective of this chapter is to develop an easily and widely applicable measure of a firm's systemic relevance, explicitly accounting for the company's interconnectedness within the financial sector. We assess companies' risk of financial distress on the basis of share price information which directly incorporates market perceptions of a firm's prospects, publicly accessible market data as well as balance sheet data. As for risk interconnectedness only dependencies in extreme tails of asset return distributions matter, we base our measure on extreme conditional quantiles of corresponding return distributions quantifying the risk of distress of individual companies and the entire system respectively. In this sense, our setting builds on the concept of conditional Value-at-Risk (VaR), which is a popular and widely accepted measure for tail risk.[4] For each firm, we identify its so-called *relevant*

[3] See also Cocco, Gomes, and Martins (2009) for parts of the financial sector in Portugal, Elsinger, Lehar, and Summer (2006) for Austria, and Degryse and Nguyen (2007) for Belgium. A rare exception is the unique data set for India with full information on the intra-banking market studied in Iyer and Peydrió (2011).

[4] Note that the VaR is a coherent risk measure in realistic market settings, i.e., in cases of return distributions with tails decaying faster than those of the Cauchy distribution, see Garcia, Renault, and Tsafack (2007). In principle, our methodology could also be adapted to other tail risk measures such as, e.g., expected shortfall. Such a setting, however, would involve additional estimation steps and complications, probably inducing an overall loss of accuracy in results given the limited amount of available data.

(tail) risk drivers as the minimal set of macroeconomic fundamentals, firm-specific characteristics and risk spillovers from competitors and other companies driving the company's VaR. Detecting with whom and how strongly any institution is connected allows us to construct a tail risk network of the financial system. A company's contribution to systemic risk is then defined as the induced total effect of an increase in its individual tail risk on the VaR of the entire system, conditional on the firm's position within the financial network as well as overall market conditions. Furthermore, by assessing a company's conditional VaR in dependence of respective tail risk drivers, we obtain a reliable measure of a company's idiosyncratic risk in the presence of network spillovers.

The underlying statistical setting is a two-stage quantile regression approach: In the first step, firm-specific VaRs are estimated as functions of firm characteristics, macroeconomic state variables as well as tail risk spillovers of other banks which are captured by loss exceedances. Hereby, the major challenge is to shrink the high-dimensional set of possible cross-linkages between all financial firms to a feasible number of *relevant* risk connections. We address this issue statistically as a model selection problem in individual institution's VaR specifications which we solve in a pre-step. In particular, we make use of novel Least Absolute Shrinkage and Selection Operator (LASSO) techniques (see Belloni and Chernozhukov (2011)) which allows us to identify the relevant tail risk drivers for each company in a fully automatic way. The resulting identified risk interconnections are best represented in terms of a network graph as illustrated in Figure 2.1 (and discussed in more detail in the remainder of the chapter) for the system of 57 of the largest U.S. financial companies. In the second step, for measuring a firm's systemic impact, we individually regress the VaR of a value-weighted index of the financial sector on the firm's estimated VaR while controlling for the pre-identified company-specific risk drivers as well as macroeconomic state variables. We derive standard errors which explicitly account for estimation errors resulting from the pre-estimation of regressors in quantile relations. As the generally available sample sizes of balance sheet and macroeconomic information make the use of large-sample inference questionable, we provide (non-standard) bootstrap methods to construct finite-sample-based parameter tests.

We determine a company's systemic risk contribution as the marginal effect of its individual VaR on the VaR of the system. In analogy to an (inverted) asset pricing relationship in quantiles we call the measure *systemic risk beta*. It corresponds to the system's marginal risk exposure due to changes in the tail of a firm's loss distribution. For comparing the systemic relevance of companies across the system, however, it is necessary to compute the induced *total* increase in systemic risk. We therefore rank companies according to their "realized" systemic risk beta corresponding to the product of a company's systemic risk beta

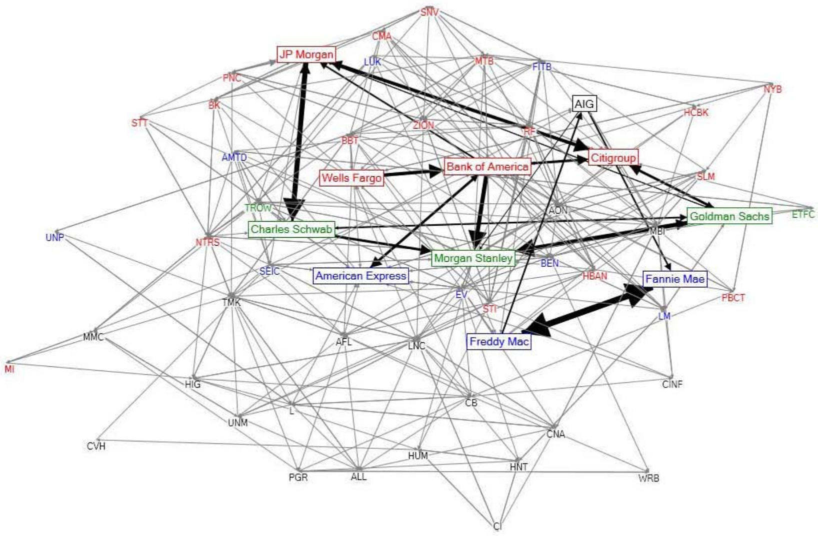

Figure 2.1: Risk network of the U.S. financial system schematically highlighting key companies in the system in 2000–2008. Details on all other firms in the system only appearing as unlabeled shaded nodes will be provided later in the chapter. Depositories are marked in red, broker dealers in green, insurance companies in black, others in blue. An arrow pointing from firm j to firm i reflects an impact of extreme returns of j on the VaR of i (VaR^i) which is identified as being relevant employing statistical selection techniques presented in the remainder of the chapter. VaRs are measured in terms of 5%-quantiles of the return distribution. The effect of j on i is measured in terms of the impact of an increase of the return X^j on VaR^i given X^j is below its 10% quantile. The size of the respective increase in VaR^i given a 1% increase of this "loss exceedance" of j is reflected by the thickness of the respective arrowhead. We distinguish between three categories: thin arrowheads display an increase up to 0.4, medium size of 0.4-0.8, and thick arrowheads of greater than 0.8. The thickness of the line of the arrow is chosen along the same categories. If arrows point in both directions, the thickness of the line corresponds to the bigger one of the two effects. The graph is constructed such that the total length of all arrows in the system is minimized. Accordingly, more interconnected firms are located in the center.

and its VaR. The systemic risk beta - and therefore also its realized version - is modeled as a function of firm-specific characteristics, such as leverage, maturity mismatch and size. Accordingly, a firm's tail risk effect on the system can vary with its economic conditions and/or its balance sheet structure changing its marginal systemic importance even though its individual risk level might be identical at different time points.

Our empirical results reveal a high degree of tail risk interconnectedness among U.S. financial institutions. In particular, we find that these network risk interconnection effects are the dominant risk drivers in individual risk. The detected channels of potential risk spillovers contain fundamental information for supervision authorities but also for company risk managers. Based on the topology of the systemic risk network, we can categorize firms into three broad groups according to their type and extent of connectedness with other companies: main risk transmitters, risk recipients and companies which both receive and transmit tail risk. From a regulatory point of view, the second group of pure risk recipients has the least systemic impact. Monitoring their condition, however, might still convey important accumulated information on potentially hidden problems in those companies which act as their risk drivers. In any case, the internal risk management of these companies should account for the possible threat induced by the large degree of dependence on others. In particular, assessing their full risk exposures requires network augmented risk measures such as, e.g., our proposed VaR specifications depending on (pre-selected) network risk drivers. The highest attention of supervision authorities should be attracted by firms which mainly act as risk drivers or are highly interconnected risk transmitters in the system. These are particularly firms in the center of the network which appear as "too interconnected to fail", but also large risk producers at the boundary which are linked to only a few but heavily connected risk transmitters. While the systemic risk network yields *qualitative* information on risk channels and roles of companies within the financial system, estimates of systemic risk betas allow to *quantify* the resulting individual systemic relevance and thus complement the full picture. Ranking companies based on (realized) systemic risk betas shows that large depositories are particularly risky. After controlling for all relevant network effects, they have the overall strongest impact on systemic risk and should be regulated accordingly. Confirming general intuition, time evolutions of (realized) systemic risk betas indicate that most companies' systemic risk contribution sharply increases during the 2007–2009 financial crisis. These effects are particularly pronounced for firms, which indeed got into financial distress during the crisis and are (ex post) identified as being clearly systemically risky by our approach. Figure 2.2 exemplarily illustrates the evolutions of their marginal systemic contributions – as reflected by systemic risk betas – as well as their exposure to idiosyncratic tail risk – as quantified by their VaR. A detailed pre-crisis case study confirms the validity of our methodology since firms such as, e.g., Lehman Brothers are ex-ante identified as being highly systemically relevant. It is well-known that their subsequent failure has indeed had a huge impact on the stability of the entire financial system. Likewise, the extensive bail-outs of American International Group (AIG), Freddie Mac and Fannie Mae can be justified given their high systemic risk betas and high interconnectedness by the end of 2007.

This chapter relates to several strands of recent empirical literature on systemic risk contributions. Closest to our work is White, Kim, and Manganelli (2012) who propose a

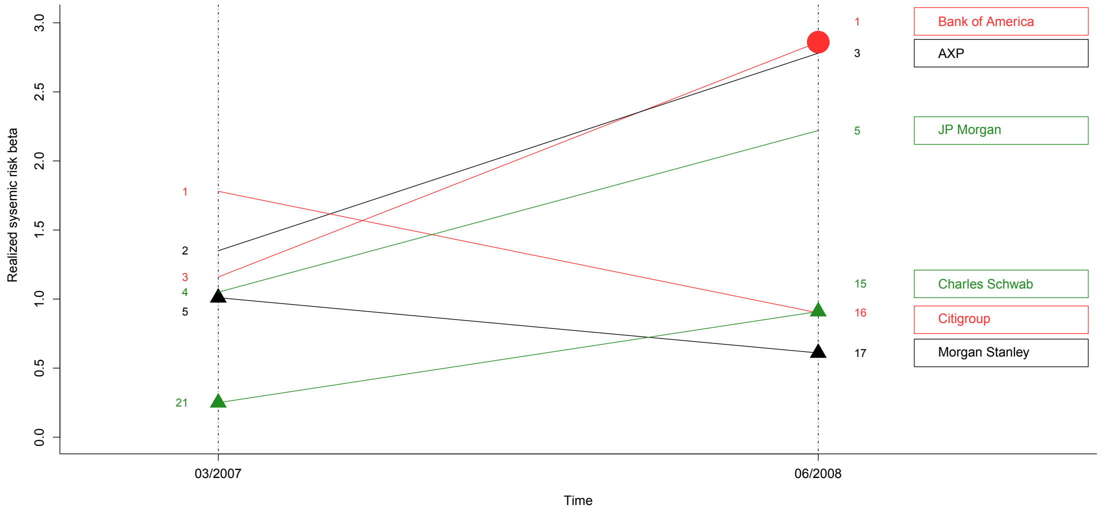

Figure 2.2: Systemic relevance of five exemplary firms in the U.S. financial system at two time points before and at the height of the financial crisis, 2008. Systemic relevance is measured in "systemic risk betas" quantifying the marginal increase of the VaR of the system given an increase in a bank's VaR while controlling for the bank's (pre-identified) risk drivers. All VaRs are computed at the 5% level and are by definition positive. We depict respective "realized" versions of the systemic risk beta corresponding to the product of a risk beta and the corresponding VaR representing a company's total effect on systemic risk. Connecting lines are just added to graphically highlight changes between the two time points but do not mark real evolutions. The size of the elements in the graph reflects the size of the VaR of the respective company at each of the two time points. We use the following scale: the element is k·standard size with $k = 1$ for $VaR \leq 0.05$, $k = 1.5$ for $VaR \in (0.05, 0.1]$, $k = 2$ for $VaR \in (0.1, 0.15]$, $k = 3$ for $VaR \in (0.2, 0.25]$ and $k = 5.5$ for $VaR \in (0.65, 0.7]$. Attached numbers inside the figure mark the position of the respective company in an overall ranking of the 57 largest U.S. financial companies for each of the two time points.

bivariate vector-autoregressive system of each company's VaR and the system VaR. They capture time variations in tail risk in a pure time series setting which however does not account for mutual dependencies and network effects. In contrast, our set-up models tail risk in dependence of economic state variables and network spillovers which automatically account for periods of turbulence when monitoring systemic relevance. Building on VaR, Adrian and Brunnermeier (2011) were the first to construct a systemic risk measure, called $CoVaR$, with balance sheet characteristics driving individual risk exposures. Note that CoVaR is conceptionally different two our two-step quantile approach and can by definition only vary through the channel of individual risk of the considered company. Moreover, network interconnections are not addressed which we identify as crucial for the performance of the model. Our work also complements papers which measure a company's systemic relevance by focusing on the size of potential bail-out costs, such as Acharya, Pedersen, Philippon, and Richardson (2012) and Brownlees and Engle (2012). Such approaches cannot detect spillover effects driven by the topology of the risk network and might underestimate the systemic importance of small but very interconnected companies. Moreover, while Brownlees and Engle (2012) study the situation of an individual firm given that the system is under distress, we investigate the reverse relation and measure the effect on the system given an individual firm is in financial trouble. Both approaches are justified as they take complementary perspectives and measure different dimensions of systemic risk. In the same way, we also complement macroeconomic approaches taking a more aggregated view as, e.g., the literature on systemic risk indicators (e.g., Segoviano and Goodhart (2009), 2009, Giesecke and Kim (2011), 2011) or papers on early warning signals (e.g., Schwaab, Koopman, and Lucas (2001), 2011, and Koopman, Lucas, and Schwaab (2011), 2011).

The remainder of the chapter is structured as follows. In Section 2.2, we briefly explain the modeling idea and describe the underlying data. Section 2.3 presents the model and estimation procedure for individual companies' VaRs, before discussing results on the financial network structure. Section 2.4 gives the second stage, the system VaR model, including estimation procedure, inference method and empirical results. In Section 2.5, we robustify and validate our results by presenting a case study of five large financial institutions that were affected by the financial crisis, and try to predict their distress and systemic relevance using only pre-crisis data. Section 2.6 concludes.

2.2 Measuring systemic relevance in a network

2.2.1 Framework

Assessing the dependence between systemic risk and firm-specific risk requires modeling regression relations in the (left) tails of respective asset return distributions, rather than in the center. This is in sharp contrast to a standard correlation analysis in (conditional) means which cannot quantify spillovers in tail situations of financial distress, and also goes beyond simple descriptive correlations between tails. Tail correlations do not allow detecting causal dependencies between tails and do not permit forecasting systemic risk contributions. We consider a stress-test-type scenario for assessing how changes in individual company-specific risk affect the risk of failure of the entire system given underlying network dependencies between institutions and market externalities at the respective point in time. Therefore, our model does not feature a general equilibrium framework, but is exclusively designed to provide a practically feasible and reliable measure of a company's marginal contribution to systemic risk in the presence of risk spillovers from other companies. These underlying network linkages between tail risks of firms in the system must be identified in a first step.

Defining the company-specific asset return as X_t^i, we measure the tail risk of a company as its conditional Value-at-Risk (VaR), $VaR_{p,t}^i$, given a set of company-specific *tail risk drivers* $\mathbf{W}_t^{(i)}$ containing network influences from other institutions in the system, i.e.,

$$\Pr(-X_t^i \geq VaR_{p,t}^i|\mathbf{W}_t^{(i)}) = \Pr(X_t^i \leq Q_{p,t}^i|\mathbf{W}_t^{(i)}) = p \tag{2.1}$$

with $VaR_{p,t}^i = VaR_{p,t}^i(\mathbf{W}_t^{(i)}) = -Q_{p,t}^i$ denoting the (negative) conditional p-quantile of X_t^i.[5] Likewise, system risk, $VaR_{p,t}^s$, is measured as the conditional VaR of the system return X_t^s obtained as the value-weighted average return of the set of all major financial companies.[6] To measure the systemic impact of company i, the system VaR is modeled in dependence of $VaR_{p,t}^i$ and additional control variables $\mathbf{V}_t$, i.e., $VaR_{p,t}^s = VaR_{p,t}^s(VaR_{p,t}^i, \mathbf{V}_t) = -Q_{p,t}^s$. Then, we define the *systemic risk beta* as the marginal effect of firm i's tail risk on the system tail risk given by

$$\frac{\partial VaR_{p,t}^s(\mathbf{V}_t, VaR_{q,t}^i)}{\partial VaR_{q,t}^i} = \beta_{p,q}^{s|i}. \tag{2.2}$$

We classify the systemic relevance of institutions according to the statistical significance of $\beta_{p,q}^{s|i}$ and the size of their total effect $\beta_{p,q}^{s|i} VaR_{q,t}^i$. We define the latter as a firm's *realized systemic risk contribution* raising with the system's marginal exposure to the company's tail risk (measured by $\beta_{p,q}^{s|i}$) and the firm's $VaR_{q,t}^i$. Changes in systemic relevance over time, however, cannot only occur through $VaR_{q,t}^i$ but also through the systemic risk beta $\beta_{p,q}^{s|i}$ which we allow to vary in firm-specific characteristics (see Section 2.4).[7] Note that this is conceptionally different to CoVaR of Adrian and Brunnermeier (2011).

As the VaR is not observable and has to be estimated, a major challenge is to select appropriate significant conditioning variables $\mathbf{W}_t^{(i)}$ yielding a flexible but still parsimonious model specification. We determine the *relevant* i-specific tail risk drivers out of a large set of potential regressors $\mathbf{W}_t$ containing lagged macroeconomic state variables $\mathbf{M}_{t-1}$, lagged

[5]Defining VaR as the *negative* p-quantile ensures that the Value-at-Risk is positive and is interpreted as a loss position.

[6]For details, see Section 2.2.2.

[7]For ease of illustration, here we skip the time index in $\beta_{p,q}^{s|i}$.

firm-specific characteristics $\mathbf{C}^i_{t-1}$, the i-specific lagged return X^i_{t-1}, and influences of all other companies apart from i, $\mathbf{E}^{-i}_t = (E^j_t)_{j \neq i}$, by a statistical selection technique as discussed in the remainder of the chapter. We find that these intra-system influences are best captured via contemporaneous loss exceedances, where the loss exceedance of a firm j is defined as $E^j_t = X^j_t 1(X^j_t \leq \hat{Q}^j_{0.1})$ and $\hat{Q}_{0.1}$ is the unconditional 10% sample quantile of X^j. Hence, company j only affects the VaR of company i if the former is under pressure. Since $\mathbf{E}^{-i}_t$ are return *realizations* and VaR^i_t is a future predicted quantity, this specification furthermore circumvents simultaneity issues. A model for VaR^i_t based on economic state variables as well as loss exceedances by construction automatically adjusts and prevails in distress scenarios under shocks in externalities. This is a clear advantage compared to pure time series approaches (cp. e.g. White, Kim, and Manganelli (2012), and Brownlees and Engle (2012)), which also drives the empirically convincing results in our validity case study in Section 2.5. These clearly reveal that systemic risk beta could have served as a valuable monitoring tool for prudent bail-out decisions of regulation authorities before the crisis.

The selection step allows identifying which (and how strongly) loss exceedances of other companies influence $VaR^i_{p,t}$ and is crucial for accounting for network dependencies between companies. As demonstrated in the sequel of the chapter, the latter are crucial for appropriately explaining individual tail risks. Moreover, identifying cross-firm dependencies for each company i is not only essential for appropriately capturing firm-specific VaRs in a first step but is also crucial for selecting necessary control variables in the estimation of $\beta^{s|i}_{p,q}$ in the second step. In particular, for an unbiased estimate of $\beta^{s|i}_{p,q}$, it is necessary to control for any tail risk drivers influencing *both* $VaR^s_{p,t}$ and VaR^i_q. Accordingly, $\mathbf{V}_t$ must contain macroeconomic state variables as well as the tail risks (represented by the VaRs) of *all* companies which are identified to influence company i. Ignoring these spillover effects would lead to a biased measure of systemic risk contribution.

The identified risk connections between all firms constitute a systemic risk network. The latter is not only a prerequisite for the quantification of marginal systemic risk contributions but contains additional valuable regulatory information on potential risk channels and specific roles of companies as risk transmitters and/or recipients. Accordingly, the following analysis consists of two steps where in the first step firm-specific VaRs and network effects are quantified (Section 2.3) before in the second step, systemic risk betas are estimated while controlling for the (pre-)identified cross-company dependencies (Section 2.4).

2.2.2 Data

Our analysis focuses on publicly traded U.S. financial institutions. The list of included companies in Table 2.A.1 comprises depositories, broker dealers, insurance companies and others.[8] To assess a firm's systemic relevance, we use publicly available market and balance sheet data. Such data constitutes a solid basis for transparent regulation since timely access on detailed information of connections between firms' assets and obligations, is very difficult and expensive to obtain – even for central banks.

Daily equity prices are obtained from Datastream and are converted to weekly log returns. To account for the general state of the economy, we use weekly observations of seven lagged macroeconomic variables $\boldsymbol{M}_{t-1}$ as suggested and used by Adrian and Brunnermeier (2011) (abbreviations as used in the remainder of the chapter are given in brackets): the implied volatility index, VIX, as computed by the Chicago Board Options Exchange (vix), a short term "liquidity spread", computed as the difference of the 3-month collateral repo rate (available on Bloomberg) and the 3-month Treasury bill rate from the Federal Reserve Bank of New York (repo), the change in the 3-month Treasury bill rate (yield3m) and the change in the slope of the yield curve, corresponding to the spread between the 10-year and 3-month Treasury bill rate (term). Moreover, we utilize the change in the credit spread between BAA rated bonds and the Treasury bill rate (both at 10 year maturity) (credit), the weekly equity market return from CRSP (marketret) and the one-year cumulative real estate sector return, computed as the value-weighted average of real estate companies available in the CRSP data base (housing).

Moreover, to capture characteristics of individual institutions predicting a bank's propensity to become financially distressed, $\boldsymbol{C}^i_{t-1}$, we follow Adrian and Brunnermeier (2011) and use (i) leverage, calculated as the value of total assets divided by total equity (in book values) (LEV), (ii) maturity mismatch, measuring short-term refinancing risk, calculated as short term debt net of cash divided by the total liabilities (MMM), (iii) the market-to-book value, defined as the ratio of the market value to the book value of total equity (BM), (iv) market capitalization, defined by the logarithm of market valued total assets (SIZE) and (v) the equity return volatility, computed from daily equity return data (VOL). The system return is chosen as the return on the financial sector index provided by Datastream. It is computed as the value-weighted average of prices of 190 U.S. financial institutions.

As balance sheets are available only on a quarterly basis, we interpolate the quarterly data to a daily level using cubic splines, and then aggregate them back to calendar weeks.

[8] Companies are distinguished according to their two-digit SIC codes, following the categorization in Acharya, Pedersen, Philippon, and Richardson (2012).

We focus on 57 financial institutions existing through the period from beginning of 2000 to end of 2008, resulting into 467 weekly observations on individual returns. This restriction has the drawback of excluding companies which defaulted during the financial crisis. Therefore, to address this issue and to validate and robustify our approach, we re-estimate the model over a sub-period ending before the financial crisis and including, among others, the investment banks Lehman Brothers and Merrill Lynch that were massively affected by the crisis.

2.3 A tail risk network

2.3.1 Measuring firm-specific tail risks

Identification of tail risk drivers

Specifying the VaR of firm i at time point $t = 1, \dots, T$ as a linear function of the i-specific tail risk drivers $\mathbf{W}_t^{(i)}$,

$$VaR_q^i = \mathbf{W}^{(i)\prime} \boldsymbol{\xi}_q^i \,, \tag{2.3}$$

yields a linear function in return quantiles

$$X_t^i = -\mathbf{W}_t^{(i)\prime} \boldsymbol{\xi}_q^i + \varepsilon_t^i, \quad \text{with} \quad Q_q(\varepsilon_t^i | \mathbf{W}_t^{(i)}) = 0. \tag{2.4}$$

If we knew the i-relevant risk drivers $\mathbf{W}^{(i)}$ selected out of $\mathbf{W}$, then, estimates $\widehat{\boldsymbol{\xi}_q^i}$ of $\boldsymbol{\xi}_q^i$ could be obtained according to standard linear quantile regression (Koenker and Bassett (1978)) by minimizing

$$\frac{1}{T} \sum_{t=1}^{T} \rho_q \left(X_t^i + \mathbf{W}_t^{(i)\prime} \boldsymbol{\xi}_q^i \right) \tag{2.5}$$

with loss function $\rho_q(u) = u(q - I(u < 0))$, where the indicator $I(\cdot)$ is 1 for $u < 0$ and zero otherwise, and

$$\widehat{VaR}_{q,t}^i = \mathbf{W}_t^{(i)\prime} \widehat{\boldsymbol{\xi}}_q^i \,. \tag{2.6}$$

However, the relevant risk drivers $\mathbf{W}^{(i)}$ for firm i are unknown and must be determined from $\mathbf{W}$ in advance. Model selection is not straightforward in the given setting as tests on the individual significance of single variables do not account for the (possibly high) collinearity between the covariates. Moreover, sequences of joint significance tests have too many possible variations to be easily checked in case of more than 60 variables. Since alternative model selection criteria, like the Bayes Information Criterion (BIC) or the Akaike Information Criterion (AIC), are not available in a quantile setting, we choose the *relevant* covariates in a data-driven way by employing a statistical shrinkage technique known as the least absolute shrinkage and selection operator (LASSO). LASSO methods are standard for high-dimensional conditional mean regression problems (see Tibshirani (1996)), and have recently been adapted to quantile regression by Belloni and Chernozhukov (2011). Accordingly, we run an l_1-penalized quantile regression and calculate for a fixed individual penalty parameter λ^i,

$$\widetilde{\boldsymbol{\xi}}^i_q = \text{argmin}_{\xi^i} \frac{1}{T} \sum_{t=1}^{T} \rho_q \left(X^i_t + \mathbf{W}'_t \xi^i \right) + \lambda^i \frac{\sqrt{q(1-q)}}{T} \sum_{k=1}^{K} \hat{\sigma}_k |\xi^i_k| \, , \tag{2.7}$$

with the set of potentially relevant regressors $\mathbf{W}_t = (W_{t,k})_{k=1}^K$, componentwise variation $\hat{\sigma}_k^2 = \frac{1}{T}\sum_{t=1}^T (W_{t,k})^2$ and the loss function ρ_q as in (2.5). The key idea is to select relevant regressors according to the absolute value of their respective estimated marginal effects (scaled by the regressor's variation) in the penalized VaR regression (2.7). Regressors are eliminated if their shrunken coefficients are sufficiently close to zero. Here, all firms in $\mathbf{W}$ with absolute marginal effects $|\widetilde{\boldsymbol{\xi}}^i|$ below a threshold $\tau = 0.0001$ are excluded keeping only the $K(i)$ remaining relevant regressors $\mathbf{W}^{(i)}$. Hence, LASSO de-selects those regressors contributing only little variation. Due to the additional penalty term in (2.7), all coefficients $\widetilde{\boldsymbol{\xi}}^i_q$ are generally downward biased in finite samples. Therefore, we re-estimate the unrestricted model (2.5) only with the selected relevant regressors $\mathbf{W}^{(i)}$ yielding the final estimates $\widehat{\boldsymbol{\xi}}^i_q$. This post-LASSO step produces finite sample estimates of coefficients $\boldsymbol{\xi}^i_q$ which are superior to the original LASSO estimates or plain quantile regression results without penalization suffering from overidentification problems (see the original paper by Belloni and Chernozhukov (2011) for consistency of the post LASSO step).

The selection of relevant risk drivers via LASSO crucially depends on the choice of the company-specific penalty parameter λ^i. The larger λ^i, the more regressors are eliminated. Conversely, in case of $\lambda^i = 0$, we are back in the standard quantile regression setting (2.5) without any de-selection. For each institution, we determine the appropriate penalty level λ^i in a completely data-driven way such that it dominates a relevant measure of noise in the sample criterion function. In particular, we use the supremum norm of a suitably rescaled gradient of the sample criterion function evaluated at the true parameter value as in Belloni and Chernozhukov (2011). In this sense, number and elements of the set of relevant risk drivers are determined only from the data without any restrictive pre-assumptions. For details on the empirical procedure we refer to Appendix 2.A.1.

Evaluating the goodness of fit of conditional VaR model specifications should take into account how well the model captures the specific percentile of the return distribution but also how well the model predicts size and frequency of losses. The latter issue cannot be captured, e.g., by quantile-based modifications of the conventional R^2. We therefore consider a VaR specification as inadequate if it either fails producing the correct empirical level of VaR exceedances but also if the sequence of exceedances is *not* independently and identically distributed over the considered time period. This proceeding ensures that VaR violations today do not contain information about VaR violations in the future and both occur according to the same distribution. This is formally tested using a likelihood ratio (LR) version of the dynamic quantile (DQ) test developed in Engle and Manganelli (2004) and described in detail in Appendix 2.A.1. Berkowitz, Christoffersen, and Pelletier (2011) show that this likelihood ratio (LR) test has superior size and power properties compared to competing conditional VaR backtesting methods which dominate plain unconditional level tests (as e.g. Kupiec (1995)).

Empirical evidence

We estimate VaR specifications with $q = 0.05$ for all companies employing the LASSO selection procedure described in Section 2.3.1.[9] Exemplary VaR^i (post-)LASSO regression results for firms in the four industrial sectors depositories, insurances, brokers and others are provided in Table 2.A.2.

The main drivers of company-specific VaRs are loss exceedances of other firms. In their presence, macroeconomic variables and firm-specific characteristics often do not have any statistically significant influence and are not selected by the LASSO procedure. In Table 2.A.2, only for Torchmark (TMK) and Regions Financial (RF) regressors other than cross-firm links are selected. In contrast, VaR specifications of Goldman Sachs (GS), Morgan Stanley (MS), JP Morgan (JPM) and AIG exclusively contain loss exceedances from other firms. Particularly the connections between close competitors, such as Goldman Sachs and Morgan Stanley and the influence of mortgage company Freddie Mac (FRE) on AIG correspond are highly plausible and are confirmed by market evidence. The relevance of cross-firm effects is additionally robustified by testing for the joint significance of the individually selected loss exceedances $\mathbf{E}_t^{-i}$. This is performed based on a quantile regression version of the F-test of linear hypothesis developed by Koenker and Bassett (1982). We find that the selected tail risk spillovers are highly significant in all but very few cases. See

[9] Due to the limited number of observations, we refrain from considering more extreme probabilities.

Table 2.A.3 for an overview of all cross-effects.

The importance of including other companies' loss exceedances as potential risk drivers for a company i is also illustrated by a simple comparison of the performance of our LASSO-selected specifications to a model of VaR^i only using macroeconomic variables as in Adrian and Brunnermeier (2011). According to the employed backtests, specifications allowing for cross-firm dependencies reveal a strong predictive ability and are significantly superior to simplistic models including macroeconomic regressors only. Figure 2.3 shows the distributions of the backtesting p-values implied by both models. Hence, inter-company linkages do not only add crucial explanatory power in VaR specifications but in fact contain the main information for explaining individual tail risk.

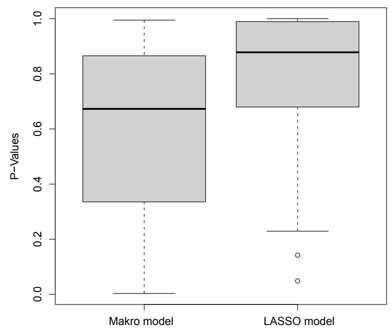

Figure 2.3: Boxplots of backtesting p-values indicating the in-sample model fit of VaR specifications including macroeconomic regressors only (left) and VaR specifications resulting from the LASSO selection procedure (2.7) (right).

Our results show that the major information about cross-company dependencies in tail risks is primarily contained in *contemporaneous* loss exceedances $\mathbf{E}_t^{-i}$. In contrast, alternative VaR specifications utilizing corresponding returns X_t^{-j} or lagged loss exceedances $\mathbf{E}_{t-1}^{-i}$ imply significantly inferior backtest performances with the regressors being mostly not significant in joint F-tests.[10] Moreover, linking VaR forecasts and thus predictions of *hypothetical* losses to already *realized* loss exceedances allows measuring mutual dependencies between companies without requiring a simultaneous system of equations in conditional quantiles. In particular, observed bi-directional relationships between conditional quantiles and realized loss exceedances of different firms (e.g., between Goldman Sachs and Morgan Stanley) do not reflect simultaneities as feedbacks are not contemporaneous: For instance,

[10] All F-test results are available upon request and omitted here for sake of brevity.

a highly negative (realized) return of company j increases the conditional loss quantile and therefore increases the VaR of firm i. However, a higher conditional VaR of i does not necessarily directly increase the absolute realized loss return of i but just makes it more likely. Avoiding an explicit treatment of simultaneities in quantiles while still addressing network dependencies is an important advantage of our approach.[11]

2.3.2 Network model and structure

We constitute a tail risk network of the system from individually selected loss exceedances reflecting cross-firm dependencies. Taking all firms as nodes in such a network, there is an influence of firm j on firm i, if E^j is LASSO-selected in (2.7) as a relevant risk externality of firm i in VaR_q^i. In particular, if E^j is part of $\mathbf{W}^{(i)}$ as its k-th component, then the corresponding coefficient $\xi_{q,k}^i$ in $\boldsymbol{\xi}_q^i$ marks the risk impact of firm j on firm i in the network. If E^j is not selected as relevant risk driver of firm i, there is no arrow from firm j to firm i.

For each company in the system, the network builds on only directly influencing and influenced firms and all other companies directly influencing the influenced firms. In the Bayesian network literature, these constitute a so-called Markov blanket assumed to contain all relevant information for predicting the node's role in the network (see Friedman, Geiger, and Goldszmidt (1997)). An overview of the identified tail risk connections between all companies is provided in Table 2.A.3, reporting which company's loss exceedance affects which others' VaR and vice versa. We observe that the number of risk connections substantially varies over the cross-section of companies. While some firms such as, e.g., Morgan Stanley, Bank of America (BAC), American Express (AXP) as well as Bank of New York Mellon (BK), are strongly inter-connected with many other companies, there are institutions, such as Fannie Mae (FNM), AIG (AIG) and a couple of further insurances revealing significantly less cross-firm dependencies. In order to effectively illustrate identified risk connections and directions, we graphically depict the resulting network of companies in Figure 2.4. The layout and allocation of the network is chosen such that the sum of cross-firm distances are minimized. Consequently, the most connected firms are located in the center of the network while the less involved companies are placed at its boundary.

[11] Econometrically it is open how to handle such a system in conditional quantiles in general. In contrast to relations in (conditional) means, it is unclear how marginal q-quantiles constitute the respective quantile in the joint distribution under appropriate independence assumptions. Only in lags, restricted to very small dimensions and under strong assumptions, solutions have been obtained via CaViAR type recursions (see White, Kim, and Manganelli (2012)).

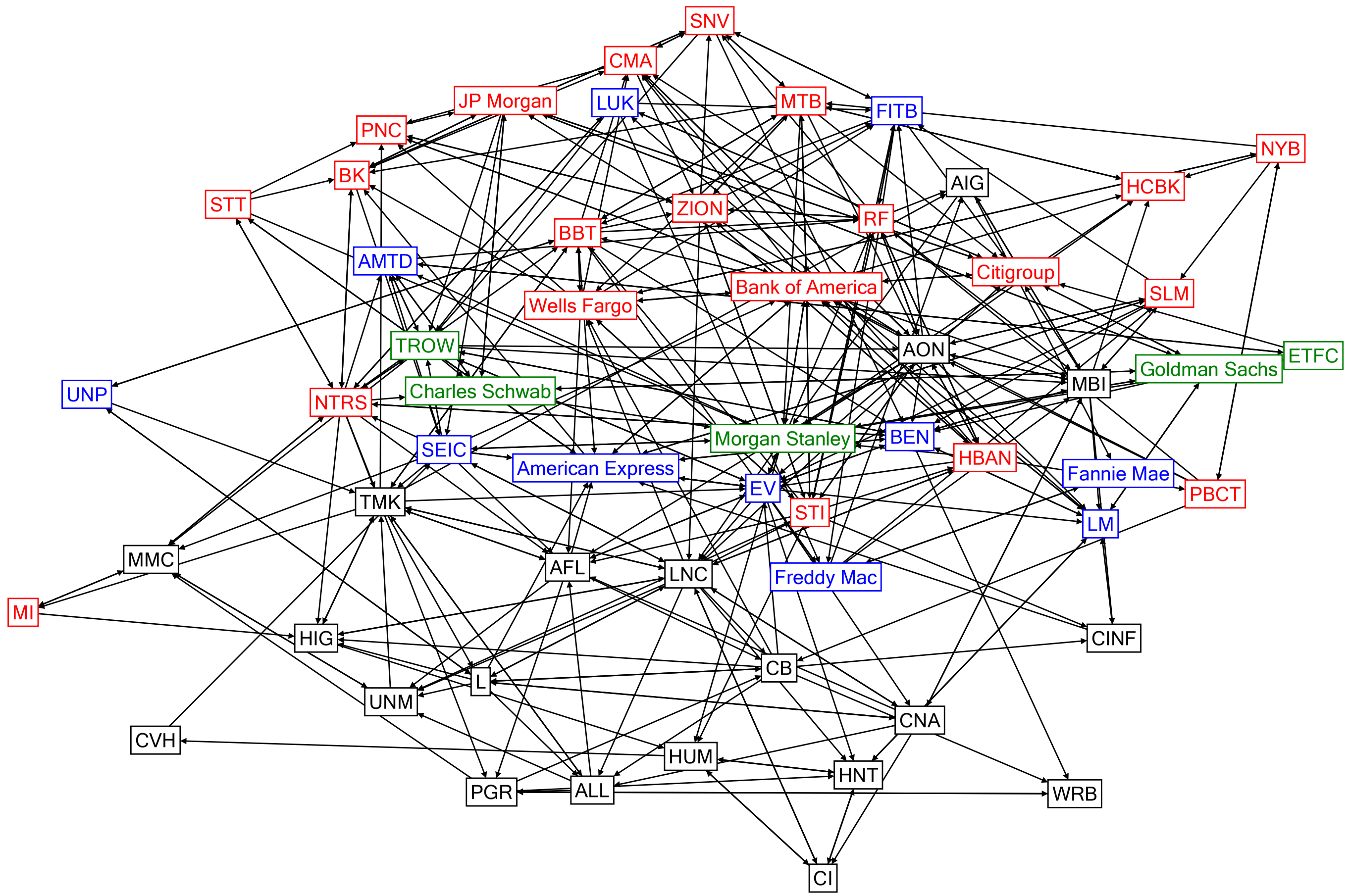

Figure 2.4: Full network graph for the system of the 57 largest financial companies in the U.S. For simplicity, arrows only mark risk spillovers effects without referring to their respective size. Otherwise arrows and colors are as defined in Figure 2.1. A complete list of firms' acronyms is contained in Table 2.A.1. The graphical allocation is obtained via the Fruchtermann-Reingold algorithm which minimizes the total length of all arrows.

The resulting network topology reveals different roles of companies within the financial network. We distinguish between three major categories: The first group contains companies with only few incoming arrows but numerous outgoing ones, which thus mainly act as risk drivers within the system. These are institutions whose potential failure might affect many others but, conversely, which are themselves relatively unaffected by the distress of other firms. Risk management of such firms can therefore be based mostly on idiosyncratic criteria without accounting too much for influences of the system. For regulatory authorities, however, a close monitoring is important as a failure of such a company can induce substantial systemic risks through multiple channels into the financial network. Our results show that only few firms belong to this category. Examples are State Street Corporation (STT), one of the top ten U.S. banks, Leucadia National Corporation (LUK), a holding company which is, among others, engaged in banking, lending and real estate, and SEI Investments Company (SEIC), a financial services firm providing products and service in asset and investment management. Financial distress of these banks obviously has widespread consequences. For instance, State Street reveals spillovers to the financial services companies American Express and Northern Trust (NTRS), the Bank of New York Mellon and Morgan Stanley. Leucadia affects Citigroup (C), one of the biggest banks in the U.S., and Freddie Mac, one of the two largest U.S. mortgage companies. Finally, SEI Investments has links to various big institutions, such as Bank of America, American Express, Morgan Stanley and the online broker TD Ameritrade (AMTD).

The second group contains companies which mainly are risk takers within the system. These companies are not necessarily systemically risky but might severely suffer from distress of others and should account for such spillovers in their internal risk management. According to Table 2.A.3 and Figure 2.4 these firms are primarily insurance companies. Examples are Cincinnati Financial Corporation (CINF), a company for property and casualty insurance, Humana Incorporation (HUM) managing health insurances or Progressive Corporation Ohio (PGR) providing automobile insurance and other property-casualty insurances.

The third group is the largest category within the network. It consists of companies which serve as both risk recipients and risk transmitters which amplify tail risk spillovers by further disseminating risk into new channels. Due to their role as risk distributors such companies are key systemic players and should be supervised accordingly. We further distinguish between strongly and less connected firms. The first subgroup is the most difficult but most important to regulate tightly. Examples are Goldman Sachs, Citigroup, Morgan Stanley, AON Corporation (AON), Bank of America, American Express, Freddie Mac as well as the insurance company MBIA (MBI), among many others. Bank of America and

Citigroup are among the five largest banks in the U.S. and reveal strong connections to various other big institutions, such as Morgan Stanley, JP Morgan, Goldman Sachs, American Express, Regions Financial and AIG. Details on the specific role of Citigroup and Morgan Stanley within the system are highlighted in Figure 2.A.1. Morgan Stanley, with strong links to many companies, such as Goldman Sachs, Bank of America and the savings bank Hudson City Bancorporation (HCBK), and the insurance company AON are examples for deeply connected firms located in the center of the network. Likewise, Freddie Mac is strongly involved and was particularly affected by the 2008 credit crunch in the mortgage sector. Accordingly, also MBIA realized severe losses during the financial crisis due to investments in mortgage backed securities.
The second subgroup might be technically easier to monitor with companies revealing risk connections with only very few other firms. Still, supervision is not less important than for the first subgroup. Examples are Fannie Mae and AIG. Fannie Mae reveals significant bilateral risk connections to its main competitor Freddie Mac. AIG holds significant positions in mortgage backed securities and as a consequence is closely connected to both Fannie Mae and Freddie Mac. Probably due to the same reason, we also observe bilateral tail risk dependencies between AIG and MBIA. Even though their number of relevant risk connections within the network is limited, such firms can still have a crucial overall impact on the system. In case of the 2008 financial crisis, the dependence between Freddie Mac and Fannie Mae as well as their interaction with AIG had severe systemic consequences.

Figure 2.A.2 reveals that it is not sufficient to focus on sector-specific subnetworks only. Indeed interconnectedness of institutions occurs to a large proportion *between* industrial sectors. In these circle layout network graphs, companies are grouped according to industries with risk outflows for each group being highlighted. We observe that tail risks of depositories, insurances and others are relatively equally distributed among all other industry groups. Depositories are most strongly connected and also reveal the strongest tail risk links among each other. This is in contrast to the other industries where cross-firm connections *within* a group are less strong. Moreover, in contrast to other industry categories, the risk outflow of broker dealers is clearly more concentrated. They particularly affect big banks such as Bank of America and Citigroup as well as financial service companies such as American Express or SEI. Only very few direct connections to insurance companies are revealed.

Besides graphical illustration and inflow-outflow categorizations, standard network characteristics can provide a more comprehensive picture of the interconnectedness and the role of each node in the system. In Figure 2.5 we depict firms' pagerank coefficient (see Brin and Page (1998)) which does not plainly count links but empirically weights their importance in an iterative scheme.[12]

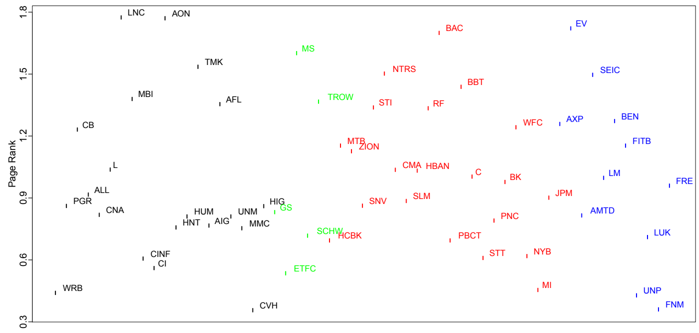

Figure 2.5: Pagerank coefficients based on the estimated tail risk network computed as in Berkhin (2005) with ordering of institutions according to sectors. Colors and acronyms are as in Figure 2.A.2.

Confirming the visual impression based on Figure 2.4, the most connected firms are Lincoln National Corporation, AON, Bank of America, TD Ameritrade and Morgan Stanley. The graph confirms our finding above that depositories tend to be slightly stronger involved than the other industry groups. Particularly insurances are seperated into a group of highly connected firms, such as Lincoln National Corp., AON and MBI, and a group of companies being less connected, such as AIG, Humana, Unum Group (UNM) and Cincinnati Financial Corp.

Note that pagerank coefficients such as other network metrics can only assess the local impact and centrality of firms in the network containing relevant but not all information for judging overall systemic relevance. Therefore, a risk network does not allow to fully quantitatively assess the systemic relevance of a financial institution. Nevertheless, the degree of firms' interconnectedness and the specific topology of the network or corresponding sub-networks allows to identify possible risk channels in the system. These interlinkages are central but not comprehensive for macroprudential regulation reflecting the particular role of a firm as risk recipient, transmitter or distributor of tail risk. To explicitly *quantify* a firm's marginal systemic relevance, we propose the concept of systemic risk betas presented

[12]The key idea is to assign a weight to each node (i.e., a company in our context) which is increasing with the number of connections to others and the relative importance thereof. The more connected a firm is, the higher its importance and thus the higher the importance of its neighbor. The computation of the pagerank coefficient can be understood as an eigenvalue problem which can solved iteratively. For more details, see Berkhin (2005).

in the following section.

2.4 Quantifying systemic risk contributions

2.4.1 Measuring systemic risk betas

Besides valuable information on financial network structures, the focus of supervision authorities is on an accurate but parsimonious measure of an institution's systemic impact. We quantify the latter as the effect of a marginal change in the tail risk of firm i on the tail risk of the system given the underlying network structure of the financial system. In order to obtain unbiased estimates of this specific marginal effect in the VaR regression of the system, however, it is sufficient to additionally only control for firms which are relevant i-specific risk drivers in the network. Conversely, variables unrelated to VaR^i do not affect firm i's systemic risk contribution.[13] Thus, a fully-fledged structural general equilibrium model is not necessary. Even if correctly specified, an equilibrium setting would be practically infeasible failing to deliver sufficiently precise estimates given the high-dimensionality and interconnectedness of the financial system on the one hand and the limited data availability on the other.

For this reason, we propose estimating systemic risk contributions based on models which are specific for each firm i as they only control for the i-specific risk drivers. Correspondingly, we estimate the firm-i-specific *systemic risk beta* $\beta^{s|i}_{q,p}$ based on a linear model for the system VaR of the form

$$VaR^s_{p,t} = \mathbf{V}_t^{(i)'} \gamma^s_p + \beta^{s|i}_{p,q} VaR^i_{q,t}, \tag{2.8}$$

where the vector of regressors $\mathbf{V}_t^{(i)} = (1, \mathbf{M}_{t-1}, \mathbf{VaR}^{(-i)}_{q,t})$ includes a constant effect, lagged macroeconomic state variables and the VaRs of all companies which are identified as risk drivers for firm i via LASSO, see Section 2.3.

The systemic risk beta $\beta^{s|i}_{p,q} = \beta^{s|i}$ of company i captures the effect of a marginal change in VaR^i_t on VaR^s_t. It can be interpreted in analogy to an inverse asset pricing relationship in quantiles, where bank i's q-th return quantile drives the p-th quantile of the system given network-specific effects and firm-specific and macroeconomic state variables.[14] Accord-

[13] See Angrist, Chernozhukov, and Fernández-Val (2006) for a simple Frisch-Waugh-type argument in quantile regressions.

[14] Note that our stress test scenario only studies the immediate effect of an exogenous risk shock in company i for the system. We do not infer anything about further steps which should then also account for converse effects of increases of system risk causing firm specific risk to raise.

ingly,

$$\bar{\beta}^{s|i}_{p,q} := \beta^{s|i}_{p,q} VaR^i_t \tag{2.9}$$

measures the full partial effect of a tail risk increase of bank i on VaR^s_t. We refer to $\bar{\beta}^{s|i}_{p,q}$ as the *realized* systemic risk contribution as it is is computed based on market realizations and is useful for real-time crisis monitoring. Moreover, scaling systemic risk betas by the corresponding VaR allows to compare systemic risk contributions cross-sectionally, and to rank banks according to their systemic relevance.

During periods of turbulence, not only banks' risk exposures change, but also their marginal importance for the system might vary. We therefore allow $\beta^{s|i}$ to be time-varying. In particular, time-variation occurs through observable factors $\mathbf{Z}^i$ characterizing a bank's propensity to get into financial distress. Accordingly, $\beta^{s|i}_t$ should be interpreted as a *conditional* measure. To limit complexity and computational burden of the model, we assume linearity of $\beta^{s|i}_{p,q,t}$ in firm-specific distress indicators $\mathbf{Z}^i_{t-1}$,

$$\beta^{s|i}_{p,q,t} = \beta^{s|i}_{0,p,q} + \mathbf{Z}^i_{t-1}{}' \boldsymbol{\eta}^{s|i}_{p,q}, \tag{2.10}$$

where $\boldsymbol{\eta}^{s|i}_{p,q}$ are the parameters driving the time-varying effects. The case of a constant systemic risk beta is obviously contained as a special case if $\boldsymbol{\eta}^{s|i}_{p,q} = 0$ and thus $\beta^{s|i}_{0,p,q} = \beta^{s|i}_{p,q,t} = \beta^{s|i}_{p,q}$.

We choose $\mathbf{Z}^i_t = C^i_t$ as the firm-specific tail risk drivers since size, leverage, maturity mismatch, book-to-market ratio and volatility might not only affect a bank's VaR, but also directly drive its marginal systemic relevance. As a consequence, systemic risk contributions of two companies with the same exposure to macroeconomic risk factors and financial network spillovers may be still different as they depend on their balance sheet structures. The significance of time variation in these quantities can then be statistically tested for (see Section 2.4.3 below).

Due to the linearity of (2.10) we can thus write the quantile model (2.8) for VaR^s_p with time-varying $\beta^{s|i}_{p,q,t}$ in the following form

$$VaR^s_{p,t} = \mathbf{V}^{(i)\prime}_t \gamma^s_p + \beta^{s|i}_{0,p,q} VaR^i_{q,t} + (VaR^i_{q,t} \cdot \mathbf{Z}^i_{t-1})' \boldsymbol{\eta}^{s|i}_{p,q} \,. \tag{2.11}$$

2.4.2 Estimation and inference

If firm specific VaRs were directly observable, the magnitude and significance of i-specific systemic risk betas could be directly inferred from the linear quantile regression (2.11) in analogy to (2.5) with the VaR defined by (2.1). However, note that the regressors VaR_t^i and $\mathbf{VaR}_{q,t}^{(-i)}$ in $\mathbf{V}^{(i)}$ are pre-estimated as they arise from the first-step quantile regressions as shown in Section 2.3. Hence, operationalizing (2.11) with $\widehat{VaR}_t^i$ and $\widehat{\mathbf{VaR}}_{q,t}^{(-i)}$ as generated regressors, yields the (second step) quantile regression,

$$\begin{aligned} X_t^s &= -\widehat{\mathbf{V}^{(i)}}_t'\gamma_p^s - \beta_{0,p,q}^{s|i}\widehat{VaR}_{q,t}^i - (\widehat{VaR}_{q,t}^i \cdot \mathbf{Z}_{t-1}^i)'\boldsymbol{\eta}_{p,q}^{s|i} + \varepsilon_t^s, \\ &\text{with} \quad Q_p(\varepsilon_t^s|\widehat{VaR}_{q,t}^i, \widehat{\mathbf{V}}_t^{(i)}, \mathbf{Z}_{t-1}^i) = 0 \,. \end{aligned} \tag{2.12}$$

Using the notation $\widehat{\mathbf{V}}^{(i)}$, we stress that some components of $\mathbf{V}^{(i)}$ are pre-estimated as $\widehat{\mathbf{VaR}}_q^{(-i)}$. Then, analogously to the first-step regressions in Section 2.3, parameter estimates are obtained via quantile regression minimizing

$$\frac{1}{T}\sum_{t=1}^{T}\rho_p\left(X_t^s + \widehat{\mathbf{V}^{(i)}}_t'\gamma_p^s + \beta_{0,p,q}^{s|i}\widehat{VaR}_{q,t}^i + (\widehat{VaR}_{q,t}^i \cdot \mathbf{Z}_{t-1}^i)'\boldsymbol{\eta}_{p,q}^{s|i}\right) \tag{2.13}$$

in the unknown parameters. Consequently, the resulting estimate of the full time-varying marginal effect $\widehat{\beta}_{p,q}^{s|i}$ in (2.10) is obtained as

$$\widehat{\beta}_{p,q,t}^{s|i} = \widehat{\beta}_{0,p,q}^{s|i} + \mathbf{Z}_{t-1}^{i}{}'\widehat{\boldsymbol{\eta}_{p,q}}^{s|i} \tag{2.14}$$

for given values $\mathbf{Z}_{t-1}^i$.

Since $VaR_{q,t}^i$ is a function of $\mathbf{W}^{(i)}$, conditional quantile independence in (2.12) is equivalent to $Q_p(\varepsilon_t^s|\mathbf{W}_t^{(i)}, \mathbf{W}_t^{(-i)}, \mathbf{Z}_{t-1}^i) = 0$ where $\mathbf{W}_t^{(-i)}$ stacks $\mathbf{W}_t^{(j)}$ for all firms relevant for company i appearing in $\widehat{\mathbf{VaR}}_{q,t}^{(-i)}$. Hence, with both quantile regression steps being linear, inserting (2.3) into (2.11) yields a full model for the system's tail risk in observable characteristics. However, direct one-step estimation is only feasible if the choice of $\mathbf{W}^{(i)}$ and thus $\mathbf{VaR}_{q,t}^{(-i)}$ is still determined in a pre-step from individual VaR regressions. Model selection based on the full model of VaR^s in observables is infeasible since correlation effects among the huge number of regressors would produce unreliable results. Furthermore, individual parameters $\beta_{0,p,q}^{s|i}$ and $\boldsymbol{\eta}_{p,q}^{s|i}$ could not be identified without additional identification condition $Q_q(\varepsilon_t^i|\mathbf{W}_t^{(i)}) = 0$, implicitly bringing back the first-step estimation. Therefore we use two-step estimation even if exact asymptotic confidence intervals are larger than for an (infeasible) single step procedure. In contrast to mean regressions, such results are non-standard in a quantile setting and are therefore provided in detail in Appendix 2.A.1. In finite samples, however, asymptotic distributions often only provide a poor approximation to the true distribution of the (scaled) difference between the estimator and the true value if sample sizes are not sufficiently large. In case of quantile regressions, this effect is even more pronounced, since valid estimates for the asymptotic variance have poor non-parametric rates and thus require even larger sample sizes to obtain the same precision.

Therefore, we suggest a procedure for testing significance and potential time-variation of $\hat{\beta}^{s|i}_{p,q,t}$ which is valid in finite samples. For a given hypothesis H_0, we use the test statistic

$$S_T = \min_{\boldsymbol{\xi}^s \in \Omega_0} \sum_{t=1}^{T} \rho_p(X_t^s - \mathbf{B}_t' \boldsymbol{\xi}^s) - \min_{\boldsymbol{\xi}^s \in \mathbb{R}^{K_B}} \sum_{t=1}^{T} \rho_p(X_t^s - \mathbf{B}_t' \boldsymbol{\xi}^s), \tag{2.15}$$

with the compound vector of all regressors in VaR^s, $\mathbf{B}_t \equiv (VaR_t^i, VaR_t^i \cdot \mathbf{Z}_{t-1}^i, \mathbf{V}_t^{(i)})$, corresponding K_B-parameter vector $\boldsymbol{\xi}^s$, and Ω_0 referring to the constrained set of parameters under H_0. This test is an adaptation to the quantile setting of a method proposed by Chen, Ying, Zhang, and Zhao (2008) for median regressions. Direct operationalization of the test is complicated by the fact that the asymptotic distribution of (2.15) involves unknown terms, and, secondly, by the nonsmooth objective function of the quantile regression, which causes inconsistency of conventional resampling techniques. Therefore, following Chen, Ying, Zhang, and Zhao (2008) we apply an adjusted bootstrap method, which is described in detail in Section 2.A.1.

2.4.3 Empirical evidence on systemic risk betas and risk rankings

We estimate systemic risk betas according to (2.12) with time variation in firm-specific characteristics (i.e. $\mathbf{Z}_t^i = C_t^i$). As in the first-step estimations, we choose $q = 0.05$, i.e., we model the loss which will not be exceeded with 95% probability. For notational convenience, we suppress the quantile index as we set $p = q$. Obtained realized systemic risk betas indeed contain information on systemic relevance beyond a company's network interconnectedness. This is illustrated in Figure 2.6 revealing only slightly positive dependencies between pagerank coefficients and realized systemic risk betas. Thus, more connected firms tend to be systemically more risky, see e.g. , Bank of America and American Express. With an R^2 of 2% in the regression, the relationship, however, is not very strong indicating that the quantification of a firm's interconnectedness is not sufficient to assess its systemic relevance which directly depends on firm-specific and macroeconomic conditions. The latter is captured by realized systemic risk contributions but not necessarily by pagerank coefficients.

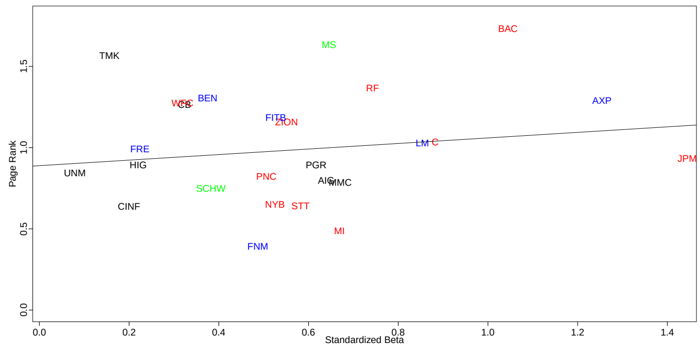

Figure 2.6: Pagerank coefficients are plotted versus realized systemic risk contributions for all companies which are classified as systemically relevant for the years 2000-2008 as in Section 2.4.3. The regression line shows only a small correlation between the pagerank coefficent and the realized beta, supported by the respective R^2 of 0.0265 of the regression. Colors and acronyms are as in Figure 2.A.2.

We statistically assess if a company's risk has a relevant direct impact on the system by testing for the significance of the respective systemic risk beta. Evaluating whether $\beta_t^{s|i} = 0$ requires testing for the joint significance of all variables driving a firm's marginal impact. Thus, we test the hypothesis

$$\mathbf{H1}: \beta_0^{s|i} = \eta_{MMM}^{s|i} = \eta_{SIZE}^{s|i} = \eta_{LEV}^{s|i} = \eta_{BM}^{s|i} = \eta_{VOL}^{s|i} = 0.$$

Whether marginal effects on the system are indeed time-varying in firm-specific characteristics can be tested by the joint hypothesis

$$\mathbf{H2}: \eta_{MMM}^{s|i} = \eta_{SIZE}^{s|i} = \eta_{LEV}^{s|i} = \eta_{BM}^{s|i} = \eta_{VOL}^{s|i} = 0.$$

If this hypothesis is not rejected, we re-specify the systemic risk beta as being constant, i.e., $\beta_t^{s|i} = \beta^{s|i}$, re-estimate the model without interaction variables and test the hypothesis

$$\mathbf{H3}: \beta^{s|i} = 0\,.$$

We find the majority of firms having a significant systemic risk beta which is classified as being time-varying in approximately 50% of all cases. In contrast, for approximately 25% of all firms we do not find systemic risk betas which are significantly different from zero. Table 2.A.4 reports the p-values of the respective underlying tests which are performed using the wild bootstrap procedure illustrated in Section 2.A.1 based on $2,000$ resamples of the test statistic.[15] We consider effects as significant if p-values are below 10%. Then, a company is defined as systemically relevant if an increase in its possible loss position, given all economic state variables and i-specific risk inflows from other companies, induces a significantly higher potential systemic loss. This requires its systemic risk beta to be significant *and* nonnegative.[16]

Table 2.A.5 lists all systemically relevant companies for the period from 2000 to 2008, ranked according to their average realized systemic risk contributions $\hat{\bar{\beta}}^{s|i}$. JP Morgan, American Express, Bank of America and Citigroup are identified as the (on average) most systemically risky companies. According to our network analysis above, these firms are categorized into the group of risk amplifiers which are strongly interconnected and should be closely supervised. To judge the validity and quality of our assessment based on market data, we compare our results with the outcomes of the Supervisory Capital Assessment Program (SCAP) conducted by the Federal Reserve in spring 2009, right after the end of our sample period. In this analysis, the Fed could draw on detailed non-public confidential balance sheet information to classify the 19 largest bank holding companies according to estimates of potential lack in capital buffer for covering risks under an adverse macro scenario. For details, see Board of Governors of the Federal Reserve System (2009). The financial institution with the biggest potential lack of capital buffer according to the SCAP, Bank of America, ranks among our highest systemically relevant companies leading the ranking in June 2008 (Table 2.A.6 b). In addition, with Citigroup, FifthThird Bancorp, Morgan Stanley, PNC, Regions Financial and Wells Fargo we identify six out of eight banks contained in our database[17] which, according to the SCAP results, were threatened by financial distress under more adverse market conditions. As we could in advance detect systemic riskiness of the majority of companies that were later found to face capital shortages in the stress test scenario of the SCAP, this confirms the quality of our method, which is entirely based on

[15]Because of multi-collinearity of time variation effects in firm characteristics for systemic risk betas, the interpretation of individual coefficients η might be misleading. Therefore, we refrain from reporting respective estimates.

[16] Since we do not impose a priori non-negativity restrictions, systemic risk betas can become negative at certain points in time. In a few cases we can directly attribute these effects to sudden time variations in one of the (interpolated) company-specific characteristics $\mathbf{Z}^i_{t-1}$ driving systemic risk betas temporarily into the negative region. These effects might be reduced by linking $\beta^{s|i}$ in (2.10) to (local) time averages of $\mathbf{Z}^i_{t-1}$. Such a proceeding would stabilize systemic risk betas but at the cost of a potentially high loss of information.

[17]Due to a lack of data, we cannot include KeyCorp and GMAC in our analysis which also have been found to be financially distressed in a critical macroeconomic environment.

only publicly available data.

Average systemic risk betas, however, only provide a rough picture of systemic importance as they aggregate companies' marginal systemic risk contributions and VaRs over time ignoring potential changes in the structure of the financial sector. In contrast, monitoring the evolution of systemic risk beta's over time provides a more informative picture on companies' specific systemic importance and yields valuable feedback from the market for forward-looking regulation. To illustrate the potential of our approach, we show the rankings at two specific time points: Table 2.A.6a gives the systemic risk ranking for the last week in March 2007, which was a relatively "calm" time before the start of the financial crisis. Table 2.A.6b, on the other hand, shows the ranking at the end of June 2008, shortly before the collapse of Lehman Brothers. Comparing the pre-crisis and post-crisis rankings, we observe clear changes. In most cases, systemic risk betas – and thus the magnitude of systemic risk contributions – significantly increased during the crisis. This is particularly observed for American Express, Bank of America, JP Morgan, Regions Financial and State Street, among others. Nevertheless, in some cases, as, e.g., for Citigroup and Morgan Stanley systemic risk contributions even declined.

During the crisis, we detect Bank of America as systemically most relevant. Our estimates indicate that its multiple risk channels in the center of the network, particularly to Morgan Stanley, American Express, Citigroup, Wells Fargo are systemically critical. Figure 2.A.3 shows that Bank of America's systemic risk beta has been relatively stable before the financial crisis but significantly dropped after the issuance of the Federal Reserve's rescue packages. Nevertheless, its VaR and thus its realized systemic risk contribution strongly increased during the crisis. Our results also identify AIG as highly systemically relevant. Before the crisis, AIG was among the largest issuers and holders of credit default swaps (CDS) and other credit securitization derivatives. Its obviously strong exposure to mortgage default risks is reflected by a strong dependence to Freddie Mac and Fannie Mae, among others, as depicted in the network graph in Figure 2.4. The high systemic relevance of AIG is illustrated in the upper part of Figure 2.A.3 depicting $\beta_t^{s|i}$, VaR_t^i and the product thereof, $\bar{\beta}_t^{s|i}$. In 2008, AIG faced tremendous write-downs which caused strong increases of the firm's VaR and the realized systemic risk contribution $\bar{\beta}_t^{s|i}$. The rescue packages from the Federal Reserve amounting to USD 150 billion (see Schich (2009)) in September 2008, however, significantly reduced the risk of both AIG's and the entire system's failure. This is indicated by the strong decline of the companies' systemic risk beta in Figure 2.A.3). Due to the forward-looking character of systemic risk betas, the (anticipated) bailout has already been incorporated in the systemic risk ranking of end of June 2009 where AIG drops out of the list of systemically relevant companies. This is induced by strong changes in the companies' book-to-market ratio driving the systemic risk beta of AIG into the negative region.

By construction, realized systemic risk contributions $\bar{\beta}_t^{s|i}$ might vary over time through two channels: a time-varying beta, $\beta_t^{s|i}$ and a time-varying Value-at-Risk, VaR_t^i. For selected companies, these effects are illustrated by Figure 2.2 in the introduction. In many cases we observe increases of realized systemic risk contributions which are mainly due to rising individual VaRs with systemic risk betas which even slightly decline from 2007 to 2008. Hence, companies' *marginal* contribution to the system VaR is widely unchanged while their exposure to idiosyncratic risk resulting from worse firm-specific and macroeconomic conditions has been dramatically increased. See, for instance, the first two companies in the 2008 ranking, Bank of America and American Express which, however, realize quite different combinations of marginal systemic contributions and idiosyncratic tail risk levels apparently facing different sources for systemic relevance. In both cases, the strong increase in VaR can be attributed to tail risk spillovers in the network, with, e.g., Bank of America being particularly affected by Citigroup and Morgan Stanley.

In several cases, increasing individual VaRs coincide with rising systemic risk betas. For instance, Wells Fargo is an example of a company which was not even identified as being systemically relevant in 2007 but faces a dramatic increase of both its systemic risk beta and its idiosyncratic tail risk making it highly systemically risky in 2008. Likewise, State Street, Progressive Ohio and Marshall & Isle (MI) face an increase of both $\beta_t^{s|i}$ and VaR_t^i. Also, sources for increasing effects can be found in the network structure. An exception is State Street which does not face significant risk spillovers from other companies and thus primarily depends on micro- and macroeconomic externalities. As a result, the company's high systemic relevance in 2008 is due to the combination of a moderately high systemic risk beta and severe idiosyncratic risk, which in turn affects balance sheets and obligations of other firms. For two central nodes in the network, Citigroup and Morgan Stanley, however, declining systemic risk betas overcompensate increasing VaRs resulting in declining systemic relevance.

The results illustrate that realized systemic risk contributions conveniently condense information on banks' systemic importance. Though, the underlying driving forces of a bank's changed systemic relevance can be quite different. Therefore, only simultaneously analyzing and monitoring (i) network effects, (ii) sensitivity to micro- and macroeconomic conditions and (iii) time-variations in systemic risk betas provide the full picture of companies' specific role in the network and thus build a solid basis for regulatory measures.

2.5 Validity: pre-crisis period

In the course of the financial crisis 2007–2009, a number of large institutions defaulted, were overtaken by others or supported by the government. As for our general empirical study, we required data for all considered institutions to be available over the entire period from beginning of 2000 to end of 2008, some of these companies could not be included. Nevertheless, to validate and robustify our findings, we perform an additional analysis by re-estimating the model for the time period of January 1, 2000, to June 30, 2007 and including the investment banks Lehman Brothers and Merrill Lynch.

Because of the shorter estimation period, differences between estimated systemic risk contributions are not as pronounced as in the analysis covering the full time period. Therefore, as a sharp ranking of companies might not be very meaningful and hard to interpret in this context, Table 2.A.7 rather categorizes firms into groups according to quartiles of the distribution of realized systemic risk betas. Accordingly, we can distinguish between four broad classes: Firstly, there are 9 companies with VaRs that significantly influence the system VaR and are among the 25% largest average realized betas. The most prominent members of this group are AIG, Lehman Brothers, Morgan Stanley, JP Morgan and Goldman Sachs. The second group comprises systemically risky companies with significant systemic impact, whose average realized betas lie in the third quartile of the distribution. According to the estimates reported in Table 2.A.7, these magnitudes reflect a comparably high systemic relevance.[18] Group 2 group contains mainly large depositories and investment banks including Bank of America, Merrill Lynch, Citigroup and Regions Financial, but also the mortage company Freddie Mac. Group 3 includes all companies with small but significant average systemic risk betas, in particular those below the median. Finally, the ones which, according to the significance test, are not considered as being systemically risky during the analyzed time period, are collected in Group 4.

In detail, we focus on four companies which were massively affected by the crisis: Lehman Brothers became insolvent on September 15, 2008, and was liquidated afterwards. Merrill Lynch announced a merger with Bank of America in September 2008, which was executed on January 1, 2009. Furthermore, excluding the crisis period itself may reveal the systemic relevance of the mortgage firm Freddie Mac, which is closely connected to the second largest real estate financing company Fannie Mae. Both were placed under conservatorship by the U.S. government during the course of the financial crisis. Finally, it is interesting to investigate the systemic riskiness of AIG, which faced major distress during the crisis and whose bailout was very expensive for the tax payers. As shown by

[18]For a better exposition, we multiply all values of realized systemic risk betas with 100.

Table 2.A.7 (with the specific companies marked in bold), all of these firms belong to the group of systemically relevant firms with high or mid-sized average systemic risk betas.

Table 2.A.8 summarizes the results of our empirical analysis for the four case study candidates using only the pre-crisis data. Our network analysis reveals that almost all of the companies are subject to loss spillovers from direct competitors: Freddie Mac is influenced by risk transmissions of Fannie Mae, and vice versa. Despite Fannie Mae's low average realized systemic risk beta, this direct bi-directional risk dependence reveals the company's systemic relevance. Merrill Lynch influences Citigroup (C). TD Ameritrade Holding (AMTD) and E Trade Financial (ETFC) are large online brokers which operate on the same market as Lehman and Merrill Lynch and are identified as significant tail risk producer and receiver, respectively. Likewise, we identify tail risk dependencies between Lehman and both Morgan Stanley and Goldman Sachs, being Lehman's main competitors and the two largest investment banks in the U.S. during the estimation period. AIG is clearly the most interconnected firm in this case study. Its VaR is affected by the tail risks of eight competing insurers: Allstate (ALL), Chubb (CB), Hartford Financial (HIG), Lincoln National Corp. (LNC), MBIA, Marsh & McLennan Inc. (MMC), and Torchmark (TMK) as well as by Lehman Brothers (LEH). There are mutual spillovers with Citigroup, ETFC, CNA, HIG, and MMC. Additionally, AIG's losses have an effect on the VaRs of another three insurance companies, Aflac (AFL), Humana (HUM) and Unum (UNM).

All four companies of interest have a significant impact on the system. Focussing particularly on Lehman Brothers and Merrill Lynch, we show the time evolution of their realized risk betas and VaRs in Figures 2.A.4 and 2.A.5, respectively. It turns out that the realized systemic risk beta of Lehman steadily increases from 2005 to 2007. Interestingly, its VaR only increases in the second half of 2005 but remains widely on the same level afterwards. Hence, its growing systemic relevance is mainly due to rising marginal effects on the system and is not reflected in Lehman's idiosyncratic risk exposure. The jumps in the VaR (and thus also in the realized risk beta) are induced by relevant loss exceedances which only occur whenever one of Lehman's tail risk drivers (e.g., Morgan Stanley) exceeds his (unconditional 10%) loss quantile. This discreteness reflects the company's tail risk sensitivity to loss exceedances of competitors.

In case of Merrill Lynch, we observe high fluctuations of the realized systemic risk beta over the analyzed time period. As for Lehman Brothers, we observe clear differences in the paths of our systemic risk measure and VaR. For example, while its VaR, apart from some fluctuations keeps returning to the same level, its realized risk beta increases by more than 100% from mid of 2006 to mid of 2007. Hence, the (realized) systemic risk beta again reveals information on the company's systemic importance which cannot be detected by an analysis of the VaR solely. This finding strongly backs the usefulness of our proposed measure.

From these results, which are produced only from pre-crisis data, we can infer that in June 2007, each of the five financial institutions of interest was classified as being relevant for the stability of the U.S. financial system. Our findings indicate, firstly, that bailouts during the crisis were justified for Fannie Mae, Freddie Mac and AIG. Also a failure of Merrill Lynch would have led to harsh systemic consequences which could be prevented by its merger with Bank of America in 2008. Secondly, the increasing systemic importance of Lehman Brothers could have been monitored and thus the impact of its bankruptcy could have been anticipated to a certain extent. The direct bi-directional linkage to JP Morgan, as well as the connections to Morgan Stanley and Goldman Sachs, which in turn are deeply interconnected, indicate a high risk for contagion as a result of Lehman's failure. Furthermore, our estimates show that Lehman's systemic risk contribution is only slightly lower than that of AIG, while it is substantially higher than that of, e.g., Freddie Mac. Given these results, bailing out the latter but not the former is not necessarily justifiable from a systemic risk management point of view. If these results had existed in advance, more effective regulatory measures could have been performed which could have helped reducing the extent of the financial crisis.

2.6 Conclusion

The worldwide financial crisis 2007–2009 has revealed that there is a need for a better understanding of systemic risk. Particularly in situations of distress, it is the interconnectedness of financial companies which plays a major role, but also challenges quantitative analysis and the construction of appropriate risk measures.

In this chapter, we propose a measure of firms' systemic relevance which accounts for dependence structures within the financial network given market externalities. Our analysis allows to statistically identify relevant channels of potential tail risk spillovers between firms, constituting the topology of the financial network. Based on these relevant company-specific risk drivers, we measure a firm's idiosyncratic tail risk by explicitly accounting for its interconnectedness with other institutions. Our measure for a company's systemic risk contribution quantifies the impact on the system's risk of distress induced by an increase in the risk of the specific company in a network setting. Both measures exclusively rely on publicly observable balance sheet and market characteristics and, thus, can be used for prudent regulatory decisions in a stress test scenario.

Our empirical results show the interconnectedness within the U.S. financial system and clearly mark channels of relevant potential risk spillovers. In particular, we can classify companies into major risk producers, transmitters or recipients within the system. Moreover, at any specific point in time, firms can be ranked according to their estimated contribution to systemic risk given their role and position in the network. Monitoring companies' systemic relevance over time, thus allows to detect those firms which are most central for the stability of the system. In a case study, we highlight that our approach could have served as a solid basis for sensible forward-looking regulation before the start of the financial crisis in 2007.

Our approach is readily extendable in several directions. In particular, although the financial system is dominated by the U.S, it truly is a global business with many firms operating internationally. Detecting inter- and intra-country risk connections and measuring firms' global systemic relevance, should be straightforward with our proposed methodology. Moreover, whenever additional (firm-specific or market-wide) information is available as, e.g., reported to central banks, it can be directly incorporated into our measurement procedure. The data-driven selection step of relevant risk drivers then determines if and how it increases the precision of results.

2.A Appendix

2.A.1 Econometric methodology

Asymptotic results for two-step quantile estimation

Under the adaptive choice of penalty parameter as described in the text, the LASSO selection method is consistent with rate $O_P(\sqrt{\frac{K(i)}{T}\log(\max(K,T))}$, and with high probability the coefficients selected of $\mathbf{W}$, contain the the true coefficients also in finite samples. These results follow directly from Belloni and Chernozhukov (2011). Furthermore, VaR^i is consistently estimated by the post-LASSO method described in the text which re-estimates the unrestricted model with $\mathbf{W}^{(i)}$. In particular, for all $q \in I$ with $I \in (0,1)$ being compact,

$$\hat{\boldsymbol{\xi}}_q^i - \boldsymbol{\xi}_q^i \leq O_P(\sqrt{\frac{K(i)}{T}\log(\max(K,T))}), \tag{2.A.1}$$

since in our setting it is safe to assume that the number of wrongly selected components of $\mathbf{W}$ is stochastically bounded by the number $K(i)$ of components of $\mathbf{W}$ contained in the true model for VaR^i (see equation (2.16) in Belloni and Chernozhukov (2011)). We write in a slight abuse of notation $Y_T \leq O_P(r_T)$, with Y_T being either $O_P(r_T)$ or even $o_P(r_T)$ for any random sequence Y_T and deterministic $r_T \to 0$. Note that in general for $T \to \infty$, both K and $K(i)$ might grow only extremely slowly in T, such that they can be treated close to being constants implying the standard oracle bound $O_P(\sqrt{\frac{\log(T)}{T}})$ in (2.A.1).

If the true model is selected, we find for the asymptotic distribution of the individual VaR estimates for any $q \in [0,1]$,[19]

$$\sqrt{\frac{1}{T}}\left(\hat{\boldsymbol{\xi}}_q^i - \boldsymbol{\xi}_q^i\right)' \to N\left(0, \frac{q(1-q)}{g^2(G^{-1}(q))}\mathbb{E}[\mathbf{W}^{(i)}\mathbf{W}^{(i)\prime}]^{-1}\right) , \tag{2.A.2}$$

where $g(G^{-1}(q))$ denotes the density of the corresponding error ε^i distribution at the qth quantile. This result is standard (see Koenker and Bassett (1978)). For the second step estimates, we derive the asymptotic distribution analogously to the two-step median results

[19] Required assumptions of Belloni and Chernozhukov (2011) and quantile analogies to Powell (1983) are fulfilled in our setting.

in Powell (1983)

$$\sqrt{\frac{K(i)}{T}}\left((\hat{\beta}^{s|i}_{0,p,q},\hat{\boldsymbol{\eta}}^{s|i}_{p,q},\hat{\gamma}^{s}_{p})' - (\beta^{s|i}_{0,p,q},\boldsymbol{\eta}^{s|i}_{p,q},\gamma^{s}_{\boldsymbol{p}})'\right) \tag{2.A.3}$$

$$\to \mathcal{N}\left(0, Q^{-1}\mathbb{E}\left[\frac{p(1-p)}{f^2(F^{-1}(p))}\rho_p(\varepsilon^s_t) - \frac{p(1-p)}{g^2(G^{-1}(p))}\beta^{s|i\,\prime}_{p,q}\left(\rho_p(\varepsilon^i_t),\rho^v_p(\mathbf{Z}_{t-1}\varepsilon^i_t)\right)\right]\right), \tag{2.A.4}$$

where in the scalar factor, $f(F^{-1}(p))$ is the density of the corresponding error ε^s at the pth quantile, the function ρ^v_p of a vector applies ρ_p to each of its components, and $\beta^{s|i}_{p,q} = (\beta^{s|i}_{0,p,q},\boldsymbol{\eta}^{s|i}_{p,q})$. The remaining main part Q in the variance is given by $Q = H'\mathbb{E}[\mathbf{AA}']H$ with $\mathbf{A} = (\mathbf{W}^{(i)}, vec(\mathbf{Z}_{t-1}\cdot\mathbf{W}^{(i)\prime}), \mathbf{VaR}^{(-i)})$. Denote by $\mathbf{I}$ and $\mathbf{0}$ identity and null matrices, respectively, and by $\mathbf{1}$ a vector of ones of appropriate dimension. Then,

$$H' = \begin{pmatrix} \mathrm{diag}(\boldsymbol{\xi}^i_{q,2}) & \mathbf{0} & \cdots\mathbf{0}\cdots & \cdots\mathbf{0}\cdots \\ \mathbf{0} & \mathrm{diag}(\boldsymbol{\xi}^i_{q,1}) & \cdots\mathbf{0}\cdots & \cdots\mathbf{0}\cdots \\ \mathbf{0} & \mathbf{0} & \mathrm{diag}(vec(\mathbf{1}_{d_z}\cdot\boldsymbol{\xi}^{i\,\prime}_q)) & \cdots\mathbf{0}\cdots \\ \mathbf{I} & \mathbf{0} & \cdots\mathbf{0}\cdots & \cdots\mathbf{0}\cdots \\ \mathbf{0} & \mathbf{0} & \cdots\mathbf{0}\cdots & \mathbf{I}_{d_{(-i)}\times d_{(-i)}} \end{pmatrix}$$

where d_Z is the dimension of Z which is 5 in our application, $d_{(-i)}$ is the dimension of $\mathbf{VaR}^{(-i)}_t$, and coefficients $\boldsymbol{\xi}^i_{q,2}$ are those components of $\boldsymbol{\xi}^i_q$ for regressors which appear both in the first and the second step. Correspondingly, $\boldsymbol{\xi}^i_{q,1}$ are coefficients of regressors which just appear in the first step of the individual VaR regression. Note that in the variance matrix there is a distinction in γ for parts of $\mathbf{V}$ which are also controls in VaR^i and $\mathbf{VaR}^{(-i)}_t$, which just appear in VaR^s.

Choice of the company-specific LASSO penalty parameter λ^i

We determine λ^i in a data-driven way following a bootstrap type procedure as suggested by Belloni and Chernozhukov (2011):

Step 1 Take T iid draws from $\mathcal{U}[0,1]$ independent of $\mathbf{W}_1,\ldots,\mathbf{W}_T$ denoted as $U_1,\ldots,U_T$. Conditional on observations of $\mathbf{W}$, calculate the corresponding value of the random variable,

$$\Lambda^i = T\max_{1\le k\le K}\frac{1}{T}\left|\sum_{t=1}^{T}\frac{W_{t,k}(q-I(U_t\le q))}{\hat{\sigma}_k\sqrt{q(1-q)}}\right|.$$

Step 2 Repeat step 1 for B=500 times generating the empirical distribution of Λ^i conditional on $\mathbf{W}$ through $\Lambda^i_1, \dots, \Lambda^i_B$. For a confidence level $\alpha \leq 1/K$ in the selection, set

$$\lambda^i = c \cdot Q(\Lambda^i, 1 - \alpha | \mathbf{W}_t),$$

where $Q(\Lambda^i, 1-\alpha|\mathbf{W}_t)$ denotes the $(1-\alpha)$-quantile of Λ^i given $\mathbf{W}_t$ and $c \leq 2$ is a constant.

The choice of α is a trade-off between a high confidence level and a corresponding high regularization bias from high penalty levels in (2.7). As in the simulation results in Belloni and Chernozhukov (2011), we choose $\alpha = 0.1$, which suffices to get optimal rates of the post-penalization estimators below. Finally, the parameter c is selected in a data-dependent way such that the in-sample predictive ability of the resulting VaR specification is maximized. (Belloni and Chernozhukov (2011) proceed in a similar way). The latter is evaluated in terms of its best backtesting performance according to the procedure described below.

Backtest for the model fit for VaR^i

As suggested by Berkowitz, Christoffersen, and Pelletier (2011), for each institution i, we measure VaR exceedances as $I^i_t \equiv I(X^i_t < -VaR^i_{q,t})$. If the chosen model is correct, then,

$$\mathbb{E}[I^i_t|\Omega_t] = q \, , \tag{2.A.5}$$

where Ω_t is the information set up to t. The VaR is estimated correctly, if independently for each day of the covered period, the probability of exceeding the VaR equals q. Similar to Engle and Manganelli (2004), Kuester, Mittnik, and Paolella (2006) and Taylor (2008), we include a constant, three lagged values of I_t and the current VaR estimate in the information set Ω_t. Then, condition (2.A.5) can be checked by estimating a logistic regression model

$$I^i_t = \alpha + \mathbf{A}'_t\boldsymbol{\theta} + U_t,$$

with covariates $\boldsymbol{A}_t = (I^i_{t-1}, I^i_{t-2}, I^i_{t-3}, \widehat{VaR}^i_{t-1})'$. Denote by $\bar{I}^i$ the sample mean of the binary response I^i_t and define $F_{log}(\cdot)$ as the cumulative distribution function of the logistic distribution. Then, under the joint hypothesis

$$\mathbf{H}_0: \ \alpha = q \text{ and } \boldsymbol{\theta}_1 = \cdots \boldsymbol{\theta}_4 = 0,$$

the asymptotic distribution of the corresponding likelihood ratio test statistic is

$$LR = -2(\ln \mathcal{L}_r - \ln \mathcal{L}_u) \overset{a}{\sim} \chi^2_5 \,. \tag{2.A.6}$$

Here, $\ln \mathcal{L}_u = \sum_{t=1}^{n} \left[I_t^i \ln F_{log}(\alpha + \mathbf{A}_t' \boldsymbol{\theta}) + (1 - I_t^i) \ln \left(1 - F_{log}(\alpha + \mathbf{A}_t' \boldsymbol{\theta})\right) \right]$ is the unrestricted log likelihood function which under $\mathbf{H}_0$ simplifies to $\ln \mathcal{L}_r = n\bar{I}^i \ln(q) + n(1 - \bar{I}^i) \ln(1-q)$.

Bootstrap procedure for the joint significance test

The asymptotic distribution of the test statistic introduced in Section 2.4.2,

$$S_T = \min_{\boldsymbol{\xi}^s \in \Omega_0} \sum_{t=1}^{T} \rho_p(X_t^s - \mathbf{B}_t' \boldsymbol{\xi}^s) - \min_{\boldsymbol{\xi}^s \in \mathbb{R}^{K_B}} \sum_{t=1}^{T} \rho_p(X_t^s - \mathbf{B}_t' \boldsymbol{\xi}^s), \tag{2.A.7}$$

involves the probability density function of the underlying error terms and is not feasible. Furthermore, bootstrapping S_T directly would yield inconsistent results. Therefore, we re-sample from the adjusted statistic

$$\begin{aligned} S_T^* \quad = \quad & \min_{\boldsymbol{\xi}^s \in \Omega_0} \sum_{t=1}^{T} w_t \rho_p(X_t^s - \mathbf{B}_t' \boldsymbol{\xi}^s) - \min_{\boldsymbol{\xi}^s \in \mathbb{R}^{K_B}} \sum_{t=1}^{T} w_t \rho_p(X_t^s - \mathbf{B}_t' \boldsymbol{\xi}^s) \\ & - \left(\sum_{t=1}^{T} w_t \rho_p(X_t^s - \mathbf{B}_t' \hat{\boldsymbol{\xi}}_c^s) - \sum_{t=1}^{T} w_t \rho_p(X_t^s - \mathbf{B}_t' \hat{\boldsymbol{\xi}}^s) \right), \end{aligned} \tag{2.A.8}$$

where $\hat{\boldsymbol{\xi}}_c^s$ denotes the constrained estimate of $\boldsymbol{\xi}^s$, and $\{w_t\}$ is a sequence of standard exponentially distributed random variables, having both mean and variance equal to one. According to Chen, Ying, Zhang, and Zhao (2008), the empirical distribution of S_T^* provides a good approximation of the distribution of S_T. Thus, if the test statistic S_T exceeds some large quantile of the re-sampling distribution of S_T^*, the null hypothesis is rejected.

The proposed testing method does not require re-sampling of observations but is entirely based on the original sample. This provides significant gains in accuracy in the two-step regression setting as opposed to standard pairwise bootstrap techniques as a further alternative. A pre-analysis shows that this wild bootstrap type procedure is valid in the presented form as any serial dependence in the data is sufficiently captured by the regressors in the reduced-form relation not requiring block-bootstrap techniques.[20]

[20] Pairwise block-bootstrap yields block lengths of one according to the standard procedure of Lahiri (2001). Results are available upon request.

2.A.2 Tables and figures

Depositories (21)	**Others (11)**	**Insurance Comp. (20)**
BB T Corp (BBT)	American Express Co (AXP)	AFLAC Inc (AFL)
Bank of New York Mellon (BK)	Eaton Vance Corp (EV)	Allstate Corp (ALL)
Bank of America Corp (BAC)	Fed. Home Loan Mortg. Corp (FRE)	American International Group (AIG)
Citigroup Inc (C)	Fed. National Mortgage Assn (FNM)	AON Corp (AON)
Comerica Inc (CMA)	Fifth Third Bancorp (FITB)	Berkley WR Corp (WRB)
Hudson City Bancorp Inc. (HCBK)	Franklin Resources Inc (BEN)	CIGNA Corp (CI)
Huntington Bancshares Inc. (HBAN)	Legg Mason Inc (LM)	C N A Financial Corp. (CNA)
JP Morgan Chase & Co (JPM)	Leucadia National Corp (LUK)	Chubb Corp (CB)
M & T Bank Corp. (MTB)	SEI Investments Company (SEIC)	Cincinnati Financial Corp (CINF)
Marshall & Ilsley Corp (MI)	TD Ameritrade Holding Corp (AMTD)	Coventry Health Care Inc (CVH)
NY Community Bankcorp (NYB)	Union Pacific Corp (UNP)	Hartford Financial (HIG)
Northern Trust Corp (NTRS)		HEALTH NET INC (HNT)
Peoples United Financial Inc. (PBCT)	**Broker-Dealers (7)**	Humana Inc (HUM)
PNC Financial Services Group (PNC)	E Trade Financial Corp (ETFC)	Lincoln National Corp. (LNC)
Financial Corp New (RF)	Goldman Sachs Group Inc (GS)	Loews Corp (L)
S L M Corp.	Lehman Brothers (LEH)*	Marsh & McLennan Inc. (MMC)
State Street Corp (STT)	Merrill Lynch (ML)*	MBIA Inc (MBI)
Suntrust Banks Inc (STI)	Morgan Stanley Dean Witter & Co (MS)	Progressive Corp Ohio (PGR)
Synovus Financial Corp (SNV)	Schwab Charles Corp New (SCHW)	Torchmark Corp (TMK)
Wells Fargo & Co (WFC)	T Rowe Price Group Inc. (TROW)	Unum Group (UNM)
Zions Bancorp (ZION)		

* included only in the case study

Table 2.A.1: List of included U.S. financial institutions in alphabetical order within sectors.

	Value	Std. Error	t-ratio	p-value
Goldman Sachs				
(Intercept)	-0.046	0.004	-12.54	0.000
Ex.C	-0.239	0.205	-1.17	0.243
Ex.JPM	-0.014	0.119	-0.121	0.904
Ex.LM	-0.215	0.111	-1.932	0.054
Ex.MS	-0.403	0.079	-5.096	0.000
Ex.SCHW	-0.282	0.244	-1.153	0.249
Morgan Stanley				
(Intercept)	-0.041	0.003	-16.017	0.000
Ex.AIG	-0.106	0.026	-4.036	0.000
Ex.AON	0.445	0.145	3.066	0.002
Ex.BAC	-0.604	0.145	-4.157	0.000
Ex.EV	-0.158	0.134	-1.179	0.239
Ex.GS	-0.634	0.121	-5.236	0.000
Ex.HBAN	-0.273	0.136	-2.006	0.045
Ex.HCBK	-0.452	0.28	-1.611	0.108
Ex.MTB	-0.269	0.193	-1.392	0.165
Ex.SCHW	-0.381	0.116	-3.294	0.001
Ex.SEIC	-0.229	0.154	-1.485	0.138
Ex.STT	-0.174	0.176	-0.986	0.325
Regions Financial				
(Intercept)	-0.004	0.004	-1.072	0.284
Ex.AMTD	-0.091	0.04	-2.274	0.023
Ex.AON	-0.256	0.086	-2.998	0.003
Ex.BBT	-0.307	0.104	-2.95	0.003
Ex.FITB	0.032	0.087	0.37	0.712
Ex.HBAN	-0.042	0.064	-0.661	0.509
Ex.PBCT	-0.307	0.085	-3.598	0
Ex.STI	-0.244	0.114	-2.137	0.033
Ex.ZION	-0.196	0.1	-1.947	0.052
BM	0.024	0.007	3.221	0.001
VOL	0.251	0.16	1.568	0.118
Fannie Mae				
(Intercept)	-0.049	0.003	-17.075	0.000
Ex.AIG	-0.227	0.231	-0.981	0.327
Ex.FRE	-1.007	0.121	-8.298	0.000

	Value	Std. Error	t-ratio	p-value
American International Group				
(Intercept)	-0.043	0.003	-14.026	0.000
Ex.FRE	-0.201	0.014	-14.033	0.000
Ex.MBI	-0.336	0.138	-2.423	0.016
Ex.RF	-0.455	0.051	-8.975	0.000
Ex.TMK	-0.813	0.721	-1.127	0.260
Torchmark				
(Intercept)	-0.019	0.003	-7.203	0
Ex.AFL	-0.332	0.169	-1.962	0.05
Ex.ALL	-0.256	0.207	-1.237	0.217
Ex.BBT	-0.296	0.223	-1.329	0.184
Ex.HIG	-0.084	0.175	-0.483	0.63
Ex.LNC	0.002	0.135	0.018	0.986
Ex.NTRS	-0.002	0.115	-0.015	0.988
Ex.SEIC	-0.243	0.12	-2.023	0.044
Ex.UNM	-0.088	0.179	-0.489	0.625
Ex.UNP	-0.242	0.242	-1	0.318
repo	0.031	0.017	1.78	0.076
JP Morgan				
(Intercept)	-0.040	0.003	-12.963	0.000
Ex.BAC	-0.229	0.133	-1.724	0.085
Ex.BK	-0.237	0.129	-1.842	0.066
Ex.C	-0.380	0.22	-1.729	0.084
Ex.GS	-0.253	0.154	-1.648	0.199
Ex.PNC	-0.274	0.077	-3.583	0.000
Ex.SCHW	-0.410	0.118	-3.472	0.001
American Express				
(Intercept)	-0.035	0.003	-11.723	0
Ex.AFL	-0.42	0.408	-1.03	0.303
Ex.BAC	-0.361	0.205	-1.757	0.08
Ex.BBT	-0.145	0.126	-1.151	0.25
Ex.BEN	-0.112	0.139	-0.808	0.42
Ex.CINF	-0.153	0.153	-0.999	0.318
Ex.EV	-0.181	0.163	-1.112	0.267
Ex.L	0.014	0.114	0.122	0.903
Ex.SEIC	-0.106	0.09	-1.186	0.236
Ex.SLM	0.073	0.067	1.09	0.276
Ex.STT	-0.351	0.159	-2.2	0.028
Ex.TROW	-0.3	0.126	-2.39	0.017

Table 2.A.2: Exemplary post-LASSO quantile regressions for VaR^i with $q = 0.05$. Regressors were selected by LASSO as outlined in Secion 2.3.1. Ex.j is the loss exceedance of company j, all other regressors are as in Section 2.2.2.

Name	Influencing companies	Influenced companies
	Broker Dealers	
ETFC	AMTD,GS,MS	AMTD,C
GS	C,JPM,LM,MS,SCHW	BEN,C,ETFC,JPM,LM,MS,SCHW
MS	AIG,AON,BAC,EV,GS,HBAN,HCBK,MTB,SCHW,SEIC,STT	AMTD,BAC,EV,GS,HUM,LNC,ETFC,SEIC
SCHW	AMTD,GS,JPM,NTRS,TROW	AMTD,MS,GS,JPM
TROW	AMTD,BEN,EV,JPM,LUK,NTRS,SEIC,SNV	AON,MBI,MMC,AMTD,AXP,BEN,EV,NTRS,SCHW
	Depositories	
BAC	AON,AXP,C,HBAN,LM,MS,MTB,PBCT,PNC,SEIC,STI,WFC	AXP,BBT,C,CMA,HCBK,JPM,LM,MBI,MS,MTB,PNC,STI,WFC
BBT	BAC,FITB,MTB,NTRS,STI,TMK,UNP,WFC	AXP,BEN,CMA,FRE,MTB,RF,TMK,UNP,WFC,ZION
BK	AXP,JPM,MTB,NTRS,SNV,STT,WFC	CMA,JPM,NTRS,SEIC,SNV
C	BAC,ETFC,FITB,GS,JPM,LNC,LUK,MBI,MTB	BAC,GS,JPM,LUK
CMA	AON,BAC,BBT,BK,HBAN,RF,SNV,WFC	AON,PNC,SNV,ZION
HBAN	AON,LNC,RF,STI,ZION	AON,BAC,CMA,EV,LNC,MS,PBCT,RF,ZION
HCBK	AON,BAC,MBI,MTB,NYB	MS,MTB
JPM	BAC,BK,C,GS,PNC,SCHW	BK,C,GS,SCHW,SEIC,TROW
MI	MMC,TMK	HIG,MMC
MTB	BAC,BBT,HCBK,NYB,SNV,ZION	AON,BAC,BBT,BK,HCBK,MS,SNV,WFC,ZION,C
NTRS	BEN,BK,LUK,MMC,SEIC,STT,TROW	AFL,AMTD,BBT,BEN,BK,HIG,MMC,PGR,SCHW,TMK,TROW,LUK,STT
NYB	PBCT,WFC	MTB,SLM,WFC,HCBK,PBCT
PBCT	HBAN,NYB	AON,BAC,CB,NYB,RF
PNC	BAC,CMA,STT,TMK,WFC,ZION	BAC,JPM,ZION
RF	AMTD,AON,BBT,FITB,HBAN,PBCT,STI,ZION	AIG,AON,CMA,EV,FITB,HBAN,MBI,SNV,STI,ZION
SLM	AON,AXP,FRE,MBI,NYB	AON,AXP,BEN,EV,FITB,MBI
SNV	BK,CMA,FITB,MTB,RF,ZION	BEN,BK,CMA,FITB,MTB,TROW
STI	AON,BAC,FITB,LNC,RF,WFC,ZION	AFL,AON,BAC,BBT,FITB,HBAN,RF,ZION,CINF,HUM,UNM,WFC
STT	AXP,NTRS	AXP,BK,NTRS,PNC,MS
WFC	BAC,BBT,CB,LNC,MTB,NYB,STI	FITB,PNC,STI,AFL,BAC,BBT,BK,CMA,NYB
ZION	BBT,CMA,HBAN,MTB,PNC,RF,STI	AON,RF,FITB,HBAN,LNC,MTB,PNC,SNV,STI
	Insurance Companies	
AFL	ALL,AON,CNA,EV,NTRS,SEIC,STI,TMK,WFC	AXP,CB,EV,PGR,TMK,UNM
AIG	FRE,MBI,RF,TMK	FNM,MBI,MS
ALL	CB,CNA,L,LNC,TMK	AFL,PGR,TMK,UNM
AON	CMA,HBAN,MBI,MTB,PBCT,RF,SLM,STI,TROW,ZION	AFL,BAC,BEN,CMA,EV,FITB,HBAN,HCBK,LM,MBI,MS,RF,SLM,STI
CB	AFL,L,LNC,PBCT,PGR	ALL,CINF,EV,HIG,L,WFC,WRB
CI	CNA,HNT,HUM,LNC	HNT,HUM,LNC
CINF	CB,MBI,STI	AXP,LM
CNA	EV,L,LNC,MBI	AFL,ALL,CI,L,LNC,MBI
CVH	HUM	SEIC
HIG	CB,L,LNC,MI,NTRS,TMK	HUM,LNC,TMK
HNT	CI,EV,HUM,LM,LNC,PGR	CI,HUM,LM
HUM	CI,HIG,HNT,MS,STI	CI,HNT
L	CB,CNA,LNC,TMK,UNP	ALL,AXP,CB,CNA,HIG,LNC,UNM,UNP
LNC	CI,CNA,EV,HBAN,HIG,L,MS,SEIC,TMK,ZION	ALL,C,CB,CNA,HBAN,HIG,HNT,L,SEIC,STI,TMK,UNM,WFC,CI
MBI	AIG,AON,BAC,BEN,CNA,FRE,RF,SLM,TROW	AIG,AON,BEN,C,CINF,HCBK,SLM,CNA,LM
MMC	MI,NTRS,PGR,SEIC,TROW,UNM	MI,NTRS,UNM
PGR	AFL,ALL,NTRS,WRB	MMC,CB,HNT,WRB
TMK	AFL,ALL,BBT,HIG,LNC,NTRS,SEIC,UNM,UNP	AFL,BBT,EV,L,LNC,MI,PNC,AIG,ALL,HIG
UNM	AFL,ALL,L,LNC,MMC,STI	TMK,MMC
WRB	BEN,CB,PGR	PGR
	Others	
AMTD	ETFC,MS,NTRS,SCHW,SEIC,TROW	ETFC,RF,SCHW,TROW
AXP	AFL,BAC,BBT,BEN,CINF,EV,L,SEIC,SLM,STT,TROW	BAC,BEN,BK,EV,SLM,STT
BEN	AON,AXP,BBT,EV,GS,LM,MBI,NTRS,SLM,SNV,TROW	AXP,EV,LM,MBI,NTRS,TROW,WRB
EV	AFL,AON,AXP,BEN,CB,HBAN,MS,RF,SEIC,SLM,TMK,TROW	AFL,AXP,BEN,CNA,FRE,HNT,LM,LNC,MS,TROW
FITB	AON,LUK,RF,SLM,SNV,STI,WFC,ZION	BBT,C,FRE,RF,SNV,STI
FNM	AIG,FRE	FRE
FRE	BBT,EV,FITB,FNM,LUK	AIG,MBI,SLM,FNM
LM	AON,BAC,BEN,CINF,EV,GS,HNT,MBI	BAC,BEN,GS,HNT,LUK
LUK	C,LM,NTRS	C,FITB,FRE,NTRS,TROW
SEIC	BK,CVH,JPM,LNC,MS	AFL,AMTD,AXP,BAC,EV,LNC,MMC,MS,NTRS,TMK,TROW
UNP	BBT,L	BBT,TMK,L

Table 2.A.3: Tail risk cross dependencies: For each company, we list loss exceedances selected by LASSO as regressors for the VaR^i-model (q=0.05) ('Influencing companies') and companies for which the respective loss exceedance has been selected ('Influenced companies').

Name	pv_{H1}	pv_{H2} (pv_{H3})
Companies with significant and time-varying $\beta_t^{s\|i}$		
AMERICAN EXPRESS	0.001	0.006
AMERICAN INTL.GP.	0.002	0.000
BANK OF AMERICA	0.002	0.001
CHARLES SCHWAB	0.019	0.013
CHUBB	0.017	0.015
CIGNA	0.001	0.013
CINCINNATI FINL.	0.010	0.004
CITIGROUP	0.026	0.066
COMERICA	0.016	0.020
FANNIE MAE	0.001	0.000
FIFTH THIRD BANCORP	0.039	0.021
FRANKLIN RESOURCES	0.028	0.030
FREDDIE MAC	0.098	0.092
HARTFORD FINL.SVS.GP.	0.001	0.001
HUDSON CITY BANC.	0.043	0.035
HUNTINGTON BCSH.	0.010	0.011
LEGG MASON	0.026	0.060
LEUCADIA NATIONAL	0.041	0.016
LINCOLN NAT.	0.062	0.026
M & T BK.	0.033	0.021
MARSH & MCLENNAN	0.003	0.002
MARSHALL & ILSLEY	0.020	0.019
MORGAN STANLEY	0.041	0.095
PNC FINANCIAL SVS. GP	0.012	0.012
PROGRESSIVE OHIO	0.007	0.003
REGIONS FINANCIAL	0.034	0.029
STATE STREET	0.054	0.049
T ROWE PRICE GP.	0.090	0.076
TORCHMARK	0.002	0.001
UNION PACIFIC	0.040	0.035
UNUM GROUP	0.079	0.097
W R BERKLEY	0.007	0.037
WELLS FARGO & CO	0.015	0.027
ZIONS BANCORP.	0.095	0.100
Companies with significant but constant $\beta^{s\|i}$		
AON	0.063	0.192 (0.135)
E TRADE FINANCIAL	0.072	0.160 (0.233)
JP MORGAN CHASE & CO.	0.014	0.237 (0.047)
NY.CMTY.BANC.	0.040	0.132 (0.088)
SEI INVESTMENTS	0.014	0.115 (0.025)
TD AMERITRADE HOLDING	0.049	0.131 (0.188)
Companies with insignificant $\beta^{s\|i}$		
AFLAC	0.220	-
ALLSTATE	0.114	-
BANK OF NEW YORK MELLON	0.199	-
BB &T	0.120	-
CNA FINANCIAL	0.410	-
COVENTRY HEALTH CARE	0.257	-
EATON VANCE NV.	0.276	-
GOLDMAN SACHS GP.	0.667	-
HEALTH NET	0.371	-
HUMANA	0.189	-
LOEWS	0.276	-
MBIA	0.235	-
NORTHERN TRUST	0.305	-
PEOPLES UNITED FINANCIAL	0.105	-
SLM	0.391	-
SUNTRUST BANKS	0.213	-
SYNOVUS FINL.	0.289	-

Table 2.A.4: p-values for the test on significance of systemic risk betas (Hypothesis H1) and for the test on constancy of systemic risk betas (Hypothesis H2). For the second panel, we include in parentheses the p-values for the test on significance of systemic risk betas in case $H1$ is rejected but $H2$ is not (Hypothesis H3).

Rank	Name	$\widehat{\widetilde{\beta}}^{s\|i}_{av} \cdot 10^2$	influencing companies
1	JP MORGAN CHASE & CO	1.41	BAC,BK,C,GS,PNC,SCHW
2	AMERICAN EXPRESS	1.22*	AFL,BAC,BBT,BEN,CINF,EV,L,SEIC,SLM,STT,TROW
3	BANK OF AMERICA	1.01*	AON,AXP,C,HBAN,LM,MS,MTB,PBCT,PNC,SEIC,STI,WFC
4	CITIGROUP	0.87*	BAC,ETFC,FITB,GS,JPM,LNC,LUK,MBI,MTB
5	LEGG MASON	0.83*	AON,BAC,BEN,CINF,EV,GS,HNT,MBI
6	REGIONS FINANCIAL	0.72*	AMTD,AON,BBT,FITB,HBAN,PBCT,STI,ZION,,
7	MARSHALL & ILSLEY	0.65*	MMC,TMK
8	MARSH & MCLENNAN	0.63*	MI,NTRS,PGR,SEIC,TROW,UNM
9	MORGAN STANLEY	0.62*	AIG,AON,BAC,EV,GS,HBAN,HCBK,MTB,SCHW,SEIC,STT
10	AMERICAN INTL.GP.	0.61*	FRE,MBI,RF,TMK
11	PROGRESSIVE OHIO	0.58*	AFL,ALL,NTRS,WRB
12	STATE STREET	0.55*	AXP,NTRS
13	ZIONS BANCORP	0.51*	BBT,CMA,HBAN,MTB,PNC,RF,STI,
14	FIFTH THIRD BANCORP	0.49*	AON,LUK,RF,SLM,SNV,STI,WFC,ZION
15	NY.CMTY.BANC.	0.49	PBCT,WFC
16	PNC FINANCIAL SVS. GP	0.47*	BAC,CMA,STT,TMK,WFC,ZION
17	FANNIE MAE	0.45*	AIG,FRE
18	FRANKLIN RESOURCES	0.34*	AON,AXP,BBT,EV,GS,LM,MBI,NTRS,SLM,SNV,TROW
19	CHARLES SCHWAB	0.33*	AMTD,GS,JPM,NTRS,TROW
20	CHUBB	0.30*	AFL,L,LNC,PBCT,PGR
21	WELLS FARGO & CO	0.28*	BAC,BBT,CB,LNC,MTB,NYB,STI
22	FREDDIE MAC	0.19*	BBT,EV,FITB,FNM,LUK
23	HARTFORD FINL.SVS.GP.	0.19*	CB,L,LNC,MI,NTRS,TMK
24	CINCINNATI FINL.	0.16*	CB,MBI,STI
25	TORCHMARK	0.12*	AFL,ALL,BBT,HIG,LNC,NTRS,SEIC,UNM,UNP,
26	UNUM GROUP	0.04*	AFL,ALL,L,LNC,MMC,STI

Table 2.A.5: Ranking of **average** systemic risk contributions based on realized systemic risk betas. The third column lists loss exceedances that are included in the respective company's VaR^i-regression. Estimation period 2000-2008 Q3. Systemic risk contributions based on time-varying betas are marked by *.

a) End of March 2007 (before the beginning of the financial crisis)

Rank	Name	$\widehat{\widehat{\beta}}^{s\|i}_{2007} \cdot 10^2$	$\hat{\beta}^{s\|i}_{2007}$	$\widehat{VaR}^i_{2007}$
1	CITIGROUP	**1.78***	0.263	0.068
2	AMERICAN EXPRESS	**1.35***	0.387	0.035
3	BANK OF AMERICA	**1.16***	0.304	0.038
4	JP MORGAN CHASE & CO.	**1.05**	0.265	0.040
5	MORGAN STANLEY	**1.01***	0.146	0.069
6	LEGG MASON	**0.98***	0.205	0.048
7	MARSH & MCLENNAN	**0.83***	0.222	0.037
8	REGIONS FINANCIAL	**0.78***	0.202	0.038
9	PNC FINANCIAL SVS. GP	**0.77***	0.248	0.031
10	CHUBB	**0.74***	0.240	0.031
11	AMERICAN INTL.GP.	**0.61***	0.143	0.043
12	FRANKLIN RESOURCES	**0.60***	0.143	0.042
13	STATE STREET	**0.51***	0.114	0.045
14	FIFTH THIRD BANCORP	**0.50***	0.104	0.048
15	PROGRESSIVE OHIO	**0.42***	0.092	0.046
16	NY.CMTY.BANC.	**0.41**	0.090	0.045
17	MARSHALL & ILSLEY	**0.40***	0.088	0.045
18	TORCHMARK	**0.39***	0.173	0.023
19	HARTFORD FINL.SVS.GP.	**0.38***	0.099	0.039
20	ZIONS BANCORP.	**0.26***	0.115	0.054
21	CHARLES SCHWAB	**0.25***	0.042	0.060
22	FREDDIE MAC	**0.23***	0.057	0.041
23	LEUCADIA NATIONAL	**0.19***	0.057	0.033
24	CINCINNATI FINL.	**0.13***	0.026	0.050
25	FANNIE MAE	**0.09***	0.019	0.049
26	UNUM GROUP	**0.23***	0.045	0.051
27	T ROWE PRICE GP.	**0.06***	0.014	0.043
28	LINCOLN NAT.	**0.04***	0.010	0.036

b) End of June 2008 (during the financial crisis)

Rank	Name	$\widehat{\widehat{\beta}}^{s\|i}_{2008} \cdot 10^2$	$\hat{\beta}^{s\|i}_{2008}$	$\widehat{VaR}^i_{2008}$
1	BANK OF AMERICA	**2.86***	0.186	0.154
2	AMERICAN EXPRESS	**2.78***	0.278	0.100
3	WELLS FARGO & CO	**2.51***	0.186	0.135
4	MARSHALL & ILSLEY	**2.31***	0.516	0.045
5	JP MORGAN CHASE & CO.	**2.22**	0.265	0.084
6	PROGRESSIVE OHIO	**1.97***	0.380	0.052
7	LEGG MASON	**1.96***	0.137	0.143
8	REGIONS FINANCIAL	**1.86***	0.107	0.173
9	MARSH & MCLENNAN	**1.76***	0.471	0.037
10	STATE STREET	**1.44***	0.171	0.084
11	NY.CMTY.BANC.	**1.12**	0.090	0.125
12	PNC FINANCIAL SVS. GP	**1.09***	0.153	0.071
13	CHUBB	**1.07***	0.176	0.061
14	TORCHMARK	**1.00***	0.177	0.057
15	CHARLES SCHWAB	**0.91***	0.149	0.060
16	CITIGROUP	**0.90***	0.072	0.124
17	MORGAN STANLEY	**0.61***	0.074	0.083
18	ZIONS BANCORP.	**0.58***	0.058	0.100
19	UNUM GROUP	**0.34***	0.033	0.104
20	UNION PACIFIC	**0.27***	0.047	0.056
21	HARTFORD FINL.SVS.GP.	**0.24***	0.012	0.201
22	FRANKLIN RESOURCES	**0.17***	0.026	0.064
23	T ROWE PRICE GP.	**0.01***	0.001	0.102

Table 2.A.6: Rankings of relevant systemic risk contributions based on estimated realized systemic risk betas (bold) at the specific point in time. Estimated systemic risk betas and VaRs are listed in addition illustrating the different sources of variation. Estimates based on time-varying betas are marked by *.

Systemic risk contributions	Companies
Group 1 'high'	**AIG**, **LEH**, MS, JPM, GS,STT, CINF, LM, PBCT
Group 2 'medium'	**FRE**, **ML**, BAC, C, RF, AXP, PNC,CNA, TROW, NTRS
Group 3 'low'	FNM, WFC, EV, TMK, BBT, AFL, HUM, MI, CMA, BK, LNC, ALL, HNT, CB, CVH, SLM, ETFC
Group 4	AMTD, AON, BEN, CI, FITB, HBAN, HCBK, HIG, L, LUK, MBI, MMC, MTB, NYB, PGR, SCHW, SEIC, SNV, STI, UNM, UNP, WRB, ZION

Table 2.A.7: Group ranking of systemic risk contributions for the pre-crisis period 2000 - mid 2007. The upper part, group 1 ('high'), contains companies with significant $\beta_t^{s|i}$ and the highest quartile of significant betas: $\hat{\hat{\beta}}_{av}^{s|i} \cdot 100 \in [0.5, 1.3]$. Group 2 refers to the third quartile ('medium') with $\hat{\hat{\beta}}_{av}^{s|i} \cdot 100 \in [0.03, 0.49]$ and Group 3 to realized systemic risk betas lower than the median value ('small'), for which $\hat{\hat{\beta}}_{av}^{s|i} \cdot 100 < 0.01$. Group 4 includes companies not determined to be systemically risky during the estimation period, i.e., those with insignificant systemic risk betas. Case study companies are marked in bold.

Name	incluenced by	main influences	sign.	av. $\widehat{\beta}_t^{s\|i} \cdot 100$	av. $\widehat{\beta}_t^{s\|i}$
FREDDIE	AON, BBT, EV, FITB, FNM, HUM, MBI	BBT, FNM	0.048	0.38	0.092*
MERRILL	AMTD, CB, CNA, HCBK, L, NYB, WRB	C	0.051	0.03	0.030*
LEHMAN	AMTD, AON, BEN, GS, JPM, LM, LUK, MI, MS	AIG, AXP, ETFC, JPM	0.041	0.79	0.176*
AIG	ALL, C, CB, CNA, ETFC, HIG, LEH, LNC, MBI, MMC, SCHW, STT, TMK	AFL, C, CNA, HIG, HUM, MMC, UNM	0.026	0.73	0.210*

* time-varying betas

Table 2.A.8: Summary of estimation and test results for the four case study companies: loss exceedances influencing each company's VaR, the most important other VaRs influenced, joint significance tests on $\beta_t^{s|i} = 0$ and estimated average systemic risk contributions and betas. Estimation period: January 2000 - June 2007.

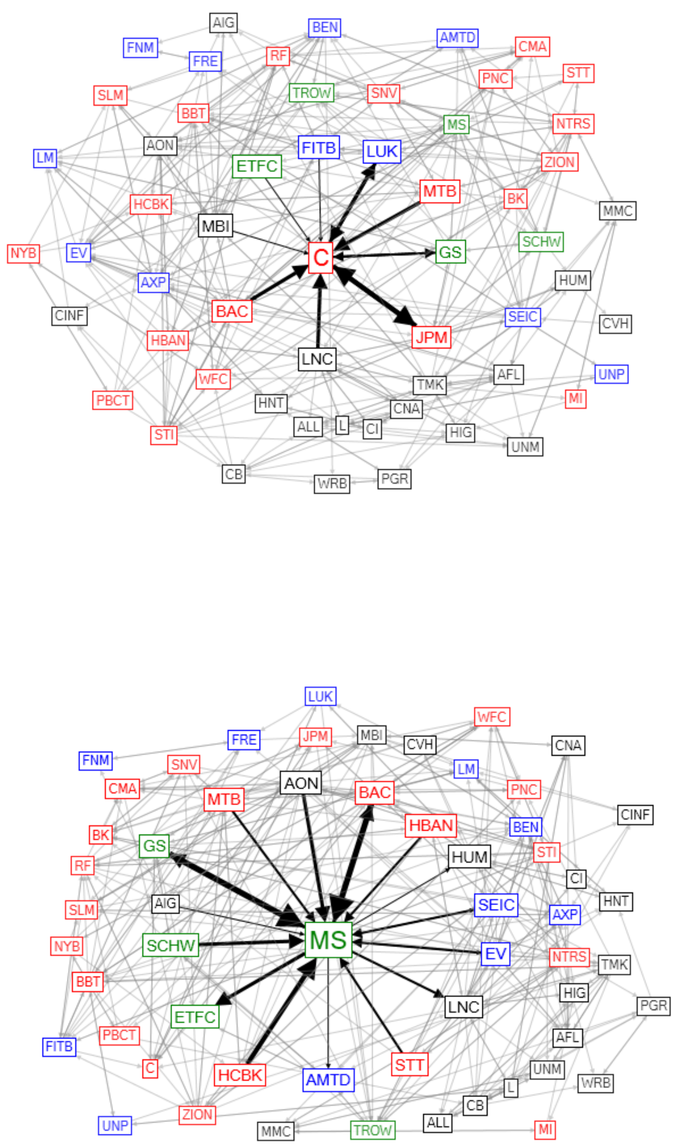

Figure 2.A.1: Full Network graphs of Citigroup (C) and Morgan Stanley (MS) highlighting risk drivers and risk recipients directly connected to the respective companies with bold arrows according to the respective size of the effect. Arrows, colors and acronyms are as in Figure 2.4. For simplicity, all other links just mark spillover effects without referring to size. The list of firm acronyms is contained in Table 2.A.1.

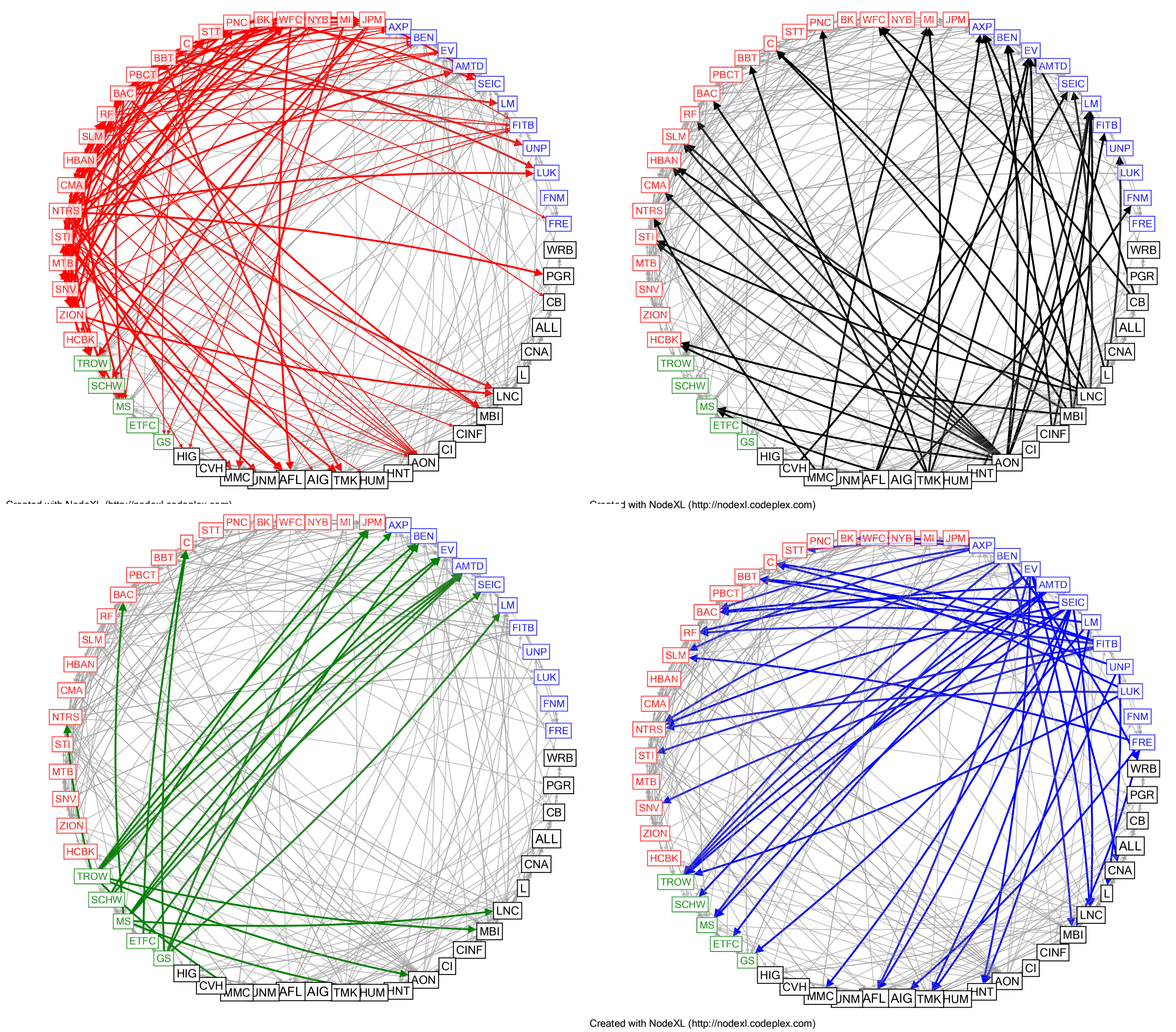

Figure 2.A.2: Network graph arranged according to industry groups highlighting the industry-specific risk spillovers from depositories (top left), insurances (top right), broker dealers (bottom left) and others (bottom right). Arrows only mark risk spillovers effects without referring to their respective size. Otherwise arrows and colors are as defined in Figure 2.1. A complete list of firms' acronyms is contained in Table 2.A.1 in the Appendix.

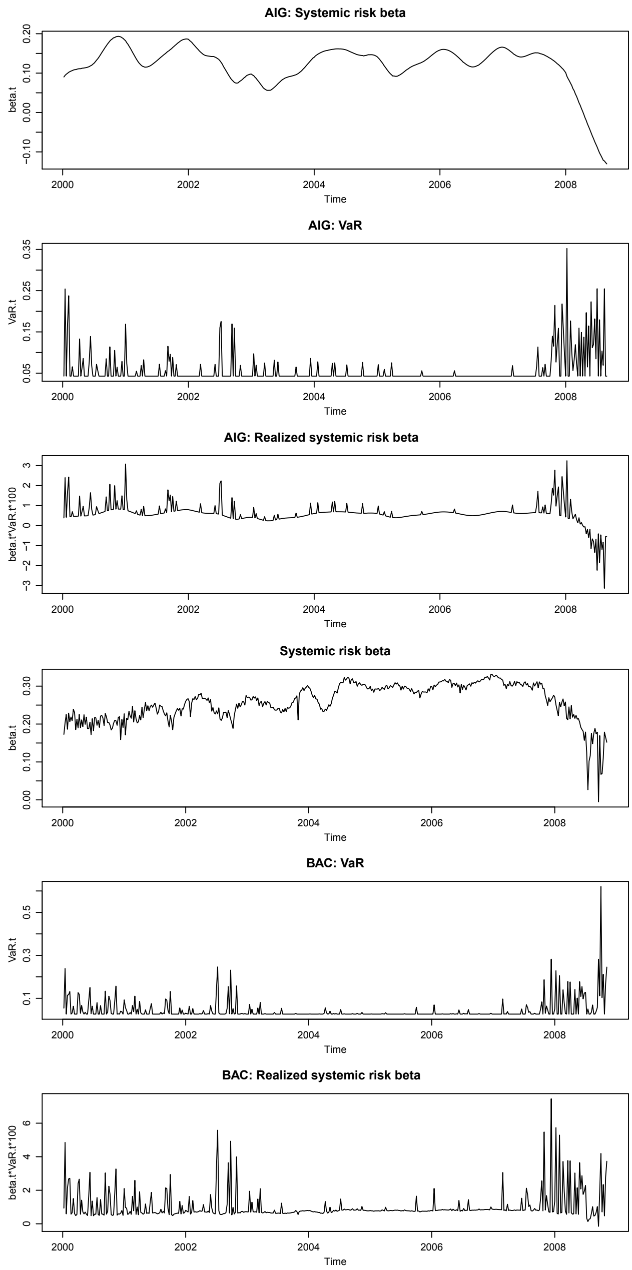

Figure 2.A.3: The upper three panels depict time-varying systemic risk betas, time-varying VaRs and the product of the two, realized systemic risk betas, for American International Group (AIG). The lower three panels show the respective three time series for Bank of America (BAC).

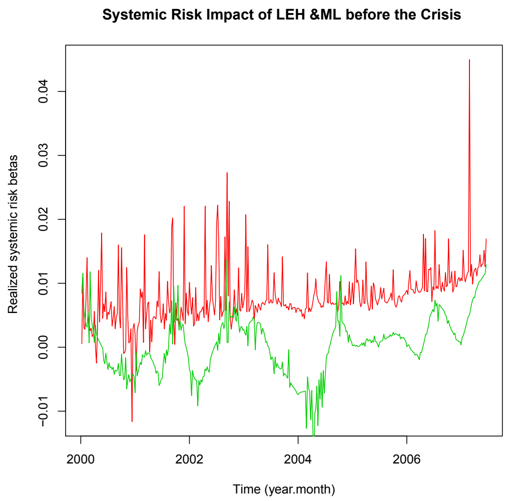

Figure 2.A.4: Realized systemic risk betas, i.e., the products of estimated systemic risk betas and individual VaRs, of Lehman Brothers (LEH, red) and Merrill Lynch (ML, green). Estimation period is the pre-crisis period, 2000 - mid 2007.

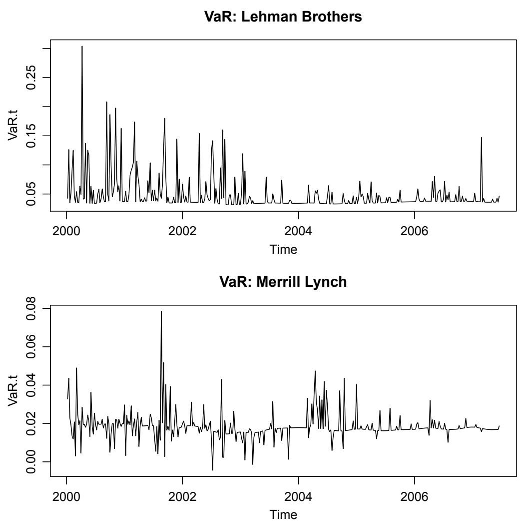

Figure 2.A.5: Estimated company-specific VaRs of Lehman Brothers (upper panel) and Merrill Lynch (lower panel). Estimation period is the pre-crisis period, 2000 - mid 2007.

Chapter 3

Forecasting systemic impact in financial networks

This chapter is based on Hautsch, Schaumburg, and Schienle (2012b).

3.1 Introduction

The breakdown risk for the financial system induced by the distress of an individual firm has long been neglected in financial regulation. Up to the financial crisis 2007–2009, this systemic risk has been exclusively attributed to the idiosyncratic risk of an institution, abstracting from the strong network cross-dependencies in the financial sector causing potential risk spillover effects. In an extensive study for the U.S. financial system, however, Chapter 2 of this thesis shows that it is mainly the interconnectedness within the financial sector that determines the systemic relevance of a particular firm. To quantify the systemic impact of an individual company, the so-called realized systemic risk beta is proposed, which is the total effect of a company's time-varying Value at Risk (VaR) on the VaR of the entire system. Firms' tail risk is determined from company-specific relevant factors among other companies' tail risks, individual balance sheet characteristics, and financial indicators, where components are selected as being "relevant" via a data-driven statistical regularization technique. The resulting individual-specific models give rise to a financial risk network, capturing exposures of financial firms towards the distress of others. These network risk spill-over channels contain important information for supervision authorities as sources for systemic risk. The data-driven determination of firms' systemic relevance from publicly available data distinguishes the realized systemic risk beta from the number of other recently proposed methods for refined measurement and prediction of systemic risk, see, e.g., Adrian and Brunnermeier (2011), White, Kim, and Manganelli (2012), Huang, Zhou, and Zhu (2009), Brownlees and Engle (2012), Acharya, Pedersen, Philippon, and Richardson (2012), Giesecke and Kim (2011), Billio, Getmansky, Lo, and Pelizzon (2012), Koopman, Lucas, and Schwaab (2011), Engle, Jondeau, and Rockinger (2012), or Schwaab, Koopman, and Lucas (2001) among many others.

Effective regulation requires models which can be used for forecasting and which are reliable even if estimation periods are short. The framework developed in Chapter 2, however, is not tailored to short-term forecasting of systemic risk and must be adapted for prediction purposes. Firstly, the systemic risk network is static, i.e., it is estimated once using the entire dataset and then forms the basis for estimation of respective time-varying realized betas. However, empirical evidence suggests that network links might change over time, especially in crisis periods. Secondly, in order to exploit additional variation, quarterly balance sheet characteristics are interpolated by cubic splines over the analyzed time period. Therefore, out-of-sample forecasting is not possible. Thirdly, the penalty parameter required for the model selection step is chosen such that a backtest criterion is optimized. VaR backtests, however, generally rely on counting and analyzing VaR exceedances, which is reasonable when the time series is long. Though, for short estimation periods, these tests should be replaced by more adequate quantile versions of F-tests.

In this chapter, we extend the framework from the previous chapter to allow for flexible systemic risk forecasting. The estimation period is shortened using rolling windows of only one year of data. This excludes influences of back-dated events on current forecasts while still pertaining sufficient prediction accuracy. The models are re-estimated each quarter, resulting in time-varying systemic risk networks. Instead of interpolating, information on firm-specific balance sheets is only updated when it is published at the end of each quarter. The model selection penalty is chosen such that the in-sample fit in the respective annual observation window is optimal. This is examined via an F-test for quantile regression. The empirical analysis investigates systemic risk in Europe. The data set covers stock prices and balance sheets of major European banks and insurance companies as well as financial indicators, including country-specific variables, during the period around the recent financial crisis. We illustrate that our approach could serve as a monitoring tool for regulators as it captures and effectively predicts systemic relevance over time.

The remainder of the chapter is structured as follows. Section 3.2 outlines the forecasting methodology and gives an algorithm for model selection and estimation of firm-specific VaRs. Furthermore, the estimation method for realized systemic risk betas is given. Section 3.3 describes the dataset, before discussing estimation results and their implications in detail in Section 3.3.2. Section 3.4 concludes.

3.2 Forecasting Methodology

Whereas Chapter 2 focuses on a single *static* network as a basis for estimating systemic impact of financial institutions, we progress by determining *time-varying* networks in a forecasting setting. These allow capturing changing risk spillover channels within the system, which are tailored to short-term forecasts from the model.

3.2.1 Time-Varying Networks

In a densely interconnected financial system, the tail risk of an institution i at a time point t is determined not only by its own balance sheet characteristics Z^i_{t-1} and general market conditions M_{t-1} but also by indications for distress in closely related banks in the system. For each bank in the system, we count a corresponding return observation as marking a distress event whenever this return is below the empirical 10% quantile. In such cases, these extreme returns might induce cross-effects on the riskiness of other banks in the system. We record these as so-called loss exceedances, i.e., the values of returns in case of an exceedance of the 10% quantile and zeros otherwise. Accordingly, the set of potential risk drivers R for a bank i therefore comprises network impacts N^{-i}_t from any other bank in the system, where each component of N^{-i}_t consists of loss exceedances for any bank but firm i in the system.

We measure tail risk by the conditional Value at Risk, VaR^i, for firm i and by VaR^s for the system, respectively. Using a post-LASSO technique, the large set of potential risk drivers $R_t = (Z^i_{t-1}, M_{t-1}, N^{-i}_t)$ for institution i can be reduced to a group of "relevant" risk drivers $R^{(i)}_t$. Selected tail-risk cross-effects from other banks in the system constitute network links from these banks to institution i. Repeating the analysis for all banks i in the system, relevant risk channels can be depicted and summarized in a respective network graph. The recent financial crisis, however, has shown that such network interconnections may change over time, as the relevance of certain institutions for the risk of others might vary substantially. Thus adequate short-run predictions of systemic importance should mainly be based on *current* dependency structures. We address this issue by a time-dependent selection of relevant risk drivers $R^{(i,t)}_t$ according to the algorithm described below. Driven by the quarterly publication frequency of companies' balance sheet information we re-evaluate the relevance of all potential risk drivers for each institution in the system at the beginning of each quarter based on data from the respective previous year and incorporate the latest balance sheet news. We therefore obtain quarterly time-varying tail risk networks which reflect the most current information of risk channels within the financial system. They are tailored for short-term quarterly predictions of the systemic riskiness of firms in the system.

With the relevant risk drivers $R^{(i,t)}$ for firm i and time t in a specific quarter, individual tail risk can be determined from observations up to one year before t as

$$\widehat{VaR}_t^i = \widehat{\xi}_0^{i,t} + \widehat{\xi}^{i,t} R_t^{(i,t)} , \tag{3.2.1}$$

where coefficients $\widehat{\xi}$ are obtained in the post-LASSO step from quantile regression of X^i on $(1, R^{(i,t)})$ as part of the procedure described below.

Algorithm for selecting relevant risk drivers and determining their effects in firms' tail risk

We adapt the data-driven procedure of Chapter 2 (see the second paragraph in Section 2.A.1), to account for time-variation in tail risk networks and marginal systemic risk contributions. Here, the automatic selection procedure is based on a sequential F-test instead of a backtest criterion. For details on the tests on joint significance in quantile regression settings, see e.g. Koenker (2005, Chapter 3) or Gutenbrunner, Jurečková, Koenker, and Portnoy (1993). Determination of relevant risk drivers $R^{(i,t_0)}$ at the beginning of a quarter t_0 uses information of observations within the previous year, the number of which we denote by τ. Hence, it is based on approximately 250 observations $R_{t_0-\tau}, \ldots, R_{t_0}$, where each R_t consists of centered observations of the potential regressors and has K dimensions. We fix a ν-equidistant grid $\Delta_c = \{c_1 > \ldots > c_l = c_1 - \nu(l-1) > c_L = 0\}$ for values of a constant c, where c_1 is chosen such that the corresponding penalty parameter is sufficiently large for selecting not more than one regressor into the model. For our purposes, we set $c_1 = 30$ and $\nu = 1$.

Step 1: For each $c \in \Delta_c$, determine the penalty parameter $\lambda_{t_0}^i(c)$ from the data in the following two sub-steps as in Belloni and Chernozhukov (2011):

Step a) Take $\tau + 1$ iid draws from $\mathcal{U}[0,1]$ independent of $R_{t_0-\tau}, \ldots, R_{t_0}$ denoted as $U_0, \ldots, U_\tau$. Conditional on observations of R, calculate

$$\Lambda_{t_0}^i = (\tau + 1) \max_{1 \leq k \leq K} \frac{1}{\tau + 1} \left| \sum_{t=0}^{\tau} \frac{R_{t_0-t,k}(q - I(U_t \leq q))}{\hat{\sigma}_k \sqrt{q(1-q)}} \right| .$$

Step b) Repeat step a) B=500 times generating the empirical distribution of $\Lambda_{t_0}^i$ conditional on R through $\Lambda_{t_0 1}^i, \ldots, \Lambda_{t_0 B}^i$. For a confidence level $\alpha = 0.1$ in the selection, set

$$\lambda^i_{t_0}(c) = c \cdot Q(\Lambda^i_{t_0}, 1-\alpha | R_{t_0-t}),$$

where $Q(\Lambda^i_{t_0}, 1-\alpha|R_{t_0-t})$ denotes the $(1-\alpha)$-quantile of $\Lambda^i_{t_0}$ given R_{t_0-t}.

Step 2: Run separate l_1-penalized quantile regressions for $\lambda^i_{t_0}(c_1)$ and $\lambda^i_{t_0}(c_2)$ from step 1 and obtain

$$\widetilde{\xi}^{it_0}_q(c) = \text{argmin}_{\xi^i} \frac{1}{\tau+1} \sum_{t=0}^{\tau} \rho_q \left(X^i_{t_0-t} + R'_{t_0-t}\xi^i \right) + \lambda^i_{t_0}(c) \frac{\sqrt{q(1-q)}}{\tau} \sum_{k=1}^{K} \hat{\sigma}_k |\xi^i_k| \,, \tag{3.2.2}$$

with the set of potentially relevant regressors $R_{t_0-t} = (R_{t_0-t,k})^K_{k=1}$, componentwise variation $\hat{\sigma}^2_k = \frac{1}{\tau+1}\sum^{\tau}_{t=0}(R_{t_0-t,k})^2$ and loss function $\rho_q(u) = u(q - I(u<0))$, where the indicator $I(\cdot)$ is 1 for $u<0$ and zero otherwise.

Step 3: Drop all components in R with absolute marginal effects $|\widetilde{\xi}^i_{t_0}(c)|$ below a threshold $\tau = 0.0001$ keeping only the $K^{it_0}(c)$ remaining relevant regressors $R^{(i,t_0)}(c)$ for $c \in \{c_1, c_2\}$. As $c_1 > c_2$, the sets of selected relevant regressors are nested $R^{(i,t_0)}(c_1) \subseteq R^{(i,t_0)}(c_2) = \{R^{(i,t_0)}(c_1), R^{(i,t_0)}(c_2 \backslash c_1)\}$. If $R^{(i,t_0)}(c_2\backslash c_1)$ is the empty set, restart Step 2 with $\lambda^i(c_2)$ and $\lambda^i(c_3)$ from Step 1. Otherwise re-estimate (3.2.2) without penalty term for the larger model c_2 only with the respective selected relevant uncentered regressors $R^{(i,t_0)}(c_2)$ and an intercept. This regression yields the post-LASSO estimates $\widehat{\xi^{it_0}_q}(c_2)$. Apply an F-test for joint significance of regressors $R^{(i,t_0)}(c_2\backslash c_1)$ at 5% level. If they are significant, restart Step 2 with $\lambda^i(c_2)$ and $\lambda^i(c_3)$ from Step 1b. Continue until additional regressors $R^{(i,t_0)}(c_{l+1}\backslash c_l)$ from penalty c_l to c_{l+1} are no longer found to be significant. Then the final model is obtained from c_l yielding the set of relevant regressors $R^{(i,t_0)}(c_2)$ with corresponding post-LASSO estimates $\widehat{\xi^{it_0}_q}(c_l)$ for the coefficients.

3.2.2 Forecasting Systemic Impact

In an interconnected financial system, we measure the systemic impact of a specific bank i as the total realized effect of its riskiness on distress of the entire financial system given network and market externalities. This can be empirically determined via

$$VaR^s_t = \alpha^{s,t} + \beta^{s|i,t}(Z^{i*}_{t-1})\widehat{VaR}^i_t + \gamma^{s,t} M_{t-1} + \theta^{s,t}\widehat{VaR}^{(-i,t)}_t \,, \tag{3.2.3}$$

where $\widehat{VaR}^{(-i)}$ comprises tail risks of all other banks in the system selected as relevant risk drivers for bank i in the corresponding network topology. The marginal effect $\beta^{s|i,t}$ of the risk of company i might vary linearly over time in selected firm-specific balance sheet characteristics Z^{i*}_{t-1}. Coefficients in (3.2.3) can be obtained via standard quantile regression analogously to (3.2.2) without penalty term. Corresponding to the one-year estimation window for the time-varying network, we also determine parameters in (3.2.3) at the beginning of each quarter, based on observations dating back no longer than one year. The systemic relevance of a company can then be predicted from the beginning of a quarter t_0 to the next quarter $t_0 + \tilde{\tau}$ as realized beta

$$\widetilde{\beta}^{s|i}_{t_0+\tilde{\tau}|t_0-} = \widehat{\beta}^{s|i,t_0}(Z^{i*}_{t_0-1})\widehat{VaR}^{i}_{t_0} \tag{3.2.4}$$

where $t-$ denotes information up to time t. Within a quarter, predictions are updated by

$$\widetilde{\beta}^{s|i}_{t+1|t-} = \widehat{\beta}^{s|i,t_0}(Z^{i*}_{t_0-1})\widehat{VaR}^{i}_{t} \tag{3.2.5}$$

for any time point $t_0 \leq t \leq t + \tilde{\tau}$.

3.3 Data and Results

3.3.1 Data

Our sample of financial firms comprises 20 European banks and insurance companies. A list can be found in Table 3.A.1. The dataset covers Europe-based banks deemed as systemically relevant by Financial Stability Board (2011), for which complete data sets over the considered period are available.[1] It includes the ten largest European banks by assets in 2010. Furthermore, six insurance companies are selected, all belonging (by assets) to the top 10 insurers in the world in 2010. The regressors explaining the individual Value at Risk (VaR^i) are selected among other companies' loss exceedances, individual balance sheet ratios, and several financial indicators, including country-specific variables.

From quarterly balance sheets obtained from Datastream/Worldscope, three key ratios are calculated: Leverage, correponding to total assets divided by total equity; maturity mismatch, the quotient of short-term debt and total debt; and size, defined as the logarithm of total assets. Furthermore, we include quarterly stock price volatility in the set of pos-

[1]Banco Espirito Santo is the only bank which is not listed by the Financial Stability Board. We include it because otherwise, financial firms from Southern Europe would be underrepresented.

sible regressors, which is estimated over the time span between quarterly reports. Instead of interpolating the data to daily values, we keep them constant until new information is published.[2]

The set of financial indicator variables contains the return on EuroStoxx 600, relative changes of the volatility index VStoxx, and returns on three major bond indices for Europe: IBOXX Sovereign, containing government bonds, iBOXX Subsovereigns, consisting of bonds issued by government owned banks, supranationals and other sub-sovereigns, and iBOXX Corporates. Furthermore, we include changes in three months Euribor, the interbank lending interest rate, and a liquidity spread between three months Eurepo, the average repo rate reflecting the cost of repurchase agreements, and the three month Bubill (German government bond rate) as proxy for the risk free rate. To capture aggregate credit quality in Europe, we also add the change in the one year and five year default probability indices from Fitch as well as the change in the five year continued series of the credit default swap index iTraxx Europe. Another two relevant economic indicators are the gold price and relative changes of the MSCI Europe Real Estate Price Index.

As proxies for the market's expectations on economic growth and to capture country-specific effects on individual VaRs, we include several ten year government bond yields (Germany, United Kingdom, Spain, United States, and Greece) as well as yield spreads (ten years minus three months yields) of German and U.S. government bonds. Finally, accounting for the global interconnectedness of financial markets, we include returns on financial sector indices, FTSE Financials Japan, Asia, and US.

When estimating systemic risk betas in the second stage, a subset of the above macro financial indicators is required as control variables. Here, we take the changes in the EuroStoxx 600 index, VStoxx, Euribor, iTraXX, the three FTSE Financial indices, the real estate index, and the spread between Eurepo and the Bubill rate.

3.3.2 Results

Time-varying tail risk networks

Having identified the tail risk drivers for each firm allows us constructing a tail risk network. Following Chapter 2, we take all firms as nodes in a network and identify a network link from firm i to firm j whenever the loss exceedance of i is selected as a tail risk driver for j. Figures 3.A.1 to 3.A.3 show the resulting systemic risk networks for the 20 financial institutions computed based on one-year rolling windows from 2006 to 2010. In order

[2]For simplicity, we assume that quarterly balance sheets become public information on fixed dates: March 31, June 30, September 30 and December 31.

to illustrate cross-country and inter-country risk channels, we order the institutions in the graph according to their (main) home countries.

We identify several risk connections which are quite stable over time and seem to be fundamental risk channels of the European financial network during the period under consideration. An interesting tail risk connection is the link between Royal Bank of Scotland (RBS) and Barclays. RBS was strongly affected by the break-down of the U.S. housing and credit markets and realized substantial write-downs in April 2008. In the beginning of 2009, RBS faced a record loss and was bailed out by the UK government which increased its stake in the company to 70 percent. Conversely, Barclays was relatively well funded until beginning of 2008 and even explored options to take over the defaulting U.S. investment bank Lehman Brothers. A further bolstering of Barclays' balance sheet was due to the raise of new capital by investors in fall 2008. Consequently, Barclays was less exposed to credit crunches and did not participate in the government's insurance schemes for toxic assets. The network analysis, however, reveals that both banks have been deeply connected. Being bi-directional before the crisis, the links became particularly pronounced and rather one-directional during the financial crisis. In particular, RBS received substantial tail risk from Barclays further increasing RBS's potential losses and making both companies systemically risky. Interestingly, the strong risk connection between Barclays and RBS vanishes in the aftermath of the financial crisis which might be a result of RBS's bailout and ongoing re-structuring in both banks.

Persistent risk connections are also identified between Deutsche Bank and various big insurance companies, particularly Allianz as well as between Deutsche Bank and Commerzbank. The latter faced significant distress due to investments in toxic assets originating from the U.S. housing market, and was the first commercial lender in Germany accepting capital injections from the government. In the beginning of 2009, Commerzbank was partly nationalised with the government taking a 25% stake. Our analysis reflects that the distress of Commerzbank also spilled over to Deutsche Bank and thus in turn to big insurances such as Allianz and Münchener Rück. Hence, governmental support of Commerzbank was an important step to reduce its systemic risk contribution. This is empirically confirmed by our analysis as we observe a declining tail risk connectedness of Commerzbank after the bailout.

Furthermore, the networks reveal persistent connections between UBS and Credit Suisse, UBS and Crédit Agricole, Agricole and Société Générale as well as Credit Suisse and Agricole. The strong interconnections between these Swiss and French banks are likely to be driven by exposure to the same toxic assets and resulting liquidity shortages stemming from the U.S. market making these banks facing common funding problems. This happened during 2008/09, where all of these banks also received substantial tail risk spillovers from other competitors. For instance, our analysis reveals that Credit Suisse was subject to tail risk inflow from Barclays and BNP Paribas which - according to the identified network connections - spilled over to the 'risk neighbors' of Credit Suisse. All of these banks received bailout packages from the Swiss and French government, respectively. As a possible consequence of these bailouts and a relaxation of the bank's funding situation in the aftermath, Credit Suisse's sensitivity to tail risk inflow from Barclays and BNP Paribas actually declined in 2009. Likewise, the Spanish bank Santander and the Portugese bank Banco Espirito Santo appear to be deeply interconnected. As discussed below, Santander serves as an originator and transmitter of systemic risk to various other companies. These dependencies become particularly visible and pronounced during and after the financial crisis and might have contributed to the instability and distress of the Spanish banking system in 2012.

Hence, though all these institutions operate on a global level, we still observe a substantial extent of persistent country-specific risk channels. These effects reflect a strong interconnectedness and consequently inherent instability of national banking systems. These within-country dependencies are complemented by cross-country linkages and industry-specific channels. Examples for the latter are tail risk connections prevailing within the insurance sector including Allianz, AXA, Aviva, Münchener Rück and Aegon. Their interconnectedness even increased during the financial crisis, causing a substantial threat for the system in case of the default of one of these companies.

Our approach, however, also captures interesting time variations in tail risk channels. In particular, in 2008/09, we observe high fluctuations of network connections. Several risk channels identified in this period seem to be rather caused by crisis-specific turbulences and consequently vanished in the aftermath. Examples are connections from Santander to HSBC, BNP Paribas, Allianz and AXA. These links make Santander systemically quite risky as the bank obviously produced and transmitted tail risk to various major players in the system. These findings are confirmed by the estimated systemic risk betas shown below. A further example is a strong connection between ING and Aviva which built up and increased through the crisis and vanished thereafter. The Dutch bank ING realized significant losses, had to cut jobs in 2009 and received capital injections from the Dutch government. Hence, our analysis shows that substantial tail risk from ING was spreading out to Aviva and in turn to other insurances.

Analyzing the pure number of outgoing tail risk connections (illustrated by the size of nodes in the network graphs), we identify Barclays, Santander, AXA, BNP Paribas, ING, Société Générale and Crédit Agricole as biggest risk transmitters within our sample. Actually, the latter four were companies which have been bailed out by their governments and got partly nationalized. Our analysis indicates that these governmental capital injections were indeed justifiable as these companies have been (and still are) in the core of the network and therefore serve as distributors and multipliers of systemic risk. According to the identified network connections, failure of one of these institutions would substantially threaten the stability of the financial system.

Systemic risk rankings

Table 3.A.2 reports systemic risk rankings for all quarters between the beginning of 2007 and the end of 2010. They are based on realized systemic risk betas at the end of the respective foregoing quarter, and therefore contain forecasts of relative systemic relevance. Prior to the estimation, we conducted a test on joint significance of VaR^i and its interaction with Z^{i*}, $i = 1, ...20$, for VaR^s, using all five years of data. Apart from two exceptions, all individual VaRs turn out to be statistically significant for the system's VaR. The two exceptions are, on the one hand, Banco Espirito Santo, which is the largest bank in Portugal, but much less internationally active than the other banks in our sample. On the other hand, Société Generale is found to be insignificant. We attribute this finding to the fact that in 2008, the bank was affected by large losses induced by the unauthorized propriety trading of one of its employees. This was a materialization of (idiosyncratic) operational risk, and may have distorted the test results concerning systemic relevance. We expect that on a longer horizon, Société Generale' systemic risk beta would be significant. In the following, however, we exclude it from the systemic risk rankings, together with Banco Espirito Santo.

It should be noted, that often differences in beta estimates between direct neighboring firms in the obtained rankings are small and thus not statistically significant. Hence, orderings in Table 3.A.2 should rather be seen as an indication for a company's relative systemic importance characterizing groups of similar relative systemic impact.[3] Figure 3.A.5 illustrates the time-varying cross-sectional distribution of the estimated betas. We observe the overall highest systemic risk betas during the height of the financial crisis. Furthermore, representatively for other firms, we depict the estimated systemic impacts of Barclays, Crédit Agricole, Santander and UBS. It turns out that the respective systemic risk betas move in locksteps before mid 2008, but strongly diverge during the crisis. Similar relationships are also shown for other companies and reflect distinct crisis-specific effects.

These effects are supported by Table 3.A.2, revealing strong variations of the relative systemic riskiness during the crisis. This is obviously induced by a severe instability of the

[3]At some time points, estimated systemic risk betas become negative. We interpret this finding as negligible systemic impacts of the respective firm in the respective quarter and therefore omit it in the ranking.

financial system during this period and is also confirmed by the high variability of network connections as discussed above. Conversely, a higher stability of systemic risk patterns over time is observed in the periods before and after the financial crisis (i.e., 2007 and 2010).

Overall, we identify BNP Paribas, HSBC and Santander as being most risky with the highest realized risk betas between 2007 and 2010. BNP Paribas was strongly affected by the credit crunch and an evaporation of liquidity in the funding market. At the end of 2008, the French government agreed to provide financial support Europe's largest bank. Our findings reflect that after the bailout, BNP's systemic riskiness was still comparably high. According to the network analysis above, this is obviously due its strong interconnectedness making BNP to one of the major originators of tail risk spillovers in 2010. In contrast, HSBC's connectedness is only moderate. However, its size and the identified tail risk connections to Barclays, BNP and Santander make it systemically quite risky. These connections became obviously quite relevant due to HSBC's heavy exposure to U.S. housing and credit markets. Consequently, the bank's distress induced by significant losses during the crisis have been spread out in the system resulting in a particularly high systemic riskiness around beginning of 2009. This is backed by the fact that HSBC had to cut a substantial amount of jobs at beginning of 2009. Our results indicate that also in the aftermath of the crisis, HSBC still remains systemically quite risky. In case of Santander, the relative systemic riskiness (compared to other banks) even tends to increase after the financial crisis (particularly in 2010). This finding might already indicate funding problems in the Spanish banking market becoming particularly evident in 2012. These results are in line with the findings of the network analysis above identifying Santander as a deeply interconnected bank being linked to several insurance companies and (particularly during the crisis) to other major players like Barclays and HSBC.

Monitoring systemic risk rankings over the course of the financial crisis provides interesting insights into the systemic importance of individual firms under extreme conditions of market distress. Four prominent examples are RBS, Barclays, Deutsche Bank and HBSC. According to the estimated systemic risk betas, we classify RBS as belonging to the most systemically risky companies in 2008. Also Barclays is identified as being systemically very relevant in several (though not all) periods in 2008/09. The identified network connections revealed that the strong connection between Barclays and RBS was obviously one driving force of the systemic relevance of both. This is also confirmed by the fact that the systemic relevance of both (as indicated by the realized betas) declined as the tail risk connection between both vanishes in 2009. Likewise, Deutsche Bank faces a steady increase of its systemic relevance in 2007 and belongs to the group of systemically most risky companies in 2008. This is confirmed by the network analysis above showing that particularly during 2008, Deutsche Bank was deeply interconnected with risk channels to various major insurance companies. Consequently, a default of Deutsche Bank would have had dramatic consequences for the insurance industry and thus the stability of the entire system. Although Deutsche Bank was not subject to any government bailouts it went through a process of substantial internal restructuring. This is confirmed by our estimates showing a decline of Deutsche Bank's systemic importance during 2009 and 2010.

Finally, for the post-crisis period, we observe a tendency for the insurance companies becoming relatively more risky. Particularly in 2010, Allianz, Aviva, Axa, Generali and Münchener Rück reveal relatively high (though not always significant) systemic risk betas. Likewise, also Société Générale and Credit Suisse are identified as systemically risky in 2010. These findings are confirmed by the network analysis showing a comparably high connectedness of Société Générale, Axa and Generali.

3.4 Conclusion

In this chapter, we propose a framework for forecasting financial institutions' marginal contribution to systemic risk based on their interconnectedness in terms of extreme downside risks. There are four major challenges in this context: Firms' (conditional) tail risks are unobserved and must be estimated from data. Determining such individual risk levels appropriately results in high-dimensional models due to the large number of potential network connections. These network dependencies, however, are inherently instable over time. Therefore forecasting stability and responsiveness require careful balancing. To tackle these issues, we adapt the two-stage quantile regression approach introduced in Chapter 2 to a rolling window out-of-sample prediction setting based on time-varying networks.

In a sample of large European banks covering the period 2007 to 2010, the adapted procedure reveals the dynamic nature of interconnectedness and corresponding risk channels in the European financial system around and during the financial crisis. The time evolution of network dependencies provides valuable insights into a bank's role in the system identifying originators and transmitters of tail risk over time. Determined relevant tail risk connections and systemic risk rankings both provide valuable input for regulation. Given the need for better and more timely market surveillance, our approach can thus serve as a useful vehicle for providing a continuous assessment of systemic risk dependencies based on market data.

3.A Appendix

Aegon (Insurance, NL)	Deutsche Bank (Bank, DE)
Allianz (Insurance, DE)	Generali (Insurance, IT)
Aviva (Insurance, UK)	HSBC (Bank, UK)
AXA (Insurance, FR)	ING Groep (Bank, NL)
Banco Espirito Santo (Bank, PT)	Lloyds Banking Group (UK)
Barclays (Bank, UK)	Muenchener Rueck (Insurance, DE)
BNP Paribas (Bank, FR)	Royal Bank of Scotland (Bank, UK)
Commerzbank (Insurance, DE)	Santander (Bank, ES)
Crédit Agricole (Bank, FR)	Société Générale (Bank, FR)
Credit Suisse (Bank, CH)	UBS (Bank, CH)

Table 3.A.1: List of included European financial institutions. As most of them provide a broad range of services, we differentiate between banks and insurance companies according to their main field of business activities. Furthermore, we state the country their headquarters are located in.

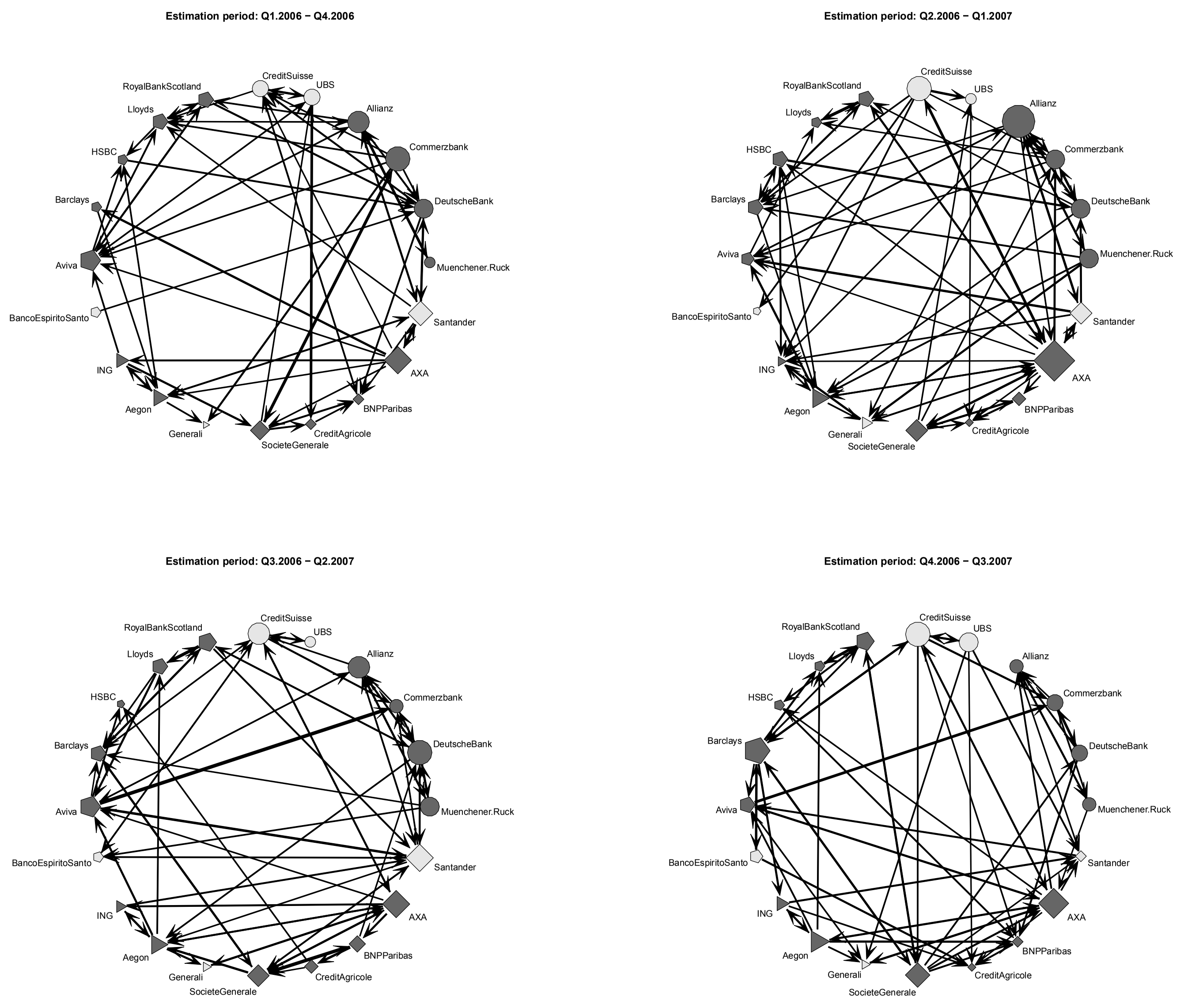

Figure 3.A.1: Estimates of yearly systemic risk network rolled over from Q4/2006 to Q3/2007.

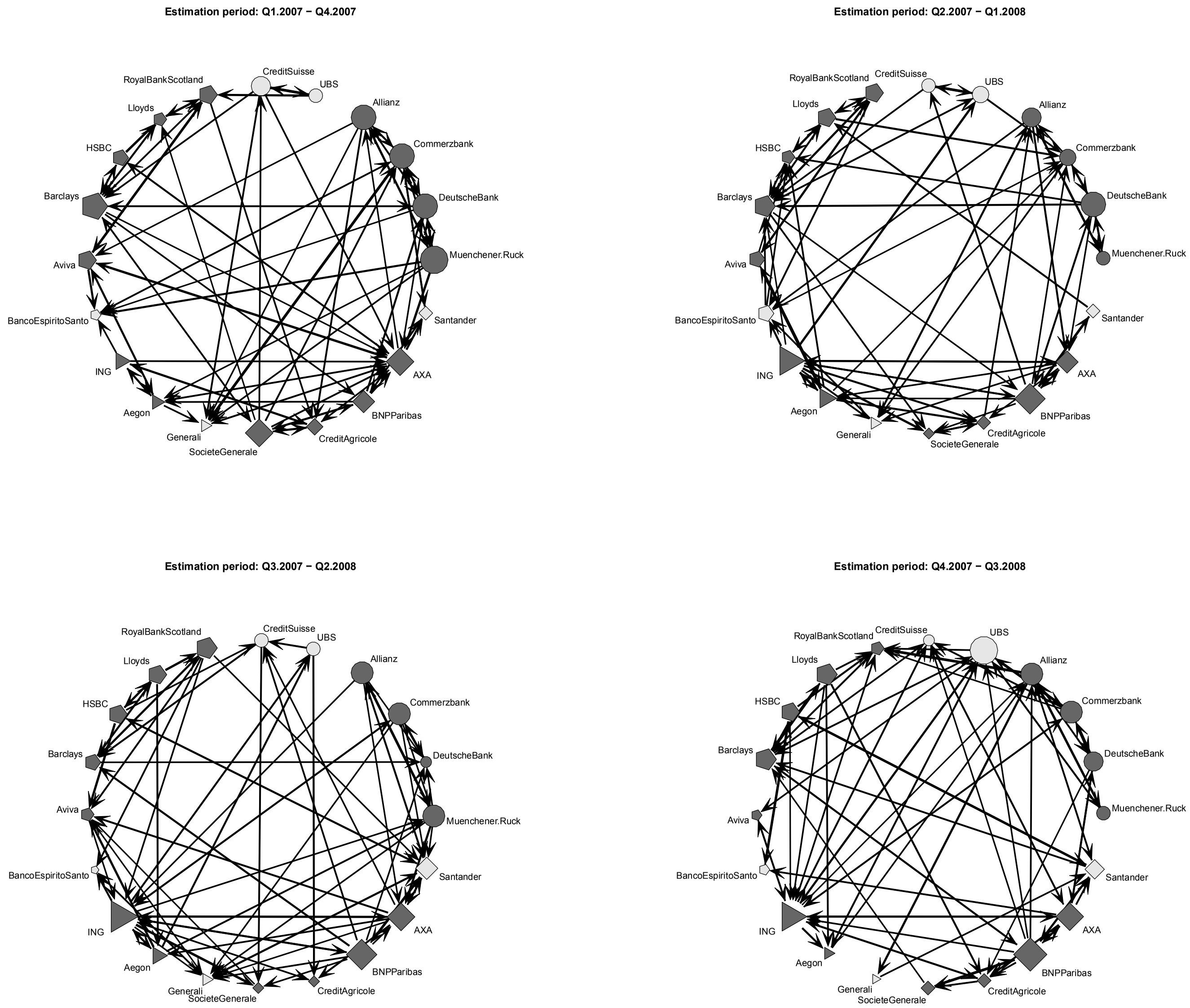

Figure 3.A.2: Estimates of yearly systemic risk network rolled over from Q4/2007 to Q3/2008.

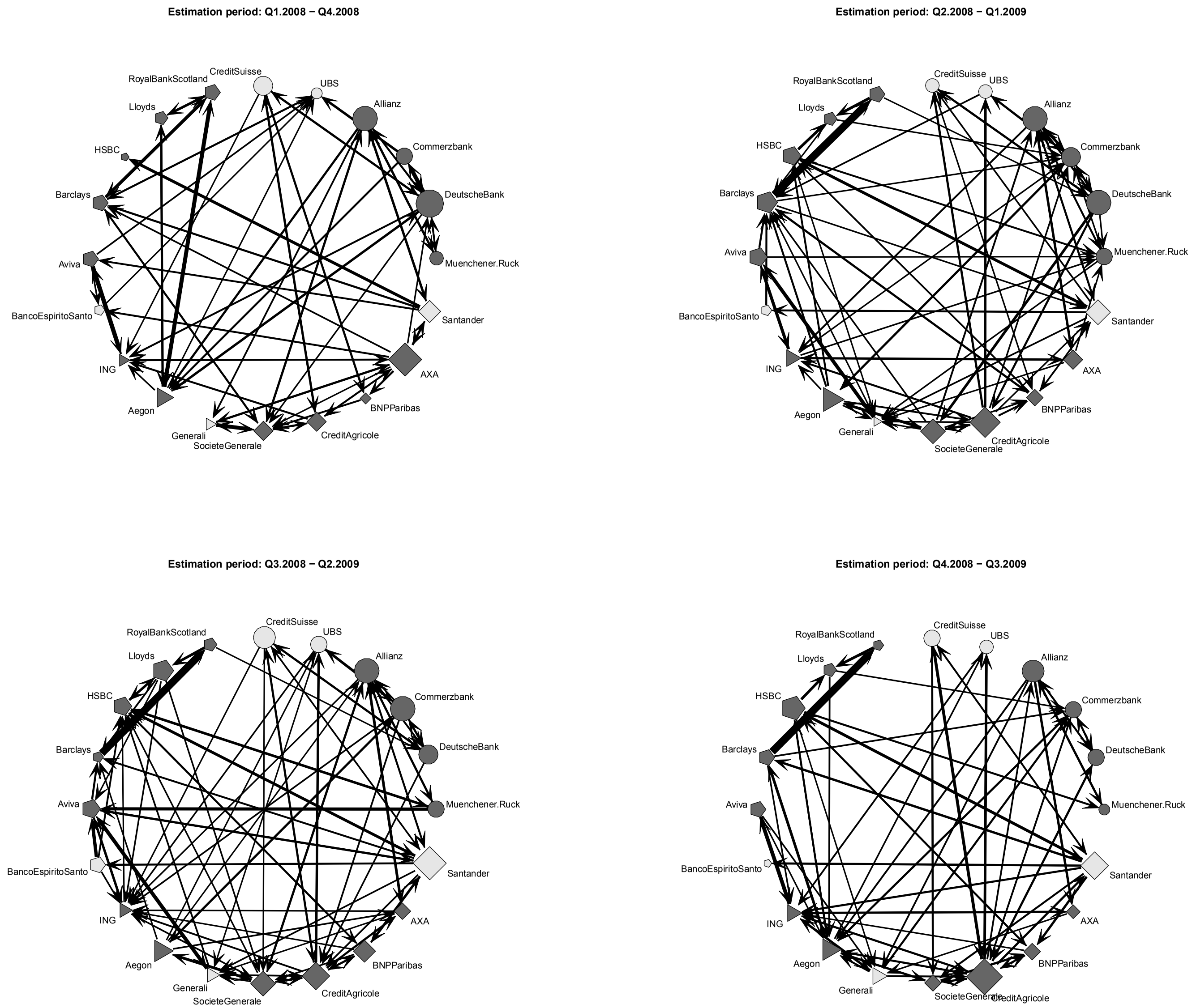

Figure 3.A.3: Estimates of yearly systemic risk network rolled over from Q4/2008 to Q3/2009.

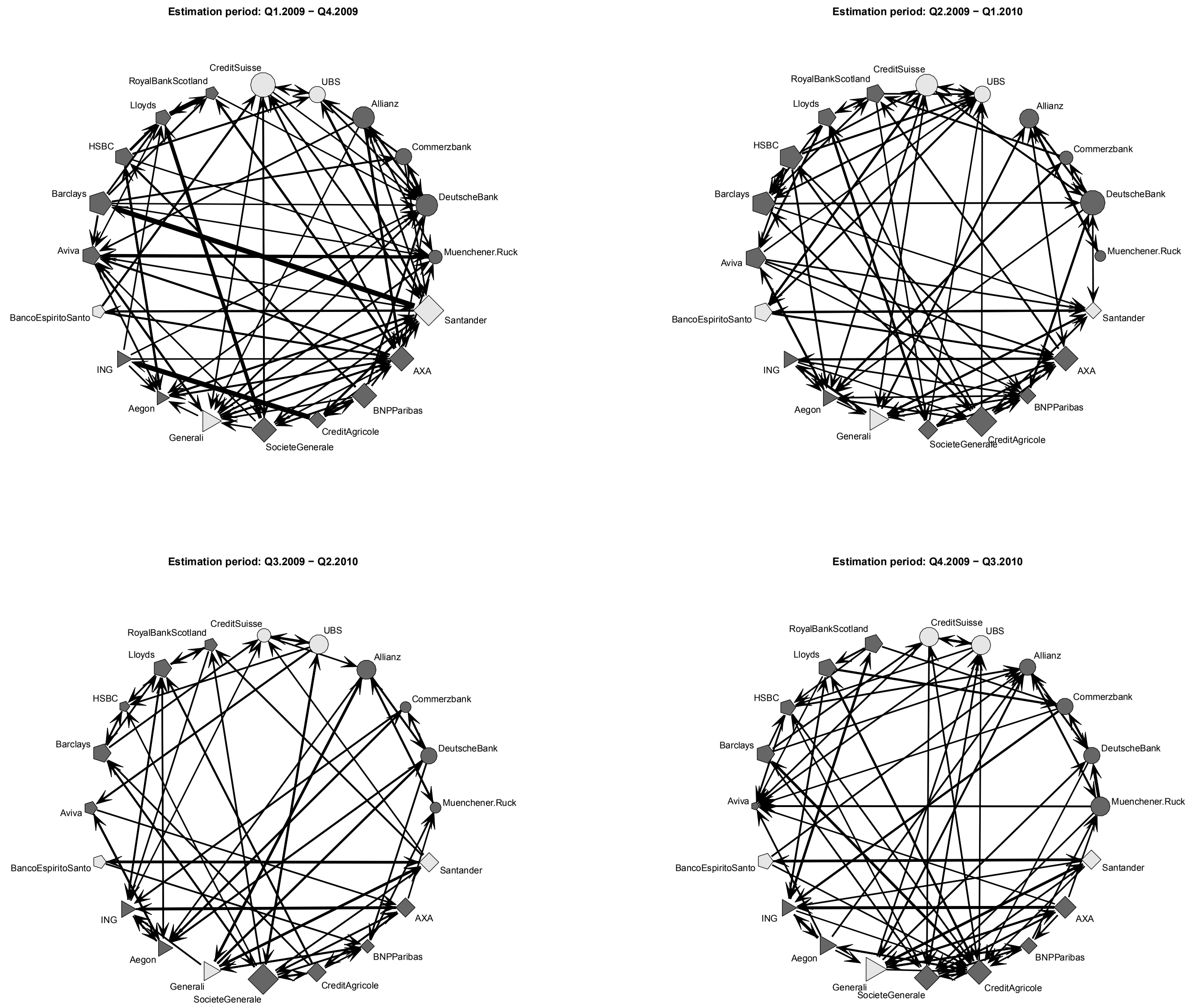

Figure 3.A.4: Estimates of yearly systemic risk network rolled over from Q4/2004 to Q3/2010.

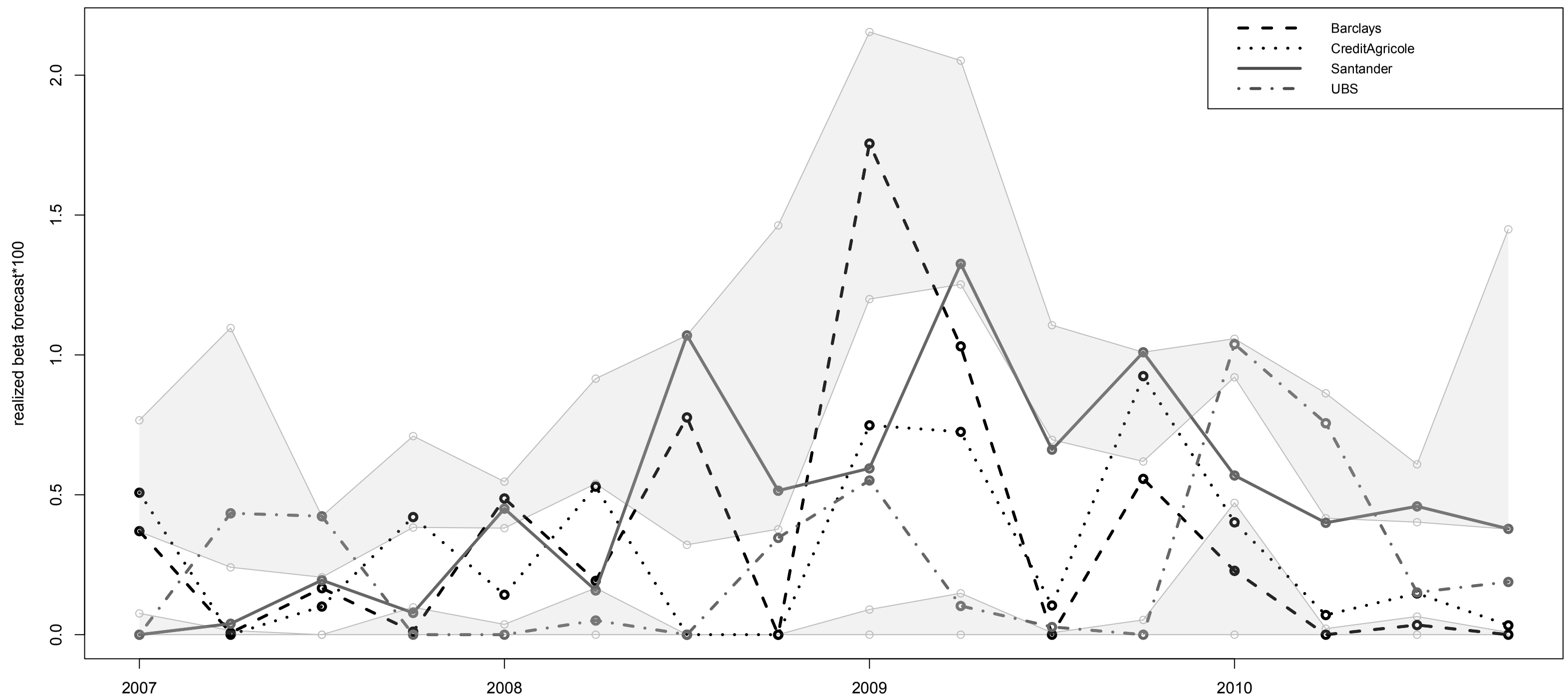

Figure 3.A.5: Illustration of time-varying risk rankings, highlighting the evolution of realized systemic risk beta forecasts $\tilde{\beta}$ of four major banks. The upper shaded area depicts the pointwise range between the maximum and the 75%-quantile of $\tilde{\beta}$ for all systemically relevant firms. The lower one marks the corresponding pointwise lower interquartile range of significant realized systemic risk beta forecasts.

Table 3.A.2: Systemic risk rankings for 2007 - 2010, based on quarterly realized beta forecasts $\widetilde{\beta}^{s|i} \cdot 100$, see equation (3.2.4).[4]

rank	name	forecast	rank	name	forecast
	Q1.2007			Q2.2007	
1	Aegon	0.7667	1	BNP Paribas	1.0964
2	Commerzbank	0.6495	2	UBS	0.434
3	Generali	0.5392	3	Aviva	0.3012
4	Credit Agricole	0.5077	4	Commerzbank	0.276
5	Barclays	0.3703	5	Deutsche Bank	0.2436
6	HSBC	0.3611	6	AXA	0.2324
7	Allianz	0.3492	7	Aegon	0.2095
8	BNP Paribas	0.3016	8	Muenchener Rueck	0.1625
9	Lloyds	0.2887	9	Allianz	0.1252
10	AXA	0.2453	10	ING	0.0914
11	Aviva	0.1888	11	Credit Suisse	0.0865
12	ING	0.163	12	Royal Bank of Scotland	0.051
13	Deutsche Bank	0.1379	13	Santander	0.0393
14	Royal Bank of Scotland	0.0556	14	Barclays	0.0067
	Q3.2007			Q4.2007	
1	UBS	0.4234	1	Deutsche Bank	0.71
2	HSBC	0.3127	2	Aviva	0.5619
3	Deutsche Bank	0.3068	3	Royal Bank of Scotland	0.5504
4	Credit Suisse	0.2296	4	Credit Agricole	0.4205
5	Generali	0.2087	5	BNP Paribas	0.3934
6	Santander	0.1947	6	Credit Suisse	0.3529
7	Barclays	0.1663	7	AXA	0.3306
8	AXA	0.1425	8	HSBC	0.3203
9	ING	0.1203	9	ING	0.3126
10	Credit Agricole	0.1007	10	Aegon	0.3104
11	Commerzbank	0.0681	11	Muenchener Rueck	0.1954
12	Lloyds	0.0672	12	Allianz	0.1594
			13	Commerzbank	0.1045
			14	Lloyds	0.0957
			15	Santander	0.0779
			16	Barclays	0.0109
	Q1.2008			Q2.2008	
1	BNP Paribas	0.5472	1	AXA	0.9152
2	Barclays	0.487	2	Royal Bank of Scotland	0.8259
3	Santander	0.4507	3	Muenchener Rueck	0.7661
4	Commerzbank	0.4375	4	Lloyds	0.5474
5	Deutsche Bank	0.3819	5	Generali	0.543
6	Royal Bank of Scotland	0.3783	6	Credit Agricole	0.5294
7	Credit Suisse	0.3508	7	BNP Paribas	0.5003
8	AXA	0.2114	8	Deutsche Bank	0.4948
9	Credit Agricole	0.1429	9	HSBC	0.4339
10	Muenchener Rueck	0.1351	10	Commerzbank	0.35
11	Allianz	0.1281	11	Aegon	0.2153
12	Lloyds	0.1148	12	Aviva	0.201
13	Aviva	0.071	13	Barclays	0.1925
14	Aegon	0.0255	14	Santander	0.1582
			15	Credit Suisse	0.1427
			16	UBS	0.0508
			17	Allianz	0.0112
	Q3.2008			Q4.2008	
1	Santander	1.07	1	HSBC	1.4631
2	Barclays	0.7768	2	Deutsche Bank	0.6341
3	Aviva	0.4461	3	Santander	0.5148
4	Credit Suisse	0.4029	4	Royal Bank of Scotland	0.4998
5	Generali	0.349	5	BNP Paribas	0.3873
6	Muenchener Rueck	0.2384	6	UBS	0.346
7	Deutsche Bank	0.2113	7	Generali	0.3118
8	HSBC	0.1727	8	Muenchener Rueck	0.2926
9	Royal Bank of Scotland	0.167	9	Lloyds	0.0985
10	ING	0.1566			
11	BNP Paribas	0.0598			

Continued on next page

[4] Avoiding multicollinearity, we include in Z^{i*} only the one component of Z^i which exhibits the lowest correlation with VaR^i in the respective interaction term in (3.2.3).

Table 3.A.2 – *Continued from previous page*

rank	name	forecast	rank	name	forecast
	Q1.2009			Q2.2009	
1	Aegon	2.1546	1	Aegon	2.0523
2	Barclays	1.7557	2	ING	1.4088
3	AXA	1.5601	3	Lloyds	1.3672
4	Aviva	1.5562	4	BNP Paribas	1.3462
5	Allianz	1.3241	5	Santander	1.3259
6	BNP Paribas	0.8262	6	Barclays	1.031
7	Credit Agricole	0.7485	7	Aviva	0.9001
8	HSBC	0.6697	8	HSBC	0.732
9	Santander	0.5945	9	Credit Agricole	0.7251
10	UBS	0.5514	10	Credit Suisse	0.4722
11	Commerzbank	0.2947	11	Muenchener Rueck	0.4417
12	Generali	0.2347	12	Allianz	0.4111
13	Credit Suisse	0.1561	13	AXA	0.2842
14	Royal Bank of Scotland	0.068	14	UBS	0.1028
15	ING	0.0455	15	Royal Bank of Scotland	0.0619
	Q3.2009			Q4.2009	
1	Commerzbank	1.1065	1	Santander	1.0097
2	Aviva	1.0086	2	Credit Agricole	0.9243
3	ING	0.8852	3	HSBC	0.8437
4	AXA	0.8303	4	BNP Paribas	0.6894
5	Lloyds	0.7041	5	Allianz	0.6225
6	BNP Paribas	0.6744	6	Royal Bank of Scotland	0.6093
7	Santander	0.6615	7	Barclays	0.5571
8	Credit Suisse	0.568	8	Lloyds	0.4588
9	Aegon	0.3393	9	ING	0.3702
10	HSBC	0.284	10	Deutsche Bank	0.3661
11	Credit Agricole	0.1044	11	AXA	0.1541
12	Royal Bank of Scotland	0.0325	12	Generali	0.0858
13	UBS	0.0276	13	Aviva	0.0699
			14	Muenchener Rueck	0.0471
	Q1.2010			Q2.2010	
1	Credit Suisse	1.058	1	Credit Suisse	0.8629
2	Lloyds	1.0418	2	UBS	0.7561
3	Generali	1.0407	3	ING	0.5004
4	UBS	1.0388	4	Aviva	0.4999
5	Aegon	0.9752	5	Generali	0.4217
6	Allianz	0.7554	6	Santander	0.4
7	AXA	0.7471	7	Royal Bank of Scotland	0.3386
8	BNP Paribas	0.6706	8	Aegon	0.2928
9	Santander	0.5692	9	Deutsche Bank	0.2234
10	Commerzbank	0.5583	10	Allianz	0.2227
11	Aviva	0.5208	11	Muenchener Rueck	0.1033
12	HSBC	0.4992	12	Credit Agricole	0.0703
13	ING	0.4722	13	AXA	0.0384
14	Deutsche Bank	0.4712	14	BNP Paribas	0.016
15	Credit Agricole	0.4019			
16	Barclays	0.2284			
17	Royal Bank of Scotland	0.1944			
	Q3.2010			Q4.2010	
1	Aviva	0.6092	1	BNP Paribas	1.4491
2	Generali	0.6008	2	Generali	0.503
3	HSBC	0.4951	3	Muenchener Rueck	0.4914
4	Santander	0.4588	4	Royal Bank of Scotland	0.4371
5	Credit Suisse	0.4493	5	Santander	0.3784
6	Muenchener Rueck	0.261	6	Aviva	0.3737
7	Aegon	0.2226	7	Allianz	0.3589
8	UBS	0.151	8	ING	0.3017
9	Credit Agricole	0.1475	9	AXA	0.2553
10	ING	0.1452	10	UBS	0.1886
11	AXA	0.1233	11	Commerzbank	0.1858
12	Allianz	0.1148	12	Aegon	0.1367
13	Commerzbank	0.0935	13	Credit Agricole	0.0334
14	BNP Paribas	0.0554			
15	Lloyds	0.0426			
16	Barclays	0.0345			
17	Royal Bank of Scotland	0.0222			

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Selbständigkeitserklärung

Ich erkläre, dass ich die vorliegende Arbeit selbständig und nur unter Verwendung der angegebenen Literatur und Hilfsmittel angefertigt habe.

Berlin, 8. Januar 2013

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