ANALYSIS AND FORECASTING OF ELECTRICITY PRICE RISKS WITH QUANTILE FACTOR MODELS

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Abstract: Forecasting quantile and value-at-risk levels for spot electricity prices is methodologically challenging because of the distinctive stochastic properties of the price density functions, volatility clustering and various exogenous factors. Despite this, accurate risk measures have considerable value in energy trading and risk management with the topic being actively researched for better techniques. We approach the problem by using a novel multifactor, dynamic, quantile regression formulation, extended to include GARCH properties. This captures the specification effects of mean reversion, spikes, time varying volatility and demonstrates how the prices of gas, coal and carbon, forecasts of demand and reserve margin in addition to price volatility influence the peak price quantiles for GB power. We show how the price elasticities for these factors vary substantially across the quantiles. We also show that a structural linear quantile regression model outperforms and easier to implement than state-of-the-art benchmark models for out-of-sample forecasts of value-at-risk.

1. INTRODUCTION

For managers involved in risk and operations, as well as for regulators concerned with market surveillance, modelling and forecasting the tails of price distributions in traded markets may often be a more crucial activity than formulating central expectations. As a task, it is certainly the more methodologically challenging. Both the relative sparseness of the data in the tails and the extreme sensitivity of the results to misspecification in the functional form of the distribution create severe difficulties. Thus, robust parametric methods for specifying predictive distributions (eg Guermat and Harris, (2001), (2002)), as well as semiparametric formulations for estimating specific quantiles (eg Engle and Manganelli (2004), Gerlach et al., (2011)), have characterised recent research. Ad hoc representations of skewed and fat-tailed properties have become particularly necessary in practice (Kuester et al. (2006)).

In the context of forecasting volatility, or Value-at-Risk (VaR) calculations, the increasing range of pragmatic approaches to this problem has inevitably led to hybrid and combining proposals in attempts to improve out-of-sample predictive performances (Taylor et al, 1998; Jeon et al, 2011). Evidently there is a need in practice to develop more accurate forecasts of tail risk rather than relying upon what the stylised theoretical models of the price formation processes generally provide.

Quantile regression, following Koenker and Basset (1978), has promised several attractive features in this respect. Firstly, it offers a semiparametric formulation of the predictive distribution, the quantiles of which can be efficiently estimated with distinct regressions. It is possible therefore to have a fine resolution of the tail characteristics in terms of empirical estimates for the required quantiles. Fundamental factors can be specified in the quantile regressions, which may exhibit different coefficients according to the quantile levels. This feature offers greater predictive insights and accuracy. Secondly, the conditional nature of the regressions is valuable for the explicit representations of varying dependencies in scenario tree creation (as may be required, for example, in stochastic optimisation models for risk management). Thirdly, the nonparametric way in which a tail distribution can be developed from separate quantile models offers an alternative perspective to multi-process modelling in the contexts where modellers may be tempted to use mixtures or regime-switching to capture different price formation process for normal and extreme events. Instead of an unobserved latent variable, such as in Markov regime switching (eg, Karakatsani et al., (2008)) and a restriction in the number of regimes, such as in smooth transition logistic regressions (eg Chen et al (2010)), quantile methods inherently associate a separate regime with each quantile.

One of the most critical areas for the use of well estimated tail probabilities is in VaR calculations, which are specified as quantiles, and have therefore motivated substantial research in finding effective quantile forecasting methods (eg Taylor (2008); Füss, (2010)). The methodological benchmark in theory appears to be the conditional autoregressive value at risk (CAViaR) approach inspired by applying GARCH ideas to quantiles, whereby the quantiles are modelled conditionally as autoregressive processes (see Engle and

Manganelli (2004)). These have proven to be very successful in predicting VaR in research studies, but (unlike GARCH models) may be difficult to apply in practice and appear to have acquired only limited implementation in financial risk management.

Spot prices in wholesale electricity markets present one of the most demanding application areas for comparative methodological research on this theme. Power prices are characterised by high volatility, positive skewness, substantial volatility clustering and large spikes. Furthermore, evidence for the substantial impact of exogenous fundamental drivers, and their nonlinear response functions, is well established (Chen et al (2010)). Forecasting tail probabilities and VaR, however, despite its widespread appeal and use in energy trading, remains under-researched and retains many open questions. Whilst the nonlinear properties of exogenous variables have motivated interest in regime switching models, their out-of sample performances have not matched their intuitive and highly significant in-sample fitting (eg Kosater et al (2006), Misorek et al., (2006)). Evidence on the value of regime-switching models for forecasting is mixed and seems to suggest that the evolutionary nature of power price formation requires more robust methods. It is an open question, therefore, whether quantile methods can be adapted to provide this capability.

Prediction of VaR out-of-sample for energy commodities can be found in Aloui (2008), Chan et al (2006), Füss et al. (2010), Giot et al (2003), and Hung et al. (2008). The majority of these studies use various GARCH models with different specifications of the innovation processes, the main conclusions being that GARCH models need to have fat tailed and possibly skewed distributions to work well. Quantile regression approaches do not yet appear to have been applied to energy commodity prices. Furthermore, it appears to follow both from the evident GARCH properties in power prices suggesting a persistence of shocks, and the in-sample regime-switching models between electricity fundamentals suggesting the need to accommodate varying effects of exogenous factors, that there is a prima facie case for examining the joint specification of both fundamental factors and volatility as explanatory factors. As Xiao and Koenker (2009) observe, this presents a complicated nonlinear quantile formulation which is elusive to conventional estimation. We adopt a two stage approach of first estimating the GARCH process with a factor model for the price levels, and then augmenting a multifactor model for quantiles with this GARCH process.

This approach is tested against a range of best-practice alternatives under the most stringent condition of the evening peak (6:30-7:00pm), which is the most volatile, skewed and spiky trading period in the British power market. The British market is a transparent one with useful exogenous day-ahead information available to market participants, and, being well-established, there is a long time series provide an extensive out-of-sample analysis of day ahead forecasts. With this methodological proposal, it is important to benchmark this quantile regression approach against the most appropriate GARCH models, typical in practice, and also the potential use of CAViaR models in this context. We provide convincing results that the quantile factor model with volatility provides better validated and more accurate forecasts than conventional and theoretical alternatives.

In the next section we provide a short review of the price formation properties in wholesale electricity, motivate the use of the exogenous variables and indicate why they may be nonlinear. Then, we describe the data, followed by the methods. In-sample and out-of-sample results are then reported and a concluding section summarises the key research contributions.

2. ELECTRICITY PRICE FORMATION FUNDAMENTALS

Electricity is a flow, rather than a stock commodity: it is produced and consumed continuously and instantaneously. Traded physical products are therefore defined and sold in the form of metered contracts for the constant delivery of a specified amount of power over a specified period of time, eg one megawatt for one hour (MWh). Most "spot" markets deal in such hourly products, although some, eg Britain and Australia, have finer granularity at half-hourly intervals. These hourly (or half-hourly) spot prices emerge either from an auction process whereby generators and retailers make offers and bids (which may be held once on the previous day to set all hourly prices for the subsequent day) or continuous trading on an exchange platform from a day ahead until a particular time before actual delivery (eg an hour in Britain). Spot price formation itself, because consumers are price inelastic in the short term and cannot store electricity once generated, is mainly a function of the demand, technology and competition amongst generators. For a particular level of demand at a particular time, there will be a stack of generating technologies available, and the market-clearing price, is usually taken as the offer price of the most expensive plant needed over that trading period. Thus, if the market were competitive, and generators offered at short-run marginal costs, market price volatility would be envisaged as the projection of demand volatility on to the supply function offered by the generators (Stoft, 2002). Given the various plant technologies available for dispatch, differentiated in terms of costs and operational constraints, this short-run supply cost function is intrinsically steeply increasing, discontinuous and convex. Hence there is an induced skewness from demand volatility into price volatility.

In the presence of these characteristics, and with the negligible demand elasticity in the short term, spot prices are sensitive to real-time uncertainties, such as demand shocks and plant outages. Thus, expectations of spot prices involve at least considerations of the underlying fuel (for short term marginal costs) and the reserve margin (for scarcity pricing above marginal cost). Moreover, as almost all electricity markets are oligopolies, at times of scarcity, when the reserve margin (available supply minus demand) is low, those generators with market power may offer and create market prices substantially above marginal costs (e.g. Wolfram (1999); Wolak and Patrick (1997)). This creates a further behavioural and possibly nonlinear element to extreme price formation. The nonlinear implications of this price formation process for the exogenous factors in a quantile regression model suggest the following functional form propositions:

1. **Demand elasticity is positive and increases nonlinearly with higher quantiles.** To the extent that the supply function is convex in available capacity, we would expect to see a positive price

coefficient to demand with higher elasticity at higher prices. Thus, one should find elasticity increases nonlinearly with higher quantiles.

2. **Reserve margin elasticity is negative and decreases nonlinearly with higher quantiles.** Scarcity induces a propensity for generators to offer at higher prices. Oligopolies characterize power markets and their market power increases with a declining reserve margin. Extreme scarcity can create a residual monopolist. Furthermore, reserve margin shocks may be due to unusual outages in generating capabilities and, once it is known to the market that a generator is short, this may induce a selling squeeze by the other generators and hence higher prices.

3. **Fuel (including carbon allowance prices) elasticities will be positive but may have nonlinear, non monotonic functional relations across quantiles.** Even though power prices in a predominantly thermal system will consist, at the market clearing price, of fuel prices including carbon costs (where levied, eg Bunn and Fezzi, 2007), plus the generator's profit margin, to the extent that changes in the fuel and carbon prices may cause a change in the merit order, a nonlinear response to fuel and carbon price changes could be expected, but this could also be non monotonic. For example if gas prices go down, gas generation will become baseload, increasing its low quantile coefficients, with gas no longer affecting the high quantile peakload prices. But if gas prices are higher than coal, and if scarcity in the carbon allowance market means that carbon prices are determined by fuel-switching from coal to gas, this may neutralize the impact of gas-coal price discrepancies. On the other hand, low carbon prices, that do not cause fuel switching, will have a decreasingly positive effect as price rise, if coal is cheaper than gas, and an increasingly positive effect if gas is cheaper than coal (since the carbon intensity of coal is higher). It is clear therefore that fuel prices will have an idiosyncratic pattern of influence on power prices that will not be linear, and indeed is likely to change over time according to movements in the coal, gas and carbon markets.

4. **Adaptive behavior (manifest as a lagged price) elasticity will be positive and nonlinear across quantiles.** Adaptive behavior will be manifest in terms of reinforcing previously successful offers.

High prices will tend to be followed with high prices. Furthermore, if there is an element of repeated gaming in power markets, as often suggested (Rothkopf, 1999), signaling between market agents will encourage this and motivate a positive coefficient for lagged prices. This will become stronger at higher quantiles, as the market becomes less competitive and gaming more possible and plausible.

3. DATA

The British electricity market, liberalised in the early 1990s, is currently competitive, and perhaps one of the most mature wholesale power markets in the world. In March 2001 the New Electricity Trading Arrangement (NETA) were implemented and introduced bilateral and voluntary forward trading in England and Wales to replace the compulsory, day-ahead uniform auction Pool that had existed since 1990. In April 2005 the British Electricity Trading and Transmission Arrangement (BETTA) extended this to include Scotland, whilst the EU carbon emissions market started at the beginning of that year. More specifically, the design of the reformed British market is based upon fully liberalised trading and plant self-scheduling, and hence, in this voluntary bilateral environment, most energy is traded with forward contracts. Close to physical delivery, agents fine tune their positions, from blocks (peak and baseload) to half-hourly resolutions, in the Power Exchanges that have emerged. These operate continuously up to 1 hour prior to each half-hourly physical delivery period, a point defined as Gate Closure, and are effectively the spot markets. After Gate Closure, the System Operator administers a market for system balancing, and invites offers and bids for load increases or decreases in real time.

In the British market, the main reference for spot trading has been the UKPX (now APX) power exchange. The spot prices are volume-weighted averages of all trades ahead of each trading period. Each day consist of 48 trading and load periods. The UKPX started its operation in March 2001. Compared to OTC contracting, UKPX is an anonymous exchange market place for trading and clearing. There are no locational prices in the British market and congestion, although dealt with in the real time system balanced by the system operator, does not contaminate the UKPX energy prices.

Our data spans 8th June 2005 to 4th September 2010. We selected the half-hourly period 38 (7pm) for this study as the most challenging time series of prices for comparative analysis. Of all periods, this exhibits highest volatility and greatest skewness. It is the evening peak when household activities and the onset of darkness create the highest prices and, if supply shortages happen, so do spikes. Figure 1 shows the development of the UKPX period 38 prices together with the day ahead spot prices of gas, coal, and carbon, as well as the day ahead demand and reserve margin forecasts as conveyed to the market by the system operator.

The dependent variable used in the analysis is:

> **UKPX Period 38 Price.** UKPX is the day-ahead and on-the-day power exchange, allowing high frequency (half hourly) trading, up to an hour before real time. Period 38 (18:30 to 19:00) represents "super" peak demand. Prices are quoted in £/MWh and represent the volume weighted prices for this period as cleared on the exchange in the preceding 24 hours.

The factors used in the analysis are all known to the market before the power exchange closes for the trading period concerned, and can therefore be considered exogenous market information for the power price formation:

> **Lagged UKPX Period 38 prices**. These are the UKPX Period 38 prices lagged by one day.
>
> **NBP Gas Price**. We use daily UK natural gas one-day forward price, from the main National Balancing Point (NBP) hub. The price is quoted in £/BTU (British Thermal Unit).
>
> **Coal price**. We use the daily steam coal Europe-ARA (Amsterdam, Rotterdam, and Antwerp) index, taking into account the $/£ rate.

Carbon emission price. We use the EEX-EU daily carbon emission allowance one year forward price taking into account the €/£ rate.

Demand forecast. This forecast is made available by the System Operator for each half-hourly trading period. In our study we use the period 38 demand forecast for the next day. Since it is released to the market at 18:17, one operational day ahead of closing prices, it reflects information available to participants and avoids the endogeneity issue of using actual demand.

Reserve margin forecast. The System Operator also makes forecasts of the available reserve margin for each half-hourly trading period. This is defined as the difference between the sum of the maximum available output capacities, as initially nominated by each generator prior to each trading period, and the demand forecast described above. In our study we use the period 38 reserve margin forecast, released at 18:16 for the next day.

Figure 1 (upper left exhibit) displays the period 38 price profile reflecting the "super peak" period with distinct demand, costs and operating constraints. The figure reveals typical spot electricity price features of spikes, mean reversion, seasonality, high (and time varying volatility). In Table 1 we present descriptive statistics that confirm these characteristics with a high price standard deviation, substantial skewness and kurtosis, rejection of normality according to the Jarque-Bera test, rejection of unit roots according to the ADF test and clear signs of positive serial correlation at different lags according to p and Q tests. We also show the empirical quantiles at 1%, 5%, 10%, 90%, 95%, and 99% levels which reveal high price risk in particular for consumers or traders having a short position in the period 38.

4. PRICE DISTRIBUTION MODELLING AND FORECASTING

In this Section we briefly describe the theoretical frameworks.

4.1 QUANTILE REGRESSION MODELS

Quantile regression methods develop explicit models for specific quantiles of the distribution of a dependent variable, using exogenous variables with different coefficients at each quantile. Quantile regression was introduced by Koenker and Bassett (1978) and is fully described in Koenker (2005) and Hao and Naiman (2007). Applications in financial risk management can be found in Alexander (2008).

Let $q \in [0,1]$ be the quantile, e.g. 1%, 5%,…., 99%. Let Y_t be the dependent variable (e.g. log of el. price) and $\mathbf{X}_t$ a d-dimensional vector of explanatory variables (e.g. log of the gas price, log of demand etc.), including a constant. The linear quantile regression model is given by

$$Y_t = \mathbf{X}_t \boldsymbol{\beta} + \varepsilon_t, \tag{4.1}$$

where the distribution of the error ε term is left unspecified. The only assumption made is that the conditional quantile function is given by

$$Q_q\left(Y_t \mid \mathbf{X}_t\right) = \mathbf{X}_t \boldsymbol{\beta}_q \tag{4.2}$$

and thus, $Q_q\left(\varepsilon_t \mid \mathbf{X}_t\right) = 0$. In standard quantile regression, as implemented in several software packages (eg Eviews, R, Stata), we find the parameter $\boldsymbol{\beta}_q$ according to the following optimization problem:

$$\arg\min_{\boldsymbol{\beta}_q} \sum_{t=1}^{T} (q - \mathbf{1}_{Y_t \le \mathbf{X}_t \boldsymbol{\beta}_q})(Y_t - \mathbf{X}_t \boldsymbol{\beta}_q)$$

where

$$\mathbf{1}_{Y_t \le \mathbf{X}_t \boldsymbol{\beta}_q} = \begin{cases} 1 & \text{if } Y_t \le \mathbf{X}_t \boldsymbol{\beta}_q \\ 0 & \text{otherwise} \end{cases}.$$

Details on standard errors for coefficients, inference and goodness of fit can be found in Koenker and Machado (1999). The linear model (4.2) represents the quantile as a linear function of only a few explanatory variables. It may be useful to have a model that incorporates the whole past information set

$\tilde{\mathbf{X}}_t$. Let $\dot{\mathbf{X}}_t$ be a subset of information variables that become available at time *t* and $\tilde{\mathbf{X}}_{t-1}$ be variables $\left\{\dot{\mathbf{X}}_j\right\}_{j=0}^{t-1}$, so that $\tilde{\mathbf{X}}_t = \left(\dot{\mathbf{X}}_t, \tilde{\mathbf{X}}_{t-1}\right)$. Chernozhukov and Umantsev (2001) formulate the following general quantile regression model

$$Q_q\left(Y_t\right) = \dot{\mathbf{X}}_t \boldsymbol{\beta}_q + \gamma_q f_1\left(\tilde{\mathbf{X}}_t, q\right) \tag{4.3}$$

$$f_1\left(\tilde{\mathbf{X}}_t, q\right) = \theta_q f_1\left(\tilde{\mathbf{X}}_{t-1}, q\right) + f_2\left(\dot{\mathbf{X}}_t, \varphi_q\right).$$

Here $f_1\left(\cdot\right)$ and $f_2\left(\cdot\right)$ represents functions of the information set. Models of this form are useful as parsimonious regressions that represent VaR/quantiles as a function of all past price information (Chernozhukov and Umantsev (2001)). One example involves the model

$$f_1\left(\tilde{X}_t, q\right) = \sigma\left(Y_t - \mu_t \mid \tilde{X}_t\right),$$

where $\sigma^2\left(Y_t - \mu_t \mid \tilde{X}_t\right)$ is the conditional variance of the de-meaned Y_t. It can take the form of an ARCH model, see Koenker and Zhao (1996). As suggested by Chernozhukov and Umantsev (2001), a simpler strategy is to first estimate $\sigma^2\left(Y_t - \mu_t \mid \tilde{X}_t\right)$ via a GARCH model and use it as a regressor in the linear model. A similar approach is also done in Xiao and Koenker (2009). They first use quantile regression to estimate the volatility, and then include lagged volatility and the lagged dependent variable as explanatory variables in the linear quantile regression model.

The CAViaR class of models also falls into this framework. Let ε_t be the de-meaned process, $\varepsilon_t = Y_t - \mu_t$. Engle and Manganelli (2004) suggest four different CAViaR specifications, all of which are first-order autoregressive. The Indirect GARCH(1,1) CAViaR model,

$$Q_q\left(\varepsilon_t\right) = \left(1 - 2I\left(q < 0.5\right)\right)\left(\alpha_1 + \alpha_2 Q_q\left(\varepsilon_{t-1}\right)^2 + \alpha_3 \varepsilon_{t-1}^2\right)^{0.5}. \tag{4.4}$$

The Symmetric Absolute Value CAViaR model,

$$Q_q(\varepsilon_t) = \alpha_1 + \alpha_2 Q_q(\varepsilon_{t-1}) + \alpha_3 |\varepsilon_{t-1}|. \tag{4.5}$$

The Asymmetric Slope CAViaR model,

$$Q_q(\varepsilon_t) = \alpha_1 + \alpha_2 Q_q(\varepsilon_{t-1}) + \alpha_3 |\varepsilon_{t-1}| I(q \geq 0.5) + \alpha_4 |\varepsilon_{t-1}| I(q < 0.5). \tag{4.6}$$

The Adaptive CAViaR model,

$$Q_q(\varepsilon_t) = Q_q(\varepsilon_{t-1}) + \alpha_1 \left[q - \left[1 + \exp\left(K\left(\varepsilon_{t-1} - Q_q(\varepsilon_{t-1}) \right) \right) \right]^{-1} \right]. \tag{4.7}$$

Here $Q_q(\varepsilon_t)$ is the *q*-percentile of the de-meaned price distribution at time *t*, *I* is a indicator function and *K* is a smoothing parameter which may be chosen or estimated. Quantile regression models are robust to distributional misspecifications, as no explicit distributional assumptions are made. Further, CAViaR models can be used when either/both error densities and volatilities are changing (Engle et al, 2004).

4.2 FULLY PARAMETRIC LOCATION-SCALE MODELS

Fully parametric models are among the most commonly used market risk measures, and are typically based on the assumption that the price distribution can be described by a parametric density function together with a model for the conditional variance. Popular specifications of the conditional density is Gaussian or skew Student-t, and a GARCH model for the conditional variance. As a methodological benchmark for the quantile regression models, we use location-scale models where the volatility dynamics follows a GARCH(1,1) structure. That is, we assume that the distribution of the dependent variable can be expressed in the form

$$Y_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t$$

$$z_t \overset{iid}{\sim} f_Z(\cdot) \tag{4.8}$$

Here μ_t is the mean of Y_t, $\sigma_t = \sqrt{\alpha_0 + \alpha_1 \sigma_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2}$, with f_Z is a zero-location, unit-scale probability density that can have additional shape parameters. The one-step forecast of the *q*-percentile of Y_t based on information up to time *t* is given by

$$Q_q\left(Y_{t+1}\right) = \widehat{\mu}_{t+1} + \widehat{\sigma}_{t+1} Q_q\left(z\right)$$

where $Q_q\left(z\right)$ is the *q*-percentile implied by f_Z. Approaches differs with respect to the specification of the conditional mean, μ_t, and conditional density f_Z. Note that the Indirect GARCH(1,1) CAViaR model (4.4) corresponds to a model on the form given by (4.8), but with zero location parameter, $\mu_t \equiv 0$, and an iid error distribution.

4.3 FORECASTING

To validate the predictive performance of the models, we consider the unconditional test of Kupiec (1995), the conditional coverage test of Christoffersen (1998) and two backtests based on regression. The Kupiec (1995) test is a likelihood ratio test designed to reveal whether the model provides the correct unconditional coverage. More precisely, let $\left\{H_t\right\}_{t=1}^{T}$ be an indicator sequence where H_t takes the value 1 if the observed price, Y_t, is below the predicted quantile, Q_t ,at time *t*

$$H_t = \begin{cases} 1 & \text{if } Y_t < Q_t \\ 0 & \text{if } \mathrm{Y}_t \geq Q_t \end{cases}. \tag{4.9}$$

Under the null hypothesis of correct unconditional coverage the test statistic is

$$LR_{uc} = -2\log\left[\frac{\left(1-\pi_{\exp}\right)^{n_0}\pi_{\exp}^{n_1}}{\left(1-\pi_{obs}\right)^{n_0}\pi_{obs}^{n_1}}\right] \overset{asy}{\sim} \chi_1^2$$

where n_1 and n_0 is the number of violations and non-violations respectively, $\pi_{\exp}$ is the expected proportion of exceedances and $\pi_{obs} = n_1 / (n_0 + n_1)$ is the observed proportion of exceedances. In the Kupiec (1995) test only the total number of ones in the indicator sequence $\{H_t\}_{t=1}^{T}$ counts, and the test does not take into account whether several quantile exceedances occur in rapid succession, or whether they tend to be isolated. Christoffersen (1998) provides a joint test for correct coverage and for detecting whether a quantile violating today has influence on the probability of a violating tomorrow. The test statistic is defined as follows:

$$LR_{cc} = -2\log\left[\frac{\pi_{\exp}^{n_1}(1-\pi_{\exp})^{n_0}}{(1-\pi_{01})^{n_{00}}\pi_{01}^{n_{10}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}}\right] \overset{asy}{\sim} \chi_2^2$$

where n_{ij} represents the number of times an observations with value *i* is followed by an observation with value *j*. $\pi_{01} = n_{01} / (n_{00} + n_{01})$ and $\pi_{11} = n_{11} / (n_{11} + n_{10})$. The LR_{cc} test is only sensitive to one violation immediately followed by another, ignoring all other patterns of clustering.

The Kupiec (1995) and Christoffersen (1998) tests only use information on past quantile violations, and therefore might not have sufficient power to detect misspecified risk models. To increase the power we may also want to consider whether violations can be predicted by including other data in the information set such as past returns, estimated volatility or the quantile estimate for the period itself. The advantage of increasing the information set is not only to increase power, but also to help us understand the areas in which the risk model is misspecified (Christoffersen (2010)). We consider the following two regression based backtests;

$$H_t = \beta_0 + \beta_1 H_{t-1} + \beta_2 H_{t-2} + \beta_3 H_{t-3} + \beta_4 H_{t-4} + \varepsilon_t \tag{4.10}$$

$$H_t = \beta_0 + \beta_1 H_{t-1} + \beta_2 H_{t-2} + \beta_3 H_{t-3} + \beta_4 H_{t-4} + \beta_5 Q_t + \varepsilon_t \,. \tag{4.11}$$

Here the indicator variable H_t is defined in Equation (4.9), and Q_t denotes the quantile estimate itself. In Equation (4.10) we test the hypotheses $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ against the alternative that at least one of $\beta_1, \beta_2, \beta_3, \beta_4$ is significant different from zero using a standard *F* test. In Equation (4.11) we test $H_0 : \beta_5 = 0$ against the alternative that quantile exceedances are linked to quantile forecast using a simple *t* test.

5. EMPIRICAL ANALYSIS

First we conduct an empirical analysis of the price sensitivities with respect to the factors described in Section 3. Then we evaluate the ability of different methods to forecast the tails of the price distribution.

5.1 IN SAMPLE MODELLING USING QUANTILE REGRESSON

We perform in-sample analysis using all data from 9th June 2005 to 4th September 2010 which consist of 1915 observations for the period 38 UK electricity prices. The elasticities of lagged prices, lagged gas/coal/carbon prices, forecast of demand and reserve margin are investigated at the quantiles 1%, 5%, 10%, 25%, 50%, 75%, 90%, 95%, and 99%.

In order to variance-stabilise the data and interpret all parameters as elasticities, we log-transform both the dependent and independent variables, as in equation (5.1).

$$\begin{aligned} Q_q\left(\ln\mathrm{P38}_t\right) = \beta_0^q + \beta_1^q \ln\mathrm{P38}_{t-1} + \beta_2^q \ln\mathrm{Gas}_{t-1} + \beta_3^q \ln\mathrm{Coal}_{t-1} + \beta_4^q \ln\mathrm{Carbon}_{t-1} + \\ \beta_5^q \ln\mathrm{Demand}_t + \beta_6^q \ln\mathrm{Reserve}_t + \beta_7^q \sigma\left(\ln\mathrm{P38}_t - \mu_t\right) \end{aligned} \tag{5.1}$$

Here $\sigma^2\left(\ln\mathrm{P38}_t - \mu_t\right)$ is the conditional variance of the de-meaned log prices, and is estimated using *Model 4* described in Section 5.2.1 below. The index *q* refers to the specific quantile. All calculations are performed in EViews (The QREG procedure).

In table 2 we show the parameter values at different quantiles for model (5.1). The explanatory power measured by Koenker and Machado (1999) pseudo R-squared is in the range of 49% to 59%.

Lagged prices. The significance and sign of the lagged electricity price is consistent with adaptive behaviour and mean-reversion. We generally (but not completely) found an increasing adaptive impact for higher prices, as proposed in section 2.

Gas prices. The gas price elasticities are generally positive in line with the previous discussion of supply function fundamentals. Using logtransform prices, the coefficients are in the range of 0.19-0.31 and all significant. There is no clear pattern in the elasticity values across the different quantiles. This is consistent with our previous proposition about the potentially non monotonic effects of fuel prices.

Coal prices. The coal price elasticities are generally positive and also consistent with the previous discussion of supply function fundamentals. The values are generally higher than gas. Low and moderate quantiles are more sensitive than high quantiles, which may reflect the fact that low prices are more likely to be driven by fuel fundamentals than high prices, where scarcity may be more determinate.

Carbon emission prices. The carbon emission price elasticities are generally positive. Their magnitudes are rather low (in the 0.01 to 0.07) range, decreasing with price levels. This is consistent with the declining impact of coal prices at higher quantiles as noted above.

Volatility. The coefficient of volatility changes sign from negative (low prices) to positive (high prices). During times of low prices, an increase in volatility tends to drive prices even lower than we can explain using fundamental factors alone. When prices are high, an increase in volatility tends to drive prices even higher. This suggests that both low and high electricity prices overshoot the fundamentals when the price uncertainty is high, which is a remarkable but plausible observation.

Demand forecast. The generally positive and increasing effects with higher prices reflect the intuitive impact of the increasing supply function discussed earlier.

Reserve margin forecast. The negative signs for margin intuitively reflect the scarcity effect that lower reserve margins produce higher prices. The effect is also increasing with higher prices as propsed in section 2. All parameters are significant.

It should be noted that other exogenous variables (such as seasonal dummies and trading volume) where tested initially but were significant. Seasonal effects are to a large extent captured in forecasted demand.

5.2 FORECASTING TAIL PROBABILITIES

VaR modelling requires accuracy in the forecasting of the tails of the price density rather than in the central body of the price distribution.

5.2.1 FULLY PARAMETRIC LOCATION SCALE-MODELS

From the fully parametric location-scale models we use two different conditional densities; Gaussian and skew Student-t. In addition, we consider two different specifications of the conditional mean; (1) μ_t is a linear function of the lagged prices, and (2) μ_t is a linear function of the lagged price, $\ln P_{t-1}$, and the factors described in Section 3.

Model 1-2:

$$\begin{aligned}\mathrm{lnP38}_t = \beta_0 + \beta_1 \mathrm{lnP38}_{t-1} + \beta_2 \mathrm{lnP38}_{t-2} + \beta_3 \mathrm{lnP38}_{t-3} + \beta_4 \mathrm{lnP38}_{t-4} + \\ \beta_5 \mathrm{lnP38}_{t-5} + \beta_6 \mathrm{lnP38}_{t-6} + \beta_7 \mathrm{lnP38}_{t-7} + \sigma_t z_{\mathrm{t}}\end{aligned}$$

Here the volatility follows a GARCH(1,1) process, $\sigma_t = \sqrt{\alpha_0 + \alpha_1 \sigma_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2}$, and the conditional density $f_Z(\cdot)$ is set to Gaussian (Model 1) and skew Student-t (Model 2), respectively.

Model 3-4:

$$\ln \text{P38}_t = \beta_0 + \beta_1 \ln\text{P38}_{t-1} + \beta_2 \ln\text{Gas}_{t-1} + \beta_3 \ln\text{Coal}_{t-1} + \beta_4 \ln\text{Carbon}_{t-1} + \beta_5 \ln\text{Demand}_t + \beta_6 \ln\text{Reserve}_t + \sigma_t z_t$$

As above, $\sigma_t = \sqrt{\alpha_0 + \alpha_1 \sigma_t^2 + \alpha_2 \varepsilon_{t-1}^2}$, and $f_Z(\cdot)$ is Gaussian (Model 3) and skew Student-t (Model 4), respectively . The estimation of models 1-4 is done in two steps. First the β -parameters are estimated using an ordinary least square (OLS) regression where log prices is regressed against explanatory variables. Then a GARCH(1,1) model is fitted to the residuals from the regression.

5.2.2 QUANTILE REGRESSION MODELS

From the quantile regression class of models we use the following functional forms:

Model 5: A linear model with only lagged prices as explanatory variables

$$Q_q\left(\ln\text{P38}_t\right) = \beta_0^q + \beta_1^q \ln\text{P38}_{t-1} + \beta_2^q \ln\text{P38}_{t-2} + \beta_3^q \ln\text{P38}_{t-3} + \beta_4^q \ln\text{P38}_{t-4} + \beta_5^q \ln\text{P38}_{t-5} + \beta_6^q \ln\text{P38}_{t-6} + \beta_7^q \ln\text{P38}_{t-7}$$

Model 6: A linear model with lagged price and fundamental explanatory variables

$$Q_q\left(\ln\text{P38}_t\right) = \beta_0^q + \beta_1^q \ln\text{P38}_{t-1} + \beta_2^q \ln\text{Gas}_{t-1} + \beta_3^q \ln\text{Coal}_{t-1} + \beta_4^q \ln\text{Carbon}_{t-1} + \beta_5^q \ln\text{Demand}_t + \beta_6^q \ln\text{Reserve}_t$$

Model 7: A linear model with lagged price, fundamental variables and volatility as explanatory variables

$$Q_q\left(\ln\text{P38}_t\right) = \beta_0^q + \beta_1^q \ln\text{P38}_{t-1} + \beta_2^q \ln\text{Gas}_{t-1} + \beta_3^q \ln\text{Coal}_{t-1} + \beta_4^q \ln\text{Carbon}_{t-1} + \beta_5^q \ln\text{Demand}_t + \beta_6^q \ln\text{Reserve}_t + \beta_7^q \sigma\left(\ln\text{P38}_t - \mu_t\right)$$

Here $\sigma^2\left(\ln\text{P38}_t - \mu_t\right)$ is the conditional variance of the de-meaned log prices, and is estimated using *Model 4* described above. That is, the estimation is performed in two steps; first a GARCH models is used to estimate the volatility, next, treating volatility as an observed variable, the linear quantile regression is estimated.

Model 8-11:

$$Q_q\left(\ln \mathrm{P38}_t\right) = \beta_0 + \beta_1 \ln \mathrm{P38}_{t-1} + \beta_2 \ln \mathrm{Gas}_{t-1} + \beta_3 \ln \mathrm{Coal}_{t-1} + \beta_4 \ln \mathrm{Carbon}_{t-1} + \beta_5 \ln \mathrm{Demand}_t + \beta_6 \ln \mathrm{Reserve}_t + Q_q\left(\varepsilon_t\right)$$

Here the error terms, ε_t , follows a CAViaR prosess. More precisely, in *Model 8* an indirect GARCH(1,1) CAViaR model is chosen for ε_t, in *Model 9* a symmetric absolute value CAViaR model, in *Model 10* a asymmetric slope CAViaR and finally *Model 11* the adaptive CAViaR model is chosen for the error term (for the adaptive model, we follow Engle and Manganelli (2004) and set K = 10).

Model 7-10 is estimated in two steps: First the β 's are estimated using OLS regression, next a CAViaR model is fitted to the residuals from the regression. Estimation of the CAViaR models is complicated by the fact that the quantiles are latent and are dependent on the unknown parameters. We use the Matlab code of Manganelli (2002) to estimate the CAViaR models. An alternative estimating strategy is Markov Chain Monte Carlo, as in Gerlach et al. (2011).

5.2.3 EMPIRICAL RESULTS

It is important to validate and compare the models in an out-of-sample forecasting context, as it is well-known that elaborate, well-specified ex-post models may not forecast better than simpler, more robust models, especially when the process being modelled is subject to continuous and abrupt structural changes. In contrast, the usefulness of well specified ex-post models is more often argued for their value in ex-post market performance analysis, e.g., market monitoring, and ex ante, for facilitating multiple scenario simulations, e.g., for risk management. The ultimate aim is of course to have a well specified model both in-sample and out-of-sample. We use two approaches for in-sample and out-of-sample:

Expanding window (EW) in sample. Here we estimate the models using the first 730 observations. We then forecast quantiles (1%, 5%, 10%, 90%, 95%, and 99%) of observation 731. Thereafter we estimate the models with the first 731 observations. We then forecast quantiles of observation 732 and so on. At the end, we estimate models with the first 1914 observations and forecast quantiles of the last observation 1915. That will leave us with 1915-730 = 1185 observations to verify tail forecasting performance.

Rolling window (RW) in sample. We start out estimating the models using the first 730 observations. We then forecast quantiles (1%, 5%, 10%, 90%, 95%, and 99%) of observation 731. Thereafter estimate the models using observation 2 to 731. We then forecast quantiles of observation 732 and so on. At the end, we estimate the models using observation 1184 to 1914 and forecast quantiles of the last observation 1915. Again, this will leave us with 1915-730 = 1185 observations to verify tail forecasting performance.

Tables 3-5 report the percentage of times the observed price is below the estimated quantile, the *p*-values for the unconditional coverage test by Kupiec (1995), the *p*-values for the conditional coverage test by Christoffersen (1998), and the *p*-values from the two different regression based tests defined by (4.10) and (4.11). We also record the average values of the quantile estimates. A good risk model method should not only pass the calibration tests described above, it should also provide narrow prediction intervals, as the width of the intervals is linked to the precision of using the method in practice. The overall results from the tests suggest that the parametric location-scale models based on Gaussian distribution are seriously flawed, failing about half of the tests. Using a skew Student-t distribution leads to clear improvements, and the models using this distribution estimated under a rolling window are generally the the best performing models. The linear quantile regression models including only lagged prices (Model 5) do not provide satisfactory forecasts of the quantiles, showing performance on par with the Gaussian based parametric location-scale model. Introducing fundamental factors in the linear quantile regression model (Model 6) significantly improves the results, underpinning the importance of these risk factors in predicting the tail probabilities of the electricity spot price. Adding volatility as an explanatory variable (Model 7) improves the tail predictions further, indicating that volatility is not adequately encapsulated through the factors in the fundamental model. The CAViaR models shows relatively good results, with the indirect GARCH (Model 8) and Asymmetric slope (Model 10) providing the best forecasts, followed by the Symmetric absolute value (Model 9) and Adaptive CAViaR (Model 11).

Going into more details for each of the tests, we find, using a 5% significance level and an expanding window, that only the linear quantile regression model with fundamental factors and volatility as explanatory variables (Model 7) provide the correct percentage of violations. Fitting the models to a rolling window generally improves the unconditional coverage, indicating that the data generating process may change over time. Using a rolling window, Model 4, 6, 7, 8 and 11 all provide correct unconditional coverage. Examining the clustering of exceedances of the quantiles, we find that the location scale GARCH models all perform very well on the regression based test defined by (4.10). The joint test of unconditional coverage and independence of Christoffersen (1998) shows less encouraging results, but this is generally caused by incorrect percentage of violations. The same conclusion can be drawn for the CAViaR models except the Adaptive model, which shows more clustering than the other CAViaR models. This is not surprising, since the Adaptive model increases the quantile by the same amount regardless of whether the size of the residual term exceeded the quantile by a small or a large margin. The linear quantile regression models, especially the model including only lagged prices, show in general somewhat more clustering than the other methods. The reason may be the same as for the Adaptive model; the quantile does not depend directly on the last residual term. We also test whether the exceedances of the quantiles are independent of the conditional quantile estimator. The results suggests that both the location scale GARCH and CAViaR models suffer from exceedances being correlated with the forecast of the quantile itself, especially in the lower quantiles (1%, 5% and 10%). The results from the linear quantile regression models are more promising, particularly for the models including fundamental information.

Turning to the width of the predictions intervals, we find that the linear fundamental quantile regression models generally give the narrowest prediction intervals. That is, on average they provide relatively high estimates of the low quantiles (1%, 5% and 10%) and relatively low estimates of the high quantiles (90%, 95% and 99%). Since the precision and value of implementing the VaR method is linked to width of the intervals, our results suggest that the linear fundamental quantile regression models generally outperform the other models when it comes to the value of using the methods in practice.

The motivation for comparing an expanding and rolling estimation is to provide some evidence whether the data generating process changes over time. The electricity price in 2010 may have different dynamics compared to the dynamics in 2005. Using both an expanding and rolling window gives some indication if the methods we are testing are sensitive to the sample size chosen. The quantile forecasts from the parametric location-scale models show generally a slight improvement using a RW compared to using an EW. For the linear quantile regression the situation appears to be the opposite, with a tendency to more clustering of exceedances using a RW. All CAViaR models except the Adaptive model seem to perform slightly better using an EW. The reason may be due to data scarcity in estimating the extreme tails of the distribution. Chernozhukov and Umantsev (2001) observe that data scarcity problems are amplified by the presence of covariates, and that point estimates provided by regression quantiles can be baised in the tails.

7. CONCLUSIONS

We have characterised the nonlinear effects of exogenous factors on peak hour wholesale electricity price formation as well as forecasting the price distribution. Using a dynamic quantile regression model with fundamental factors and volatility as explanatory variables, we capture effects such as mean reversion, spikes, time varying volatility, and at the same time, estimate the rather complex relationships of this price to fundamentals. We demonstrate how lagged prices, prices of gas, coal and carbon, forecasts of demand and reserve margin in addition to price volatility influence the peak price distribution. In general we find positive elasticities of gas, coal and carbon prices with no distinct pattern over the quantiles. The elasticity of demand is positive with increased effect as prices gets higher. The elasticity of reserve margin is negative with increased effect as prices gets higher. Volatility is mainly found to have an effect on extreme low and high prices. An increase in volatility drives low prices lower and high prices higher, suggesting that high price uncertainty combined low/high electricity prices leads prices to overshoot the fundamentals.

We have also shown that the linear quantile regression models, taking into account the nonlinear effects of exogenous factors, outperform the state-of-the-art CAViaR and GARCH models in out-of-sample forecasting of the price distribution quantiles. For example, using an expanding window and a stringent significance level of 1%, the multifactor with volatility quantile regression models are the only ones passing all the validation tests and providing the most accurate forecasts. They are also easier to implement than some of the other state-of-the-art benchmark methods

In summary, we have analysed a practical and validated multifactor quantile approach for predicting the electricity price distribution where market participants (with long/short positions) are able to analyse how various risk factors affect low/high prices. This fundamental model can also be used for accurate day-ahead nonparametric density estimation (via multiple quantiles) of the spot price distributions. This is valuable for producers, retailers, and speculators in determining optimal strategies for short-term operations, hedging and trading.

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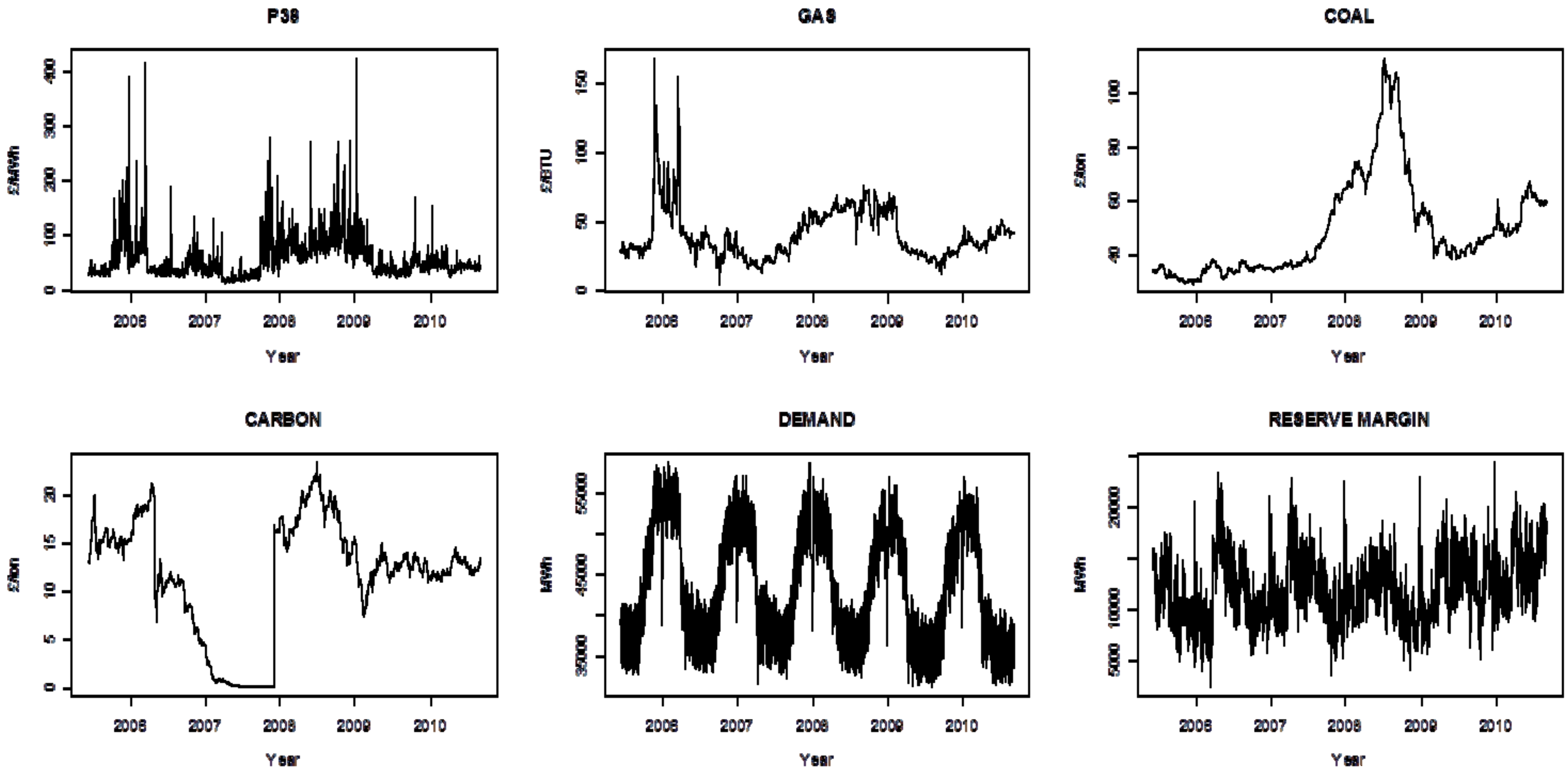

Figure 1. Price of UKPX period 38 (18:30-19:00) in £/MWh, UK day ahead forward gas price (£/BTU) from the National Balancing Point, Daily Steam Coal Europe-ARA index (translated into £/ton), EEX-EU Carbon emission price daily spot price (translated into £/ton), The UK national demand forecast for period 38 from the system operator (MWh), the UK national forecast of reserve margin for period 38 from the system operator (MWh). The data spans from 8th June 2005 to 4th September 2010 (1915 observations altogether).

Statistics	Mean	Med	Min	Max	Std	Skew	Kurt	JB	ADF	ρ_1	ρ_{10}	Q(10)	Quantiles	1 %	5 %	10 %	90 %	95 %	99 %
P_t	58.79	46.93	13.22	421.72	37.54	2.92	18.92	23040	-6.93	0.71	0.52	3268	P_t	18.18	24.12	28.89	98.38	130.07	194.06
$\ln P_t$	3.93	3.85	2.85	6.04	0.52	0.49	3.19	79	-4.55	0.84	0.72	5586	$\ln P_t$	2.90	3.18	3.36	4.59	4.87	5.27

Table 1. UKPX period 38 prices. The table shows the mean, median, min, max, standard deviation, skewness, excess kurtosis, Jarque-Bera, Augmented Dickey Fuller with constant and control lags according to the SIC criteria, autocorrelation at lag 1 and 10 and Ljung-Box statistics with 10 lags. We also show the empirical 1%,5%,10%,90%,95%, and 99% quantiles. Critical values at 1% level for JB is 9.21, for ADF-test -3.43, and for LB(10) 23.21.

Quantile	lag P38	Gas	Coal	Carbon	Demand	Reserve Margin	Volatility	R^2-adjusted
1 %	0.22***	0.31***	0.31***	0.07***	0.08	-0.37***	-0.22***	49.0
5 %	0.28***	0.27***	0.33***	0.06***	0.23***	-0.28***	-0.11***	53.6
10 %	0.31***	0.27***	0.31***	0.05***	0.24***	-0.27***	-0.02	55.2
25 %	0.38***	0.23***	0.30***	0.04***	0.30***	-0.27***	-0.02	57.5
50 %	0.47***	0.19***	0.27***	0.03***	0.25***	-0.35***	0.06**	58.9
75 %	0.55***	0.20***	0.22***	0.02***	0.26***	-0.46***	0.02	58.2
90 %	0.59***	0.20***	0.16***	0.01	0.29***	-0.54***	0.04	58.9
95 %	0.50***	0.20***	0.22***	0.02***	0.34**	-0.63***	0.21***	59.8
99 %	0.35**	0.26*	0.30	0.02	0.42	-0.86***	0.16	58.3

Table 2. Quantile regression results. The *, ** and *** indicates significance at the 10%, 5% or 1% level, respectively.

		Expanding Window						Rolling Window					
Model	Quantile	Violations	LR_{UC}	LR_{CC}	REG_{Hit}	REG_{Var}	$\overline{\text{Quantile}}$	Violations	LR_{UC}	LR_{CC}	REG_{Hit}	REG_{Var}	$\overline{\text{Quantile}}$
Gaussian,	1	0,00 %	**NA**	**0,000**	**NA**	0,056	3,422	0,00 %	**NA**	**0,000**	**NA**	0,054	3,432
lag prices	5	1,35 %	**0,000**	**0,000**	0,921	0,776	3,594	1,94 %	**0,000**	**0,000**	0,672	0,559	3,603
(Model 1)	10	5,06 %	**0,000**	**0,000**	0,469	**0,003**	3,686	5,91 %	**0,000**	**0,000**	0,158	**0,000**	3,695
	90	89,11 %	0,315	0,242	0,745	0,143	4,335	89,62 %	0,665	0,695	0,889	0,093	4,338
	95	93,16 %	**0,006**	**0,020**	**0,039**	**0,040**	4,427	92,74 %	**0,001**	**0,003**	0,238	0,091	4,429
	99	97,47 %	**0,000**	**0,000**	0,644	0,561	4,599	97,38 %	**0,000**	**0,000**	0,787	0,522	4,601
Skew	1	0,42 %	**0,024**	0,076	0,999	**0,048**	3,498	0,25 %	**0,002**	**0,009**	1,000	0,147	3,509
Student-t,	5	4,05 %	0,122	0,214	0,814	**0,045**	3,665	4,39 %	0,260	0,503	0,782	0,257	3,669
lag prices	10	9,03 %	0,258	0,432	0,433	**0,001**	3,737	9,45 %	0,526	0,540	0,613	**0,001**	3,740
(Model 2)	90	88,35 %	0,065	**0,031**	0,543	0,400	4,325	88,61 %	0,117	0,239	0,873	0,371	4,334
	95	93,67 %	**0,043**	0,080	0,065	0,148	4,457	94,01 %	0,128	0,130	0,178	**0,055**	4,464
	99	98,90 %	0,741	0,811	0,968	0,996	4,783	98,99 %	0,965	0,875	0,977	0,841	4,776
Gaussian,	1	0,51 %	0,059	0,163	0,998	**0,023**	3,461	0,68 %	0,232	0,461	0,994	**0,004**	3,471
fundamental	5	2,19 %	**0,000**	**0,000**	**0,012**	**0,000**	3,620	2,45 %	**0,000**	**0,000**	0,071	**0,000**	3,630
(Model 3)	10	6,41 %	**0,000**	**0,000**	0,976	**0,000**	3,705	6,33 %	**0,000**	**0,000**	0,647	**0,000**	3,714
	90	90,04 %	0,961	0,397	0,297	**0,023**	4,304	90,55 %	0,526	0,635	0,471	**0,003**	4,312
	95	94,43 %	0,377	0,593	0,918	0,268	4,389	94,43 %	0,377	0,316	0,216	0,322	4,396
	99	97,72 %	**0,000**	**0,001**	0,771	0,926	4,548	97,55 %	**0,000**	**0,000**	0,359	0,335	4,555
Skew	1	1,01 %	0,965	0,875	0,970	**0,000**	3,520	1,10 %	0,741	0,811	0,217	**0,001**	3,537
Student-t,	5	3,71 %	**0,034**	**0,018**	0,210	**0,000**	3,669	4,98 %	0,973	0,784	0,884	**0,000**	3,683
fundamental	10	8,44 %	0,066	**0,015**	0,970	**0,000**	3,739	10,30 %	0,736	0,072	0,185	**0,000**	3,751
(Model 4)	90	89,87 %	0,885	0,334	0,380	**0,011**	4,303	90,04 %	0,961	0,586	0,464	**0,026**	4,308
	95	95,36 %	0,567	0,754	0,870	0,506	4,415	95,44 %	0,478	0,705	0,353	0,371	4,422
	99	98,82 %	0,542	0,694	0,950	0,898	4,667	99,07 %	0,802	0,866	0,980	0,811	4,680

Table 3: Forecasting results out of sample for GARCH models with expanding window (left) and rolling window (right). Table shows the *p*-values of the backtests defined in Section 4.3. Reg_{Hit} and REG_{Var} are the regression based tests given by Eq. (4.10) and (4.11), respectively. Bold type *p*-values are significant at the 5% level. $\overline{\text{Quantile}}$ denotes the average value of the quantile estimates.

		Expanding Window						Rolling Window					
Model	Quantile	Violations	LR_{UC}	LR_{CC}	REG_{Hit}	REG_{Var}	$\overline{\text{Quantile}}$	Violations	LR_{UC}	LR_{CC}	REG_{Hit}	REG_{Var}	$\overline{\text{Quantile}}$
Linear quantile	1	0,68 %	0,232	0,461	**0,000**	**0,007**	3,393	1,01 %	0,965	0,875	**0,001**	**0,001**	3,563
regression,	5	2,87 %	**0,000**	**0,001**	**0,030**	**0,006**	3,598	3,54 %	**0,016**	**0,003**	**0,001**	0,059	3,707
lag prices	10	6,24 %	**0,000**	**0,000**	**0,001**	0,402	3,697	8,02 %	**0,019**	**0,000**	**0,000**	0,122	3,768
(Model 5)	90	91,90 %	**0,025**	**0,022**	0,354	0,308	4,357	91,81 %	**0,032**	0,069	0,169	0,911	4,301
	95	96,20 %	**0,048**	0,132	0,992	0,423	4,520	95,95 %	0,122	0,221	0,748	0,163	4,403
	99	98,90 %	0,741	0,297	**0,002**	**0,023**	4,872	98,99 %	0,965	0,875	0,112	**0,008**	4,697
Linear quantile	1	0,42 %	**0,024**	0,076	0,999	0,350	3,519	1,43 %	0,158	**0,026**	**0,001**	0,112	3,418
regression,	5	3,88 %	0,067	0,058	0,081	0,118	3,694	5,40 %	0,532	**0,021**	**0,000**	0,096	3,615
fundamental	10	8,10 %	**0,025**	**0,022**	0,057	**0,041**	3,755	10,97 %	0,272	**0,003**	**0,000**	0,071	3,708
(Model 6)	90	88,69 %	0,141	0,136	0,284	0,098	4,285	89,70 %	0,736	0,834	0,368	0,320	4,362
	95	94,26 %	0,254	0,433	0,962	0,158	4,394	94,60 %	0,532	0,553	0,439	0,128	4,515
	99	98,73 %	0,377	0,273	0,119	0,431	4,647	98,65 %	0,250	0,409	0,921	**0,006**	4,854
Linear quantile	1	0,76 %	0,385	0,635	**0,011**	0,276	3,559	1,35 %	0,250	0,232	0,169	0,151	3,581
regression,	5	4,30 %	0,260	0,262	**0,043**	0,095	3,698	5,99 %	0,128	0,064	**0,001**	0,105	3,710
fundamental and	10	8,52 %	0,083	0,099	0,201	**0,040**	3,757	10,55 %	0,532	0,388	**0,002**	**0,044**	3,767
volatility	90	89,03 %	0,272	0,202	0,374	0,174	4,288	89,45 %	0,532	0,687	0,534	0,224	4,299
(Model 7)	95	94,77 %	0,716	0,654	0,435	0,206	4,398	94,94 %	0,921	0,520	0,830	0,271	4,417
	99	98,73 %	0,377	0,551	0,385	0,507	4,647	98,40 %	0,055	**0,017**	**0,028**	**0,002**	4,690

Table 4: Forecasting results out of sample for the linear quantile regression models with expanding window (left) and rolling window (right). Table shows the *p*-values of the backtests defined in Section 4.3. Reg_{Hit} and REG_{Var} are the regression based tests given by Eq. (4.10) and (4.11), respectively. Bold type *p*-values are significant at the 5% level. $\overline{\text{Quantile}}$ denotes the average value of the quantile estimates.

Model	Quantile	Expanding Window: Violations	LR_{UC}	LR_{CC}	REG_{Hit}	REG_{Var}	$\overline{\text{Quantile}}$	Rolling Window: Violations	LR_{UC}	LR_{CC}	REG_{Hit}	REG_{Var}	$\overline{\text{Quantile}}$
Indirect	1	1,18 %	0,542	0,694	0,951	**0,001**	3,522	1,52 %	0,095	**0,003**	0,900	**0,000**	3,533
GARCH(1,1)	5	3,71 %	**0,034**	0,086	0,760	**0,000**	3,673	5,40 %	0,532	**0,031**	0,255	**0,000**	3,692
CAViaR	10	8,44 %	0,066	0,145	0,982	**0,000**	3,740	8,95 %	0,219	**0,036**	0,120	0,308	3,751
(Model 8)	90	89,79 %	0,809	0,471	0,606	0,195	4,301	90,30 %	0,734	0,714	**0,027**	0,186	4,309
	95	94,77 %	0,716	0,878	0,500	0,556	4,410	95,27 %	0,662	0,341	**0,046**	0,777	4,419
	99	98,90 %	0,741	0,811	0,963	0,977	4,647	98,57 %	0,158	0,183	**0,003**	0,602	4,671
Symmetric absolute	1	1,35 %	0,250	0,409	0,462	**0,001**	3,543	1,69 %	**0,030**	0,060	**0,022**	**0,000**	3,553
value CAViaR,	5	3,88 %	0,067	0,143	0,432	**0,000**	3,677	5,49 %	0,450	0,532	0,372	**0,000**	3,690
(Model 9)	10	7,85 %	**0,011**	**0,035**	0,988	**0,000**	3,741	8,19 %	**0,032**	**0,049**	**0,038**	**0,000**	3,749
	90	89,96 %	0,961	0,730	0,435	0,337	4,294	89,70 %	0,736	0,768	0,207	0,758	4,300
	95	94,26 %	0,254	0,433	0,766	0,923	4,405	94,26 %	0,254	0,283	0,138	0,768	4,403
	99	98,90 %	0,741	0,297	0,219	0,328	4,669	98,73 %	0,377	0,551	0,301	0,785	4,672
Asymmetric slope	1	0,93 %	0,802	0,223	0,076	**0,002**	3,533	1,77 %	**0,016**	**0,037**	**0,043**	**0,000**	3,559
CAViaR,	5	3,80 %	**0,048**	0,113	0,395	**0,000**	3,673	5,15 %	0,816	0,818	0,375	**0,000**	3,687
(Model 10)	10	8,19 %	**0,032**	0,086	0,540	**0,000**	3,749	10,21 %	0,809	0,470	0,248	**0,000**	3,752
	90	89,79 %	0,809	0,471	0,203	0,333	4,298	89,03 %	0,272	0,453	0,260	0,683	4,295
	95	95,44 %	0,478	0,406	0,110	0,225	4,413	95,02 %	0,973	0,789	0,455	0,547	4,418
	99	98,90 %	0,741	0,811	0,963	0,846	4,719	98,57 %	0,158	0,284	0,902	0,587	4,692
Adaptive CAViaR,	1	0,68 %	0,232	0,461	0,994	**0,000**	3,385	1,01 %	0,965	0,875	0,972	**0,000**	3,480
(Model 11)	5	3,12 %	**0,002**	**0,005**	0,221	**0,000**	3,651	4,14 %	0,160	0,285	0,722	**0,000**	3,686
	10	7,68 %	**0,006**	**0,014**	**0,103**	**0,000**	3,736	9,11 %	0,303	0,494	0,143	**0,000**	3,757
	90	90,30 %	0,734	0,162	**0,001**	**0,005**	4,293	90,13 %	0,884	0,069	**0,000**	**0,028**	4,293
	95	96,29 %	**0,034**	**0,026**	0,100	0,337	4,461	95,44 %	0,478	0,239	**0,008**	0,271	4,423
	99	99,16 %	0,579	0,781	**0,036**	0,629	4,764	98,90 %	0,741	0,811	**0,040**	0,949	4,720

Table 5: Forecasting results out of sample for the CAViaR models with expanding window (left) and rolling window (right). Table shows the *p*-values of the backtests defined in Section 4.3. Reg_{Hit} and REG_{Var} are the regression based tests given by Eq. (4.10) and (4.11), respectively. Bold type *p*-values are significant at the 5% level. $\overline{\text{Quantile}}$ denotes the average value of the quantile estimates.