

DS 2006 Midterm 2

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NOTE: The .rmd version of the file is available here: [\(link\)](#)

Instructions

Please prepare responses/solutions for the following questions.

Allowable resources

You may use the solutions you've prepared for the prep questions during the exam. You are welcome to use your homeworks, deliverables, or class notes. You are permitted to access the internet for publicly available content. You are not allowed to communicate with anyone via the internet or any other means during the exam. This includes, but is not limited to:

- No messaging, emailing, or using social media to contact others.
- No posting questions or seeking answers on forums, chat rooms, chat bots (including large language models like ChatGPT), or any collaborative platforms.
- No sharing or discussing exam content with peers through any online or electronic medium.

You may **NOT** discuss any aspect of the exam or prep questions with anyone other than the instructor or TA. You may **NOT** share code or documents.

Submission instructions

1. Within your course repo, create a folder called **Midterm2**
2. Within the folder, create the script file **exam.rmd** with your solutions. Create a rendered report in **.pdf** output.
3. Add, commit, and push to your repo on github.com.

Questions

Exam questions are organized into sections corresponding to the learning outcomes of the course.

Section 1. Tools of the data scientists

Learning objective: Use the tools of data scientists

Learning objective: Implement best programming/coding practices

- 1.1 [5 pts] Write your name at the prompt above (line 6 in the script).
- 1.2 [5 pts] When you are done with the exam, please render this report as a pdf document.

1.3 [5 pts] The following is a schematic of a project folder, with subfolders and files.

```
project
|
|---code
|     script.rmd
|
|---data
|     survey-responses.csv
|
|---docs
```

Supposing the `code` subfolder is the designated working directory, write the command to be included in the `script.rmd` file which will read the `survey-responses.csv` data, avoiding absolute file paths?

```
survey <- read.csv("~/data/survey-responses.csv")
```

Section 2. Probability & Diagnostics

Learning objective: Compare and contrast different definitions of probability, illustrating differences with simple examples

Learning objective: Express the rules of probability verbally, mathematically, and computationally.

Learning objective: apply cross table framework to the special case of binary outcomes

2.1 [5 pts] In a particular town, 1% of all individuals have a certain rare disease. There's a test for this disease that correctly identifies a sufferer 99% of the time (true positive rate) but also falsely identifies the disease in 2% of the healthy population (false positive rate).

HINT: False positive rate = $P(T+ | D-)$

Complete the following table of cell, row, and column probabilities based on the information about the prevalence, true positive rate, and false positive rate. You are welcome to use an excel spreadsheet ([link](#)) which will automatically create the table for any combination of prevalence, sensitivity, and specificity. (It might save time to insert a screen shot of the table rather than manually creating the table.)

	Disease +	Disease -	
Test +			
→ cell		.0198	.000198
→ row	.99	.01	
→ col		.02	
Test -			
→ cell		.97	
→ row			
→ col		.98	
	.01	.99	1

set specificity to .98 set sensitivity to .99

			D+	D-			
	T +	cell	0.010	0.010	0.020		
		row	0.497	0.503			
		col	0.980	0.010			
	T -	cell	0.000	0.980	0.980		
		row	0.000	1.000			
		col	0.020	0.990			
			0.010	0.990			

2.2 [5 pts] Suppose that a new test is developed and approved 98% true positive rate and 1% false positive rate. As a consumer, which would you prefer? Be specific about your reasoning and the quantities that you are using to make a decision.

			D+	D-			
T +	cell	0.010	0.020	0.030			
	row	0.333	0.667				
	col	0.990	0.020				
T -	cell	0.000	0.970	0.970			
	row	0.000	1.000				
	col	0.010	0.980				
		0.010	0.990				

2.3 [10 pts] An audit of an email filtering system resulted in a dataset of 10000 emails, each manually verified as spam or not spam. In addition to the type of email, the dataset indicates if the filter sent the email to the inbox or the junk folder.

The following command reads the dataset into memory. From the data, generate an estimate of the positive predictive value and the negative predictive value of the spam filter.

HINT: Specificity in this example = $P(\text{Junk folder}|\text{Spam})$

```
d1 <- readRDS(url("https://tgstewart.cloud/spam-data.RDS"))
table(d1)
```

```
##           Folder
## Type      Inbox  Junk
## Not spam  7589   422
## Spam      197  1792
```

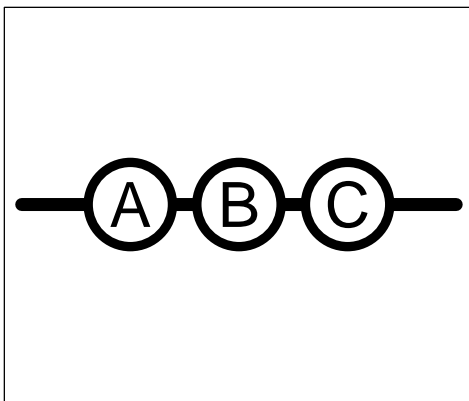
Section 3. Simulation

Learning objective: Use probability models to build simulations of complex real world processes to answer research questions

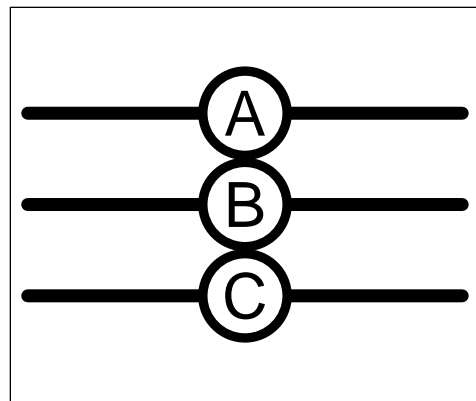
3.1 [5 pts] Consider two systems of three components (say A, B, and C). In the first, a failure of any component leads to a failure of the entire system. In the second, the components are redundant, and a failure only occurs if all three components fail.

pdf
2

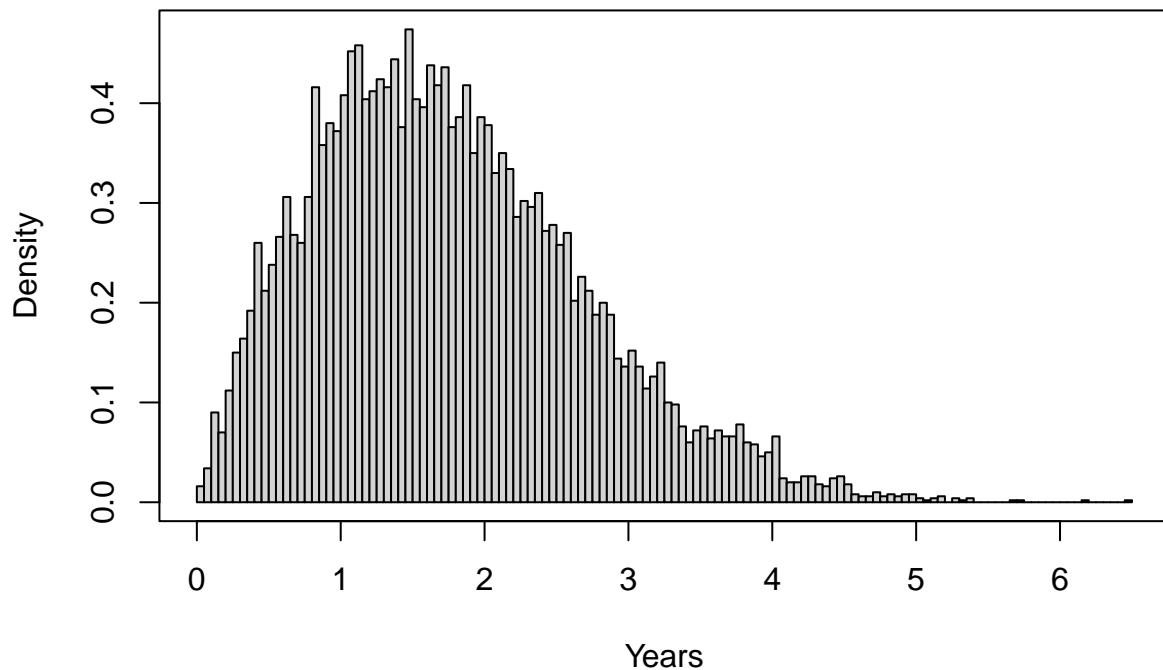
Sequential



Parallel



Suppose the failure time of an individual component is a random variable with the following distribution.



The following function `sysfail` generates replicates of the time to system failure (years) for the sequential and parallel systems. The input parameter `R` is the number of replicates that the function will return.

```
sysfail <- function(R){
  A <- array(rweibull(R*3,2,2),dim = c(R,3))
  data.frame(sequential = apply(A,1,min), parallel = apply(A,1,max))
}
```

The following provides an estimate of how much longer the parallel system will last compared to the sequential system by simulating 25 different system failures.

```
set.seed(230583)
a1 <- sysfail(25)
a2 <- colMeans(a1)
diff(a2)
```

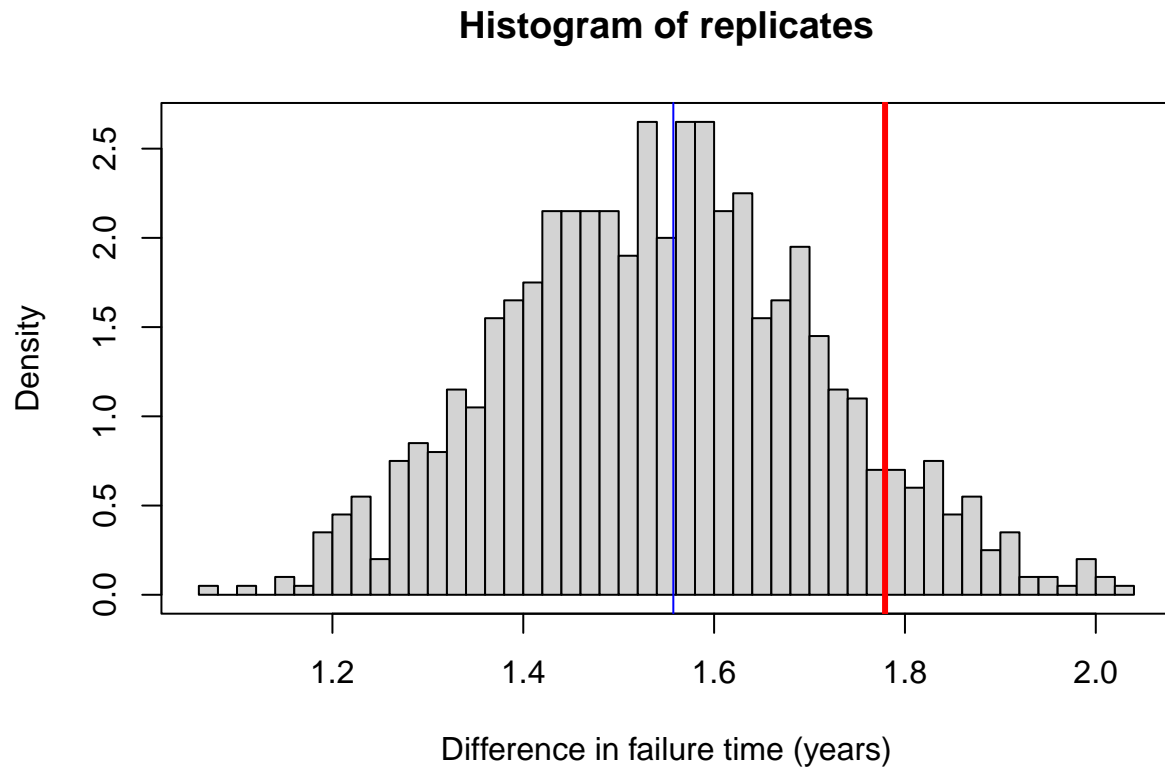
```
## parallel
## 1.779168
```

The calculated difference is based on pseudo-random data. The process can be repeated many times. The following code creates 1000 estimates. The redline refers to the single estimate generated above.

```
R <- 25
replicates <- replicate(1000, sysfail(R) |> colMeans() |> diff())
hist(replicates, breaks = 50, freq = FALSE, xlab = "Difference in failure time (years)"); box()
```



```
abline(v=diff(a2), lwd = 3, col = "red")
mean_reps <- mean(replicates <- replicate(1000, sysfail(R) |> colMeans() |> diff()))
abline(v=mean_reps, col = "blue")
```



Add to the figure blue reference line for the mean of the 1000 estimates. (Simply edit the code chunk above. You do not need to create a second copy.)

3.2 [5 pts] What is the range (min and max) of the 1000 values you generated for the improved failure time estimate?

```
#min and max  
min(replicates)
```

```
## [1] 1.01974
```

```
max(replicates)
```

```
## [1] 2.181636
```

3.3 [5 pts] What is the average absolute error of the 1000 estimates? Use the mean calculated in 3.1 as the “true” value.

```
absolute_error <- function(out, mean_reps){  
  sum_of_error <- 0  
  for(i in 1:length(out)){  
    sum_of_error <- sum_of_error + abs(out[i] - mean_reps)  
  }  
  absolute_error <- sum_of_error / length(out)  
}  
abs_err <- absolute_error(replicates, mean_reps)  
  
print(abs_err)
```

```
## parallel  
## 0.1391898
```

3.4 [5 pts] What is the average relative error of the 1000 estimates? Use the mean calculated in 3.1 as the “true” value.

```
rel_err <- abs_err/mean_reps  
print(rel_err)
```

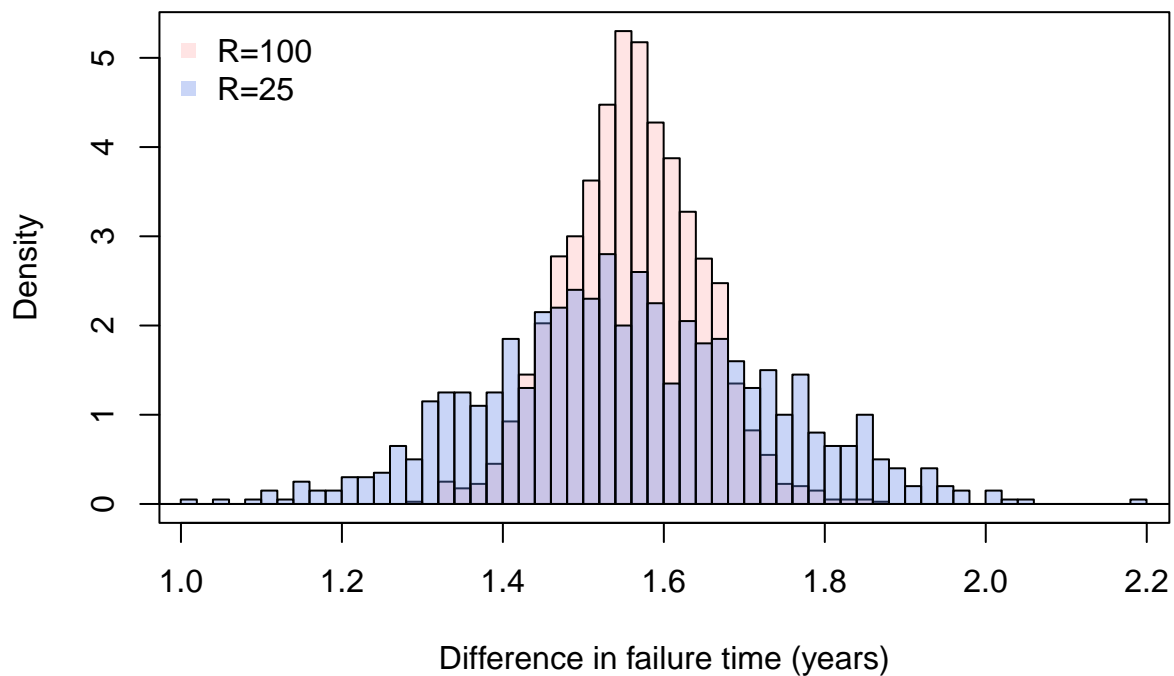
```
## parallel  
## 0.08938152
```

3.5 [5 pts] If you wanted to reduced the error by half, how many replicates (R) would you need to use?
you would want to use 100 replicates to half the error.

3.6 [5 pts] Generate a plot of overlapping histograms to show the difference between R=25 and your R from the previous problem.

```
R2 <- 100 # Change this
replicates2 <- replicate(2000, sysfail(R2) |> colMeans() |> diff())

b1 <- seq(min(c(replicates,replicates2))-0.1, max(c(replicates,replicates2))+0.1,by=0.02)
hist(replicates2, breaks = b1, col = "#ffabab50", freq = FALSE, main = "", xlim = range(replicates), xlab = "Difference in failure time (years)", legend = "topleft", legend = c(paste0("R=",R2),"R=25"), col = c("#ffabab50","#6488ea59"),bty = "n", pch = 1)
hist(replicates, breaks = b1, add=TRUE, col = "#6488ea59", freq = FALSE)
box()
```



3.7 [5 pts] Calculate the average absolute error of the 1000 estimates generated with the new choice of R? Did it change as you expected it to?

The absolute error is half with the new R, which is what we expected when we multiplied the R by 4.

```
mean_reps2 <- mean(replicates2)

absolute_error <- function(out, mean_reps2){
  sum_of_error <- 0
  for(i in 1:length(out)){
    sum_of_error <- sum_of_error + abs(out[i] - mean_reps)
  }
  absolute_error <- sum_of_error / length(out)
}
abs_err <- absolute_error(replicates2, mean_reps2)

print(abs_err)
```

```
## parallel
## 0.06522258
```

Section 4. Confounding vs Causal Pathway

Learning objective: define/describe confounding variables, Simpson's paradox, DAGs, and the causal pathway

4.1 [10 pts] The following function generates data from a cohort of individuals who agreed to be studied about heart disease. In the dataset, there is exercise level at age 20 (below average, above average), blood pressure at age 25 (low, normal, high), and heart disease at age 30 (present, absent).

Generate 10000 draws from the function and create the cross table for exercise level and heart disease. Calculate a summary of the effect of exercise by calculating the risk ratio:

$$RR = \frac{P(\text{heart disease present} | \text{below average exercise})}{P(\text{heart disease present} | \text{above average exercise})}$$

It

```
heart_data <- function(R){
  ex <- rbinom(R,1,.5)
  bp <- rnorm(R,-ex+1/2,1)
  bp <- cut(bp,c(-Inf,-1,1,Inf), labels = FALSE)
  hd <- 1*(rnorm(R,bp-3,1.8)>0)
  data.frame(
    exercise = factor(ex,0:1,c("below average","above average"))
    , blood_pressure = factor(bp,1:3,c("low","normal","high"))
    , heart_disease = factor(hd, 0:1, c("absent","present"))
  )
}

set.seed(20240329); d1 <- heart_data(1000000)

calc_delta <- function(df){
  num_ex <- 0
  ex_and_hd <- 0

  for(i in 1:nrow(df)){
    if(df[i,1] == "exercise"){
      num_ex <- num_ex + 1
      if(df[i,3] == "heart disease"){
        ex_and_hd <- ex_and_hd + 1
      }
    }
  }
  p_hd_given_ex = ex_and_hd / num_ex

  num_noex <- 0
  noex_and_nohd <- 0

  for(i in 1:nrow(df)){
    if(df[i,1] == "noexercise"){
      num_noex <- num_noex + 1
      if(df[i,3] == "nohd"){
        noex_and_nohd <- noex_and_nohd + 1
      }
    }
  }
}
```



```

    }
  }
}
p_hd_given_ex = ex_and_hd / num_ex

delta = p_hd_given_ex - noex_and_nohd
}

Ra <- 1000
exer <- heart_data(Ra)

print(exer)

```

```

##           exercise blood_pressure heart_disease
## 1    above average      normal      absent
## 2    below average      normal      absent
## 3    above average      normal      present
## 4    above average      normal      absent
## 5    above average      normal      absent
## 6    below average      high       present
## 7    below average      normal      absent
## 8    below average      normal      present
## 9    below average      high       present
## 10   above average      normal      absent
## 11   below average      normal      absent
## 12   below average      normal      absent
## 13   below average      normal      absent
## 14   below average      low        absent
## 15   above average      normal      absent
## 16   below average      high       absent
## 17   above average      normal      present
## 18   above average      low        absent
## 19   above average      low        absent
## 20   below average      normal      absent
## 21   above average      low        absent
## 22   below average      high       present
## 23   below average      high       present
## 24   above average      low        present
## 25   below average      normal      present
## 26   above average      normal      absent
## 27   below average      normal      absent
## 28   below average      normal      present
## 29   above average      normal      present
## 30   above average      normal      present
## 31   above average      normal      absent
## 32   above average      low        absent
## 33   above average      low        absent
## 34   above average      normal      absent
## 35   above average      low        absent
## 36   below average      low        absent
## 37   above average      normal      absent
## 38   above average      normal      present
## 39   below average      high       absent

```

## 40	below average	normal	present
## 41	below average	normal	absent
## 42	below average	normal	absent
## 43	below average	high	absent
## 44	below average	normal	absent
## 45	below average	normal	present
## 46	above average	normal	absent
## 47	above average	low	present
## 48	above average	normal	absent
## 49	above average	normal	absent
## 50	above average	normal	present
## 51	below average	high	present
## 52	above average	low	present
## 53	above average	normal	present
## 54	above average	normal	present
## 55	below average	normal	absent
## 56	below average	normal	absent
## 57	below average	normal	present
## 58	below average	normal	present
## 59	above average	normal	absent
## 60	below average	low	absent
## 61	above average	normal	absent
## 62	above average	normal	present
## 63	above average	high	present
## 64	above average	normal	absent
## 65	below average	high	absent
## 66	below average	normal	absent
## 67	above average	normal	absent
## 68	below average	normal	present
## 69	below average	low	absent
## 70	below average	low	absent
## 71	above average	normal	absent
## 72	below average	normal	absent
## 73	above average	low	absent
## 74	above average	normal	absent
## 75	above average	low	absent
## 76	below average	high	present
## 77	below average	normal	absent
## 78	above average	low	absent
## 79	above average	normal	absent
## 80	below average	normal	present
## 81	above average	low	absent
## 82	above average	normal	present
## 83	below average	high	absent
## 84	below average	normal	absent
## 85	above average	high	absent
## 86	below average	normal	absent
## 87	above average	low	absent
## 88	below average	normal	present
## 89	below average	normal	absent
## 90	below average	high	present
## 91	above average	low	absent
## 92	above average	normal	absent
## 93	above average	normal	present

## 94	below average	normal	present
## 95	above average	normal	present
## 96	below average	high	present
## 97	above average	low	absent
## 98	below average	high	present
## 99	above average	normal	absent
## 100	below average	high	absent
## 101	below average	low	absent
## 102	below average	normal	present
## 103	below average	high	absent
## 104	below average	normal	present
## 105	below average	normal	absent
## 106	below average	normal	present
## 107	above average	normal	absent
## 108	above average	low	present
## 109	below average	normal	absent
## 110	below average	normal	absent
## 111	below average	low	absent
## 112	below average	high	absent
## 113	below average	normal	present
## 114	above average	normal	absent
## 115	below average	normal	absent
## 116	below average	normal	absent
## 117	above average	low	absent
## 118	above average	low	absent
## 119	below average	normal	absent
## 120	above average	normal	absent
## 121	above average	normal	present
## 122	above average	normal	absent
## 123	below average	high	absent
## 124	above average	low	absent
## 125	above average	normal	absent
## 126	above average	high	present
## 127	above average	low	absent
## 128	above average	low	absent
## 129	above average	low	present
## 130	above average	normal	absent
## 131	above average	normal	absent
## 132	below average	normal	absent
## 133	above average	low	absent
## 134	below average	normal	present
## 135	above average	low	absent
## 136	above average	normal	absent
## 137	above average	low	absent
## 138	above average	low	absent
## 139	above average	normal	absent
## 140	below average	high	absent
## 141	above average	normal	absent
## 142	below average	normal	absent
## 143	above average	normal	absent
## 144	below average	normal	present
## 145	below average	normal	absent
## 146	above average	normal	absent
## 147	below average	normal	absent

## 148	above average	normal	present
## 149	below average	high	absent
## 150	above average	normal	absent
## 151	above average	normal	absent
## 152	below average	normal	absent
## 153	above average	high	absent
## 154	below average	normal	absent
## 155	below average	normal	present
## 156	below average	high	present
## 157	below average	high	present
## 158	below average	low	absent
## 159	below average	high	present
## 160	below average	normal	present
## 161	below average	low	absent
## 162	above average	normal	absent
## 163	above average	normal	present
## 164	above average	low	absent
## 165	above average	normal	absent
## 166	above average	low	present
## 167	below average	normal	absent
## 168	above average	normal	absent
## 169	above average	normal	present
## 170	below average	normal	absent
## 171	above average	high	present
## 172	above average	low	present
## 173	above average	normal	absent
## 174	below average	normal	absent
## 175	below average	normal	absent
## 176	below average	normal	absent
## 177	below average	normal	present
## 178	above average	normal	absent
## 179	below average	high	absent
## 180	below average	normal	present
## 181	above average	normal	present
## 182	below average	normal	absent
## 183	below average	normal	absent
## 184	below average	high	present
## 185	above average	normal	absent
## 186	below average	high	absent
## 187	above average	normal	present
## 188	above average	normal	absent
## 189	above average	normal	present
## 190	above average	normal	absent
## 191	above average	normal	absent
## 192	above average	low	absent
## 193	below average	high	present
## 194	below average	normal	absent
## 195	below average	normal	present
## 196	below average	high	absent
## 197	above average	low	absent
## 198	below average	normal	absent
## 199	above average	low	absent
## 200	above average	normal	absent
## 201	above average	normal	present

## 202	above average	normal	absent
## 203	above average	normal	absent
## 204	above average	low	absent
## 205	below average	high	absent
## 206	above average	normal	absent
## 207	above average	normal	absent
## 208	above average	low	absent
## 209	below average	normal	absent
## 210	above average	low	present
## 211	above average	normal	absent
## 212	above average	normal	absent
## 213	below average	normal	absent
## 214	above average	normal	absent
## 215	below average	high	absent
## 216	above average	low	absent
## 217	above average	normal	present
## 218	above average	normal	absent
## 219	below average	normal	absent
## 220	below average	high	present
## 221	below average	normal	absent
## 222	above average	normal	absent
## 223	above average	normal	absent
## 224	below average	normal	absent
## 225	above average	normal	absent
## 226	above average	normal	absent
## 227	below average	low	present
## 228	below average	normal	absent
## 229	below average	normal	present
## 230	below average	low	absent
## 231	above average	normal	absent
## 232	below average	normal	absent
## 233	below average	high	present
## 234	below average	normal	absent
## 235	above average	normal	absent
## 236	below average	normal	absent
## 237	below average	high	present
## 238	above average	low	absent
## 239	below average	normal	absent
## 240	below average	high	present
## 241	above average	normal	absent
## 242	above average	low	absent
## 243	below average	normal	absent
## 244	above average	normal	absent
## 245	above average	normal	absent
## 246	above average	normal	absent
## 247	above average	normal	present
## 248	above average	high	present
## 249	above average	low	absent
## 250	above average	high	absent
## 251	below average	high	absent
## 252	below average	normal	absent
## 253	below average	high	present
## 254	below average	normal	present
## 255	above average	low	absent

## 256	below average	normal	absent
## 257	above average	low	absent
## 258	below average	high	absent
## 259	below average	high	absent
## 260	above average	normal	absent
## 261	above average	normal	present
## 262	below average	high	present
## 263	below average	normal	absent
## 264	below average	low	absent
## 265	above average	normal	absent
## 266	above average	high	present
## 267	below average	high	absent
## 268	below average	high	present
## 269	above average	normal	absent
## 270	above average	low	absent
## 271	below average	high	absent
## 272	above average	low	absent
## 273	above average	low	absent
## 274	below average	normal	present
## 275	below average	normal	absent
## 276	below average	normal	absent
## 277	above average	low	absent
## 278	above average	low	absent
## 279	above average	normal	absent
## 280	above average	normal	absent
## 281	above average	low	present
## 282	below average	normal	present
## 283	below average	normal	absent
## 284	below average	normal	absent
## 285	above average	normal	present
## 286	below average	normal	present
## 287	above average	high	present
## 288	below average	normal	absent
## 289	above average	normal	absent
## 290	above average	high	absent
## 291	below average	normal	absent
## 292	above average	low	absent
## 293	below average	normal	present
## 294	below average	normal	absent
## 295	above average	normal	absent
## 296	above average	normal	absent
## 297	below average	high	present
## 298	above average	normal	present
## 299	above average	high	present
## 300	above average	normal	present
## 301	below average	high	absent
## 302	above average	low	absent
## 303	above average	normal	absent
## 304	below average	high	absent
## 305	above average	normal	absent
## 306	below average	high	absent
## 307	above average	normal	absent
## 308	below average	normal	absent
## 309	above average	low	absent

## 310	above average	normal	present
## 311	above average	normal	absent
## 312	above average	normal	absent
## 313	below average	normal	present
## 314	below average	high	present
## 315	above average	normal	absent
## 316	above average	normal	absent
## 317	below average	high	absent
## 318	above average	low	absent
## 319	above average	low	absent
## 320	above average	normal	present
## 321	below average	normal	absent
## 322	above average	normal	absent
## 323	below average	high	absent
## 324	above average	high	present
## 325	below average	normal	present
## 326	above average	normal	absent
## 327	below average	normal	absent
## 328	below average	normal	absent
## 329	above average	normal	absent
## 330	above average	normal	absent
## 331	above average	normal	absent
## 332	above average	normal	absent
## 333	below average	low	absent
## 334	below average	high	absent
## 335	below average	high	present
## 336	above average	low	present
## 337	above average	normal	absent
## 338	below average	normal	absent
## 339	below average	normal	absent
## 340	below average	normal	absent
## 341	below average	high	present
## 342	above average	low	absent
## 343	above average	normal	absent
## 344	above average	normal	absent
## 345	below average	normal	present
## 346	above average	normal	present
## 347	below average	normal	present
## 348	below average	normal	present
## 349	below average	low	absent
## 350	above average	low	absent
## 351	above average	low	present
## 352	below average	normal	absent
## 353	below average	normal	absent
## 354	above average	low	present
## 355	below average	normal	absent
## 356	above average	normal	absent
## 357	above average	low	present
## 358	above average	normal	absent
## 359	below average	normal	absent
## 360	below average	normal	absent
## 361	above average	normal	present
## 362	below average	high	absent
## 363	below average	high	present

## 364	below average	high	absent
## 365	below average	high	absent
## 366	below average	high	absent
## 367	above average	normal	absent
## 368	below average	normal	absent
## 369	above average	normal	absent
## 370	below average	low	present
## 371	above average	low	absent
## 372	below average	high	present
## 373	below average	normal	absent
## 374	above average	normal	absent
## 375	below average	high	absent
## 376	above average	normal	absent
## 377	below average	normal	absent
## 378	below average	normal	absent
## 379	below average	normal	absent
## 380	above average	low	absent
## 381	above average	normal	absent
## 382	below average	high	present
## 383	below average	normal	present
## 384	above average	high	present
## 385	below average	high	present
## 386	above average	low	absent
## 387	below average	normal	present
## 388	below average	high	present
## 389	below average	normal	absent
## 390	below average	normal	present
## 391	above average	normal	present
## 392	above average	low	absent
## 393	above average	low	absent
## 394	below average	normal	absent
## 395	below average	high	present
## 396	above average	low	present
## 397	below average	normal	absent
## 398	above average	low	absent
## 399	below average	normal	absent
## 400	above average	low	present
## 401	above average	normal	absent
## 402	below average	low	present
## 403	below average	normal	absent
## 404	above average	normal	absent
## 405	below average	normal	absent
## 406	below average	high	absent
## 407	below average	normal	present
## 408	above average	normal	absent
## 409	below average	high	present
## 410	above average	normal	absent
## 411	above average	normal	absent
## 412	above average	low	absent
## 413	below average	low	absent
## 414	below average	normal	present
## 415	above average	low	absent
## 416	below average	normal	absent
## 417	above average	normal	absent

## 418	above average	low	absent
## 419	below average	normal	absent
## 420	below average	normal	absent
## 421	above average	normal	present
## 422	above average	low	absent
## 423	below average	normal	absent
## 424	below average	normal	absent
## 425	below average	normal	absent
## 426	below average	high	absent
## 427	below average	high	absent
## 428	below average	normal	absent
## 429	below average	high	absent
## 430	above average	low	absent
## 431	above average	normal	present
## 432	below average	normal	absent
## 433	above average	high	present
## 434	below average	normal	absent
## 435	above average	normal	absent
## 436	below average	normal	absent
## 437	below average	normal	absent
## 438	below average	normal	present
## 439	below average	normal	absent
## 440	above average	normal	absent
## 441	above average	normal	present
## 442	below average	normal	absent
## 443	below average	normal	absent
## 444	below average	normal	present
## 445	below average	normal	absent
## 446	below average	normal	absent
## 447	above average	low	absent
## 448	below average	low	absent
## 449	above average	normal	absent
## 450	above average	normal	present
## 451	below average	normal	absent
## 452	above average	high	present
## 453	above average	normal	present
## 454	above average	normal	absent
## 455	below average	normal	absent
## 456	below average	normal	absent
## 457	above average	normal	absent
## 458	above average	normal	present
## 459	above average	normal	present
## 460	above average	low	absent
## 461	below average	high	present
## 462	below average	high	absent
## 463	above average	high	present
## 464	above average	normal	absent
## 465	above average	normal	present
## 466	above average	normal	present
## 467	above average	normal	absent
## 468	above average	normal	present
## 469	below average	high	present
## 470	below average	high	present
## 471	above average	normal	present

## 472	below average	normal	absent
## 473	below average	normal	absent
## 474	below average	normal	present
## 475	below average	normal	absent
## 476	above average	low	absent
## 477	above average	normal	present
## 478	above average	normal	present
## 479	above average	normal	present
## 480	below average	normal	present
## 481	above average	normal	absent
## 482	below average	normal	absent
## 483	above average	low	absent
## 484	below average	high	present
## 485	above average	normal	absent
## 486	below average	high	absent
## 487	above average	normal	present
## 488	above average	normal	present
## 489	below average	high	absent
## 490	below average	normal	absent
## 491	above average	low	absent
## 492	above average	normal	present
## 493	below average	normal	absent
## 494	below average	normal	absent
## 495	below average	normal	absent
## 496	below average	high	present
## 497	above average	low	absent
## 498	below average	normal	present
## 499	below average	high	present
## 500	above average	normal	absent
## 501	below average	normal	present
## 502	above average	normal	absent
## 503	below average	high	present
## 504	below average	high	present
## 505	above average	normal	absent
## 506	below average	normal	present
## 507	above average	normal	absent
## 508	below average	normal	absent
## 509	below average	normal	present
## 510	above average	normal	present
## 511	above average	normal	absent
## 512	above average	normal	absent
## 513	above average	low	absent
## 514	below average	high	absent
## 515	above average	normal	absent
## 516	below average	normal	present
## 517	below average	normal	absent
## 518	above average	low	present
## 519	below average	normal	absent
## 520	above average	low	present
## 521	below average	high	absent
## 522	above average	low	absent
## 523	above average	normal	absent
## 524	above average	normal	present
## 525	below average	high	absent

## 526	above average	low	present
## 527	below average	high	present
## 528	below average	normal	absent
## 529	above average	normal	absent
## 530	above average	normal	absent
## 531	above average	normal	absent
## 532	below average	normal	absent
## 533	below average	high	absent
## 534	below average	normal	absent
## 535	below average	high	absent
## 536	below average	normal	absent
## 537	below average	high	present
## 538	above average	normal	present
## 539	above average	normal	absent
## 540	below average	high	present
## 541	above average	normal	absent
## 542	above average	low	absent
## 543	below average	normal	absent
## 544	below average	normal	absent
## 545	above average	normal	absent
## 546	below average	normal	absent
## 547	above average	low	absent
## 548	below average	high	present
## 549	below average	high	present
## 550	above average	high	present
## 551	above average	low	absent
## 552	above average	low	absent
## 553	below average	normal	absent
## 554	above average	low	absent
## 555	above average	normal	absent
## 556	above average	high	present
## 557	below average	normal	absent
## 558	above average	normal	absent
## 559	below average	normal	present
## 560	above average	normal	absent
## 561	below average	high	present
## 562	above average	normal	present
## 563	below average	normal	absent
## 564	above average	low	absent
## 565	below average	high	absent
## 566	above average	high	absent
## 567	below average	normal	absent
## 568	below average	normal	absent
## 569	below average	normal	absent
## 570	below average	normal	absent
## 571	below average	normal	absent
## 572	below average	high	present
## 573	above average	low	absent
## 574	below average	normal	absent
## 575	below average	normal	absent
## 576	below average	low	absent
## 577	above average	high	absent
## 578	below average	normal	present
## 579	below average	normal	present

## 580	above average	normal	absent
## 581	below average	normal	absent
## 582	below average	normal	present
## 583	above average	normal	absent
## 584	above average	normal	absent
## 585	above average	low	absent
## 586	above average	normal	present
## 587	below average	normal	absent
## 588	above average	low	absent
## 589	below average	normal	absent
## 590	above average	low	absent
## 591	below average	normal	absent
## 592	above average	normal	absent
## 593	above average	normal	absent
## 594	below average	low	absent
## 595	above average	normal	absent
## 596	above average	normal	absent
## 597	above average	low	absent
## 598	above average	normal	absent
## 599	above average	normal	present
## 600	above average	normal	present
## 601	above average	low	absent
## 602	below average	normal	present
## 603	below average	high	absent
## 604	above average	low	absent
## 605	below average	normal	absent
## 606	below average	normal	absent
## 607	above average	low	absent
## 608	above average	low	absent
## 609	above average	normal	absent
## 610	above average	low	absent
## 611	below average	normal	present
## 612	above average	normal	absent
## 613	above average	normal	present
## 614	below average	normal	absent
## 615	above average	low	absent
## 616	above average	low	absent
## 617	above average	normal	present
## 618	below average	normal	absent
## 619	below average	high	present
## 620	above average	normal	absent
## 621	above average	low	absent
## 622	above average	low	absent
## 623	above average	low	present
## 624	below average	normal	absent
## 625	below average	normal	absent
## 626	above average	normal	absent
## 627	above average	normal	present
## 628	below average	normal	absent
## 629	below average	normal	absent
## 630	below average	normal	absent
## 631	below average	high	present
## 632	below average	high	absent
## 633	above average	high	absent

## 634	below average	normal	absent
## 635	below average	high	present
## 636	below average	high	absent
## 637	below average	high	present
## 638	below average	high	present
## 639	above average	low	absent
## 640	below average	normal	absent
## 641	above average	normal	absent
## 642	below average	high	present
## 643	above average	normal	present
## 644	below average	normal	absent
## 645	below average	low	absent
## 646	above average	normal	present
## 647	above average	high	absent
## 648	below average	high	present
## 649	above average	normal	present
## 650	below average	high	present
## 651	above average	normal	absent
## 652	above average	normal	present
## 653	below average	normal	absent
## 654	below average	high	present
## 655	above average	low	absent
## 656	above average	normal	absent
## 657	below average	high	absent
## 658	below average	normal	present
## 659	above average	normal	present
## 660	above average	high	present
## 661	above average	normal	absent
## 662	above average	normal	absent
## 663	above average	normal	absent
## 664	above average	normal	present
## 665	above average	low	absent
## 666	above average	low	present
## 667	below average	high	absent
## 668	below average	normal	absent
## 669	below average	normal	absent
## 670	below average	normal	absent
## 671	below average	high	absent
## 672	below average	high	absent
## 673	above average	normal	present
## 674	above average	low	absent
## 675	above average	normal	absent
## 676	below average	high	present
## 677	above average	normal	absent
## 678	above average	normal	present
## 679	above average	normal	absent
## 680	above average	low	present
## 681	below average	normal	present
## 682	above average	normal	absent
## 683	above average	low	absent
## 684	below average	high	absent
## 685	below average	normal	absent
## 686	above average	normal	absent
## 687	below average	normal	absent

## 688	below average	normal	present
## 689	below average	normal	absent
## 690	below average	low	present
## 691	above average	normal	absent
## 692	below average	normal	absent
## 693	below average	normal	absent
## 694	below average	normal	absent
## 695	above average	normal	absent
## 696	below average	high	present
## 697	below average	normal	absent
## 698	below average	high	absent
## 699	below average	high	present
## 700	below average	high	absent
## 701	above average	normal	absent
## 702	below average	normal	present
## 703	above average	normal	absent
## 704	above average	low	absent
## 705	below average	normal	absent
## 706	above average	low	absent
## 707	below average	high	present
## 708	below average	normal	absent
## 709	below average	normal	absent
## 710	below average	normal	absent
## 711	below average	high	absent
## 712	above average	normal	present
## 713	above average	low	absent
## 714	above average	normal	absent
## 715	below average	normal	present
## 716	below average	normal	absent
## 717	below average	normal	present
## 718	above average	normal	present
## 719	below average	high	present
## 720	above average	normal	present
## 721	below average	normal	absent
## 722	above average	normal	absent
## 723	below average	high	absent
## 724	below average	normal	present
## 725	above average	normal	absent
## 726	above average	normal	absent
## 727	above average	normal	absent
## 728	below average	low	present
## 729	above average	normal	absent
## 730	above average	normal	present
## 731	below average	normal	absent
## 732	below average	high	present
## 733	below average	normal	present
## 734	above average	low	absent
## 735	above average	normal	absent
## 736	below average	normal	absent
## 737	below average	high	present
## 738	below average	normal	absent
## 739	above average	normal	present
## 740	below average	normal	absent
## 741	below average	low	absent

## 742	above average	normal	absent
## 743	below average	high	present
## 744	below average	normal	absent
## 745	below average	high	absent
## 746	above average	high	present
## 747	below average	normal	absent
## 748	below average	normal	absent
## 749	below average	normal	absent
## 750	above average	low	absent
## 751	below average	high	absent
## 752	below average	high	present
## 753	above average	low	absent
## 754	above average	low	absent
## 755	below average	high	present
## 756	above average	low	present
## 757	below average	normal	present
## 758	below average	normal	absent
## 759	above average	normal	absent
## 760	below average	high	present
## 761	below average	normal	present
## 762	below average	normal	present
## 763	above average	normal	present
## 764	above average	normal	absent
## 765	below average	high	absent
## 766	below average	high	present
## 767	below average	normal	present
## 768	below average	high	present
## 769	below average	normal	present
## 770	above average	normal	present
## 771	below average	normal	absent
## 772	below average	high	present
## 773	above average	high	absent
## 774	above average	normal	present
## 775	below average	normal	absent
## 776	below average	normal	present
## 777	below average	normal	absent
## 778	above average	normal	present
## 779	below average	normal	absent
## 780	above average	normal	absent
## 781	above average	normal	absent
## 782	below average	normal	present
## 783	above average	high	present
## 784	below average	normal	absent
## 785	below average	high	absent
## 786	below average	normal	absent
## 787	above average	normal	absent
## 788	above average	low	absent
## 789	above average	high	present
## 790	below average	high	present
## 791	above average	low	absent
## 792	above average	normal	absent
## 793	above average	normal	absent
## 794	below average	low	present
## 795	below average	high	absent

## 796	above average	normal	absent
## 797	below average	low	absent
## 798	above average	normal	absent
## 799	below average	low	absent
## 800	below average	normal	absent
## 801	above average	low	absent
## 802	above average	normal	absent
## 803	below average	normal	absent
## 804	above average	normal	present
## 805	below average	normal	absent
## 806	below average	normal	absent
## 807	above average	normal	absent
## 808	below average	normal	absent
## 809	below average	high	absent
## 810	above average	low	absent
## 811	below average	normal	absent
## 812	below average	normal	present
## 813	below average	low	absent
## 814	below average	normal	absent
## 815	above average	high	absent
## 816	above average	normal	absent
## 817	above average	normal	present
## 818	below average	high	present
## 819	above average	normal	absent
## 820	below average	normal	present
## 821	below average	normal	absent
## 822	below average	normal	absent
## 823	below average	high	present
## 824	above average	normal	absent
## 825	below average	normal	present
## 826	above average	normal	absent
## 827	above average	normal	present
## 828	below average	high	present
## 829	below average	normal	absent
## 830	above average	low	absent
## 831	above average	normal	absent
## 832	below average	high	present
## 833	above average	normal	present
## 834	below average	normal	absent
## 835	above average	low	absent
## 836	above average	normal	absent
## 837	below average	normal	absent
## 838	above average	low	absent
## 839	above average	normal	absent
## 840	above average	normal	absent
## 841	below average	normal	absent
## 842	below average	high	absent
## 843	below average	high	present
## 844	below average	normal	absent
## 845	below average	normal	absent
## 846	above average	normal	absent
## 847	below average	high	present
## 848	above average	low	present
## 849	above average	low	present

## 850	below average	normal	present
## 851	above average	normal	present
## 852	below average	high	present
## 853	below average	high	absent
## 854	above average	low	absent
## 855	below average	high	absent
## 856	above average	low	absent
## 857	below average	normal	absent
## 858	above average	low	absent
## 859	above average	low	absent
## 860	above average	normal	absent
## 861	above average	low	absent
## 862	above average	normal	absent
## 863	above average	high	absent
## 864	below average	normal	absent
## 865	above average	normal	present
## 866	above average	normal	absent
## 867	above average	normal	present
## 868	below average	normal	absent
## 869	above average	normal	absent
## 870	below average	normal	absent
## 871	below average	normal	absent
## 872	above average	normal	absent
## 873	above average	normal	absent
## 874	below average	normal	absent
## 875	below average	normal	absent
## 876	below average	normal	absent
## 877	below average	normal	absent
## 878	below average	normal	absent
## 879	below average	high	absent
## 880	above average	normal	absent
## 881	above average	normal	present
## 882	above average	normal	absent
## 883	below average	normal	present
## 884	above average	normal	present
## 885	above average	low	absent
## 886	above average	low	absent
## 887	above average	normal	present
## 888	above average	low	absent
## 889	above average	high	present
## 890	above average	low	absent
## 891	below average	normal	present
## 892	above average	normal	absent
## 893	above average	normal	present
## 894	below average	high	absent
## 895	above average	low	absent
## 896	below average	high	present
## 897	above average	normal	present
## 898	below average	high	present
## 899	above average	normal	absent
## 900	below average	high	absent
## 901	below average	normal	absent
## 902	below average	normal	absent
## 903	above average	normal	absent

## 904	above average	low	absent
## 905	below average	normal	absent
## 906	above average	high	present
## 907	below average	normal	absent
## 908	below average	normal	present
## 909	above average	normal	present
## 910	below average	high	present
## 911	below average	normal	absent
## 912	below average	normal	absent
## 913	below average	normal	absent
## 914	above average	normal	absent
## 915	above average	normal	present
## 916	below average	normal	absent
## 917	below average	normal	present
## 918	below average	normal	present
## 919	below average	normal	present
## 920	above average	normal	absent
## 921	below average	high	absent
## 922	above average	normal	absent
## 923	below average	normal	absent
## 924	above average	high	absent
## 925	below average	normal	absent
## 926	above average	normal	present
## 927	below average	normal	present
## 928	below average	high	present
## 929	above average	high	absent
## 930	above average	high	absent
## 931	below average	normal	present
## 932	below average	high	present
## 933	above average	normal	absent
## 934	below average	normal	absent
## 935	below average	high	present
## 936	below average	normal	absent
## 937	below average	normal	absent
## 938	below average	high	present
## 939	below average	normal	absent
## 940	below average	normal	present
## 941	below average	normal	present
## 942	below average	normal	absent
## 943	below average	normal	absent
## 944	above average	low	absent
## 945	above average	low	absent
## 946	above average	normal	absent
## 947	above average	normal	absent
## 948	below average	normal	absent
## 949	above average	normal	present
## 950	below average	normal	present
## 951	above average	normal	absent
## 952	below average	normal	present
## 953	above average	normal	absent
## 954	below average	high	absent
## 955	above average	normal	present
## 956	below average	high	present
## 957	above average	normal	present

## 958	below average	low	absent
## 959	above average	high	absent
## 960	above average	low	present
## 961	below average	high	present
## 962	above average	normal	absent
## 963	above average	normal	absent
## 964	below average	normal	absent
## 965	below average	high	absent
## 966	above average	normal	absent
## 967	below average	normal	absent
## 968	above average	normal	absent
## 969	above average	low	present
## 970	below average	high	absent
## 971	below average	normal	present
## 972	below average	normal	present
## 973	above average	low	absent
## 974	above average	normal	absent
## 975	above average	normal	present
## 976	above average	normal	present
## 977	above average	normal	absent
## 978	above average	normal	absent
## 979	above average	normal	absent
## 980	below average	normal	absent
## 981	below average	normal	absent
## 982	below average	normal	present
## 983	below average	normal	absent
## 984	above average	normal	absent
## 985	above average	normal	absent
## 986	above average	low	absent
## 987	above average	normal	absent
## 988	below average	high	present
## 989	below average	low	absent
## 990	below average	low	present
## 991	above average	high	absent
## 992	below average	high	present
## 993	above average	normal	present
## 994	below average	normal	absent
## 995	below average	high	present
## 996	below average	normal	present
## 997	below average	normal	absent
## 998	above average	normal	absent
## 999	below average	normal	present
## 1000	above average	normal	present

```
delta <- calc_delta(exer)
print(delta)
```

```
## [1] NaN
```

4.2 [10 pts] Stratify the table by blood pressure. As in the previous problem, calculate the same treatment effect in each strata.

4.3 [5 pts] Based on the summary of the treatment effect that you observed in the combined and stratified tables, is exercise associated with lower rates of heart disease?

It would depend on whether or not the values were positive or negative.

4.4 [5 pts] Which measure of treatment effect is most persuasive? The combined estimate or the stratified estimates? Which estimate(s) should you rely on? Explain why, creating a DAG to represent relationship between the variables.

assuming the stratified tables reveal information, a DAG could look like:

Exercise amount \rightarrow Blood pressure (D) \rightarrow heart disease (R) | $+$ \rightarrow
heart disease (R)