TASI Lectures on Flavor Physics

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These notes overlap with lectures given at the TASI summer schools in 2014 and 2011, as well as at the European School of High Energy Physics in 2013. This is primarily an attempt at transcribing my handwritten notes, with emphasis on topics and ideas discussed in the lectures. It is not a comprehensive introduction or review of the field, nor does it include a complete list of references. I hope, however, that someone may find it useful to better understand the reasons for excitement about recent progress and future opportunities in flavor physics.

Preface

There are many books and reviews on flavor physics (e.g., Refs. [1; 2; 3; 4; 5; 6; 7; 8; 9]). The main points I would like to explain in these lectures are:

- CP violation and flavor-changing neutral currents (FCNC) are sensitive probes of short-distance physics, both in the standard model (SM) and in beyond standard model (BSM) scenarios.
- The data taught us a lot about not directly seen physics in the past, and are likely crucial to understand LHC new physics (NP) signals.
- In most FCNC processes BSM/SM $\sim \mathcal{O}(20\%)$ is still allowed today, the sensitivity will improve to the few percent level in the future.
- Measurements are sensitive to very high scales, and might find unambiguous signals of BSM physics, even outside the LHC reach.
- There is interesting interplay of theory and experimental progress, many open questions and interesting problems yet to be solved.

Flavor physics is interesting because there is a lot we do not understand yet. The "standard model flavor puzzle" refers to our lack of understanding why and how the 6 quark and 6 lepton flavors differ, why masses and quark mixing are hierarchical, but lepton mixing is not. The "new physics flavor puzzle" is the tension between the relatively low scale required to solve the fine tuning problem (also suggested by the WIMP paradigm), and the high scale that is seemingly required to suppress the non-SM contributions to flavor changing processes. If there is NP at the TeV scale, we need to understand why and in what way its flavor structure is non-generic.

The key questions and prospects that make the future interesting are [7]

- What is the achievable experimental precision?
  The LHCb, Belle II, NA62, KOTO, $\mu \to e\gamma$, $\mu 2e$, etc., experiments will improve the sensitivity in many modes by orders of magnitude.
- What are the theoretical uncertainties?
  In many key measurements, the theory uncertainty is well below future experimental sensitivity; while in some cases theoretical improvements are needed (so you can make an impact!).
- How large deviations from SM can we expect due to TeV-scale NP?
  New physics with generic flavor structure is ruled out; observable effects near current bounds are possible, many models predict some.
- What will the measurements teach us?
  In all scenarios there is complementarity with high-p_T measurements, and synergy in understanding the structure of any NP seen.

Another simple way to get a sense of (a lower bound on) the next 10–15 years of B physics progress is to consider the expected increase in data,

$$\frac{(\text{LHCb upgrade})}{(\text{LHCb } 1\,\text{fb}^{-1})} \sim \frac{(\text{Belle II data set})}{(\text{Belle data set})} \sim \frac{(\text{2009 BaBar data set})}{(\text{1999 CLEO data set})} \sim 50\,.$$

This will yield a $\sqrt[4]{50} \sim 2.5$ increase in sensitivity to higher mass scales, even just by redoing existing measurements. More data has always motivated new theory ideas, yielding even faster progress. This is a comparable increase in reach as going from LHC7–8 $\to$ LHC13–14.

Outline The topics these lectures will cover include a brief introduction to flavor physics in the SM, testing the flavor structure in neutral meson mixing and CP violation, examples of how to get theoretically clean information on short-distance physics. After a glimpse at the ingredients of the SM CKM fit, we discuss how sizable new physics contributions are still allowed in neutral meson mixing, and how this will improve in the future. Then we explain some implications of the heavy quark limit, tidbits of heavy quark symmetry, the operator product expansion and inclusive decays, to try to give an impression of what makes some hadronic physics tractable. The last lecture discusses some topics in TeV-scale flavor physics, top quark physics, Higgs flavor physics, bits of the interplay between searches for supersymmetry and flavor, and comments on minimal flavor violation. Some questions one may enjoy thinking about are in the footnotes.

1. Introduction to Flavor Physics and CP Violation

Most of the experimentally observed particle physics phenomena are consistent with standard model (SM). Evidence that the minimal SM is incomplete come from the lack of a dark matter candidate, the baryon asymmetry of the Universe, its accelerating expansion, and nonzero neutrino masses. The baryon asymmetry and neutrino mixing are certainly connected to CP violation and flavor physics, and so may be dark matter. The hierarchy problem and seeking to identify the particle nature of dark matter strongly motivate TeV-scale new physics.

Studying flavor physics and CP violation provide a rich program to probe the SM and search for NP, with sensitivity to very high, $1-10^5$ TeV, scales, depending on details of the models. As we shall see, the sensitivity to BSM contributions to dimension-6 four-quark operator contributions to K, D, B_d, and B_s mixing (with coefficients $1/\Lambda^2$), correspond, roughly, to the $\Lambda \sim 10^2 - 10^5$ TeV scales (see Table 1 and related discussion below).

How this sensitivity comes about and how it can be improved, requires going into the details of a variety of flavor physics measurements.

Baryon asymmetry requires CP violation beyond SM The baryon asymmetry of the Universe is the measurement of

$$\frac{n_B - n_{\bar{B}}}{s} \approx 10^{-10}\,, \tag{1}$$

where n_B ($n_{\bar{B}}$) is the number density of (anti-)baryons and s is the entropy density. This means that 10^{-6} seconds after the Big Bang, when the temperature was $T > 1\,\text{GeV}$, and quarks and antiquarks were in thermal equilibrium, there was a corresponding asymmetry between quarks and antiquarks. Sakharov pointed out [10] that for a theory to generate such an asymmetry in the course of its evolution from a hot Big Bang (assuming inflation washed out any possible prior asymmetry), it must contain:

(1) baryon number violating interactions;
(2) C and CP violation;
(3) deviation from thermal equilibrium.

Interestingly, the SM contains 1–2–3, but (i) CP violation is too small, and (ii) the deviation from thermal equilibrium is too small at the electroweak phase transition. The SM expectation is many orders of magnitude below the observation, due to the suppression of CP violation by

$$\big[\Pi_{u_i \neq u_j}(m_{u_i}^2 - m_{u_j}^2)\big]\big[\Pi_{d_i \neq d_j}(m_{d_i}^2 - m_{d_j}^2)\big]/m_W^{12}\,, \tag{2}$$

and m_W indicates a typical weak interaction scale here.[a]

Therefore, CP violation beyond the SM must exist. While this argument does not tell us the scale of the corresponding new physics, it motivates searching for new sources of CP violation. (It may occur only in flavor-diagonal processes, such as EDMs, or only in the lepton sector, as in leptogenesis.) In any case, we want to understand the microscopic origin of CP violation, and how precisely we can test those we can measure.

Equally important is that almost all TeV-scale new physics models contain new sources of CP violation. Baryogenesis at the electroweak scale may still be viable, and the LHC will probe the remaining parameter space.

The SM and flavor The SM is defined by the gauge interactions,

$$SU(3)_c \times SU(2)_L \times U(1)_Y\,, \tag{3}$$

the particle content, i.e., three generations of the fermion representations,

$$Q_L(3,2)_{1/6}, \quad u_R(3,1)_{2/3}, \quad d_R(3,1)_{-1/3}, \quad L_L(1,2)_{-1/2}, \quad \ell_R(1,1)_{-1}\,, \tag{4}$$

and electroweak symmetry breaking. A condensate $\langle\phi\rangle = \binom{0}{v/\sqrt{2}}$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$, the dynamics of which we now know is well approximated by a seemingly elementary SM-like scalar Higgs field.

The kinetic terms in the SM Lagrangian are

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}\sum_{\rm groups}(F_{\mu\nu}^a)^2 + \sum_{\rm rep's}\overline{\psi}\, i\not{D}\,\psi\,. \tag{5}$$

These are always CP conserving, as long as we neglect a possible $F\widetilde{F}$ term. This is the "strong CP problem" [11], the solution of which is also an open question. The Higgs terms,

$$\mathcal{L}_{\rm Higgs} = |D_\mu\phi|^2 + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2\,, \tag{6}$$

[a] Why is this suppression a product of all up and down quark mass differences, while fewer factors of mass splittings suppress CP violation in hadron decays and meson mixings?

are CP conserving in the SM, but can be CP violating with an extended Higgs sector (already with two Higgs doublets; need three if natural flavor conservation is imposed [12]). Finally, the Yukawa couplings are,

$$\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} - Y^u_{ij}\,\overline{Q^I_{Li}}\,\widetilde{\phi}\,u^I_{Rj} - Y^\ell_{ij}\,\overline{L^I_{Li}}\,\phi\,\ell^I_{Rj} + \text{h.c.} \tag{7}$$

The $Y^{ij}_{u,d}$ are 3×3 complex matrices, i,j are generation indices, $\widetilde{\phi} = i\sigma_2\phi^*$.

After electroweak symmetry breaking, Eq. (7) gives quark mass terms,

$$\begin{aligned}\mathcal{L}_{\text{mass}} =& -\overline{d^I_{Li}}\,(M_d)_{ij}\,d^I_{Rj} - \overline{u^I_{Li}}\,(M_u)_{ij}\,u^I_{Rj} + \text{h.c.} \\ =& -\big(\overline{d^I_L}V^\dagger_{dL}\big)\big(V_{dL}M_dV^\dagger_{dR}\big)\big(V_{dR}\,d^I_R\big) \\ & -\big(\overline{u^I_L}V^\dagger_{uL}\big)\big(V_{uL}M_uV^\dagger_{uR}\big)\big(V_{uR}\,u^I_R\big) + \text{h.c.},\end{aligned} \tag{8}$$

where $M_f = (v/\sqrt{2})\,Y^f$. The last two lines show the diagonalization of the mass matrices necessary to obtain the physical mass eigenstates,

$$M^{\text{diag}}_f \equiv V_{fL}\,M_f\,V^\dagger_{fR}\,, \qquad f_{Li} \equiv V^{ij}_{fL}\,f^I_{Lj}\,, \qquad f_{Ri} \equiv V^{ij}_{fR}\,f^I_{Rj}\,, \tag{9}$$

where $f = u,d$ denote up- and down-type quarks. The diagonalization is different for u_{Li} and d_{Li}, which are in the same $SU(2)_L$ doublet,

$$\begin{pmatrix} u^I_{Li} \\ d^I_{Li} \end{pmatrix} = (V^\dagger_{uL})_{ij} \begin{pmatrix} u_{Lj} \\ (V_{uL}V^\dagger_{dL})_{jk}\,d_{Lk} \end{pmatrix}. \tag{10}$$

The "misalignment" between these two transformations,

$$V_{\text{CKM}} \equiv V_{uL}V^\dagger_{dL}\,, \tag{11}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. By virtue of Eq. (11), it is unitary.

Eq. (10) shows that the charged current weak interactions, which arise from the $\overline{\psi}\,i\not{D}\,\psi$ terms in Eq. (5), become non-diagonal in the mass basis

$$-\frac{g}{2}\,\overline{Q^I_{Li}}\,\gamma^\mu W^a_\mu \tau^a Q^I_{Li} + \text{h.c.} \;\Rightarrow\; -\frac{g}{\sqrt{2}}\,\big(\overline{u_L},\,\overline{c_L},\,\overline{t_L}\big)\gamma^\mu W^+_\mu\,V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}, \tag{12}$$

where $W^\pm_\mu = (W^1_\mu \mp W^2_\mu)/\sqrt{2}$. Thus, charged current weak interactions change flavor, and this is the only flavor changing interaction in the SM.

In the absence of Yukawa couplings, the SM has a global $[U(3)]^5$ symmetry ($[U(3)]^3$ in the quark and $[U(3)]^2$ in the lepton sector), rotating the 3 generations of the 5 fields in Eq. (4). This is broken by the Yukawa interactions in Eq. (7). In the quark sector the breaking is

$$U(3)_Q \times U(3)_u \times U(3)_d \;\to\; U(1)_B\,, \tag{13}$$

In the lepton sector, we do not yet know if $U(3)_L \times U(3)_\ell$ is fully broken.

Flavor and CP violation in the SM Since the Z couples flavor diagonally,[b] there are no tree-level flavor-changing neutral currents, such as $K_L \to \mu^+\mu^-$. This led GIM [13] to predict the existence of the charm quark. Similarly, $K^0 - \overline{K}^0$ mixing vanishes at tree-level, which allowed the prediction of m_c [14; 15] before the discovery of the charm quark. In the previous examples, because of the unitarity of the CKM matrix,

$$V_{ud}\, V_{us}^* + V_{cd}\, V_{cs}^* + V_{td}\, V_{ts}^* = 0\,. \tag{14}$$

Expanding the loop functions, e.g., in a FCNC kaon decay amplitude,

$$V_{ud}\, V_{us}^*\, f(m_u) + V_{cd}\, V_{cs}^*\, f(m_c) + V_{td}\, V_{ts}^*\, f(m_t)\,, \tag{15}$$

the result is always proportional to the up-quark mass-squared differences,

$$\frac{m_i^2 - m_j^2}{m_W^2}\,. \tag{16}$$

So FCNCs probe directly the differences between the generations.

One can also see that CP violation is related to irremovable phases of Yukawa couplings. Starting from a term in Eq. (7),

$$Y_{ij}\, \overline{\psi_{Li}}\, \phi\, \psi_{Rj} + Y_{ij}^*\, \overline{\psi_{Rj}}\, \phi^\dagger\, \psi_{Li} \;\stackrel{CP}{\longrightarrow}\; Y_{ij}\, \overline{\psi_{Rj}}\, \phi^\dagger\, \psi_{Li} + Y_{ij}^*\, \overline{\psi_{Li}}\, \phi\, \psi_{Rj}\,. \tag{17}$$

The two expressions are identical if and only if a basis for the quark fields can be chosen such that $Y_{ij} = Y_{ij}^*$, i.e., that Y_{ij} are real.

Counting flavor parameters Most parameters of the SM (and also of many of its extensions) are related to flavor. In the CKM matrix, due to unitarity, 9 complex elements depend on 9 real parameters. Of these 5 phases can be absorbed by redefining the quark fields, leaving 4 physical parameters, 3 mixing angles and 1 CP violating phase. This is the only source of CP violation in flavor changing transitions in the SM.

A more general way to account for all flavor parameters is to consider that the two Yukawa matrices, $Y_{i,j}^{u,d}$ in Eq. (7), contain 18 real and 18 imaginary parameters. They break the global $[U(3)]^3 \to U(1)_B$, see Eq. (13), so there are 26 broken generators (9 real and 17 imaginary). This leaves 10 physical quark flavor parameters: 9 real ones (the 6 quark masses and 3 mixing angles) and 1 complex CP violating phase.[c]

[b] Show that there are no tree-level flavor-changing Z couplings in the SM. What if, besides doublets, there were a left-handed $SU(2)$ singlet quark field as well?

[c] Show that for N generations, the CKM matrix depends on $N(N-1)/2$ mixing angles and $(N-1)(N-2)/2$ CP violating phases. So the 2-generation SM conserves CP.

Neutrino masses How does lepton flavor differ? With the particle content in Eq. (4), it is not possible to write down a renormalizable mass term for neutrinos. It would require introducing a $\nu_R(1,1)_0$ field, a singlet under all SM gauge groups, to be light, which is unexpected. Such a particle is sometimes called a sterile neutrino, as it has no SM interactions. Whether there are such fields can only be decided experimentally.

Viewing the SM as a low energy effective theory, there is a single type of dimension-5 gauge invariant term made of SM fields,

$$\mathcal{L}_Y = -\frac{Y_\nu^{ij}}{\Lambda_{\rm NP}}\, L_{Li}^I\, L_{Lj}^I\, \phi\, \phi\,. \tag{18}$$

This term gives rise to neutrino masses and also violates lepton number. Its suppression cannot be the electroweak scale, $1/v$ (instead of $1/\Lambda_{\rm NP}$), because such a term in the Lagrangian cannot be generated from SM fields at arbitrary loop level, or even nonperturbatively. [Eq. (18) violates $B-L$, which is an accidental symmetry of the SM that is not anomalous.] The above mass term is called a Majorana mass, as it couples $\overline{\nu_L}$ to $(\nu_L)^c$ instead of ν_R [the latter occurs for Dirac mass terms, see Eq. (8)]. The key distinction is whether lepton number is violated or conserved. In the presence of Eq. (18) and the charged lepton Yukawa coupling in the last term in Eq. (7), the global $U(3)_L \times U(3)_\ell$ symmetry is completely broken, and the counting of lepton flavor parameters is[d]

$$(12 + 18 \text{ couplings}) - (18 \text{ broken sym.}) \;\Rightarrow\; 12 \text{ physical parameters}\,. \tag{19}$$

These are the 6 masses, 3 mixing angles, and 3 CP violating phases, of which one is the analog of the CKM phase measurable in oscillation experiments, while two additional “Majoran phases” only contribute to lepton number violating processes, such as neutrinoless double beta decay.[e]

The CKM matrix Quark mixing is observed to be approximately flavor diagonal. The Wolfenstein parameterization conveniently exhibits this,

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \ldots\,, \tag{20}$$

[d]Show that the Yukawa matrix in Eq. (18) is symmetric, $Y_\nu^{ij} = Y_\nu^{ji}$. Derive that for N such generations there are $N(N-1)/2$ CP violating phases.
[e]Can you think of ways to get sensitivity to another linear combination of the two CP violating Majorana phases, besides the one that enters neutrinoless double beta decay?

where $\lambda \simeq 0.23$ may be viewed as an "expansion parameter". It is a useful book-keeping of the magnitudes of the CKM matrix elements, but it hides which combination of CKM elements are phase convention independent. Sometimes it can be useful to think of V_{ub} and V_{td} as the ones with $\mathcal{O}(1)$ CP violating phases, but it is important that any CP violating observable in the SM must depend on at least four CKM elements.[f]

In any case, the interesting question is not primarily measuring CKM elements, but testing how precisely the SM description of flavor and CP violation holds. This can be done by "redundant" measurements, which in the SM relate to some combination of flavor parameters, but are sensitive to different BSM physics, thus testing for (in)consistency. Since there are many experimental constraints, a simple way to compare different measurements can be very useful. Recall that CKM unitarity implies

$$\sum_k V_{ik} V_{jk}^* = \sum_k V_{ki} V_{kj}^* = \delta_{ij} \,, \tag{21}$$

and the 6 vanishing relations can be represented as triangles in a complex plane. The most often used such "unitarity triangle" (shown in Fig. 1) arises from the scalar product of the 1st and 3rd columns,

$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td}\, V_{tb}^* = 0 \,. \tag{22}$$

(Unitarity triangles constructed from neighboring columns or rows are "squashed".) We define the α, β, γ angles of this triangle, and two more,

$$\begin{aligned} &\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right), \\ &\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right), \quad \beta_K \equiv \arg\left(-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right). \end{aligned} \tag{23}$$

On different continents the $\phi_1 = \beta$, $\phi_2 = \alpha$, $\phi_3 = \gamma$, and/or the $\phi_s = -2\beta_s$ notations are used. Here β_s (β_K), of order λ^2 (λ^4), is the small angle of a "squashed" unitarity triangle obtained by multiplying the 2nd column of the CKM matrix with the 3rd (1st) column.

The magnitudes of CKM elements determine the sides of the unitarity triangle. They are mainly extracted from semileptonic and leptonic K and B decays, and $B_{d,s}$ mixing. Any constraint which renders the area of the unitarity triangle nonzero, such as angles, has to measure CP violation. Some of the most important constraints are shown in Fig. 2, together with the CKM fit in the SM. (Using $\bar\rho$, $\bar\eta$ instead of ρ, η simply corresponds to a small modification of the parameterization, to keep unitarity exact.)

[f] Prove this statement. Are there constraints on which four?

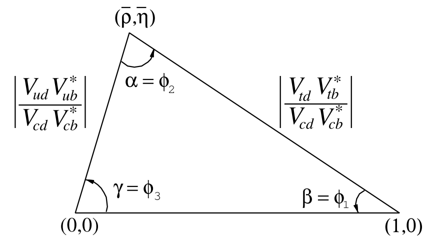


Fig. 1. The unitarity triangle.

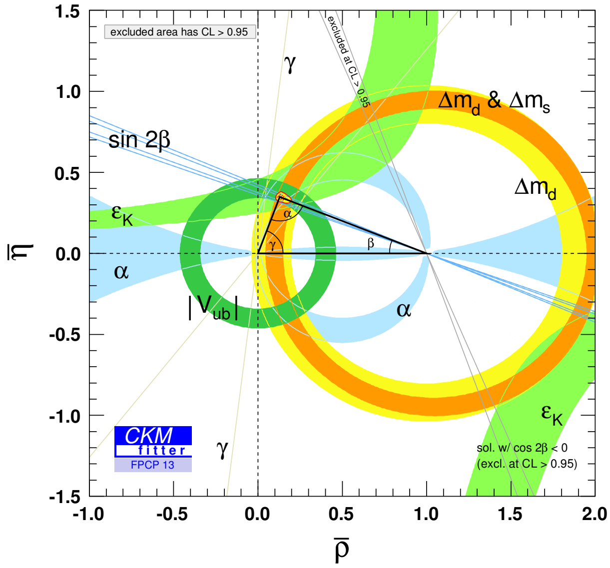


Fig. 2. The SM CKM fit, and individual constraints (colored regions show 95% CL).

The low energy EFT viewpoint At the few GeV scale, relevant for B, D, and some K decays, all flavor changing processes (whether tree or loop level) are mediated by dozens of higher dimension local operators. They arise from integrating out heavy particles, W and Z bosons and the t quark in the SM, or not yet observed heavy states (see Fig. 3). Since the coefficients of a large number of operators depend on just a few parameters in the SM, there are many correlations between decays of hadrons containing s, c, b quarks, which NP may violate. From this point of view there is no difference between flavor-changing neutral currents and $\Delta F = 1$ processes, as all flavor changing transitions are generated at scales $\gg m_{s,c,b}$. Measur-

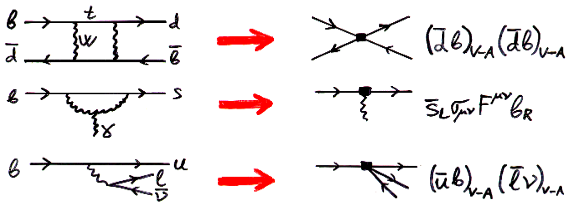


Fig. 3. Diagrams at the electroweak scale (left) and operators at the scale m_b (right).

ing such observables, one can test the SM in many ways by asking (i) does NP modify the coefficients of dimension-6 operators? (ii) does NP generate operators absent in the SM (e.g., right-handed couplings)?

Neutral meson mixing Let us first sketch a back-of-an-envelope estimate of the mass difference in $K^0-\overline{K}^0$ mixing. In the SM,

$$\Delta m_K \sim \alpha_w^2\, |V_{cs}V_{cd}|^2\, \frac{m_c^2 - m_u^2}{m_W^4}\, f_K^2\, m_K\,. \tag{24}$$

The result is suppressed by CKM angles, a loop factor, the weak coupling, and the GIM mechanism. If a heavy particle, X, contributes $\mathcal{O}(1)$ to Δm_K,

$$\left|\frac{\Delta m_K^{(X)}}{\Delta m_K^{(\rm exp)}}\right| \sim \left|\frac{g^2\, \Lambda_{\rm QCD}^3}{M_X^2\, \Delta m_K^{(\rm exp)}}\right| \quad \Rightarrow \quad M_X \gtrsim g \times 2 \cdot 10^3\, \text{TeV}\,. \tag{25}$$

So even TeV-scale particles with loop-suppressed couplings $[g \sim \mathcal{O}(10^{-3})]$ can give observable effects. This illustrates that flavor physics measurements indeed probe the TeV scale if NP has SM-like flavor structure, and much higher scales if the NP flavor structure is generic.

A more careful evaluation of the bounds in all four neutral meson systems are shown in Table 1. If $\Lambda = \mathcal{O}(1\,\text{TeV})$ then $C \ll 1$, and if $C = \mathcal{O}(1)$ then $\Lambda \gg 1\,\text{TeV}$. The bounds are weakest for $B_{(s)}$ mesons, as mixing is the least suppressed in the SM in that case. The bounds on many NP models are the strongest from Δm_K and ϵ_K, since so are the SM suppressions. These are built into NP models since the 1970s, otherwise the models are immediately excluded. In the SM, larger FCNCs and CP violating effects occur in B mesons, which can be measured precisely. In many BSM models the 3rd generation is significantly different than the first two, motivated by the large top Yukawa, and may give larger signals in the B sector.

Few more words on kaons With recent lattice QCD progress on B_K and f_K [16], ϵ_K has become a fairly precise constraint on the SM. However,

Table 1. Bounds on some $\Delta F = 2$ operators, $(C/\Lambda^2)\,\mathcal{O}$, with $\mathcal{O}$ given in the first column. The bounds on Λ assume $C = 1$, the bounds on C assume $\Lambda = 1\,\text{TeV}$. (From Ref. [17].)

Operator	Bound on $\Lambda\,[\text{TeV}]$ $(C=1)$		Bound on C $(\Lambda = 1\,\text{TeV})$		Observables
	Re	Im	Re	Im	
$(\bar s_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar s_R\, d_L)(\bar s_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar c_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar c_R\, u_L)(\bar c_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar b_L\gamma^\mu d_L)^2$	6.6×10^2	9.3×10^2	2.3×10^{-6}	1.1×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar b_R\, d_L)(\bar b_L d_R)$	2.5×10^3	3.6×10^3	3.9×10^{-7}	1.9×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar b_L\gamma^\mu s_L)^2$	1.4×10^2	2.5×10^2	5.0×10^{-5}	1.7×10^{-5}	Δm_{B_s}; $S_{\psi\phi}$
$(\bar b_R\, s_L)(\bar b_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	Δm_{B_s}; $S_{\psi\phi}$

ϵ'_K is notoriously hard to calculate, involving cancellation between two comparable terms, each with sizable uncertainties. (Lattice QCD calculations of the hadronic matrix elements for ϵ'_K may be reliably computed in the future.) At present, we cannot prove nor rule out that a large part of the observed value of ϵ'_K is due to BSM. Thus, to test CP violation, one had to consider other systems; it was realized in the 1980s that many precise measurements of CP violation are possible in B decays.

In the kaon sector, precise calculations of rare decays involving neutrinos (see Fig. 4) are possible, and the SM predictions are

$$\mathcal{B}(K^+\to\pi^+\nu\bar\nu) = (7.8\pm0.8)\times10^{-11}, \quad \mathcal{B}(K_L^0\to\pi^0\nu\bar\nu) = (2.4\pm0.4)\times10^{-11}. \tag{26}$$

The K_L^0 decay is CP violating, and therefore it is under especially good theoretical control, since it is determined by the top quark loop contributions, and CP conserving charm quark contributions are absent (which enter $K^+\to\pi^+\nu\bar\nu$ and are subject to some hadronic uncertainties).

Our current knowledge from 7 events at E787/E949 is $\mathcal{B}(K\to\pi^+\nu\bar\nu) = (17.3^{+11.5}_{-10.5})\times10^{-11}$, whereas in the K_L mode the bound is many times the SM rate. NA62 at CERN aims to measure the K^+ rate with 10% uncertainty, and will start to have dozens of events in 2015. The K_L mode will probably be first observed by the KOTO experiment at J-PARC.

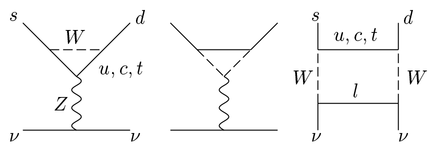


Fig. 4. Diagrams contributing to $K\to\pi\nu\bar\nu$ decay.

2. Theory of Some Important B Decays

Studying FCNC and CP violation is particularly interesting in B meson decays, because many measurements are possible with clean interpretations.

The main theoretical reasons are: (i) t quark loops are neither GIM nor CKM suppressed; (ii) large CP violating effects possible; (iii) some of the hadronic physics is understandable model independently ($m_b \gg \Lambda_{\rm QCD}$).

The main experimental reasons: (i) long B lifetime (small $|V_{cb}|$); (ii) the $\Upsilon(4S)$ is a clean source of B-s in e^+e^- colliders; (iii) for B_d, $\Delta m/\Gamma = \mathcal{O}(1)$.

Neutral meson mixing formalism Similar to neutral kaons, there are two neutral B^0 meson flavor eigenstates,

$$|B^0\rangle = |\bar{b}\, d\rangle\,, \qquad |\overline{B}{}^0\rangle = |b\, \bar{d}\rangle\,. \tag{27}$$

Their time evolutions are described by the Schrödinger equation,

$$i\, \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} |B^0(t)\rangle \\ |\overline{B}{}^0(t)\rangle \end{pmatrix} = \Big(M - \frac{i}{2}\, \Gamma\Big) \begin{pmatrix} |B^0(t)\rangle \\ |\overline{B}{}^0(t)\rangle \end{pmatrix}, \tag{28}$$

where the mass mixing matrix, M, and the decay mixing matrix, Γ, are 2×2 Hermitian matrices. CPT invariance implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The heavier and lighter mass eigenstates are the eigenvectors of $M - i\Gamma/2$,

$$|B_{H,L}\rangle = p\, |B^0\rangle \mp q\, |\overline{B}{}^0\rangle\,, \tag{29}$$

and their time dependence is

$$|B_{H,L}(t)\rangle = e^{-(im_{H,L} + \Gamma_{H,L}/2)t}\, |B_{H,L}\rangle\,. \tag{30}$$

where $\Delta m \equiv m_H - m_L$ and $\Delta\Gamma = \Gamma_L - \Gamma_H$. This defines Δm to be positive, but the sign of $\Delta\Gamma$ is physical. Note that $m_{H,L}$ ($\Gamma_{H,L}$) are not the eigenvalues of M (Γ).[g] The off-diagonal elements, M_{12} and Γ_{12}, arise from virtual and on-shell intermediate states, respectively. M_{12} is dominated in the SM by top quark box diagrams in Fig. 5, hence it is determined by short

[g]Derive that the time evolutions of mesons that are B^0 and $\overline{B}{}^0$ at $t=0$ are given by

$$|B^0(t)\rangle = g_+(t)\, |B^0\rangle + \frac{q}{p}\, g_-(t)\, |\overline{B}{}^0\rangle\,, \qquad |\overline{B}{}^0(t)\rangle = \frac{p}{q}\, g_-(t)\, |B^0\rangle + g_+(t)\, |\overline{B}{}^0\rangle\,, \tag{31}$$

where, denoting $m = (m_H + m_L)/2$ and $\Gamma = (\Gamma_H + \Gamma_L)/2$,

$$\begin{aligned} g_+(t) &= e^{-it(m - i\Gamma/2)} \bigg(\cosh\frac{\Delta\Gamma\, t}{4}\, \cos\frac{\Delta m\, t}{2} - i \sinh\frac{\Delta\Gamma\, t}{4}\, \sin\frac{\Delta m\, t}{2} \bigg)\,, \\ g_-(t) &= e^{-it(m - i\Gamma/2)} \bigg(- \sinh\frac{\Delta\Gamma\, t}{4}\, \cos\frac{\Delta m\, t}{2} + i \cosh\frac{\Delta\Gamma\, t}{4}\, \sin\frac{\Delta m\, t}{2} \bigg)\,. \end{aligned} \tag{32}$$

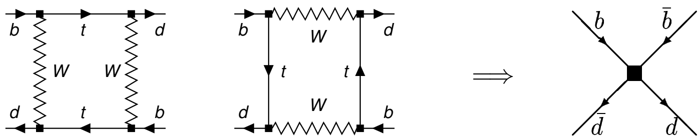


Fig. 5. Left: box diagrams that give rise to the $B^0 - \overline{B}^0$ mass difference; Right: operator in the effective theory below m_W whose B meson matrix element determines Δm_B.

Table 2. Orders of magnitudes of the SM predictions for mixing parameters. The uncertainty of $(|q/p| - 1)_D$ is especially large.

meson	$x = \Delta m/\Gamma$	$y = \Delta\Gamma/(2\Gamma)$	$\lvert q/p\rvert - 1$
K	1	1	10^{-3}
D	10^{-2}	10^{-2}	10^{-3}
B_d	1	10^{-2}	10^{-4}
B_s	10^{1}	10^{-1}	10^{-5}

distance physics and is calculable with good accuracy, and sensitive to high scales. (This is a complication for D mixing: the W can always be shrunk to a point, but the d and s quarks in the box diagrams cannot, so long distance effects are important.) On the other hand, Γ_{12} is determined by on-shell physical states to which both B^0 and $\overline{B}^0$ can decay, corresponding to c and u quarks in the box diagrams.

The solution of the eigenvalue equation is

$$(\Delta m)^2 - \frac{(\Delta\Gamma)^2}{4} = 4\,|M_{12}|^2 - |\Gamma_{12}|^2\,, \qquad \Delta m\,\Delta\Gamma = -4\,\mathrm{Re}(M_{12}\Gamma_{12}^*)\,,$$
$$\frac{q}{p} = -\frac{\Delta m + i\,\Delta\Gamma/2}{2M_{12} - i\,\Gamma_{12}} = -\frac{2M_{12}^* - i\,\Gamma_{12}^*}{\Delta m + i\,\Delta\Gamma/2}\,. \tag{33}$$

Physical observables measurable in neutral meson mixing are

$$x = \frac{\Delta m}{\Gamma}\,, \qquad y = \frac{\Delta\Gamma}{2\Gamma}\,, \qquad \left|\frac{q}{p}\right| - 1\,. \tag{34}$$

The orders of magnitudes of the SM predictions are shown in Table 2. That $x \neq 0$ is established in the K, B, and B_s mixing; $y \neq 0$ in the K, D, and B_s mixing; $|q/p| \neq 1$ in K mixing. The significance of $x_D \neq 0$ is $\sim 2\sigma$, and in $B_{d,s}$ mixing there is an unconfirmed DØ signal for $|q/p| \neq 1$; more below.

Simpler approximate solutions can be obtained expanding about the limit $|\Gamma_{12}| \ll |M_{12}|$. This is a good approximation in both B_d and B_s systems. $|\Gamma_{12}| < \Gamma$ always holds, because Γ_{12} arises from decays to final states common to B^0 and $\overline{B}^0$. For B_s mixing the world average is $\Delta\Gamma_s/\Gamma_s = 0.138 \pm 0.012$ [18], while $\Delta\Gamma_d$ is expected to be ~ 20 times smaller and is not yet measured. Up to higher order terms in $|\Gamma_{12}/M_{12}|$, Eqs. (33) become

$$\Delta m = 2\,|M_{12}|\,, \qquad \Delta\Gamma = -2\,\frac{\mathrm{Re}(M_{12}\Gamma_{12}^*)}{|M_{12}|}\,,$$
$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}\left(1 - \frac{1}{2}\,\mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}}\right), \tag{35}$$

where we kept the second term in q/p, as it will be needed later.

***CP* violation in decay** This is any form of CP violation that cannot be absorbed in a neutral meson mixing amplitude (also called direct CP violation). It can occur in any hadron decay, as opposed to those specific to neutral mesons discussed below. For a given final state, f, the $B \to f$ and $\overline{B} \to \overline{f}$ decay amplitudes can, in general, receive several contributions

$$A_f = \langle f|\mathcal{H}|B\rangle = \sum_k A_k\, e^{i\delta_k}\, e^{i\phi_k}, \qquad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle = \sum_k A_k\, e^{i\delta_k}\, e^{-i\phi_k}. \tag{36}$$

There are two types of complex phases. Complex parameters in the Lagrangian which enter a decay amplitude also enter the CP conjugate amplitude but in complex conjugate form. In the SM such "weak phases", ϕ_k, only occur in the CKM matrix. Another type of phases are due to absorptive parts of decay amplitudes, and give rise to CP conserving "strong phases", δ_k. These arise from on-shell intermediate states rescattering into the desired final state, and they are the same for an amplitude and its CP conjugate. The individual phases δ_k and ϕ_k are convention dependent, but the phase differences, $\delta_i - \delta_j$ and $\phi_i - \phi_j$, and therefore $|\overline{A}_{\overline{f}}|$ and $|A_f|$, are physical. Clearly, if $|\overline{A}_{\overline{f}}| \neq |A_f|$ then CP is violated; this is called CP violation in decay, or direct CP violation.[h]

There are many observations of direct CP violation by now. While some give strong constraints on NP that does not contain all the SM suppressions (e.g., ϵ'_K, the first direct CP violation measured with high significance), at present no single direct CP violation measurement gives a precise test of the SM, due to the lack of reliable calculations of relevant strong phases. For all observations of direct CP violation [viewed in itself, see caveat near Eq. (42)], it is possible that, say, half of the measured value is from BSM. For ϵ'_K, lattice QCD may yield progress in the future. In certain B decays we may better understand the implications of the heavy quark limit; so far $A_{K^+\pi^0} - A_{K^+\pi^-} = 0.12 \pm 0.02$ [18], the "$K\pi$ puzzle", is poorly understood.

[h]Derive that direct CP violation requires interference of at least two contributing amplitudes with different strong and weak phases, $|\overline{A}|^2 - |A|^2 = 4A_1A_2\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)$.

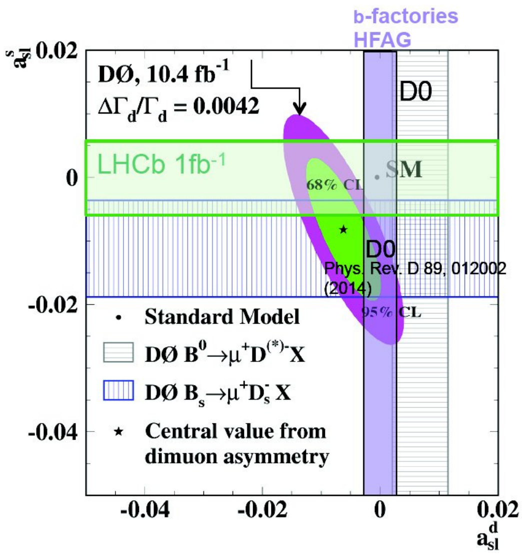


Fig. 6. Status of $A_{\rm SL}$ measurements (from M. Artuso, talk at FPCP 2014). The DØ result is in a 3.6σ tension with the SM expectation.

***CP* violation in mixing** If CP were conserved, the mass and CP eigenstates would coincide, and the mass eigenstates would be proportional to $|B^0\rangle \pm |\overline{B}^0\rangle$, up to phases, corresponding to $|q/p| = 1$ and $\arg(M_{12}/\Gamma_{12}) = 0$. If $|q/p| \neq 1$, then CP is violated. This is called CP violation in mixing. It follows from Eq. (29) that $\langle B_H|B_L\rangle = |p|^2 - |q|^2$, so if CP is violated in mixing, the physical states are not orthogonal. (This again illustrates that CP violation is a quantum mechanical effect, impossible in a classical system.) The simplest example is the CP asymmetry in semileptonic decay of neutral mesons to "wrong sign" leptons (Fig 6 summarizes the data),

$$A_{\rm SL}(t) = \frac{\Gamma(\overline{B}^0(t) \to \ell^+ X) - \Gamma(B^0(t) \to \ell^- X)}{\Gamma(\overline{B}^0(t) \to \ell^+ X) + \Gamma(B^0(t) \to \ell^- X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \simeq \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}}\,. \tag{37}$$

To obtain the right-hand side, use Eqs. (29) and (30) for the time evolution, and Eq. (35) for $|q/p|$. In kaon decays this asymmetry is measured [19], in agreement with the SM prediction, $4\,\mathrm{Re}\,\epsilon_K$. In B_d and B_s decays the asymmetry is expected to be [20]

$$A^d_{\rm SL} \approx -4 \times 10^{-4}\,, \qquad A^s_{\rm SL} \approx 2 \times 10^{-5}\,. \tag{38}$$

The calculation of $\mathrm{Im}(\Gamma_{12}/M_{12})$ requires calculating inclusive nonleptonic decay rates, which can be addressed using an operator product expansion in the $m_b \gg \Lambda_{\rm QCD}$ limit. Such a calculation has sizable hadronic uncertainties, the details of which would lead to a long discussion. The constraints on new physics are significant nevertheless [21], as the m_c^2/m_b^2 suppression of $A_{\rm SL}$ in the SM can be avoided in the presence of new physics.

CP violation in the interference of decay with and without mixing
A third type of CP violation is possible when both B^0 and $\overline{B}^0$ can decay to a final state, f. In the simplest cases, when f is a CP eigenstate, define

$$\lambda_f = \frac{q}{p}\,\frac{\overline{A}_f}{A_f}\,. \tag{39}$$

If there is no direct CP violation in a given mode, then $\overline{A}_f = \eta_f\,\overline{A}_{\overline{f}}$, where $\eta_f = \pm 1$ is the CP eigenvalue of f [$+1$ (-1) for CP-even (-odd) states]. This is useful, because A_f and $\overline{A}_{\overline{f}}$ are related by CP transformation. If CP is conserved, then not only $|q/p| = 1$ and $|\overline{A}_{\overline{f}}/A_f| = 1$, but the relative phase between q/p and $\overline{A}_f/A_f$ also vanishes, hence $\lambda_f = \pm 1$.

The experimentally measurable CP violating observable is[i]

$$\begin{aligned} a_f &= \frac{\Gamma[\overline{B}^0(t)\to f] - \Gamma[B^0(t)\to f]}{\Gamma[\overline{B}^0(t)\to f] + \Gamma[B^0(t)\to f]} \\ &= -\frac{(1-|\lambda_f|^2)\cos(\Delta m\,t) - 2\,{\rm Im}\,\lambda_f \sin(\Delta m\,t)}{1+|\lambda_f|^2} \\ &\equiv S_f \sin(\Delta m\,t) - C_f \cos(\Delta m\,t)\,, \end{aligned} \tag{40}$$

where we have neglected $\Delta\Gamma$ (important in the B_s system). The last line defines the S and C coefficients, which are fit to the experimental data (see Fig. 8). If ${\rm Im}\lambda_f \neq 0$, then CP violation arises in the interference between the decay $B^0 \to f$, and mixing followed by decay, $B^0 \to \overline{B}^0 \to f$.

This asymmetry can be nonzero if any type of CP violation occurs. In particular, in both the B_d and B_s systems $\big||q/p| - 1\big| < \mathcal{O}(10^{-2})$ model independently, and it is much smaller in the SM [see, Eq. (38)]. If, in addition, amplitudes with a single weak phase dominate a decay, then $|\overline{A}_f/A_f| \simeq 1$, and $\arg(\overline{A}_f/A_f)$ is just (twice) the weak phase, determined by short-distance physics. It is then possible that ${\rm Im}\lambda_f \neq 0$, $|\lambda_f| \simeq 1$, and although we cannot compute the decay amplitude, we can extract the weak phase difference between $B^0 \to f$ and $B^0 \to \overline{B}^0 \to f$ in a theoretically clean way from the measurement of

$$a_f = {\rm Im}\lambda_f \sin(\Delta m\,t)\,. \tag{41}$$

[i] Derive the CP asymmetry in Eq. (40 using Eq. (31)). For extra credit, keep $\Delta\Gamma \neq 0$.

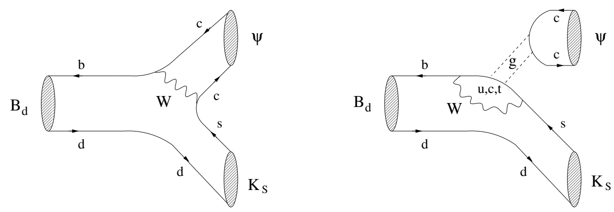


Fig. 7. "Tree" (left) and "penguin" (right) contributions to $B \to \psi K_S$ (from Ref. [22]).

There is an interesting subtlety. Considering two final states, it is possible that direct CP violation in each channel, $|\lambda_{f_1}| - 1$ and $|\lambda_{f_2}| - 1$, are unmeasurably small, but direct CP violation is detectable nevertheless. If

$$\eta_{f_1}\mathrm{Im}(\lambda_{f_1}) \neq \eta_{f_2}\mathrm{Im}(\lambda_{f_2})\,, \tag{42}$$

then CP violation must occur outside the mixing amplitude, even though it may be invisible in the data on any one final state.

$\sin 2\beta$ from $B \to \psi K_{S,L}$ This is one of the cleanest examples of CP violation in the interference between decay with and without mixing, and one of the theoretically cleanest measurements of a CKM parameter.

There are "tree" and "penguin" contributions to $B \to \psi K_{S,L}$, with different weak and strong phases (see Fig. 7). The tree contribution is dominated by $b \to c\bar{c}s$ transition, while there are penguin contributions with three different combinations of CKM elements,

$$\overline{A}_T = V_{cb}V_{cs}^*\,T_{c\bar{c}s}\,, \qquad \overline{A}_P = V_{tb}V_{ts}^*\,P_t + V_{cb}V_{cs}^*\,P_c + V_{ub}V_{us}^*\,P_u\,. \tag{43}$$

(P_u can be defined to absorb the $V_{ub}V_{us}^*\,T_{u\bar{u}s}$ "tree" contribution.) We can rewrite the decay amplitude using $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* + V_{ub}V_{us}^* = 0$ to obtain

$$\begin{aligned}\overline{A} &= V_{cb}V_{cs}^*\,(T_{c\bar{c}s} + P_c - P_t) + V_{ub}V_{us}^*\,(P_u - P_t)\\ &\equiv V_{cb}V_{cs}^*\,T + V_{ub}V_{us}^*\,P\,,\end{aligned} \tag{44}$$

where the second line defines T and P. Since $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \approx 0.02$, the T amplitude with $V_{cb}V_{cs}^*$ weak phase dominates. Thus,

$$\lambda_{\psi K_{S,L}} = \mp\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right) = \mp e^{-2i\beta}\,, \tag{45}$$

and so $\mathrm{Im}\lambda_{\psi K_{S,L}} = \pm\sin 2\beta$. The first term is the SM value of q/p in B_d mixing, the second is $\overline{A}/A$, the last one is p/q in the K^0 system, and $\eta_{\psi K_{S,L}} = \mp 1$. Note that without $K^0 - \overline{K}^0$ mixing there would be no interference between $\overline{B}^0 \to \psi\overline{K}^0$ and $B^0 \to \psi K^0$. The accuracy of the

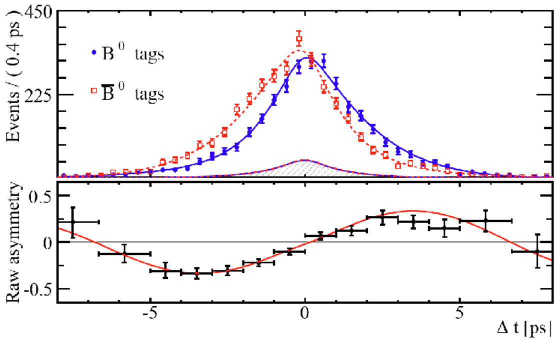


Fig. 8. Time dependence of tagged $B \to \psi K$ decays (top), and CP asymmetry (below).

relation between $\lambda_{\psi K_{S,L}}$ and $\sin 2\beta$ depends on model dependent estimates of $|P/T|$, which are below unity, so one expects it to be of order

$$\left| \frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} \frac{P}{T} \right| \lesssim 10^{-2} . \tag{46}$$

The absence of detectable direct CP violation does not in itself bound this. To fully utilize future LHCb and Belle II data, better estimates are needed.

The first evidence for CP violation outside the kaon sector was the BaBar and Belle measurements of $S_{\psi K}$. The current world average is [18]

$$\sin 2\beta = 0.682 \pm 0.019 . \tag{47}$$

This is consistent with other constraints, and showed that CP violation in quark mixing is an $\mathcal{O}(1)$ effect, which is simply suppressed in K decays by small flavor violation suppressing the 3rd generation's contributions.

$\phi_s \equiv -2\beta_s$ from $B \to \psi\phi$ The analogous CP asymmetry in B_s decay, sensitive to BSM contributions to $B_s - \overline{B}_s$ mixing, is $B_s \to \psi\phi$. Since the final state consists of two vector mesons, it is a combination of CP-even ($L = 0, 2$) and CP-odd ($L = 1$) partial waves. What is actually measured is the time dependent CP asymmetry for each CP component of the $\psi K^+ K^-$ and $\psi\pi^+\pi^-$ final states. The SM prediction is suppressed compared to β by λ^2, and is rather precise, $\beta_s = 0.0182^{+0.0007}_{-0.0006}$ [23]. The latest LHCb result using $3\,\text{fb}^{-1}$ data is [24] (Fig. 9 shows all measurements)

$$\phi_s \equiv -2\beta_s = -0.010 \pm 0.039 , \tag{48}$$

which has an uncertainty approaching that of 2β, suggesting that the "room for new physics" in B_s mixing is no longer larger than in B_d (more below).

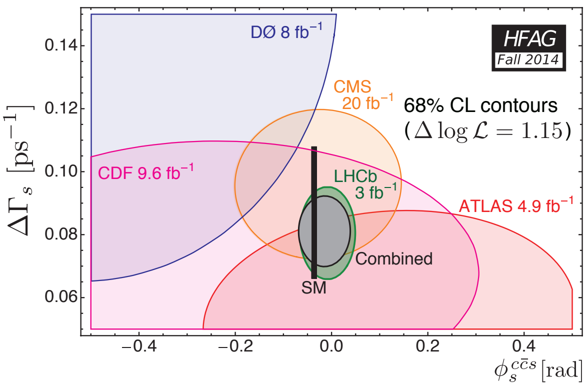


Fig. 9. Measurements of CP violation in $B_s \to \psi\phi$ and $\Delta\Gamma_s$ (from Ref. [18]).

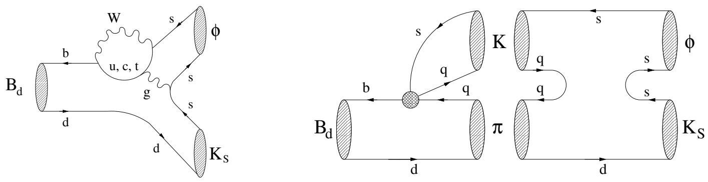


Fig. 10. "Penguin" (left) and "tree" (right) contributions to $B \to \phi K_S$ (from Ref. [22]).

"Penguin-dominated" measurements of $\beta_{(s)}$ Time dependent CP violation in $b \to s$ dominated decays is a sensitive probe of new physics. Tree-level contributions to $b \to s\bar{s}s$ transition are expected to be small, and the penguin contributions to $B \to \phi K_S$ (left diagram in Fig. 10) are

$$\overline{A}_P = V_{cb}V_{cs}^* \left(P_c - P_t\right) + V_{ub}V_{us}^* \left(P_u - P_t\right). \tag{49}$$

Due to $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \approx 0.02$ and expecting $|P_c - P_t|/|P_u - P_t| = \mathcal{O}(1)$, the $B \to \phi K_S$ amplitude is also dominated by a single weak phase, $V_{cb}V_{cs}^*$. Therefore, the theory uncertainty relating $S_{\phi K_S}$ to $\sin 2\beta$ is small, although larger than in $B \to \psi K_S$. There is also a "tree" contribution from $b \to u\bar{u}s$ followed by $u\bar{u} \to s\bar{s}$ rescattering (right diagram in Fig. 10). This amplitude is proportional to the suppressed CKM combination, $V_{ub}V_{us}^*$, and it is actually not separable from $P_u - P_t$. Unless its matrix element is largely enhanced, it should not upset the $\mathrm{Im}\lambda_{\phi K_S} = \sin 2\beta + \mathcal{O}(\lambda^2)$ expectation in the SM. Similar reasons make many other modes, such as $B \to \eta^{(\prime)} K_S$, $B_s \to \phi\phi$, etc., interesting and promising to study.

The determinations of γ and α By virtue of Eq. (23), γ does not depend on CKM elements involving the top quark, so it can be measured in tree-level B decays. This is an important distinction from α and β, and implies that γ is less likely to be affected by BSM physics.

Most measurements of γ utilize the fact that interference of $B^- \to D^0 K^-$ ($b \to c\bar{u}s$) and $B^- \to \overline{D}^0 K^-$ ($b \to u\bar{c}s$) transitions can be studied in final states accessible in both D^0 and $\overline{D}^0$ decays [25]. (A notable exception is the measurement from the four time dependent $\overline{B}_s$ and $B_s \to D_s^\pm K^\mp$ rates, possible at LHCb.) It is possible to measure the B and D decay amplitudes, their relative strong phases, and the weak phase γ from the data. There are many variants, based on different D decay channels [26; 27; 28; 29; 30; 31]. The best current measurement comes from $D^0, \overline{D}^0 \to K_S\,\pi^+\pi^-$ [30; 31], in which case both amplitudes are Cabibbo allowed, and the analysis can be optimized by studying the Dalitz plot dependence of the interference. The world average of all γ measurements is [23]

$$\gamma = \left(73.2^{+6.3}_{-7.0}\right)^\circ. \tag{50}$$

Most importantly, the theory uncertainty in the SM measurement is smaller than the accuracy of any planned or imaginable future experiment.

The measurements usually referred to as determining α, measure $\pi - \beta - \gamma$, the third angle of the unitarity triangle in any model in which the unitarity of the 3×3 CKM matrix is maintained. These measurements are in time dependent CP asymmetries in $B \to \pi\pi$, $\rho\rho$, and $\rho\pi$ decays. In these decays the $b \to u\bar{u}d$ "tree" amplitudes are not much larger than the $b \to \sum_q q\bar{q}d$ "penguin" contributions, which have different weak phases.[j] The tree contributions change isospin by $\Delta I = 3/2$ or $1/2$, while the penguin contribution is $\Delta I = 1/2$ only. It is possible to use isospin symmetry of the strong interaction to isolate CP violation in the $\Delta I = 3/2$ channel, eliminating the penguin contributions [32; 33; 34], yielding [23]

$$\alpha = \left(87.7^{+3.5}_{-3.3}\right)^\circ. \tag{51}$$

Thus, the measurements of α are sensitive to new physics in $B^0 - \overline{B}^0$ mixing and via possible $\Delta I = 5/2$ new physics amplitudes [35].

New physics in B_d and B_s mixing Although the SM CKM fit in Fig. 2 shows impressive and nontrivial consistency, the implications of the level of agreement are often overstated. Allowing new physics contributions, there are a larger number of parameters related to CP and flavor violation, and

[j] Show that if the "tree" amplitudes dominated these decays then $\lambda_{\pi\pi}^{(\mathrm{tree})} = e^{2i\alpha}$.

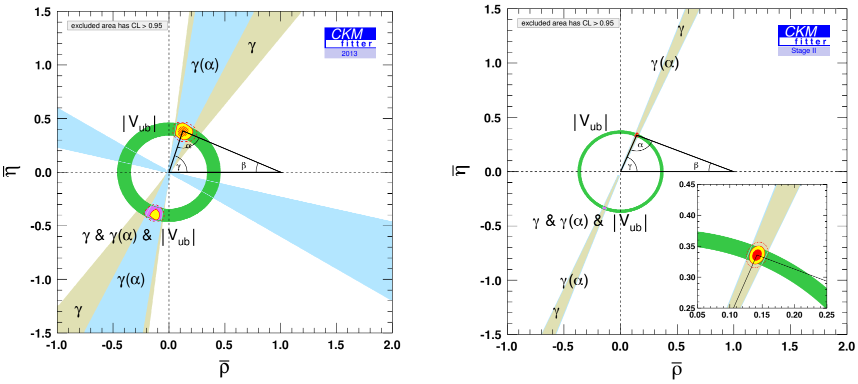


Fig. 11. Constraints on $\bar{\rho}-\bar{\eta}$, allowing new physics in the $B_{d,s}$ mixing amplitudes. Left plot shows the current constraints, right plot is the expectation using $50\,\mathrm{ab}^{-1}$ Belle II and $50\,\mathrm{fb}^{-1}$ LHCb data. Colored regions show 95% CL, as in Fig. 2. (From Ref. [36].)

the fits become less constraining. This is shown in the left plot in Fig. 11 where the allowed region is indeed significantly larger than in Fig. 2 (the 95% CL combined fit regions are indicated on both plots).

It has been known for decades that the mixing of neutral mesons is particularly sensitive to new physics, and probe some of the highest scales. In a large class of models, NP has a negligible impact on tree-level SM transition, and the 3×3 CKM matrix remains unitary. (In such models $\alpha+\beta+\gamma=\pi$ is maintained, and independent measurements of $\pi-\beta-\alpha$ and γ can be averaged.) We can parameterize the NP contributions to neutral meson mixing as

$$M_{12} = M_{12}^{\rm SM} (1 + h_q\, e^{2i\sigma_q})\,, \qquad q = d, s\,. \tag{52}$$

The constraints on h_q and σ_q in the B_d^0 and B_s^0 systems are shown in the top and bottom rows of Fig. 12, respectively.

For example, if NP modifies the SM operator describing B mixing, by

$$\frac{C_q^2}{\Lambda^2}\, (\bar{b}_L \gamma^\mu q_L)^2\,, \tag{53}$$

then one finds

$$h \simeq \frac{|C_q|^2}{|V_{tb}^* V_{tq}|^2} \left(\frac{4.5\,\mathrm{TeV}}{\Lambda}\right)^2 . \tag{54}$$

We can then translate the plots in Fig. 12 to the scale of new physics probed. The summary of expected sensitivities are shown in Table 3. The

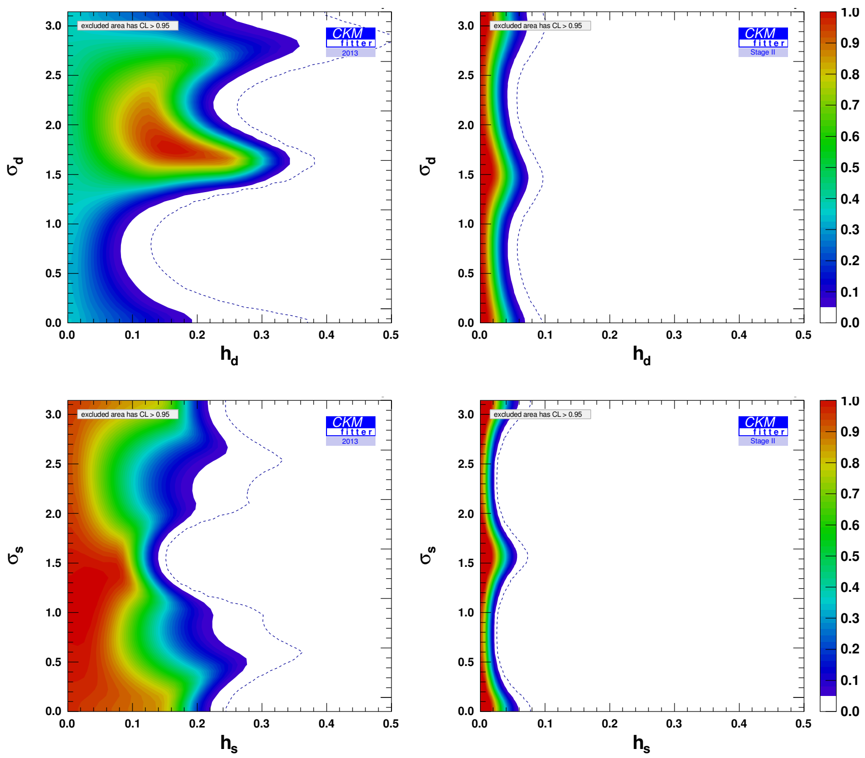


Fig. 12. Constraints on the $h_d - \sigma_d$ (top row) and $h_s - \sigma_s$ parameters (bottom row). Left plots show the current constraints, right plots show those estimated to be achievable using $50\,\mathrm{ab}^{-1}$ Belle II and $50\,\mathrm{fb}^{-1}$ LHCb data. Colored regions show 2σ with the colors indicating CL as shown, while the dashed lines show 3σ. (From Ref. [36].)

sensitivities, even with SM-like loop- and CKM-suppressed coefficients, are comparable to the scales probed by the LHC.

Table 3. The scale of the operator in Eq. (53) probed by B_d^0 and B_s^0 mixings with $50\,\mathrm{ab}^{-1}$ Belle II and $50\,\mathrm{fb}^{-1}$ LHCb data. The differences due to CKM-like hierarchy of couplings and/or loop suppression is indicated. (From Ref. [36].)

Couplings	NP loop order	Scales (TeV) probed by	
		B_d mixing	B_s mixing
$\lvert C_q\rvert = \lvert V_{tb}V_{tq}^*\rvert$ (CKM-like)	tree level	17	19
	one loop	1.4	1.5
$\lvert C_q\rvert = 1$ (no hierarchy)	tree level	2×10^3	5×10^2
	one loop	2×10^2	40

3. Some Implications of the Heavy Quark Limit

We have not directly discussed so far that most quark flavor physics processes (other than top quark decays) involve strong interactions in a regime where perturbation theory is not (or not necessarily) reliable. The running of the QCD coupling at lowest order is

$$\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \dfrac{\alpha_s}{2\pi}\,\beta_0 \ln \dfrac{\mu}{\Lambda}}\,, \tag{55}$$

where $\beta_0 = 11 - 2n_f/3$ and n_f is the number of light quark flavors. Even in B decays, the typical energy scale of certain processes can be a fraction of m_b, possibly around or below a GeV. The ways I know how to deal with this in a tractable way are (i) symmetries of QCD, exact, or approximate in some limits (CP invariance, heavy quark symmetry, chiral symmetry); (ii) the operator product expansion (for inclusive decays); (iii) lattice QCD (for certain hadronic matrix elements). An example of (i) is the determination of $\sin 2\beta$ from $B \to \psi K_S$, see Eq. (46). So is the determination of $|V_{cb}|$ from $B \to D^*\ell\bar\nu$, see Eq. (73) below. An example of (ii) is the analysis of inclusive $B \to X_s\gamma$ decay rates discussed below, which provides some of the strongest constraints on many TeV-scale BSM scenarios.

The role of (strong interaction) model independent measurements cannot be overstated. To establish that a discrepancy between experiment and theory is a sign of new physics, model independent predictions are crucial. Results that rely on modeling nonperturbative strong interaction effects will not disprove the SM. Most model independent predictions are of the form,

$$\text{Observable} = (\text{calculable terms}) \times \left\{ 1 + \sum_{i,k} \big[(\text{small parameters})_i\big]^k \right\}, \tag{56}$$

where the small parameters can be Λ_{QCD}/m_b, $m_s/\Lambda_{\chi\text{SB}}$, $\alpha_s(m_b)$, etc. For the purpose of these lectures, strong interaction model independent means that the theoretical uncertainty is suppressed by small parameters, so that theorists argue about $\mathcal{O}(1)\times$(small numbers) instead of $\mathcal{O}(1)$ effects. There are always theoretical uncertainties suppressed by some (small parameter)^n, which cannot be calculated from first principles. If the goal is to test the SM, one must assign $\mathcal{O}(1)$ uncertainties in such terms.

In addition, besides formal suppressions of certain corrections in some limits, experimental guidance is always needed to establish how well an expansion works; for example, f_π, m_ρ, and m_K^2/m_s are all of order Λ_{QCD}, but their numerical values span an order of magnitude.

Heavy quark symmetry (HQS) In hadrons composed of heavy quarks the dynamics of QCD simplifies. Mesons containing a heavy quark – heavy antiquark pair, $Q\overline{Q}$, form positronium-type bound states, which become perturbative in $m_Q \gg \Lambda_{\rm QCD}$ limit [37]. In mesons composed of a heavy quark, Q, and a light antiquark, $\bar q$ (and gluons and $q\bar q$ pairs), the heavy quark acts as a static color source with fixed four-velocity, v^μ, and the wave function of the light degrees of freedom (the "brown muck") become insensitive to the spin and mass (flavor) of the heavy quark, resulting in heavy quark spin-flavor symmetries [38].

The physical picture is similar to atomic physics, where simplifications occur due to the fact that the electron mass, m_e, is much smaller than the nucleon mass, m_N. The analog of flavor symmetry is that isotopes have similar chemistry, because the electrons' wave functions become independent of m_N in the $m_N \gg m_e$ limit. The analog of spin symmetry is that hyperfine levels are almost degenerate, because the interaction of the electron and nucleon spin diminishes in the $m_N \gg m_e$ limit.

Spectroscopy of heavy-light mesons The spectroscopy of heavy hadrons simplifies due to heavy quark symmetry. We can write the angular momentum of a heavy-light meson as $J = \vec s_Q + \vec s_l$, where $\vec s_l$ is the total angular momentum of the light degrees of freedom. Angular momentum conservation, $[\vec J, \mathcal{H}] = 0$, and heavy quark symmetry, $[\vec s_Q, \mathcal{H}] = 0$, imply $[\vec s_l, \mathcal{H}] = 0$. In the $m_Q \gg \Lambda_{\rm QCD}$ limit, the spin of the heavy quark and the total angular momentum of light degrees of freedom are separately conserved, modified only by subleading interactions suppressed by $\Lambda_{\rm QCD}/m_Q$.

Thus, hadrons containing a single heavy quark can be labeled with s_l, and for any value of s_l there are two (almost) degenerate states with total angular momentum $J_\pm = s_l \pm \frac12$. (An exception occurs for the lightest baryons containing a heavy quark, when $s_l = 0$, and there is a single state with $J = \frac12$, the Λ_b and Λ_c.) The ground state mesons with $Q\bar q$ flavor quantum numbers contain light degrees of freedom with spin-parity $s_l^{\pi_l} = \frac12^-$, giving a doublet containing a spin zero and spin one meson. For $Q = c$ these are the D and D^*, while $Q = b$ gives the B and B^* mesons.

The mass splittings between the doublets, Δ_i, are of order $\Lambda_{\rm QCD}$, and are the same in the B and D sectors at leading order in $\Lambda_{\rm QCD}/m_Q$, as illustrated in Fig. 13. The mass splittings within each doublet are of order $\Lambda^2_{\rm QCD}/m_Q$. This is supported by experimental data; e.g., for the $s_l^{\pi_l} = \frac12^-$ ground state doublets $m_{D^*} - m_D \approx 140\,{\rm MeV}$ while $m_{B^*} - m_B \approx 45\,{\rm MeV}$, and their ratio, 0.3, is consistent with m_c/m_b.

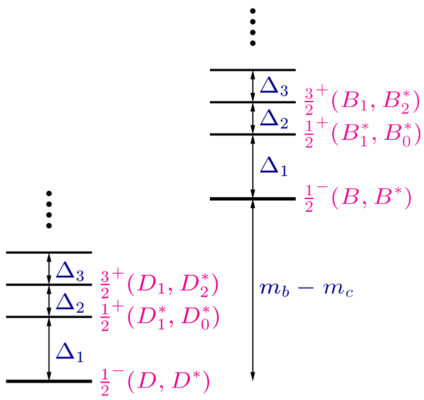


Fig. 13. Spectroscopy of B and D mesons. For each doublet level, the spin-parity of the light degrees of freedom, $s_l^{\pi_l}$, and the names of the physical states are indicated.

Let us mention a puzzle. The mass splitting of the lightest vector and pseudoscalar mesons being $\mathcal{O}(\Lambda_{\rm QCD}^2/m_Q)$ implies that $m_V^2 - m_P^2$ is approximately constant. This argument relies on $m_Q \gg \Lambda_{\rm QCD}$. The data are

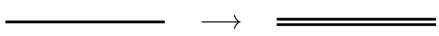
$$\begin{aligned} m_{B^*}^2 - m_B^2 &= 0.49\,{\rm GeV}^2\,, & m_{B_s^*}^2 - m_{B_s}^2 &= 0.50\,{\rm GeV}^2\,, \\ m_{D^*}^2 - m_D^2 &= 0.54\,{\rm GeV}^2\,, & m_{D_s^*}^2 - m_{D_s}^2 &= 0.58\,{\rm GeV}^2\,, \\ m_\rho^2 - m_\pi^2 &= 0.57\,{\rm GeV}^2\,, & m_{K^*}^2 - m_K^2 &= 0.55\,{\rm GeV}^2\,. \end{aligned} \tag{57}$$

It is not understood why the light meson mass splittings in the last line are so close numerically. (It is expected in the nonrelativistic constituent quark model, which fails to account for several properties of these mesons.) There must be something more going on than heavy quark symmetry, and if this was its only prediction, we could not say that there is strong evidence that it is useful. So in general, to understand a theory, it is not only important how well it works, but also how it breaks down outside its range of validity.

Heavy quark effective theory (HQET) The consequences of heavy quark symmetry and the corrections to the symmetry limit can be studied by constructing an effective theory which makes the consequences of heavy quark symmetry explicit. The heavy quark in a heavy-light meson is almost on-shell, so we can expand its momentum as $p_Q^\mu = m_Q v^\mu + k^\mu$, where $|k| = \mathcal{O}(\Lambda_{\rm QCD})$ and $v^2 = 1$. Expanding the heavy quark propagator,

$$\frac{i}{\not p - m_Q} = \frac{i(\not p + m_Q)}{p^2 - m_Q^2} = \frac{i(m_Q \not v + \not k + m_Q)}{2m_Q\, v\cdot k + k^2} = \frac{i}{v\cdot k}\,\frac{1+\not v}{2} + \dots\,. \tag{58}$$

it becomes independent of the heavy quark mass, a manifestation of heavy quark flavor symmetry. Hence the Feynamn rules simplify,

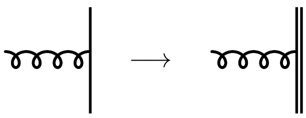


$$\frac{i}{\not{p} - m_Q} \qquad \longrightarrow \qquad \frac{i}{v \cdot k}\, P_+(v)\,, \tag{59}$$

where $P_\pm = (1 \pm \not{v})/2$ are projection operators, and the double line denotes the heavy quark propagator. In the rest frame of the heavy quark, $P_+ = (1 + \gamma^0)/2$ projects onto the heavy quark (rather than anti-quark) components. The coupling of a heavy quark to gluons simplifies due to

$$P_+ \gamma^\mu P_+ = P_+ v^\mu P_+ = v^\mu P_+\,, \tag{60}$$

hence we can replace



$$ig\gamma^\mu\, \frac{\lambda^a}{2} \qquad \longrightarrow \qquad igv^\mu\, \frac{\lambda^a}{2}\,. \tag{61}$$

The lack of γ matrix is a manifestation of heavy quark spin symmetry.

To derive the effective Lagrangian of HQET, it is convenient to decompose the four-component Dirac spinor as

$$Q(x) = e^{-im_Q v\cdot x} \big[Q_v(x) + \mathcal{Q}_v(x)\big]\,, \tag{62}$$

where

$$Q_v(x) = e^{im_Q v\cdot x}\, P_+(v)\, Q(x)\,, \qquad \mathcal{Q}_v(x) = e^{im_Q v\cdot x}\, P_-(v)\, Q(x)\,. \tag{63}$$

The $e^{im_Q v\cdot x}$ factor subtracts $m_Q v$ from the heavy quark momentum. At leading order only Q_v contributes, and the effects of $\mathcal{Q}_v$ are suppressed by powers of $\Lambda_{\rm QCD}/m_Q$. The heavy quark velocity, v, acts as a label of the heavy quark fields [39], because v cannot be changed by soft interactions. In terms of these fields the QCD Lagrangian simplifies,

$$\mathcal{L} = \bar{Q}(i\not{D} - m_Q)Q = \bar{Q}_v i\not{D} Q_v + \ldots = \bar{Q}_v(iv\cdot D)Q_v + \ldots\,, \tag{64}$$

where the ellipses denote terms suppressed by powers of $\Lambda_{\rm QCD}/m_Q$. The absence of any Dirac matrix is a consequence of heavy quark symmetry, which imply that the heavy quark's propagator and its coupling to gluons are independent of the heavy quark spin. This effective theory provides a framework to calculate perturbative $\mathcal{O}(\alpha_s)$ corrections and to parameterize nonperturbative $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ terms.

Semileptonic $B \to D^{(*)}\ell\bar{\nu}$ decays and $|V_{cb}|$ Heavy quark symmetry is particularly predictive for these decays. In the $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, the configuration of the brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin. So when the weak current changes suddenly (on a time scale $\ll \Lambda_{\rm QCD}^{-1}$) the flavor $b \to c$, the momentum $\vec{p}_b \to \vec{p}_c$, and possibly flips the spin, $\vec{s}_b \to \vec{s}_c$, the brown muck only feels that the four-velocity of the static color source changed, $v_b \to v_c$. Therefore, the matrix elements that describe the transition probabilities from the initial to the final state are independent of the Dirac structure of weak current, and can only depend on a scalar quantity, $w \equiv v_b \cdot v_c$.

The ground-state pseudoscalar and vector mesons for each heavy quark flavor (the spin symmetry doublets $D^{(*)}$ and $B^{(*)}$) can be represented by a "superfield", combining fields with different spins, that has the right transformation property under heavy quark and Lorentz symmetry,

$$\mathcal{M}_v^{(Q)} = \frac{1+\not{v}}{2}\Big[\gamma^\mu M_\mu^{*(Q)}(v,\varepsilon) - i\gamma_5 M^{(Q)}(v)\Big]. \tag{65}$$

The $B^{(*)} \to D^{(*)}$ matrix element of any current can be parameterized as

$$\langle M^{(c)}(v')\,|\,\bar{c}_{v'}\,\Gamma\, b_v\,|\,M^{(b)}(v)\rangle = \mathrm{Tr}\Big[F(v,v')\,\bar{\mathcal{M}}_{v'}^{(c)}\,\Gamma\,\mathcal{M}_v^{(b)}\Big]. \tag{66}$$

Because of heavy quark symmetry, there cannot be other Dirac matrices between the $\bar{\mathcal{M}}_{v'}^{(c)}$ and $\mathcal{M}_v^{(b)}$ fields. The most general form of F is

$$F(v,v') = f_1(w) + f_2(w)\not{v} + f_3(w)\not{v}' + f_4(w)\not{v}\not{v}'. \tag{67}$$

As stated above, $w \equiv v\cdot v'$ is the only possible scalar, related to $q^2 = m_B^2 + m_{D^{(*)}}^2 - 2m_B m_{D^{(*)}} w$. Using $\mathcal{M}_v^{(Q)} = P_+(v)\mathcal{M}_v^{(Q)}P_-(v)$, we can write

$$\begin{aligned} F &\doteq P_-(v)FP_-(v') = \big[f_1(w) - f_2(w) - f_3(w) + f_4(w)\big]P_-(v)P_-(v') \\ &= \xi(w)\,P_-(v)\,P_-(v') \doteq \xi(w)\,. \end{aligned} \tag{68}$$

This defines the Isgur-Wise function, $\xi(w)$, and $\doteq$ denotes relations valid when evaluated inside the trace in Eq. (66).

Since only weak interactions change b-quark number, the matrix element of $\bar{b}\gamma_0 b$, the b-quark number current, is $\langle B(v)|\bar{b}\gamma_0 b|B(v)\rangle = 2m_B v_0$. Comparing it with the result obtained using Eq. (66),

$$\langle B(v)|\bar{b}\gamma_\mu b|B(v)\rangle = 2m_B v_\mu\,\xi(1)\,, \tag{69}$$

implies that $\xi(1) = 1$. That is, at $w = 1$, the "zero recoil" point, when the $D^{(*)}$ is at rest in the rest-frame of the decaying B meson, the configuration of the brown muck does not change at all, and heavy quark symmetry

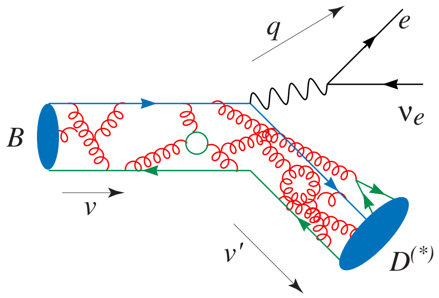


Fig. 14. Illustration of strong interactions parameterized by the Isgur-Wise function.

determines the hadronic matrix element (see Fig. 14). Moreover, the six form factors that describe semileptonic $B \to D^{(*)}\ell\bar\nu$ decays are related to this universal function, which contains all the low energy nonperturbative hadronic physics relevant for these decays.[k]

The determination of $|V_{cb}|$ from $B \to D^{(*)}\ell\bar\nu$ decays use fits to the decay distributions to measure the rates near zero recoil, $w = 1$. The rates can be schematically written as

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar\nu)}{\mathrm{d}w} = (\text{calculable})\, |V_{cb}|^2 \begin{cases} (w^2-1)^{1/2}\, \mathcal{F}_*^2(w), & \text{for } B \to D^*, \\ (w^2-1)^{3/2}\, \mathcal{F}^2(w), & \text{for } B \to D\,. \end{cases} \tag{72}$$

Both $\mathcal{F}(w)$ and $\mathcal{F}_*(w)$ are equal to the Isgur-Wise function in the $m_Q \to \infty$ limit, and $\mathcal{F}_{(*)}(1) = 1$ is the basis for a model independent determination of $|V_{cb}|$. There are calculable corrections in powers of $\alpha_s(m_{c,b})$, as well as terms suppressed by $\Lambda_{\rm QCD}/m_{c,b}$, which can only be parameterized, and

[k]Using only Lorentz invariance, six form factors parameterize $B \to D^{(*)}\ell\bar\nu$ decay,

$$\begin{aligned} \langle D(v')|V_\nu|B(v)\rangle &= \sqrt{m_B m_D}\, \big[h_+\,(v+v')_\nu + h_-\,(v-v')_\nu\big], \\ \langle D^*(v')|V_\nu|B(v)\rangle &= i\sqrt{m_B m_{D^*}}\, h_V\, \epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}v'^\beta v^\gamma, \\ \langle D(v')|A_\nu|B(v)\rangle &= 0, \\ \langle D^*(v')|A_\nu|B(v)\rangle &= \sqrt{m_B m_{D^*}}\, \big[h_{A_1}\,(w+1)\epsilon^*_\nu - h_{A_2}\,(\epsilon^*\cdot v)v_\nu - h_{A_3}\,(\epsilon^*\cdot v)v'_\nu\big], \end{aligned} \tag{70}$$

where $V_\nu = \bar c\gamma_\nu b$, $A_\nu = \bar c\gamma_\nu\gamma_5 b$, and h_i are functions of w. Show that this is indeed the most general form of these matrix elements, and at leading order in $\Lambda_{\rm QCD}/m_Q$,

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)\,, \qquad h_-(w) = h_{A_2}(w) = 0\,. \tag{71}$$

that is where hadronic uncertainties enter. Schematically,

$$\mathcal{F}_*(1) = 1_{\text{(Isgur-Wise)}} + c_A(\alpha_s) + \frac{0_{\text{(Luke)}}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \dots\,,$$

$$\mathcal{F}(1) = 1_{\text{(Isgur-Wise)}} + c_V(\alpha_s) + \frac{\text{(lattice or models)}}{m_{c,b}} + \dots\,. \qquad (73)$$

The absence of the $\mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b})$ term for $B \to D^*\ell\bar{\nu}$ at zero recoil is a consequence of Luke's theorem [40]. Calculating corrections to the heavy quark limit in these decays is a vast subject. Heavy quark symmetry also has model independent predictions for B decays to excited D mesons [41]. It is due to heavy quark symmetry that the recently observed anomalies in the $B \to D^{(*)}\tau\bar{\nu}$ branching ratios [42] are under good theoretical control.

Inclusive semileptonic decays and $B \to X_s\gamma$ Instead of identifying all final state particles in a decay, sometimes it is useful to sum over final state hadrons which can be produced by strong interactions, subject to constraints determined by short distance physics, e.g., the energy of a photon or a charged lepton. Although hadronization is nonperturbative, it occurs on much longer distance (and time) scales than the underlying weak decay. Typically we are interested in a quark-level transition, such as $b \to c\ell\bar{\nu}$, $b \to s\gamma$, etc., and we would like to extract from the data short distance parameters, $|V_{cb}|$, $C_7(m_b)$, etc. To do this, we need to relate the quark-level operators to the measurable decay rates.

For example, consider inclusive semileptonic $b \to c$ decay mediated by

$$O_{\text{sl}} = -\frac{4G_F}{\sqrt{2}}\, V_{cb}\, (J_{bc})^\alpha\, (J_{\ell\nu})_\alpha\,, \qquad (74)$$

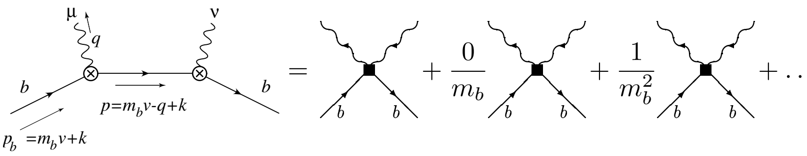
where $J_{bc}^\alpha = (\bar{c}\,\gamma^\alpha P_L\, b)$ and $J_{\ell\nu}^\beta = (\bar{\ell}\,\gamma^\beta P_L\, \nu)$. The decay rate is given by the square of the matrix element, integrated over phase space, and summed over final states,

$$\Gamma(B \to X_c\ell\bar{\nu}) \sim \sum_{X_c} \int \text{d[PS]}\, \big|\langle X_c\ell\bar{\nu}|O_{\text{sl}}|B\rangle\big|^2. \qquad (75)$$

Since leptons have no strong interaction, the phase space factorizes to $B \to X_c W^*$ and a perturbatively calculable leptonic part, $W^* \to \ell\bar{\nu}$. The nontrivial part is the hadronic tensor,

$$\begin{aligned} W^{\mu\nu} &= \sum_{X_c} (2\pi)^3\, \delta^4(p_B - q - p_X)\, \big|\langle B|J_{bc}^{\mu\dagger}|X_c\rangle\, \langle X_c|J_{bc}^\nu|B\rangle\big|^2 \\ &= \frac{1}{\pi}\, \text{Im} \int \text{d}x\, e^{-iq\cdot x}\, \langle B|\, T\big\{J_{bc}^{\mu\dagger}(x)\, J_{bc}^\nu(0)\big\}\, |B\rangle\,, \end{aligned} \qquad (76)$$

where the second line is obtained using the optical theorem, and T denotes here the time ordered product of the operators. It is this time ordered product that can be expanded in an operator product expansion (OPE) [43; 44; 45; 46]. In the $m_b \gg \Lambda_{\rm QCD}$ limit, the time ordered product is dominated by short distances, $x \ll \Lambda_{\rm QCD}^{-1}$, and one can express the hadronic tensor $W^{\alpha\beta}$ as a sum of local operators. Schematically,



(77)

This is analogous to the multipole expansion. At leading order in $\Lambda_{\rm QCD}/m_b$ the lowest dimension operator is $\bar b\,\Gamma\, b$, where Γ is some (process-dependent) Dirac matrix. Its matrix element is determined by the b quark content of the initial state using Eqs. (66) and (69); therefore, inclusive B decay rates in the $m_b \gg \Lambda_{\rm QCD}$ limit are equal to the b quark decay rates. Subleading effects are parameterized by matrix elements of operators with increasing number of derivatives, which are sensitive to the distribution of chromomagnetic and chromoelectric fields. There are no $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ corrections, because the B meson matrix element of any dimension-4 operator vanishes, $\langle B(v)|\, \bar Q_v^{(b)}\, iD_\alpha \Gamma\, Q_v^{(b)} |B(v)\rangle = 0$. The leading nonperturbative effects, suppressed by $\Lambda_{\rm QCD}^2/m_b^2$, are parameterized by two HQET matrix elements, denoted by $\lambda_{1,2}$. This is the basis of the model independent determinations of m_b and $|V_{cb}|$ from inclusive semileptonic B decays.

Some important applications, such as $B \to X_s\gamma$ [47] or $B \to X_u \ell\bar\nu$, are more complicated. Near boundaries of phase space, the energy release to the hadronic final state may not be large. One can think of the OPE as an expansion in the residual momentum of the b quark, k, shown in Eq. (77),

$$\frac{1}{(m_b v + k - q)^2 - m_q^2} = \frac{1}{[(m_b v - q)^2 - m_q^2] + [2k\cdot(m_b v - q)] + k^2}\,. \tag{78}$$

For the expansion in k to converge, the final state phase space can only be restricted in a way that allows hadronic final states, X, to contribute with

$$m_X^2 - m_q^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2\,. \tag{79}$$

In $B \to X_s\gamma$ when an experimental lower cut is imposed on E_γ to reject backgrounds, the left-most inequality can be violated. The same occurs in $B \to X_u\ell\bar\nu$ when experimental cuts are used to suppress $B \to X_c\ell\bar\nu$ backgrounds. If the right-most inequality in Eq. (79) is satisfied, a more complicated OPE in terms of nonlocal operators is still possible [48; 49].

4. Top, Higgs, and New Phyics Flavor

The scale of new physics In the absence of direct observation of BSM particles so far, viewing the standard model as a low energy effective theory, the search for new physics amounts to seeking evidence for higher dimension operators invariant under the SM gauge symmetries.

Possible dimension-6 operators include baryon and lepton number violating operators, such as $\frac{1}{\Lambda^2}QQQL$. Limits on the proton lifetime imply $\Lambda \gtrsim 10^{16}$ GeV. Non-SM flavor and CP violation could arise from $\frac{1}{\Lambda^2}Q\bar{Q}Q\bar{Q}$. The bounds on the scale of such operators are $\Lambda \gtrsim 10^{4\cdots7}$ GeV, depending on the generation index of the quark fields. Precision electroweak measurements constrain operators of the form $\frac{1}{\Lambda^2}(\phi D_\mu \phi)^2$ to have $\Lambda \gtrsim 10^{3\cdots4}$ GeV. These constraints are remarkable, because flavor, CP, and custodial symmetry are broken by the SM itself, so it is unlikely for new physics to have a symmetry reason to avoid introducing additional contributions.

As mentioned earlier, there is a single type of gauge invariant dimension-5 operators made of SM fields, which give rise to neutrino masses, see Eq. (18). The observed neutrino mass square differences hint at scales $\Lambda > 10^{10}$ GeV for these $\frac{1}{\Lambda}(L\phi)^2$ type operators (in many models $\Lambda \sim 10^{15}$ GeV). Such mass terms violate lepton number. It is an experimental question to determine the nature of neutrino masses, which is what makes the search for neutrinoless double beta decay (and determining the neutrino mass hierarchy) so important.

Charged lepton flavor violation (CLFV) The SM with vanishing neutrino masses would have predicted lepton flavor conservation. We now know that this is not the case, hence there is no reason to impose it on possible new physics scenarios. In particular, if there are TeV-scale new particles that carry lepton number (e.g., sleptons), then they have their own mixing matrices, which could give rise to CLFV signals. While the one-loop SM contributions to processes such as $\mu \to e\gamma$ are suppressed by the neutrino mass-squared differences[1], the NP contributions have a-priori no such suppressions, other than the somewhat heavier scales and being generated at one-loop in most BSM scenarios.

Within the next decade, the CLFV sensitivity will improve by about 4 orders of magnitude, corresponding to an increase in the new physics scale probed by an order of magnitude, possibly the largest such gain in sensitivity achievable soon. If any CLFV signal is discovered, we would want

[1]Estimate the $\mu \to e\gamma$ rate in the SM.

to measure many processes to map out the underlying patters, including $\mu \to e\gamma$, $\mu \to 3e$, $\tau \to e\gamma$, $\tau \to 3e$, $\tau \to \mu\gamma$, $\tau \to 3\mu$, etc.

Electric dipole moments (EDM) As discussed earlier, the experimental bound on the neutron EDM imply that a possible dimension-4 term in the SM Lagrangian, $\theta_{\rm QCD} F\widetilde{F}/(16\pi^2)$, has a coefficient $\theta_{\rm QCD} \lesssim 10^{-10}$. While there are plausible explanations [11], we do not yet know the resolution with certainty. Neglecting this term, CP violation in the CKM matrix only gives rise to quark EDMs at three-loop order, and lepton EDMs at four-loop level, resulting in EDMs below near future experimental sensitivities. On the other hand, new physics (e.g., supersymmetry) could generate both quark and lepton EDMs at the one-loop level, so even if the scale of new physics is $10-100\,\text{TeV}$, observable effects could arise.

Top quark flavor physics Well before the LHC turned on, it was already certain that it was going to be a top quark factory; the HL-LHC is expected to produce a few times 10^9 $t\bar{t}$ pairs. In the SM, top quarks almost exclusively decay to Wb, as $\big||V_{tb}| - 1\big| \approx 10^{-3}$. The current bounds on FCNC top decays are at the 10^{-3} level, and the ultimate LHC sensitivity is expected to reach the 10^{-5} to 10^{-6} level, depending on the decay mode. The SM rates are much smaller[m], so observation of any FCNC top decay signal would be clear evidence for new physics.

There is obvious complementarity between FCNC searches in the top sector and low energy flavor physics bounds. Since t_L is in the same $SU(2)$ doublet as b_L, several operators have correlated effects in t and b decays. For some operators, mainly those involving left-handed quark fields, the low energy constraints already exclude a detectable LHC signal, whereas other operators may still have large enough coefficients to yield detectable effects in top FCNCs at the LHC (see, e.g., Ref. [50]).

The $t\bar{t}$ forward-backward asymmetry provided a clear example recently of the interplay between flavor physics and anomalies in the high energy collider data (even those that may seem little to do with flavor at first). The CDF measurement in 2011, $A^{\rm FB}_{t\bar{t}}(m_{t\bar{t}} > 450\,\text{GeV}) = 0.475 \pm 0.114$ [51], was stated to be 3.4σ above the NLO SM prediction. At the LHC, the same underlying physics would produce a rapidity asymmetry.[n] It became quickly apparent that models that could account for this signal faced severe

[m]Estimate the $t \to cZ$ and $t \to c\gamma$ branching ratios in the SM.
[n]Show that if in $t\bar{t}$ production at the Tevatron more t goes in the p than in the $\bar{p}$ direction, then at the LHC the mean magnitude of t quark rapidity is greater than that of $\bar{t}$.

flavor constraints. This provides an example (with hundreds of papers in the literature) that flavor physics will likely be crucial to understand what the explanation of a high-p_T LHC anomaly can be, and also what it cannot be. By now this excitement has subsided, because the significance of the Tevatron anomaly decreased and because the LHC has not seen any anomalies in the top production data predicted by most models (see, e.g., Ref. [52]) built to explain the Tevatron signal.

Higgs flavor physics With the discovery of a SM-like Higgs boson at the LHC, it is now clear that the LHC is also a Higgs factory. Understanding the properties of this particle entails both the precision measurements of its observed (and not yet seen) couplings predicted by the SM, and the search for possible decays forbidden in the SM.

The source of Higgs flavor physics, obviously, is the same set of Yukawa couplings whose structure and consequences we also seek to understand in low energy flavor physics measurements. While in terms of SUSY model building $m_h \approx 125\,\text{GeV}$ is challenging to understand, this mass allows experimentally probing many Higgs production and decay channels. The fact that ultimately the LHC will be able to probe Higgs production via (i) gluon fusion ($gg \to h$), (ii) vector boson fusion ($q\bar{q} \to q\bar{q}h$), (iii) W/Z associated production ($q\bar{q} \to hZ$ or hW), (iv) b/t associated production ($gg \to hb\bar{b}$ or $ht\bar{t}$) sensitively depend on the Yukawa couplings and m_h.[o]

If we allow new physics to contribute to Higgs-related processes, which is especially well motivated for loop-induced production (e.g., the dominant $gg \to h$) and decay (e.g., $h \to \gamma\gamma$) channels, then the first evidence for non-universal Higgs couplings to fermions was the bound on $h \to \mu^+\mu^-$ below $10 \times$ (SM prediction), combined with the observations of $h \to \tau^+\tau^-$ at the SM level, implicitly bounding $\mathcal{B}(h \to \mu^+\mu^-)/\mathcal{B}(h \to \tau^+\tau^-) \lesssim 0.03$.

There is an obvious interplay between the search for flavor non-diagonal Higgs decays and indirect bounds from flavor changing quark transitions and bounds on CLFV in the lepton sector. For example, $y_{e\mu} \neq 0$ would generate a one-loop contribution to $\mu \to e\gamma$, $y_{uc} \neq 0$ would generate D^0–$\overline{D}^0$ mixing, etc. [53]. In some cases the flavor physics constraints imply that there is no chance to detect a particular flavor violating Higgs decay, while signals in some modes may be above future direct search sensitivities. The interplay between measurements and constraints on flavor-diagonal and flavor-changing Higgs decay modes can provide additional insight on which flavor models are viable (see, e.g., Ref [54]).

[o] How would Higgs production and decay change if m_t were, say, $50\,\text{GeV}$?

Supersymmetry and flavor While I hope the LHC will discover something unexpected, of the known BSM scenarios, supersymmetry is particularly interesting, and its signals have been worked out in great detail. The minimal supersymmetric standard model (MSSM) contains 44 CP violating phases and 80 other CP conserving flavor parameters [55].[p] It has long been known that flavor physics (neutral meson mixings, ϵ'_K, $\mu \to e\gamma$, $B \to X_s\gamma$, etc.) imposes strong constraints on the SUSY parameter space. The MSSM also contains flavor diagonal CP violation (in addition to $\theta_{\rm QCD}$), and the constraints from the bounds on electric dipole moments are fairly strong on these phases if the mass scale is near 1 TeV.

As an example, consider the $K_L - K_S$ mass difference. The squark–gluino box contribution compared to the data contains terms, roughly,

$$\frac{\Delta m_K^{(\rm SUSY)}}{\Delta m_K^{(\rm exp)}} \sim 10^4 \left(\frac{1\,\mathrm{TeV}}{\tilde m}\right)^2 \left(\frac{\Delta \tilde m^2}{\tilde m^2}\right)^2 \mathrm{Re}\big[(K_L^d)_{12}(K_R^d)_{12}\big], \tag{80}$$

where K_L^d (K_R^d) are the mixing matrices in the gluino couplings to left-handed (right-handed) down quarks and their scalar partners [3]. The constraint from ϵ_K corresponds to replacing $10^4\,\mathrm{Re}\big[(K_L^d)_{12}(K_R^d)_{12}\big]$ with $10^6\,\mathrm{Im}\big[(K_L^d)_{12}(K_R^d)_{12}\big]$. The simplest supersymmetric frameworks with parameters in the ballpark of $\tilde m = \mathcal{O}(1\,\mathrm{TeV})$, $\Delta\tilde m^2/\tilde m^2 = \mathcal{O}(0.1)$, and $(K_{L,R}^d)_{ij} = \mathcal{O}(1)$ are excluded by orders of magnitude.

There are several ways to address the supersymmetric flavor problems. There are classes of models that suppress each of the terms in Eq. (80): (i) heavy squarks, when $\tilde m \gg 1\,\mathrm{TeV}$ (e.g., split SUSY); (ii) universality, when $\Delta\tilde m^2_{\tilde Q,\tilde D} \ll \tilde m^2$ (e.g., gauge mediation); (iii) alignment, when $(K_{L,R}^d)_{12} \ll 1$ (e.g., horizontal symmetry). All viable models incorporate some of these ingredients in order not to violate the experimental bounds. Conversely, if SUSY is discovered, mapping out its flavor structure will help to answer important questions about even higher scales, e.g., the mechanism of SUSY breaking, how it is communicated to the MSSM, etc.

A special role in constraining SUSY models is played by $D^0 - \overline{D}^0$ mixing, which was the first observed FCNC process in the up-quark sector. It is a special probe of BSM physics, because it is the only neutral meson system in which mixing is generated by intermediate down-type quarks in the SM, or intermediate up-type squarks in SUSY. The constraints are thus complementary to FCNC processes involving K and B mesons. $D^0 - \overline{D}^0$ mixing and FCNC in the up-quark sector are particularly important in constraining scenarios utilizing quark-squark alignment [56; 57].

[p]Check this [55], using the counting of couplings and broken global symmetries.

Another important implication for SUSY searches is that the constraints on squark masses imposed by the LHC data is significantly affected by the level of (non-)degeneracy of squarks required to satisfy flavor physics constraints. Most SUSY searches assume that the first two generation squarks, $\tilde{u}_{L,R}$, $\tilde{d}_{L,R}$, $\tilde{s}_{L,R}$, $\tilde{c}_{L,R}$, are all degenerate, which increases signal cross sections and affects signal/background estimates. Relaxing this assumption consistent with flavor bounds [57], results in substantially weaker squark mass limits, as low as around the 500 GeV scale [58].

It is apparent from the above discussion that there is a tight interplay between the implications of the non-observation of new physics at the LHC so far, and the non-observation of deviations from the SM in flavor physics. If there is new physics at the TeV scale, which we hope the LHC will discover in its next run, then we know already that its flavor structure must be rather non-generic to suppress FCNCs, and the combination of all data will contain plenty of additional information about the structure of new physics. The higher the scale of new physics, the less severe the flavor constraints are. If NP is beyond the reach of the LHC, flavor physics experiments may still observe robust deviations from the SM predictions, which would point to an upper bound on the next scale to probe.

Minimal flavor violation (MFV) The standard model without Yukawa couplings has a global $[U(3)]^5$ symmetry ($[U(3)]^3$ in the quark and $[U(3)]^2$ in the lepton sector), rotating the 3 generations of the 5 fields in Eq. (4). This is broken by the Yukawa interactions in Eq. (7). One may view the Yukawa couplings as spurions, which transform under $[U(3)]^5$ in a way that makes the Lagrangian invariant, and then the global flavor symmetry is broken by the background values of the Yukawas. BSM scenarios in which there are no new sources of flavor violation beyond the Yukawa matrices are called minimal flavor violation [59; 60; 61]. Since the SM breaks the $[U(3)]^5$ flavor symmetry already, MFV gives a framework to characterize "minimal reasonable" deviations from the SM predictions.

Let us focus on the quark sector. Under $U(3)_Q \times U(3)_u \times U(3)_d$ the transformation properties are

$$Q_L(3,1,1)\,, \quad u_R(1,3,1)\,, \quad d_R(1,1,3)\,, \qquad Y_u(3,\bar{3},1)\,, \quad Y_d(3,1,\bar{3})\,. \tag{81}$$

One can choose a basis in which

$$Y_d = \mathrm{diag}(y_d\,, y_s\,, y_b)\,, \qquad Y_u = V_{\mathrm{CKM}}^\dagger\, \mathrm{diag}(y_u\,, y_c\,, y_t)\,. \tag{82}$$

To generate a flavor-changing transition, requires constructing $[U(3)]^3$ singlet terms which connect the required fields. For example, in the down-quark sector, the simplest terms are [61]

$$\bar{Q}_L Y_u Y_u^\dagger Q_L\,, \qquad \bar{d}_R Y_d^\dagger Y_u Y_u^\dagger Q_L\,, \qquad \bar{d}_R Y_d^\dagger Y_u Y_u^\dagger Y_d d_R\,. \tag{83}$$

A useful feature of this approach is that it allows EFT-like analyses.

Consider $B \to X_s\gamma$ as an example. We are interested in the magnitude of a possible NP contribution to the Wilson coefficient of the operator $\frac{X}{\Lambda}(\bar{s}_L\sigma_{\mu\nu}F^{\mu\nu}b_R)$. A term $\bar{Q}_L b_R$ is not invariant under $[U(3)]^3$. A term $\bar{Q}_L Y_d\, d_R$ is $[U(3)]^3$ invariant, but it is diagonal, so it only connects same generation fields. The first non-vanishing contribution comes from $\bar{Q}_L Y_u Y_u^\dagger Y_d\, d_R$, which has a $V_{tb}V_{ts}^*\, y_t^2 y_b(\bar{s}_L b_R)$ component. We learn that in MFV models, in general, $X \propto y_b V_{tb} V_{ts}^*$, as is the case in the SM.

Thus, in MFV models, most flavor changing operators "automatically" have their SM-like suppressions, proportional to the same CKM elements, quark masses from chirality flips, etc. Therefore, the scale of MFV models can be $\mathcal{O}(1\,\text{TeV})$ without violating flavor physics bounds, thus solving the new physics flavor puzzle. Originally introduced for technicolor models [59], gauge mediated supersymmetry breaking provides another well known scenario in which MFV is expected to be a good approximation.

MFV models have important implications for new particle searches, too. Since the only quark flavor changing parameters are the CKM elements, and the ones that couple the third generation to the lighter ones are very small, in MFV models new particles that decay to a single final quark (and other particles) decay to either a third generation quark or to a first two generation quark, but (to a good approximation) not to both [62].

The MFV ansatz can be incorporated in models that do not specifically contain flavor breaking unrelated to the Yukawa couplings. MFV is not expected to be an exact symmetry, but it may be a useful organizing principle to understand details of the new physics we soon hope to get a glimpse of.

5. Summary

An essential feature of flavor physics is its ability to probe very high scales, beyond the masses of particles that can be produced on-shell in colliders. Flavor physics can also teach us about properties of TeV-scale new physics, which cannot be learned from the direct production of new particles.

Some of the main points I tried to explain in these lectures were:

- Flavor-changing neutral currents and meson mixing probe scales well above the masses of particles colliders can produce, and provide strong constraints on TeV-scale new physics.

- CP violation is always the result of interference phenomena, without a classical analog.
- The KM phase has been established as the dominant source of CP violation in flavor changing processes.
- Tremendous progress will continue: until ~ 10 years ago, more than $\mathcal{O}(1)$ deviations from the SM were possible, at present $\mathcal{O}(20\%)$ corrections to most FCNC processes are still allowed, and in the future few % sensitivities will be reached.
- The future goal is not measuring SM parameters better, but to search for corrections to the SM, and to learn about NP as much as possible.
- Direct information on new particles and their influence on flavor changing processes will both be crucial to understand the underlying physics.
- The sensitivity of future experiments in a number of important processes is only limited by statistics, not theory.
- The interesting (and fun) interplay between theoretical and experimental developments in flavor physics will continue.

At present, both direct production and flavor physics experiments only give bounds on new physics. These constraints imply that if new physics is accessible at the LHC, it is likely to have flavor suppression factors similar to the SM ones. In many models (e.g., the MSSM), measurements or bounds on FCNC transitions constrain the product of certain mass splittings times mixing parameters divided by the square of the new physics scale. If the LHC discovers new physics, then in principle the mass splittings and mixing parameters can be measured separately. If flavor physics experiments establish a deviation from the SM in a related process, the combination of LHC and flavor data can be very powerful to discriminate between models, and ultimately, the consistency of measurements would tell us that we understand the flavor structure of new physics and how the new physics flavor puzzle is solved. The present situation and an (optimistic) future scenario are shown in Fig. 15. Let us hope that we shall have the privilege to think about such questions, motivated by data, in the coming years.

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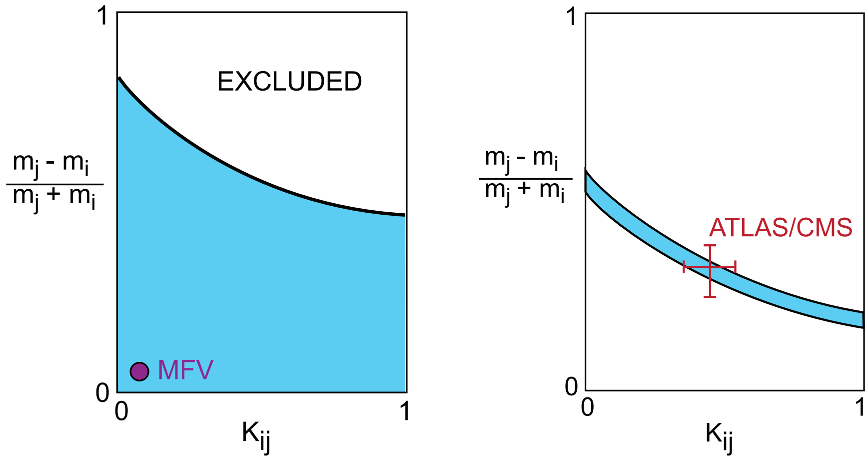

Fig. 15. Schematic description of the constraints on the mass splitting, $(m_i - m_j)/(m_i + m_j)$, and mixing angle, K_{ij}, between squarks (or sleptons). Left: typical constraint from not observing deviations from the SM. The fact that $\mathcal{O}(1)$ splittings and mixings are excluded constitutes the new physics flavor puzzle. Right: possible future scenario where ATLAS/CMS measurements fit flavor physics signals of NP. (From Ref. [7].)

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