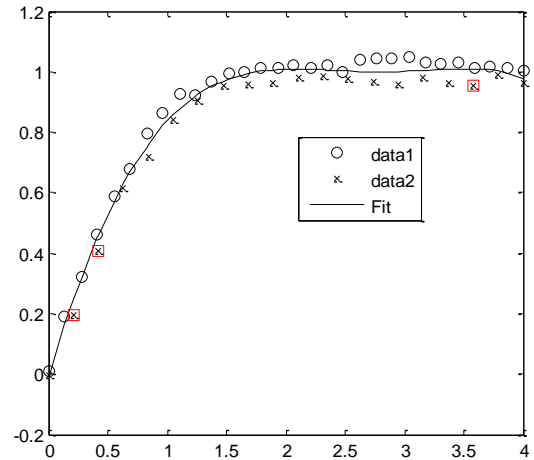


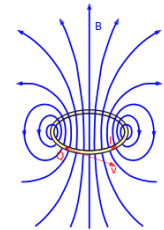
Problem 1 (6 points)

- Read the measurement data *data1.txt* and *data2.txt* and plot them with legend.
- Form a common data set from both measurements and draw in a 5th degree polynomial with minimum squared error to the data set.
- Mark the three points with the largest deviation from the polynomial.



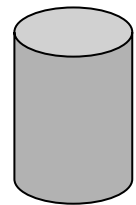
Problem 2 (2 points)

The function $Hloop(R, r, z)$ calculates the magnetic field H of a circular conductor loop with radius R at axial distance z and radial distance r from the center. Calculate the magnetic flux Φ through this conductor loop ($R=10\text{cm}$ and wire diameter $d=2\text{mm}$) by integrating the flux density B enclosed by the inner edge of the conductor.



Problem 3 (3 points)

A closed cylindrical can made of sheet metal is to hold a volume of one liter. What dimensions (diameter, height) must the can have so that as little sheet metal as possible has to be used?



Problem 4 (4 points)

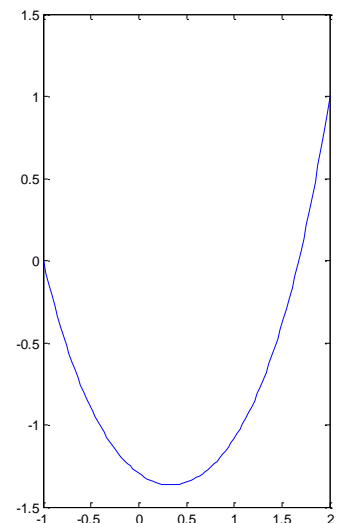
Draw the curve of a hanging chain, of length $l=5$, attached at point $P_1(x_1=-1, y_1=0)$ and at point $P_2(x_2=2, y_2=1)$.

The equation of the chain line is

$$y(x) = a \cdot \cosh\left(\frac{x-x_0}{a}\right) + y_0$$

To determine the three unknown parameters x_0 , y_0 and a , three equations can be set up. Two equations result from the two suspension points.

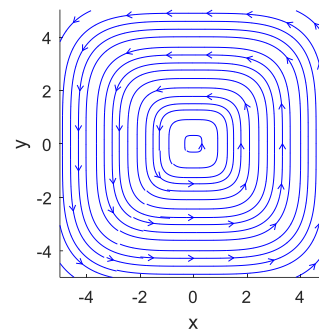
The third one reads: $l = 2a \sinh\left(\frac{x_2-x_1}{2a}\right)$



Problem 5 (3 points)

Plot the following vector field H in the xy -plane:

$$H_x = \frac{-y^3}{r^5} \quad H_y = \frac{x^3}{r^5} \quad r = \sqrt{x^2 + y^2}$$

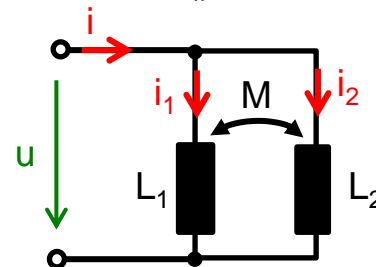


Problem 6 (3 points)

Determine the equivalent inductance L_e for two coupled inductors in parallel.

In addition to Kirchhoff's current law, Kirchhoff's voltage law applies:

$$U = L_1 \dot{I}_1 + M \dot{I}_2 = L_2 \dot{I}_2 + M \dot{I}_1$$



Give an analytical expression for L_e as a function of L_1 , L_2 , and M .

Problem 7 (7 points)

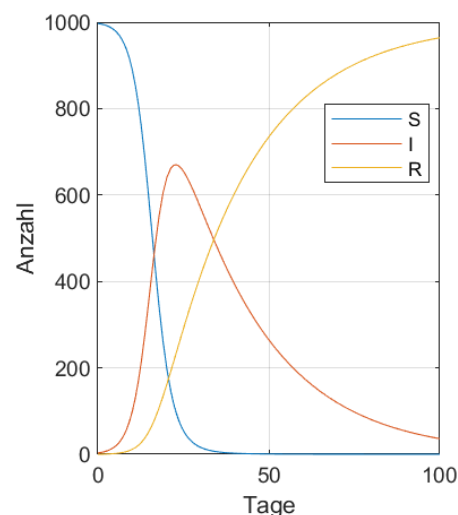
The SIR model distinguishes between three groups of individuals: At time t , $S(t)$ denotes the number of healthy individuals not immune to the disease (susceptible individuals), $I(t)$ the number of infectious individuals, and $R(t)$ the number of removed individuals. The latter occurs either by recovery with acquired immunity to the disease or by death. Furthermore, let $N=1000$ be the total number of individuals, that is: $N = I+S+R = \text{const.}$ The infection process is described by the following three equations:

$$\frac{dS}{dt} = -\beta \frac{SI}{N} \quad \frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I \quad \frac{dR}{dt} = \gamma I$$

Where $\beta=0.4$ is the infection rate and $\gamma=0.04$ is the immunization rate per day.

- Simulate the course of infection over 100 days if there are $I(0)=3$ infected persons at the beginning.
- After 15 days, all intensive care beds are occupied and a lock-down is imposed. This measure reduces the infection rate to $\beta = 0.04$. Extend your model to include a temporary lock-down. How long does this have to last for the beds to be sufficient: $I(t>15) \leq I(15)$?

days =



problem 8 ChatGPT (5 points)

The following is a question for ChatGPT. Take a position on the matlab solution proposed by the AI on the next page. What is good/bad, right/wrong?

A capacitor with capacitance C carries charge Q and discharges through a resistor R and an inductor L . How do I calculate the time dependent current through the three components in series connection?

To calculate the time dependent current through the capacitor, resistor, and inductor in series connection, you can solve the differential equations for the three components and combine the results. The differential equation for the capacitor is:

$$C \cdot dQ/dt = I$$

where Q is the charge of the capacitor, and I is the current through the capacitor. The differential equation for the resistor is:

$$R \cdot I = U$$

where R is the resistance and U is the voltage across the resistance. The differential equation for the inductance is:

$$L \cdot dI/dt = U$$

The solutions of the differential equations depend on the initial values and the boundary conditions. It is possible to solve using analytical and numerical means. A simpler solution is to use simulation software such as LTSpice. It allows you to simulate circuits with different components and initial values, and to view the evolution of current and other signals over time.

How can I solve these differential equations with Matlab?

In MATLAB you can solve the differential equations with the function "ode45".

"ode45" is a built-in solver in MATLAB that solves the differential equations numerically. Here is an example of how to solve the differential equations for the capacitor, resistor and inductor in series connection using "ode45":

Write a function that represents the right-hand side of the differential equations. This function takes the current time, the current variable values and the parameters of the circuit as input and returns the derivatives of the variables as output.

```
function dy = diffeq(t, y, R, L, C)
    I = y(1);
    Q = y(2);
    dy = [1/L*(y(3)-R*I); 1/C*I; 0];
end
```

Define the initial values for the variables, the initial values of time, and the parameters of the circuit.

```
R = 1;
L = 1;
C = 1;
y0 = [0; 0; 1];
tspan = [0, 10];
```

Call the function "ode45" to solve the differential equations.

```
[t, y] = ode45(@(t, y) diffeq(t, y, R, L, C), tspan, y0);
```

Plot the results to show the time history of the current through the circuit.

```
plot(t, y(:, 1))
xlabel('time')
ylabel('current')
```

Note that the results of "ode45" are only an approximation of the solutions, which depend on the accuracy of the numerical methods used and the step size of the integration. To get higher accuracy, you can adjust the options of ode45.