

Simulation of an electrically assisted bicycle using the Energetic Macroscopic Representation

Goal of the exercise

- From theory to implementation
 - Modelling
 - From the cyclist to the bicycle
 - Representation using EMR
 - Deduction of an Inverse Based Control
 - SIMULATION
 - To see if it works as expected

Outline

- Introduction
- Modelling and representation of the system
- Identification of the Inverse Based Control
- Implementation using Matlab/Simulink

Introduction

- System to model



Introduction

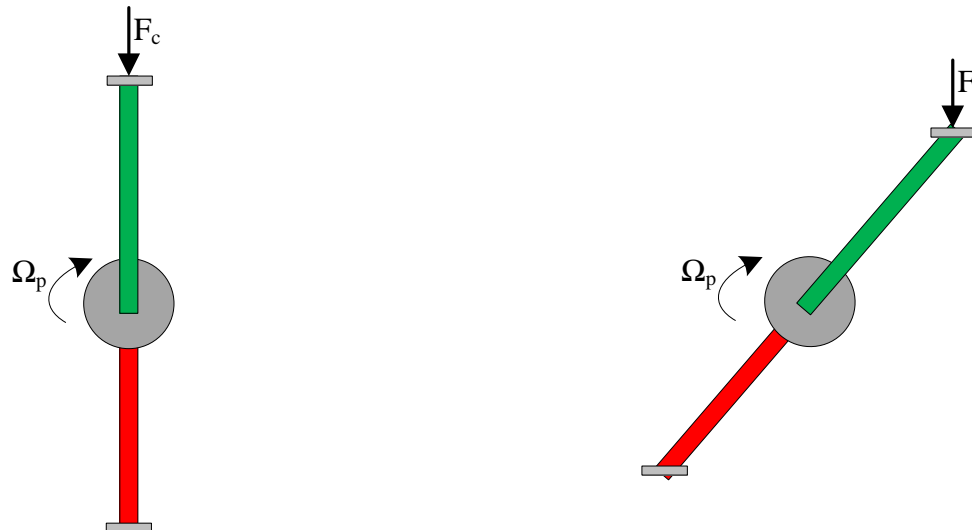
- Objectives
 - Identify the potential of electrically assisted bicycle in helping people to choose efficient transportation means (and good for health)
 - Identify the constraints applied on the electrical part of the system
 - Analysis of the working principle for such systems
- Assumptions
 - Damping systems are not considered
 - Cyclist is supposed to be a rigid mass (!), always sited.
 - No modification of mass repartition during acceleration or braking.
 - Inertias of all rotating masses are neglected
 - No friction losses in the pedalboard, wheel axes or in the transmission
 - Contact wheel/ground without loss (no slip phenomena)
 - No pedal binding

Modelling and representation

- Divide and conquer: sub-systems to consider
 - Cyclist
 - Batteries (1)
 - Electrical assistance (2)
 - Pedals (3)
 - Pedal axis (4)
 - Transmission/Reduction (Pedal axis, gear wheels and chain) (5)
 - Free wheeling system on rear wheel (6)
 - Disc brakes on the rear and front wheels (7)
 - Frame of bike, saddle and handlebars
 - Environment

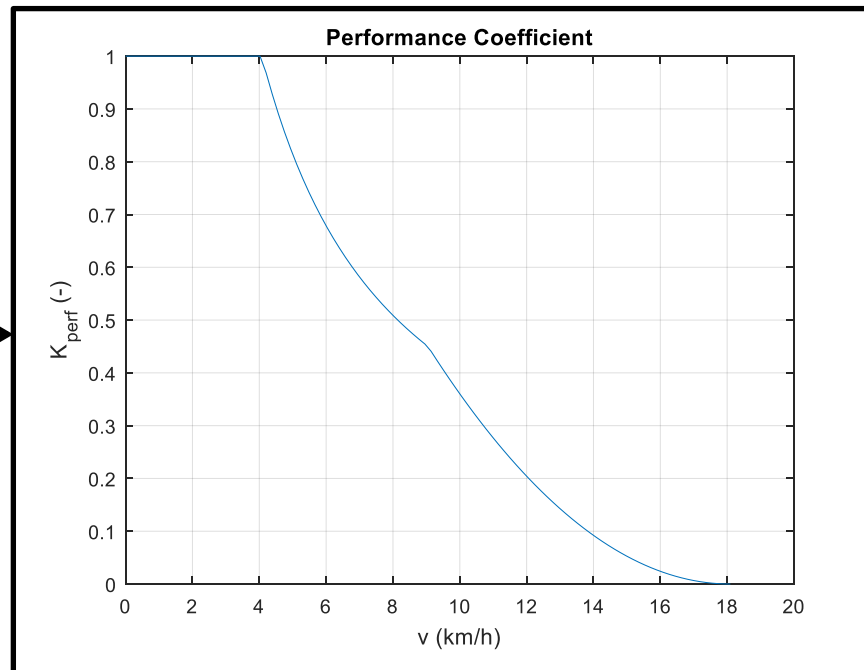
Modelling and representation

- Model of a cyclist
 - From a pure static point of view, a human being can support at least its weight, up to twice its weight or more.... (!?)
 - For a cyclist, who does not want to force too much, assumption will be made that the maximum weight applied on the descending pedal will be defined between 50% and 75% of its own weight (up to you to decide). There is no pedal binding: no force applied on the ascending pedal.
 - The force on the pedals is always vertically applied



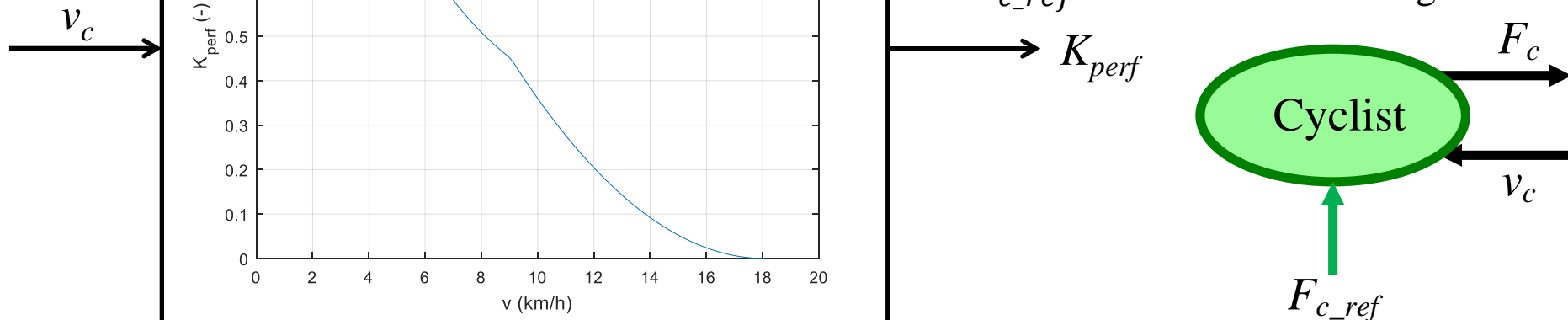
Modelling and representation

- Model and representation of a cyclist
 - Movement of the foot are such as it is circular
 - Defined by its peripheral velocity v_c
 - Introducing K_{perf} coefficient of performance:
 - $K_{perf} = 1$: the cyclist can apply the force he wants (!)
 - $K_{perf} < 1$: limitation of performances at high peripheral velocity.



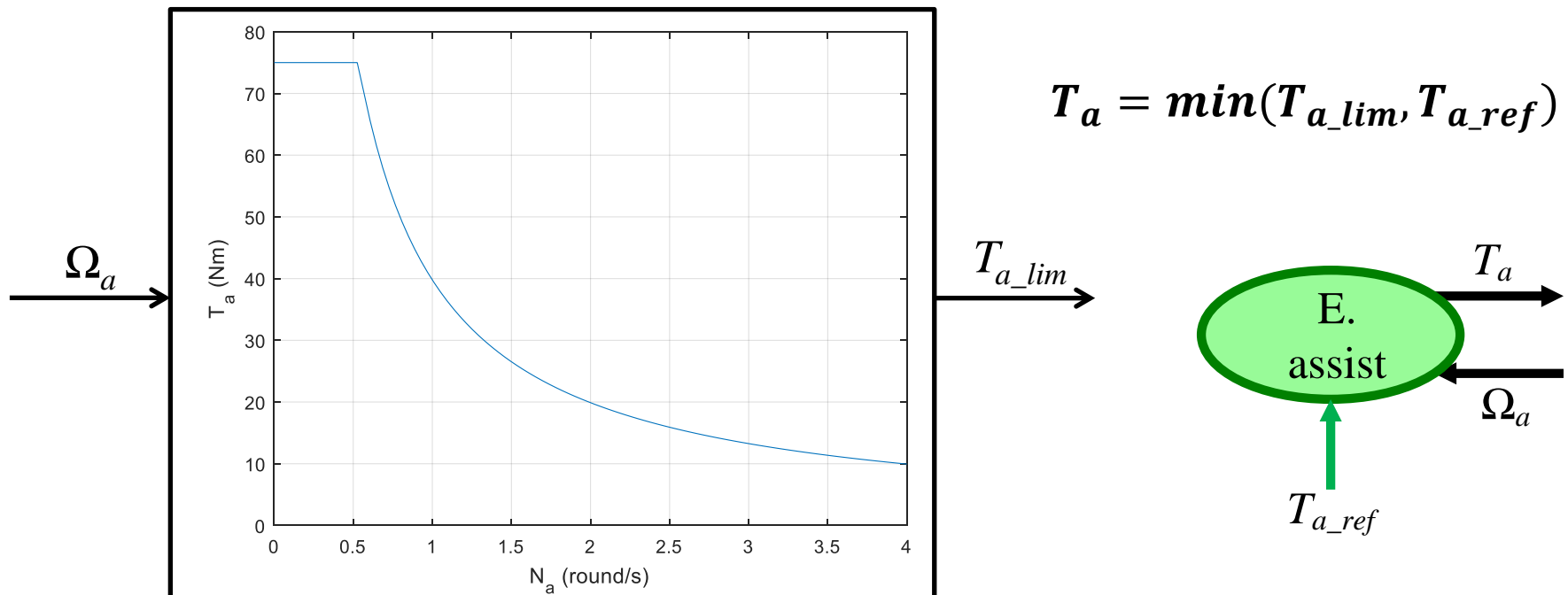
$$F_c = K_{perf} \cdot F_{c_ref}$$

- With F_{c_ref} the force the cyclist wishes to apply
- F_{c_ref} is limited to $0.75 \cdot M \cdot g$



Modelling and representation

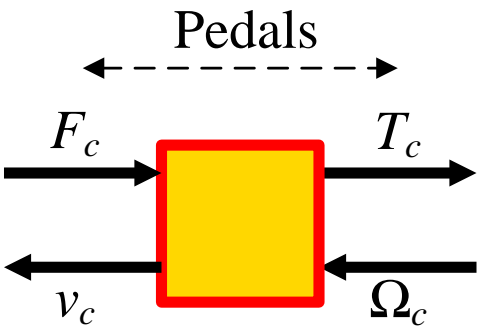
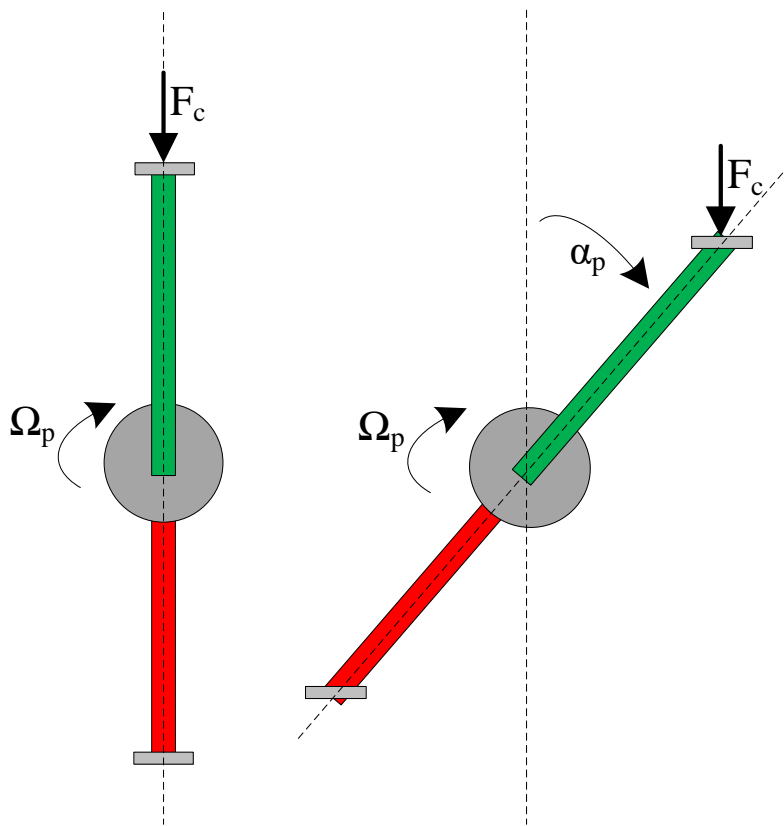
- Model and representation of the electrical assistance
 - In a first approach, considered as a source.
 - Maximum torque applied at the pedal axis: $T_{a_max}=75Nm$
 - Maximum Power: $P_{a_max}=250W$
 - Applies the torque T_a on the pedal axis, upon request,



Modelling and representation

- Model and representation of the pedals
 - Radius R_p , convert the force F_c from the cyclist to the torque T_c applied to the pedal axis.

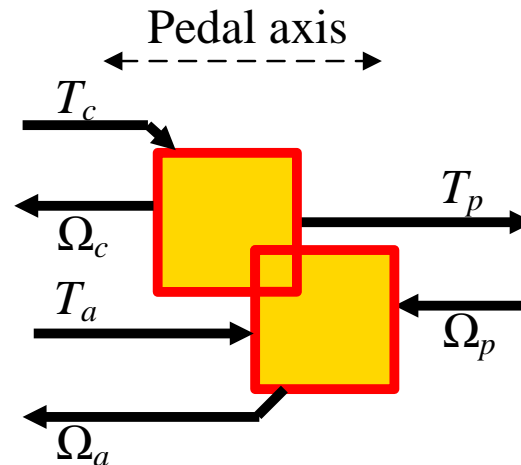
$$\left\{ \begin{aligned} T_c &= R_p \cdot F_c \cdot |\sin(\alpha_c)| \text{ with } \alpha_c = \int \Omega_c \cdot dt \\ \Omega_c &= \frac{1}{R_p} \cdot v_c \end{aligned} \right.$$



Modelling and representation

- Model and representation of the pedal axis
 - Coupling of the torques from the pedals and the electrical assistance

$$\begin{cases} T_p = T_c + T_a \\ \Omega_p = \Omega_c = \Omega_a \end{cases}$$



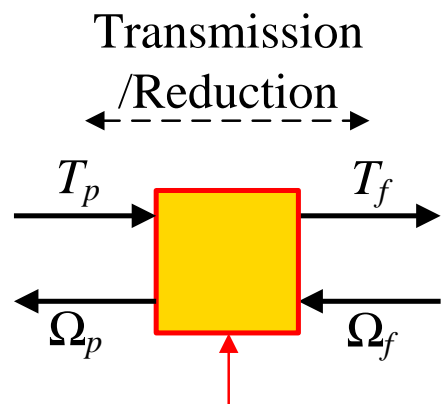
Modelling and representation

- Model and representation of Transmission/Reduction (Pedal axis, gear wheels and chain)
 - The cyclist can use 10 speeds, defined by the conversion ratio K_{vit} :

K_{vit}	36/18	34/18	30/18	28/18	25/18	22/18	20/18	17/18	14/18	11/18
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- The torque applied on the rear wheel free-wheeling system, and the angular velocity are defined by:

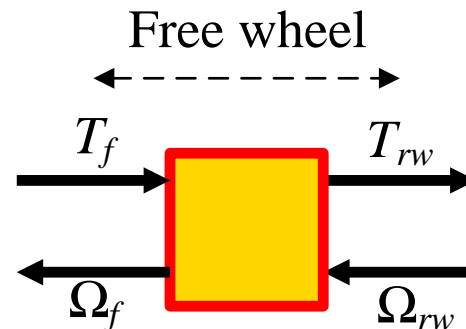
$$\begin{cases} T_f = K_{vit} \cdot T_p \\ \Omega_p = K_{vit} \cdot \Omega_f \end{cases}$$



Modelling and representation

- Model and representation of the free wheeling system on rear wheel
 - This mechanism forbids the cyclist to apply negative torque on the rear wheel
 - This mechanism allows the cyclist to stop pushing the pedals even if the cycle velocity is positive
 - The torque applied on the rear wheel free-wheeling system will be transmitted to the rear wheel on the only condition it is positive.

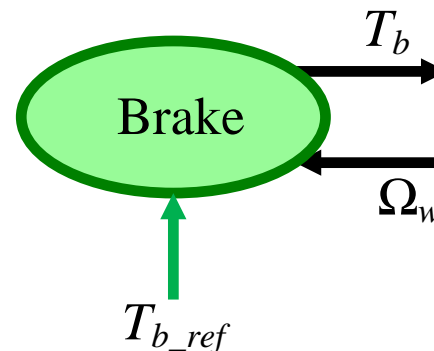
$$\begin{cases} T_{rw} = T_f \\ \Omega_{rw} = \Omega_f \end{cases} \text{ if } T_f > 0 \qquad \begin{cases} T_{rw} = 0 \\ \Omega_f = 0 \end{cases} \text{ if } T_f \leq 0$$



Modelling and representation

- Model and representation of the disc brakes on the rear and front wheels:
 - In a first approach, considered as sources
 - As a simplification, one will consider that the brakes generate a torque directly on each wheel axis.
 - The braking torque is directly imposed by the cyclist from the brake pedals.
 - By convention, this torque will be in any case null or negative.

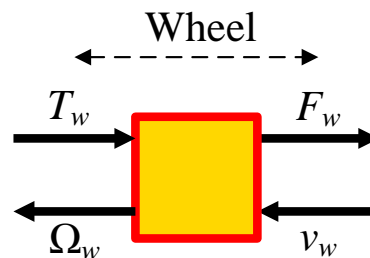
$$T_b = T_{b_ref} \text{ with } T_{b_ref} \leq 0$$



Modelling and representation

- Model and representation of the wheels:
 - Of course, there are two wheels:
 - Rear wheel: its axis adda torques from the free-wheeling system and from the rear brake
 - Front wheel: only the front brake will apply a torque
 - Convert torque into force and linear velocity into angular velocity
 - Radius R_w

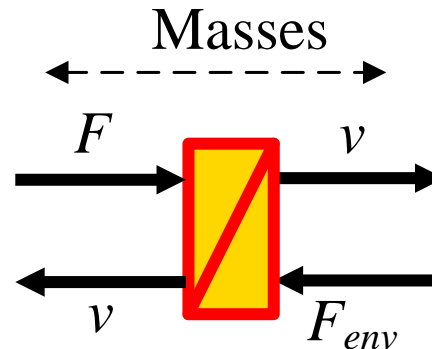
$$\begin{cases} F_w = \frac{1}{R_w} T_w \\ v_w = R_w \cdot \Omega_w \end{cases}$$



Modelling and representation

- Model and representation of the Frame of bike, saddle and handlebars:
 - The two wheels are ideally linked by the ground:
 - They have the same linear velocity, and add the force F they apply.
 - Assuming that M is the mass of the cyclist, M_b is the mass of the cycle
 - For a velocity v :

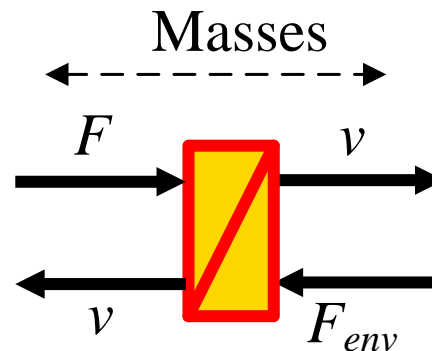
$$(M + M_b) \cdot \frac{dv}{dt} = F - F_{env}$$



Modelling and representation

- Model and representation of the environnement:
 - The two wheels are ideally linked by the ground:
 - They have the same linear velocity, and add the force F they apply.
 - Assuming that M is the mass of the cyclist, M_b is the mass of the cycle
 - For a velocity v :

$$(M + M_b) \cdot \frac{dv}{dt} = F - F_{env}$$

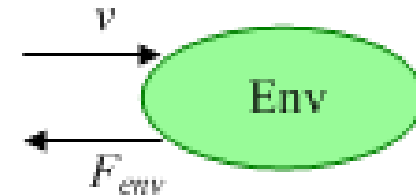


Modelling and Representation

- Model of the environment

- Model

- F_{aero} : aerodynamic resistance
- F_{roll} : rolling resistance
- F_{grade} : grade resistance

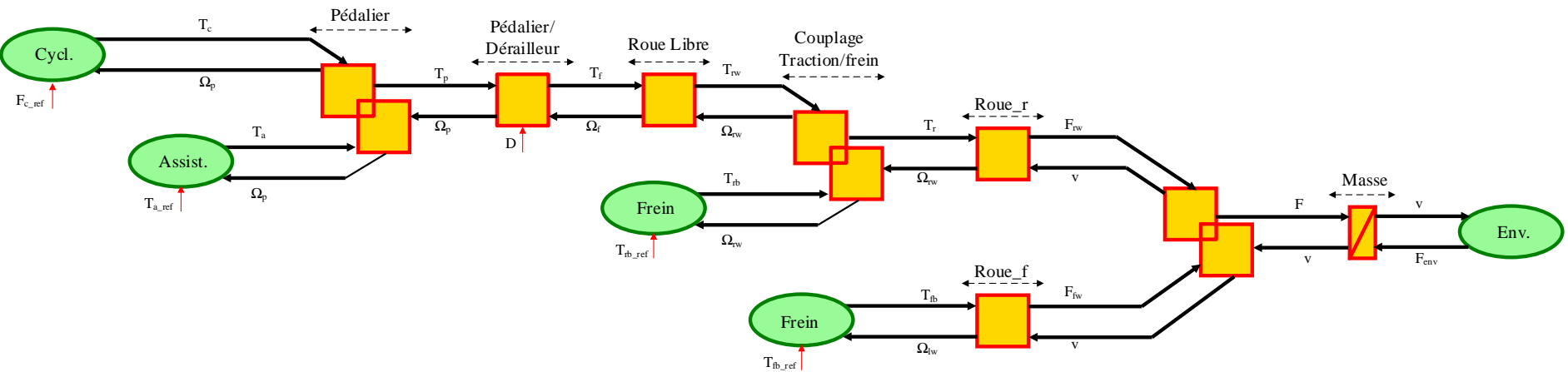


$$\begin{cases} F_{aero} = \frac{1}{2} \rho_{air} S_{CX} \cdot v^2 \\ F_{roll} = C_{rr} (M + M_b) \cdot g \cdot \cos \alpha \\ F_{grade} = (M + M_b) \cdot g \cdot \sin \alpha \end{cases}$$

**If α small
($h/L < 20\%$)**

$$\begin{cases} F_{aero} = \frac{1}{2} \rho_{air} S_{CX} \cdot v^2 \\ F_{roll} = C_{rr} (M + M_b) \cdot g \\ F_{grade} = (M + M_b) \cdot g \cdot \frac{h}{L} \end{cases}$$

EMR of the bike



Activity 1

- Define the Inversed Based Control of the system
 - On the paper
 - Implementation on Matlab/Simulink
 - Tests on a dedicated track (try to respect a 15km/h speed)
 - With/without electrical assistance

