Robotics Assignment #02

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27. April 2015

Task 2.1. 1) The manipulator transformation is a series of multiple rotational $Rot_z(\theta_i)$ and translational $Trans_{x_i}(a_i)$ transformations. That means, ${}^0T_3 = {}^0A_1{}^1A_2{}^2A_3$ is given by

$$\begin{aligned}
{}^0A_1 &= Rot_z(\theta_1) \cdot Trans_{x_1}(a_1) \\
{}^1A_2 &= Rot_z(\theta_2) \cdot Trans_{x_2}(a_2) \\
{}^2A_3 &= Rot_z(\theta_3) \cdot Trans_{x_3}(a_3)
\end{aligned}$$

where $Rot_z(\theta_i) = \begin{bmatrix} C_i & -S_i & 0 & 0 \\ S_i & C_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $Trans_{x_i}(a_i) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

With this in mind, we can calculate the partial homogeneous transformations:

$${}^0A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & C_1a_1 \\ S_1 & C_1 & 0 & S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2a_2 \\ S_2 & C_2 & 0 & S_2a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & C_3a_3 \\ S_3 & C_3 & 0 & S_3a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For intermediate results, we first calculate ${}^0T_2 = {}^0A_1{}^1A_2$ and then ${}^0T_3 = {}^0T_2{}^2A_3$.

$$\begin{aligned}
{}^0T_2 &= \begin{bmatrix} C_1 & -S_1 & 0 & C_1a_1 \\ S_1 & C_1 & 0 & S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & C_2a_2 \\ S_2 & C_2 & 0 & S_2a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_1C_2 - S_1S_2 & -C_1S_2 - S_1C_2 & 0 & C_1C_2a_2 - S_1S_2a_2 + C_1a_1 \\ S_1C_2 + C_1S_2 & -S_1S_2 + C_1C_2 & 0 & S_1C_2a_2 + C_1S_2a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & C_{1+2}a_2 + C_1a_1 \\ S_{1+2} & C_{1+2} & 0 & S_{1+2}a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^0T_3 &= \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & C_{1+2}a_2 + C_1a_1 \\ S_{1+2} & C_{1+2} & 0 & S_{1+2}a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & C_3a_3 \\ S_3 & C_3 & 0 & S_3a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_{1+2}C_3 - S_{1+2}S_3 & -C_{1+2}S_3 - S_{1+2}C_3 & 0 & C_{1+2}C_3a_3 - S_{1+2}S_3a_3 + C_{1+2}a_2 + C_1a_1 \\ S_{1+2}C_3 + C_{1+2}S_3 & -S_{1+2}S_3 + C_{1+2}C_3 & 0 & S_{1+2}C_3a_3 + C_{1+2}S_3a_3 + S_{1+2}a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

$$= \begin{bmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & C_{1+2+3}a_3 + C_{1+2}a_2 + C_1a_1 \\ S_{1+2+3} & C_{1+2+3} & 0 & S_{1+2+3}a_3 + S_{1+2}a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Because the relation $\theta_1+\theta_2+\theta_3 = 180°$ is given, we can simplify the results. For $\cos(180°) = -1$ and $\sin(180°) = 0$:

$$^0T_3 = \begin{bmatrix} -1 & 0 & 0 & -a_3 + C_{1+2}a_2 + C_1a_1 \\ 0 & -1 & 0 & S_{1+2}a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Furthermore, $\cos(\alpha) = -\cos(180° - \alpha)$ and $\sin(\alpha) = \sin(180° - \alpha)$ for $\alpha \in [0°, 180°]$. These facts result in the given transformation matrix:

$$^0T_3 = \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -1 & 0 & S_1a_1 + S_3a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) To rotate around x_0, we need to use Euler angles and append the rotation transformation matrix to the left side of $^0T_3$, because we need to rotate the manipulator before the other transformation can be started.

$$\begin{aligned} {^0T_3}' &= Rot_{x_0}(\theta_0) \cdot {^0T_3} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_0 & -S_0 & 0 \\ 0 & S_0 & C_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -1 & 0 & S_1a_1 + S_3a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -C_0 & -S_0 & C_0S_1a_1 + C_0S_3a_2 \\ 0 & -S_0 & C_0 & S_0S_1a_1 + S_0S_3a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

For the second rotation around x_3 by θ_4, we append the rotation matrix to the right side of $^0T_3$.

$$\begin{aligned} {^0T_3}'' &= {^0T_3} \cdot Rot_{x_3}(\theta_4) \\ &= \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -1 & 0 & S_1a_1 + S_3a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_4 & -S_4 & 0 \\ 0 & S_4 & C_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -C_4 & S_4 & S_1a_1 + S_3a_2 \\ 0 & S_4 & C_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Task 2.2. 1)

2)

Task 2.3. 1)

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