# Robotics Assignment #01

Paul Bienkowski, Konstantin Kobs

20. April 2015

**Task 1.1.** Assuming a right handed coordinate system with the z-axis going upwards towards point E, we have, considering the limitations on the edges being parallel to certain axes, a few permutations for the positions of the vertices A through D. We chose the following base positions:

$$A = \begin{pmatrix} -5 \\ -5 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 5 \\ 5 \\ 0 \end{pmatrix}, D = \begin{pmatrix} -5 \\ 5 \\ 0 \end{pmatrix}, E = \begin{pmatrix} 0 \\ 0 \\ 30 \end{pmatrix}$$

We have 3 different transformation (rotation) matrices for the three steps:

$$R_1 \approx \begin{bmatrix} 0.7071 & -0.7071 & 0 & 0 \\ 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_2 \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.8660 & -0.5 & 0 \\ 0 & 0.5 & 0.8660 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_3 \approx \begin{bmatrix} 0.8660 & 0 & -0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0.5 & 0 & 1 & 0.8660 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1) In this case we use Euler angles, so we need to left multiply the given transformation matrices. We get the following transformation matrix/transformed vertices:

$$M_1 = R_3 R_2 R_1 \approx \begin{bmatrix} 0.4356 & -0.7891 & -0.4330 & 0 \\ 0.6124 & 0.6124 & -0.5 & 0 \\ 0.6597 & -0.0474 & 0.75 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 \approx \begin{pmatrix} 1.7678 \\ -6.1237 \\ -3.0619 \end{pmatrix} B_1 \approx \begin{pmatrix} 6.1237 \\ -0.0000 \\ 3.5355 \end{pmatrix} C_1 \approx \begin{pmatrix} -1.7678 \\ 6.1237 \\ 3.0619 \end{pmatrix} D_1 \approx \begin{pmatrix} -6.1237 \\ 0.0000 \\ -3.5355 \end{pmatrix} E_1 \approx \begin{pmatrix} -12.9904 \\ -15.0000 \\ 22.5000 \end{pmatrix}$$

2) In this case we use Gimbal angles, we need to right multiply the given transformation matrices. We get the following transformation matrix/transformed vertices:

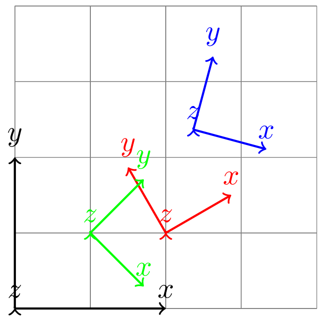
$$M_2 = R_1 R_2 R_3 \approx \begin{bmatrix} 0.7891 & -0.6124 & -0.0474 & 0.0000 \\ 0.4356 & 0.6124 & -0.6597 & 0.0000 \\ 0.4330 & 0.5000 & 0.7500 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

$$A_1 \approx \begin{pmatrix} -0.8839 \\ -5.2398 \\ -4.6651 \end{pmatrix}, B_1 \approx \begin{pmatrix} 7.0076 \\ -0.8839 \\ -0.3349 \end{pmatrix}, C_1 \approx \begin{pmatrix} 0.8839 \\ 5.2398 \\ 4.6651 \end{pmatrix}, D_1 \approx \begin{pmatrix} -7.0076 \\ 0.8839 \\ 0.3349 \end{pmatrix}, E_1 \approx \begin{pmatrix} -1.4210 \\ -19.7922 \\ 22.5000 \end{pmatrix}$$

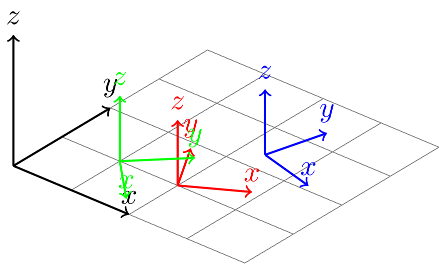
**Task 1.2.**

1) To calculate the homogeneous transformation from A to C, or short ${}^{A}T_{C}$, we need to right multiply ${}^{A}T_{B}$ and ${}^{B}T_{C}$. Intuitively we transform all points in A to B and then to C. That's why we need to right multiply the matrices. So the transformation ${}^{A}T_{C}$ is unambiguously defined.

2) Since both transformations contain only rotations around z-axis and translations on the xy-plane, we can visualize using a perspective that neglects the z-axis as follows:




Here is a 3D visualization with a grid on the xy-plane:




**Task 1.3.**

1) Examples for $(\phi, \theta, \psi)$ are:

   i. $(90°, -20°, 30°)$ in $ZY'X''$ means a $90°$ rotation around the z-axis, a rotation by $-20°$ around the newly defined y-axis and a $30°$ rotation around the new frame's x-axis.

   ii. $(-15°, 100°, 12°)$ in $XZ'Y''$ means a $-15°$ rotation around the x-axis, a rotation by $100°$ around the newly defined z-axis and a $12°$ rotation around the new frame's y-axis.

   iii. $(30°, 10°, -50°)$ in $YX'Y''$ means a $30°$ rotation around the y-axis, a rotation by $10°$ around the newly defined x-axis and a $-50°$ rotation around the new frame's y-axis.

2) If we rotate an object around one axis, the coordinate system changes in the other two axes, but not in the rotated one. Rotating again around the same axis is senseless, because we could have achieved that with the first rotation. Hence, we have 3 choices for the first axis, and 2 choices for each remaining axis, resulting in the following number of rotation sequences:

$$n_{\text{sequences}} = 3 \cdot 2 \cdot 2 = 12$$