Robotics Assignment #01

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Task 1.1. **Note:** All the code for calculating the results can be found at `http://tinyurl.com/qbmvazo`. The given points are $A = (5, -5, 0), B = (5, 5, 0), C = (-5, 5, 0), D = (-5, -5, 0), E = (0, 0, 20)$

1) In this case we use Euler-angles, so we need to left multiply the given transformation matrices. We get the following transformation matrix:

$$M \approx \begin{bmatrix} 0.78914913099243 & -0.61237243569579 & 0.65973960844117 & 0 \\ -0.78914913099243 & 0.61237243569579 & -0.65973960844117 & 0 \\ 0.43301270189222 & 0.5 & 0.75 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After multiplying it with the given points, we get

$$\begin{aligned} A' &\approx (7.0076078334411, -7.0076078334411, -0.3349364905389), \\ B' &\approx (0.88388347648318, -0.88388347648318, 4.6650635094611), \\ C' &\approx (-7.0076078334411, 7.0076078334411, 0.3349364905389), \\ D' &\approx (-0.88388347648318, 0.88388347648318, -4.6650635094611), \\ E' &\approx (19.792188253235, -19.792188253235, 22.5) \end{aligned}$$

2) In this case we use Gimbal-angles, we need to right multiply the given transformation matrices. We get the following transformation matrix:

$$M' \approx \begin{bmatrix} 0.43559574039916 & -0.43559574039916 & 0.43301270189222 & 0 \\ -0.61237243569579 & 0.61237243569579 & -0.5 & 0 \\ 0.047367172745376 & -0.047367172745376 & 0.75 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After multiplying it with the given points, we get

$$\begin{aligned} A'' &\approx (4.3559574039916, -6.1237243569579, 0.47367172745376), \\ B'' &\approx (4.4408920985006 \cdot 10^{-16}, 4.4408920985006 \cdot 10^{-16}, 5.5511151231258 \cdot 10^{-16}), \\ C'' &\approx (-4.3559574039916, 6.1237243569579, -0.47367172745376), \\ D'' &\approx (-4.4408920985006 \cdot 10^{-16}, -4.4408920985006 \cdot 10^{-16}, -5.5511151231258 \cdot 10^{-16}), \\ E'' &\approx (12.990381056767, -15, 22.5) \end{aligned}$$

Task 1.2.

1) To calculate the homogeneous transformation from A to C, or short ${}^{A}T_{C}$, we need to right multiply ${}^{A}T_{B}$ and ${}^{B}T_{C}$. Intuitively we transform all points in A to B and then to C. That's why we need to right multiply the matrices. So the transformation ${}^{A}T_{C}$ is unambiguously defined.

2)

Task 1.3. 1) Examples for ϕ, θ, ψ are:

i. $90°, -20°, 30°$ in $ZY'X''$ means a 90° rotation around the z-axis, a rotation by $-20°$ around the newly defined y-axis and a 30° rotation around the new frame's x-axis.

ii. $-15°, 100°, 12°$ in $XZ'Y''$ means a $-15°$ rotation around the x-axis, a rotation by 100° around the newly defined z-axis and a 12° rotation around the new frame's y-axis.

iii. $30°, 10°, -50°$ in $YX'Y''$ means a 30° rotation around the y-axis, a rotation by 10° around the newly defined x-axis and a $-50°$ rotation around the new frame's y-axis.

2) If we rotate an object around one axis, the coordinate system changes in the other two axis, but not in the rotated one. Rotating again around the same axis is senseless, because we could have achieved that with the first rotation. So we have the following number of rotation sequences:

$$n_{\text{sequences}} = 3 \cdot 2 \cdot 2 = 12$$