

Robotics Assignment #02

Paul Bienkowski, Konstantin Kobs

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Task 2.1. 1) The manipulator transformation is a series of multiple rotational $Rot_z(\theta_i)$ and translational $Trans_{x_i}(a_i)$ transformations. That means, ${}^0T_3 = {}^0A_1 {}^1A_2 {}^2A_3$ is given by

$${}^0A_1 = Rot_z(\theta_1) \cdot Trans_{x_1}(a_1)$$

$${}^1A_2 = Rot_z(\theta_2) \cdot Trans_{x_2}(a_2)$$

$${}^2A_3 = Rot_z(\theta_3) \cdot Trans_{x_3}(a_3)$$

$$\text{where } Rot_z(\theta_i) = \begin{bmatrix} C_i & -S_i & 0 & 0 \\ S_i & C_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } Trans_{x_i}(a_i) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

With this in mind, we can calculate the partial homogeneous transformations:

$${}^0A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & C_1 a_1 \\ S_1 & C_1 & 0 & S_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2 a_2 \\ S_2 & C_2 & 0 & S_2 a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & C_3 a_3 \\ S_3 & C_3 & 0 & S_3 a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For intermediate results, we first calculate ${}^0T_2 = {}^0A_1 {}^1A_2$ and then ${}^0T_3 = {}^0T_2 {}^2A_3$.

$$\begin{aligned} {}^0T_2 &= \begin{bmatrix} C_1 & -S_1 & 0 & C_1 a_1 \\ S_1 & C_1 & 0 & S_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & C_2 a_2 \\ S_2 & C_2 & 0 & S_2 a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} C_1 C_2 - S_1 S_2 & -C_1 S_2 - S_1 C_2 & 0 & C_1 C_2 a_2 - S_1 S_2 a_2 + C_1 a_1 \\ S_1 C_2 + C_1 S_2 & -S_1 S_2 + C_1 C_2 & 0 & S_1 C_2 a_2 + C_1 S_2 a_2 + S_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & C_{1+2} a_2 + C_1 a_1 \\ S_{1+2} & C_{1+2} & 0 & S_{1+2} a_2 + S_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ {}^0T_3 &= \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & C_{1+2} a_2 + C_1 a_1 \\ S_{1+2} & C_{1+2} & 0 & S_{1+2} a_2 + S_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & C_3 a_3 \\ S_3 & C_3 & 0 & S_3 a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} C_{1+2} C_3 - S_{1+2} S_3 & -C_{1+2} S_3 - S_{1+2} C_3 & 0 & C_{1+2} C_3 a_3 - S_{1+2} S_3 a_3 + C_{1+2} a_2 + C_1 a_1 \\ S_{1+2} C_3 + C_{1+2} S_3 & -S_{1+2} S_3 + C_{1+2} C_3 & 0 & S_{1+2} C_3 a_3 + C_{1+2} S_3 a_3 + S_{1+2} a_2 + S_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & C_{1+2+3}a_3 + C_{1+2}a_2 + C_1a_1 \\ S_{1+2+3} & C_{1+2+3} & 0 & S_{1+2+3}a_3 + S_{1+2}a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Because the relation $\theta_1 + \theta_2 + \theta_3 = 180^\circ$ is given, we can simplify the results. For $\cos(180^\circ) = -1$ and $\sin(180^\circ) = 0$:

$${}^0T_3 = \begin{bmatrix} -1 & 0 & 0 & -a_3 + C_{1+2}a_2 + C_1a_1 \\ 0 & -1 & 0 & S_{1+2}a_2 + S_1a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Furthermore, $\cos(\alpha) = -\cos(180^\circ - \alpha)$ and $\sin(\alpha) = \sin(180^\circ - \alpha)$ for $\alpha \in [0^\circ, 180^\circ]$. These facts result in the given transformation matrix:

$${}^0T_3 = \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -1 & 0 & S_1a_1 + S_3a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 2) To rotate around x_0 , we need to use Euler angles and append the rotation transformation matrix to the left side of 0T_3 , because we need to rotate the manipulator before the other transformation can be started.

$$\begin{aligned} {}^0T_3' &= Rot_{x_0}(\theta_0) \cdot {}^0T_3 \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_0 & -S_0 & 0 \\ 0 & S_0 & C_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -1 & 0 & S_1a_1 + S_3a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -C_0 & -S_0 & C_0S_1a_1 + C_0S_3a_2 \\ 0 & -S_0 & C_0 & S_0S_1a_1 + S_0S_3a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

For the second rotation around x_3 by θ_4 , we append the rotation matrix to the right side of 0T_3 .

$$\begin{aligned} {}^0T_3'' &= {}^0T_3 \cdot Rot_{x_3}(\theta_4) \\ &= \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -1 & 0 & S_1a_1 + S_3a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_4 & -S_4 & 0 \\ 0 & S_4 & C_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & 0 & C_1a_1 - C_3a_2 - a_3 \\ 0 & -C_4 & S_4 & S_1a_1 + S_3a_2 \\ 0 & S_4 & C_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Task 2.2. 1)
2)

Task 2.3.

Link i	θ_i	α_{i-1}	a_{i-1}	d_i
1	θ_1	0°	L_1	0
2	θ_2	0°	L_2	0
3	θ_3	0°	L_3	0

Task 2.4. 1)
2)