Robotics Assignment #04

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Task 4.1. To calculate the Jacobian matrix, we need the position of the end-effector based on the joint angles and the orientation of the end-effector. To get the position, we need to determine the homogeneous transformation from the base to the end-effector point. We can reuse the general transformation matrix given in assignment 2 task 1, because these manipulators are the same. Only a_i needs to be changed to l_i. Therefore we have

$$^0T_3 = \begin{pmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & C_{1+2+3}l_3 + C_{1+2}l_2 + C_1 l_1 \\ S_{1+2+3} & C_{1+2+3} & 0 & S_{1+2+3}l_3 + S_{1+2}l_2 + S_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From this matrix, we can easily extract the position of the end-effector, which is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} C_{1+2+3}l_3 + C_{1+2}l_2 + C_1 l_1 \\ S_{1+2+3}l_3 + S_{1+2}l_2 + S_1 l_1 \\ 0 \end{pmatrix}$$

The orientation of the end-effector is easy to determine. Because the manipulator just moves in the (x, y)-plane, we can set the orientation vector to

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \theta_1 + \theta_2 + \theta_3 \end{pmatrix}$$

with ω denotes the angular velocity. Now we can calculate the Jacobian matrices J_v and J_w and from this the resulting Jacobian matrix J, which are all partial derivatives of both determined vectors:

$$\begin{aligned} J_v &= \begin{pmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3} \end{pmatrix} \\ &= \begin{pmatrix} -S_{1+2+3}l_3 - S_{1+2}l_2 - S_1 l_1 & -S_{1+2+3}l_3 - S_{1+2}l_2 & -S_{1+2+3}l_3 \\ C_{1+2+3}l_3 + C_{1+2}l_2 + C_1 l_1 & C_{1+2+3}l_3 + C_{1+2}l_2 & C_{1+2+3}l_3 \\ 0 & 0 & 0 \end{pmatrix} \\ J_w &= \begin{pmatrix} \frac{\partial \omega_x}{\partial \theta_1} & \frac{\partial \omega_x}{\partial \theta_2} & \frac{\partial \omega_x}{\partial \theta_3} \\ \frac{\partial \omega_y}{\partial \theta_1} & \frac{\partial \omega_y}{\partial \theta_2} & \frac{\partial \omega_y}{\partial \theta_3} \\ \frac{\partial \omega_z}{\partial \theta_1} & \frac{\partial \omega_z}{\partial \theta_2} & \frac{\partial \omega_z}{\partial \theta_3} \end{pmatrix} \end{aligned}$$

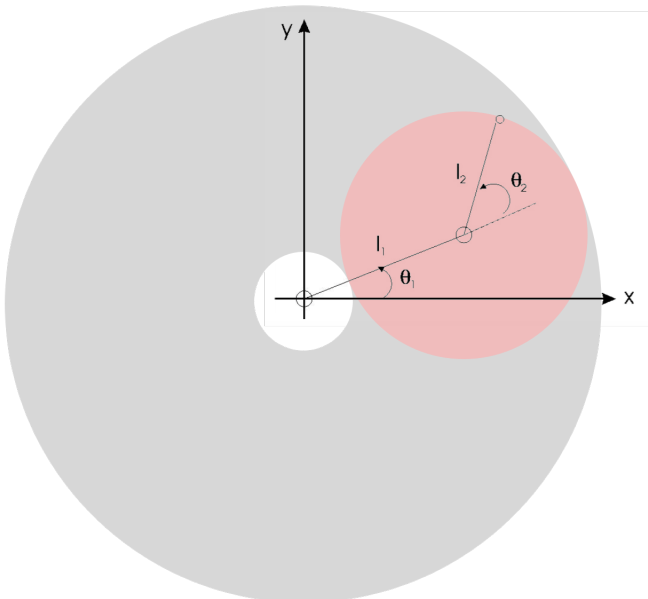
$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$J = \begin{pmatrix} J_v \\ J_w \end{pmatrix}$$

$$= \begin{pmatrix} -S_{1+2+3}l_3 - S_{1+2}l_2 - S_1 l_1 & -S_{1+2+3}l_3 - S_{1+2}l_2 & -S_{1+2+3}l_3 \\ C_{1+2+3}l_3 + C_{1+2}l_2 + C_1 l_1 & C_{1+2+3}l_3 + C_{1+2}l_2 & C_{1+2+3}l_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Task 4.2.

1) The visualization can be found in the following figure. The grey area that looks like a DVD is the reachable workspace of the arm. The red area depicts the space the second link can reach.



2)
3)
4)

Task 4.3.

Task 4.4.