Elementary Concepts in Linear Algebra

Number Representations

1

a) Convert $1\,110\,000\,101$ from base 2 into base 10 by hand.

1

b) Convert 852 from base 10 into base 2 by hand.

3

c) Repeat the above base 2 conversions by utilizing R.

1

d) Convert $756\,001$ from base 8 into base 10 by hand.

1

e) Convert 9530 from base 10 into base 8 by hand.

3

f) Repeat the above base 2 conversions by utilizing R.

10

g) Write a function `dec_to_bin` that takes a decimal number as input and outputs its binary representation in a vector of 1s and 0s. Instead of using the build-in conversion from R, implement this with help of an appropriate loop. Then test its correctness by calculating the binary representation of 852.

Polynomials

4

h) Write a function `my_power` in R that calculates the exponentiation of two numbers b^n using a for-loop. Assume that $n \in \mathbb{N}$. Finally, use this function to calculate 7^3 and 5^0.

4

i) Create a function `horner` that evaluates a polynomial at given point $x_0 = 3$ by utilizing the Horner scheme for a given vector of polynomial coefficients. For example, the polynomial $2x^4 - 4x^3 - 5x^2 + 7x + 11$ corresponds to a vector representation `c(2, -4, -5, 7, 11)` in R.

Linear Algebra

2

j) Calculate the dot product for the following two vectors by hand and with the help of R.

$$\boldsymbol{x} = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{y} = \begin{bmatrix} 8 \\ 7 \\ -3 \end{bmatrix}.$$

2

k) Write a function `dot_product` in R to calculate the dot product of the vectors from the previous exercise. Use a for loop to calculate the result.

2

l) Write a function `mult_matrix_vector` in R to multiply a matrix by a vector. Use a for loop to calculate the result. Apply your function to A and $\boldsymbol{x}$ defined by

$$A = \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ 0 & 12 \end{bmatrix} \quad \text{and} \quad \boldsymbol{x} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

and then compare to the built-in functionality within R.

2

m) Write a function `mult_matrix_matrix` in R to multiply two matrices. Use a for loop to calculate the result. Apply your function to A and B with

$$A = \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ 0 & 12 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 2 & 5 & 1 \\ -2 & 0 & 4 & 1 \end{bmatrix}.$$

Hint: The result should be $AB = \begin{bmatrix} 6 & 8 & 32 & 7 \\ -17 & -2 & 23 & 6 \\ -24 & 0 & 48 & 12 \end{bmatrix}$.

3

n) Consider the vector

$$\boldsymbol{x} = \begin{bmatrix} -3 \\ -4 \\ -1 \\ 0 \\ 2 \end{bmatrix}.$$

What are its L^1-, L^2 and 4-norm?

2

o) Calculate the inverse (or pseudoinverse, depending on what is necessary) for the following matrix. Double-check subsequently.

$$A = \begin{bmatrix} 3 & 7 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 8 \\ 3 & 12 \end{bmatrix}$$

1

p) Find the determinant, eigenvalues and eigenvectors to the matrix

$$A = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}$$

by hand.

1

q) Calculate the determinates, eigenvalues and eigenvectors of the two matrices

$$A_1 = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

with the help of R.

1

r) Are the following two matrices positive or negative definite?

$$A_1 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$