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1 Atkinson Index

1.1 Definition

For the purpose of measuring credit portfolio or market portfolio **Concentration Risk**, income inequality or diversity, the **Atkinson Index** is a parametric family of indexes which, given an index parameter *epsilon* is defined as follows

1.2 Details

If we have n exposures (alternatively values / income measurements) E_i summing up to a total value of

$$E_T = \sum_{i=1}^n E_i$$

where each observation's fraction is defined as

$$w_i = E_i / E_T$$

Then the Atkinson index is given by^[1]

$$A_{\epsilon} = \begin{cases} 1 - n^{\epsilon} / (n^{\epsilon} - 1) \left(\sum_{i=1}^n w_i^{1-\epsilon} \right)^{1/(1-\epsilon)} & \text{for } 0 \leq \epsilon < 1 \\ 1 - n \sum_{i=1}^n w_i \log w_i & \text{for } \epsilon = 1 \end{cases}$$

- The index parameter epsilon can take any positive value
- The index varies between zero (homogeneous observations) and one (perfect concentration)

1.3 Usage

None

1.4 Variations

None

1.5 Issues and Challenges

None

1.6 Implementation

Open Source implementations of the Atkinson index are available in

- the R package [IC2](#)
- the Python library [Concentration Library](#)

1.7 See Also

- [Wikipedia](#)

1.8 References

1. ? A. Atkinson, "On the measurement of inequality", Journal of Economic Theory 2, 244-263 (1970)

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2 Berger-Parker Index

2.1 Definition

Berger-Parker Index for the purpose of measuring credit portfolio or market **Concentration Risk**, diversity or inequality metrics the Berger-Parker Index is a measure of the contribution of the largest entity of exposure to the total portfolio exposure - essentially the fraction.

It is the special case $k=1$ of the **Concentration Ratio**. It is the fraction of the population formed by the most abundant category.

2.2 Formula

The concentration ratio is simply the percentage of portfolio exposure by the n largest exposures. Assuming the exposure data are given and ordered, calculation is easy.

$$BP = CR_1 = w_1$$

where w_i is simply the share of the i -th exposure in the total portfolio exposure.

2.3 Variations

None

2.4 Issues and Challenges

Although the data requirements for the Berger-Parker calculation seem to be moderate at first sight, they often turn out to be difficult to meet in practice. They require the aggregation of all exposures to the same borrower for the whole business entity, be it a bank or a banking group. A heterogeneous IT environment can already present a serious technical obstacle to this aggregation. Furthermore, large borrowers, which are actually the most relevant entities for measuring name concentration, can themselves be complex structures that are not always easy to identify as belonging to the same borrower entity.

2.5 Implementation

Open Source implementations of the Berger-Parker are available in

- the R package **IC2**
- the Python library **Concentration Library**

2.6 See Also

None

2.7 References

{{#set:Has Formula = True | Has SourceCode = True | Has Object = False | Has Lambda = False | Field Type= Documentation}}

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3 Category:Concentration Measurement

Articles related to [Concentration Measurement](#) in general

4 Concentration Ratio

4.1 Definition

For the purpose of measuring credit portfolio or market **Concentration Risk**, diversity or inequality metrics the **concentration ratio** is a measure of the contribution of a given set of exposures to the total portfolio exposure - essentially the portfolio fraction. For example the fraction of exposure contributed by the top ten largest clients, the contribution to the market capitalization by the top five ten largest listed companies etc.

4.2 Details

The concentration ratio is simply the percentage of portfolio exposure by the n largest exposures. Assuming the exposure data are given, calculation is easy.

The ratio associated with the contribution of k entities (in a portfolio or sample of size $n > k$) is denoted as

$$CR_k = \sum_{i=1}^k w_i \quad \{\displaystyle CR_k = \sum_{i=1}^k w_i\}$$

where w_i is simply the share of the i -th exposure in the total portfolio exposure.

4.2.1 Berger Parker Index (special case)

In **diversity studies**, the special case $k=1$ is denoted as the Berger-Parker index. It is the fraction of the population formed by the most abundant category.

4.3 Usage

Along with the **Herfindahl-Hirschman Index** (HHI) the concentration ratio is a tool typically used by competition authorities to measure market concentration.

These tools are useful also in the context of analysing portfolio concentrations, for example credit or market risk concentrations.

The concentration ratio is a "relative" measure, i.e., all exposures are normalized as fractions of total portfolio exposure, therefore two portfolios with same index are deemed to be equally concentrated irrespective of their absolute total size.

4.3.1 Common Ratios

There is no intrinsic rule to select the number n of exposures to assess. Some commonly used examples include

- The $n=1$ Concentration Ratio measures the total contribution of the largest exposure (equivalent to the Berger-Parker index).
- The $n=4$ Concentration Ratio measures the total contribution of the four largest exposures.
- The $n=8$ Concentration Ratio measures the total contribution of the eight largest exposures.
- The $n=20$ Concentration Ratio measures the total contribution of the twenty largest exposures.
- The $n=50$ Concentration Ratio measures the total contribution of the fifty largest exposures.

The case $n=20$ is important for the purposes of credit risk concentration analysis because it is used in the **Large Exposures Framework**

4.3.2 Interpretation and Concentration levels

Concentration ratios can range in value from 0 to 100 percent.

Whether a given result indicates a concentrated portfolio cannot be assessed beforehand but only in relation with typical portfolios, standard market practices etc. or other metrics. For example the **Large Exposures Framework** links the size of the top 20 large exposures to the capital of the firm.

Concentration levels can be characterized as *no*, *low* or *medium* to *high* to "total" concentration based on subjective thresholds.

4.3.3 Advantages

Key advantages of the concentration ratio index include

- Its conceptual and computational simplicity. Indeed even a casual glance at the sorted and normalized (divided by total) exposures provides already an intuitive feeling as to the degree of concentration in a given portfolio and
- The relatively moderate data requirements, i.e. only the exposure size per borrower (but see qualification below)

4.3.4 Disadvantages

- The definition of the concentration ratio must pick a set of representative entities (the number n) to assess concentration and does not use the contribution of all exposures. A portfolio may look acceptable for a given n , but less so for another choice.
- The index suffers from a one-dimensional view of concentration risk, namely it is assumed that relative size is the only factor worthy of consideration. In reality there may be a wide range of other factors
 1. Exposure size may not be obvious to define
 2. Risks associated with different exposures can be very different
 3. There may be varying degrees of dependency among different exposures
- It is difficult to objectively characterize the concentration thresholds.

The [Herfindahl-Hirschman Index](#) and other indexes provide more complete picture of concentration than does the concentration ratio, thereby mitigating the first of the above disadvantages

4.4 Variations

None

4.5 Issues and Challenges

Although the data requirements for the concentration ratio calculation seem to be moderate at first sight, they often turn out to be difficult to meet in practice. They require the aggregation of all exposures to the same borrower for the whole business entity, be it a bank or a banking group. A heterogeneous IT environment can already present a serious technical obstacle to this aggregation. Furthermore, large borrowers, which are actually the most relevant entities for measuring name concentration, can themselves be complex structures that are not always easy to identify as belonging to the same borrower entity.

4.6 Implementation

Open Source implementations of the Concentration Ratio are available in

- the R package [IC2](#)
- the Python library [Concentration Library](#)

4.7 See Also

None

4.8 References

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5 Country Risk Concentration

5.1 Definition

Country Risk Concentration

6 Credit Risk Concentration

6.1 Definition

Credit Risk Concentration refers to disproportionately large risk exposure to specific credit risks (as opposed to a diversified risk profile).

Regulatory frameworks generally recognize the following specific **concentrations risks**:

- Name Concentration
- Sector Concentration
- Geographic Concentration
- Product Concentration

6.2 Data Requirements

Measuring credit risk concentration requires detailed information about **Exposure** (loan level data, accurate sector assignment etc.)

6.3 Calculation

There is a large variety of approaches for measuring Concentration Risk. At its simplest it can be calculated using a "concentration ratio" which explains what percentage of the outstanding total risk is represented by the largest exposure. For example, if a bank has 5 outstanding loans, with four of equal value and the fifth having twice the value, then the concentration ratio is 1/3. Slightly more sophisticated concentration indicators are e.g. the **Herfindahl-Hirschman Index** and the **Gini Index**

Various other considerations may enter into concentration when applying risk analysis in real world applications, most notably:

- the definition of individual exposures and the **related entity** aggregation of exposure
- the definition of **industrial sectors** and allocation of portfolio to sectors
- the relative riskiness of exposures expressed e.g., in terms of the **probability of default** or the **expected loss**

6.4 Mitigation

Credit concentration risk can be controlled with risk management tools such as:

- Individual **limits** for name concentration
- Higher level **industry and country limits**
- Hedging of exposures
- Outright sales of exposures

6.5 Issues and Challenges

- There can be more esoteric forms of concentration risk, for example **product concentration** which may overlap in part with other forms of concentration
-

7 Ellison-Glaeser Index

7.1 Definition

The **Ellison-Glaeser Index (EG)** is an index developed for the assessment of industrial agglomeration^[1]

7.2 Details

The calculation process for the EG index is detailed here (the terminology and steps are adjusted versus the original publication to typical **Concentration Risk** applications)

7.2.1 Exposures

We have N exposures E_k summing up to a total exposure of

$$E_T = \sum_{k=1}^N E_k$$

Each exposure is associated with one industry (business sector) and one geography. The total HHI index capturing single exposure concentration across the entire portfolio is

$$H = \sum_{k=1}^N \left(\frac{E_k}{E_T} \right)^2$$

7.2.2 Business Sectors

We have I industries (business sectors) $i = 1, 2, \dots, I$. Each industry comprises of N^i exposures. The total exposure per industry is

$$E^{\bullet i} = \sum_{k=1}^{N^i} E_k$$

where the bullet denotes the implied summation over all areas. The fraction of each exposure within an industry is

$$z_{ki} = \frac{E_k}{E^{\bullet i}}$$

The HHI index *within* each industry is defined as

$$H^i = \sum_{k=1}^{N^i} z_{ki}^2$$

The fraction of each industry as part of the total exposure is

$$w_i = \frac{E^{\bullet i}}{E_T}$$

The HHI index capturing concentration in the distribution of the different industry sectors

$$H^I = \sum_{i=1}^I (w_i)^2 H^i$$

The aggregate HHI is connected with the industry specific indexes via

$$H = \sum_{i=1}^I w_i^2 H^i$$

7.2.3 Geographic Areas

We have M areas (countries or regions) $a = 1, 2, \dots, M$. Each area comprises of N^a exposures. The total exposure per area is

$$E^{\bullet a} = \sum_{k=1}^{N^a} E_k$$

The fraction of each area as part of the total exposure is

$$x^a = \frac{E^a}{E^T} \quad \{\displaystyle x^a = \frac{E^{\{a\}}{E_{\{T\}}}\}$$

The geographic concentration is captured by an HHI type metric

$$H^G = \sum_{a=1}^M x^a \quad \{\displaystyle H^G = \sum_{a=1}^M x^a\}$$

7.2.4 Raw Concentration Metric

Within each area, an industry comprises of N^i exposures, summing up to a total of E^i for each industry / area combination. The allocation of industry exposure to areas is given by

$$s^i = \frac{E^i}{E^T} \quad \{\displaystyle s^i = \frac{E^i}{E^{\{T\}}}\}$$

An (ad-hoc) metric capturing industry exposure concentration per area

$$G^i = \sum_{a=1}^M (s^i - x^a)^2 = \sum_{a=1}^M (s^i - \frac{E^i}{E^T})^2 \quad \{\displaystyle G^i = \sum_{a=1}^M (s^i - x^a)^2 = \sum_{a=1}^M (\frac{E^i}{E^{\{T\}}} - \frac{E^{\{a\}}{E^{\{T\}}})^2\}$$

This metric is zero when industrial exposure in an area as a fraction of the total in that industry is the same as the fraction of all industrial exposure in that area over all exposure.

7.2.5 EG Index per industry

The EG industrial concentration index (per industry i) is given by

$$\gamma^i = \frac{G^i}{(1 - H^G)} - H^i \quad \{\displaystyle \gamma^i = \frac{G^i}{(1 - H^G)} - H^i\}$$

7.3 Usage

None

7.4 Variations

None

7.5 Issues and Challenges

- The setup for the calculation of the EG index implies that the geographical distribution of exposures is given (i.e. it does not measure geographic distribution per se, but the relative concentration of industrial exposure *given* the overall geographical distribution)
- The EG index is sensitive to the definition of geographical regions

7.6 Implementation

Open Source implementations of the Ellison-Glaeser index are available in

- The Python library [Concentration Library](#)
- The Python [spatial analysis library PySAL](#)

7.7 See Also

None

7.8 References

1. ? Ellison, Glaeser, Geographic Concentration in U.S. Manufacturing Industries: A Dartboard Approach, 1997

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8 Generalized Entropy Index

8.1 Definition

For the purpose of measuring concentration, the **Generalized Entropy Index** is measure of concentration that draws from concepts of information theory

8.2 Details

If the total exposure is

$$E_T = \sum_{i=1}^n E_i \quad \{\displaystyle E_T = \sum_{i=1}^n E_i\}$$

and the fractional exposures w_i $\{\displaystyle w_i\}$ are defined as

$$w_i = E_i / E_T \quad \{\displaystyle w_i = \frac{E_i}{E_T}\}$$

Then the Generalized Entropy index is defined as

$$GE(\alpha) = \begin{cases} \frac{1}{n\alpha(\alpha-1)} \sum_{i=1}^n (n w_i^\alpha - n), & \alpha \neq 0, 1, \log(n) + \sum_{i=1}^n w_i \log(w_i), & \alpha = 1, -\log(n) - \frac{1}{n} \sum_{i=1}^n \log(w_i), & \alpha = 0. \end{cases} \quad \{\displaystyle GE(\alpha) = \begin{cases} \frac{1}{n\alpha(\alpha-1)} \sum_{i=1}^n \left(n w_i^\alpha - n \right), & \alpha \neq 0, 1, \log(n) + \sum_{i=1}^n w_i \log(w_i), & \alpha = 1, -\log(n) - \frac{1}{n} \sum_{i=1}^n \log(w_i), & \alpha = 0. \end{cases}\}$$

8.2.1 Relation with the Theil Index

The [Theil Index](#) is the Generalized Entropy Index for $\alpha = 1$

8.3 Usage

None

8.4 Variations

None

8.5 Issues and Challenges

None

8.6 Implementation

Open Source implementations of the Generalized Entropy index are available in

- the R package [lneq](#)
- the Python library [Concentration Library](#)

8.7 See Also

- [Shannon Index](#)

8.8 References

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9 Category:Geographic Concentration

Articles about Geographic Concentration

10 Geographic Concentration Measurement

10.1 Definition

Geographic Concentration Measurement denotes the quantitative assessment of the degree to which a portfolio may be excessively concentrated in a particular geography or region. When the geography in question is a single country we have a more specific form of **Country Risk Concentration Measurement**.

10.2 Benchmarks

For any quantification of risks, it is convenient to have quantitative benchmarks, for example to measure the distance from a neutral reference state of no concentration or full diversification. Some concentration/diversification indicators can be defined at portfolio level, providing synthetic measures of geographic concentration.

10.3 Concentration Ratio

- The simplest method to quantify geographic concentration is computing the share of exposure (EAD) held by the k largest geographies or regions in the portfolio relative to total exposure.
- Weaknesses of this index are that the choice of k is arbitrary (e.g., country level, cluster of countries etc.) and the index does not use all the information available
- In place of EAD, exposures can be measured as product of $EAD \cdot LGD$, thus considering the expected severity of losses which can actually differentiate the effective contribution to credit risk

10.4 Gini Index and the Lorenz Curve

The standard Gini index can also be used to measure geographic concentration

- The index G varies between 0 (perfect equality of exposures) to 1 for perfect inequality (limit in which one region accounts for the whole exposure and the others tend to zero).
- The index is sensitive to inhomogeneity of exposures but not to exposure number.

10.5 Herfindhahl-Hirschman Index

The standard HHI index can also be used to measure geographic concentration

- The index reflects both geographic heterogeneity and number of recognized regions (e.g. it tends to zero for many granular regions)

10.6 Ellison and Glaeser Index

The **Ellison-Glaeser Index** (and related variations) are indexes that have been developed in the context of assessing industrial agglomeration^[1]

10.7 Issues and Challenges

- Geographic Regions are somewhat arbitrary specifications. Sovereign entities included within each region are usually also very relevant from a risk perspective
- The interaction of sectoral and geographic concentration is a complex problem that is still not adequately understood

10.8 References

1. ? Ellison, Glaeser, Geographic Concentration in U.S. Manufacturing Industries: A Dartboard Approach, 1997

11 Gini Index

11.1 Definition

For the purpose of measuring concentration, the **Gini Index** (also Gini coefficient) is an index defined in terms of the Lorenz curve of distribution values.

11.2 Details

More precisely, if we have n values E_i summing up to a total value of

$$E_T = \sum_{i=1}^n E_i$$

and the fractional value w_i is defined as

$$w_i = E_i / E_T$$

Then the Gini index is defined as the area under the Lorenz curve which is geometrically reduced to

$$G = 1 + \frac{1}{n} \sum_{i=1}^n (1 - 2i) w_i$$

11.3 Alternative Formula

Gini's *absolute mean difference* is defined as

$$\Delta = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n |E_i - E_j|$$

The relative mean difference is defined as Δ / μ where $\mu = E_T / n$

The Gini index is equivalently given by

$$G = \Delta / \mu$$

11.4 Usage

None

11.5 Variations

None

11.6 Issues and Challenges

NB: Sometimes the formula appears also with the opposite sign!

11.7 Implementation

Open Source implementations of the Gini index are available in

- the R package [Ineq](#)
- the Python library [Concentration Library](#)

11.8 See Also

- [Hall-Tideman Index](#)

11.9 References

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12 Granularity Adjustment

12.1 Definition

The **Granularity Adjustment** (GA) for the ASRF model is an approximation formula for calculating the capital needed to cover the risk arising from the potential default of large borrowers.

12.2 Context

The granularity adjustment is an extension of the ASRF model which forms the theoretical basis of the Internal Ratings-Based (IRB) approaches of Basel II/Basel III. Through this adjustment, single Name Concentration is integrated into the ASRF model thereby making capital requirements more risk sensitive.

The ASRF model assumes that portfolios are fully diversified with respect to individual borrowers, so that risk capital depends only on Systematic Risk. Hence, the IRB formula omits the contribution of the residual Idiosyncratic Risk to the required capital.

A granularity adjustment that incorporates name concentration in the IRB model was already included in the Second Consultative Paper of Basel II and was later refined by the work of Martin and Wilde ^[1] and further in Gordy and Lütkebohmert ^[2].

Given a portfolio of N borrowers, Gordy and Lütkebohmert developed a simplified formula for an add-on to the capital for unexpected loss (UL capital) in a single-factor model. The simplified formula follows from the ?full? granularity adjustment of Gordy and Lütkebohmert if quadratic terms are dropped. An alternative interpretation is to assume that any idiosyncratic risk in recovery rates that is still explicitly captured by the ?full? adjustment formula is eliminated by diversification.

12.3 Issues and Challenges

- The simplicity of the granularity adjustment formula comes at the price of a potential model error. The reason is that the granularity adjustment, unlike the IRB model, was developed in a CreditRisk+ setting, and the CreditRisk+ model differs in the tail of the loss distribution from the IRB model. For this reason, the formula comes close to, but is not fully consistent, with the IRB model.
- More broadly the approximation is only valid within a narrow and fairly simplistic view of credit portfolios

12.4 References

1. ? Richard Martin and Tom Wilde. Unsystematic credit risk. Risk, 15(11):123-128, November 2002
2. ? Michael B. Gordy and Eva Lutkebohmert. Granularity adjustment for Basel II. January 2010

13 Hall-Tideman Index

13.1 Definition

The **Hall-Tideman** index is a metric useful for the purpose of measuring [Concentration Risk](#) diversity or inequality metrics. An alternative name is Rosenbluth Index.

13.2 Details

More precisely, if we have n values E_i summing up to a total of

$$E_T = \sum_{i=1}^n E_i$$

and each value fraction is defined as

$$w_i = \frac{E_i}{E_T}$$

Then the HTI index is defined as

$$HTI = \frac{1}{2 \sum_{i=1}^n w_i - 1}$$

13.3 See Also

- The [Gini Index](#) is a related measure

13.4 Implementation

Open Source implementations of the HTI index are available in

- the Python library [Concentration Library](#)

13.5 References

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14 Hannah Kay Index

14.1 Definition

For the purpose of measuring concentration (e.g., name, sector or geographic concentration), the **Hannah-Kay Index** is defined as the sum product of relative portfolio shares of the exposures raised to a desired exponent (power).

14.2 Details

More precisely, if we have n exposures E_i summing up to a total exposure of

$$E_T = \sum_{i=1}^n E_i$$

where each exposure fraction is defined ^[1] as

$$w_i = E_i / E_T$$

Then the Hannah-Kay index is defined as

$$HK_a = \begin{cases} (\sum_{i=1}^n w_i^a)^{1/(a-1)} & \text{for } 0 \leq a \neq 1 \\ \exp(-\sum_{i=1}^n w_i \log w_i) & \text{for } a = 1 \end{cases}, \quad 0 \leq a \neq 1$$

14.3 Usage

None

14.4 Variations

None

14.5 Issues and Challenges

None

14.6 Implementation

Open Source implementations of the Hannah-Kay index are available in

- the Python library [Concentration Library](#)

14.7 See Also

None

14.8 References

1. ? O.Bajo R. Salas, "Inequality Foundations of Concentration Measures

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15 Herfindahl-Hirschman Index

15.1 Definition

For the purpose of measuring credit portfolio or market **Concentration Risk** (e.g., name, sector or geographic risk), diversity or inequality metrics, the **Herfindahl-Hirschman Index (HHI)** is defined as the sum of all squared *relative* portfolio shares of the exposures.

15.2 Details

More precisely, if we have n exposures E_i summing up to a total exposure of

$$E_T = \sum_{i=1}^n E_i$$

and each exposure fraction is defined as

$$w_i = \frac{E_i}{E_T}$$

Then the HHI index is defined as

$$HHI = \sum_{i=1}^n w_i^2$$

Assuming the exposure data are given, it is easy to calculate the HHI by first forming the fractional (normalized) exposures and then computing and summing up their squares. By construction, the HHI is restricted between zero and one, with one corresponding to full concentration to one counterparty. Sometimes the inverse of the index is also used.

15.3 Usage

15.3.1 Interpretation and Concentration levels

Whether a given result indicates a concentrated portfolio cannot be assessed beforehand but only in relation with typical portfolios, standard market practices etc. or other metrics. Concentration levels can be characterized as *no*, *low* or *medium* to *high* to "total" concentration based on subjective thresholds.

For example in the context of corporate sector market share analysis an HHI below 0.15 is deemed "not concentrated," between 0.15 and 0.25 as "moderately concentrated," and above 0.25 as highly concentrated (monopolistic / oligopolistic). These thresholds might not be appropriate to use in the context for financial risk concentrations.

15.3.2 Connection with risk based measures

In a risk based framework for concentration management, the HHI is linked to portfolio volatility

15.3.3 Advantages

Key advantages of the HHI index include

- Computational simplicity
- Relatively moderate data requirements (but see qualification below)
- Unambiguous definition: Compared to the simpler **Concentration Ratio** measures the HHI index does not require specifying which fraction of the portfolio to focus

15.3.4 Disadvantages

The difficulty to interpret the index on an absolute basis is already mentioned. The HHI can be misleading as a risk measure because of several further limitations:

1. It does not consider the borrower's credit quality. An exposure to a Aaa-rated borrower, for example, is treated in the same way as an exposure to a B-rated borrower.

2. It does not account for credit risk dependencies between borrowers. Two large exposures to borrowers belonging to the same supply chain and located in the same town are treated in the same way as two large exposures to borrowers in completely unrelated industry sectors and located on different continents.
3. It does not account for the nature of the exposure. A long dated exposure in a derivatives portfolio can lead to substantial mark-to-market losses if the counterparty deteriorates, even if there is no default event
4. It does not enable to express a risk appetite statement around concentration. There is no obvious implied stress level linked to a particular HHI value.

15.4 Variations

The HHI index has been independently discovered and used in various fields with other names and / or simple transformations. The following are all alternative names

- Gibbs?Martin index
- Blau index
- Gini?Simpson index

The following transformations of the HHI are in common use

- Square root
- Inverse
- Unit Complement ($1 - \text{HHI}$)

15.4.1 Calculation in presence of granular exposures

When the exposure data contain a large amount of very small exposures it may be acceptable and computationally advantageous to compute the index assuming the small exposures do not contribute to concentration. This is done simply by adding a total granular exposure E_G so that

$$E_T = \sum_{i=1}^n E_i + E_G$$

Hence while small exposures are added and increase the total exposure, they are assumed by construction to have zero contribution to the HHI summation

15.5 Issues and Challenges

- The first limitation can be addressed by a rating-weighted HHI. The squared relative exposure shares are weighted in the aggregation by a numeric borrower rating, thereby giving more weight to borrowers with a lower credit quality. See also the link of HHI with risk based measures below.
- The second limitation, the ignorance of default dependencies, cannot be remedied in a similarly simple way. As a rating-weighted HHI provides a ranking in terms of name concentration, it can be useful for a comparison across credit portfolios; yet, it fails to produce a capital figure for low granularity

15.6 Implementation

Open Source implementations of the Herfindahl-Hirschman index are available in

- the R package [lneq](#)
- the Python library [Concentration Library](#)
- [A Spreadsheet Implementation](#)

15.6.1 Implementation Challenges

Although the data requirements for the HHI calculation seem to be moderate at first sight, they often turn out to be difficult to meet in practice. They require nothing less than the aggregation of all exposures to the same borrower for the whole business entity, be it a bank or a banking group. A heterogeneous IT environment can already present a serious technical obstacle to this aggregation. Furthermore, large borrowers, which are actually the most relevant entities for measuring name concentration, can themselves be complex structures that are not always easy to identify as belonging to

the same borrower entity.

15.7 See Also

None

15.8 References

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16 Hoover Index

16.1 Definition

For the purpose of measuring name, sector or geographic concentration, the **Hoover Index** is a simple index defined in terms of the absolute deviation from the mean (The L^1 norm).

16.2 Details

More precisely, if we have n exposures E_i summing up to a total exposure of

$$E_T = \sum_{i=1}^n E_i$$

an average exposure

$$E_m = E_T / n$$

and fractional exposures w_i are defined as

$$w_i = E_i / E_T$$

Then the Hoover index is defined as

$$H = \frac{1}{2} \sum_{i=1}^n |w_i - E_m| = \frac{1}{2} \sum_{i=1}^n |w_i - \frac{1}{n}|$$

16.3 Usage

None

16.4 Variations

None

16.5 Issues and Challenges

None

16.6 Implementation

Open Source implementations of the Hoover index are available in

- the Python library [Concentration Library](#)

16.7 See Also

None

16.8 References

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17 Kolm Index

17.1 Definition

The *Kolm Index* is a concentration index useful for the purpose of measuring name, sector or geographic concentration

17.2 Details

If we have n exposures E_i summing up to a total exposure of

$$E_T = \sum_{i=1}^n E_i$$

average exposure

$$\mu = E_T / n$$

and the fractional exposures w_i are defined as

$$w_i = E_i / E_T$$

Then the Kolm index is defined as

$$K = \frac{1}{a} \left(\log \left(\sum_{i=1}^n e^{a[w_i - \mu]} \right) - \log(n) \right)$$

17.3 Usage

None

17.4 Variations

None

17.5 Issues and Challenges

None

17.6 Implementation

Open Source implementations of the Kolm index are available in

- the R package [Ineq](#)
- the Python library [Concentration Library](#)

17.7 See Also

None

17.8 References

{{#set:Has Formula = True | Has SourceCode = True | Has Object = False | Has Lambda = False | Field Type= Documentation}}

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18 Large Exposures Framework

18.1 Definition

The Large Exposures Framework is a BIS **Limit framework** for measuring and controlling large credit exposures proposed in March 2013 and adopted as a standard in April 2014.

18.2 History

The Basel Committee issued its first guidance on credit exposures in 1991.

The large exposures framework complements the risk-based capital standard (Basel II / III) because the latter is not designed specifically to protect banks from large losses resulting from the sudden default of a single counter-party.

In particular, the minimum capital requirements (Pillar 1) of the Basel capital framework implicitly assume that a bank holds infinitely granular portfolios, i.e., no form of concentration risk is considered in calculating capital requirements. This is explained in more detail in the **Basel II Model** article.

Contrary to this simplification, idiosyncratic risk due to large exposures to individual counter-parties may be present in banks' portfolios. Although through the supervisory review process (SREP) and the Basel II (Pillar 2) concentration risk adjustments could be made to mitigate this risk, these adjustments are not harmonized across jurisdictions, and may not be able to control extraordinary losses from a single counter-party default.

To serve as a backstop to risk-based capital requirements, the large exposures framework is designed so that the maximum possible loss a bank could incur if a single counter-party or group of connected counter-parties were to suddenly fail would not endanger the bank's survival as a going concern.

The Large Exposures framework is an example of a range of **model free** risk management tools (**LCR, NSFR**) that were introduced by regulators worldwide on the wake of the financial crisis of 2008.

The implementation date of the large exposures framework is 1st of January 2019.

18.3 Definition of large exposure:

A risk position (sum of all exposures to a counterparty or to a group of connected counterparties) that compares to more than 10 per cent of the eligible capital base (Tier 1 Capital). Hence if the capital base is K , a risk position E_i is classified as a large exposure if $E_i > 0.10 * K$.

18.4 Main provisions

- Large exposures as per the above definition must be reported to the bank supervisors.
- The largest 20 exposures need to be reported anyway
- Standalone Banks not belonging to a Group and Banking Groups have a limit on any single large exposure that is no more than 25% of available eligible capital base. Hence if $E_i < 0.25 * K$ for all large exposures.
- Exposures to large systemic institutions are singled out for a separate limit. The so called SIFI limit applied to a G-SIB's exposure to another G-SIB should be between no more than 15% of the eligible capital base (Tier 1).

18.5 Issues and Challenges

- The limit is rather arbitrary and does not have any adjustment for counterparty risk or likelihood of joint default between counterparties (if they are not connected).
- The main data challenge concerns the accurate aggregation of exposures to distinct counterparties across a potentially large banking group. This can be a significant challenge especially for more complex exposures:
 - ♦ Derivative contracts, where the exposure may vary according to market factors
 - ♦ Structured products, where the exposure may be as part of an underlying portfolio
- The intuitive justification for the framework is that enforcement of such a limit would require four distinct counterparty credit events before Tier 1 capital is exhausted. The implication is that such an event has a probability within the risk appetite of the regulatory framework.
- Besides the risk profile of the large exposures themselves, the credit risk of the remaining portfolio is also ignored. E.g., a credit event in mortgage bank is highly likely to coincide with increased losses and capital depletion in direct retail exposures.

18.6 References

- BIS:Supervisory framework for measuring and controlling large exposures
 - BIS:Measuring and controlling large credit exposures
-

19 Margalef Index

19.1 Definition

The **Margalef Index** (D) is a simple metric useful for the purpose of measuring diversity.

19.2 Details

The index is only applicable for categorical data where all observations can be classified into a finite number of categories (species, types etc)

More precisely, if we have n observations values and a total of S categories, then the D index is defined as

$$D = \frac{S-1}{\ln N}$$

19.3 Implementation

Open Source implementations of the Margalef index are available in

- the Python library [Concentration Library](#)
- the EcolndR R package

19.4 See Also

- [Menhinick Index](#)

19.5 References

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20 Menhinick Index

20.1 Definition

The **Menhinick Index** is a simple metric useful for the purpose of measuring diversity.

20.2 Details

The index is only applicable for categorical data where all observations can be classified into a finite number of categories (species, types etc)

More precisely, if we have n observations values and a total of S categories, then the D index is defined as

$$I = \frac{S}{\sqrt{N}}$$

20.3 Implementation

Open Source implementations of the Menhinick index are available in

- the Python library [Concentration Library](#)
- the EcolndR R package

20.4 See Also

- [Margalef Index](#)

20.5 References

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21 Name Concentration

21.1 Definition

Name Concentration (also *Single Name Concentration* or *Single Borrower Concentration*) is a form of **Credit Risk Concentration**, describing a condition in which a **Credit Portfolio** has a material share allocated to a single **counterparty** or a group of **Related Counterparties** linked by specific ties (e.g. corporate group).

What constitutes a "material share" must be defined in context: For example in relation to total assets, to available **Risk Capital** etc.

21.2 Factors

The degree of name concentration (and associated risk) depends on various characteristics of the portfolio:

- the number of counterparties of the portfolio (the concentration risk is generally higher for a lower number of counterparties)
- the heterogeneity of the exposure size (the risk is higher when some exposures dominate)
- the underlying **Credit Risk** of the counterparties (large counterparties of poor credit being key drivers)
- the **Credit Dependency** between exposures in the portfolio

21.3 Mitigation

Effective handling of name concentration requires:

- proper identification (aggregation) of exposures
- measuring concentration using appropriate metrics
- a framework for monitoring and reporting name concentrations
- applying mitigation actions and/or other management actions in accordance with that framework

21.4 Issues and Challenges

- Name concentration requires a valid aggregation of exposure to a counterparty. This task can have many gray areas in reality there can be a wide range of legal and economic dependencies between legal entities
- Despite the long standing recognition of this risk, a consistent interpretation and measurement is still lacking, although there are a number of tools available

21.5 See Also

- **Single Obligor Exposure**

21.6 References

- **BIS:Studies on credit risk concentration**
 - **OCC:Concentrations of Credit**
-

22 Category:Name Concentration

Articles covering the issue of credit name concentration (identification, measurement, management)

23 Name Concentration Measurement

23.1 Definition

The quantitative assessment of the degree to which a portfolio may be excessively concentrated in a particular **Exposure / Counterparty**

23.2 Methodologies

For any quantification of risks, it is convenient to have quantitative benchmarks, for example to measure the distance from a neutral reference state of no concentration or full diversification. Some concentration/diversification indicators can be simply defined at portfolio level, providing synthetic measures of credit risk name concentration.

23.2.1 Concentration ratio

- The simplest method to quantify concentration is computing the share of exposure (EAD) held by the k largest customers in the portfolio relative to total exposure.
- Weaknesses of the index are that the choice of k is arbitrary and the index does not use all the information available
- In place of EAD, exposures can be measured as product of $EAD \cdot LGD$, thus considering the expected severity of losses which can actually differentiate the effective contribution to credit risk.

23.2.2 Gini Index and the Lorenz Curve

- The index G varies between 0 (perfect equality of exposures) to 1 for perfect inequality (limit in which one obligor accounts for the whole exposure and the others tend to zero).
- The index is sensitive to inhomogeneity of exposures but not to exposure number.

23.2.3 Herfindahl-Hirschman Index

- The index reflects both exposures heterogeneity and their number (e.g. in tends to zero with n for homogeneous pool).
-

24 Product Concentration

24.1 Definition

Product Concentration in the context of Risk Management is a form of **Credit Risk Concentration**. It arises when a material share of a **Credit Portfolio** is allocated to a lending product or group of related products that exhibit correlated behaviour because of product features (such as reference to interest rates, foreign currency rates etc.)

24.2 Usage

What constitutes a "material share" of the portfolio must be defined in context: For example in relation to total assets, to available **risk capital** etc. Product concentration depends on various characteristics of the portfolio:

- the number of distinctly different products represented in the portfolio
- the degree of **credit dependency** between obligors

Recent examples of product concentration are mortgage portfolios, consumer lending portfolios with FX dependency etc.

Effective handling of product concentration requires:

- proper identification of product features that may lead to correlated client behavior
- applying mitigation actions and/or other management actions

24.3 Issues and Challenges

- Product concentration is a relatively newcomer in the concentration risk vocabulary. A consistent interpretation and measurement is still lacking, although there are a number of tools available
- Typical concentration risk measures such as the HHI may not be applicable due to the lack of easy classification of products and their corresponding risks

24.4 References

- **EBA:Guidelines on concentration risk**
-

25 Sector Concentration

25.1 Definition

Sector concentration is a form of **Credit Risk Concentration**. It arises when a material share of a **Credit Portfolio** is allocated to

- a single **Business Sector** or
- a group of related sectors linked by strong economic ties

25.2 Usage

What constitutes a "material share" of the portfolio must be defined in context: For example in relation to total assets, to available **risk capital** etc.

Sector concentration depends on various characteristics of the portfolio:

- the number of sectors represented in the portfolio (the concentration risk is generally higher for a lower number of sectors)
- the heterogeneity of the exposure size (the risk is higher when some sectors dominate)
- the underlying average **credit risk** of the sectors (large sectors with poorer average credit being key contributors)
- the **credit dependency** between sectors in the portfolio (sectors that tend to under-perform together)

Effective handling of sector concentration requires:

- proper identification (aggregation) of sector exposures on the basis of business activity
- measuring concentration using appropriate metrics
- a framework for monitoring and reporting sector concentrations against a **limit framework**
- applying mitigation actions and/or other management actions in accordance with that management framework

25.3 Issues and Challenges

- Sector concentration requires a valid aggregation of exposure to a sector. This task can have many gray areas: E.g., entities may be active in many sectors and a precise allocation is difficult
- Despite the long standing recognition of this risk, a consistent interpretation and measurement is still lacking, although there are a number of tools available

25.4 References

- **BIS:Studies on credit risk concentration**
- **EBA:Guidelines on concentration risk**
- **OCC:Concentrations of Credit**

26 Category:Sector Concentration

Articles about Sector Concentration

27 Sector Concentration Measurement

27.1 Definition

Sector Concentration Measurement is the quantitative assessment of the degree to which a portfolio may be concentrated in a set of particular business sectors.

27.2 Approaches

For any quantification of risks, it is convenient to have quantitative benchmarks, for example to measure the distance from a neutral reference state of no concentration or full diversification.

Some concentration/diversification indicators can be simply defined at portfolio level, providing synthetic measures of credit risk sector concentration.

27.2.1 Concentration Ratio

Concentration Index is the simplest method to quantify concentration is computing the share of **Exposure** (EAD) held by the k largest sectors in the portfolio relative to total exposure. A weaknesses of the index is that the choice of k is arbitrary and the index does not use all the information available. Instead of using of EAD, exposures can be measured as product of EAD*LGD, thus considering the expected severity of losses which can actually differentiate the effective contribution to credit risk.

27.2.2 Gini Index and the Lorenz Curve

The **Gini Index** G varies between 0 (perfect equality of exposures) to 1 for perfect inequality (limit in which one sector accounts for the whole exposure and the others tend to zero). The index is sensitive to inhomogeneity of exposures but not to exposure number.

27.2.3 Herfindahl-Hirschman Index

The **Herfindahl-Hirschman Index** reflects both exposures heterogeneity and their number (e.g. in tends to zero with n for homogeneous pool).

27.3 Issues and Challenges

27.4 References

28 Shannon Index

28.1 Definition

For the purpose of measuring name or sector concentration, the **Shannon Index** (also entropy index) is defined as the sum product of relative portfolio shares of the exposures, times the natural logarithm of the exposures.

28.2 Details

More precisely, if we have n exposures E_i summing up to a total exposure of

$$E_T = \sum_{i=1}^n E_i$$

where each exposure fraction is defined as

$$w_i = \frac{E_i}{E_T}$$

then the Shannon index is defined as

$$S = - \sum_{i=1}^n w_i \ln w_i$$

28.3 Usage

None

28.4 Variations

None

28.5 Issues and Challenges

None

28.6 Implementation

Open Source implementations of the Shannon index are available in

- the Python library [Concentration Library](#)

28.7 See Also

- [Generalized Entropy Index](#)
- [Theil Index](#)

28.8 References

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29 Theil Index

29.1 Definition

For the purpose of measuring inequality or concentration, the **Theil Index** (also entropy index) is defined as the sum product of relative values of the distribution, times the natural logarithm of the same values.

29.2 Details

More precisely, if we have n values E_i summing up to a total value of

$$E_T = \sum_{i=1}^n E_i$$

where each fraction is defined as

$$w_i = \frac{E_i}{E_T}$$

then the Theil index is defined as

$$T = \sum_{i=1}^n w_i \ln w_i$$

29.3 Usage

None

29.4 Variations

None

29.5 Issues and Challenges

None

29.6 Implementation

Open Source implementations of the Theil index are available in

- the Python library [Concentration Library](#)

29.7 See Also

- [Generalized Entropy Index](#)
- [Shannon Index](#)

29.8 References

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