## Question



**Coherent orthogonal binary FSK modulation is used to transmit two equiprobable symbol waveforms $s_1(t) = \alpha \cos 2\pi f_1 t$ and $s_2(t) = \alpha \cos 2\pi f_2 t$, where $\alpha$ = 4 mV. Assume an AWGN channel with two-sided noise power spectral density $\frac{N_0}{2} = 0.5 \times 10^{-12}\ W/Hz$. Using an optimal receiver and the relation $Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^\infty e^{-u^2/2} du$, the bit error probability for a data rate of 500 kbps is**

This question was previously asked in

**GATE EC 2014 Official Paper: Shift 2**



1. Q(2)
2. $Q\left(2\sqrt{2}\right)$
3. Q(4)
4. $Q\left(4\sqrt{2}\right)$



**Answer** (Detailed Solution Below)

Option 3 : Q(4)

## Detailed Solution

**Concept:**

FSK Modulation:

In FSK: transmission of 1 is represented as:

$s_1(t) = A_c \cos 2\pi f_H t$

Transmission of 0 is represented as:

$s_2(t) = A_c \cos 2\pi f_L t$

and the Bit error probability $= Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$

Where $E_d$ is the energy of $s_1(t) - s_2(t)$

$$E_d = \int_0^{T_b} \{s_1(t) - s_2(t)\}^2\, dt$$

$$E_d = \int_0^{T_b} s_1^2(t)dt + \int_0^{T_b} s_2^2(t)dt - 2\int_0^{T_b} s_1(t) - s_2(t)dt$$

Since $s_1(t)$ & $s_2(t)$ are orthogonal, we cn write:

$$\therefore \int_0^{T_b} s_1(t) - s_2(t)dt = 0$$

$$E_d = \int_0^{T_b} s_1^2(t)dt + \int_0^{T_b} s_2^2(t)dt$$

$$E_d = \frac{A_c^2 T_b}{2} + \frac{A_c^2 T_b}{2} = A_c^3 T_b$$

$$BER = Q\left(\sqrt{\frac{A_c^2 T_b}{2N_0}}\right)$$

**Analysis:**

Given:

$A_c = \alpha = 4$ mV

$$\frac{N_0}{2} = 0.5 \times 10^{-12}\ w/Hz$$

$N_0 = 10^{-12}$ w/Hz

$$T_b = \frac{1}{R_b} = \frac{1}{800 \times 10^3}$$

$T_b = 0.2 \times 10^{-5}$

$T_b = 2 \times 10^{-6}$ sec.

$$BER = Q\left(\sqrt{\frac{A_c^2 T_b}{2N_0}}\right)$$

$$BER = Q\left(\sqrt{\frac{(4 \times 10^{-3})^2 \times 2 \times 10^{-6}}{2 \times 10^{-12}}}\right)$$

$$BER = Q\left(\sqrt{\frac{16 \times 10^{-6} \times 2 \times 10^{-6}}{2 \times 10^{-12}}}\right)$$

$$BER = Q\left(\sqrt{16}\right)$$

BER = Q(4)