

Projection of the risk free curve and recalibration inside OSEM

The purpose of this workbook is to showcase the calibration methodology inside OSEM. The key to the calibration is the Smith Wilson algorithm commonly used in insurance. This algorithm allows a continuous approximation of arbitrary maturities based on a subset of available maturities.

```
In [1]:  
import numpy as np  
import pandas as pd  
import matplotlib.pyplot as plt  
import matplotlib.ticker as mtick
```

```
In [2]:  
from ImportData import import_SWEiopa  
from CurvesClass import Curves
```

Importing external files

The parameters and the current risk free curve are provided as input:

- Parameters.csv; TBC
- EIOPA_param_file.csv; TBC
- EIOPA_curves_file.csv; TBC

Import Param file

```
In [3]:  
paramfile = pd.read_csv("Input\Parameters.csv")  
paramfile.index = paramfile["Parameter"]  
del paramfile["Parameter"]
```

Read base EIOPA curve from file

```
In [4]:  
selected_param_file = paramfile.loc["EIOPA_param_file"][0]  
selected_curves_file = paramfile.loc["EIOPA_curves_file"][0]  
country = paramfile.loc["country"][0]  
  
[maturities_country, curve_country, extra_param, Qb] = import_SWEiopa(selected_param_
```

Initiate Curve object

```
In [5]: # ultimate forward rate  
ufr = extra_param["UFR"]/100  
# Numeric precision of the optimisation  
Precision = float(paramfile.loc["Precision"][0])  
  
# Targeted distance between the extrapolated curve and the ultimate forward rate at 1 basis point  
Tau = float(paramfile.loc["Tau"][0])# 1 basis point  
  
MD = paramfile.loc["Modelling_Date"]  
  
In [6]: Curves = Curves(ufr, Precision, Tau, MD, country)  
  
In [8]: Curves.SetObservedTermStructure(maturity_vec=curve_country.index.tolist(), yield_vec=  
  
In [9]: ProjYear = 0  
# Number of projection years  
N = int(paramfile.loc["n_proj_years"][0])
```

Calculate 1 year forward rates

The forward rates will be used to calculate forward spot curves

```
In [11]: Curves.CalcFwdRates()
```

```
In [12]: display(Curves.fwd_rates)
```

Forward	
0	NaN
1	1.031582
2	1.027909
3	1.026170
4	1.026146
...	...
145	1.034191
146	1.034211
147	1.034231
148	1.034251
149	1.034271

150 rows × 1 columns

Forward spot rates

$$y_i(t - i) = \prod_i^t (1 + fw_{EIOPA}(t))^{\frac{1}{t-i}}$$

In [13]: `Curves.ProjectForwardRate(N)`

In [15]: `display(Curves.r_obs)`

	Yield	Yield year1	Yield year2	Yield year3	Yield year4	Yield year5	Yield year6	Yield year7	Yield year8	Yield year9
0	0.03472	0.031582	0.027909	0.026170	0.026146	0.026663	0.027041	0.027560	0.028290	0.028950
1	0.03315	0.029745	0.027039	0.026158	0.026404	0.026852	0.027301	0.027925	0.028620	0.028010
2	0.03140	0.028552	0.026741	0.026326	0.026617	0.027088	0.027631	0.028267	0.028104	0.029452
3	0.03009	0.027951	0.026722	0.026505	0.026853	0.027389	0.027960	0.027968	0.029161	0.029882
4	0.02930	0.027693	0.026786	0.026716	0.027140	0.027701	0.027782	0.028841	0.029563	0.029543
...
145	0.03274	0.032739	0.032757	0.032801	0.032856	NaN	NaN	NaN	NaN	NaN
146	0.03275	0.032749	0.032768	0.032811	NaN	NaN	NaN	NaN	NaN	NaN
147	0.03276	0.032760	0.032778	NaN						
148	0.03277	0.032770	NaN							
149	0.03278	NaN								

150 rows × 50 columns

Calculate the alpha parameter

Calibration of the alpha parameter for the base curve using the bisection algorithm.

In [16]: `alphaoptimized = [Curves.BisectionAlpha(0.05, 0.5, Curves.m_obs["Maturity"], Curves.r_obs["Yield year"])]`

```
if "Yield year" in Curves.alpha.columns:
    Curves.alpha["Yield year"] = alphaoptimized
else:
    Curves.alpha = Curves.alpha.join(pd.Series(data=None, index=None, name="Yield year"))
    Curves.alpha["Yield year"] = alphaoptimized
```

In [17]: `Curves.alpha`

Out[17]: `Yield year`

0	0.5
----------	-----

Calibrate first calibration vector b

```
In [18]:  
bCalibrated = Curves.SWCalibrate(Curves.r_obs["Yield"], Curves.m_obs["Maturity"], Curves.ufr)  
bCalibrated = np.append(bCalibrated,np.repeat(np.nan, ProjYear))
```

```
In [19]:  
if "Yield year" in Curves.b.columns:  
    Curves.b["Yield year"] = bCalibrated  
else:  
    Curves.b = Curves.b.join(pd.Series(data=None, index=None, name="Yield year", dtype=object))  
    Curves.b["Yield year"] = bCalibrated
```

Calibrate every yield curve

Parameters that stay the same

Calibrate time 0 curve

Calculated implied observations for the first year of projection

```
In [20]:  
ProjYear = 1  
NameOfYear = "Yield year"+str(ProjYear)
```

```
In [21]:  
r_obs = np.transpose(np.array(Curves.r_obs[NameOfYear]))[:-ProjYear]
```

Calculated implied maturities for the first year of projection

```
In [22]:  
m_obs = np.transpose(np.array(range(1,r_obs.size+1)))
```

Calibrate

```
In [23]:  
alphaoptimized = [Curves.BisectionAlpha(0.05, 0.5, m_obs, r_obs, Curves.ufr, Curves.tau)]
```

```
In [24]:  
if NameOfYear in Curves.alpha.columns:  
    Curves.alpha[NameOfYear] = alphaoptimized  
else:  
    Curves.alpha = Curves.alpha.join(pd.Series(data=None, index=None, name=NameOfYear))  
    Curves.alpha[NameOfYear] = alphaoptimized
```

```
In [25]:  
bCalibrated = Curves.SWCalibrate(r_obs, m_obs, Curves.ufr, Curves.alpha[NameOfYear])  
bCalibrated = np.append(bCalibrated,np.repeat(np.nan,ProjYear))
```

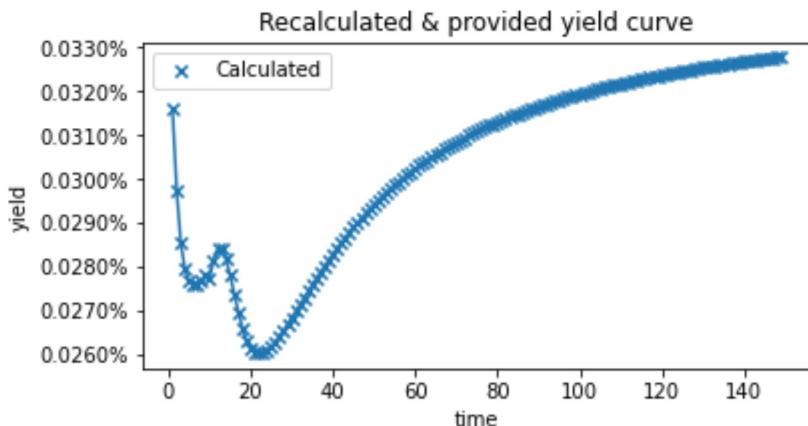
```
In [26]:  
if NameOfYear in Curves.b.columns:  
    Curves.b[NameOfYear] = bCalibrated  
else:  
    Curves.b = Curves.b.join(pd.Series(data= None, index=None, name=NameOfYear, dtype= None))  
    Curves.b[NameOfYear] = bCalibrated
```

```
In [27]:  
Curves.b[NameOfYear][:-1]
```

```
Out[27]:  
0      -0.080601  
1       0.015151  
2       0.007984  
3       0.010251  
4      -0.007252  
      ...  
144     6.988021  
145    -1.911537  
146     0.574049  
147    -0.314029  
148     0.266142  
Name: Yield year1, Length: 149, dtype: float64
```

```
In [28]:  
r_Obs_Est = Curves.SWExtrapolate(m_obs, m_obs, Curves.b[NameOfYear][:-1], Curves.ufr)
```

```
In [29]:  
fig, ax1 = plt.subplots(1,1)  
ax1.scatter(m_obs, r_obs, label="Calculated", marker="x")  
ax1.plot(m_obs, r_Obs_Est)  
ax1.set_ylabel("yield")  
ax1.set_title('Recalculated & provided yield curve')  
ax1.set_xlabel("time")  
ax1.legend()  
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())  
fig.set_figwidth(6)  
fig.set_figheight(3)  
plt.show()
```



Repeat for all

In [30]:

```

for iYear in range(2,50):
    ProjYear = iYear
    NameOfYear = "Yield year"+str(ProjYear)
    r_Obs = np.transpose(np.array(Curves.r_obs[NameOfYear]))[:-ProjYear]
    M_Obs = np.transpose(np.array(range(1,r_Obs.size+1)))
    alphaoptimized = [Curves.BisectionAlpha(0.05, 0.5, M_Obs, r_Obs, Curves.ufr, Curves.b)]
    if NameOfYear in Curves.alpha.columns:
        Curves.alpha[NameOfYear] = alphaoptimized
    else:
        Curves.alpha = Curves.alpha.join(pd.Series(data=None, index=None, name=NameOfYear))
        Curves.alpha[NameOfYear] = alphaoptimized
    bCalibrated = Curves.SWCalibrate(r_Obs, M_Obs, Curves.ufr, Curves.alpha[NameOfYear])
    bCalibrated = np.append(bCalibrated,np.repeat(np.nan,ProjYear))
    if NameOfYear in Curves.b.columns:
        Curves.b[NameOfYear] = bCalibrated
    else:
        Curves.b = Curves.b.join(pd.Series(data= None,index=None, name=NameOfYear,dtype='float'))
        Curves.b[NameOfYear] = bCalibrated
    r_Obs_Est = Curves.SWExtrapolate(M_Obs, M_Obs, Curves.b[NameOfYear])[:- (ProjYear)]

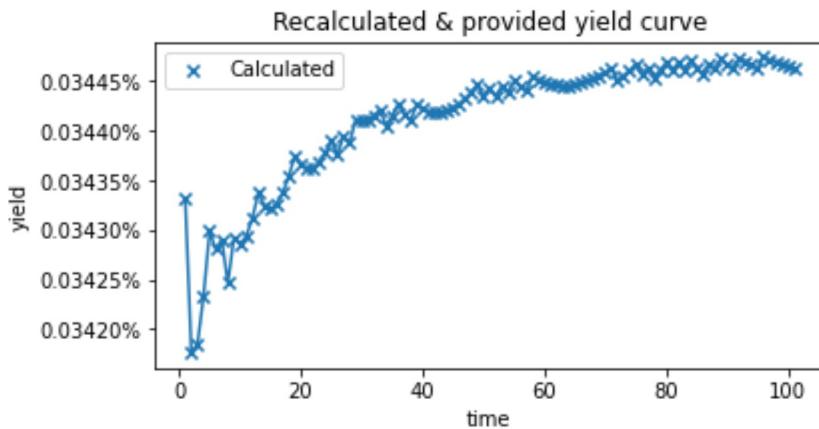
```

In [31]:

```

fig, ax1 = plt.subplots(1,1)
ax1.scatter(M_Obs, r_Obs, label="Calculated", marker="x")
ax1.plot(M_Obs, r_Obs_Est)
ax1.set_ylabel("yield")
ax1.set_title('Recalculated & provided yield curve')
ax1.set_xlabel("time")
ax1.legend()
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(6)
fig.set_figheight(3)
plt.show()

```



Saving calibrated results

In [32]:

```
Curves.b.to_csv("Intermediate/b.csv")
```

In [33]:

```
Curves.alpha.to_csv("Intermediate/alpha.csv")
```

```
In [34]: Curves.r_obs
```

	Yield	Yield year1	Yield year2	Yield year3	Yield year4	Yield year5	Yield year6	Yield year7	Yield year8	Yield year9
0	0.03472	0.031582	0.027909	0.026170	0.026146	0.026663	0.027041	0.027560	0.028290	0.028950
1	0.03315	0.029745	0.027039	0.026158	0.026404	0.026852	0.027301	0.027925	0.028620	0.028010
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145	0.03274	0.032739	0.032757	0.032801	0.032856	NaN	NaN	NaN	NaN	NaN
146	0.03275	0.032749	0.032768	0.032811	NaN	NaN	NaN	NaN	NaN	NaN
147	0.03276	0.032760	0.032778	NaN						
148	0.03277	0.032770	NaN							
149	0.03278	NaN								

150 rows × 50 columns

```
In [35]: Curves.m_obs.to_csv("Intermediate/M_Obs.csv")
```

```
In [36]: Curves.fwd_rates.to_csv("Intermediate/FwdRates.csv")
```

```
In [37]: Qb.to_csv("Intermediate/Qb_0.csv")
```