ESG FRAMEWORK STOCHASTIC RISK NEUTRAL_OPEN

February 10, 2025

1 STOCHASTIC SCENARIO TESTS

This test checks the consistency of a market-consistent and risk-neutral output from an Economic Scenario Generator (ESG). The goal of this script is to:

- Verify if average performance of the stochastic output is consistent with the deterministic equivalent as implied by the risk-neutral property
- Preform a series of Martingale tests using multiple investment strategies

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Description of the stochastic scenario tests 1. Average of the stochastic deflator is equal to the deterministic deflator 2. Martingale test for a Forward return with `1`-year spread strategy using sport rates 3. Martingale test for a Buy-and-hold strategy with sport rates 4. Martingale test for a Buy-and-hold strategy with a total-return index

Limitations of the implementation This implementation has the following limitations:

- The data is assumed to have a monthly granularity
- This implementation only looks at a subset of all scenarios
- Only one currency at a time is checked
- Only one set of deterministic/stochastic scenario pairs are used
- Only one total-return index is used
- Only two spot rates with term `1`-and `2`-years are considered

The accounting convention used is that all months are assumed to be of equal length and represent exactly `1/12` of a year.

Note that the Extra content sections contain plots for the spot curve with up to the `5`-year term.

Success criteria definition The following success criteria is proposed:

- Difference between the average of stochastic deflator curve and the deterministic deflator is never larger than 300bps
- Average difference between the average of stochastic deflator curve and the deterministic deflator is never larger than 500bps
- The maximum difference between the theoretical and observed return of a Martingale strategy is not larger than 100bps
- Average difference between the theoretical and observed return of a Martingale strategy is not larger than 50bps

[509]:
```
test_threshold_max = 0.03
test_threshold_mean = 0.05
```

[510]:
```
test_martingale_threshold_max = 0.01
test_martingale_threshold_mean = 0.005
```

[511]:
```
def success_test(test_statistics, threshold_max, threshold_mean):
    out1 = False
    out2 = False
    if max(test_statistics)<threshold_max:
        print("Test passed")
        out1 = True
    else:
        print("Test failed")

    if np.mean(test_statistics)<threshold_mean:
        print("Test passed")
        out2 = True
    else:
        print("Test failed")
    return [out1, out2]
```

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1.1 Data requirements

To perform this test, the following data was provided:

- Deterministic curves: File:TEST_DATA_DETERMINISTIC.csv
- Stochastic scenario: File:TEST_DATA_STOCHASTIC.csv

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1.2 External dependencies

This implementation uses three well established Python packages widely used in the financial industry. Pandas (https://pandas.pydata.org/docs/), Numpy (https://numpy.org/doc/), and Matplotlib (https://matplotlib.org/stable/index.html).

[514]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick
%matplotlib inline
```

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1.2.1 Importing data

Two input files are used in this test.

[516]:
```
file_det = 'TEST_DATA_DETERMINISTIC.csv'
file_stoch = 'TEST_DATA_STOCHASTIC.csv'
```

[517]:
```
data_det_raw = pd.read_csv(file_det, sep=',', index_col=0)
```

[518]:
```
data_stoch_raw = pd.read_csv(file_stoch, sep=',', index_col=0, low_memory=False)
```

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1.3 Data manipulation

This step converts the raw data from the input file into a format that can be digested by the testing framework.

To uniquely identify the correct scenarios, 4 header types are used: 1. Description ex. DEFLATOR ASSET, SPOT RATE, … 2. Term ex. 1, 2, 3 , … 3. Currency ex. EUR, USD, GBP, … 4. Scenario number ex. 0, 1, 2, …

The dataset of both deterministic and stochastic scenarios is filtered for desired curves. The filtering is done using the filter_scenario_file() function to avoid duplication of code:

[522]:
```
def filter_scenario_file(data, mydescription, myterm, mycurrency):
    description_row = data.iloc[0,:]
    term_row =  data.iloc[1,:]
    currency_row = data.iloc[2,:]
    true_description = description_row.eq(mydescription, level=None)
    true_term = term_row.eq(myterm, level=None)
    true_currency = currency_row.eq(mycurrency, level=None)
    out = true_description * true_term * true_currency
    out = out.tolist()
    return out
```

RUN PARAMETER
Currency selection

[524]:
```python
currency = "EUR"
```

In both files, the currency denominated deflator asset can be identified by:

[527]:
```python
deflator_variable = "DEFLATOR ASSET"
deflator_term = "0"
```

The currency denominated spot rates can be identified by:

[529]:
```python
spot_variable = "SPOT RATE"
```

Note that the term of the spot curve can vary between the tests.

The currency denominated total-return index can be identified by:

[532]:
```python
index_variable = "INDEX"
index_term = "0"
```

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1.3.1 Deterministic scenario preparation

This subsection splits the raw file containing the deterministic scenarios into two tables. The first one contains the headers that are used to identify individual scenarios. The second one contains the numeric data.

[535]:
```python
data_det_headers = data_det_raw.iloc[:4,:]
```

Numeric data is extracted and transposed such that the time runs across rows as is expected by the testing framework.

[537]:
```python
data_det = data_det_raw.iloc[4:,:].astype(float)
```

VISUAL INSPECTION
Deterministic headers and scenario data

[571]:
```python
data_det_headers.head()
```

[571]:
```
                    DEFLATOR     SPOT_1     SPOT_2     SPOT_3     SPOT_4  \
VARIABLE_KEY
DESCRIPTION   DEFLATOR ASSET  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                       0          1          2          3          4
CURRENCY                 EUR        EUR        EUR        EUR        EUR
SCENARIO                   0          0          0          0          0

                 SPOT_5  INDEX
VARIABLE_KEY
DESCRIPTION   SPOT RATE  INDEX
```

```
TERM                   5      0
CURRENCY             EUR    EUR
SCENARIO               0      0
```

[572]:
```
data_det.head()
```

[572]:
```
              DEFLATOR    SPOT_1    SPOT_2    SPOT_3    SPOT_4    SPOT_5  \
VARIABLE_KEY
30/09/2022    1.000000  0.025484  0.028942  0.029624  0.030070  0.030430
31/10/2022    0.998170  0.026134  0.029381  0.029853  0.030273  0.030593
30/11/2022    0.996310  0.026804  0.029787  0.030077  0.030467  0.030751
31/12/2022    0.994414  0.027485  0.030155  0.030293  0.030650  0.030900
31/01/2023    0.992476  0.028166  0.030483  0.030499  0.030821  0.031041

                 INDEX
VARIABLE_KEY
30/09/2022    1.000000
31/10/2022    1.001861
30/11/2022    1.003674
31/12/2022    1.005618
31/01/2023    1.007612
```

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1.3.2 Stochastic scenarios preparation

This subsection splits the raw file containing the stochastic scenarios into two tables. One with descriptive headers and the other one with numeric data.

[576]:
```
data_stoch_headers = data_stoch_raw.iloc[:4,:]
```

[577]:
```
data_stoch = data_stoch_raw.iloc[4:,:].astype(float)
```

VISUAL INSPECTION
Stochastic headers and scenario data

[590]:
```
data_stoch_headers.head()
```

[590]:
```
                    DEFLATOR      SPOT_1      SPOT_2      SPOT_3      SPOT_4  \
VARIABLE_KEY
DESCRIPTION   DEFLATOR ASSET  SPOT RATE   SPOT RATE   SPOT RATE   SPOT RATE
TERM                       0           1           2           3           4
CURRENCY                 EUR         EUR         EUR         EUR         EUR
SCENARIO                   1           1           1           1           1

                 SPOT_5  INDEX      DEFLATOR.1   SPOT_1.1   SPOT_2.1  …  \
```

```
VARIABLE_KEY                                                                 …
DESCRIPTION    SPOT RATE  INDEX  DEFLATOR ASSET  SPOT RATE  SPOT RATE  …
TERM                   5      0               0          1          2  …
CURRENCY             EUR    EUR             EUR        EUR        EUR  …
SCENARIO               1      1               2          2          2  …

             SPOT_4.498 SPOT_5.498 INDEX.498    DEFLATOR.499 SPOT_1.499  \
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE      INDEX  DEFLATOR ASSET  SPOT RATE
TERM                   4          5          0               0          1
CURRENCY             EUR        EUR        EUR             EUR        EUR
SCENARIO             499        499        499             500        500

             SPOT_2.499 SPOT_3.499 SPOT_4.499 SPOT_5.499 INDEX.499
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE      INDEX
TERM                   2          3          4          5          0
CURRENCY             EUR        EUR        EUR        EUR        EUR
SCENARIO             500        500        500        500        500

[4 rows x 3500 columns]
```

[591]:
```
data_stoch.head()
```

[591]:
```
              DEFLATOR    SPOT_1    SPOT_2    SPOT_3    SPOT_4    SPOT_5  \
VARIABLE_KEY
30/09/2022    1.000000  0.025484  0.028942  0.029624  0.030070  0.030430
31/10/2022    0.998170  0.025951  0.029216  0.029703  0.030137  0.030470
30/11/2022    0.996326  0.026256  0.029277  0.029602  0.030025  0.030339
31/12/2022    0.994477  0.024064  0.026857  0.027118  0.027591  0.027951
31/01/2023    0.992825  0.024050  0.026521  0.026690  0.027154  0.027509

                 INDEX  DEFLATOR.1  SPOT_1.1  SPOT_2.1  …  SPOT_4.498  \
VARIABLE_KEY                                            …
30/09/2022    1.000000    1.000000  0.025484  0.028942  …    0.030070
31/10/2022    1.004252    0.998170  0.024965  0.028254  …    0.029700
30/11/2022    1.013114    0.996408  0.023438  0.026530  …    0.021411
31/12/2022    1.019324    0.994793  0.023999  0.026794  …    0.022387
31/01/2023    1.022011    0.993146  0.026649  0.029053  …    0.025759

              SPOT_5.498  INDEX.498  DEFLATOR.499  SPOT_1.499  SPOT_2.499  \
VARIABLE_KEY
30/09/2022      0.030430   1.000000      1.000000    0.025484    0.028942
31/10/2022      0.030045   0.989699      0.998170    0.022864    0.026205
30/11/2022      0.021956   0.997647      0.996583    0.019003    0.022206
31/12/2022      0.022887   1.006041      0.995337    0.014633    0.017664
31/01/2023      0.026152   1.007956      0.994471    0.017642    0.020277
```

```
             SPOT_3.499  SPOT_4.499  SPOT_5.499  INDEX.499
VARIABLE_KEY
30/09/2022     0.029624    0.030070    0.030430   1.000000
31/10/2022     0.026774    0.027288    0.027698   1.011132
30/11/2022     0.022726    0.023335    0.023829   1.007644
31/12/2022     0.018178    0.018892    0.019486   1.009948
31/01/2023     0.020618    0.021247    0.021761   1.011125

[5 rows x 3500 columns]
```

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1.4 Currency and indices selection

Each data file contains multiple currencies and curves. In this section, the required curves are extracted. The scope is described in the Limitation section. For each run, only a single currency is chosen.

[600]:
```
data_det_headers.head()
```

[600]:
```
                     DEFLATOR     SPOT_1     SPOT_2     SPOT_3     SPOT_4  \
VARIABLE_KEY
DESCRIPTION    DEFLATOR ASSET  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                        0          1          2          3          4
CURRENCY                  EUR        EUR        EUR        EUR        EUR
SCENARIO                    0          0          0          0          0

                  SPOT_5  INDEX
VARIABLE_KEY
DESCRIPTION    SPOT RATE  INDEX
TERM                   5      0
CURRENCY             EUR    EUR
SCENARIO               0      0
```

Deflator asset; deterministic scenario The deflator asset is represented as a curve of prices of a risk-free bond with a notional value of `1`, issued at time `0` and maturing at time `t`:

$$ZCB_{DET,RF}(0,t)$$

Where: - `DET` denotes that this Zero Coupon Bond (ZCB) was priced using the deterministic curve

- `RF` denotes that this Zero Coupon Bond represents a risk-free asset (deflator)
- `0` is the Issue time
- `t` is the Maturity time

[603]:
```
locationindex = filter_scenario_file(\
                    data_det_headers, deflator_variable, deflator_term, \
                        currency)
```

[606]:
```
data_det_headers.iloc[:,locationindex]
```

[606]:
```
                     DEFLATOR
VARIABLE_KEY
DESCRIPTION    DEFLATOR ASSET
TERM                        0
CURRENCY                  EUR
SCENARIO                    0
```

[607]:
```
deflator_det = data_det.iloc[:, locationindex]
```

Spot rate with a 1-year term; deterministic scenario The spot rate with term of 1-year is one of the simulated financial curves. It is represented as a series of annualized returns. The value at time `t` represents the return of investing `1` unit of currency at time `t` until time `t+12` (months):

$$r_{DET,T1}(t)$$

Where:

- `DET` denotes that this is a return calculated using the deterministic scenario curve
- `T1` denotes that this return is calculated using the spot curve with a `1`-year term
- `t` is the time of investment

[615]:
```
locationindex = filter_scenario_file(data_det_headers, spot_variable, \
                                  "1", currency)
```

[616]:
```
data_det_headers.iloc[:,locationindex]
```

[616]:
```
                  SPOT_1
VARIABLE_KEY
DESCRIPTION    SPOT RATE
TERM                   1
CURRENCY             EUR
SCENARIO               0
```

[617]:
```
spot_1_det = data_det.iloc[:, locationindex]
```

Total-return index; deterministic scenario The total-return indices represent the second kind of simulated financial curves. It is represented as the value of an index with a starting value of `100` at modelling time `0`.

It represents the value of the normalized index at time `t`:

$$I_{DET,1}(t)$$

Where:

- DET denotes that this is a return calculated using the deterministic scenario curve
- 1 denotes that this total-return index represents the first hypothetical assets
- t is the Modelling time

Note that the index is normalized to the starting value of 1 currency unit.

[620]:
```
locationindex = filter_scenario_file(data_det_headers, \
                    index_variable, index_term, currency)
```

[621]:
```
data_det_headers.iloc[:,locationindex]
```

[621]:
```
              INDEX
VARIABLE_KEY
DESCRIPTION   INDEX
TERM              0
CURRENCY        EUR
SCENARIO          0
```

[622]:
```
index_det =  data_det.iloc[:, locationindex]
```

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1.4.1 Stochastic scenarios filter

Deflator asset; stochastic scenarios The deflator asset is represented as the price curve of a risk-free bond with a notional value of 1, issued at time 0 and maturing at time t. The curves from the stochastic scenarios file do not have the subscript DET, but instead the stochastic scenario number i. This is represented as:

$$ZCB_{i,RF}(0,t)$$

Where:

- i denotes that this is a Zero Coupon Bond (ZCB) was priced using the i-th stochastic scenario
- RF denotes that this Zero Coupon Bond represents a risk-free asset (deflator)
- 0 is the Issue time
- t is the Maturity time

[630]:
```
locationindex = filter_scenario_file(data_stoch_headers, \
    deflator_variable, deflator_term, currency)
```

[631]:
```
data_stoch_headers.iloc[:,locationindex]
```

[631]:
```
                   DEFLATOR      DEFLATOR.1      DEFLATOR.2      DEFLATOR.3  \
VARIABLE_KEY
DESCRIPTION  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                      0               0               0               0
CURRENCY                EUR             EUR             EUR             EUR
SCENARIO                  1               2               3               4

                 DEFLATOR.4      DEFLATOR.5      DEFLATOR.6      DEFLATOR.7  \
VARIABLE_KEY
DESCRIPTION  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                      0               0               0               0
CURRENCY                EUR             EUR             EUR             EUR
SCENARIO                  5               6               7               8

                 DEFLATOR.8      DEFLATOR.9  …    DEFLATOR.490  \
VARIABLE_KEY                                 …
DESCRIPTION  DEFLATOR ASSET  DEFLATOR ASSET  …  DEFLATOR ASSET
TERM                      0               0  …               0
CURRENCY                EUR             EUR  …             EUR
SCENARIO                  9              10  …             491

               DEFLATOR.491    DEFLATOR.492    DEFLATOR.493    DEFLATOR.494  \
VARIABLE_KEY
DESCRIPTION  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                      0               0               0               0
CURRENCY                EUR             EUR             EUR             EUR
SCENARIO                492             493             494             495

               DEFLATOR.495    DEFLATOR.496    DEFLATOR.497    DEFLATOR.498  \
VARIABLE_KEY
DESCRIPTION  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                      0               0               0               0
CURRENCY                EUR             EUR             EUR             EUR
SCENARIO                496             497             498             499

               DEFLATOR.499
VARIABLE_KEY
DESCRIPTION  DEFLATOR ASSET
TERM                      0
CURRENCY                EUR
SCENARIO                500

[4 rows x 500 columns]
```

[632]:
```
deflator = data_stoch.iloc[:,locationindex]
```

VISUAL INSPECTION
Stochastic deflator asset headers and scenarios

[634]:
```
data_stoch_headers.iloc[:,locationindex]
```

[634]:
```
                    DEFLATOR      DEFLATOR.1      DEFLATOR.2      DEFLATOR.3  \
VARIABLE_KEY
DESCRIPTION   DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                       0               0               0               0
CURRENCY                 EUR             EUR             EUR             EUR
SCENARIO                   1               2               3               4

                  DEFLATOR.4      DEFLATOR.5      DEFLATOR.6      DEFLATOR.7  \
VARIABLE_KEY
DESCRIPTION   DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                       0               0               0               0
CURRENCY                 EUR             EUR             EUR             EUR
SCENARIO                   5               6               7               8

                  DEFLATOR.8      DEFLATOR.9  …    DEFLATOR.490  \
VARIABLE_KEY                                  …
DESCRIPTION   DEFLATOR ASSET  DEFLATOR ASSET  …  DEFLATOR ASSET
TERM                       0               0  …               0
CURRENCY                 EUR             EUR  …             EUR
SCENARIO                   9              10  …             491

                DEFLATOR.491    DEFLATOR.492    DEFLATOR.493    DEFLATOR.494  \
VARIABLE_KEY
DESCRIPTION   DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                       0               0               0               0
CURRENCY                 EUR             EUR             EUR             EUR
SCENARIO                 492             493             494             495

                DEFLATOR.495    DEFLATOR.496    DEFLATOR.497    DEFLATOR.498  \
VARIABLE_KEY
DESCRIPTION   DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET  DEFLATOR ASSET
TERM                       0               0               0               0
CURRENCY                 EUR             EUR             EUR             EUR
SCENARIO                 496             497             498             499

                DEFLATOR.499
VARIABLE_KEY
DESCRIPTION   DEFLATOR ASSET
TERM                       0
CURRENCY                 EUR
SCENARIO                 500
```

```
[4 rows x 500 columns]
```

[635]:
```python
deflator.head(4)
```

[635]:
```
             DEFLATOR  DEFLATOR.1  DEFLATOR.2  DEFLATOR.3  DEFLATOR.4  \
VARIABLE_KEY
30/09/2022   1.000000    1.000000    1.000000    1.000000    1.000000
31/10/2022   0.998170    0.998170    0.998170    0.998170    0.998170
30/11/2022   0.996326    0.996408    0.995904    0.996705    0.996666
31/12/2022   0.994477    0.994793    0.993855    0.994944    0.994744

             DEFLATOR.5  DEFLATOR.6  DEFLATOR.7  DEFLATOR.8  DEFLATOR.9  …  \
VARIABLE_KEY                                                             …
30/09/2022     1.000000    1.000000    1.000000    1.000000    1.000000  …
31/10/2022     0.998170    0.998170    0.998170    0.998170    0.998170  …
30/11/2022     0.995663    0.996545    0.996361    0.996389    0.996160  …
31/12/2022     0.993562    0.994015    0.994973    0.994567    0.994002  …

             DEFLATOR.490  DEFLATOR.491  DEFLATOR.492  DEFLATOR.493  \
VARIABLE_KEY
30/09/2022       1.000000      1.000000      1.000000      1.000000
31/10/2022       0.998170      0.998170      0.998170      0.998170
30/11/2022       0.996747      0.995999      0.996333      0.996731
31/12/2022       0.995077      0.993976      0.994701      0.995361

             DEFLATOR.494  DEFLATOR.495  DEFLATOR.496  DEFLATOR.497  \
VARIABLE_KEY
30/09/2022       1.000000      1.000000      1.000000      1.000000
31/10/2022       0.998170      0.998170      0.998170      0.998170
30/11/2022       0.996151      0.996685      0.996308      0.996043
31/12/2022       0.995770      0.995182      0.994799      0.994050

             DEFLATOR.498  DEFLATOR.499
VARIABLE_KEY
30/09/2022       1.000000      1.000000
31/10/2022       0.998170      0.998170
30/11/2022       0.996366      0.996583
31/12/2022       0.995295      0.995337

[4 rows x 500 columns]
```

Spot rate with a 1-year term; stochastic scenarios The spot rate with term of 1-year is represented as a series of returns. The value at time `t` represents the return of investing `1` at time `t` until time `t+12` (months) in scenario `i`.

$$r_{i,T1}(t)$$

Where:

- `i` denotes that this is the return calculated using the `i`-th stochastic scenario
- `T1` denotes that this return is calculated using the spot curve with a `1`-year term
- `t` is the time of investment

[639]:
```
locationindex = filter_scenario_file(data_stoch_headers,\
                        spot_variable, "1", currency)
```

[640]:
```
spot_1 = data_stoch.iloc[:,locationindex]
```

VISUAL INSPECTION
Stochastic spot rate with 1-year term; headers and scenarios

[642]:
```
data_stoch_headers.iloc[:,locationindex]
```

[642]:
```
                  SPOT_1   SPOT_1.1   SPOT_1.2   SPOT_1.3   SPOT_1.4  \
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                   1          1          1          1          1
CURRENCY             EUR        EUR        EUR        EUR        EUR
SCENARIO               1          2          3          4          5

                SPOT_1.5   SPOT_1.6   SPOT_1.7   SPOT_1.8   SPOT_1.9  …  \
VARIABLE_KEY                                                           …
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  …
TERM                   1          1          1          1          1  …
CURRENCY             EUR        EUR        EUR        EUR        EUR  …
SCENARIO               6          7          8          9         10  …

              SPOT_1.490 SPOT_1.491 SPOT_1.492 SPOT_1.493 SPOT_1.494  \
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                   1          1          1          1          1
CURRENCY             EUR        EUR        EUR        EUR        EUR
SCENARIO             491        492        493        494        495

              SPOT_1.495 SPOT_1.496 SPOT_1.497 SPOT_1.498 SPOT_1.499
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                   1          1          1          1          1
CURRENCY             EUR        EUR        EUR        EUR        EUR
SCENARIO             496        497        498        499        500
```

```
[4 rows x 500 columns]
```

[643]:

```python
spot_1.head()
```

[643]:

```
               SPOT_1  SPOT_1.1  SPOT_1.2  SPOT_1.3  SPOT_1.4  SPOT_1.5  \
VARIABLE_KEY
30/09/2022   0.025484  0.025484  0.025484  0.025484  0.025484  0.025484
31/10/2022   0.025951  0.024965  0.031043  0.021404  0.021871  0.033967
30/11/2022   0.026256  0.023438  0.028679  0.025185  0.027129  0.029316
31/12/2022   0.024064  0.023999  0.031913  0.029466  0.026192  0.024812
31/01/2023   0.024050  0.026649  0.033978  0.029959  0.036508  0.022529

             SPOT_1.6  SPOT_1.7  SPOT_1.8  SPOT_1.9  …  SPOT_1.490  \
VARIABLE_KEY                                         …
30/09/2022   0.025484  0.025484  0.025484  0.025484  …    0.025484
31/10/2022   0.023326  0.025541  0.025197  0.027956  …    0.020897
30/11/2022   0.034493  0.020698  0.025932  0.029994  …    0.024095
31/12/2022   0.039323  0.023184  0.024342  0.030728  …    0.019358
31/01/2023   0.045296  0.012303  0.029277  0.031384  …    0.020451

             SPOT_1.491  SPOT_1.492  SPOT_1.493  SPOT_1.494  SPOT_1.495  \
VARIABLE_KEY
30/09/2022     0.025484    0.025484    0.025484    0.025484    0.025484
31/10/2022     0.029902    0.025875    0.021099    0.028071    0.021642
30/11/2022     0.028359    0.023632    0.020486    0.008710    0.022090
31/12/2022     0.031593    0.021744    0.029503    0.006404    0.022279
31/01/2023     0.037064    0.031838    0.031011    0.013253    0.020579

             SPOT_1.496  SPOT_1.497  SPOT_1.498  SPOT_1.499
VARIABLE_KEY
30/09/2022     0.025484    0.025484    0.025484    0.025484
31/10/2022     0.026177    0.029374    0.025478    0.022864
30/11/2022     0.022156    0.027993    0.016918    0.019003
31/12/2022     0.023948    0.027462    0.018420    0.014633
31/01/2023     0.019591    0.021532    0.022537    0.017642

[5 rows x 500 columns]
```

Spot rate with a 2-year term; stochastic scenarios The spot rate with term of 2 years is represented as a series of annualized returns. The value at time `t` represents the annualized return of investing `1` unit of currency at time `t` until time `t+24` (months) in scenario `i`. This is represented as:

$$r_{i,T2}(t)$$

Where:

- `i` denotes that this is the return calculated using the `i`-th stochastic scenario
- `T2` denotes that this return is calculated using the spot curve with a 2-year term
- `t` is the time of investing

[647]:
```
locationindex = filter_scenario_file(data_stoch_headers, \
                                     spot_variable, "2", currency)
```

[648]:
```
spot_2 = data_stoch.iloc[:,locationindex]
```

VISUAL INSPECTION
Stochastic spot rate with 2-year term; headers and scenarios

[654]:
```
data_stoch_headers.iloc[:,locationindex]
```

[654]:
```
                  SPOT_2   SPOT_2.1   SPOT_2.2   SPOT_2.3   SPOT_2.4  \
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                   2          2          2          2          2
CURRENCY             EUR        EUR        EUR        EUR        EUR
SCENARIO               1          2          3          4          5

                SPOT_2.5   SPOT_2.6   SPOT_2.7   SPOT_2.8   SPOT_2.9  …  \
VARIABLE_KEY                                                          …
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  …
TERM                   2          2          2          2          2  …
CURRENCY             EUR        EUR        EUR        EUR        EUR  …
SCENARIO               6          7          8          9         10  …

              SPOT_2.490 SPOT_2.491 SPOT_2.492 SPOT_2.493 SPOT_2.494  \
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                   2          2          2          2          2
CURRENCY             EUR        EUR        EUR        EUR        EUR
SCENARIO             491        492        493        494        495

              SPOT_2.495 SPOT_2.496 SPOT_2.497 SPOT_2.498 SPOT_2.499
VARIABLE_KEY
DESCRIPTION    SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE  SPOT RATE
TERM                   2          2          2          2          2
CURRENCY             EUR        EUR        EUR        EUR        EUR
SCENARIO             496        497        498        499        500

[4 rows x 500 columns]
```

[655]:
```
spot_2.head()
```

[655]:
```
                SPOT_2   SPOT_2.1  SPOT_2.2  SPOT_2.3  SPOT_2.4  SPOT_2.5  \
VARIABLE_KEY
30/09/2022    0.028942  0.028942  0.028942  0.028942  0.028942  0.028942
31/10/2022    0.029216  0.028254  0.034180  0.024781  0.025237  0.037030
30/11/2022    0.029277  0.026530  0.031639  0.028232  0.030128  0.032259
31/12/2022    0.026857  0.026794  0.034505  0.032121  0.028931  0.027586
31/01/2023    0.026521  0.029053  0.036191  0.032277  0.038656  0.025039

              SPOT_2.6  SPOT_2.7  SPOT_2.8  SPOT_2.9  …  SPOT_2.490  \
VARIABLE_KEY                                          …
30/09/2022    0.028942  0.028942  0.028942  0.028942  …    0.028942
31/10/2022    0.026655  0.028815  0.028480  0.031170  …    0.024287
30/11/2022    0.037305  0.023859  0.028961  0.032920  …    0.027170
31/12/2022    0.041724  0.025999  0.027128  0.033350  …    0.022270
31/01/2023    0.047213  0.015074  0.031613  0.033665  …    0.023014

              SPOT_2.491  SPOT_2.492  SPOT_2.493  SPOT_2.494  SPOT_2.495  \
VARIABLE_KEY
30/09/2022      0.028942    0.028942    0.028942    0.028942    0.028942
31/10/2022      0.033067    0.029142    0.024484    0.031282    0.025014
30/11/2022      0.031326    0.026718    0.023651    0.012170    0.025215
31/12/2022      0.034193    0.024596    0.032158    0.009642    0.025118
31/01/2023      0.039197    0.034107    0.033302    0.016000    0.023138

              SPOT_2.496  SPOT_2.497  SPOT_2.498  SPOT_2.499
VARIABLE_KEY
30/09/2022      0.028942    0.028942    0.028942    0.028942
31/10/2022      0.029436    0.032552    0.028754    0.026205
30/11/2022      0.025280    0.030970    0.020173    0.022206
31/12/2022      0.026744    0.030169    0.021356    0.017664
31/01/2023      0.022176    0.024067    0.025046    0.020277

[5 rows x 500 columns]
```

Extra content

[657]:
```
locationindex = filter_scenario_file(data_stoch_headers, spot_variable, "3",␣
  ↪currency)
spot_3 = data_stoch.iloc[:,locationindex]
```

[658]:
```
locationindex = filter_scenario_file(data_stoch_headers, spot_variable, "4",␣
  ↪currency)
spot_4 = data_stoch.iloc[:,locationindex]
```

[659]:
```
locationindex = filter_scenario_file(data_stoch_headers, spot_variable, "5",␣
  ↪currency)
```

```
spot_5 = data_stoch.iloc[:,locationindex]
```

Total-return index stochastic scenarios The total-return indices are represented as the value of an index at time `t` with a starting value of `1` units of currency at modelling time `0` in scenario `i`.

$$I_{i,1}(t)$$

Where:

- `i` denotes that this index is calculated using the `i`-th stochastic scenario
- `1` denotes that this total-return index represents the first hypothetical asset
- `t` is the Modelling time

[671]:
```
locationindex = filter_scenario_file(data_stoch_headers, index_variable,␣
↪index_term, currency)
```

[672]:
```
first_index = data_stoch.iloc[:,locationindex]
```

VISUAL INSPECTION
Total-return index; headers and scenarios

[674]:
```
data_stoch_headers.iloc[:,locationindex]
```

[674]:
```
                INDEX INDEX.1 INDEX.2 INDEX.3 INDEX.4 INDEX.5 INDEX.6 INDEX.7  \
VARIABLE_KEY
DESCRIPTION     INDEX   INDEX   INDEX   INDEX   INDEX   INDEX   INDEX   INDEX
TERM                0       0       0       0       0       0       0       0
CURRENCY          EUR     EUR     EUR     EUR     EUR     EUR     EUR     EUR
SCENARIO            1       2       3       4       5       6       7       8

               INDEX.8 INDEX.9  … INDEX.490 INDEX.491 INDEX.492 INDEX.493  \
VARIABLE_KEY                    …
DESCRIPTION      INDEX   INDEX  …     INDEX     INDEX     INDEX     INDEX
TERM                 0       0  …         0         0         0         0
CURRENCY           EUR     EUR  …       EUR       EUR       EUR       EUR
SCENARIO             9      10  …       491       492       493       494

               INDEX.494 INDEX.495 INDEX.496 INDEX.497 INDEX.498 INDEX.499
VARIABLE_KEY
DESCRIPTION        INDEX     INDEX     INDEX     INDEX     INDEX     INDEX
TERM                   0         0         0         0         0         0
CURRENCY             EUR       EUR       EUR       EUR       EUR       EUR
SCENARIO             495       496       497       498       499       500
```

```
[4 rows x 500 columns]
```

[675]:
```
first_index.head()
```

[675]:
```
                INDEX   INDEX.1   INDEX.2   INDEX.3   INDEX.4   INDEX.5  \
VARIABLE_KEY
30/09/2022   1.000000  1.000000  1.000000  1.000000  1.000000  1.000000
31/10/2022   1.004252  0.991676  0.996692  1.009442  1.006826  0.998891
30/11/2022   1.013114  0.998047  1.004273  1.013157  0.995257  0.998347
31/12/2022   1.019324  1.000392  1.007218  1.019881  0.995524  0.989838
31/01/2023   1.022011  1.004860  1.006790  1.023282  1.002880  0.989899

              INDEX.6   INDEX.7   INDEX.8   INDEX.9  …  INDEX.490  \
VARIABLE_KEY                                         …
30/09/2022   1.000000  1.000000  1.000000  1.000000  …   1.000000
31/10/2022   1.015911  0.987568  1.006754  1.008715  …   0.998135
30/11/2022   1.019634  0.987449  1.009823  1.019360  …   1.000675
31/12/2022   1.012374  0.975855  1.005642  1.009754  …   1.009533
31/01/2023   1.021796  0.980949  1.011963  1.019708  …   1.011067

             INDEX.491  INDEX.492  INDEX.493  INDEX.494  INDEX.495  \
VARIABLE_KEY
30/09/2022    1.000000   1.000000   1.000000   1.000000   1.000000
31/10/2022    0.994408   1.013690   0.991886   1.006662   1.009607
30/11/2022    1.003585   1.008598   0.999374   1.016761   1.001836
31/12/2022    1.011131   1.027138   1.015096   1.023693   1.016197
31/01/2023    1.017271   1.028211   1.010807   1.021912   1.017094

             INDEX.496  INDEX.497  INDEX.498  INDEX.499
VARIABLE_KEY
30/09/2022    1.000000   1.000000   1.000000   1.000000
31/10/2022    1.009335   0.992735   0.989699   1.011132
30/11/2022    1.001443   0.997197   0.997647   1.007644
31/12/2022    0.991262   0.995540   1.006041   1.009948
31/01/2023    0.994203   1.001523   1.007956   1.011125

[5 rows x 500 columns]
```

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1.4.2 Stochastic deflator performance calculation

To calculate the performance of the deflator asset in different time increments, first the monthly return at each time increment `t` in each scenario `i` is calculated:

$$r_{i,RF}^{1M}(t) = \frac{ZCB_{i,RF}(0,t-1)}{ZCB_{i,RF}(0,t)} - 1$$

Annualized (12 month) performance can be calculated as the 12-month rolling product of the monthly capitalisation factor:

$$r_{i,RF}(t) = \prod_{j=t}^{t+12} (1 + r_{i,RF}^{1M}(j)) - 1$$

Where:

- $ZCB_{i,RF}(0,t)$ is the value of a ZCB issued at time `0` with a Maturity at time `t` priced using the `i`-th scenario
- $r_{i,RF}^{1M}(t)$ is the 1-month return of the deflator asset at time `t` under the `i`-th scenario
- $r_{i,RF}(t)$ is the yearly (12-month) return of the deflator asset between times `t` and `t+12` under the `i`-th scenario

[704]:
```
r_d_1_t = deflator.shift()/ deflator # 1+ monthly return
```

[705]:
```
r_d_12_t = r_d_1_t.rolling(window=12).apply(np.prod, raw=True) - 1 # annual␣
↪return
```

[706]:
```
r_d_12_t = r_d_12_t.iloc[12:,:] # remove NaN (13) entries;
```

[707]:
```
r_d_12_t.index = r_d_1_t.index[:-12] # Corrected index dates
```

VISUAL INSPECTION
Deflator asset monthly and annual performance

[732]:
```
r_d_1_t.head()
```

[732]:
```
              DEFLATOR  DEFLATOR.1  DEFLATOR.2  DEFLATOR.3  DEFLATOR.4  \
VARIABLE_KEY
30/09/2022         NaN         NaN         NaN         NaN         NaN
31/10/2022    1.001833    1.001833    1.001833    1.001833    1.001833
30/11/2022    1.001851    1.001768    1.002275    1.001470    1.001509
31/12/2022    1.001859    1.001623    1.002062    1.001770    1.001932
31/01/2023    1.001664    1.001658    1.002318    1.002115    1.001842

              DEFLATOR.5  DEFLATOR.6  DEFLATOR.7  DEFLATOR.8  DEFLATOR.9  …  \
VARIABLE_KEY                                                              …
30/09/2022           NaN         NaN         NaN         NaN         NaN  …
31/10/2022      1.001833    1.001833    1.001833    1.001833    1.001833  …
30/11/2022      1.002518    1.001631    1.001816    1.001787    1.002018  …
31/12/2022      1.002115    1.002545    1.001395    1.001832    1.002171  …
31/01/2023      1.001726    1.002932    1.001590    1.001688    1.002220  …

              DEFLATOR.490  DEFLATOR.491  DEFLATOR.492  DEFLATOR.493  \
VARIABLE_KEY
```

```
30/09/2022          NaN          NaN          NaN          NaN
31/10/2022     1.001833     1.001833     1.001833     1.001833
30/11/2022     1.001428     1.002180     1.001844     1.001444
31/12/2022     1.001678     1.002035     1.001641     1.001376
31/01/2023     1.001269     1.002292     1.001469     1.002118

             DEFLATOR.494  DEFLATOR.495  DEFLATOR.496  DEFLATOR.497  \
VARIABLE_KEY
30/09/2022            NaN           NaN           NaN           NaN
31/10/2022       1.001833      1.001833      1.001833      1.001833
30/11/2022       1.002027      1.001490      1.001869      1.002135
31/12/2022       1.000383      1.001510      1.001517      1.002005
31/01/2023       1.000175      1.001515      1.001654      1.001948

             DEFLATOR.498  DEFLATOR.499
VARIABLE_KEY
30/09/2022            NaN           NaN
31/10/2022       1.001833      1.001833
30/11/2022       1.001811      1.001592
31/12/2022       1.001076      1.001252
31/01/2023       1.001191      1.000871

[5 rows x 500 columns]
```

[734]:
```
r_d_12_t.head()
```

[734]:
```
             DEFLATOR  DEFLATOR.1  DEFLATOR.2  DEFLATOR.3  DEFLATOR.4  \
VARIABLE_KEY
30/09/2022   0.020827    0.026285    0.034641    0.022062    0.032137
31/10/2022   0.022666    0.027230    0.038219    0.021999    0.034811
30/11/2022   0.024252    0.028432    0.041514    0.022603    0.037150
31/12/2022   0.025756    0.030051    0.044917    0.022805    0.039650
31/01/2023   0.028001    0.031371    0.047627    0.022665    0.042654

             DEFLATOR.5  DEFLATOR.6  DEFLATOR.7  DEFLATOR.8  DEFLATOR.9  …  \
VARIABLE_KEY                                                             …
30/09/2022     0.020858    0.038673    0.018733    0.029472    0.022307  …
31/10/2022     0.020307    0.040746    0.019605    0.029984    0.020637  …
30/11/2022     0.019316    0.043069    0.020066    0.031586    0.018773  …
31/12/2022     0.018710    0.045469    0.020515    0.032690    0.017706  …
31/01/2023     0.018060    0.047791    0.020101    0.033502    0.016815  …

             DEFLATOR.490  DEFLATOR.491  DEFLATOR.492  DEFLATOR.493  \
VARIABLE_KEY
30/09/2022       0.019504      0.036817      0.032964      0.018647
31/10/2022       0.020299      0.039327      0.034376      0.017440
30/11/2022       0.020958      0.041613      0.035730      0.016307
```

```
31/12/2022        0.021372      0.044232      0.037461      0.015837
31/01/2023        0.022236      0.046724      0.039662      0.015525

              DEFLATOR.494  DEFLATOR.495  DEFLATOR.496  DEFLATOR.497  \
VARIABLE_KEY
30/09/2022        0.020413      0.023793      0.020776      0.022708
31/10/2022        0.021587      0.025199      0.021842      0.023527
30/11/2022        0.022378      0.026491      0.022654      0.024252
31/12/2022        0.025352      0.028517      0.023763      0.024909
31/01/2023        0.028822      0.031275      0.024574      0.026397

              DEFLATOR.498  DEFLATOR.499
VARIABLE_KEY
30/09/2022        0.018307      0.010119
31/10/2022        0.018646      0.008269
30/11/2022        0.019662      0.006607
31/12/2022        0.021212      0.005481
31/01/2023        0.022691      0.004750

[5 rows x 500 columns]
```

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2 Test 1; Mean of stochastic deflator is equal to the deterministic

The objective of this test is to compare the average performance of the stochastic deflation asset and the deterministic deflator.

The two test statistics used are:

$$S^1_{AVERAGE} = \frac{1}{T-12}\sum_{t=0}^{T-12}\frac{1}{N}\sum_{i=0}^{N}\left|ZCB_{i,RF}(0,t) - ZCB_{DET,RF}(0,t)\right|$$

$$S^1_{MAX} = \max_t \frac{1}{N}\left|\sum_{i=0}^{N} ZCB_{i,RF}(0,t) - ZCB_{DET,RF}(0,t)\right|$$

Where:

- N ... is the number scenarios
- T ... is the number of time increments
- $ZCB_{i,RF}(0,t)$ ... is the deflator asset between time `0` and `t`, priced using the stochastic deflator scenario `i`
- $ZCB_{DET,RF}(0,t)$ ... is the deflator asset between time `0` and `t` priced using the deterministic scenario

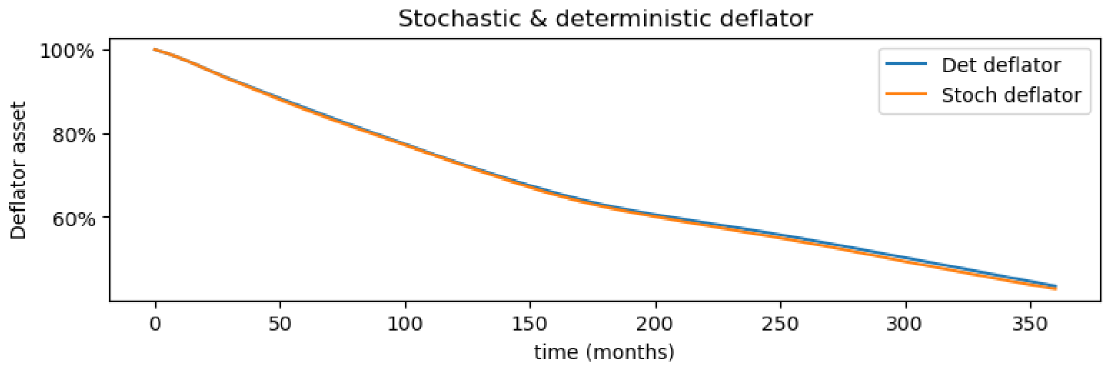
- $S^1_{AVERAGE}$ ... is the average test statistics for the first test
- S^1_{MAX} ... is the maximum deviation test statistics for the first test

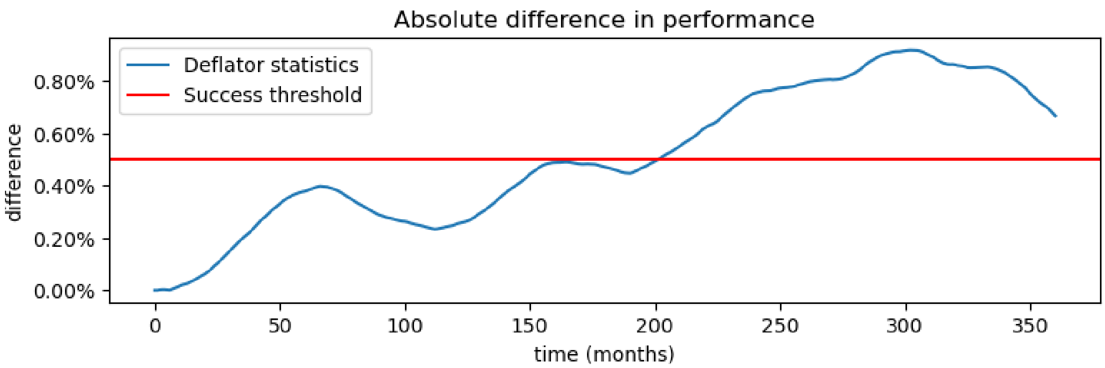
[759]:
```
deflator_stoch_mean = pd.DataFrame(deflator.mean(axis=1),columns=["Deflator␣
↪mean"])
```

[760]:
```
test_statistics_1 = np.abs(deflator_det.values-deflator_stoch_mean.values)
```

VISUAL INSPECTION
Deflator asset test

[762]:
```
x_data_label = range(0,deflator_det.shape[0],1)
fig, (ax1, ax2) = plt.subplots(2,1)
ax1.plot(x_data_label, deflator_det.values*100, color='tab:blue',label="Det␣
↪deflator")
ax1.plot(x_data_label, deflator_stoch_mean.values*100, color='tab:
↪orange',label="Stoch deflator")
ax1.legend()
ax1.set_xlabel("time (months)")
ax1.set_ylabel("Deflator asset")
ax1.set_title('Stochastic & deterministic deflator')

ax2.plot(x_data_label, test_statistics_1*100, label="Deflator statistics")
ax2.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
↪threshold")
ax2.legend()
ax2.set_xlabel("time (months)")
ax2.set_ylabel("difference")
ax2.set_title('Absolute difference in performance')
fig.tight_layout(h_pad=2)

ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
ax2.yaxis.set_major_formatter(mtick.PercentFormatter())

fig.set_figwidth(8)
fig.set_figheight(6)

plt.show()
```





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2.1 Test 1; Success criteria

The successful application of the success criteria marks the completion of the test

[775]:
```
result1 = success_test(test_statistics_1, test_threshold_max,␣
↪test_threshold_mean)
```

```
Test passed
Test passed
```

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2.2 Spot curve cumulative performance

The spot curve time-series contains annualized returns starting at every month. To calculate the cumulative performance for longer time periods, three extra steps must be performed:

1. Separate non-chained returns
2. Convert the annualized return into the actual return for the entire period
3. Aggregate the chained returns through time

The function spot_cumulative() applies the 3 steps above. This example is limited to terms 1 and 2 with terms 3 to 10 calculated in the Extra content section. It is possible to extend the methodology to higher terms.

[778]:
```python
def spot_cumulative(spot, term):
    out = (1+spot).pow(term)
    for iRow in range(term*12,out.shape[0]):
        out.iloc[iRow, :] = out.iloc[iRow, :]*out.iloc[iRow-term*12, :]
    out = out.iloc[:(-term*12),:]
    return out
```

[779]:
```python
spot_1_cum = spot_cumulative(spot_1, 1)
```

[780]:
```python
spot_2_cum = spot_cumulative(spot_2, 2)
```

Extra content

[782]:
```python
spot_3_cum = spot_cumulative(spot_3, 3)
spot_4_cum = spot_cumulative(spot_4, 4)
spot_5_cum = spot_cumulative(spot_5, 5)
```

VISUAL INSPECTION
Spot rate cumulative performance

[791]:
```python
spot_1_cum.head()
```

[791]:
```
               SPOT_1  SPOT_1.1  SPOT_1.2  SPOT_1.3  SPOT_1.4  SPOT_1.5  \
VARIABLE_KEY
30/09/2022   1.025484  1.025484  1.025484  1.025484  1.025484  1.025484
31/10/2022   1.025951  1.024965  1.031043  1.021404  1.021871  1.033967
30/11/2022   1.026256  1.023438  1.028679  1.025185  1.027129  1.029316
31/12/2022   1.024064  1.023999  1.031913  1.029466  1.026192  1.024812
31/01/2023   1.024050  1.026649  1.033978  1.029959  1.036508  1.022529

             SPOT_1.6  SPOT_1.7  SPOT_1.8  SPOT_1.9  …  SPOT_1.490  \
VARIABLE_KEY                                         …
30/09/2022   1.025484  1.025484  1.025484  1.025484  …    1.025484
31/10/2022   1.023326  1.025541  1.025197  1.027956  …    1.020897
30/11/2022   1.034493  1.020698  1.025932  1.029994  …    1.024095
31/12/2022   1.039323  1.023184  1.024342  1.030728  …    1.019358
31/01/2023   1.045296  1.012303  1.029277  1.031384  …    1.020451

             SPOT_1.491  SPOT_1.492  SPOT_1.493  SPOT_1.494  SPOT_1.495  \
VARIABLE_KEY
30/09/2022     1.025484    1.025484    1.025484    1.025484    1.025484
31/10/2022     1.029902    1.025875    1.021099    1.028071    1.021642
30/11/2022     1.028359    1.023632    1.020485    1.008710    1.022090
31/12/2022     1.031593    1.021744    1.029503    1.006404    1.022279
31/01/2023     1.037064    1.031838    1.031011    1.013253    1.020579
```

```
              SPOT_1.496  SPOT_1.497  SPOT_1.498  SPOT_1.499
VARIABLE_KEY
30/09/2022      1.025484    1.025484    1.025484    1.025484
31/10/2022      1.026177    1.029373    1.025478    1.022864
30/11/2022      1.022156    1.027993    1.016918    1.019003
31/12/2022      1.023948    1.027462    1.018420    1.014633
31/01/2023      1.019591    1.021532    1.022537    1.017642

[5 rows x 500 columns]
```

[792]:
```
spot_2_cum.head()
```

[792]:
```
                SPOT_2  SPOT_2.1  SPOT_2.2  SPOT_2.3  SPOT_2.4  SPOT_2.5  \
VARIABLE_KEY
30/09/2022    1.058721  1.058721  1.058721  1.058721  1.058721  1.058721
31/10/2022    1.059285  1.057306  1.069527  1.050177  1.051110  1.075431
30/11/2022    1.059411  1.053763  1.064278  1.057262  1.061163  1.065559
31/12/2022    1.054435  1.054305  1.070200  1.065274  1.058698  1.055934
31/01/2023    1.053745  1.058949  1.073692  1.065596  1.078806  1.050705

              SPOT_2.6  SPOT_2.7  SPOT_2.8  SPOT_2.9  …  SPOT_2.490  \
VARIABLE_KEY                                          …
30/09/2022    1.058721  1.058721  1.058721  1.058721  …    1.058721
31/10/2022    1.054021  1.058461  1.057771  1.063311  …    1.049165
30/11/2022    1.076002  1.048287  1.058760  1.066923  …    1.055077
31/12/2022    1.085189  1.052674  1.054992  1.067813  …    1.045036
31/01/2023    1.096655  1.030376  1.064225  1.068463  …    1.046557

              SPOT_2.491  SPOT_2.492  SPOT_2.493  SPOT_2.494  SPOT_2.495  \
VARIABLE_KEY
30/09/2022      1.058721    1.058721    1.058721    1.058721    1.058721
31/10/2022      1.067228    1.059132    1.049568    1.063542    1.050653
30/11/2022      1.063634    1.054150    1.047862    1.024487    1.051066
31/12/2022      1.069555    1.049797    1.065349    1.019377    1.050866
31/01/2023      1.079930    1.069377    1.067713    1.032255    1.046812

              SPOT_2.496  SPOT_2.497  SPOT_2.498  SPOT_2.499
VARIABLE_KEY
30/09/2022      1.058721    1.058721    1.058721    1.058721
31/10/2022      1.059738    1.066164    1.058334    1.053097
30/11/2022      1.051200    1.062898    1.040754    1.044905
31/12/2022      1.054203    1.061247    1.043169    1.035641
31/01/2023      1.044843    1.048714    1.050720    1.040965

[5 rows x 500 columns]
```

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3 Martingale tests

To evaluate the market-consistency, IVASS regulation 18 article 57 recommends to perform the Martingale test on asset classes used in the calibration process of the ESG and for simple financial strategies. The following asset classes are covered:

- Spot curves
- Total-return indices

Article 57 recommends using multiple simple investment strategies. Two are used in this example:

- Forward return with a spread of 1 year
- Buy-and-hold strategy

3.1 Forward return with a spread of 1 year investment strategy

Forward return with a spread of `1`-year strategy consists of buying and selling fixed income assets in a way that the return is equal to a forward rate with constant maturity. In this example, a constant maturity of `1`-year is implemented by buying (long position) a ZCB with a maturity of `K+1` years and selling (short position) a ZCB with a maturity of `K`-years.

$$ZCB_{i,TK}(t) = \frac{1}{\left(1 + r_{i,TK}(t)\right)^{K}}$$

$$Perf^{FR}_{i,TK,TK+1}(t) = \frac{ZCB_{i,TK}(t)}{ZCB_{i,TK+1}(t)} =$$

$$= \frac{\frac{1}{\left(1+r_{i,TK}(t)\right)^{K}}}{\frac{1}{\left(1+r_{i,TK+1}(t)\right)^{K+1}}} = \frac{\left(1 + r_{i,TK+1}(t)\right)^{K+1}}{\left(1 + r_{i,TK}(t)\right)^{K}}$$

Therefore:

$$DPerf^{FR}_{i,TK,TK+1}(t) = \frac{Perf^{FR}_{i,TK,TK+1}(t)}{(1 + r_{i,RF}(t + 12 * K))}$$

is a Martingale.

The test verifies the following asymptotic identity:

$$E^{Q}[Perf^{FR}_{i,TK,TK+1}(t)] =^{?} 1$$

Where:

- Q is the numeraire used in this test which is the 1-year forward rate of the deflator

The function forward_strategy_test() performs the beforementioned steps for any sequential spot rate terms

[801]:
```
def forward_strategy_test(annualdef, spot, spotnext, term):
    if term>1:
        annualdef= annualdef.iloc[:-(term-1)*12,:]

    perf = ((1+spotnext).pow(term+1))/((1+spot).pow(term)).values
    perf_shifted = perf.shift(term*12)
    Mspot = perf_shifted.iloc[:-term*12,:].values/(1+annualdef).values
    Mspot = pd.DataFrame(Mspot)
    Mspot = Mspot.iloc[term*12:,:]
    return (Mspot - 1).mean(axis=1)
```

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3.2 Test 2; Forward return with a spread of 1-year investment strategy

Forward return with a spread of 1-year strategy using 1- and 2-year spot rates consists of buying and selling ZCB bonds with 1- and 2-year term. In this example, this consists of buying (long position) a ZCB with a maturity of 2 years and selling (short position) a ZCB with a maturity of 1 year.

$$ZCB_{i,T1}(t) = \frac{1}{(1 + r_{i,T1}(t))}$$

$$ZCB_{i,T2}(t) = \frac{1}{(1 + r_{i,T2}(t))^2}$$

$$Perf^{FR}_{i,T1,T2}(t) = \frac{(1 + r_{i,T2}(t))^2}{(1 + r_{i,T1}(t))}$$

The strategy consists of selling the 1-year ZCB bond and using the proceeds to buy a ZCB bond with a maturity of 2 years.

In a risk-neutral scenario this strategy can be replicated by depositing one unit at year 1 into the deflator asset and withdrawing it at time 2.

Therefore:

$$DPerf^{FR}_{i,T1,T2}(t) = \frac{Perf^{FR}_{i,T1,T2}(t)}{(1 + r_{i,RF}(t + 12))}$$

The test verifies the following asymptotic identity:

$$E^Q[Perf^{FR}_{i,T1,T2}(t)] =^? 1$$

Where:

- Q is the numeraire used in this test which is the 1-year forward rate of the deflator

The two test statistics used are:

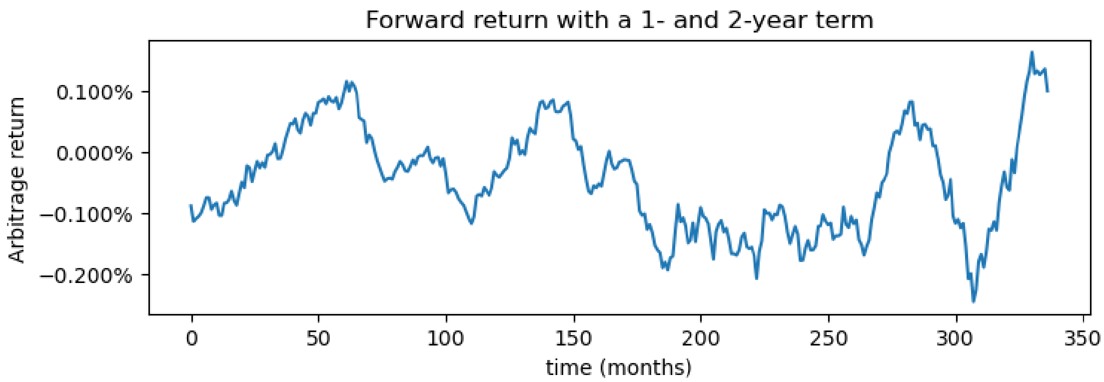
$$S_{AVERAGE} = \frac{1}{T-12} \sum_{t=0}^{T-12} \frac{1}{N} \sum_{i=0}^{N} \left| DPerf_{i,T1,T2}^{FR}(t) - 1 \right|$$

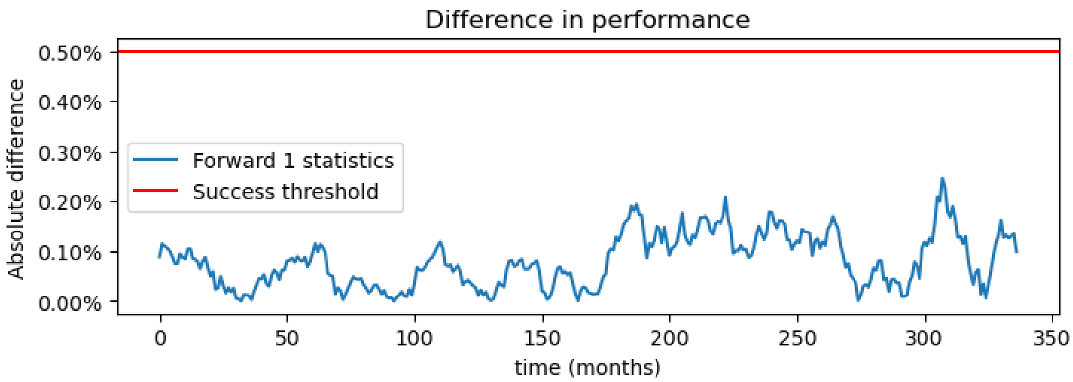
$$S_{MAX} = \max_{t} \frac{1}{N} \left| \sum_{i=0}^{N} (DPerf_{i,T1,T2}^{FR}(t) - 1) \right|$$

[809]:
```
test_statistics_2 = forward_strategy_test(r_d_12_t, spot_1, spot_2, 1)
```

VISUAL INSPECTION
Forward return strategy using the 1- and 2-year spot rate

[811]:
```
x_data_label = range(0,test_statistics_2.shape[0],1)
fig, (ax1,ax2) = plt.subplots(2,1)
ax1.plot(x_data_label, test_statistics_2.values*100, color='tab:blue')

ax1.set_xlabel("time (months)")
ax1.set_ylabel("Arbitrage return")
ax1.set_title('Forward return with a 1- and 2-year term')

ax1.yaxis.set_major_formatter(mtick.PercentFormatter())

ax2.plot(x_data_label, np.abs(test_statistics_2)*100, label="Forward 1␣
↪statistics")
ax2.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
↪threshold")
ax2.legend()
ax2.set_xlabel("time (months)")
ax2.set_ylabel("Absolute difference")
ax2.set_title('Difference in performance')
fig.tight_layout(h_pad=2)

ax2.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(8)
fig.set_figheight(6)

plt.show()
```





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3.3 Test 2; Success criteria

The successful application of the success criteria marks the completion of the test

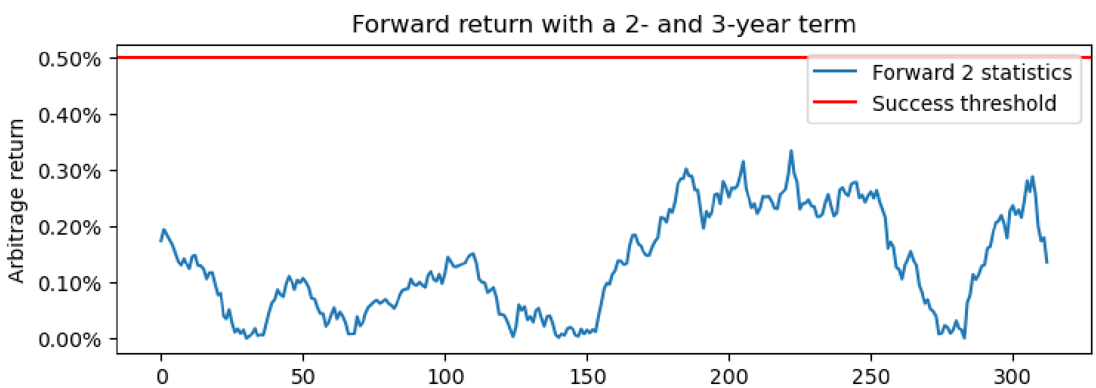
[814]:
```
result2 = success_test(np.abs(test_statistics_2),␣
↪test_martingale_threshold_max, test_martingale_threshold_mean)
```

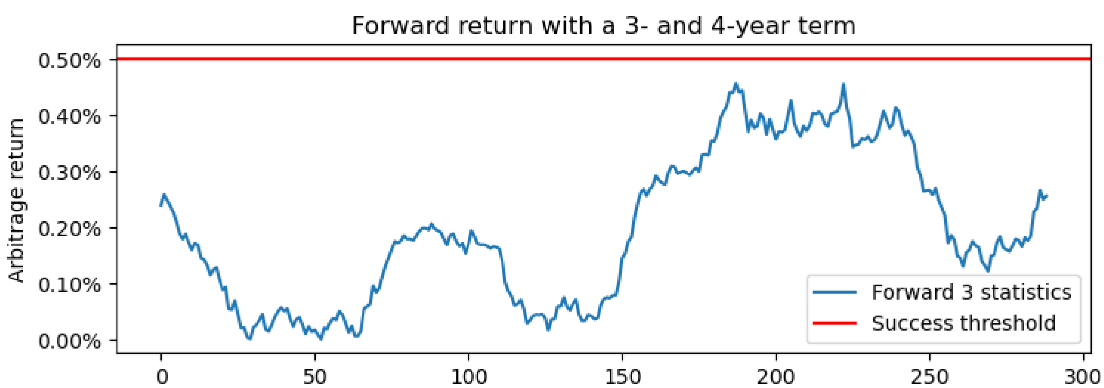
```
Test passed
Test passed
```

EXTRA CONTENT

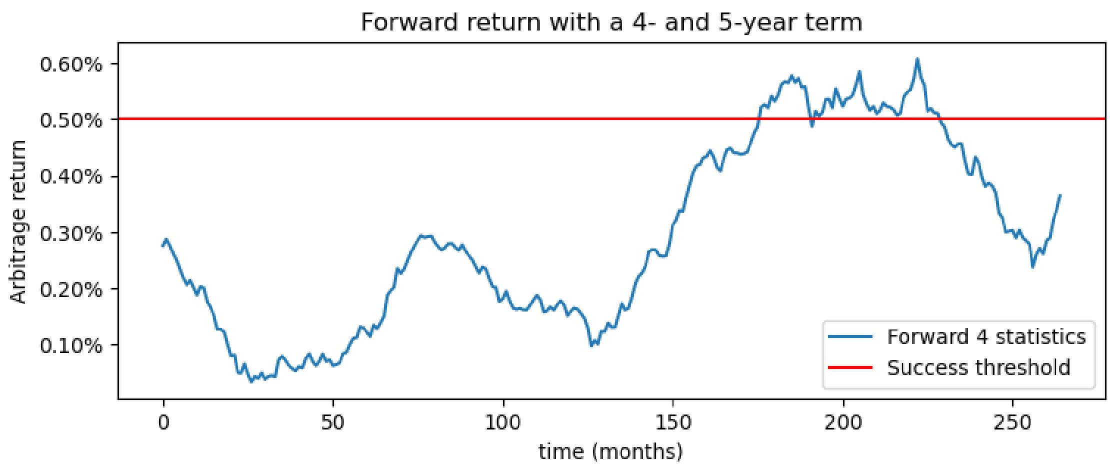
[816]:
```
test2_diff_2 = forward_strategy_test(r_d_12_t, spot_2, spot_3, 2)
test2_diff_3 = forward_strategy_test(r_d_12_t, spot_3, spot_4, 3)

x_data_label_2 = range(0,test2_diff_2.shape[0],1)
x_data_label_3 = range(0,test2_diff_3.shape[0],1)

fig, (ax1, ax2) = plt.subplots(2,1)
ax1.plot(x_data_label_2, np.abs(test2_diff_2).values*100, color='tab:blue',␣
↪label="Forward 2 statistics")
```

```
ax1.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")
ax1.set_title('Forward return with a 2- and 3-year term')
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
ax1.set_ylabel("Arbitrage return")
ax1.legend()
ax2.plot(x_data_label_3, np.abs(test2_diff_3).values*100, color='tab:blue',␣
 ↪label="Forward 3 statistics")
ax2.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")
ax2.yaxis.set_major_formatter(mtick.PercentFormatter())
ax2.set_title('Forward return with a 3- and 4-year term')
ax2.set_ylabel("Arbitrage return")
ax2.legend()
fig.tight_layout(h_pad=2)
fig.set_figwidth(8)
fig.set_figheight(6)

plt.show()
```





[817]:
```python
test2_diff_4 = forward_strategy_test(r_d_12_t, spot_4, spot_5, 4)

x_data_label_4 = range(0,test2_diff_4.shape[0],1)

fig, ax3 = plt.subplots(1,1)

ax3.plot(x_data_label_4, np.abs(test2_diff_4).values*100, color='tab:blue',␣
 ↪label="Forward 4 statistics")
ax3.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")
ax3.yaxis.set_major_formatter(mtick.PercentFormatter())
ax3.set_ylabel("Arbitrage return")
ax3.set_title('Forward return with a 4- and 5-year term')
ax3.set_xlabel("time (months)")
ax3.legend()
fig.tight_layout(h_pad=2)
fig.set_figwidth(8)
fig.set_figheight(3)

plt.show()
```



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3.4 Test 3; Buy-and-hold investment strategy

The Buy-and-hold strategy consists of investing all funds into ZCBs of a selected maturity, holding them until maturity and rolling all proceeds to buy another round of ZCB with the same selected maturity.

The strategy consists of investing for example `1` unit of currency in month `k` and withdrawing the accrued payoff at time T.

k is one of the first $K * 12$ months where K is the term of the spot rate curve used.

This payoff can be replicated by investing the initial amount in a risk-free asset represented by the deflator asset at time k and withdrawing everything at the end time T.

If the strategy has a term of K years, $K * 12$ separate starting points need to be tested (to cover the entire curve). For each starting point, the performance can be calculated as:

$$Perf^{BH}_{i,TK,k}(t`) = \prod_{j=0}^{t`} \left((1 + r_{i,TK}(k + 12Kj)\right)^{K}$$

Where $t` = 1, 2, 3, ...$. Note that $t`$ represents year multiples of the selected term K.

Therefore:

$$DPerf^{BH}_{i,TK,k}(t`) = Perf^{BH}_{i,TK,k}(t`) * ZCB_{i,RF}(0, k + 12Kt`)$$

is a Martingale and therefore the test verifies the following asymptotic identity:

$$E^{Q_k}[Perf^{BH}_{i,TK,k}(t`)] =^{?} 1$$

Where:

- Q is the numeraire used in this test which is the terminal value of the deflator asset starting at time k

The function deflator_normalisation() normalizes the deflator asset such that the resulting series has a value of 1 at starting month k.

[823]:
```
def deflator_normalisation(deflator, n):
    out = deflator.copy()
    for iRow in range(0,out.shape[0]):
        out.iloc[iRow,:] = out.iloc[iRow,:]/deflator.iloc[iRow % (n*12),:]
 ↪#ToDo modulo in python
    out = out.iloc[n*12:,:]
    return out
```

The function month_decomposition() separates the time-series that starts at each month in a separate row, making it possible to compare them.

[825]:
```
def month_decomposition(data, term):
    years = int(np.floor(data.shape[0]/(term*12)))-1
    out = np.zeros((years,term*12))

    for iStart in range(0,years*(term*12),1):
        position = iStart%(term*12)
        time = int(np.floor(iStart/(term*12)))
        out[time,position] = data[iStart]
```

```
    return out
```

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Test 3.1;Spot rate 1-year term

$$Perf^{BH}_{i,T1,k}(t`) = \prod_{j=0}^{t`} \left(1 + r_{i,T1}(k + 12 * j)\right)$$

Where $t` = 1, 2, 3, ...$ and $k = 0, 1, 2, ..., 11$

Therefore

$$DPerf^{BH}_{i,T1,k}(t`) = Perf^{BH}_{i,T1,k}(t`) * ZCB_{i,RF}(0, k + t` * 12)$$

is a Martingale and therefore the test verifies the following asymptotic identity

$$E^{Q_k}[Perf^{BH}_{i,T1,k}(t`)] =^? 1$$

The two test statistics used are:

$$S_{AVERAGE} = \frac{1}{T`} \sum_{t`=0}^{T`} \frac{1}{N} |\sum_{i=0}^{N} (DPerf^{BH}_{i,T1,k}(t`) - 1)|$$

$$S_{MAX} = \max_{t`} \frac{1}{N} |\sum_{i=0}^{N} (DPerf^{BH}_{i,T1,k}(t`) - 1)|$$

[829]:
```
deflator_1 = deflator_normalisation(deflator, 1)
MPerf1 = pd.DataFrame(spot_1_cum.values*deflator_1.values)
```

[830]:
```
test_statistics_3_1 = (MPerf1 - 1).mean(axis=1)
```

VISUAL INSPECTION
1-year Buy-and-hold strategy test

[832]:
```
MPerf1.head()
```

[832]:

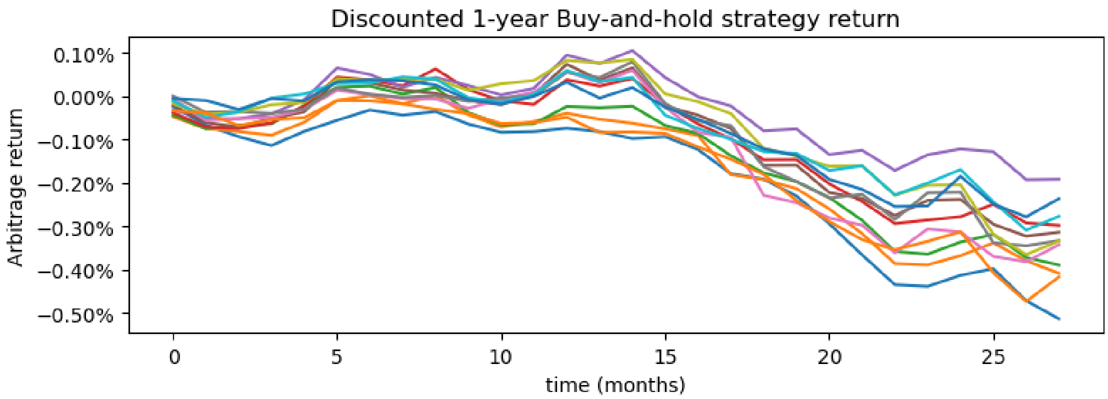
	0	1	2	3	4	5	6 \
0	1.004562	0.999219	0.991150	1.003348	0.993555	1.004531	0.987302
1	1.003213	0.997795	0.993088	0.999417	0.987495	1.013388	0.983262
2	1.001956	0.995144	0.987677	1.002524	0.990338	1.009810	0.991778
3	0.998350	0.994125	0.987555	1.006513	0.987055	1.005991	0.994121
4	0.996157	0.995422	0.986971	1.007133	0.994106	1.004390	0.997618

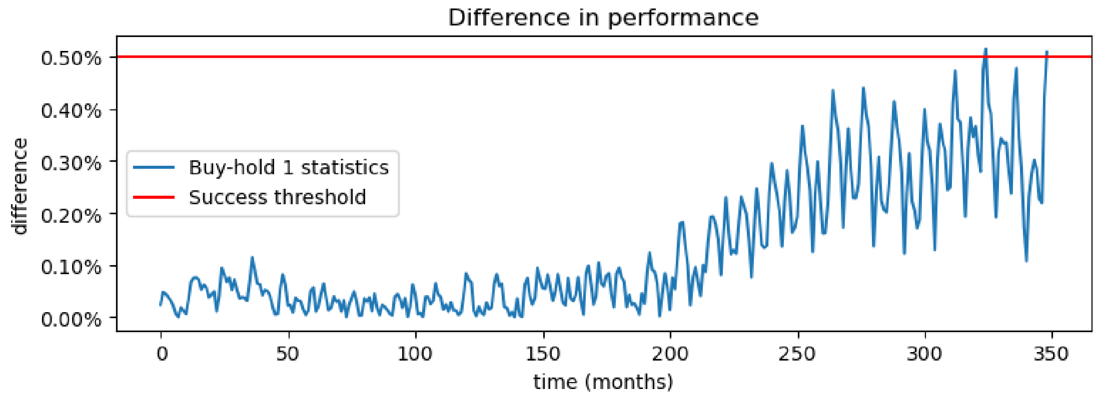
```
        7         8         9  …       490       491       492       493  \
```

```
0  1.006627  0.996127  1.003108  …  1.005866  0.989069  0.992759  1.006712
1  1.005822  0.995352  1.007171  …  1.000586  0.990931  0.991782  1.003596
2  1.000620  0.994519  1.011014  …  1.003072  0.987275  0.988319  1.004111
3  1.002615  0.991917  1.012795  …  0.998028  0.987896  0.984851  1.013453
4  0.992355  0.995912  1.014328  …  0.998254  0.990771  0.992474  1.015249

        494       495       496       497       498       499
0  1.004969  1.001652  1.004612  1.002714  1.007048  1.015211
1  1.006347  0.996530  1.004242  1.005712  1.006707  1.014475
2  0.986631  0.995712  0.999514  1.003652  0.997309  1.012315
3  0.981520  0.993935  1.000180  1.002491  0.997266  1.009101
4  0.984867  0.989628  0.995137  0.995261  0.999849  1.012831

[5 rows x 500 columns]
```

[833]:
```
plot_strategy_1 = month_decomposition(test_statistics_3_1, 1)
```

[834]:
```
x_data_label_1 = range(0,plot_strategy_1.shape[0],1)
x_data_label_2 = range(0,test_statistics_3_1.shape[0],1)

fig, (ax1,ax2) = plt.subplots(2,1)
ax1.plot(x_data_label_1, plot_strategy_1*100)
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())
ax1.set_xlabel("time (months)")
ax1.set_ylabel("Arbitrage return")
ax1.set_title('Discounted 1-year Buy-and-hold strategy return')

ax2.plot(x_data_label_2, np.abs(test_statistics_3_1)*100, label="Buy-hold 1␣
 ↪statistics")
ax2.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")

ax2.set_xlabel("time (months)")
ax2.set_ylabel("difference")
ax2.set_title('Difference in performance')
fig.tight_layout(h_pad=2)
ax2.legend()
ax2.yaxis.set_major_formatter(mtick.PercentFormatter())

fig.set_figwidth(8)
fig.set_figheight(6)

fig.tight_layout(h_pad=2)

plt.show()
```





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3.5 Test 3.1; Success criteria

The successful application of the success criteria marks the completion of the test

[836]:
```
result31 = success_test(np.abs(test_statistics_3_1),␣
↪test_martingale_threshold_max, test_martingale_threshold_mean)
```

```
Test passed
Test passed
```

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Test 3.2; Spot rate 2-year term

$$Perf^{BH}_{i,T2,k}(t`) = \prod_{j=0}^{t`} \left((1 + r_{i,T2}(k + 24j)\right)^2$$

Where $t` = 1, 2, 3, ...$ and $k = 0, 1, 2, ..., 23$.

Therefore

$$DPerf^{BH}_{i,T2,k}(t`) = Perf^{BH}_{i,T2,k}(t`) * ZCB_{i,RF}(0, k + 24t`)$$

is a Martingale and therefore the test verifies the following asymptotic identity:

$$E^{Q_k}[Perf^{BH}_{i,T2,k}(t`)] =^? 1$$

The two test statistics used are:

$$S_{AVERAGE} = \frac{1}{T-24} \sum_{t=0}^{T} |\frac{1}{N} \sum_{i=0}^{N} (DPerf^{BH}_{i,T2,k}(t) - 1)|$$

$$S_{MAX} = \max_t |\frac{1}{N} \sum_{i=0}^{N} (DPerf^{BH}_{i,T2,k}(t) - 1)|$$

[841]:
```
deflator_2 = deflator_normalisation(deflator, 2)
MPerf2 = pd.DataFrame(spot_2_cum.values*deflator_2.values)
```

[842]:
```
test_statistics_3_2 = (MPerf2 - 1).mean(axis=1)
```

VISUAL INSPECTION
2-year Buy-and-hold strategy test

[844]:
```
MPerf2.head()
```

[844]:

	0	1	2	3	4	5	6	\
0	0.987804	0.995772	0.958710	1.023029	0.966309	1.024074	0.957349	
1	0.986066	0.992620	0.965574	1.017481	0.956770	1.039609	0.950018	
2	0.984107	0.987466	0.958412	1.026432	0.963630	1.030374	0.966157	
3	0.977909	0.985546	0.961410	1.036239	0.960092	1.020917	0.971313	
4	0.975116	0.987340	0.962555	1.039016	0.976373	1.015152	0.978943	

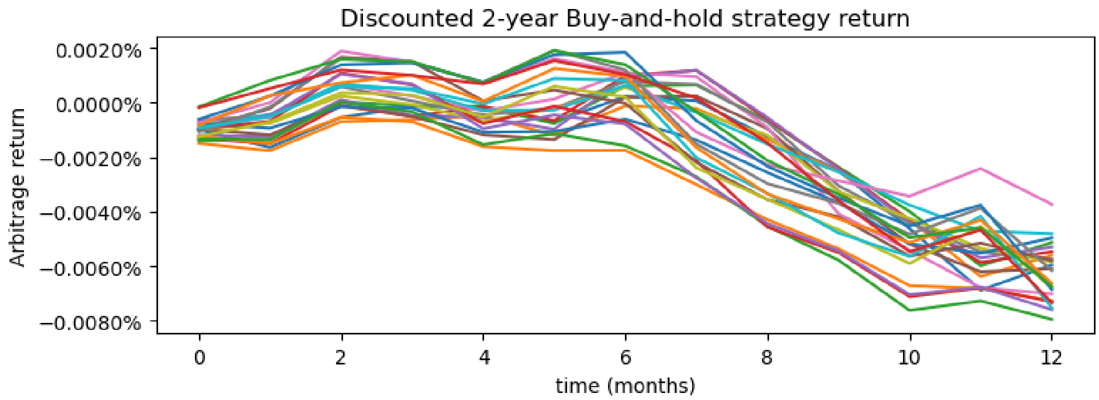
	7	8	9	…	490	491	492	493	\
0	1.010823	1.005664	1.022176	…	1.017128	0.972413	0.986842	1.015954	
1	1.010024	1.005465	1.026817	…	1.008156	0.978269	0.985488	1.006890	
2	1.000004	1.007448	1.031612	…	1.013869	0.973820	0.978974	1.004639	
3	1.003374	1.004735	1.033812	…	1.004674	0.978008	0.972903	1.020778	
4	0.981117	1.014926	1.036062	…	1.005777	0.986106	0.988721	1.023611	

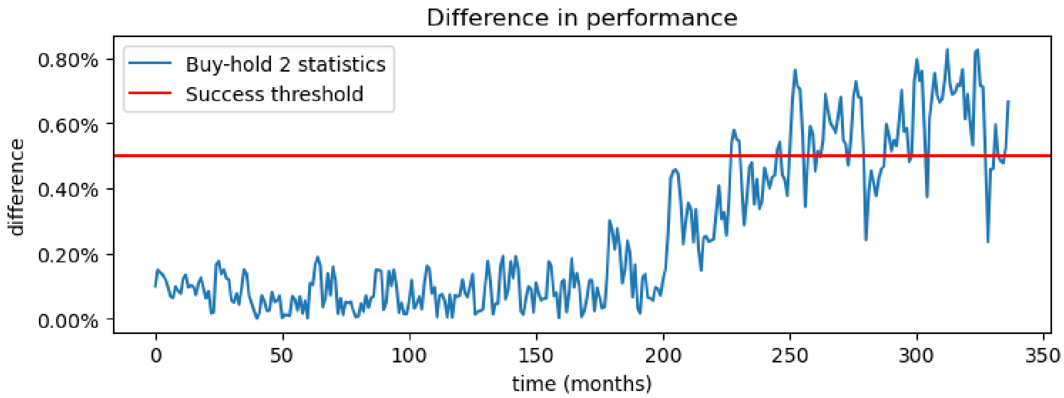
	494	495	496	497	498	499
0	0.993610	0.990125	0.999217	0.996569	1.014536	1.044291
1	0.995958	0.981113	0.998096	1.003136	1.013737	1.040211
2	0.957384	0.979795	0.988150	0.999873	0.995640	1.034115
3	0.948895	0.977734	0.988632	0.997856	0.996031	1.026409

```
4  0.957155  0.972586  0.977612  0.986324  1.001435  1.032183

[5 rows x 500 columns]
```

[845]:
```
plot_strategy_2 = month_decomposition(test_statistics_3_2, 2)
```

[846]:
```
x_data_label_1 = range(0,plot_strategy_2.shape[0],1)
x_data_label_2 = range(0,test_statistics_3_2.shape[0],1)

fig, (ax1, ax2) = plt.subplots(2,1)
ax1.plot(x_data_label_1, plot_strategy_2)
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())

ax1.set_xlabel("time (months)")
ax1.set_ylabel("Arbitrage return")
ax1.set_title('Discounted 2-year Buy-and-hold strategy return')

ax2.plot(x_data_label_2, np.abs(test_statistics_3_2)*100, label="Buy-hold 2␣
 ↪statistics")
ax2.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")

ax2.set_xlabel("time (months)")
ax2.set_ylabel("difference")
ax2.set_title('Difference in performance')
fig.tight_layout(h_pad=2)
ax2.legend()
ax2.yaxis.set_major_formatter(mtick.PercentFormatter())

fig.set_figwidth(8)
fig.set_figheight(6)

fig.tight_layout(h_pad=2)

plt.show()
```





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3.6 Test 3.2; Success criteria

The successful application of the success criteria marks the completion of the test

[848]:
```
result32 = success_test(np.abs(test_statistics_3_2),␣
↪test_martingale_threshold_max, test_martingale_threshold_mean)
```

```
Test passed
Test passed
```

3.6.1 Extra content

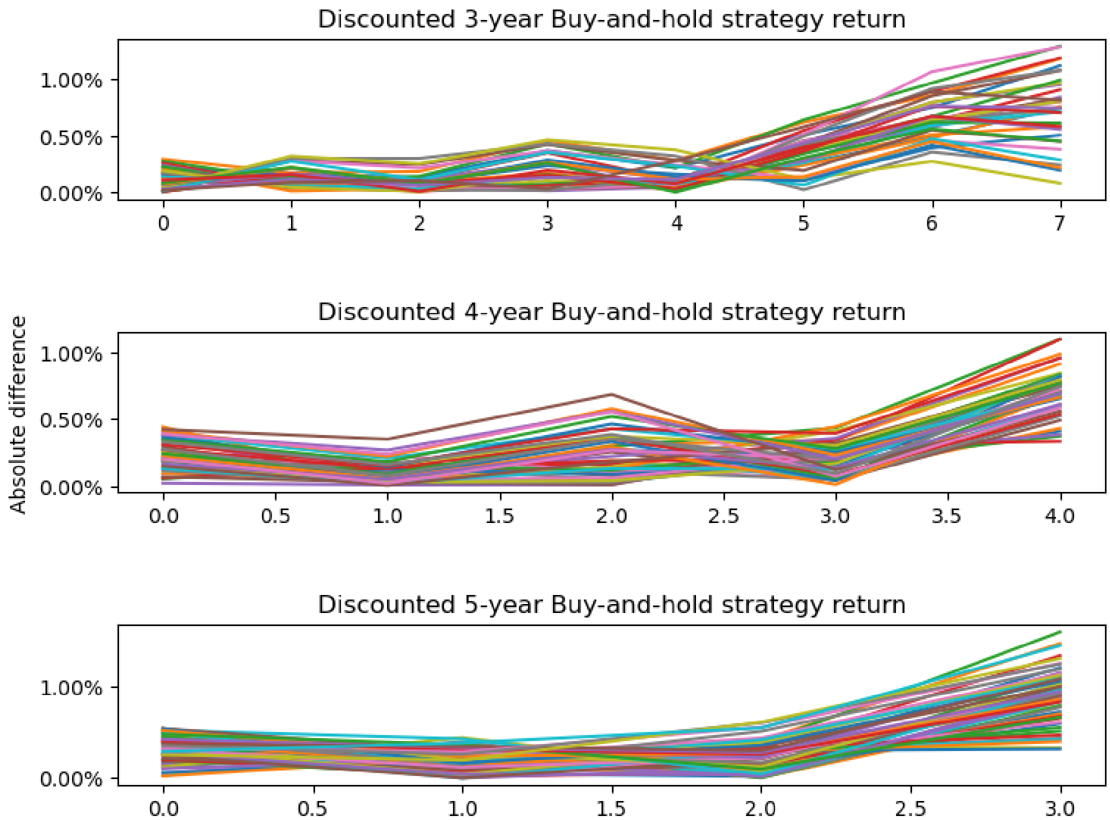
[850]:
```
deflator_3 = deflator_normalisation(deflator, 3)
MPerf3 = pd.DataFrame(spot_3_cum.values*deflator_3.values)
test3_diff_3 = (MPerf3 - 1).mean(axis=1)
plot_strategy_3 = month_decomposition(test3_diff_3, 3)
```

[851]:
```
deflator_4 = deflator_normalisation(deflator, 4)
MPerf4 = pd.DataFrame(spot_4_cum.values*deflator_4.values)
test3_diff_4 = (MPerf4 - 1).mean(axis=1)
```

```
plot_strategy_4 = month_decomposition(test3_diff_4, 4)
```

[852]:
```
deflator_5 = deflator_normalisation(deflator,5)
MPerf5 = pd.DataFrame(spot_5_cum.values*deflator_5.values)
test3_diff_5 = (MPerf5 - 1).mean(axis=1)
plot_strategy_5 = month_decomposition(test3_diff_5, 5)
```

[853]:
```
x_data_label_3 = range(0,plot_strategy_3.shape[0],1)
x_data_label_4 = range(0,plot_strategy_4.shape[0],1)
x_data_label_5 = range(0,plot_strategy_5.shape[0],1)

fig, (ax1, ax2, ax3) = plt.subplots(3,1)

ax1.plot(x_data_label_3, np.abs(plot_strategy_3)*100)
ax1.set_title('Discounted 3-year Buy-and-hold strategy return')
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())

ax2.plot(x_data_label_4, np.abs(plot_strategy_4)*100)
ax2.yaxis.set_major_formatter(mtick.PercentFormatter())
ax2.set_ylabel("Absolute difference")
ax2.set_title('Discounted 4-year Buy-and-hold strategy return')

ax3.plot(x_data_label_5, np.abs(plot_strategy_5)*100)
ax3.yaxis.set_major_formatter(mtick.PercentFormatter())
ax3.set_title('Discounted 5-year Buy-and-hold strategy return')

fig.tight_layout(h_pad=2)

fig.set_figwidth(8)
fig.set_figheight(6)

plt.show()
```



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3.7 Test 4; Total-return index

The total-return index in a risk-neutral framework has on average the same performance as the stochastic deflator asset.

$$DPerf^{I}_{i,1}(t) = I_{i,1}(t) * ZCB_{i,RF}(0,t)$$

Therefore, the performance of the index is a Martingale if the deflator asset is used as the numeraire:

$$E^{Q}[I_{i,1}(t)] =^{?} 1$$

Where:

- Q is the numeraire used in this test which is the deflator asset at time `t`

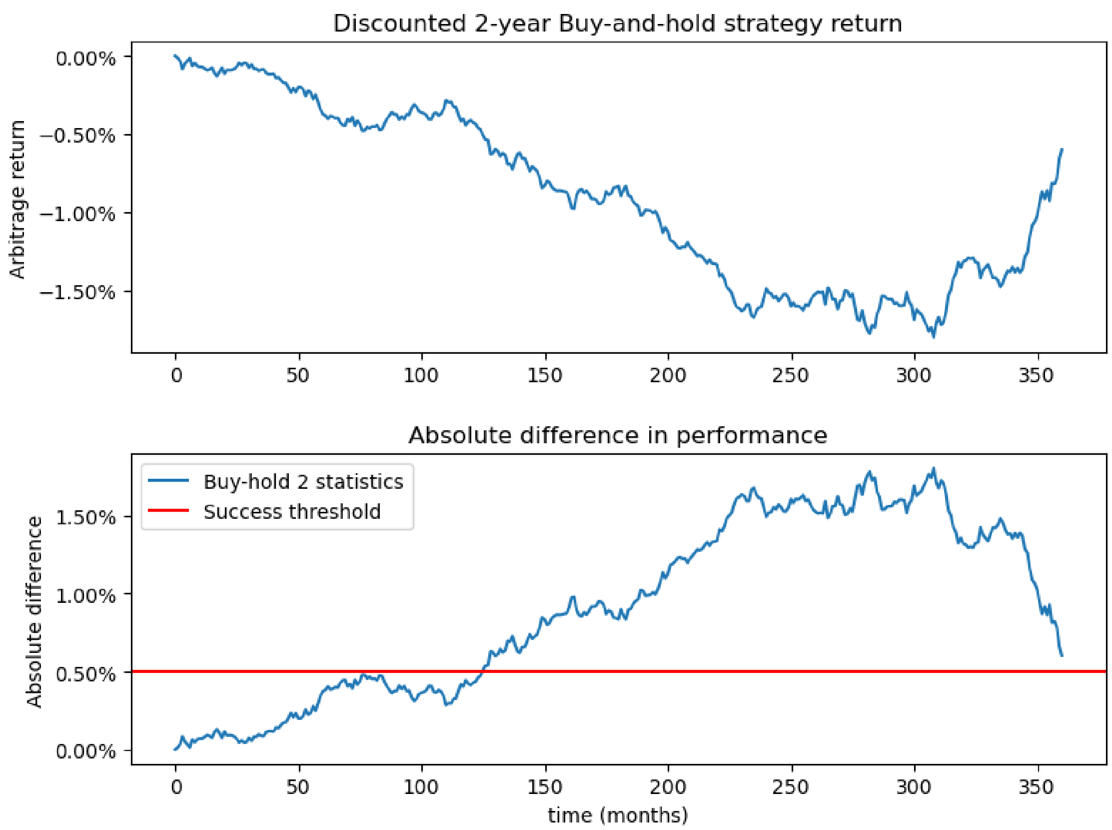
The two test statistics used are:

$$S_{AVERAGE} = \frac{1}{T}\sum_{t=0}^{T}\frac{1}{N}|\sum_{i=0}^{N} DPerf_{i,1}^{I}(t) - 1|$$

$$S_{MAX} = \max_{t}\frac{1}{N}|\sum_{i=0}^{N}(DPerf_{i,1}^{I}(t) - 1)|$$

[858]:
```
test_statistics_4 = pd.DataFrame(first_index.values * deflator.values).
 ↪mean(axis=1)-1
```

[860]:
```
x_data_label = range(0,test_statistics_4.shape[0],1)

fig, (ax1,ax2) = plt.subplots(2,1)
ax1.plot(x_data_label, test_statistics_4.values*100, color='tab:blue')

ax1.set_ylabel("Arbitrage return")
ax1.set_title('Discounted 2-year Buy-and-hold strategy return')
ax1.yaxis.set_major_formatter(mtick.PercentFormatter())

ax2.plot(x_data_label, np.abs(test_statistics_4)*100, label="Buy-hold 2␣
 ↪statistics")
ax2.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")

ax2.set_xlabel("time (months)")
ax2.set_ylabel("Absolute difference")
ax2.set_title('Absolute difference in performance')
fig.tight_layout(h_pad=2)
ax2.legend()
ax2.yaxis.set_major_formatter(mtick.PercentFormatter())

fig.set_figwidth(8)
fig.set_figheight(6)

fig.tight_layout(h_pad=2)

plt.show()
```



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3.8 Test 4; Success criteria

The successful application of the success criteria marks the completion of the test.

[862]:
```
result4 = success_test(np.abs(test_statistics_4),␣
    ↪test_martingale_threshold_max, test_martingale_threshold_mean)
```

```
Test failed
Test failed
```

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3.9 Conclusion

This script conducted tests on the output of the stochastic scenario output. The files selected are:

[864]:
```
file_det
```

[864]: 'TEST_DATA_DETERMINISTIC.csv'

[865]:
```
file_stoch
```

[865]: 'TEST_DATA_STOCHASTIC.csv'

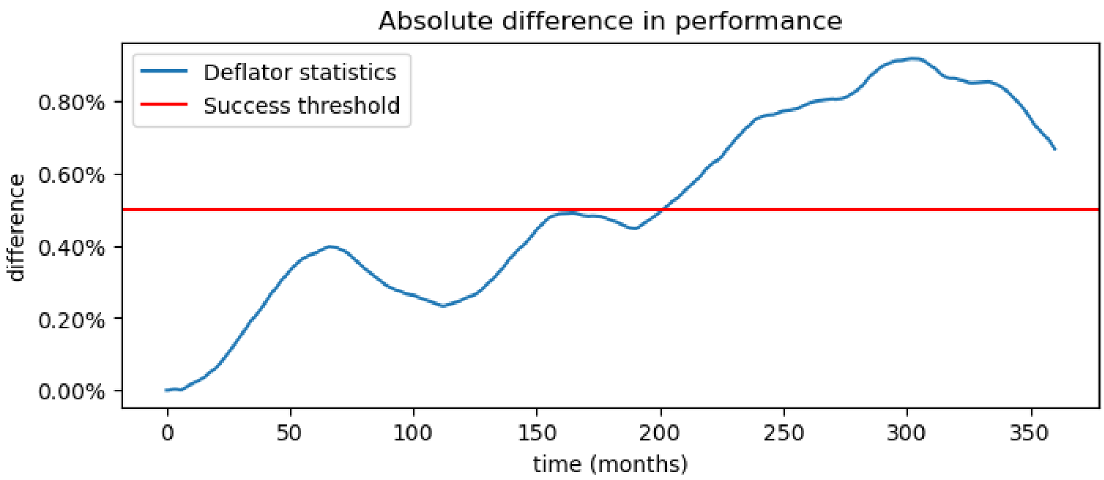
The currency selected for this run is

```
[867]: currency
```

[867]: 'EUR'

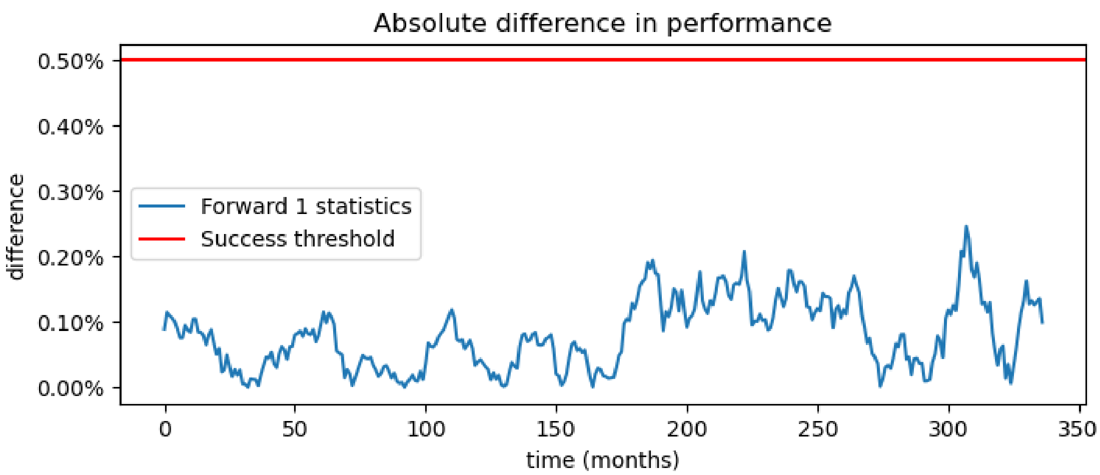
Test 1 compares the average performance of the stochastic deflator asset compared with the deterministic deflator asset.

```
[869]: x_data_label = range(0,deflator_det.shape[0],1)
fig, ax = plt.subplots(1,1)
ax.plot(x_data_label, test_statistics_1*100, label="Deflator statistics")
ax.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")

ax.set_xlabel("time (months)")
ax.set_ylabel("difference")
ax.set_title('Absolute difference in performance')
ax.legend()
ax.yaxis.set_major_formatter(mtick.PercentFormatter())

fig.set_figwidth(8)
fig.set_figheight(3)

plt.show()
```



Test 2 performs a Martingale test on a specific investment strategy using spot curves.The Forward return with a spread of 1-year strategy using 1- and 2-year spot rates consists of buying and selling ZCB bonds with 1- and 2-year term. In this example, this consists of buying (long position) a ZCB with a maturity of 2 years and selling (short position) a ZCB with a maturity of 1 year.

[871]:
```
x_data_label = range(0,test_statistics_2.shape[0],1)
fig, ax = plt.subplots(1,1)
ax.plot(x_data_label, np.abs(test_statistics_2)*100, label="Forward 1␣
 ↪statistics")
ax.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")
ax.legend()
ax.set_xlabel("time (months)")
ax.set_ylabel("difference")
ax.set_title('Absolute difference in performance')

ax.yaxis.set_major_formatter(mtick.PercentFormatter())
fig.set_figwidth(8)
fig.set_figheight(3)

plt.show()
```

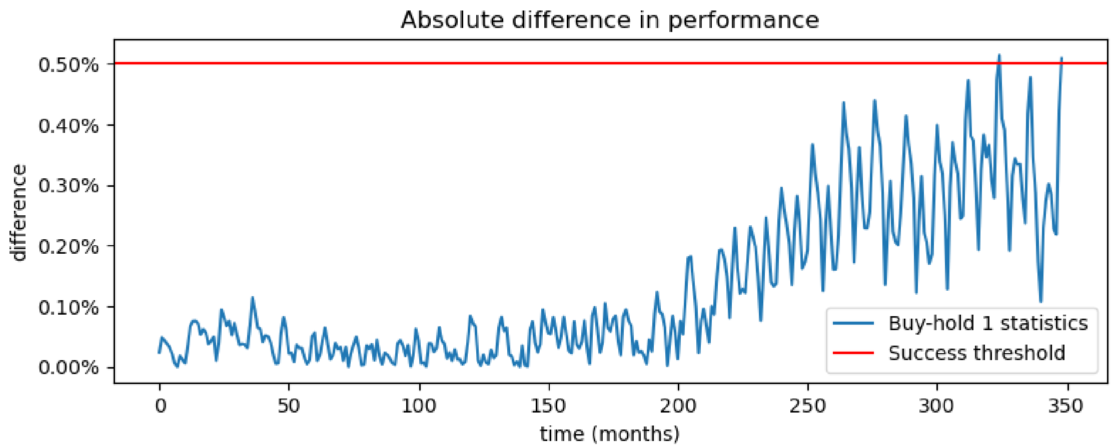


Tests 3.1 and 3.2 perform a Martingale test on the second financial strategy using spot rate curves. This Buy-and-hold strategy consists of investing all funds into ZCBs of a selected maturity, holding them until maturity and rolling all proceeds to buy another round of ZCB with the same selected maturity.

Test 3.1 uses the 1-year spot curve.

[874]:
```
x_data_label_2 = range(0,test_statistics_3_1.shape[0],1)

fig, ax = plt.subplots(1, 1)

ax.plot(x_data_label_2, np.abs(test_statistics_3_1)*100, label="Buy-hold 1␣
 ↪statistics")
```

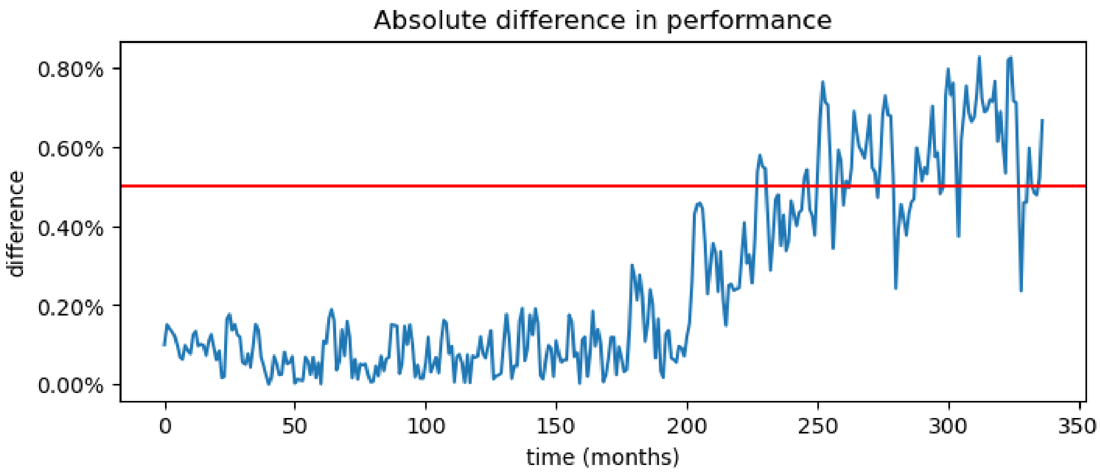
```
ax.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
↪threshold")
ax.legend()
ax.set_xlabel("time (months)")
ax.set_ylabel("difference")
ax.set_title('Absolute difference in performance')
fig.tight_layout(h_pad=2)

ax.yaxis.set_major_formatter(mtick.PercentFormatter())

fig.set_figwidth(8)
fig.set_figheight(3)

plt.show()
```



Test 3.2 uses the 2-year spot curve.

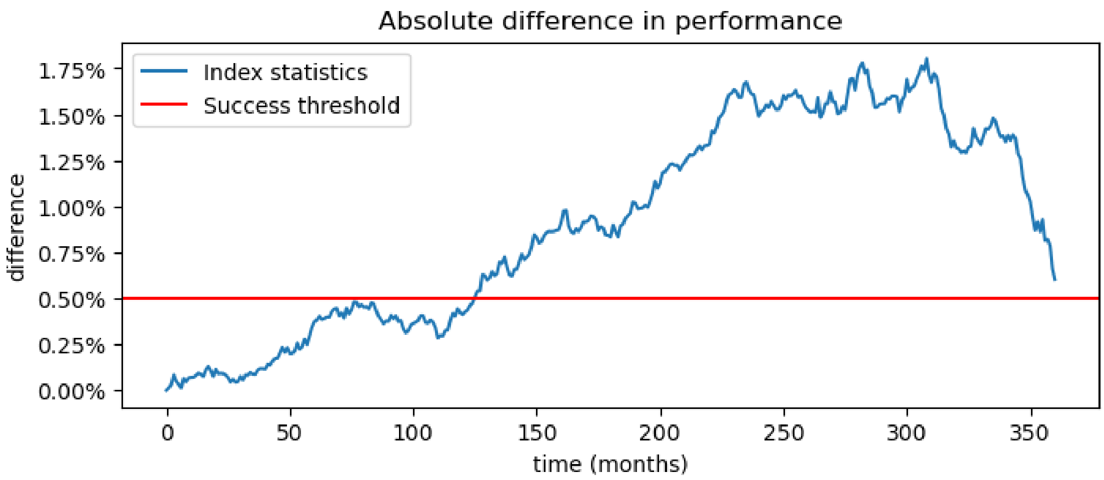
[876]:
```
x_data_label_2 = range(0,test_statistics_3_2.shape[0],1)

fig, ax = plt.subplots(1,1)

ax.plot(x_data_label_2, np.abs(test_statistics_3_2)*100, label="Buy-hold 2␣
↪statistics")
ax.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
↪threshold")

ax.set_xlabel("time (months)")
ax.set_ylabel("difference")
ax.set_title('Absolute difference in performance')

ax.yaxis.set_major_formatter(mtick.PercentFormatter())
```

```
fig.set_figwidth(8)
fig.set_figheight(3)

plt.show()
```



Test 4 performs the Martingale test using the total-return index. This test checks if the performance of the index using the deflator as numeraire is a Martingale.

[878]:
```
x_data_label = range(0,test_statistics_4.shape[0],1)

fig, ax = plt.subplots(1,1)

ax.plot(x_data_label, np.abs(test_statistics_4)*100, label="Index statistics")
ax.axhline(y = test_martingale_threshold_mean*100, color = 'r', label="Success␣
 ↪threshold")

ax.set_xlabel("time (months)")
ax.set_ylabel("difference")
ax.set_title('Absolute difference in performance')
ax.legend()
ax.yaxis.set_major_formatter(mtick.PercentFormatter())

fig.set_figwidth(8)
fig.set_figheight(3)

plt.show()
```



Given the success criteria, the tests show the following success rate:

[881]:
```
pd.DataFrame(data = [result1, result2, result31, result32, result4], columns =␣
↪["Mean test","Max test"],  \
             index= ["Deflator/deterministic","Martingale Forward return 12",␣
↪"Martingale Buy-and-hold 1",\
                     "Martingale Buy-and-hold 2","Martingale Total-return␣
↪index"])
```

[881]:

	Mean test	Max test
Deflator/deterministic	True	True
Martingale Forward return 12	True	True
Martingale Buy-and-hold 1	True	True
Martingale Buy-and-hold 2	True	True
Martingale Total-return index	False	False