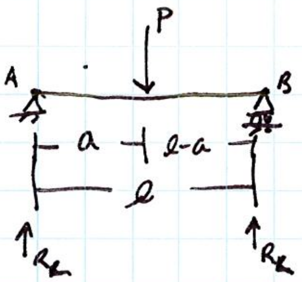
TIMOSHENK BM - POINT LOAD ANYWHERE IN SPAN

$$M_x = -EI\frac{\partial \theta}{\partial x} \qquad V_x = kAG\left(-\theta + \frac{\partial \Delta}{\partial x}\right)$$



$+\uparrow \Sigma V = 0 = R_L + R_R - P$

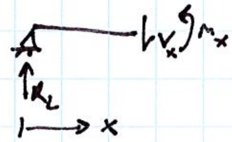
$R_R = P - R_L$

$\circlearrowleft \Sigma M_B = 0 = R_L \cdot l - P\cdot(l-a)$

$R_L = \frac{P(l-a)}{l} = \underline{\underline{P - \frac{Pa}{l}}}$

$R_R = P - P + \frac{Pa}{l} = \underline{\underline{+\frac{Pa}{l}}}$

$0 \le a$



$V_x = R_L$

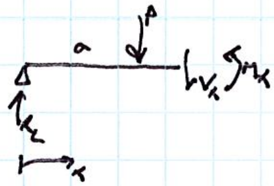
$M_x = R_L \cdot x$

$0 \le a$

$R_L = kAG\left(-\theta + \frac{\partial \Delta}{\partial x}\right)$ ①

$\Rightarrow \quad R_L x = -EI\frac{\partial \theta}{\partial x}$ ②

$a \le x \le l$



$V_x = R_L - P = -R_R$

$M_x = R_L \cdot x - P(x-a)$

$= \cancel{Px} - \frac{Pax}{l} - \cancel{Px} + Pa$

$= \frac{-Pax}{l} + Pa$

$a < x \le l$

$-R_R = kAG\left(-\theta + \frac{\partial \Delta}{\partial x}\right)$ ③

$\Rightarrow \quad \frac{-Pax}{l} + Pa = -EI\frac{\partial \theta}{\partial x}$ ④

② $\int -\frac{R_L x}{EI} = \theta -$

$-\frac{R_L x^2}{2EI} + C_1 = \theta$

① ~~$R_L \frac{R_L x^2 \cdot kAG}{EI}$~~

$\int \frac{R_L}{kAG} + \frac{R_L x^2}{2EI} + C_1 = \int \frac{\partial \Delta}{\partial x}$

$\frac{R_L x}{kAG} + \frac{R_L x^3}{6EI} + C_1 x + C_2 = \Delta$

④ $\int \frac{Pax}{EIl} - \frac{Pa}{EI} = \theta$

$\frac{Pax^2}{2EIl} - \frac{Pax}{EI} + C_3 = \theta$

③ $\int -\frac{R_R}{kAG} + \frac{Pax^2}{2EIl} - \frac{Pax}{EI} + C_3 \, dx = \Delta$

$-\frac{R_R x}{kAG} + \frac{Pax^3}{6EIl} - \frac{Pax^2}{2EI} + C_3 x + C_4 = \Delta$

$\Delta = 0 \quad , x = 0$

$$\boxed{C_2 = 0}$$

$\Delta = 0 \quad , x = l$

$$\frac{-R_R l}{kAG} + \frac{Pa\,l^{\cancel{3}\,2}}{6EI\cancel{l}} - \frac{Pa\,l^2}{2EI} + C_3 l + C_4 = 0$$

$$\frac{-R_L l}{kAG} - \frac{Pa\,l^2}{3EI} + C_3 l + C_4 = 0$$

$$\boxed{C_4 = \frac{+R_L l}{kAG} + \frac{Pa\,l^2}{3EI} - C_3 l \qquad \Rightarrow \frac{Pa}{kAG} + \cancel{\frac{Pa\,l^2}{3EI}} - \frac{Pa^3}{6EI} - \cancel{\frac{Pa\,l^2}{3EI}} = \underline{\frac{Pa}{kAG} - \frac{Pa^3}{6EI}}}$$

θ = CONSTANT @ $x = a$

$$\frac{-R_L a^2}{2EI} + C_1 = \frac{Pa^3}{2EIl} - \frac{Pa^2}{EI} + C_3$$

$$\frac{-Pa^2}{2EI} + \cancel{\frac{Pa^3}{2EIl}} + C_1 = \cancel{\frac{Pa^3}{2EIl}} - \frac{Pa^2}{EI} + C_3$$

$$\boxed{C_1 = \frac{-Pa^2}{2EI} + C_3 \qquad \Rightarrow \frac{-Pa^2}{2EI} + \underline{\frac{Pa^3}{6EIl}} + \frac{Pal}{3EI}}$$

Δ = CONSTANT @ $x = a$

$$\frac{R_L a}{kAG} - \frac{R_L a^3}{6EI} + C_1 a = \frac{-R_R a}{kAG} + \frac{Pa^4}{6EIl} - \frac{Pa^3}{2EI} + C_3 a + C_4$$

$$\cancel{\frac{Pa}{kAG}} - \cancel{\frac{Pa^2}{l\,kAG}} - \frac{Pa^3}{6EI} + \cancel{\frac{Pa^4}{6EIl}} - \cancel{\frac{Pa^3}{2EI}} + \cancel{C_3 a} = \cancel{\frac{-Pa^2}{l\,kAG}} + \cancel{\frac{Pa^4}{6EIl}} - \cancel{\frac{Pa^3}{2EI}} + \cancel{C_3 a} + \cancel{\frac{Pa}{kAG}} + \frac{Pa\,l^2}{3EI} - C_3 l$$

$$-\frac{Pa^3}{6EI} - \frac{Pa\,l^2}{3EI} = -C_3 l$$

$$\boxed{\underline{\frac{Pa^3}{6EIl}} + \frac{Pal}{3EI} = C_3}$$

$\underline{a = \frac{\ell}{2} \; ; \; x = \frac{\ell}{2}}$

$R_L = P - \frac{P}{2} = P/2$

**TIMOSHENKO & YOUNG
MID SPAN POINT
LOAD EXAMPLE**

$$C_4 = \frac{P\ell}{2kAG} - \frac{P\ell^3}{48EI}$$

$$C_3 = \frac{P\ell^{3^2}}{48EI\ell} + \frac{P\ell^2}{6EI} = \frac{3P\ell^2}{16EI}$$

$$C_1 = \frac{-P\ell^2}{8EI} + \frac{3P\ell^2}{16EI} = \frac{P\ell^2}{16EI}$$

EQ. (1)

$$\frac{R_L\ell}{2kAG} - \frac{R_L\ell^3}{48EI} + \frac{P\ell^3}{32EI} + 0 = \Delta$$

$$\frac{P\ell}{2kAG} - \frac{P\ell}{4kAG} - \frac{P\ell^3}{96EI} + \frac{P\ell^3}{32EI} = \Delta$$

$$\boxed{\frac{P\ell}{4kAG} + \frac{P\ell^3}{48EI} = \Delta}$$

EQ. (3)

$$\frac{-R_R\ell}{2kAG} + \frac{P\ell^3}{96EI} - \frac{P\ell^3}{16EI} + \frac{P\ell^3}{96EI} + \frac{P\ell^3}{12EI} + \frac{P\ell}{2kAG} - \frac{P\ell^3}{48EI} = \Delta$$

$$\frac{-P\ell}{4kAG} + \frac{P\ell^3}{48EI} + \frac{P\ell}{2kAG} = \Delta$$

$$\boxed{\frac{P\ell}{4kAG} + \frac{P\ell^3}{48EI} = \Delta}$$

= ✓

$I = A r^2$ or $A = \frac{I}{r^2}$

$r^2 = \frac{h^2}{12}$ ← For Rectangle

$A = \frac{12I}{h^2}$

$$\frac{P\ell^3 h^2}{4k\,12\,I\,G\,\ell^2} + \frac{P\ell^3}{48EI} = \Delta$$

$$\frac{P\ell^3 h^2}{48kG\ell^2 I} + \frac{P\ell^3}{48EI}$$

$$\frac{P\ell^3}{48I}\left(\frac{h^2}{kGI\ell^2} + \frac{1}{EI}\right) \Rightarrow \frac{P\ell^3}{48EI}\left(\underset{\Delta_s}{\frac{h^2E}{kG\ell^2}} + \underset{\Delta_b}{1}\right)$$

For $k = \frac{2}{3}$, $E/G = 2.5$

$$\hookrightarrow \frac{P\ell^3}{48EI}\left(\underset{\Delta_s}{3.75\frac{h^2}{\ell^2}} + \underset{\Delta_b}{1}\right)$$

UNIT FORCE METHOD

$$\frac{1}{kAG}\int_0^{\ell} V v \, dx$$

REAL

$a = \ell/2$

$P = P$

$$V_x = \begin{cases} P - \dfrac{Pa}{\ell} \\ -\dfrac{Pa}{\ell} \end{cases} = \begin{cases} P - \dfrac{P}{2} = P/2 \\ -P/2 \end{cases}$$

VIRTUAL

$a = \ell/2$

$P = 1$

$$v_x = \begin{cases} 1 - 1/2 = 1/2 \\ -1/2 \end{cases}$$

$$\cancel{\frac{1}{kAG}\int_0^{\ell} \frac{P}{2} - \frac{Pa}{2\ell}}$$

$$\frac{1}{kAG}\int_0^{\ell/2} \frac{P}{2}\left(\frac{1}{2}\right) dx + \frac{1}{kAG}\int_{\ell/2}^{\ell} -\frac{P}{2}\left(-\frac{1}{2}\right) dx = 1 \cdot \Delta$$

$$\frac{1}{kAG}\left[\frac{Px}{4}\right]_0^{\ell/2} + \frac{1}{kAG}\left[\frac{P}{4}x\right]_{\ell/2}^{\ell} = 1 \cdot \Delta$$

$$\cancel{\frac{P\ell}{8kAG}} + \frac{P\ell}{4kAG} - \cancel{\frac{P\ell}{8kAG}} = 1 \cdot \Delta$$

$$\frac{P\ell}{4kAG} = 1 \cdot \Delta$$