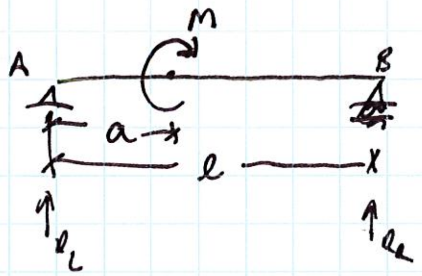
TIMOSHENKO BM - POINT MOMENT

$$M_x = -EI\frac{\partial\theta}{\partial x} \qquad V_x = kAG\left(-\theta + \frac{d\Delta}{dx}\right)$$



$$\uparrow_+ \Sigma V = R_L + R_R = 0 \Rightarrow R_L = -R_R \Rightarrow R_R = \frac{M}{l}$$

$$\Sigma M_B = 0 = M + R_L l$$

$$R_L = \frac{-M}{l}$$

$0 \le x \le a$

$$V_x = \frac{-M}{l} = kAG\left(-\theta + \frac{d\Delta}{dx}\right) \quad ①$$

$$M_x = \frac{-Mx}{l} = -EI\frac{\partial\theta}{\partial x} \quad ②$$

$a \le x \le l$

$$V_x = \frac{-M}{l} = kAG\left(-\theta + \frac{\partial\Delta}{\partial x}\right) \quad ③$$

$$M_x = \frac{-Mx}{l} + M = -EI\frac{\partial\theta}{\partial x} \quad ④$$

②
$$\int \frac{Mx}{EIl}\,dx = \int d\theta$$

$$\frac{Mx^2}{2EIl} + C_1 = \theta$$

①
$$\int \frac{-M}{kAGl} + \frac{Mx^2}{2EIl} + C_1\,dx = \int d\Delta$$

$$\frac{-Mx}{kAGl} + \frac{Mx^3}{6EIl} + C_1x + C_2 = \Delta$$

④
$$\int \frac{+Mx}{EIl} - \frac{M}{EI}\,dx = \int d\theta$$

$$\frac{Mx^2}{2EIl} - \frac{Mx}{EI} + C_3 = \theta$$

③
$$\int \frac{-M}{kAGl} + \frac{Mx^2}{2EIl} - \frac{Mx}{EI} + C_3\,dx = \int d\Delta$$

$$\frac{-Mx}{kAGl} + \frac{Mx^3}{6EIl} - \frac{Mx^2}{2EI} + C_3x + C_4 = \Delta$$

$\Delta = 0 \;@\; x = 0$

$$\boxed{C_2 = 0}$$

$\Delta = 0 \;@\; x = l$

$$\frac{-M}{kAG} + \frac{Ml^2}{6EI} - \frac{Ml^2}{2EI} + C_3l + C_4 = 0 \Rightarrow C_4 = \frac{M}{kAG} + \frac{Ml^2}{3EI} - C_3l$$

$\underline{\Theta = \text{CONSTANT} \quad @ \ x = a}$

$$\frac{Ma^2}{2EI\ell} + C_1 = \frac{Mx^2}{2EI\ell} - \frac{Ma}{EI} + C_3$$

$$C_1 = \frac{-Ma}{EI} + C_3 \quad \Rightarrow \quad \boxed{C_1 = \frac{-Ma}{EI} - \frac{Ma^2}{2EI\ell} + \frac{M}{kAG\ell} + \frac{M\ell}{3EI}}$$

$\underline{\Delta = \text{CONSTANT} \quad @ \ x = a}$

$$\cancel{\frac{-Ma}{kAG\ell}} + \cancel{\frac{Ma^3}{6EI\ell}} + C_1 a = \cancel{\frac{-Ma}{kAG\ell}} + \cancel{\frac{Mx^3}{6EI\ell}} - \frac{Ma^2}{2EI} + \cancel{C_3 a} + \frac{M}{kAG} + \frac{M\ell^2}{3EI} - C_3\ell$$

$$-\frac{Ma^2}{EI} + \cancel{C_3 a} = -\frac{Ma^2}{2EI} + \frac{M}{kAG} + \frac{M\ell^2}{3EI} - C_3\ell$$

$$\frac{Ma^2}{2EI} - \frac{M}{kAG} - \frac{M\ell^2}{3EI} = -C_3\ell$$

$$\boxed{-\frac{Ma^2}{2EI\ell} + \frac{M}{kAG\ell} + \frac{M\ell}{3EI} = C_3}$$

$$C_4 = \cancel{\frac{M}{kAG}} + \cancel{\frac{M\ell^2}{3EI}} + \frac{Ma^2}{2EI} - \cancel{\frac{M}{kAG}} - \cancel{\frac{M\ell^2}{3EI}}$$

$$\boxed{C_4 = \frac{Ma^2}{2EI}}$$

$\underline{0 \leq x \leq a}$

$$① \quad \Delta = \cancel{\frac{-Mx}{kAG\ell}} + \frac{Mx^3}{6EI\ell} - \frac{Max}{EI} - \frac{Ma^2x}{2EI\ell} + \cancel{\frac{Mx}{kAG\ell}} + \frac{M\ell x}{3EI}$$

$$= \frac{Mx^3}{6EI\ell} - \frac{Max}{EI} - \frac{Ma^2x}{2EI\ell} + \frac{M\ell x}{3EI}$$

$$② \quad \Theta = \frac{Mx^2}{2EI\ell} - \frac{Ma}{EI} - \frac{Ma^2}{2EI\ell} + \frac{M}{kAG\ell} + \frac{M\ell}{3EI}$$

$\underline{a \leq x \leq \ell}$

$$③ \quad \Delta = \cancel{\frac{-Mx}{kAG\ell}} + \frac{Mx^3}{6EI\ell} - \frac{Mx^2}{2EI} - \frac{Ma^2x}{2EI\ell} + \cancel{\frac{Mx}{kAG\ell}} + \frac{M\ell x}{3EI} + \frac{Ma^2}{2EI}$$

$$= \frac{Mx^3}{6EI\ell} - \frac{Mx^2}{2EI} - \frac{Ma^2x}{2EI\ell} + \frac{M\ell x}{3EI} + \frac{Ma^2}{2EI}$$

← NOTE: NO $\frac{C}{kAG}$ TERMS $\therefore$ Δ DUE TO SHEAR = 0 FOR POINT MOMENT.

$$④ \quad \Theta = \frac{Mx^2}{2EI\ell} - \frac{Mx}{EI} - \frac{Ma^2}{2EI\ell} + \frac{M}{kAG\ell} + \frac{M\ell}{3EI}$$

* NOTE: SIGN CHOSEN FOR DIRECTION OF M RESULTS IN $+\Delta$ IN THE $-Y$ DIRECTION