

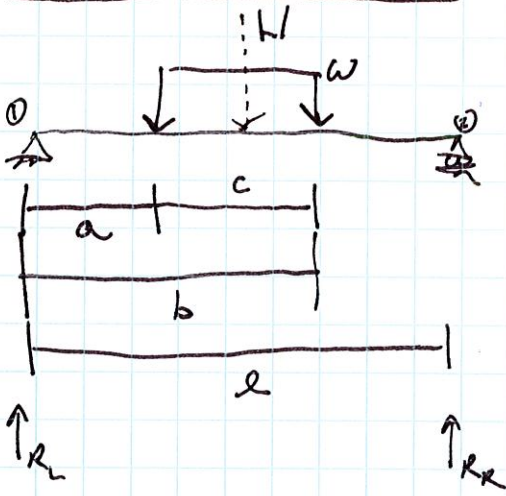
TIMOSHENKO BM - UDL

$$M_x = -EI \frac{d\theta}{dx}$$

$$V_x = kAG (-\theta + \frac{d\Delta}{dx})$$

$$W = w \cdot c = w(b-a) = wb - wa$$

$$c = b - a$$



$$\uparrow + \Sigma V = 0 = R_L + R_R - wb + wa$$

$$R_R = wb - wa - R_L = \frac{-a^2 w}{2L} + \frac{b^2 w}{2L}$$

$$\circlearrowleft \Sigma M_{\odot} = 0 = R_L \cdot l - (wb - wa)(l - \frac{a+b}{2})$$

$$R_L = \frac{a^2 w}{2L} + \frac{abw}{L} + \frac{b^2 w}{2L} - bw$$

$$= \frac{a^2 w}{2L} + \frac{abw}{L} + \frac{b^2 w}{2L} - bw$$

$$= \frac{a^2 w}{2L} - \frac{b^2 w}{2L} - wa + wb$$

$$\underline{x \leq a}$$

$$V_x = R_L = \frac{a^2 w}{2L} - \frac{b^2 w}{2L} - wa + wb = kAG (-\theta + \frac{d\Delta}{dx}) \quad (1)$$

$$M_x = R_L x = \frac{a^2 wx}{2L} - \frac{b^2 wx}{2L} - wax + wbx = -EI \frac{d\theta}{dx} \quad (2)$$

$$\underline{a \leq x \leq b}$$

$$V_x = R_L - wx + wa = \frac{a^2 w}{2L} - \frac{b^2 w}{2L} + wb - wx = kAG (-\theta + \frac{d\Delta}{dx}) \quad (3)$$

$$M_x = R_L x - \frac{wx^2}{2} + wax + C_1 = \frac{a^2 wx}{2L} - \frac{b^2 wx}{2L} + wbx - \frac{wx^2}{2} + C_1 = -EI \frac{d\theta}{dx} \quad (4)$$

$$\underline{b \leq x \leq l}$$

$$V_x = R_L - W = R_R = \frac{a^2 w}{2L} + \frac{b^2 w}{2L} = kAG (-\theta + \frac{d\Delta}{dx}) \quad (5)$$

$$M_x = -R_L x + C_2 = \frac{a^2 wx}{2L} + \frac{b^2 wx}{2L} + C_2 = -EI \frac{d\theta}{dx} \quad (6)$$

$$\underline{x = a \quad M = \text{CONSTANT}}$$

$$\frac{a^3 w}{2L} - \frac{ab^2 w}{2L} - a^2 w + abw = \frac{a^3 w}{2L} - \frac{ab^2 w}{2L} + abw - \frac{a^2 w}{2} + C_1$$

$$C_1 = -a^2 w + \frac{a^2 w}{2} = -\frac{a^2 w}{2}$$

$$\underline{x = L \quad M = 0}$$

$$\frac{a^2 w L}{2L} + \frac{b^2 w L}{2L} + C_2 = 0 \Rightarrow C_2 = -\frac{a^2 w}{2} + \frac{b^2 w}{2}$$

$$(2) \int \frac{-a^2 wx}{2EI L} + \frac{b^2 wx}{2EI L} + \frac{wax}{EI} - \frac{wbx}{EI} dx = \int d\theta$$

$$-\frac{a^2 wx^2}{4EI L} + \frac{b^2 wx^2}{4EI L} + \frac{wax^2}{2EI} - \frac{wbx^2}{2EI} + C_3 = \theta$$

$$(1) \int \frac{a^2 w}{2LkAG} - \frac{b^2 w}{2LkAG} - \frac{wa}{kAG} + \frac{wb}{kAG} - \frac{a^2 wx^2}{4EI L} + \frac{b^2 wx^2}{4EI L} + \frac{wax^2}{2EI} - \frac{wbx^2}{2EI} + C_3 dx = d\Delta$$

$$\frac{a^2 wx}{2LkAG} - \frac{b^2 wx}{2LkAG} - \frac{wax}{kAG} + \frac{wbx}{kAG} - \frac{a^2 wx^3}{12EI L} + \frac{b^2 wx^3}{12EI L} + \frac{wax^3}{6EI} - \frac{wbx^3}{6EI} + C_3 x + C_4 = \Delta$$

$$(4) \int \frac{-a^2 wx}{2EI L} + \frac{b^2 wx}{2EI L} - \frac{wbx}{EI} + \frac{wx^2}{2EI} + \frac{a^2 w}{2EI} dx = \int d\theta$$

$$-\frac{a^2 wx^2}{4EI L} + \frac{b^2 wx^2}{4EI L} - \frac{wbx^2}{2EI} + \frac{wx^3}{6EI} + \frac{a^2 wx}{2EI} + C_5 = \theta$$

$$(3) \int \frac{a^2 w}{2LkAG} - \frac{b^2 w}{2LkAG} + \frac{wb}{kAG} - \frac{wx}{kAG} - \frac{a^2 wx^2}{4EI L} + \frac{b^2 wx^2}{4EI L} - \frac{wbx^2}{2EI} + \frac{wx^3}{6EI} + \frac{a^2 wx}{2EI} + C_5 dx = \int d\Delta$$

$$\frac{a^2 wx}{2LkAG} - \frac{b^2 wx}{2LkAG} + \frac{wbx}{kAG} - \frac{wx^2}{2kAG} - \frac{a^2 wx^3}{12EI L} + \frac{b^2 wx^3}{12EI L} - \frac{wbx^3}{6EI} + \frac{wx^4}{24EI} + \frac{a^2 wx^2}{4EI} + C_5 x + C_6 = \Delta$$

$$(6) \int \frac{-a^2 wx}{2EI L} + \frac{b^2 wx}{2EI L} + \frac{a^2 w}{2EI} - \frac{b^2 w}{2EI} dx = \int d\theta$$

$$-\frac{a^2 wx^2}{4EI L} + \frac{b^2 wx^2}{4EI L} + \frac{a^2 wx}{2EI} - \frac{b^2 wx}{2EI} + C_7 = \theta$$

$$(5) \int \frac{+a^2 w}{2LkAG} - \frac{b^2 w}{2LkAG} + \frac{a^2 wx^2}{4EI L} + \frac{b^2 wx^2}{4EI L} + \frac{a^2 wx}{2EI} - \frac{b^2 wx}{2EI} + C_7 dx = \int d\Delta$$

$$\frac{+a^2 wx}{2LkAG} - \frac{b^2 wx}{2LkAG} + \frac{a^2 wx^3}{12EI L} + \frac{b^2 wx^3}{12EI L} + \frac{a^2 wx^2}{4EI} - \frac{b^2 wx^2}{4EI} + C_7 x + C_8 = \Delta$$

$$\Delta = 0 \text{ e } x = 0$$

$$C_4 = 0$$

$$\Delta = 0 \text{ e } x = L$$

$$\frac{a^2 w L}{2kAG} - \frac{b^2 w L}{2kAG} - \frac{a^2 w L^2}{12EI} + \frac{b^2 w L^2}{12EI} + \frac{a^2 w L^2}{4EI} - \frac{b^2 w L^2}{4EI} + C_7 L + C_8 = 0$$

$$C_8 = -\frac{a^2 w}{2kAG} + \frac{b^2 w}{2kAG} - \frac{a^2 w L^2}{6EI} + \frac{b^2 w L^2}{6EI} - C_7 L$$

$$\Theta = \text{CONSTANT} \text{ e } x = a$$

$$\frac{-a^4 w}{4EI} + \frac{a^2 b^2 w}{4EI} + \frac{w a^3}{2EI} - \frac{w b a^2}{2EI} + C_3 = \frac{-a^4 w}{4EI} + \frac{a^2 b^2 w}{4EI} - \frac{w b a^2}{2EI} + \frac{w b a^3}{6EI} + \frac{w a^3}{2EI} + C_5$$

$$C_3 = \frac{a^3 w}{6EI} + C_5$$

$$\Delta = \text{CONSTANT} \text{ e } x = a$$

$$\frac{a^3 w}{2kAG} - \frac{b^2 w}{2kAG} - \frac{a^2 w}{kAG} + \frac{a b w}{kAG} - \frac{a^5 w}{12EI} + \frac{a^3 b^2 w}{12EI} + \frac{a^4 w}{6EI} - \frac{a^3 b w}{6EI} + \frac{a^4 w}{6EI} + C_5 a =$$

$$\frac{a^3 w}{2kAG} - \frac{b^2 w}{2kAG} + \frac{a b w}{kAG} - \frac{a^2 w}{2kAG} - \frac{a^5 w}{12EI} + \frac{a^3 b^2 w}{12EI} - \frac{a^3 b w}{6EI} + \frac{a^4 w}{24EI} + \frac{a^4 w}{4EI} + \frac{C_5 a}{5} + C_6$$

$$-\frac{a^2 w}{2kAG} + \frac{a^4 w}{24EI} = C_6$$

$$\Theta = \text{CONSTANT} \text{ e } x = b$$

$$\frac{-a^2 b^2 w}{4EI} + \frac{b^4 w}{4EI} - \frac{w b^3}{2EI} + \frac{w b^3}{6EI} + \frac{a^2 b w}{2EI} + C_5 = \frac{-a^2 b^2 w}{4EI} + \frac{b^4 w}{4EI} + \frac{a^2 b w}{2EI} - \frac{b^2 w}{2EI} + C_7$$

$$C_5 + \frac{b^3 w}{6EI} = C_7$$

$$\Delta = \text{CONSTANT } e \cdot x = b$$

$$\frac{a^2 b^3}{2EI KAG} - \frac{b^3}{2EI KAG} + \frac{\omega b^2}{KAG} - \frac{\omega b^2}{2EI KAG} - \frac{a^2 b^3}{12EI L} + \frac{b^3}{12EI L} - \frac{\omega b^4}{6EI} + \frac{\omega b^4}{24EI} + \frac{a^2 b^3}{4EI} + \cancel{C_6} + C_6 = \frac{-a^2 \omega}{2EI KAG} + \frac{a^4 \omega}{24EI}$$

$$\frac{a^2 b^3}{2EI KAG} - \frac{b^3}{2EI KAG} - \frac{a^2 b^3}{12EI L} + \frac{b^3}{12EI L} + \frac{a^2 b^3}{4EI} - \frac{b^4}{4EI} + C_7 b + C_8 \rightarrow C_5 b + \frac{b^4 \omega}{6EI}$$

$$\frac{\omega b^2}{2EI KAG} - \frac{b^4 \omega}{24EI} - \frac{a^2 \omega}{2EI KAG} + \frac{a^4 \omega}{24EI} = C_8$$

$$\frac{\omega b^3}{2EI KAG} - \frac{a^2 \omega}{2EI KAG} - \frac{b^4 \omega}{24EI L} + \frac{a^4 \omega}{24EI L} = \frac{-a^2 \omega}{2EI KAG} + \frac{b^2 \omega}{2EI KAG} - \frac{a^2 \omega L}{6EI} + \frac{b^2 \omega L}{6EI} - C_7$$

$$C_7 = \frac{-a^2 \omega L}{6EI} + \frac{b^2 \omega L}{6EI} + \frac{b^4 \omega}{24EI L} - \frac{a^4 \omega}{24EI L}$$

$$C_5 = \frac{-a^2 \omega L}{6EI} + \frac{b^2 \omega L}{6EI} + \frac{b^4 \omega}{24EI L} - \frac{a^4 \omega}{24EI L} - \frac{b^3 \omega}{6EI}$$

$$C_3 = \frac{-a^2 \omega L}{6EI} + \frac{b^2 \omega L}{6EI} + \frac{b^4 \omega}{24EI L} - \frac{a^4 \omega}{24EI L} - \frac{b^3 \omega}{6EI} + \frac{a^3 \omega}{6EI}$$