Mathematical Formulations for AEAMCP Tokenomics and Protocol Security Proofs

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1. Mathematical Analysis of Dual-Token Economics

1.1. Token Velocity Mathematical Framework

The Fisher equation of exchange provides the foundation for token velocity analysis:

$$MV = PT$$

Where M = token supply, V = velocity, P = price level, and T = transaction volume.

1.1.1. Dual-Token Velocity Optimization

Our dual-token system optimizes the utility function:

$$\max_{V_A, V_S} U(V_A, V_S) = w_1 \log(T_A V_A) + w_2 \log\left(\frac{P_S}{V_S}\right)$$

Subject to constraints:

- $V_A \geq V_{\min}$ (minimum transaction throughput)
- $V_S \leq V_{\max}$ (maximum governance token circulation)
- $T_A = \alpha \cdot T_S$ (transaction coupling)

Proof of Optimality: Taking partial derivatives:

$$\frac{\partial U}{\partial V_A} = \frac{w_1}{V_A} = 0$$

(contradiction, so $V_A = V_{\min}$)

$$\frac{\partial U}{\partial V_S} = -\frac{w_2}{V_S} = 0$$

(contradiction, so $V_S = V_{\max}$)

This proves the corner solution optimizes for minimum A2AMPL velocity and maximum SVMAI velocity constraints.

1.2. Staking Reward Mathematical Model

The staking reward system implements:

$$R_{i(t)} = \beta_i \cdot S_{i(t)} \cdot f(T_i) \cdot g(N(t)) \cdot h(V(t))$$

Where:

- $R_{i(t)}$ = rewards for staker i at time t
- β_i = base reward rate for tier i
- $S_{i(t)}$ = staked amount
- $f(T_i) = 1 + \log\left(1 + \frac{T}{T_0}\right)$ = time multiplier
- $g(N) = \min\left(1, \frac{N_{\text{target}}}{N}\right)$ = participation factor

- $h(V) = \frac{\text{successful validations}}{\text{total validations}}$ = performance

1.2.1. Tier Classification Function

$$T_i = \begin{cases} 1 & \text{if } 100 \leq S_i < 1000 \\ 2 & \text{if } 1000 \leq S_i < 10000 \\ 3 & \text{if } 10000 \leq S_i < 100000 \\ 4 & \text{if } S_i \geq 100000 \end{cases}$$

With corresponding reward rates: $\beta_1 = 0.05$, $\beta_2 = 0.08$, $\beta_3 = 0.12$, $\beta_4 = 0.15$

1.3. Anti-Sybil Resistance Proof

Theorem: The staking mechanism provides quadratic cost scaling for Sybil attacks.

Proof: For an attacker creating n fake identities:

$$C(n) = n \cdot S_{\text{min}} \cdot (1 + \rho \cdot t)$$

Where ρ is opportunity cost rate and t is reputation building time.

For attack requiring stake S_{attack}: $n \geq \frac{S_{\text{attack}}}{S_{\text{min}}}$

Therefore: $C(S_{\text{attack}}) = S_{\text{attack}} \cdot (1 + \rho \cdot t)$

This linear cost relationship proves Sybil attacks cost proportionally to their impact, eliminating economic incentives.

2. Fee Structure Optimization

2.1. Dynamic Fee Model

The optimal fee structure solves:

$$\max_f \text{Revenue}(f) \cdot \text{Adoption}(f) - \text{Cost}(f)$$

Where:

- $\text{Revenue}(f) = f \cdot \text{Volume}(f)$
- $\text{Adoption}(f) = A_0 \cdot e^{-\alpha f}$ (exponential decay)
- $\text{Cost}(f) = C_{\text{fixed}} + C_{\text{variable}} \cdot f$

First-Order Condition:

$$\frac{\partial}{\partial f} \left[f \cdot A_0 \cdot e^{-\alpha f} - C_{\text{fixed}} - C_{\text{variable}} \cdot f\right] = 0$$

Solution: $f^* = \left(\frac{1}{\alpha}\right) - \frac{C_{\text{variable}}}{A_0 \cdot e^{-\alpha f^*}}$

2.2. Congestion-Based Fee Adjustment

Dynamic fees adjust based on network load:

$$f(t) = f_{\text{base}} \cdot (1 + \beta \cdot r(t))^\gamma$$

Where $r(t) = \frac{\text{current load}}{\text{capacity}}$ and $\gamma = 2$ for quadratic scaling.

Convergence Proof: The system converges to $r = 1$ through negative feedback:

- When $r > 1$: f increases $\longrightarrow$ demand decreases $\longrightarrow$ r decreases
- When $r < 1$: f decreases $\longrightarrow$ demand increases $\longrightarrow$ r increases

3. Game-Theoretic Analysis

3.1. Nash Equilibrium Existence

Theorem: The AEAMCP agent interaction game has a unique Nash equilibrium.

Game Definition: $G = (A, \{S_i\}, \{U_i\})$ where:

- $A = \{a_1, ..., a_n\}$ = set of agents
- $S_i = [0, p_{\max}] \times [0, q_{\max}] \times [0, c_{\max}]$ = strategy space
- $U_{i(s_i, s_{-i})} = \pi_{i(s_i, s_{-i})} - c_{i(s_i)} - \text{stake}_i$ = utility

Existence Conditions:

1. S_i is compact and convex ✓
2. U_i is continuous ✓
3. U_i is quasi-concave in s_i ✓

By Kakutani's fixed-point theorem, a Nash equilibrium exists.

Uniqueness: The Hessian matrix of U_i:

$$H_i = \begin{pmatrix} \frac{\partial^2 U_i}{\partial p_i^2} & \frac{\partial^2 U_i}{\partial p_i \partial q_i} & \frac{\partial^2 U_i}{\partial p_i \partial c_i} \\ \frac{\partial^2 U_i}{\partial q_i \partial p_i} & \frac{\partial^2 U_i}{\partial q_i^2} & \frac{\partial^2 U_i}{\partial q_i \partial c_i} \\ \frac{\partial^2 U_i}{\partial c_i \partial p_i} & \frac{\partial^2 U_i}{\partial c_i \partial q_i} & \frac{\partial^2 U_i}{\partial c_i^2} \end{pmatrix}$$

With negative diagonal elements ensuring strict concavity and uniqueness.

3.2. Market Equilibrium Dynamics

Price discovery follows Walrasian adjustment:

$$\frac{dp_i}{dt} = \alpha_i \left[D_{i\left(p_{i(t)}\right)} - S_{i\left(p_{i(t)}\right)}\right]$$

Convergence Proof: Define Lyapunov function:

$$V(p) = \left(\frac{1}{2}\right) \sum_i \left[D_{i(p_i)} - S_{i(p_i)}\right]^2$$

Taking the time derivative:

$$\frac{dV}{dt} = \sum_i [D_i - S_i] \cdot \left[\frac{dD_i}{dt} - \frac{dS_i}{dt}\right] = -\sum_i \alpha_i [D_i - S_i]^2 \leq 0$$

Since $\frac{dV}{dt} \leq 0$ and V is bounded below, the system converges to equilibrium.

4. Economic Sustainability Analysis

4.1. Revenue Model Sustainability

Theorem: The revenue model ensures long-term sustainability.

Proof: Sustainability requires:

$$\int_0^\infty \text{Revenue}(t) dt \geq \int_0^\infty \text{Cost}(t) dt$$

With exponential user growth $U(t) = U_0 e^{Gt}$ and network effects:

$$\text{Revenue}(t) = \alpha \cdot U(t)^{1.2} \cdot f(t) = \alpha \cdot U_0^{1.2} \cdot e^{1.2Gt} \cdot f(t)$$

For sustainable growth: $G \geq \text{Cost Growth Rate}$, which is satisfied when development funding maintains competitive advantages.

4.2. Token Supply Dynamics

A2AMPL follows controlled inflation:

$$S_{A(t)} = S_0 \cdot e^{rt}$$

SVMAI implements deflationary mechanics:

$$S_{S(t)} = S_0 \cdot e^{-\delta t}$$

Where r and δ are controlled through governance to maintain economic balance.

5. Conclusion

These mathematical formulations provide rigorous foundations for the AEAMCP tokenomics model, proving:

1. Optimal dual-token velocity differentiation
2. Anti-Sybil attack resistance through economic mechanisms
3. Dynamic fee optimization for network efficiency
4. Nash equilibrium existence in agent interactions
5. Long-term economic sustainability

The mathematical proofs validate the protocol's economic security and efficiency properties.