LEIBNIZ INSTITUTE of PHOTONIC TECHNOLOGY

Lichtwerkstatt – Workshop:
Inline Holographical Microscope

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Holography


BP-6M
NOKIA


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EURO
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SECURE
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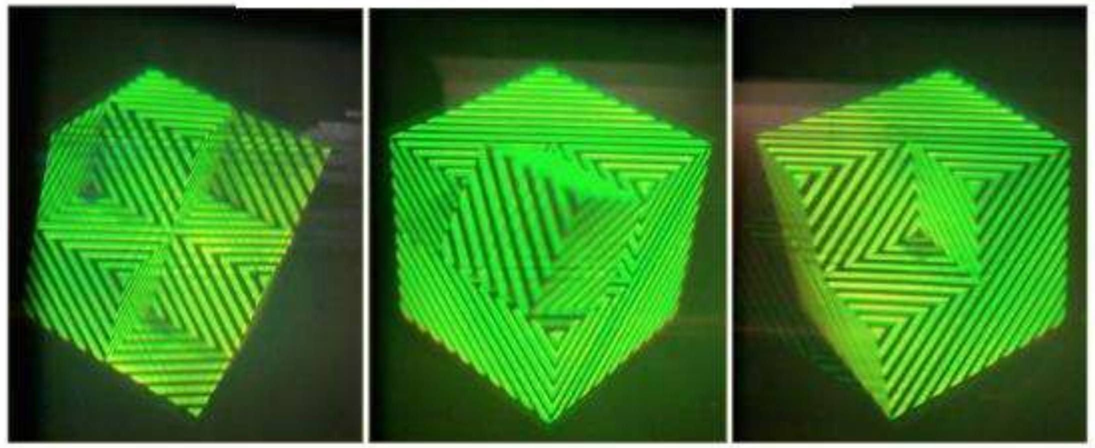


Holography

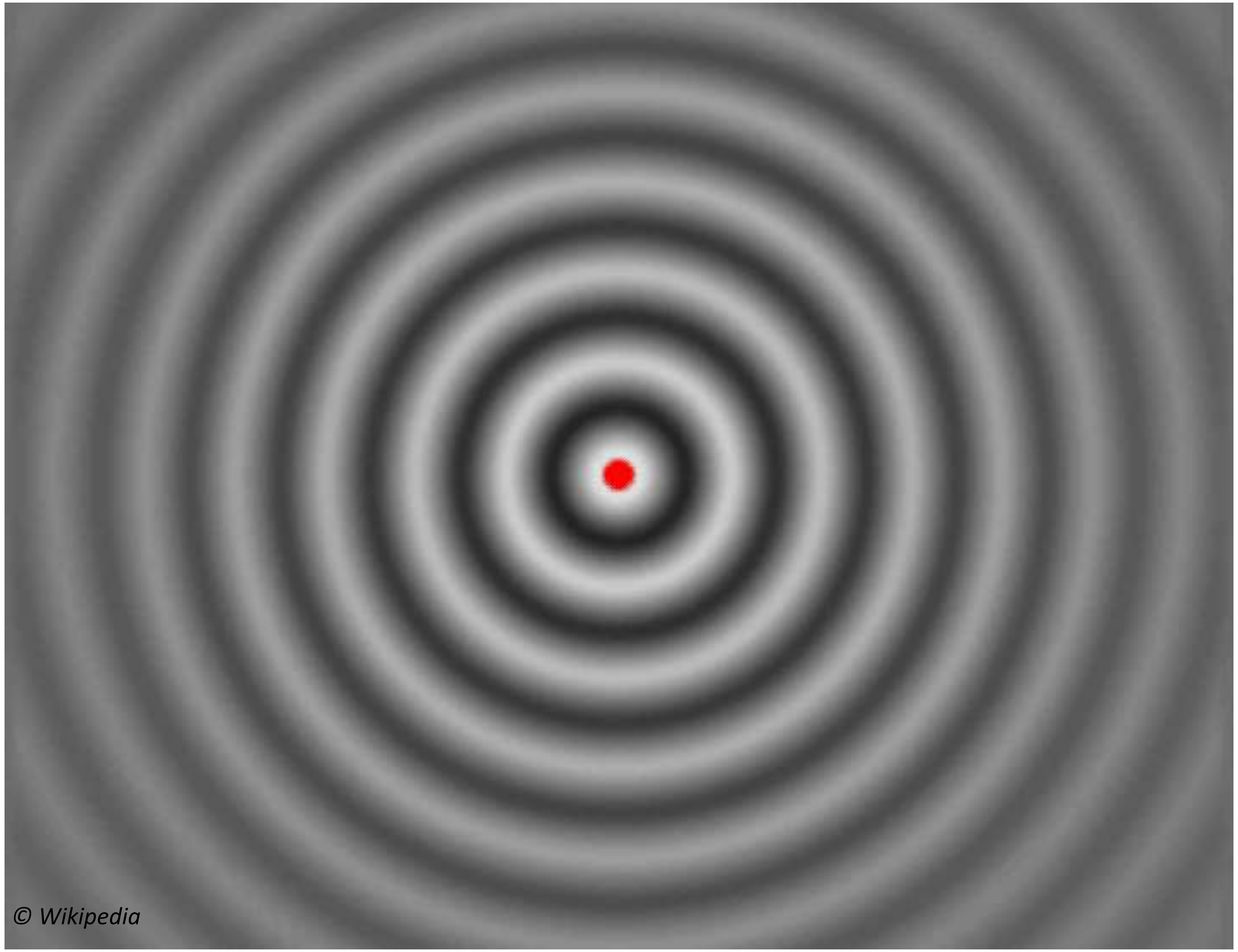
- Reconstructs the original wavefront in amplitude and phase
- Comparison: photo reconstructs only the intensity
- Has any number of perspectives
- Allows free focusing through the image
- Discovered by Dennis Gabor (Hungary, 1947)

What is light?

What is Light?

Throw a stone into the water…

…a wave will appear.

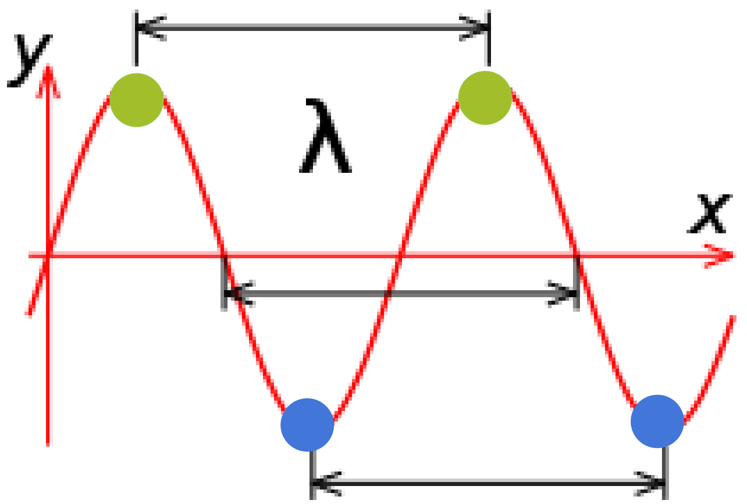


"Pumping" the water creates a continuous wave

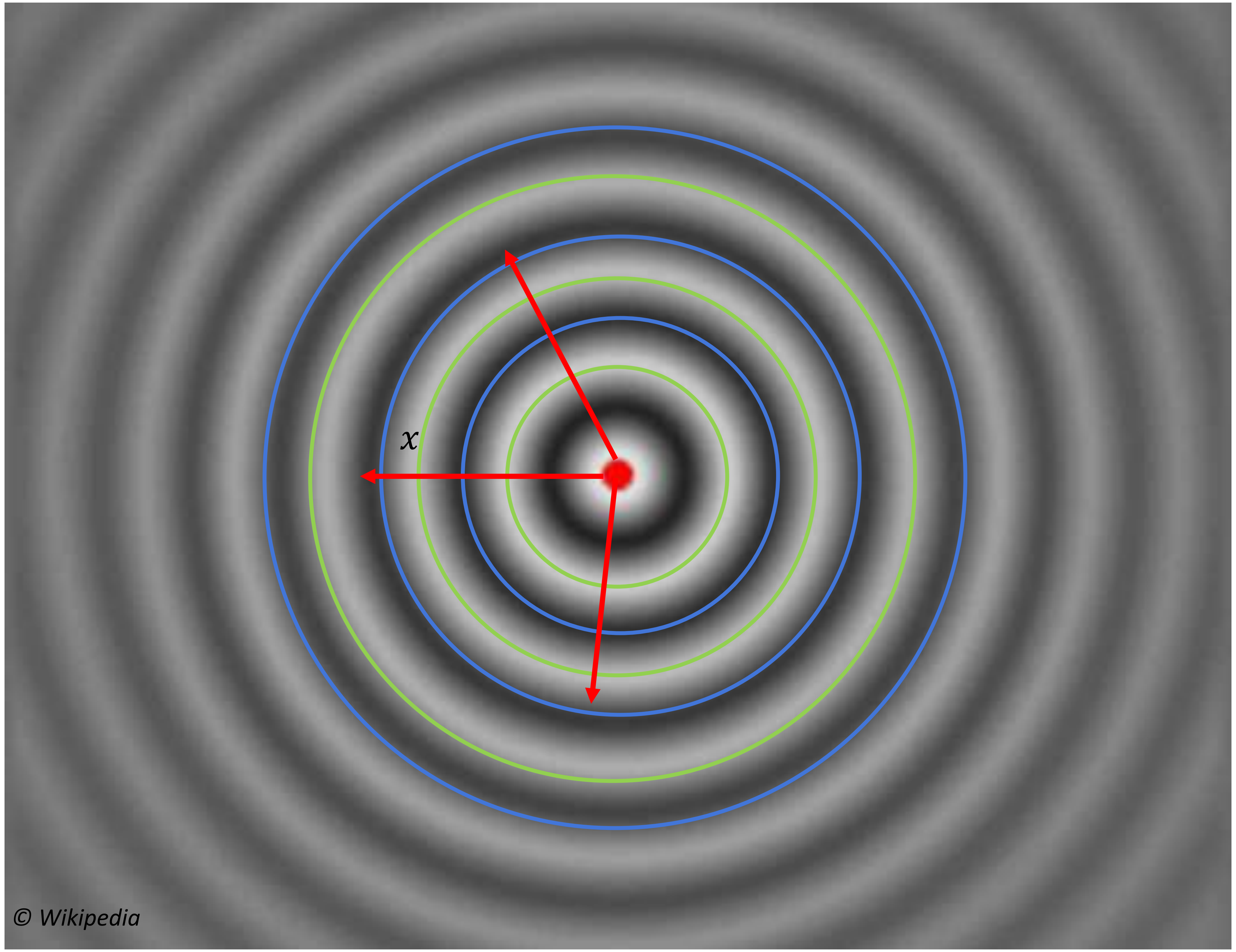
What is Light?

Wave:

$$y(k) = y_0 \cdot \sin\left(kx - \frac{2\pi c}{\lambda}t + \phi\right) + y_b$$



- *Light waves propagate like sine-waves in space (3D)*
- *Ray Optics represents light waves as arrows*
- *Rays are always perpendicular to the wavefront*
- ***MAXIMA***
- ***MINIMA***
- ***DIRECTION***



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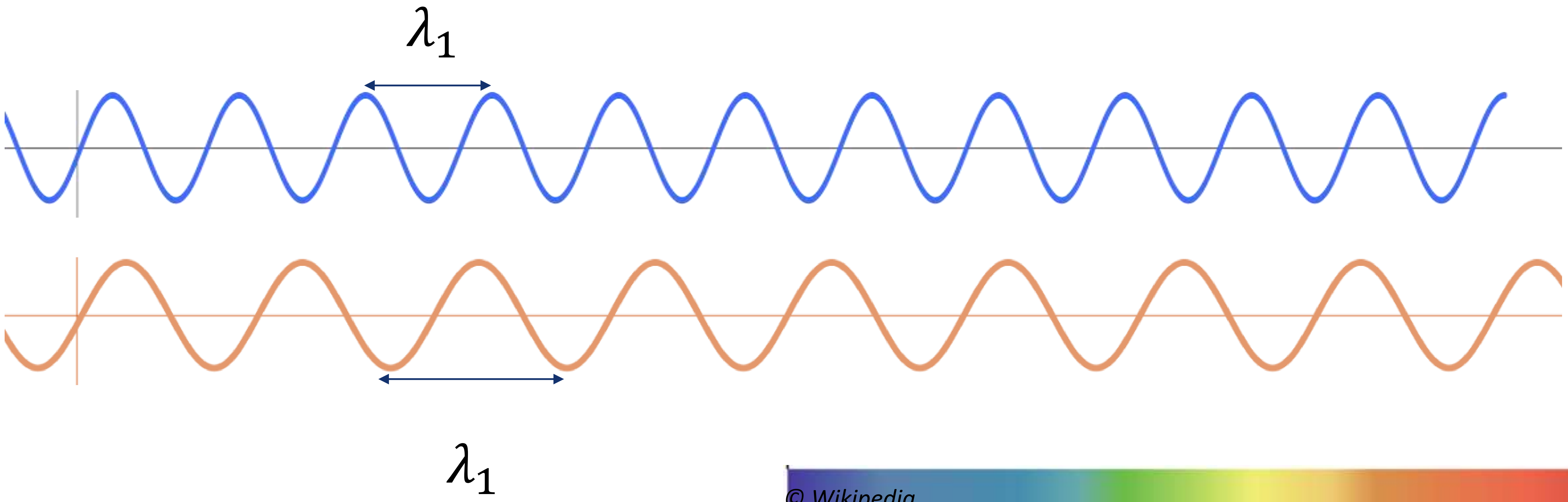
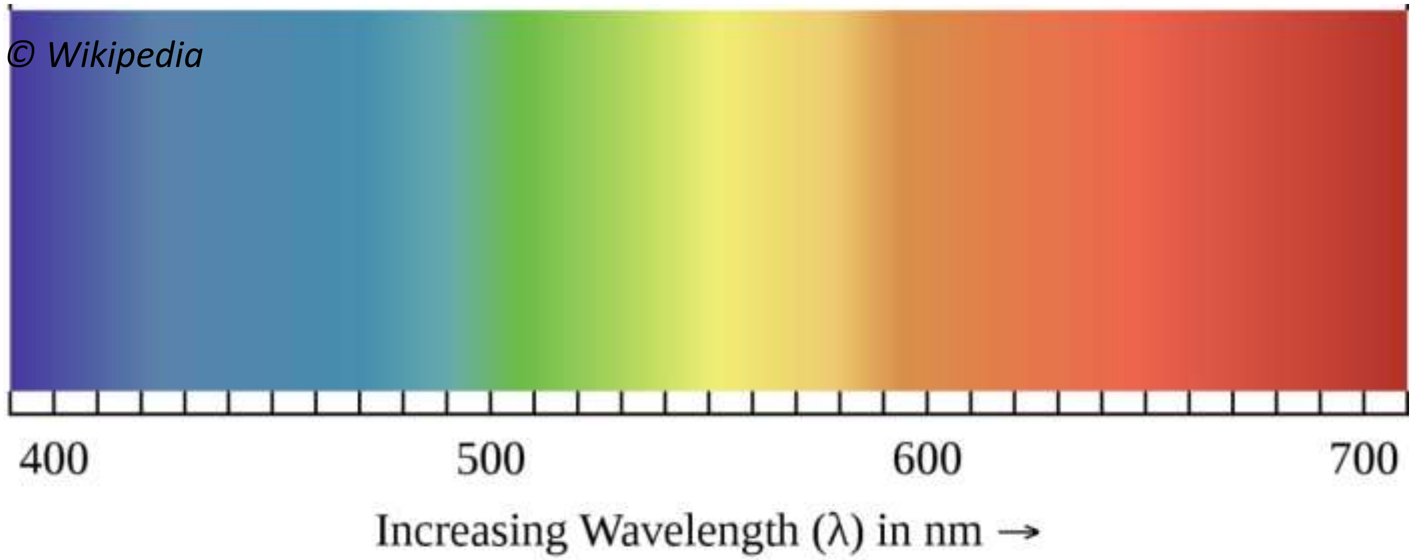
What is Light? WAVELENGTH

Wave:

$$y(k) = y_0 \cdot \sin\left(kx - \boxed{\frac{2\pi c}{\lambda}} t + \phi\right) + y_b$$

Colour!



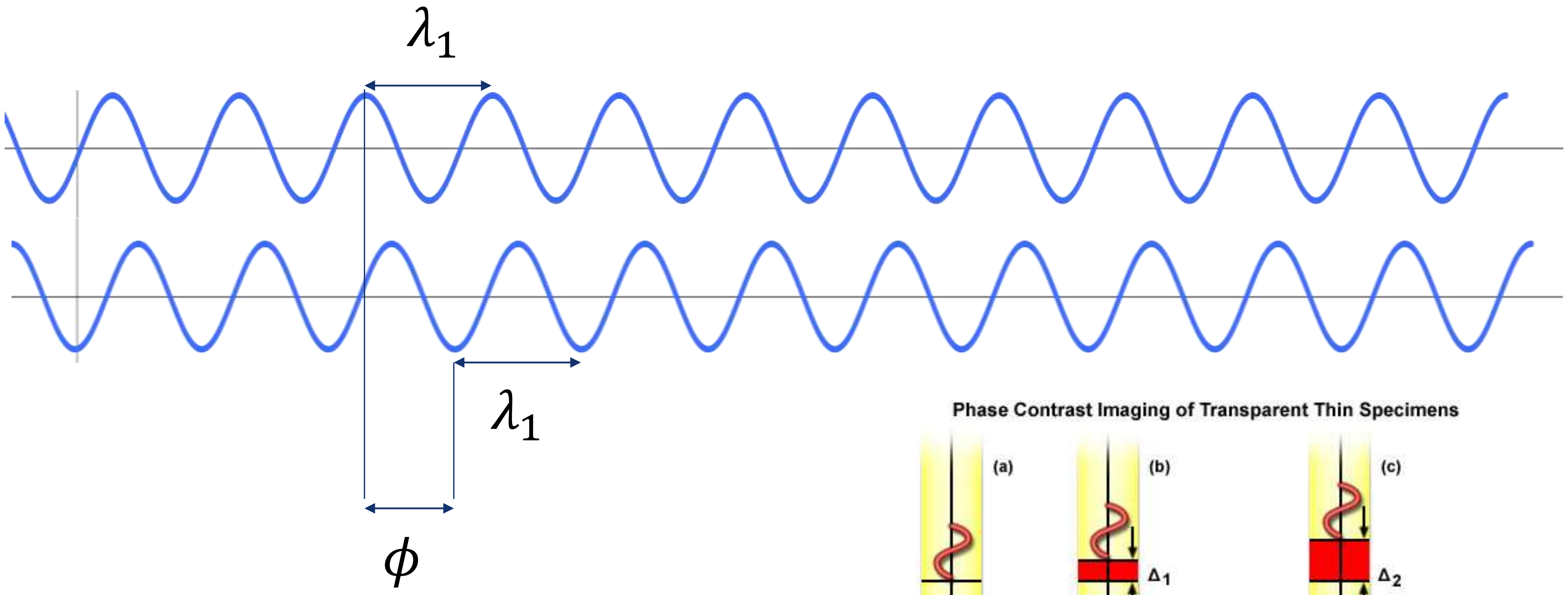


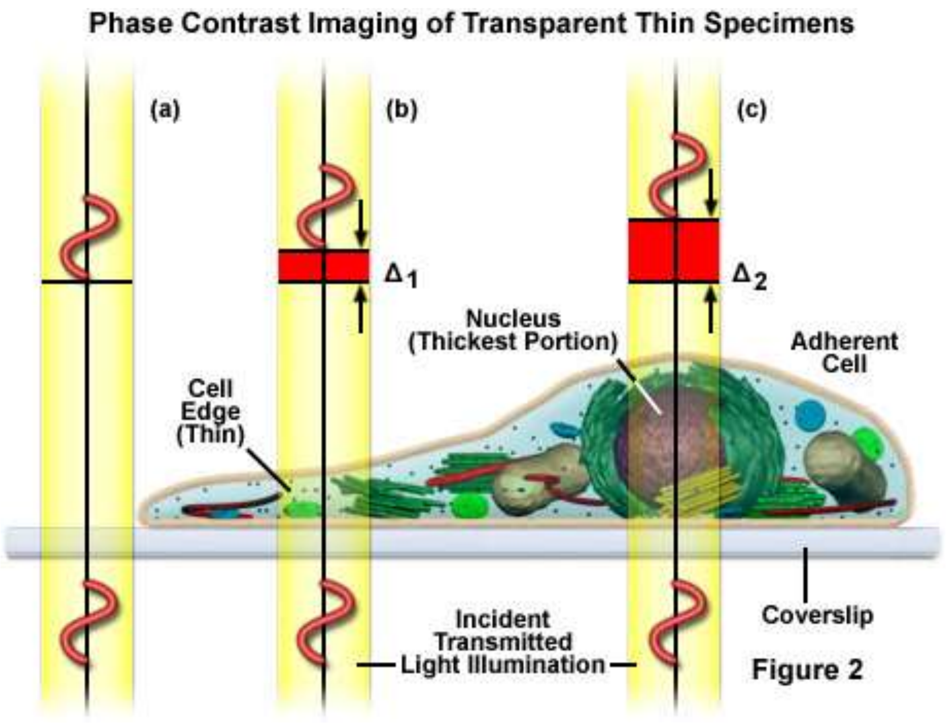
What is Light? PHASE

Wave:

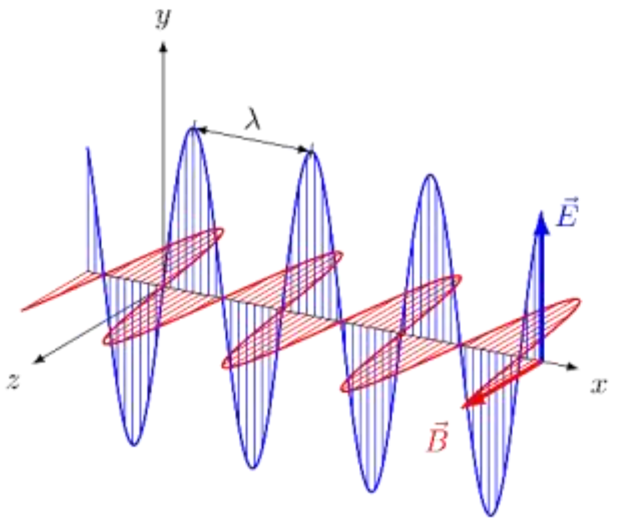
$$y(k) = y_0 \cdot \sin\left(kx - \frac{2\pi c}{\lambda} t + \boxed{\phi}\right) + y_b$$

Phase!

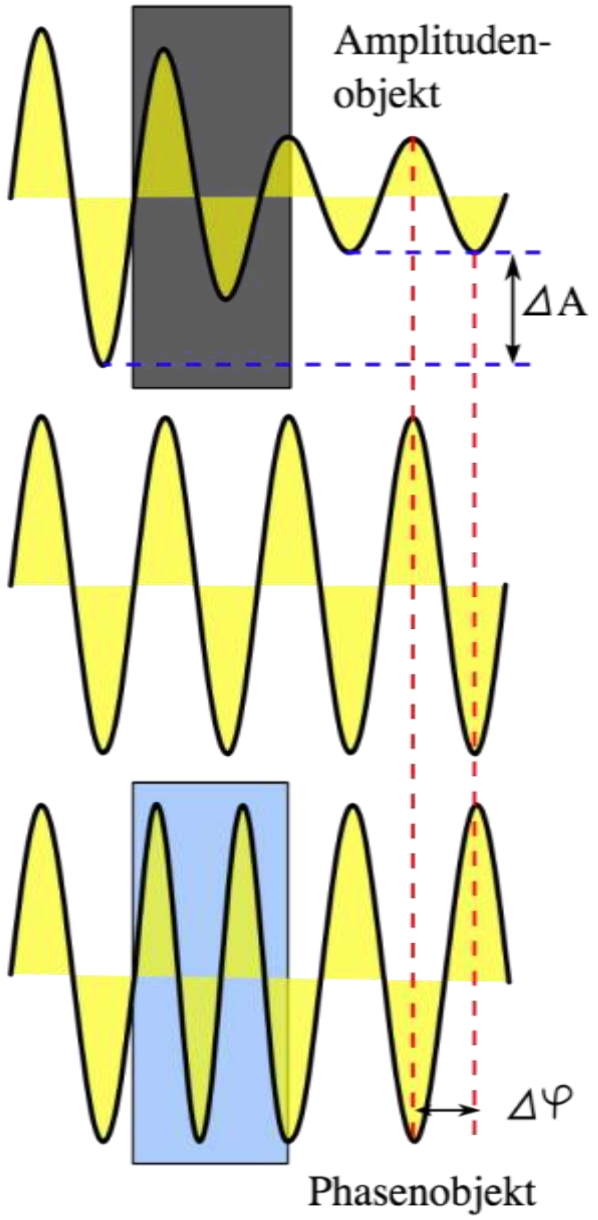




What is Light? AMPLITUDE and INTENSITY



Light as a wave in space (Vacuum)



Light as a wave in space (with Media)

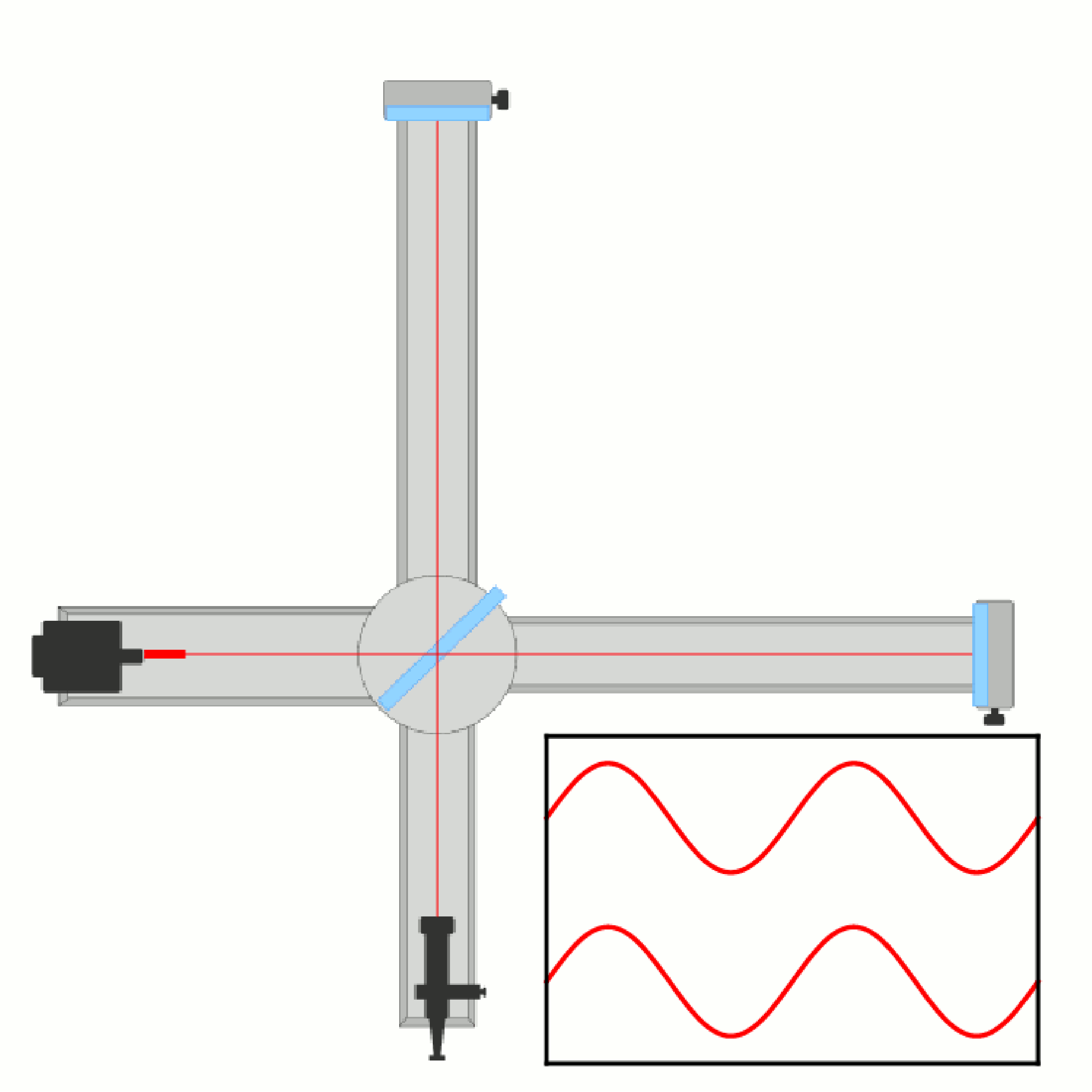
Electromagnetic Wave:

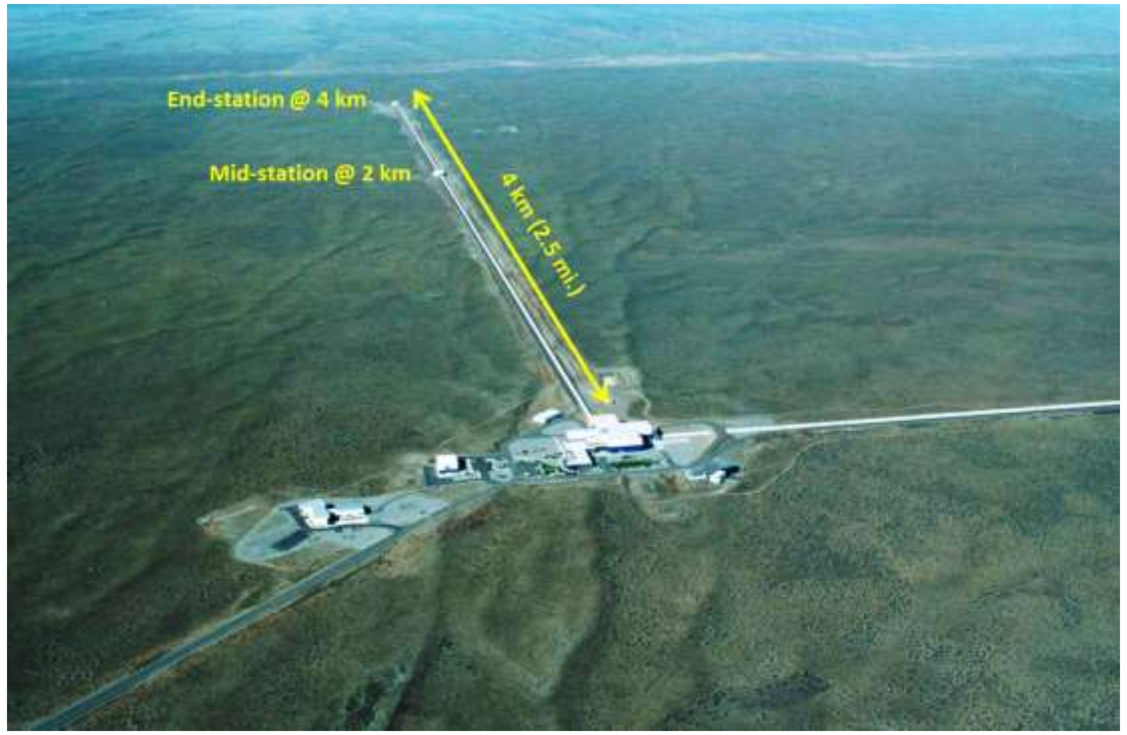
$$E(x,y) = A(x,y) \cdot \exp\big(-i\,\phi(x,y)\big)$$

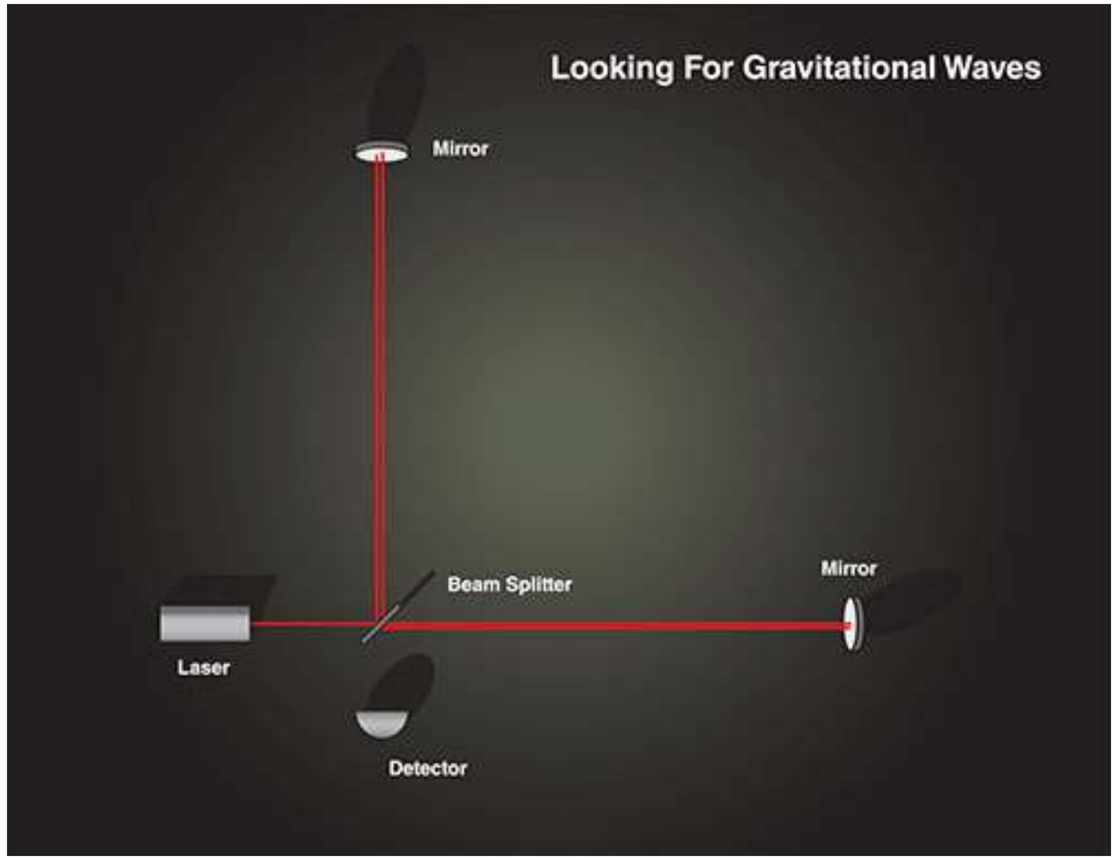
Intensity (Irradiance):

$$I = E \cdot E^* = |E|^2$$

Interferomter – Hands on experiment?







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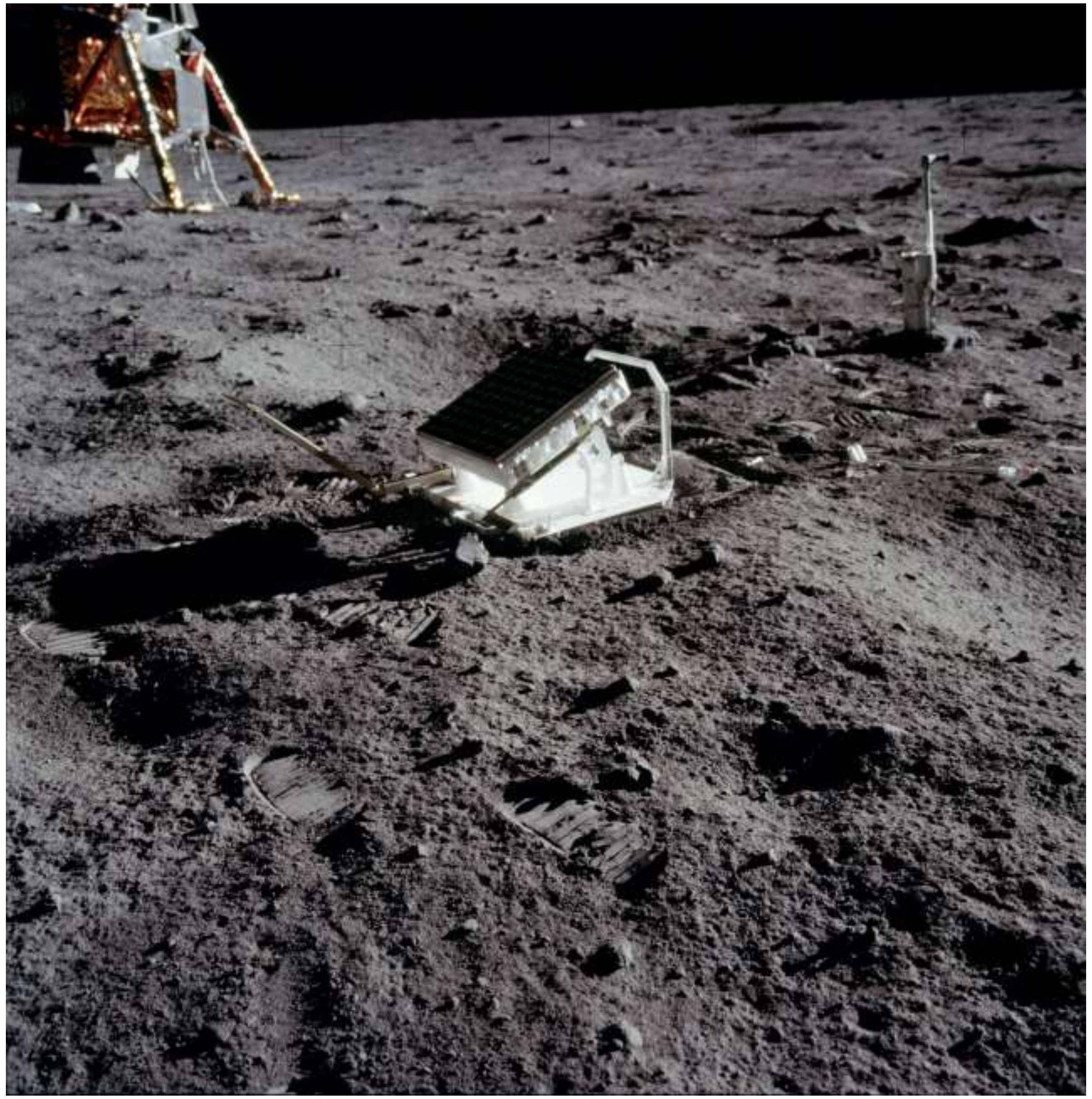
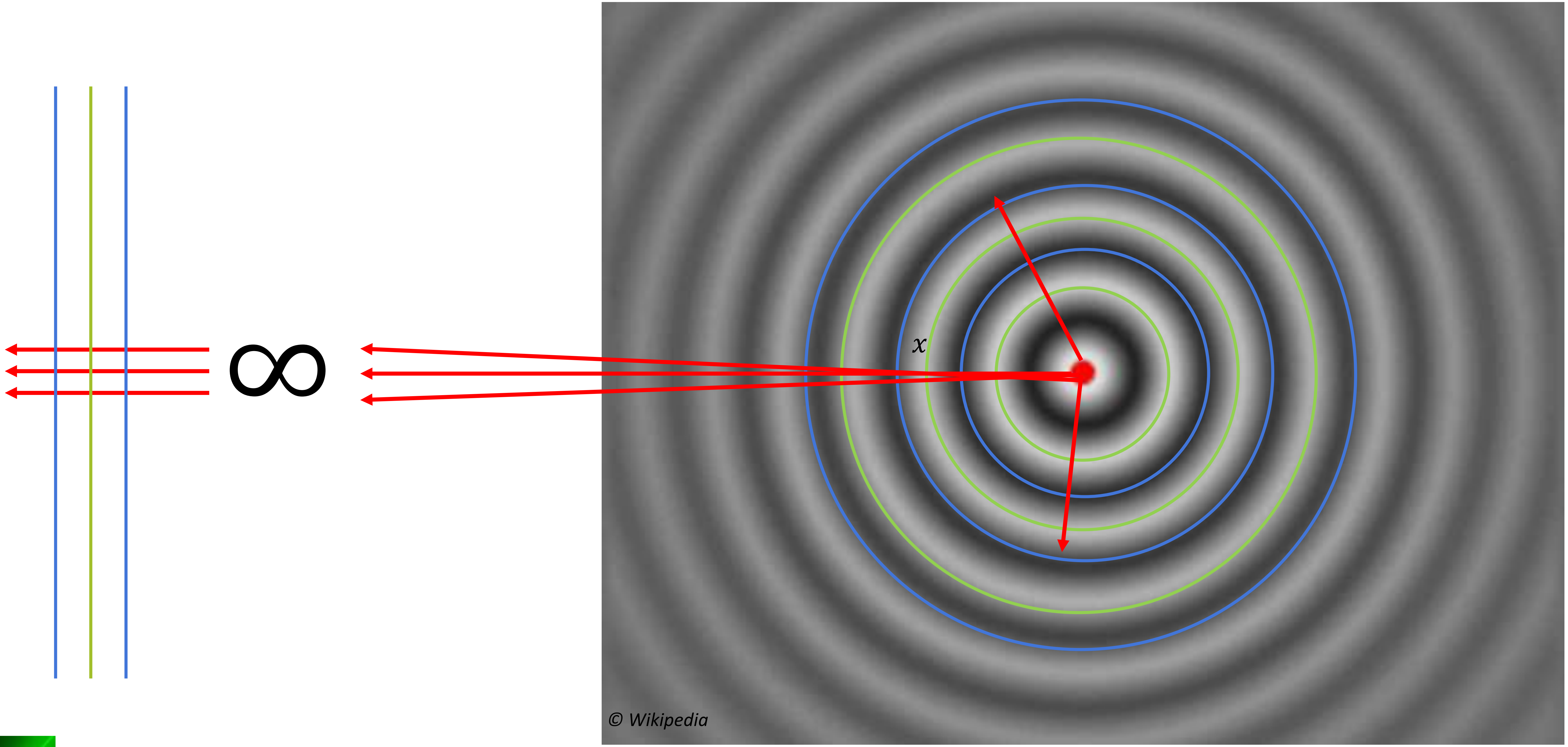
How to measure the distance to the moon



What is light?

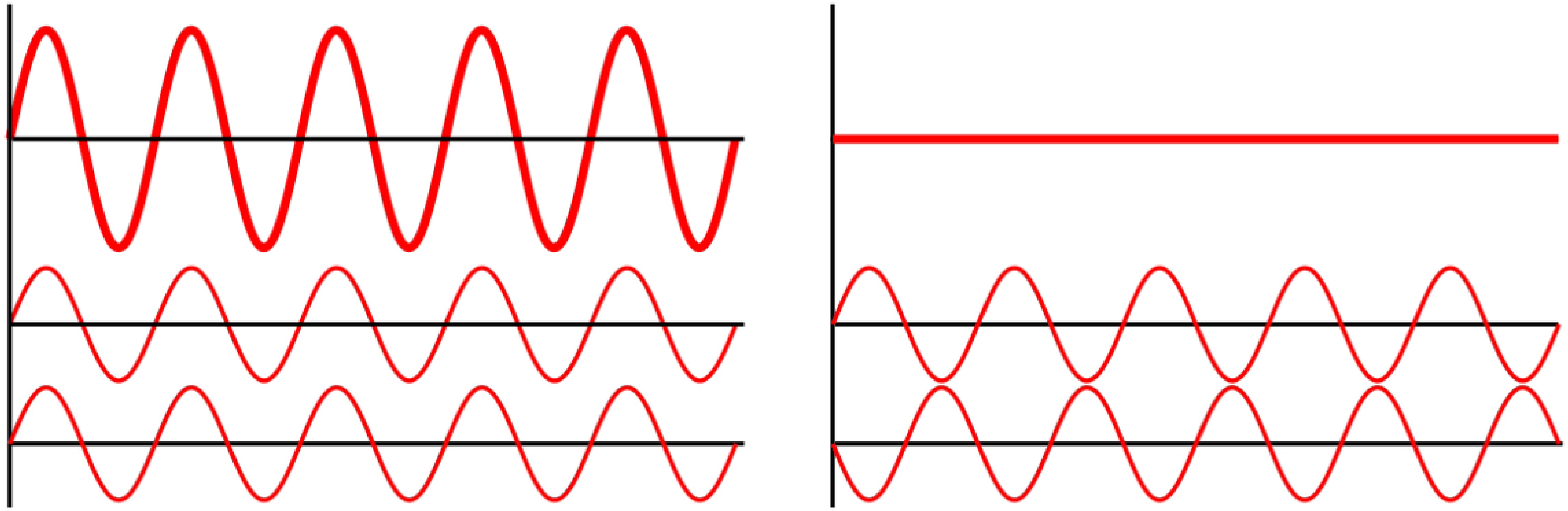


Plane Wave
=> LASER

vs.

Spherical Wave
=> LED

Holography – Interference



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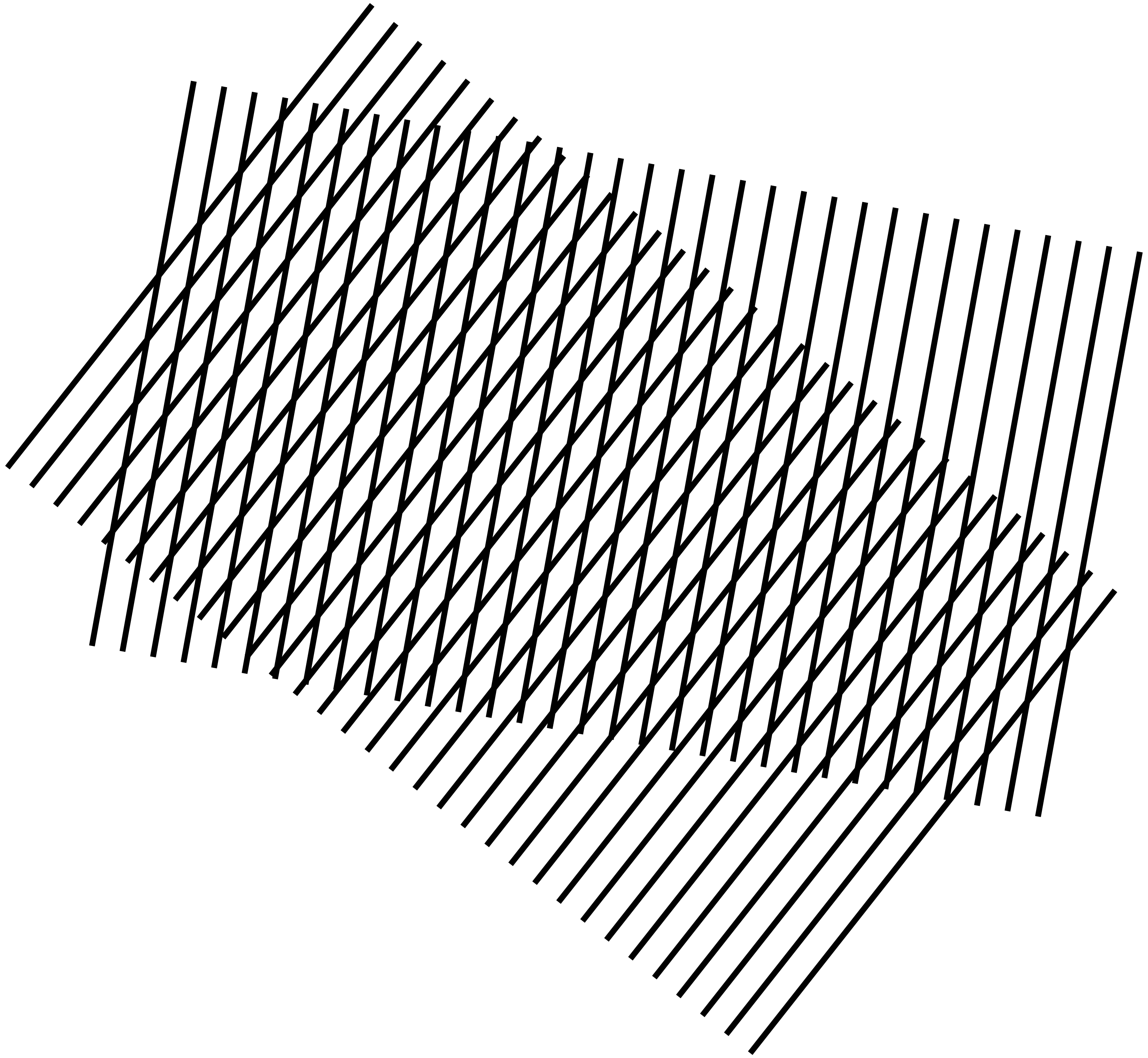
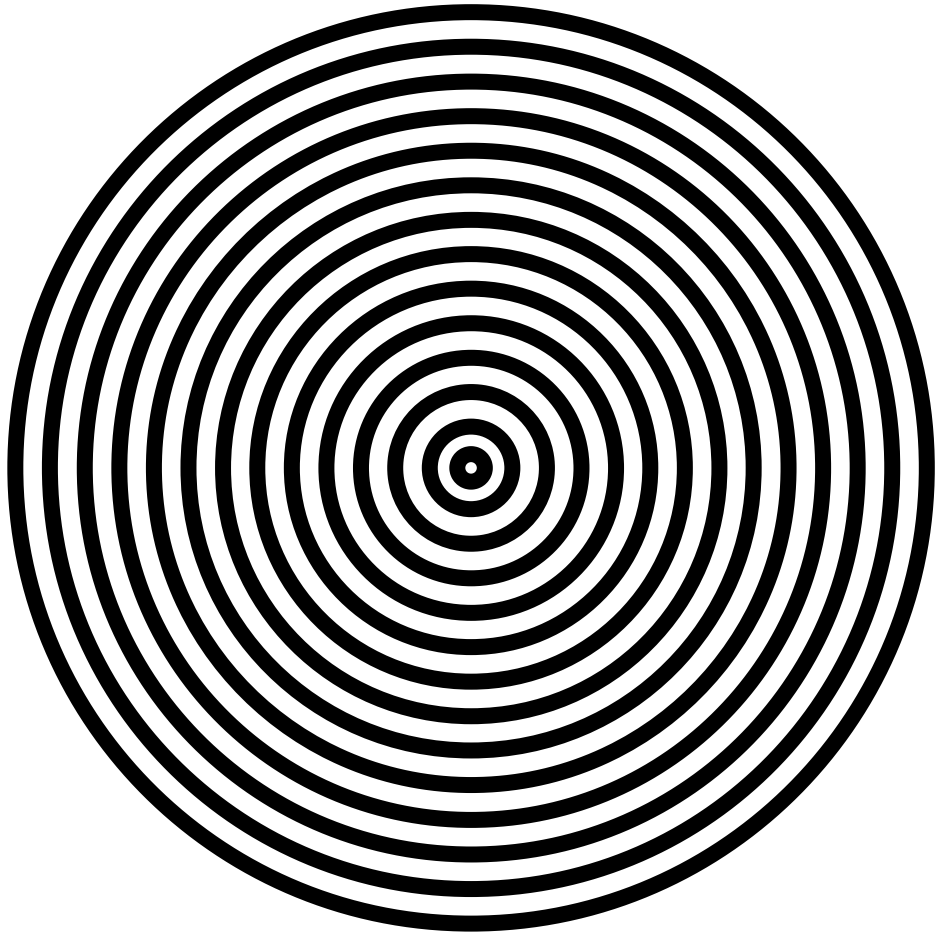
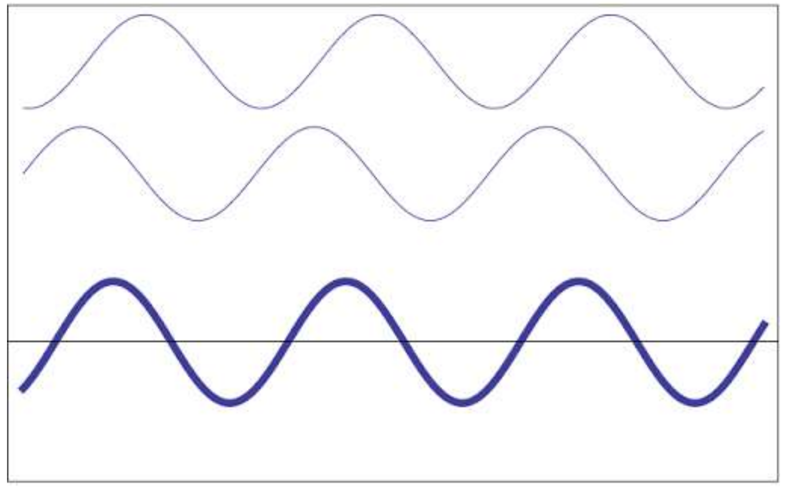
Interference: Wave 1 + Wave 2







Wave 1

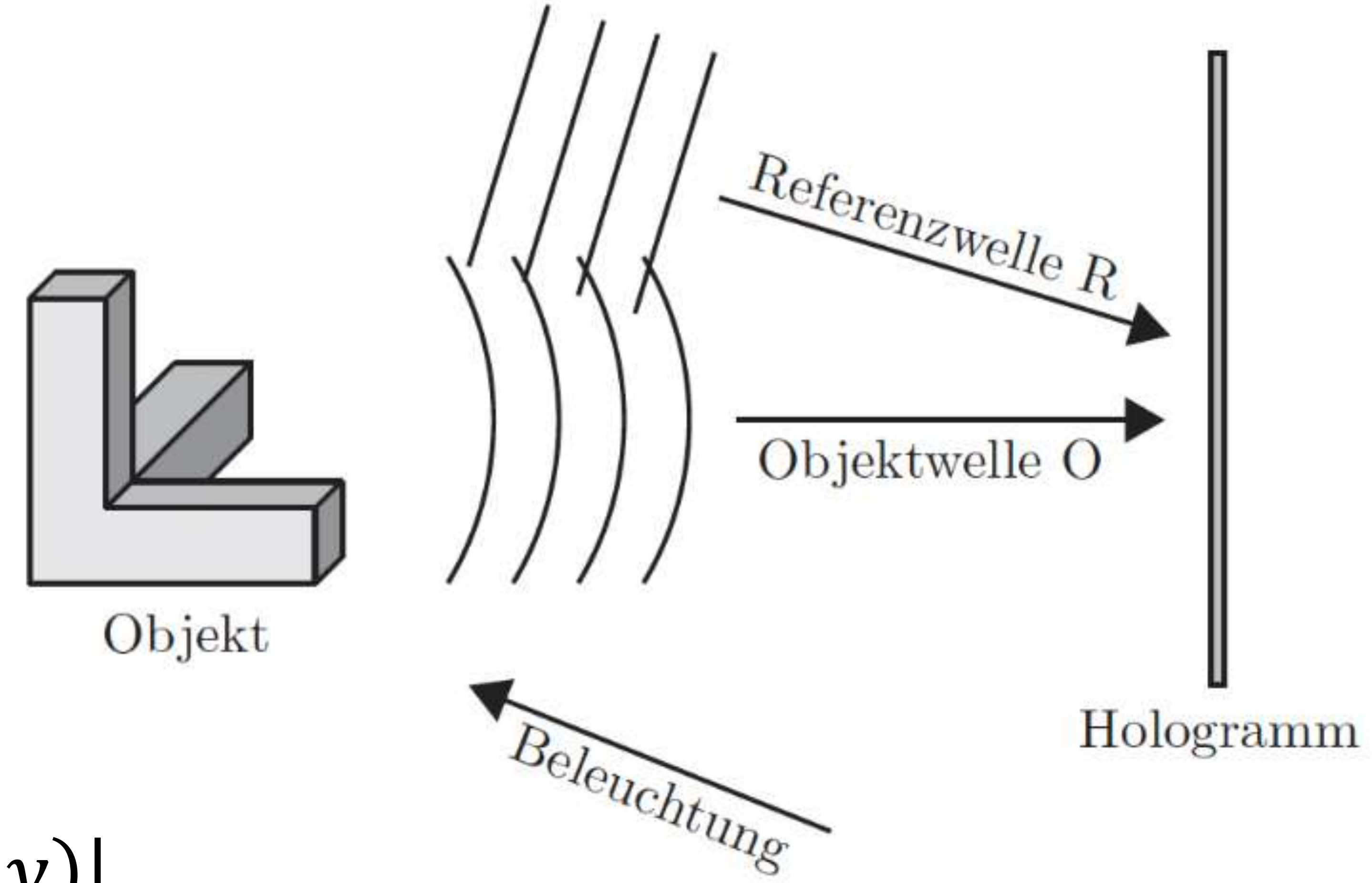
Wave 2

Wave 1 + Wave 2

Holography - Acquisition

Interference of two "waves"

$$E(x,y) = R(x,y) + O(x,y)$$



Detectors only see intensity!

$$I(x,y) = |E(x,y)|^2 = |R(x,y) + O(x,y)|^2$$

$$= |R(x,y) + O(x,y)|^* \cdot |R(x,y) + O(x,y)|$$

$$= R^*(x,y)R(x,y) + R^*(x,y)O(x,y) + O^*(x,y)R(x,y) + O^*(x,y)O(x,y)$$

$$= R^*R + R^*O + O^*R + O^*O$$

Holography - Reconstruction

Illuminate the hologram with the reference wave

$$E(x,y) = I(x,y) \cdot R(x,y)$$

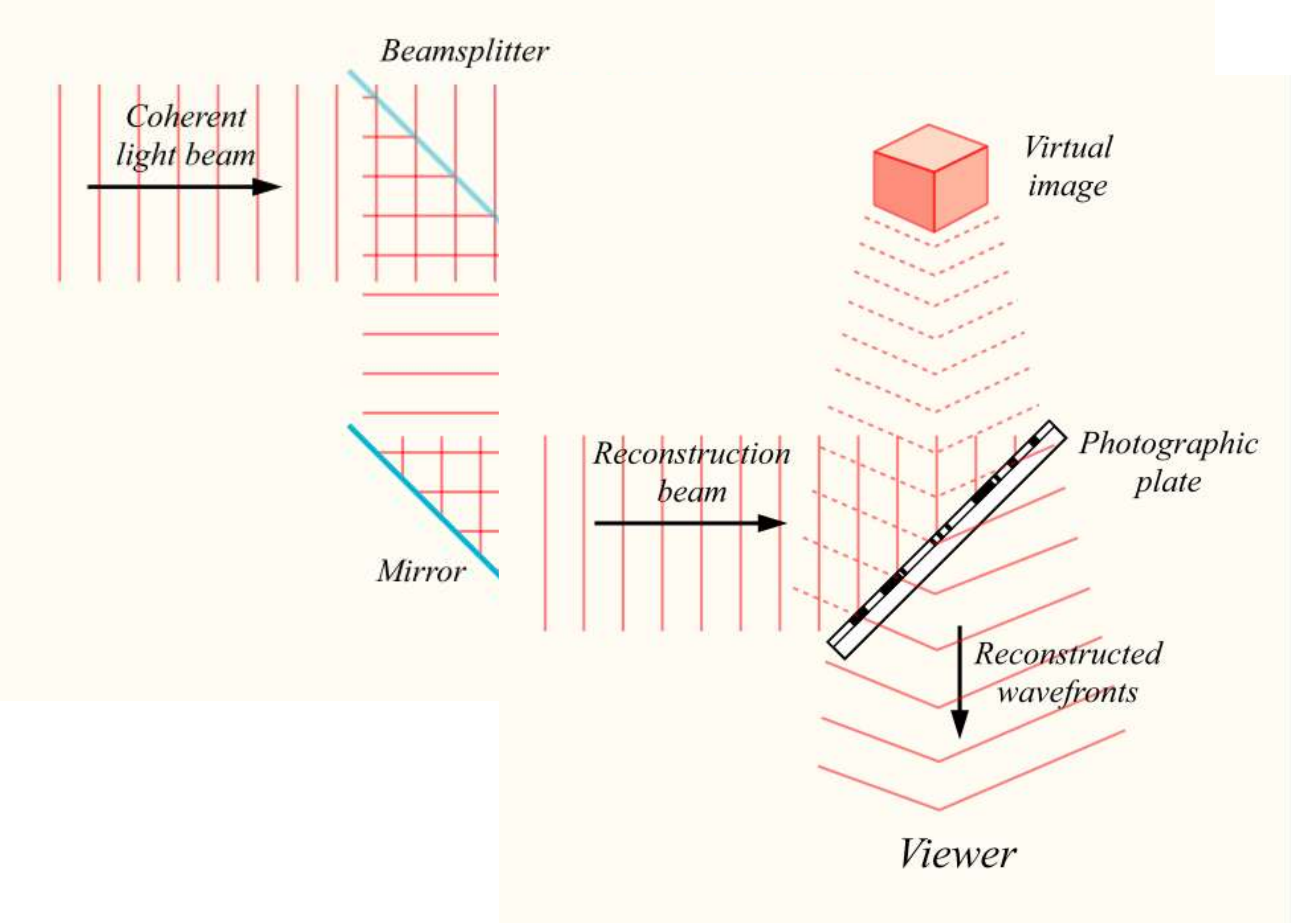
$$I = R^*R + R^*O + O^*R + O^*O$$

$$E = R \cdot (R^*R + R^*O + O^*R + O^*O)$$

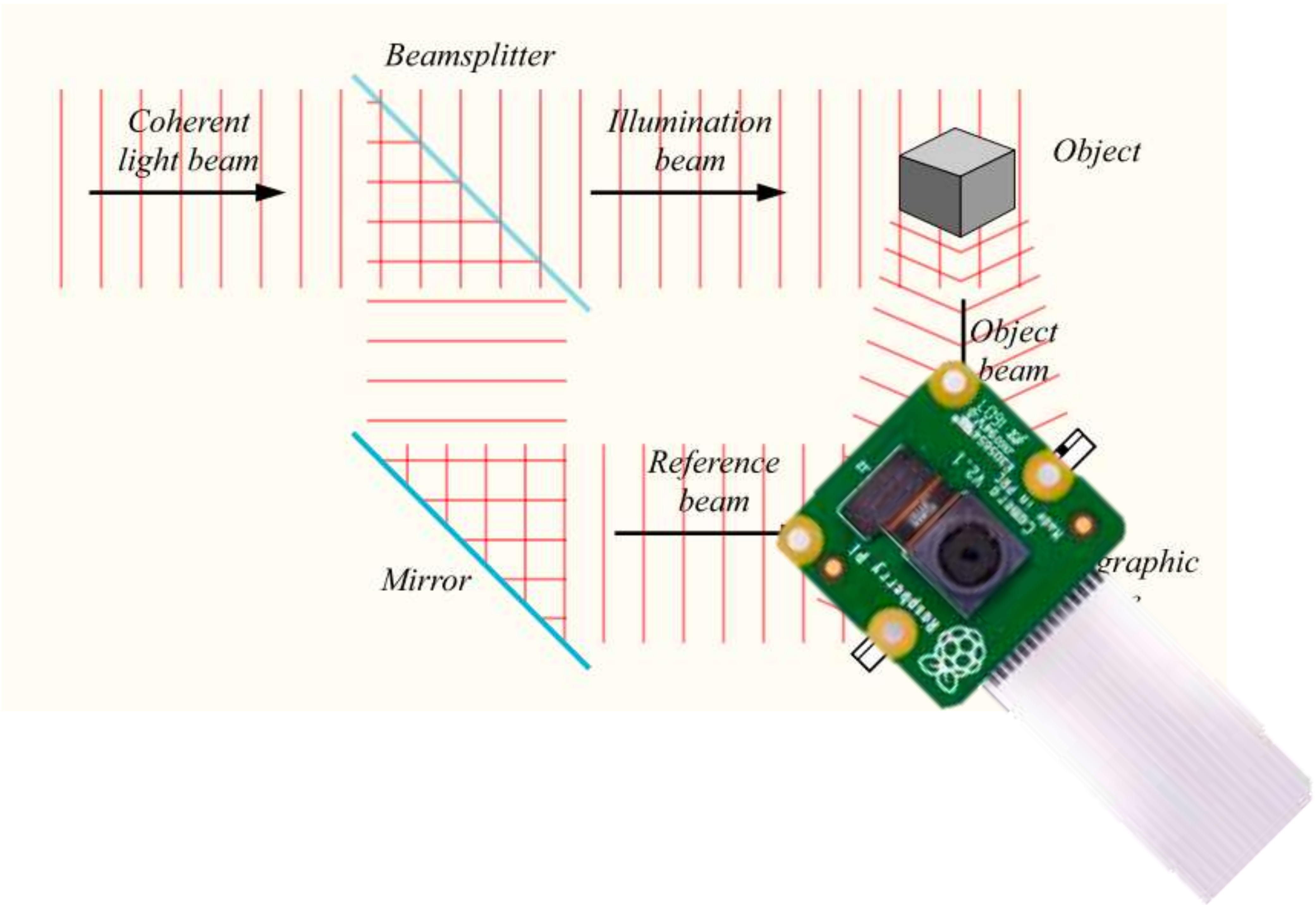
$E = R \cdot (R^2 + O^2)$	broadened 0th diffraction order
$+\vert\mathrm{R}\vert^2 \cdot O$	1st diffraction order, virtual/orthoscopic image
$+\mathrm{R}^2 \cdot O^*$	-1st diffraction order, real/pseudoscopic image

Interpretation O^*: Phase is conjuagated $\phi \to -\phi$

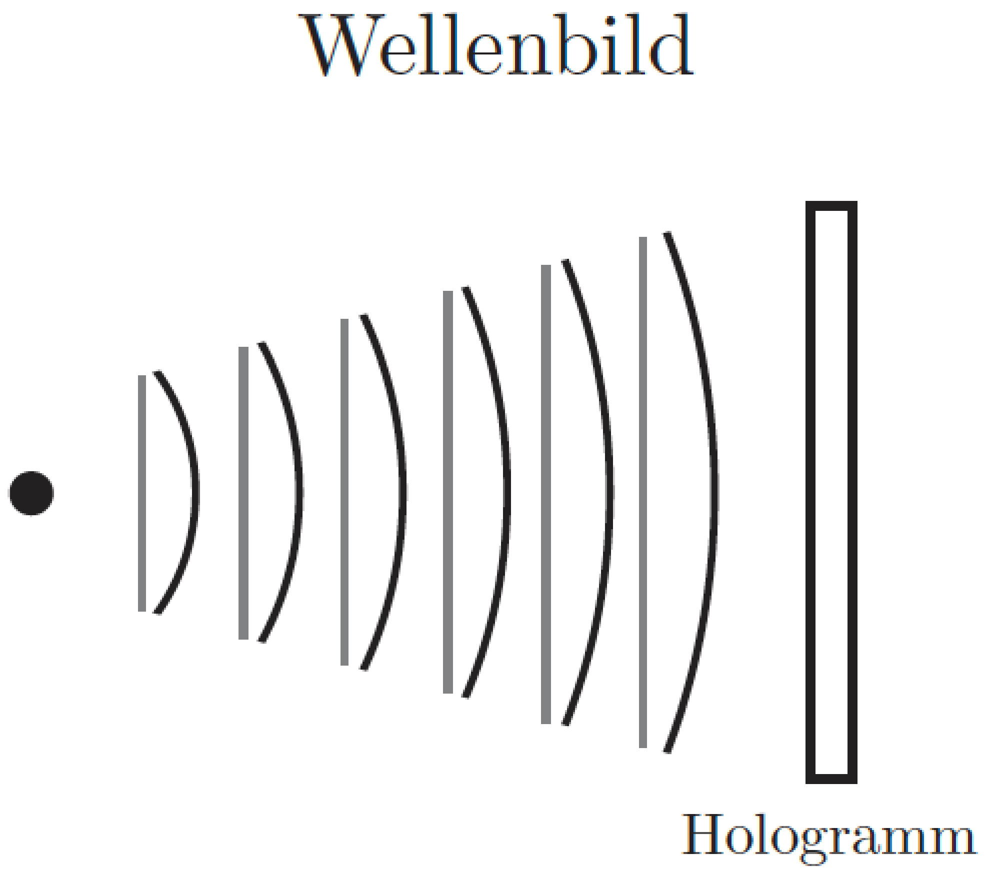
Holography – Recording and Reconstruction

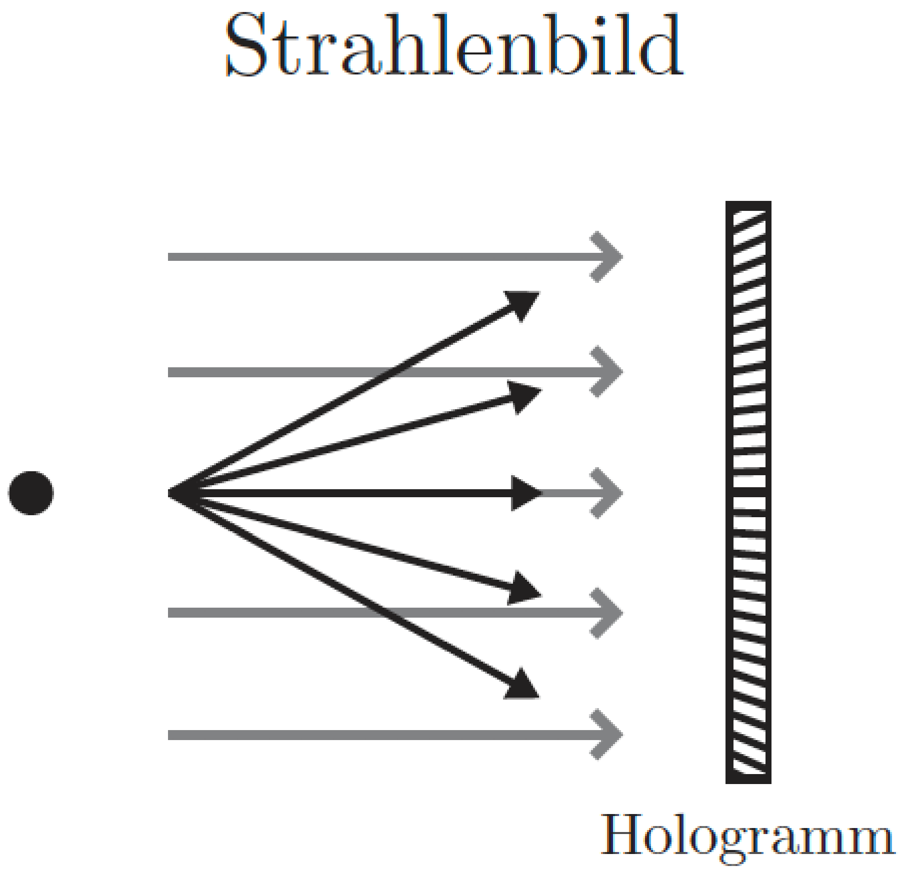


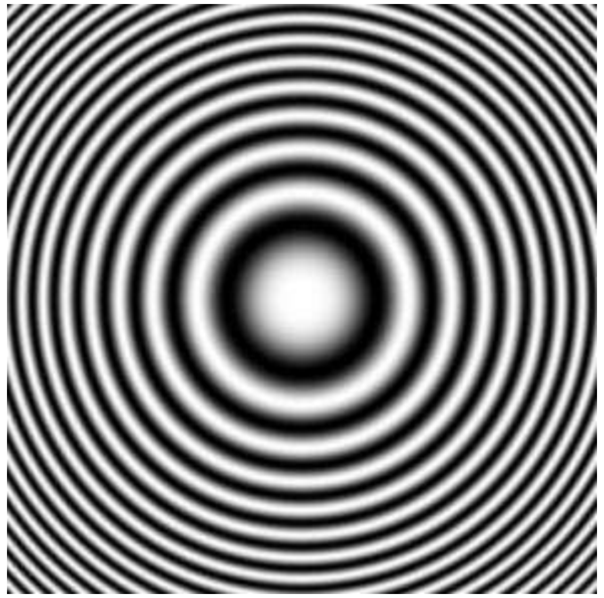
Holography – Recording and Reconstruction (Digital)



Holography – Zoneplate

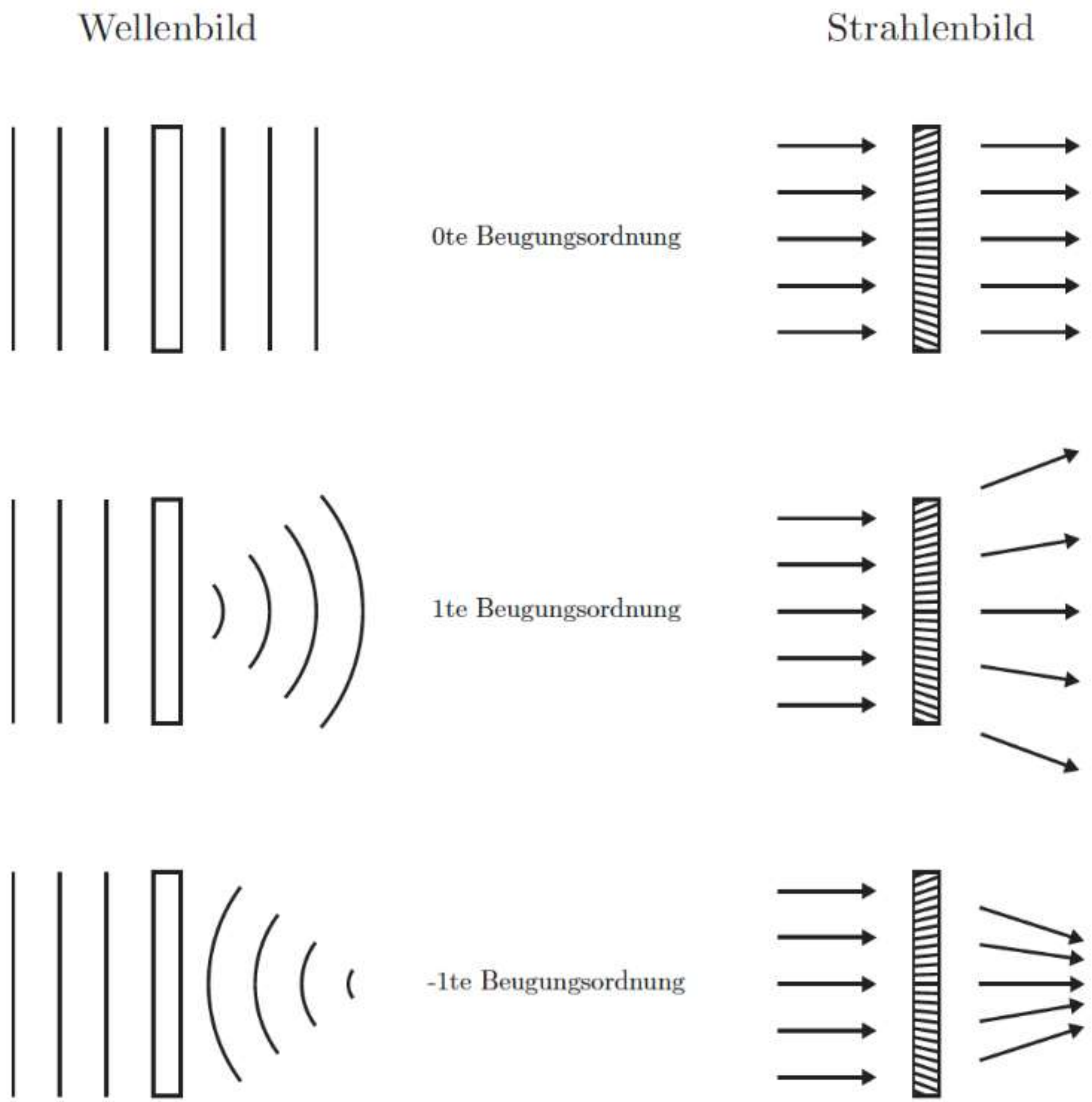


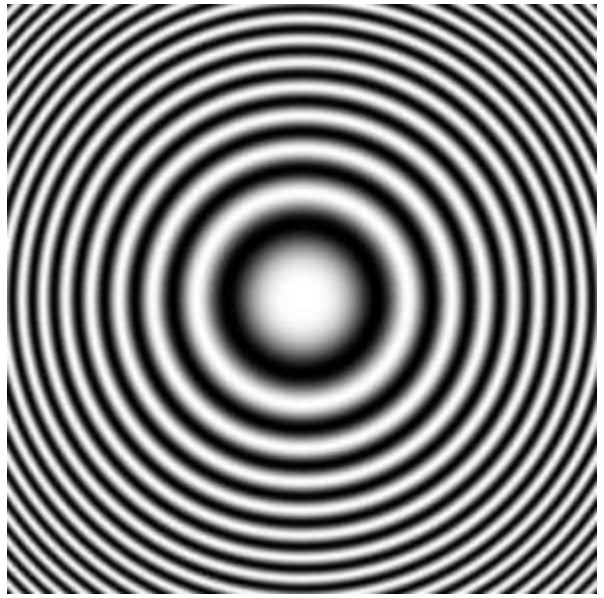




Hologram of a point

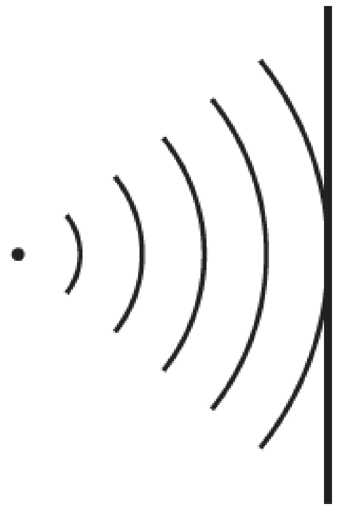
Holography – Zoneplate

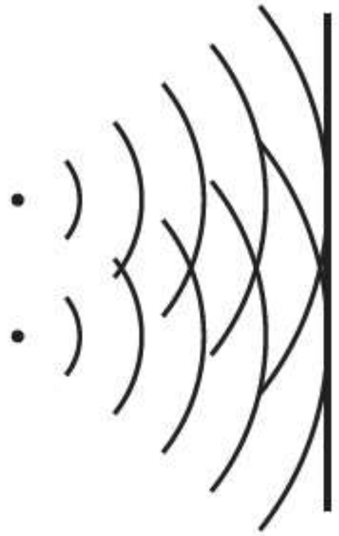


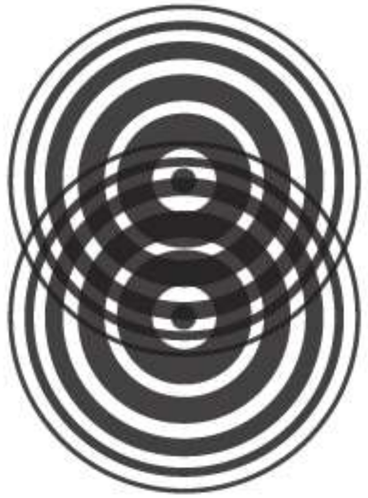


Illumination with a plane wave
=> Focussing

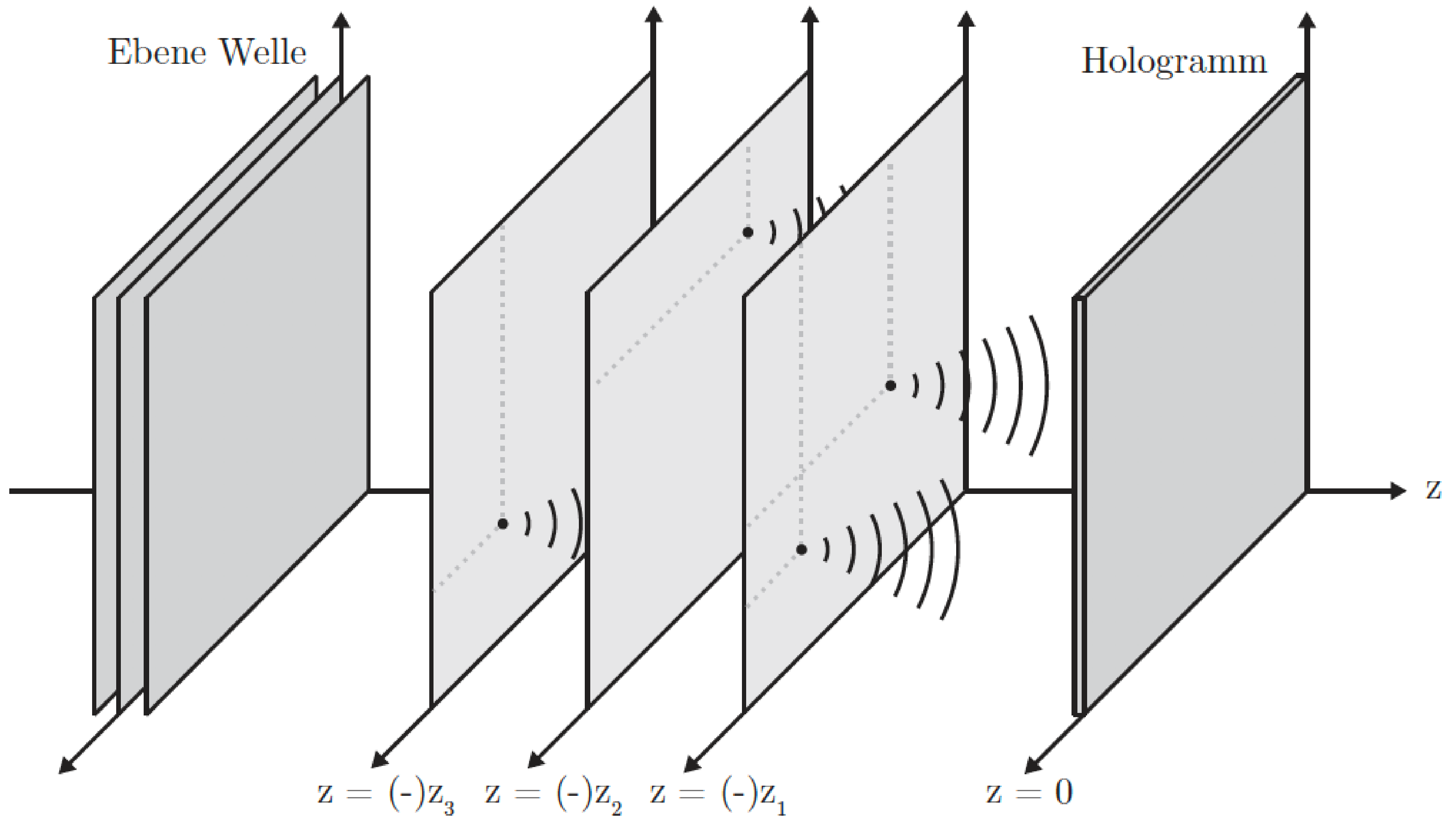
Holography – Multiple Zoneplates

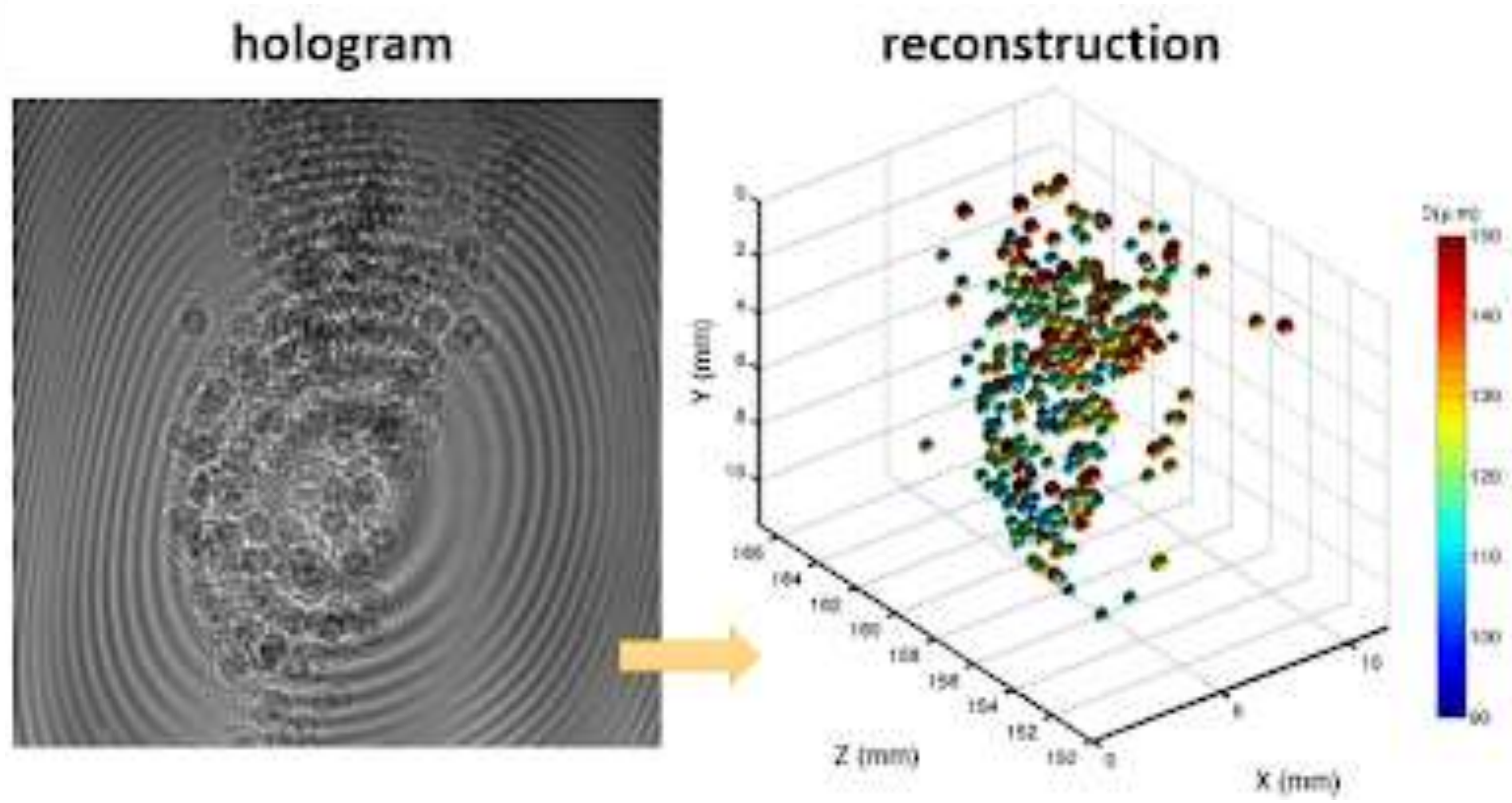






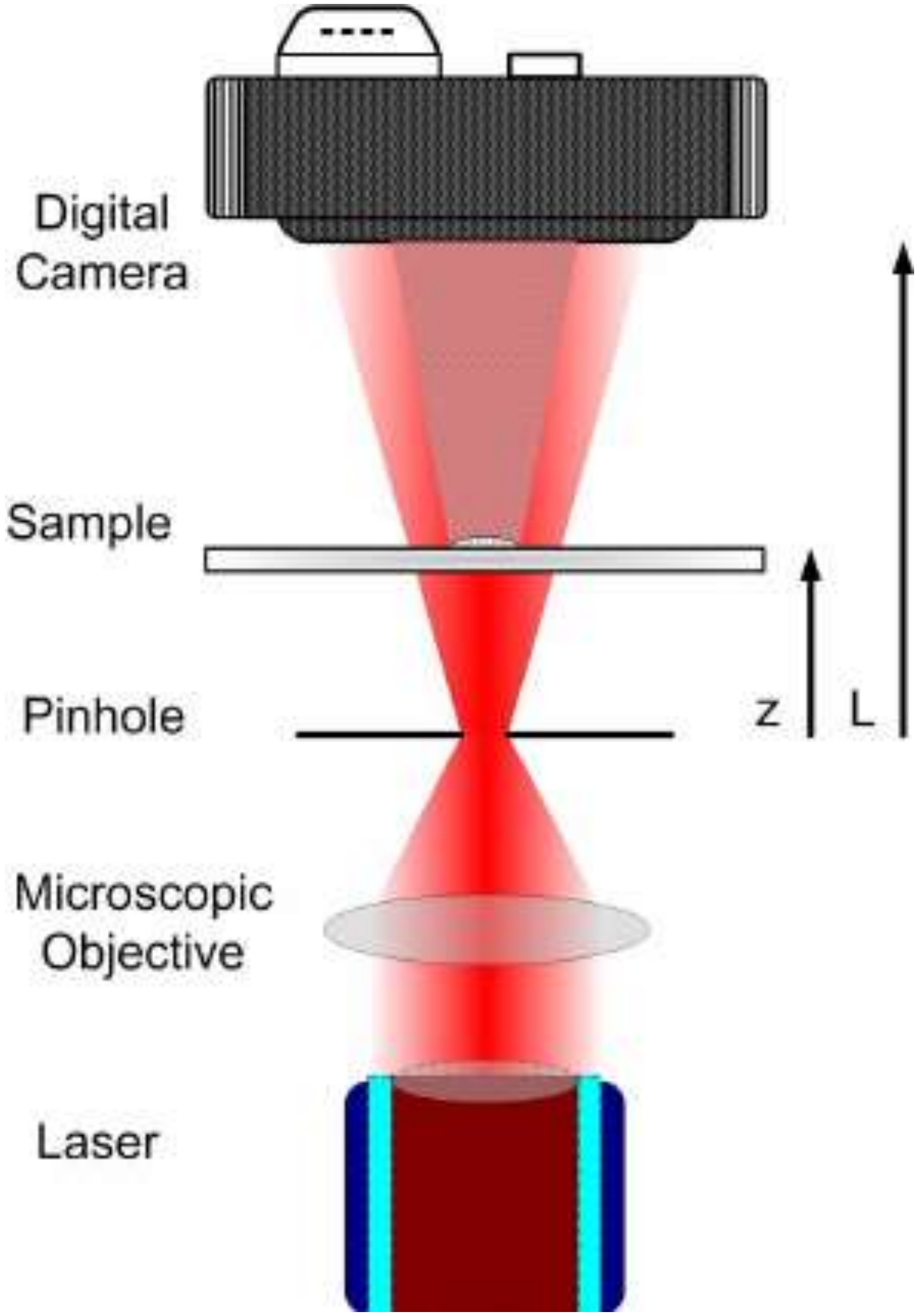
Holography – Multiple Zoneplates in 3D

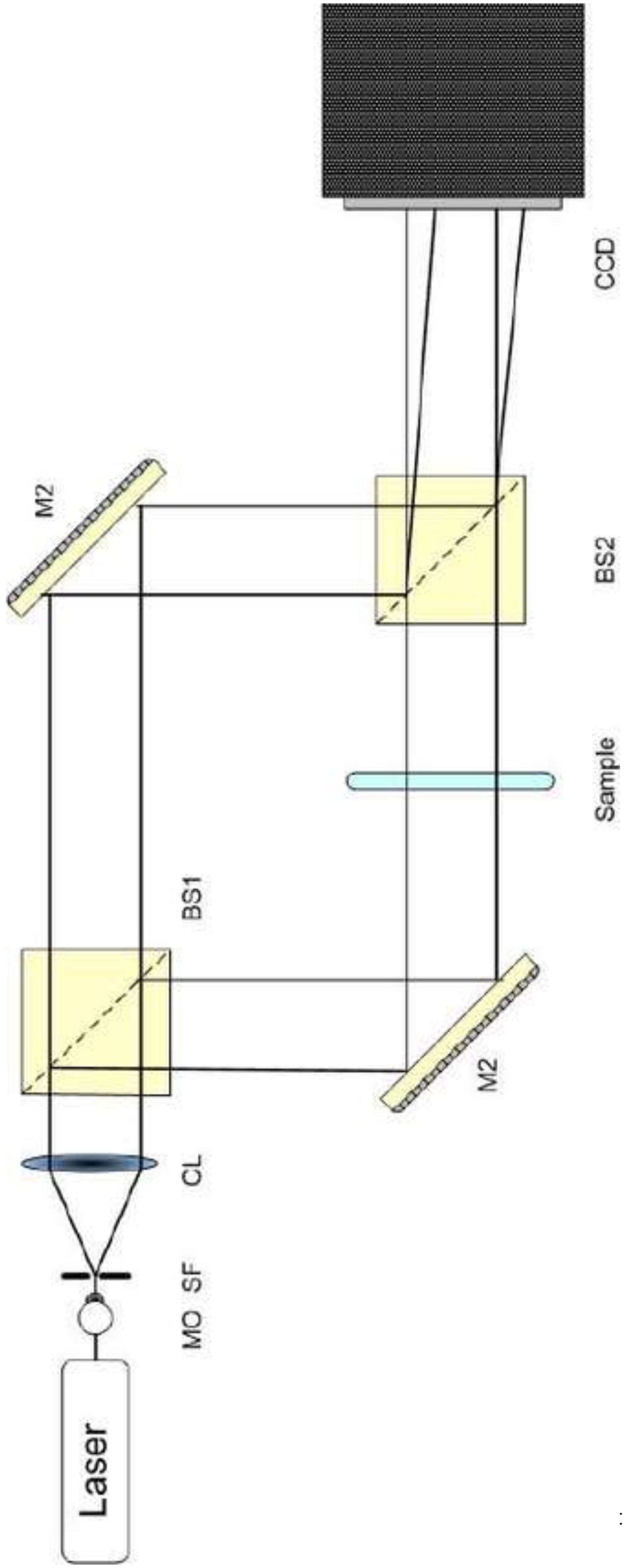




Lei Tian, J. C. Petruccelli, and G. Barbastathis, OSA Digital holography and Three-Dimensional Imaging, paper DTh4A.4, 2013

Holography – Inline vs. Off-axis Setup





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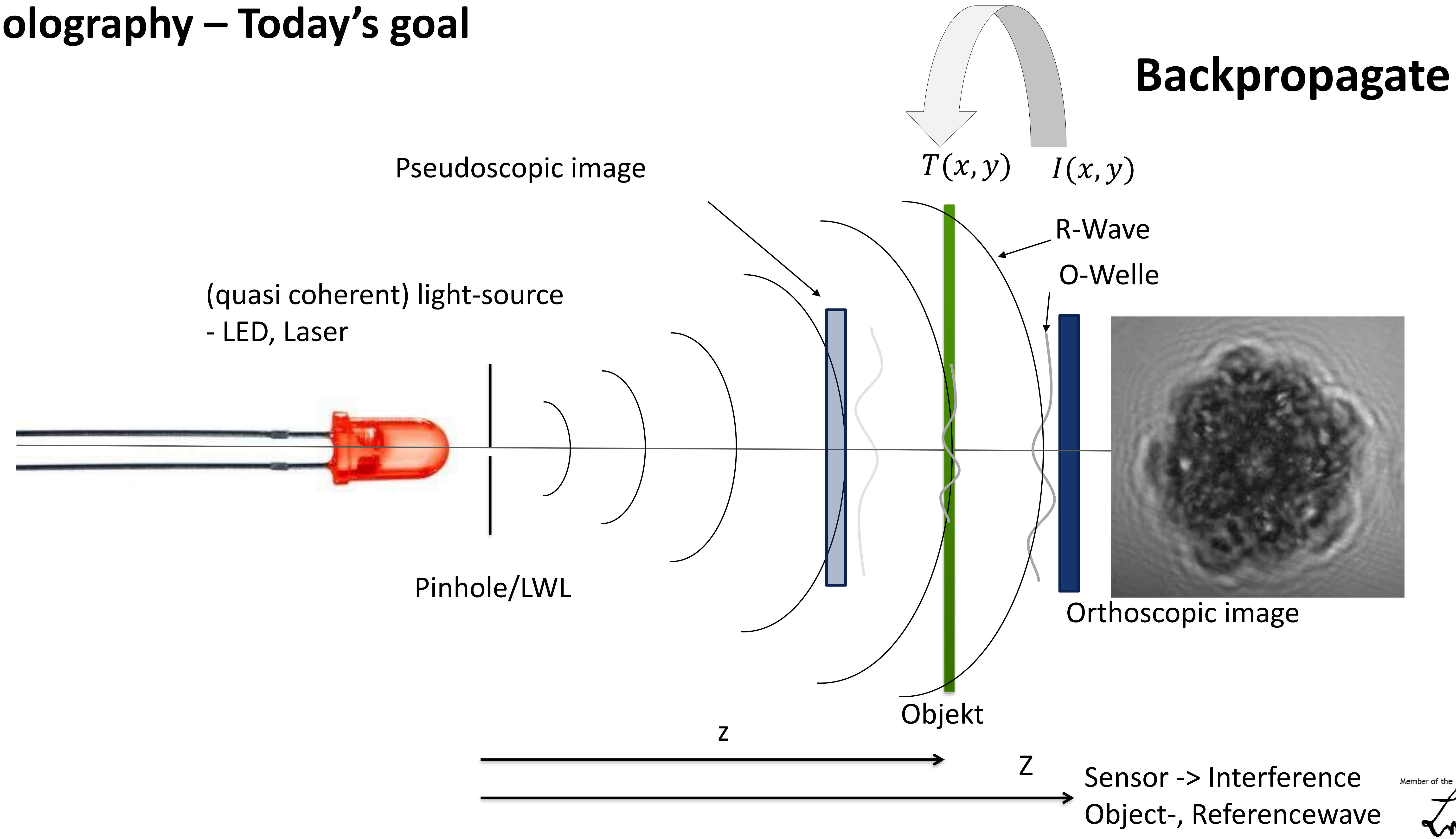
Holography – Today's goal



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COMPUTATIONAL TOOLS

Spherical Wav

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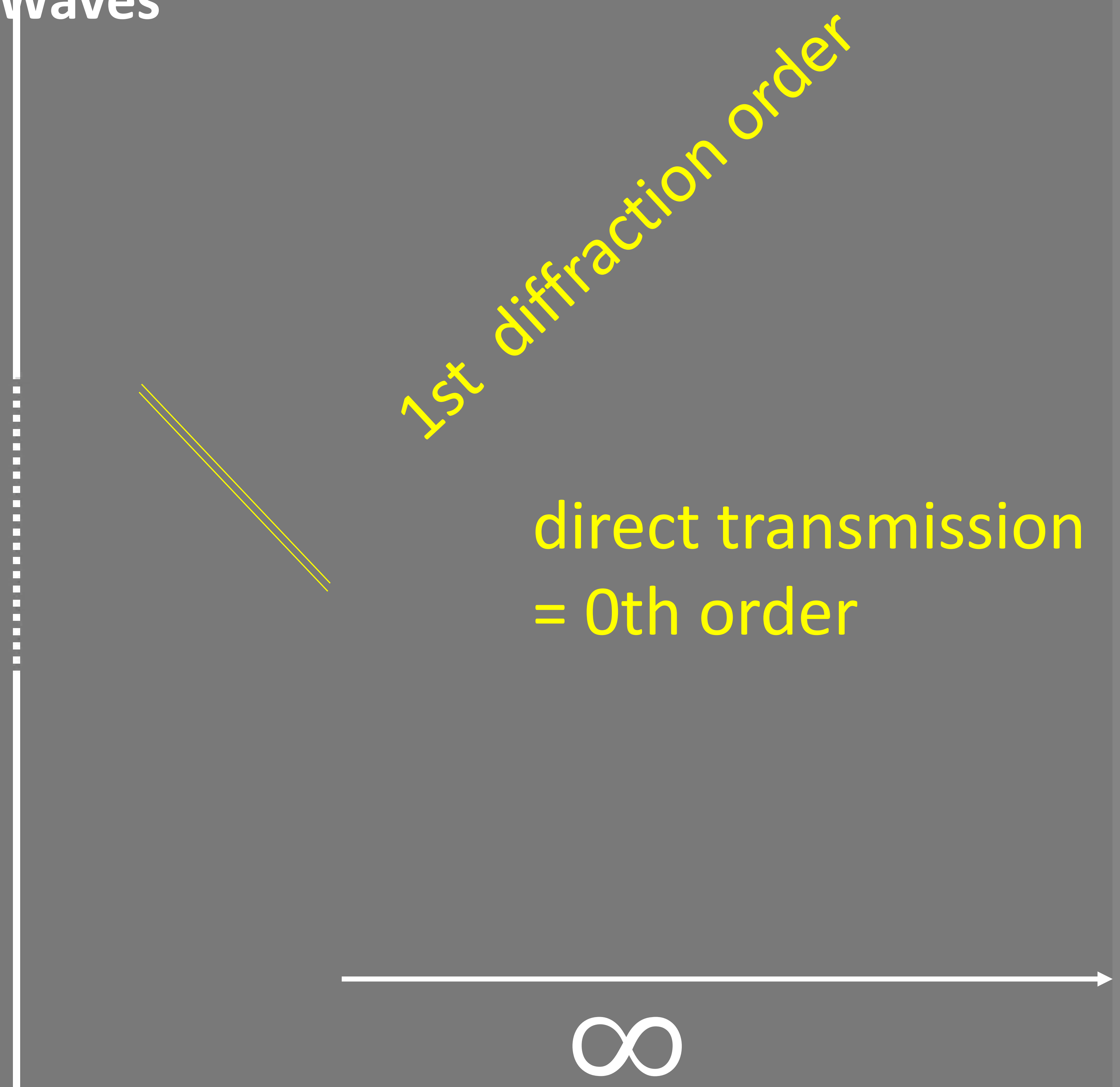
Multiple Spherical Waves



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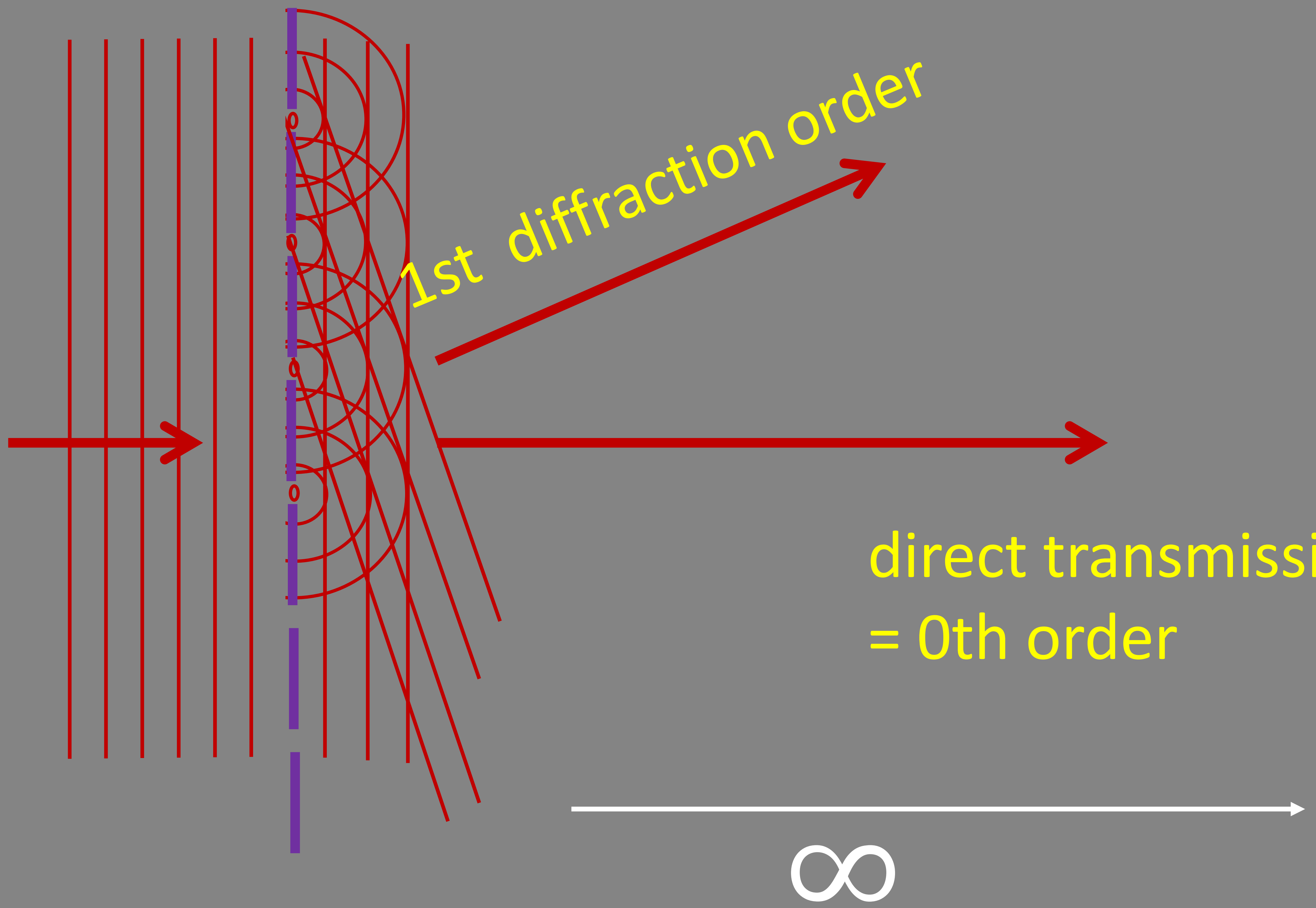
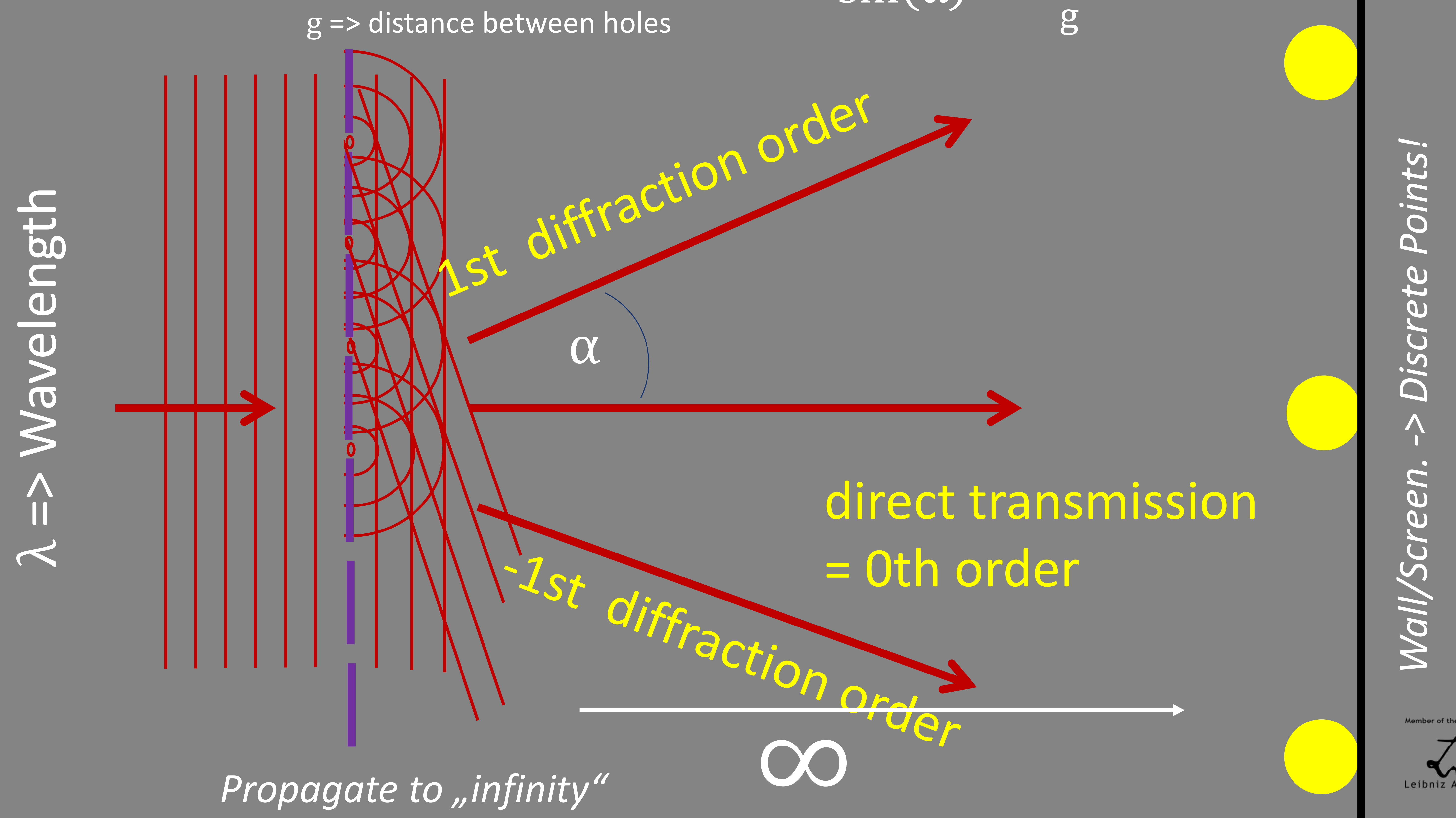
Multiple Spherical Waves - Grating



Multiple Spherical Waves - Grating

$$\sin(\alpha) = \frac{\lambda \cdot m}{g}$$



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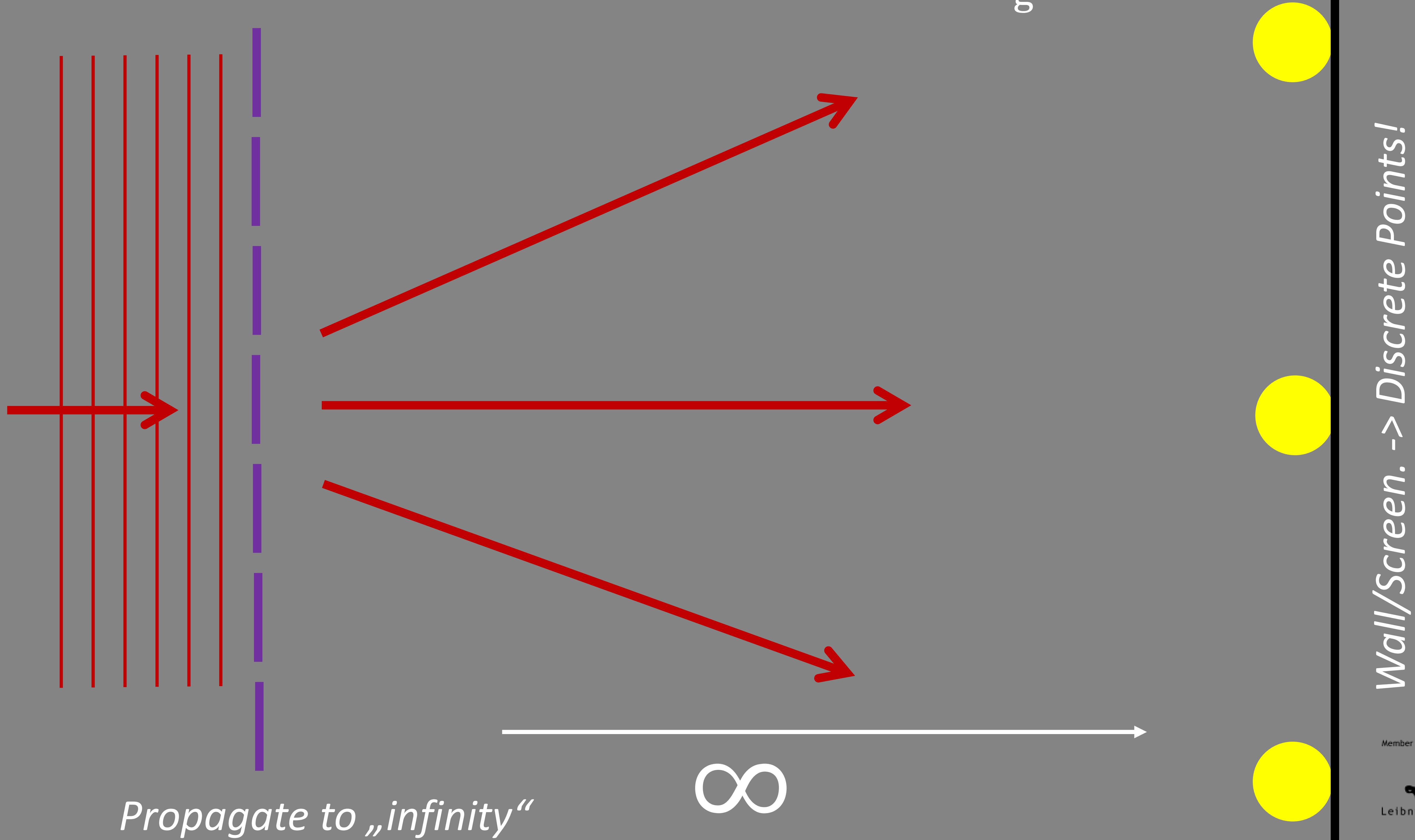
Multiple Spherical Waves - Grating

$$\sin(\alpha) = \frac{\lambda \cdot m}{g}$$



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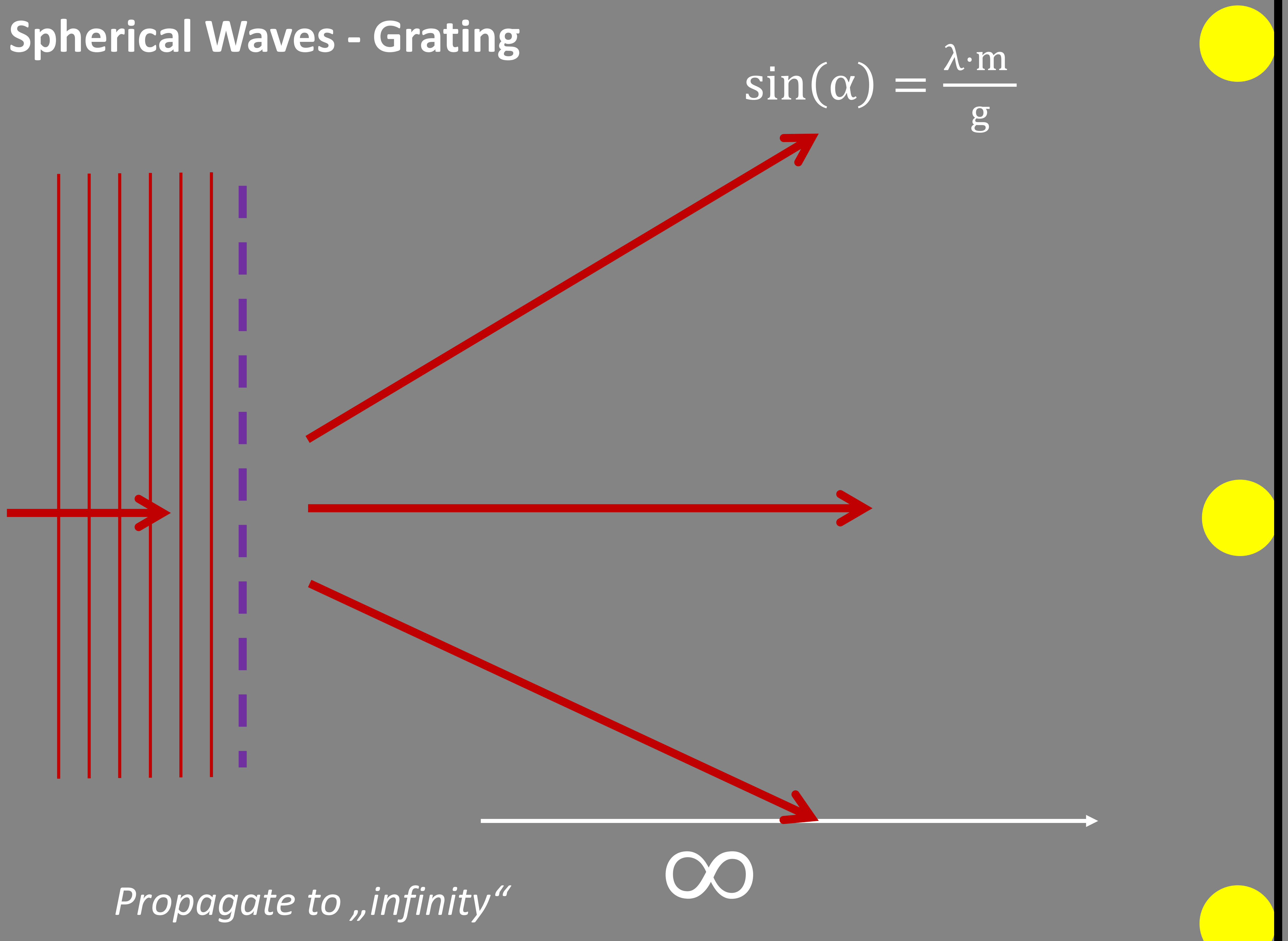
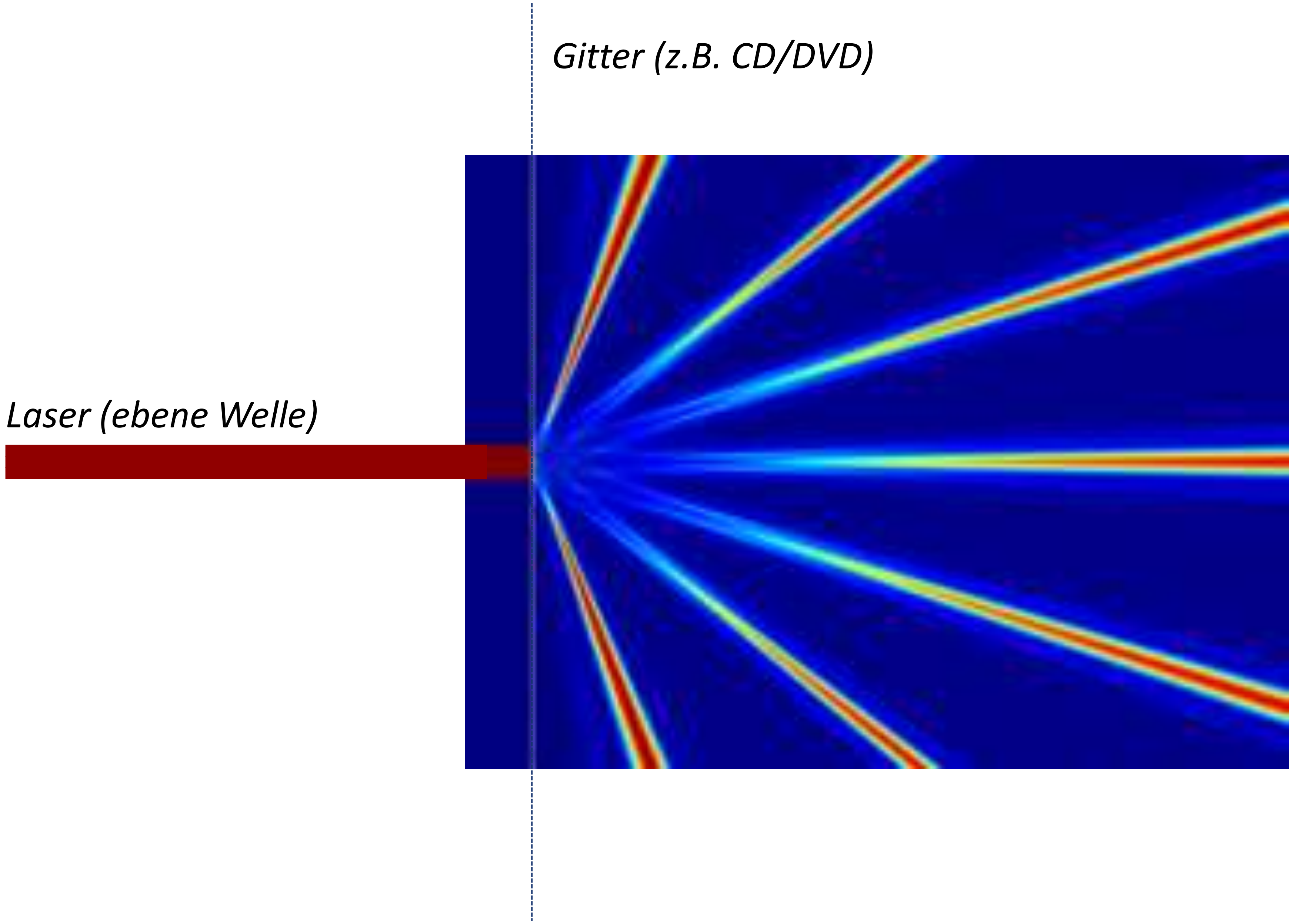
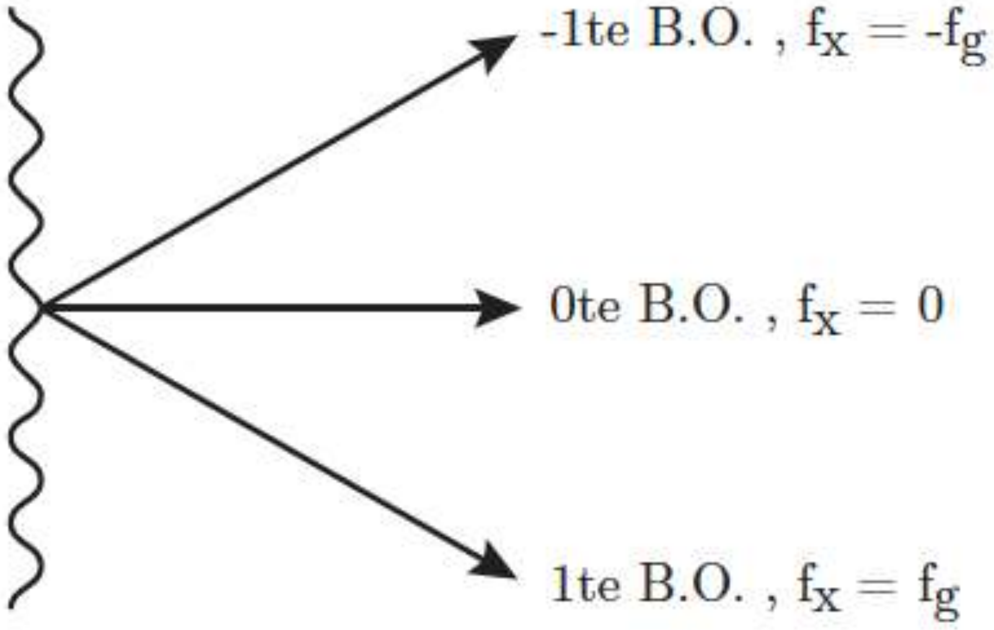
Multiple Spherical Waves - Grating

$$\sin(\alpha) = \frac{\lambda \cdot m}{g}$$



Holography – Diffraction





$$\sin(\alpha) = \frac{\lambda \cdot m}{g}$$

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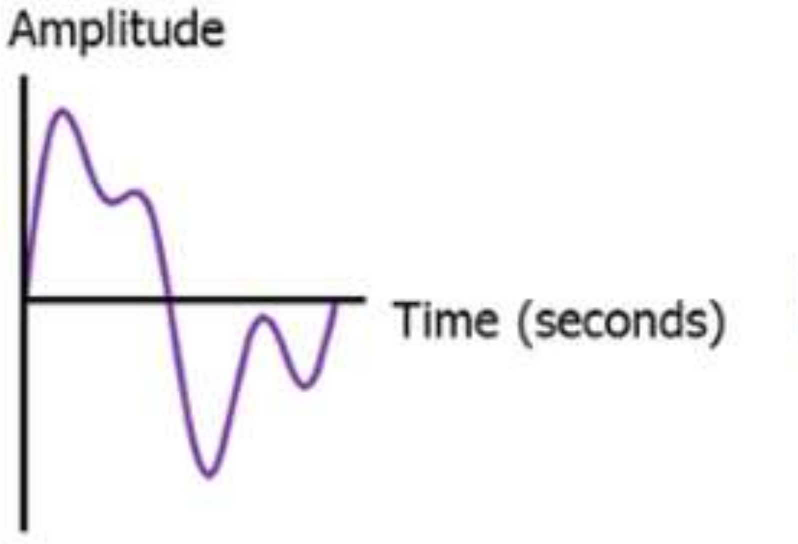
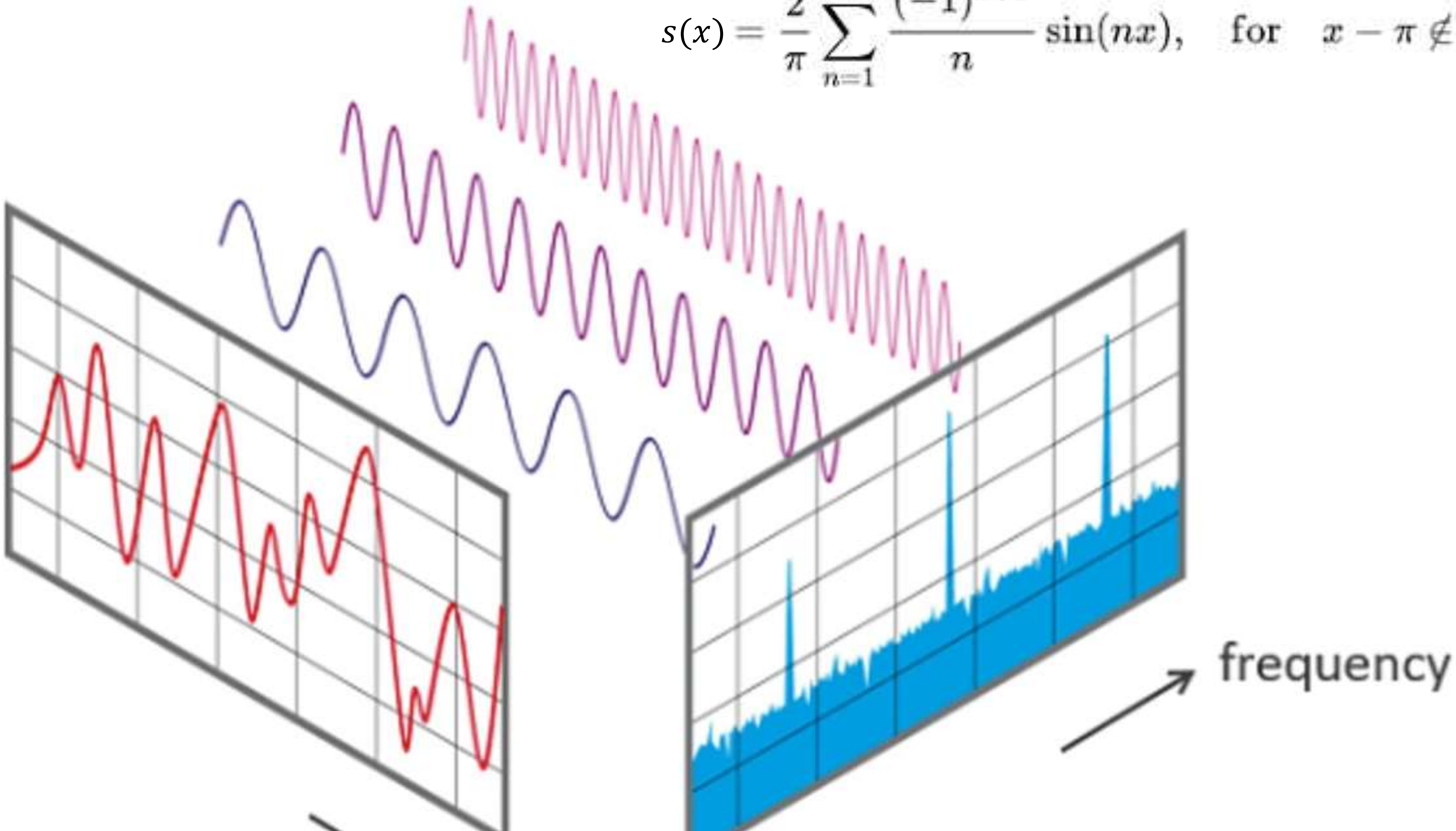
Mix of frequencies?

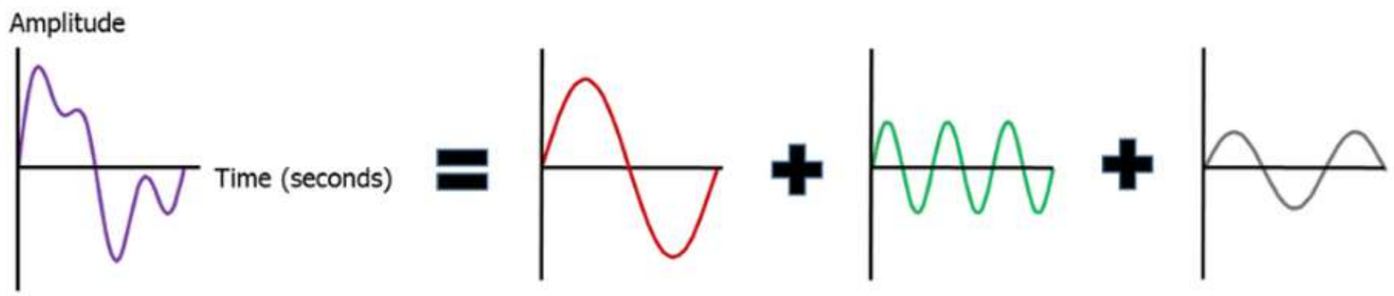


Bring infinity a little bit closer

$$s(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(nx), \quad \text{for} \quad x-\pi \notin 2\pi\mathbb{Z}.$$



Joseph Fourier



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Fourier Transform

Fourier Transformation

$g(x, y)$ is some signal in space

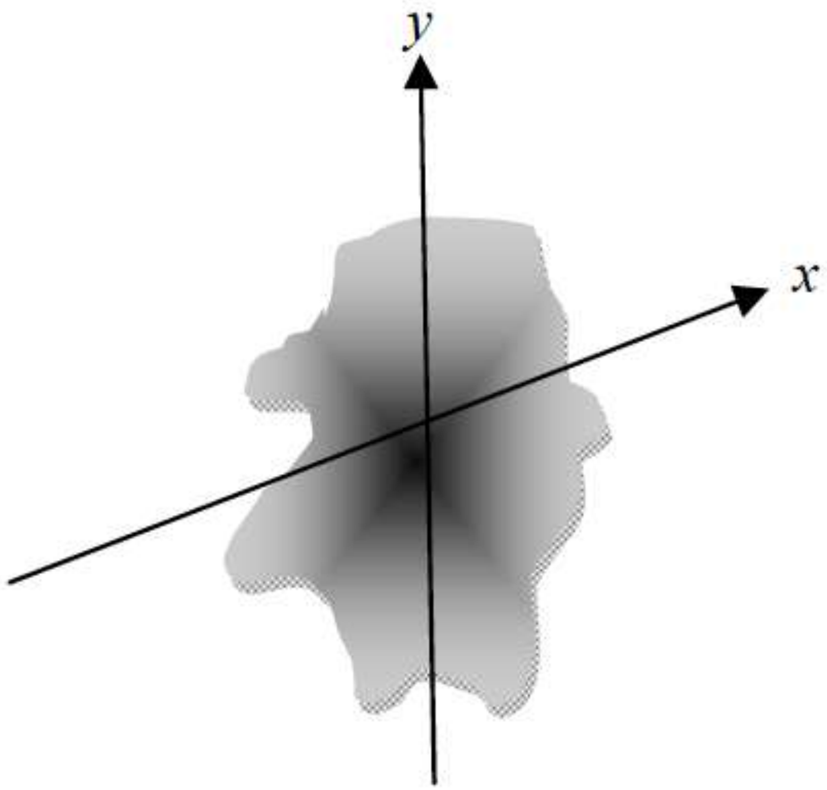
$$G(x, y) = \int_{-\infty}^{\infty} g(x, y) \cdot \exp(-2\pi i\, (v_x x + v_y y))\mathrm{dx\, dy}$$

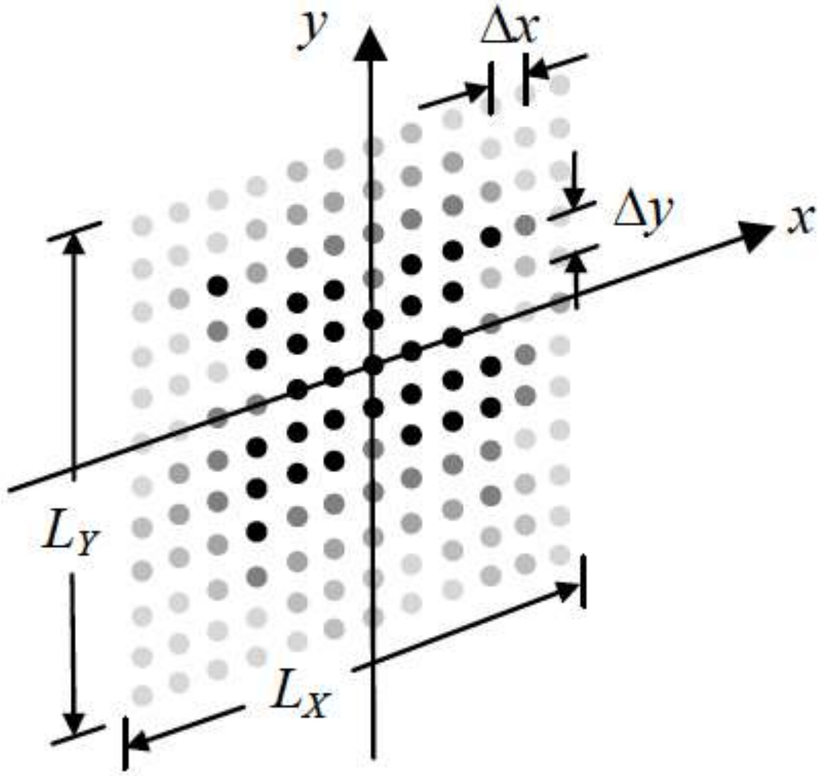
Inverse Fourier Transformation

$G(x, y)$ is some signal in frequency space

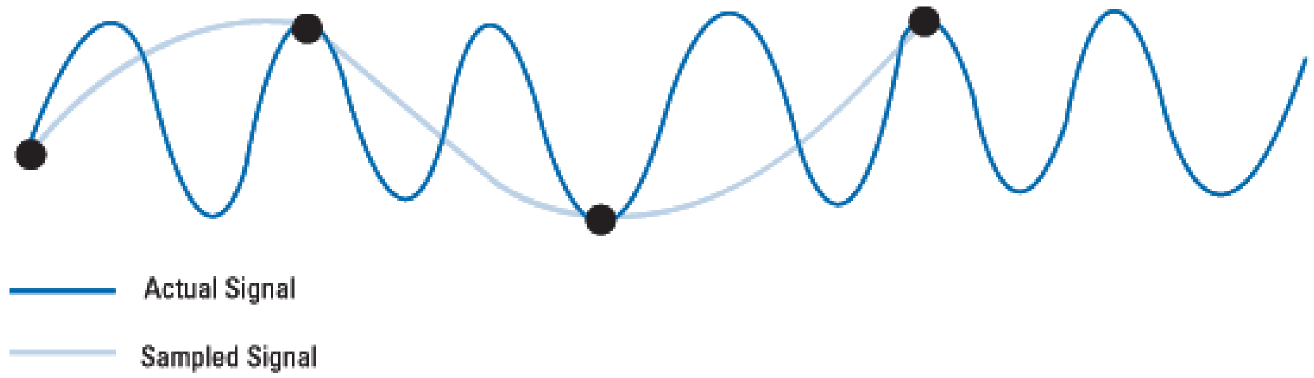
$$\mathrm{g}(x, y) = \int_{-\infty}^{\infty} G(x, y) \cdot \exp(2\pi i\, (v_x x + v_y y))\mathrm{dx\, dy}$$

Sampling



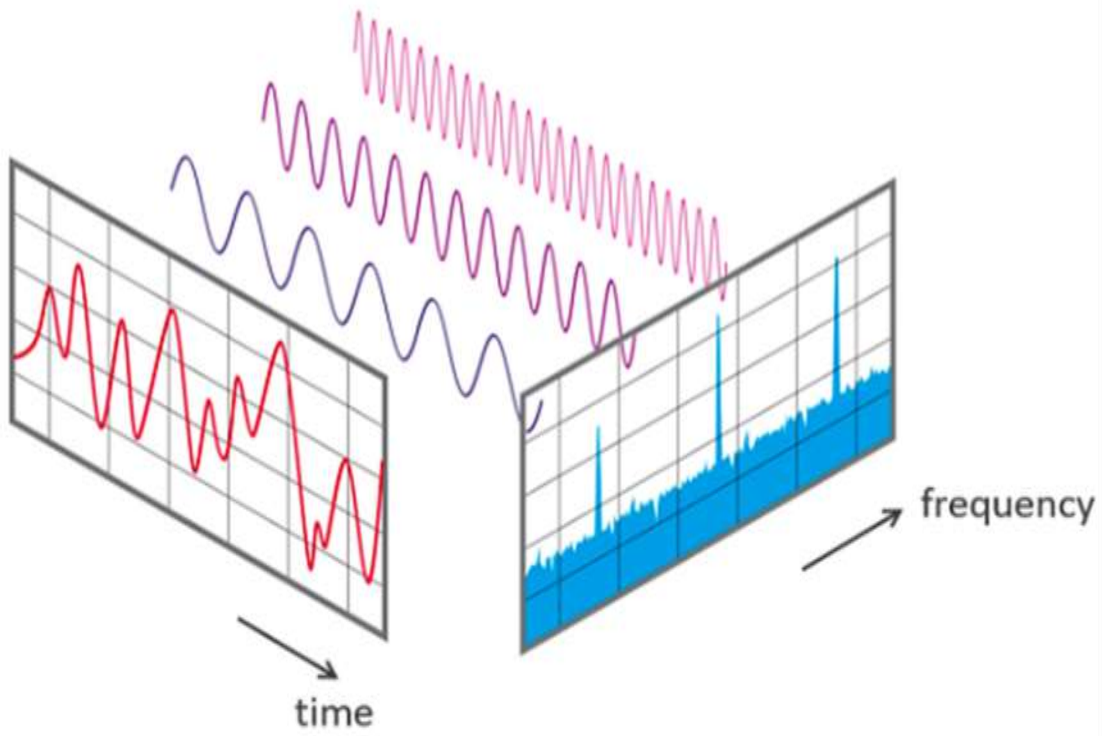


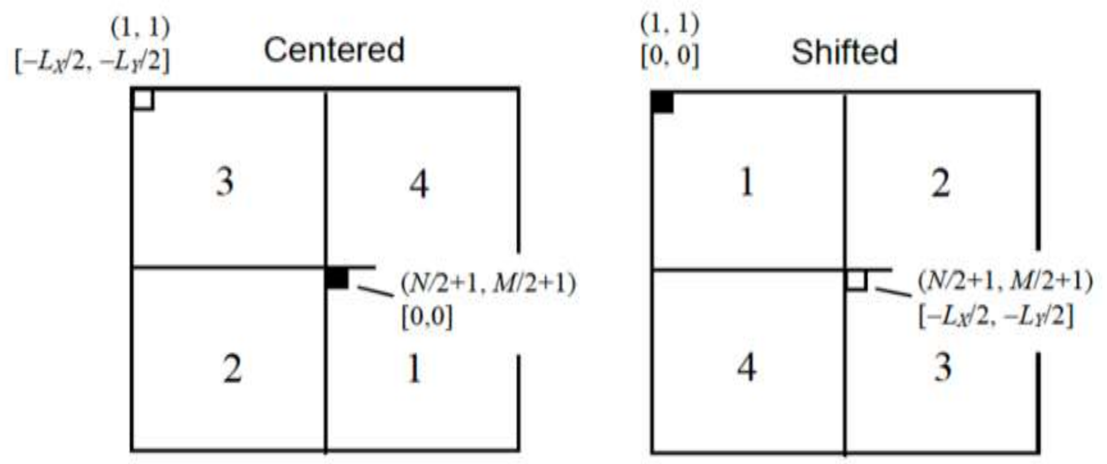
- sample interval $\Delta x/\Delta y$
- m/n pixels in x/y
- Nyquist-Shannon criteria:
  - $\nu_{Nx} = \frac{1}{2\Delta x}$



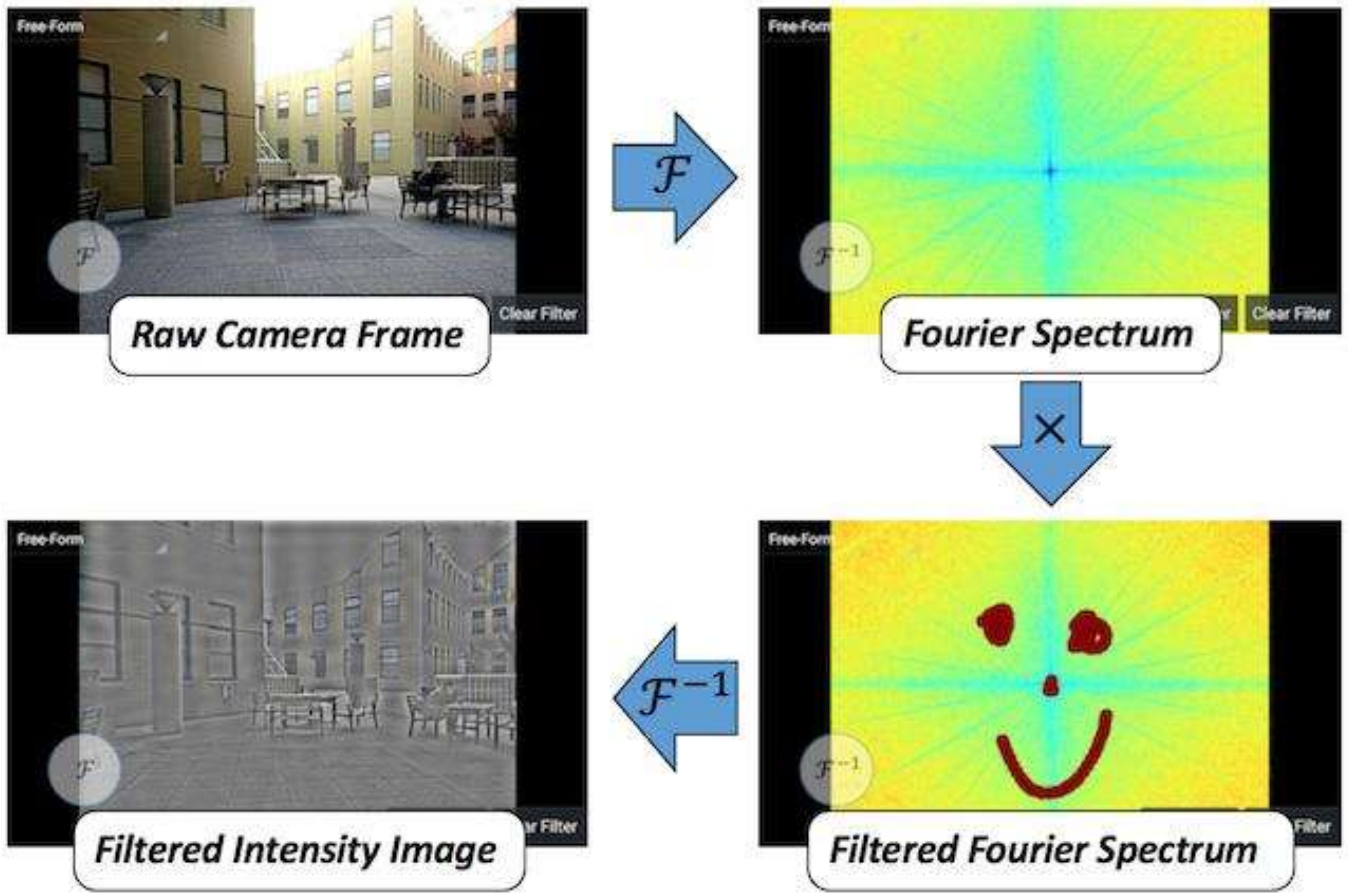
Fast Fourier Transform (FFT)

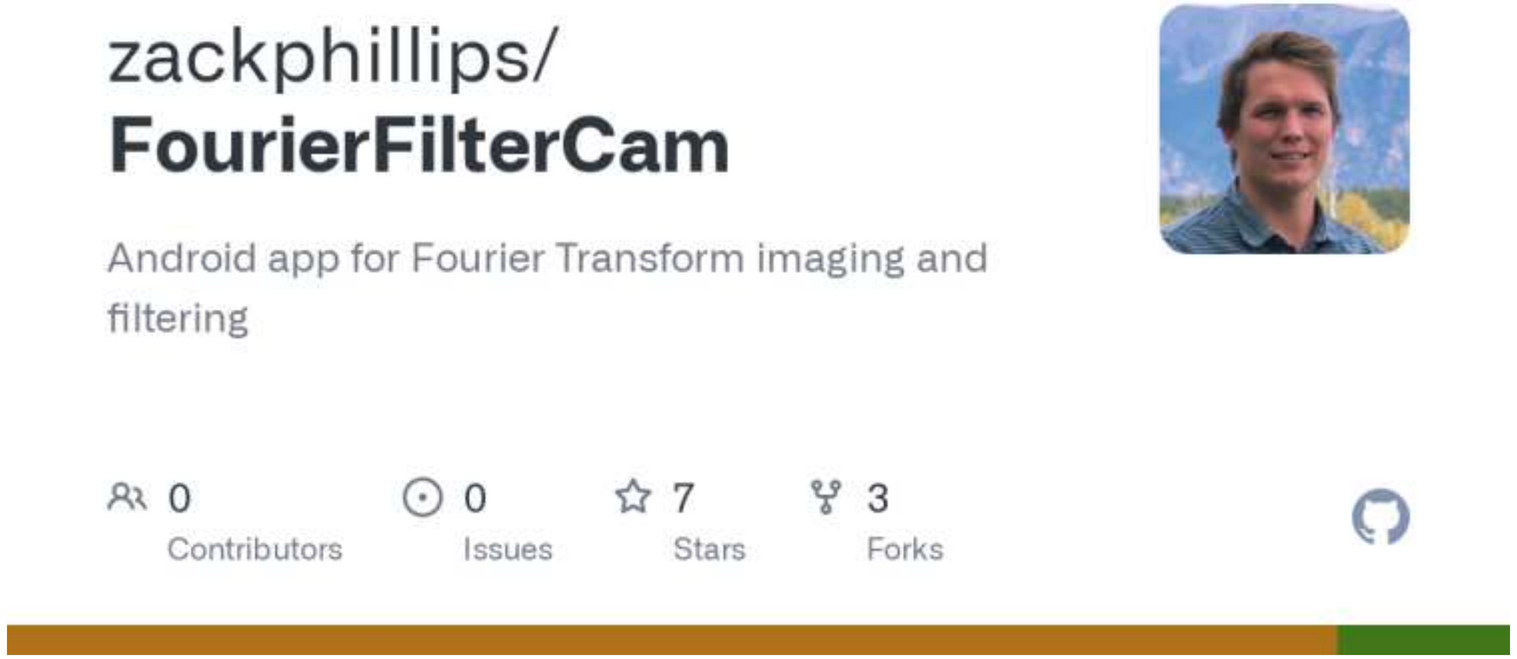
- Fourier Analysis instead of Fourier Transform
- Fourier Analysis is carried out on a discrete grid (i.e. matrix) with constant grating constant:
  - $\Delta f_x = \frac{1}{M\Delta x}, M =$ number of sampling points in xx
- $G_k(m,n) = \sum^{\frac{M}{2}-1}{}_M \sum^{\frac{M}{2}-1}{}_M \, g(m,n)_n \exp\left(-2\pi i \left(\frac{pm}{M} + \frac{qn}{N}\right)\right)$





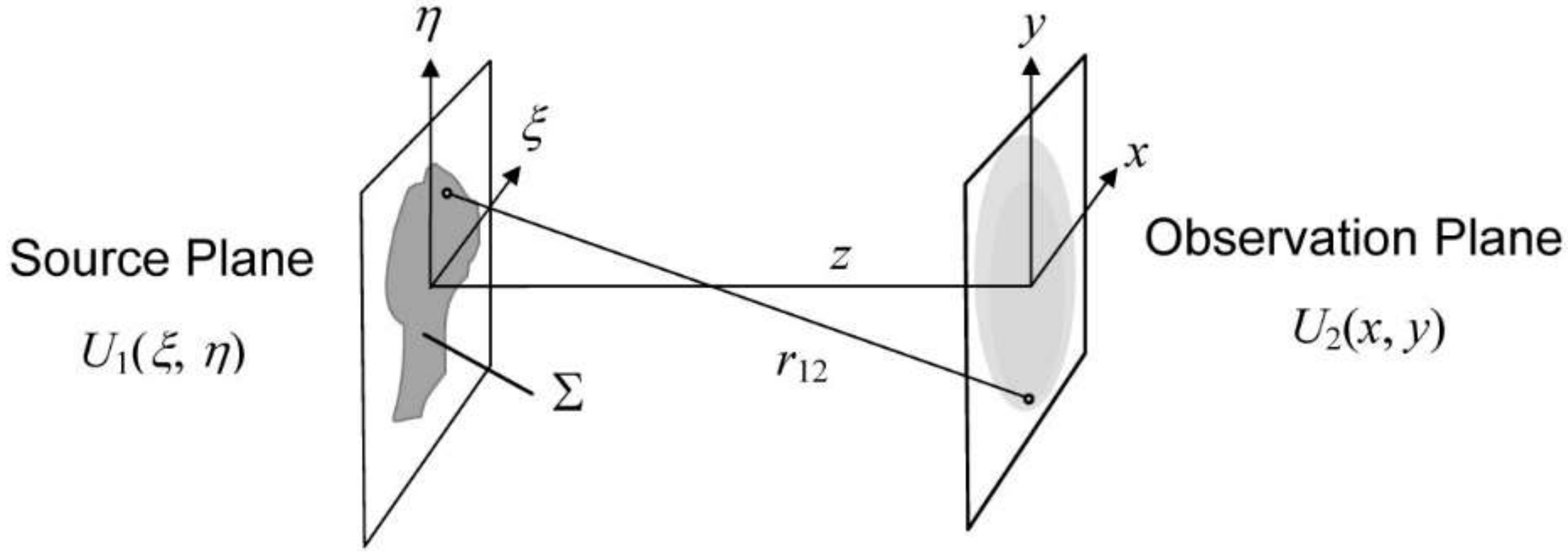
Fourier Filter Cam for your phone!





https://github.com/zackphillips/FourierFilterCam

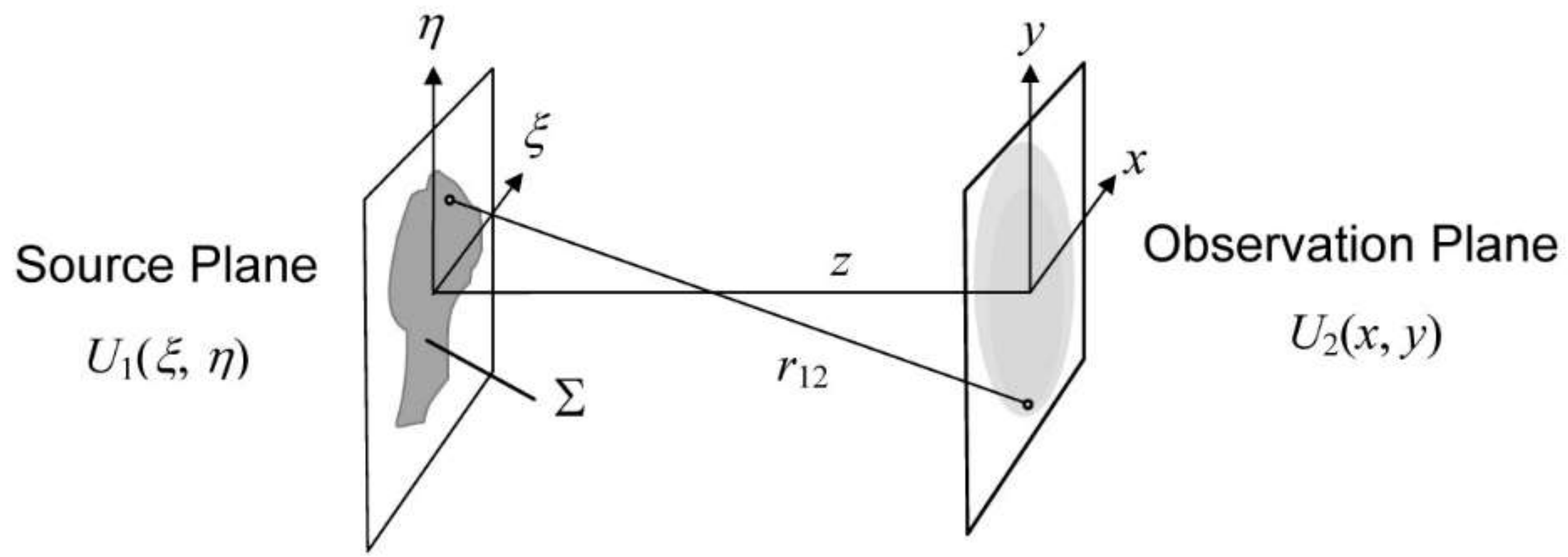
Holography – Propagation; Rayleigh Sommerfeld Approximation



$$U_2(x,y) = \frac{z}{i\lambda}\iint_{\Sigma} \frac{U_1(\epsilon,\eta)\exp(i\,kr_{12})}{r_{12}^2}d\xi d\eta$$

- $r_{12} = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2}$
- Propagate all points on the source plane that act as point-sources (spherical waves) and integrate/sum on the observation plane (detector) coherently (phase matters!)
- $r \gg \lambda$

Holography – Propagation; Fresnel Approximation



$$U_2(x,y) = \frac{\exp(ikz)}{i\lambda z} \iint U_1(\epsilon,\eta) \exp\left(i \frac{k}{2z}\left((x-\xi)^2 + (y-\eta)^2\right)\right) d\xi d\eta$$

- Simplification: approdimate square-root for r_{12}
- Now: Convolution of signal with Fresnel kernel (in FT-space: multiply with chirp function)

$$U_2(x,y) = \frac{\exp(ikz)}{i\lambda z} \exp\left(i \frac{k}{2z}(x^2+y^2)\right)$$

$$\iint U_1(\epsilon,\eta) \exp\left(i \frac{k}{2z}\left((\xi)^2 + (\eta)^2\right)\right) \exp\left(-i\left(\frac{2\pi}{\lambda z}(x\xi + y\eta)\right)\right) d\xi d\eta$$

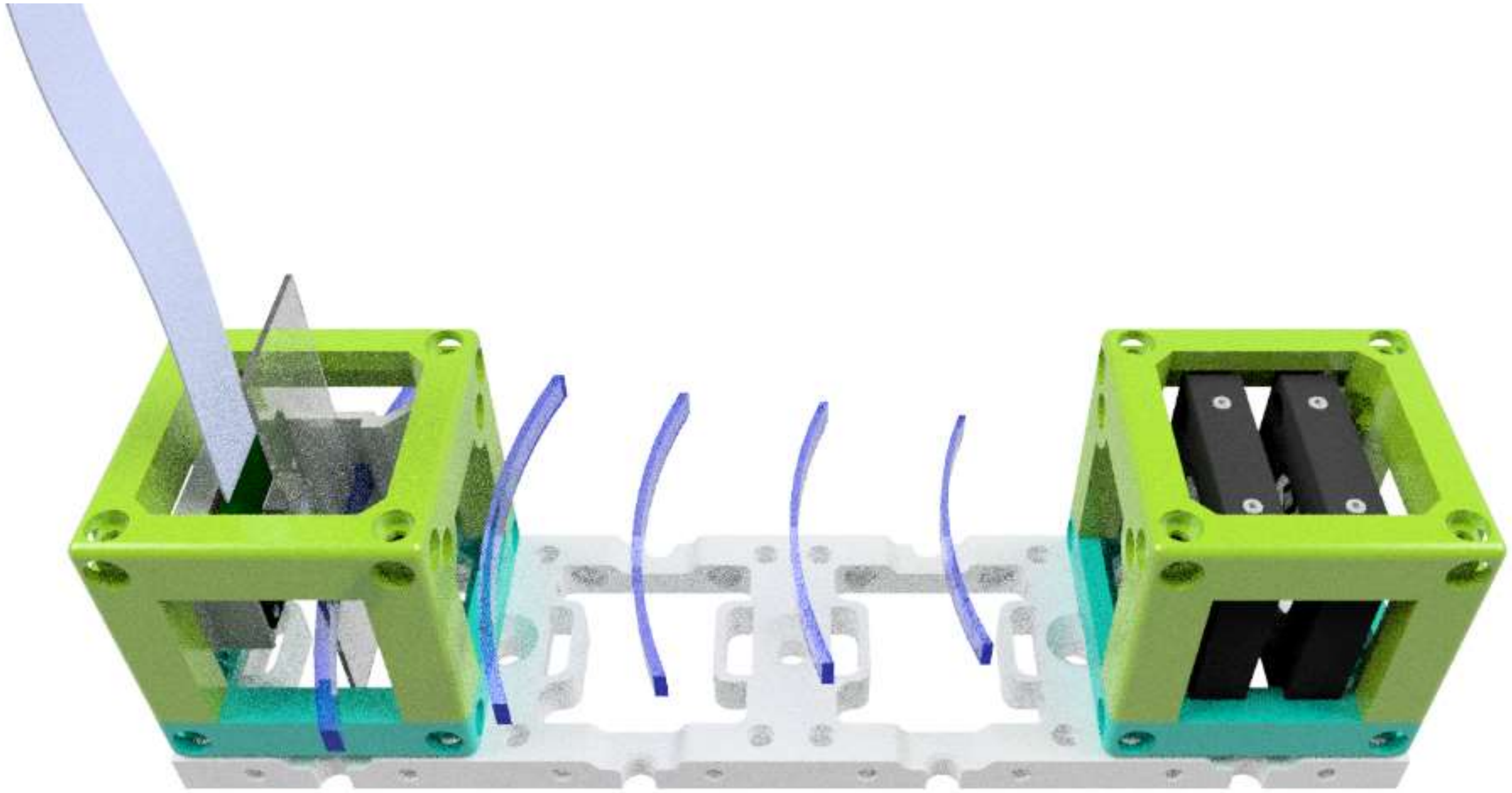
Holography – Propagation; Fresnel Approximation

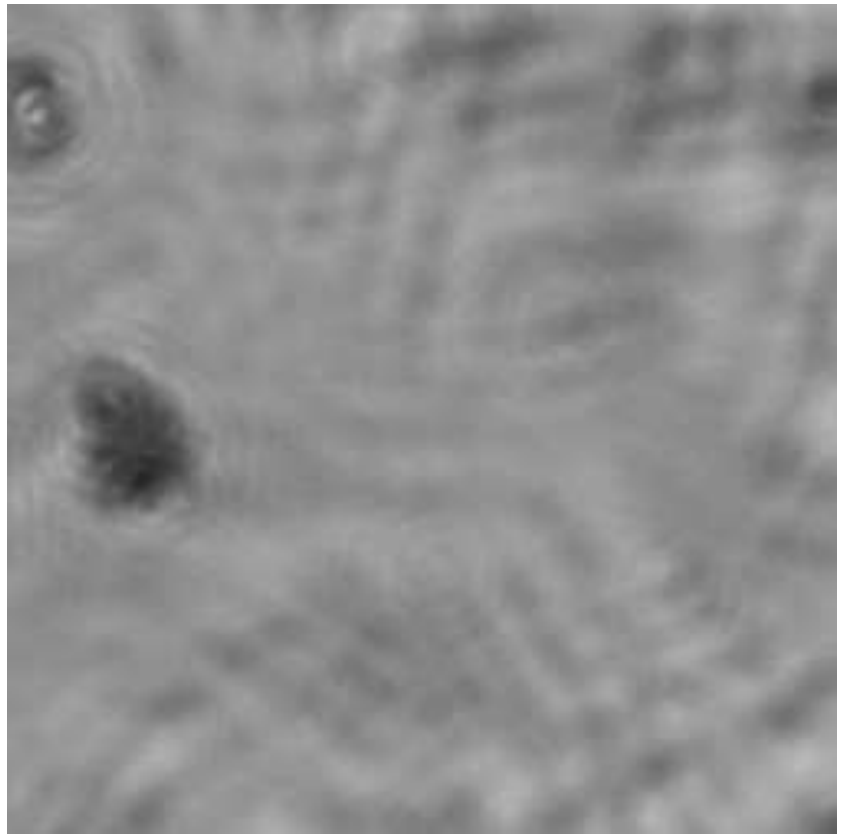
- Fresnel propagation as a convolution in real-space
- Or: multiplication in frequency-space :

$$U_2(x,y) = \mathfrak{I}^{-1}\{\mathfrak{I}\{U_1(x,y\}H_f(f_x,f_y)\}$$

$$H_f\left(f_x,f_y\right) = \exp(i2\pi z)\cdot\exp\left(-i\pi\lambda z\left(f_x^2+f_y^2\right)\right)$$

Holographie – Heutiges Ziel



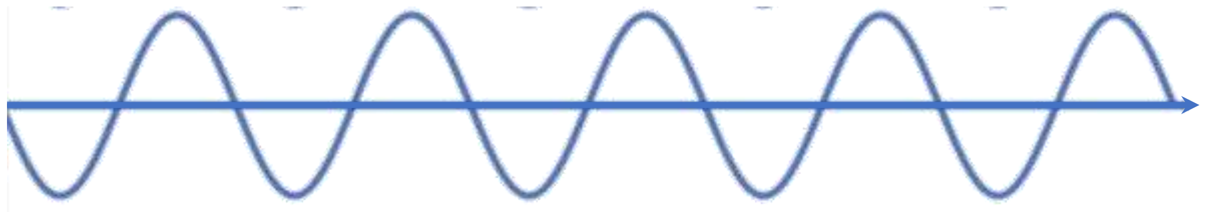


Inline Holography – Lensless Imaging

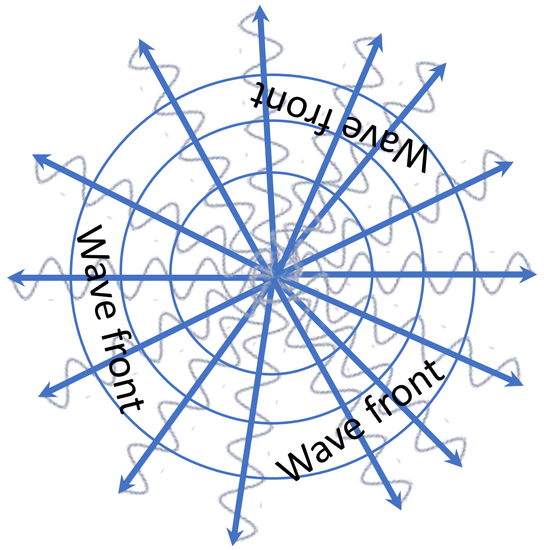
Interference and Diffreaction cannot be explained using Ray Optics

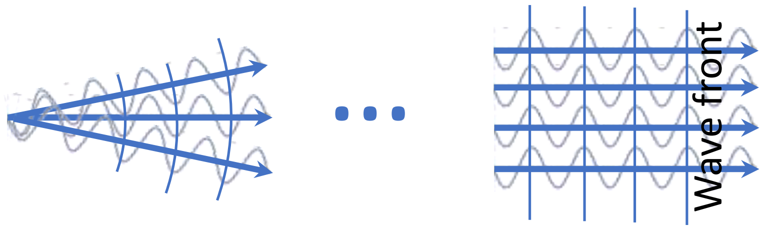
Wave Optics describes light as an electromagnetic wave

Light ray



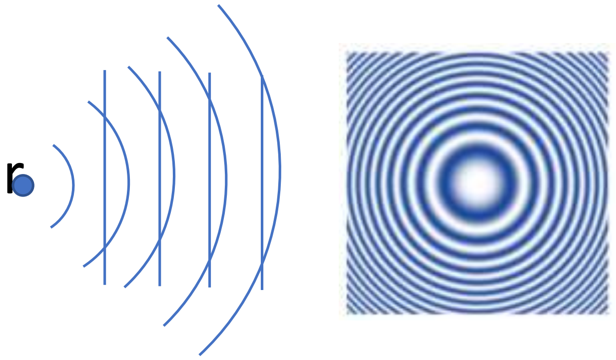
Spherical wave propagates from a point light source.



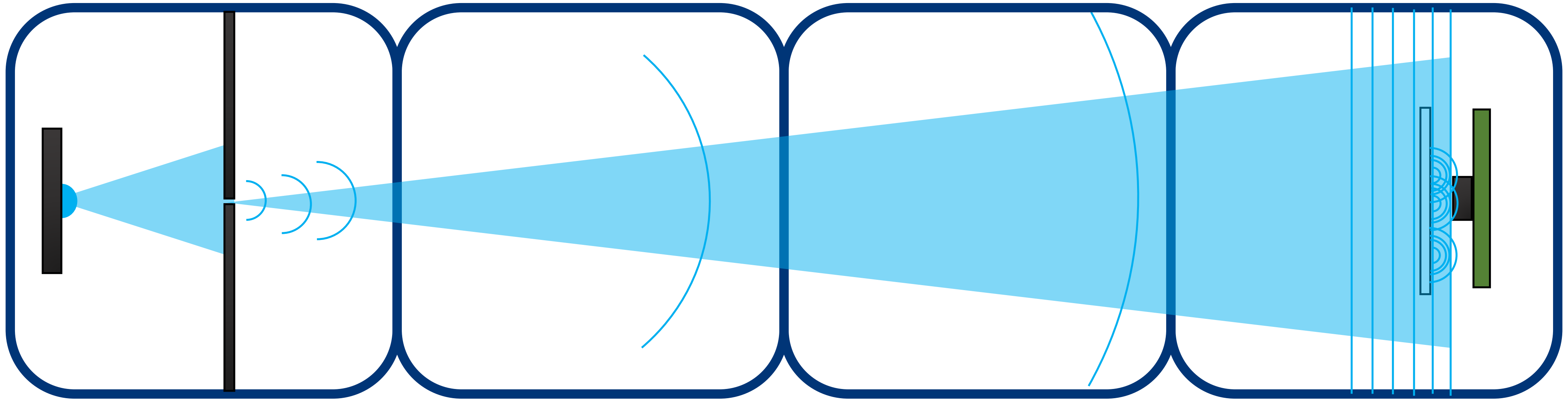


Far away from the light source the spherical waves look like plane waves.

Interference – when the waves meet each other



Hologram of a point-like object



Hologram of Diatoms?

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