

# Flux Balance Analysis - part 2

**Author(s): Ronan Fleming, Ines Thiele, University of Galway; Jeff Orth, University of California, San Diego.**

**Reviewer(s):**

## Table of Contents

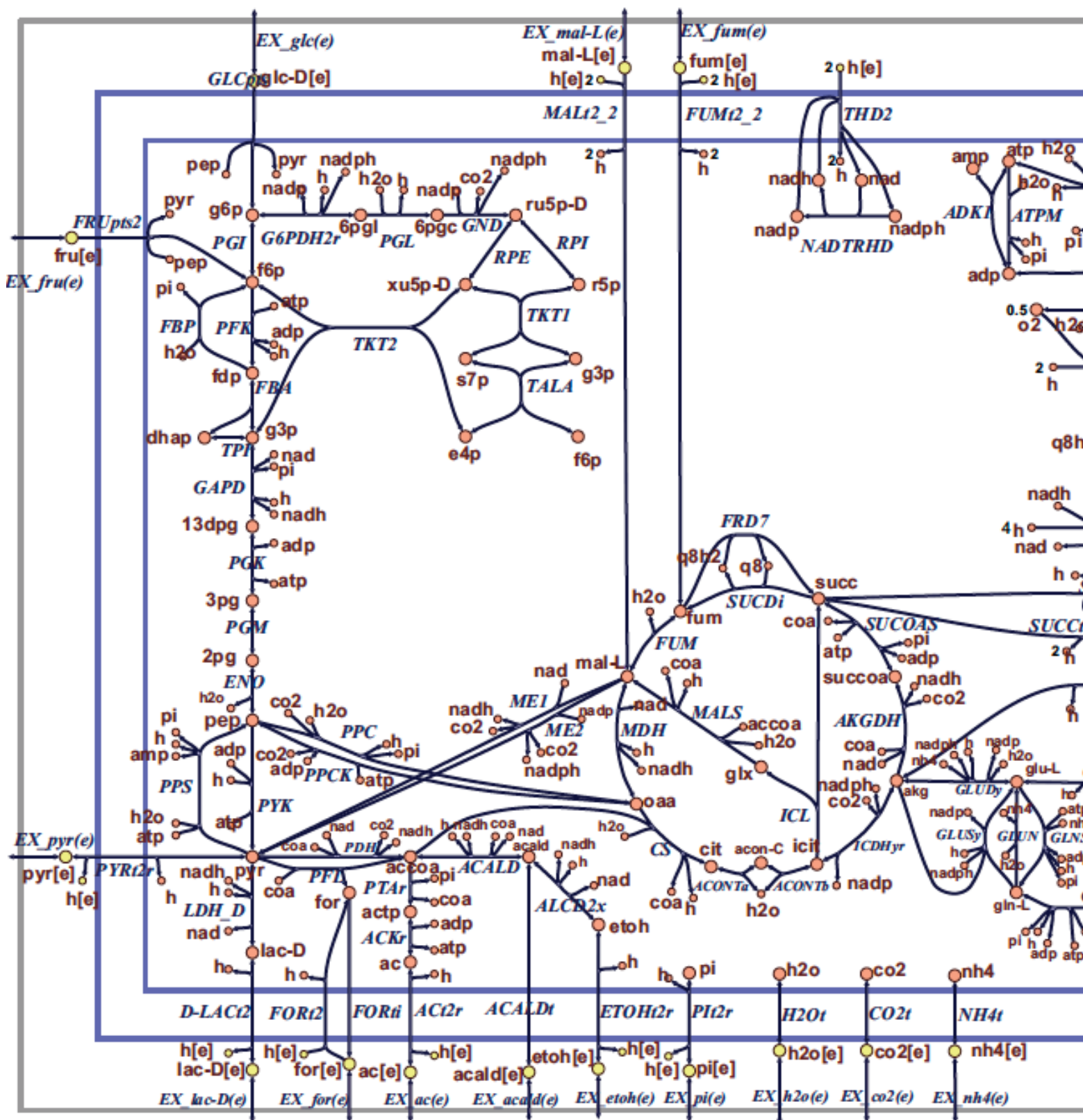
Author(s): Ronan Fleming, Ines Thiele, University of Galway; Jeff Orth, University of California, San Diego.....	1
Reviewer(s): .....	1
INTRODUCTION.....	2
E. coli core model.....	2
MATERIALS - EQUIPMENT SETUP.....	4
PROCEDURE.....	4
Load E. coli core model.....	4
Checking the non-trivial constraints on a model.....	6
What are the default constraints on the model? .....	6
Hint: printConstraints.....	6
Example 5. Robustness analysis.....	6
What is the effect of varying glucose uptake rate on growth? At what uptake rates does glucose alone constrain growth?.....	6
Hint: fix the oxygen uptake rate to 17 mmol gDW-1 hr-1.....	6
What is the growth rate as a function of oxygen uptake rate with glucose uptake held constant?.....	8
Hint: fix glucose uptake at 10 mmol gDW-1 hr-1.....	8
Example 6. Simulating gene knockouts.....	9
Which genes are required to be knocked out to eliminate flux through the phosphofructokinase (PFK) reaction? .....	9
Hint: study the model.....	9
What is the growth rate of E. coli when the gene b1852 (zwf) is knocked out?.....	10
Hint: deleteModelGenes.....	10
Knockout every pairwise combination of the 136 genes in the E. coli core model and display the results.....	10
Hint: doubleGeneDeletion.....	10
Example 7. Which genes are essential for which biomass precursor? .....	11
Which genes are essential for synthesis of each biomass precursor?.....	11
Hint: addDemandReaction.....	11
Which biomass precursors cannot be net synthesised by the E. coli core model? What does this say about the E. coli core model itself?.....	13
Hint: think about steady state versus mass balance.....	13
Example 8. Which non-essential gene knockouts have the greatest effect on the network flexibility?.....	13
What reactions are increased and decreased in flux span in response to deletion of these 6 genes? {'b0114', 'b0723', 'b0726', 'b0767', 'b1602', 'b1702'} With reference to a particular example, what is the biochemical interpretation of an increased flux span in response to a gene deletion? .....	14
Hint: fluxVariability, hist.....	14
TIMING.....	15
Acknowledgments.....	15
REFERENCES.....	15

## INTRODUCTION

In this tutorial, Flux Balance Analysis (FBA) is introduced using the E. coli core model, with functions in the COBRA Toolbox v3.0 [2].

### **E. coli core model**

A map of the E. coli core model is shown in Figure 1.



**Figure 1 Map of the core *E. coli* metabolic network.** Orange circles represent cytosolic metabolites, yellow circles represent extracellular metabolites, and the blue arrows represent reactions. Reaction name abbreviations are uppercase (blue) and metabolite name abbreviations are lowercase (rust colour). This flux map was drawn using SimPheny and edited for clarity with Adobe Illustrator.

## MATERIALS - EQUIPMENT SETUP

Please ensure that all the required dependencies (e.g. , `git` and `curl`) of The COBRA Toolbox have been properly installed by following the installation guide [here](#). Please ensure that the COBRA Toolbox has been initialised (`tutorial_initialize.mlx`) and verify that the pre-packaged LP and QP solvers are functional (`tutorial_verify.mlx`).

## PROCEDURE



























### Load E. coli core model

The most direct way to load a model into The COBRA Toolbox is to use the `readCbModel` function. For example, to load a model from a MAT-file, you can simply use the filename (with or without file extension).

```
fileName = 'ecoli_core_model.mat';  
if ~exist('modelOri','var')  
    modelOri = readCbModel(fileName);  
end  
%backward compatibility with primer requires relaxation of upper bound on  
%ATPM  
modelOri = changeRxnBounds(modelOri,'ATPM',1000,'u');  
model = modelOri;
```

model

1x1 struct with 28 fields

Field ▲	Value	Size
 S	72x95 sparse do...	72x95
 mets	72x1 cell	72x1
 b	72x1 double	72x1
 csense	72x1 char	72x1
 rxns	95x1 cell	95x1
 lb	95x1 double	95x1
 ub	95x1 double	95x1
 c	95x1 double	95x1
 osenseStr	'max'	1x3
 genes	137x1 cell	137x1
 rules	95x1 cell	95x1
 metCharges	72x1 int32	72x1
 metFormulas	72x1 cell	72x1
 metNames	72x1 cell	72x1
 metInChIString	72x1 cell	72x1
 metKEGGID	72x1 cell	72x1
 metChEBIID	72x1 cell	72x1
 metPubChemID	72x1 cell	72x1
 grRules	95x1 cell	95x1
 rxnGeneMat	95x137 sparse d...	95x137
 rxnConfidence...	95x1 double	95x1
 rxnNames	95x1 cell	95x1
 rxnNotes	95x1 cell	95x1
 rxnECNumbers	95x1 cell	95x1
 rxnReferences	95x1 cell	95x1
 subSystems	95x1 cell	95x1

The meaning of each field in a standard model is defined in the [standard COBRA model field definition](#).

In general, the following fields should always be present:

- **S**, the stoichiometric matrix
- **mets**, the identifiers of the metabolites
- **b**, Accumulation (positive) or depletion (negative) of the corresponding metabolites. 0 Indicates no concentration change.
- **csense**, indicator whether the b vector is a lower bound ('G'), upper bound ('L'), or hard constraint 'E' for the metabolites.
- **rxns**, the identifiers of the reactions
- **lb**, the lower bounds of the reactions
- **ub**, the upper bounds of the reactions
- **c**, the linear objective
- **genes**, the list of genes in your model
- **rules**, the Gene-protein-reaction rules in a computer readable format present in your model.
- **osenseStr**, the objective sense either 'max' for maximisation or 'min' for minimisation

## Checking the non-trivial constraints on a model

What are the default constraints on the model?

Hint: `printConstraints`

```
printConstraints(model, -1000, 1000)
```

```
MinConstraints:
ATPM      8.39
EX_glc(e) -10
maxConstraints:
```

## Example 5. Robustness analysis

Another method that uses FBA to analyze network properties is robustness analysis [14]. In this method, the flux through one reaction is varied and the optimal objective value is calculated as a function of this flux. This reveals how sensitive the objective is to a particular reaction. There are many insightful combinations of reaction and objective that can be investigated with robustness analysis. One example is examination of the effects of nutrient uptake on growth rate.

**What is the effect of varying glucose uptake rate on growth? At what uptake rates does glucose alone constrain growth?**

Hint: fix the oxygen uptake rate to 17 mmol gDW-1 hr-1

To determine the effect of varying glucose uptake on growth, first use `changeRxnBounds` to set the oxygen uptake rate (`EX_o2(e)`) to 17 mmol gDW-1 hr-1, a realistic uptake rate. Then, use a for loop to set both the upper and lower bounds of the glucose exchange reaction to values between 0 and -20 mmol gDW-1 hr-1, and use `optimizeCbModel` to perform FBA with each uptake rate. Be sure to store each growth rate in a vector or other type of Matlab list. The COBRA Toolbox also contains a function for performing robustness analysis (`robustnessAnalysis`) that can perform these functions. The full code to perform this robustness analysis is:

```

model = modelOri;
model = changeRxnBounds(model, 'EX_o2(e)', -17, 'b');
glucoseUptakeRates = zeros(21,1);
growthRates = zeros(21,1);
for i = 0:20
    model = changeRxnBounds(model, 'EX_glc(e)', -i, 'b');
    glucoseUptakeRates(i+1)=-i;
    FBAsolution = optimizeCbModel(model, 'max');
    growthRates(i+1) = FBAsolution.f;
end

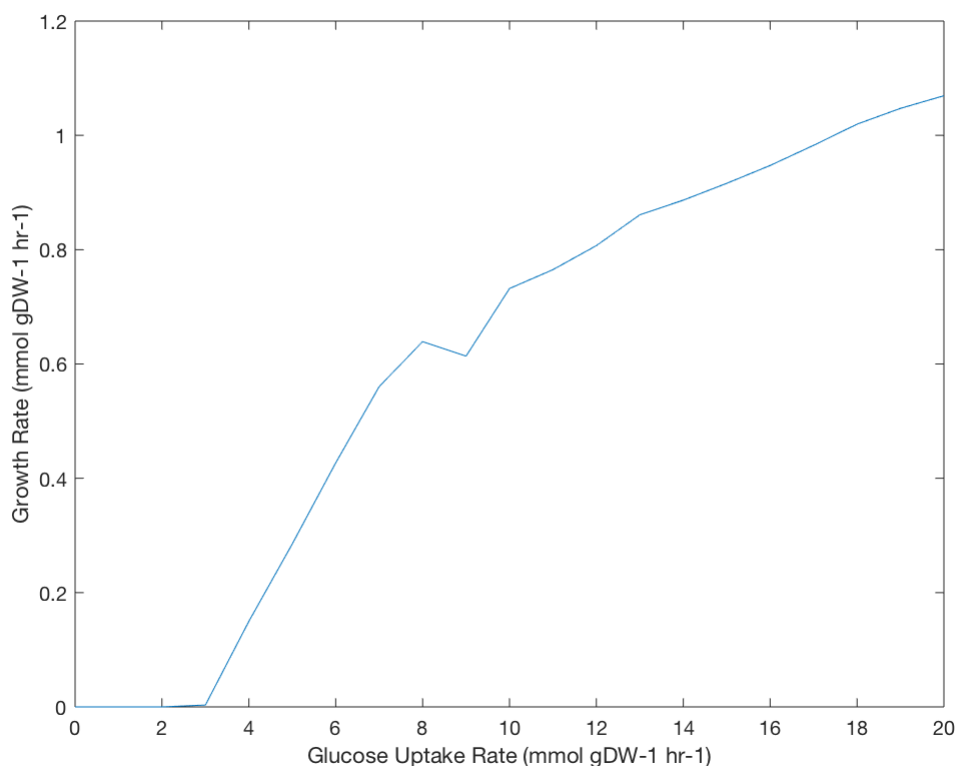
```

The results can then be plotted:

```

figure;
plot(-glucoseUptakeRates,growthRates,'-')
xlabel('Glucose Uptake Rate (mmol gDW-1 hr-1)')
ylabel('Growth Rate (mmol gDW-1 hr-1)')

```



**Figure 5** Robustness analysis for maximum growth rate while varying glucose uptake rate with oxygen uptake fixed at 17 mmol gDW<sup>-1</sup> hr<sup>-1</sup>.

As expected, with a glucose uptake of 0 mmol gDW<sup>-1</sup> hr<sup>-1</sup>, the maximum possible growth rate is 0 hr<sup>-1</sup>. Growth remains at 0 hr<sup>-1</sup> until a glucose uptake rate of about 2.83 mmol gDW<sup>-1</sup> hr<sup>-1</sup>, because with such a small amount of glucose, the system cannot make 8.39 mmol gDW<sup>-1</sup> hr<sup>-1</sup> ATP to meet the default lower bound of the ATP maintenance reaction (ATPM). This reaction simulates the consumption of ATP by non-growth associated processes such as maintenance of electrochemical gradients across the cell membrane. Once enough glucose

is available to meet this ATP requirement, growth increases rapidly as glucose uptake increases. At an uptake rate of about 7.59 mmol gDW-1 hr-1, growth does not increase as rapidly. It is at this point that oxygen and not glucose limits growth. Excess glucose cannot be fully oxidized, so the fermentation pathways are used.

### What is the growth rate as a function of oxygen uptake rate with glucose uptake held constant?

#### Hint: fix glucose uptake at 10 mmol gDW-1 hr-1

The oxygen uptake rate can also be varied with the glucose uptake rate held constant. With glucose uptake fixed at 10 mmol gDW-1 hr-1, growth rate increases steadily as oxygen uptake increases (Figure 6). At an oxygen uptake of about 21.80 mmol gDW-1 hr-1, growth actually begins to decrease as oxygen uptake increases. This is because glucose becomes limiting at this point, and glucose that would have been used to produce biomass must instead be used to reduce excess oxygen. In the previous example, growth rate continues to increase once oxygen become limiting because *E. coli* can grow on glucose without oxygen. In this example, *E. coli* cannot grow with oxygen but not glucose (or another carbon source), so growth decreases when excess oxygen is added.

```
model = modelOri;
model = changeRxnBounds(model, 'EX_glc(e)', -10, 'b');
oxygenUptakeRates = zeros(25,1);
growthRates = zeros(25,1);
for i = 0:25
    model = changeRxnBounds(model, 'EX_o2(e)', -i, 'b');
    oxygenUptakeRates(i+1)=-i;
    FBAsolution = optimizeCbModel(model, 'max');
    growthRates(i+1) = FBAsolution.f;
end
figure;
plot(-oxygenUptakeRates, growthRates, '-')
xlabel('Oxygen Uptake Rate (mmol gDW-1 hr-1)')
ylabel('Growth Rate (mmol gDW-1 hr-1)')
```



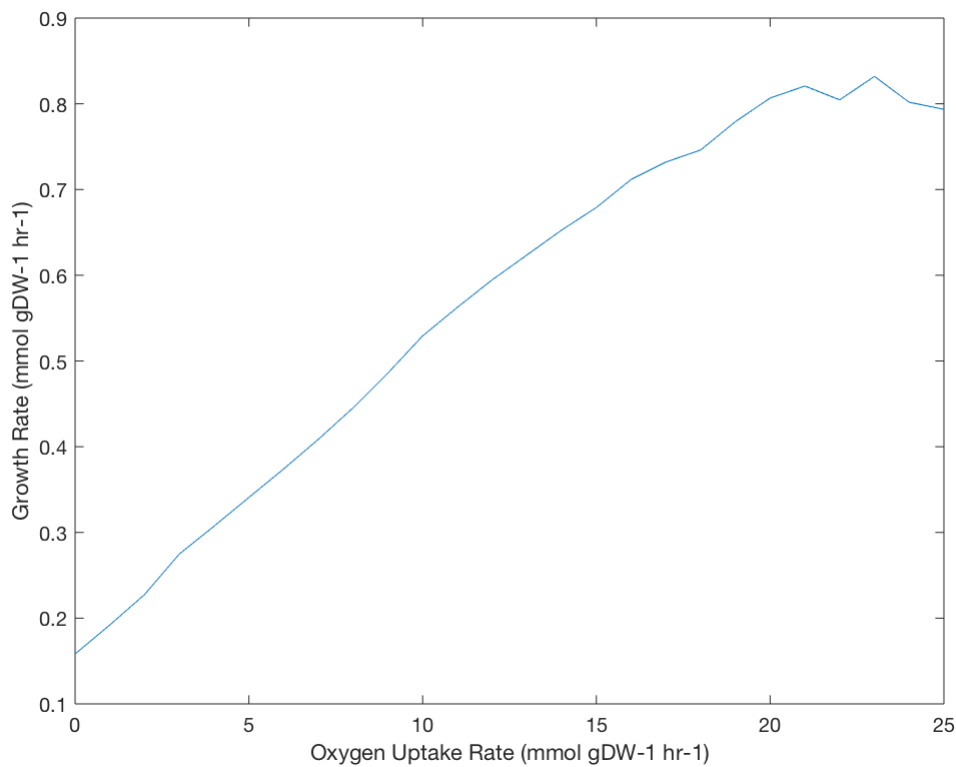


Figure 6 Robustness analysis for maximum growth rate while varying oxygen uptake rate with glucose uptake fixed at 10 mmol gDW-1 hr-1.

## Example 6. Simulating gene knockouts

Just as growth in different environments can be simulated with FBA, gene knockouts can also be simulated by changing reaction bounds. To simulate the knockout of any gene, its associated reaction or reactions can simply be constrained to not carry flux. By setting both the upper and lower bounds of a reaction to 0 mmol gDW-1 hr-1, a reaction is essentially knocked out, and is restricted from carrying flux. The E. coli core model, like many other constraint-based models, contains a list of gene-protein-reaction interactions (GPRs), a list of Boolean rules that dictate which genes are connected with each reaction in the model. When a reaction is catalyzed by isozymes (two different enzymes that catalyze the same reaction), the associated GPR contains an “or” rule, where either of two or more genes may be knocked out but the reaction will not be constrained.

**Which genes are required to be knocked out to eliminate flux through the phosphofructokinase (PFK) reaction?**

**Hint: study the model**

The GPR for phosphofructokinase (PFK) is “b1723 (pfkB) or b3916 (pfkA),” so according to this Boolean rule, both pfkB and pfkA must be knocked out to restrict this reaction. When a reaction is catalyzed by a protein with multiple essential subunits, the GPR contains an “and” rule, and if any of the genes are knocked out the reaction will be constrained to 0 flux. Succinyl-CoA synthetase (SUCOAS), for example, has the GPR “b0728 (sucC) and b0729 (sucD),” so knocking out either of these genes will restrict this reaction. Some reactions

are catalyzed by a single gene product, while others may be associated with ten or more genes in complex associations.

### What is the growth rate of *E. coli* when the gene b1852 (zwf) is knocked out?

**Hint:** `deleteModelGenes`

The COBRA Toolbox contains a function called `deleteModelGenes` that uses the GPRs to constrain the correct reactions. Then FBA may be used to predict the model phenotype with gene knockouts. For example, growth can be predicted for *E. coli* growing aerobically on glucose with the gene b1852 (zwf), corresponding to the reaction glucose-6-phosphate dehydrogenase (G6PDH2r), knocked out. The FBA predicted growth rate of this strain is 1.6329 hr<sup>-1</sup>, which is slightly lower than the growth rate of 1.6531 hr<sup>-1</sup> for wild-type *E. coli* because the cell can no longer use the oxidative branch of the pentose phosphate pathway to generate NADPH. Using FBA to predict the phenotypes of gene knockouts is especially useful in predicting essential genes. When the gene b2779 (eno), corresponding to the enolase reaction (ENO), is knocked out, the resulting growth rate on glucose is 0 hr<sup>-1</sup>. Growth is no longer possible because there is no way to convert glucose into TCA cycle intermediates without this glycolysis reaction, so this gene is predicted to be essential. Because FBA can compute phenotypes very quickly, it is feasible to use it for large scale computational screens for gene essentiality, including screens for two or more simultaneous knockouts.

### Knockout every pairwise combination of the 136 genes in the *E. coli* core model and display the results.

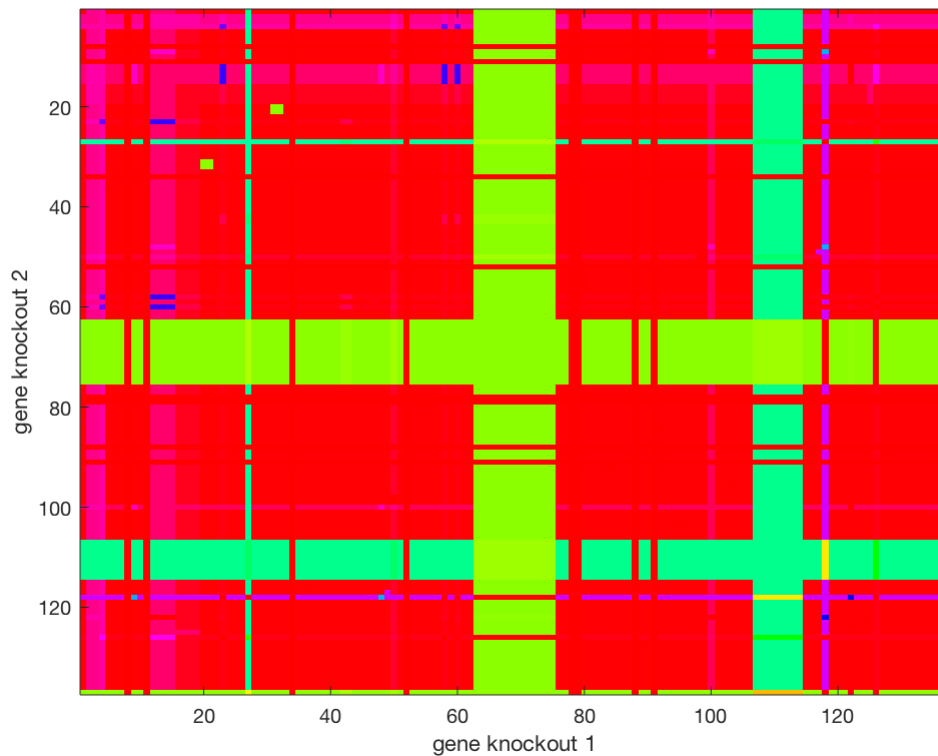
**Hint:** `doubleGeneDeletion`

Figure 8 shows the results of a double knockout screen, in which every pairwise combination of the 136 genes in the *E. coli* core model were knocked out. The code to produce this figure is:

```
model = modelOri;
[grRatio,grRateKO,grRateWT] = doubleGeneDeletion(model);
```

```
Single deletion analysis to remove lethal genes
100%      [.....]
131 non-lethal genes
Double gene deletion analysis
Total of 8515 pairs to analyze
Double gene deletion analysis in progress ...
Perc complete      CPU time
```

```
figure;
imagesc(grRatio);
colormap('hsv')
xlabel('gene knockout 1')
ylabel('gene knockout 2')
```



**Figure 8** Gene knockout screen on glucose under aerobic conditions. Each of the 136 genes in the core *E. coli* model were knocked out in pairs, and the resulting relative growth rates were plotted. In this figure, genes are ordered by their b number. Some genes are always essential, and result in a growth rate of 0 when knocked out no matter which other gene is also knocked out. Other genes form synthetic lethal pairs, where knocking out only one of the genes has no effect on growth rate, but knocking both out is lethal. Growth rates in this figure are relative to wild-type.

## Example 7. Which genes are essential for which biomass precursor?

Earlier, we saw how to determine essential genes for growth. Here we will repeat the same analysis but instead of optimizing for the biomass production, we will optimize for the synthesis of all biomass precursors individually.

### Which genes are essential for synthesis of each biomass precursor?

**Hint:** `addDemandReaction`

Therefore, we have to add a demand reaction for each biomass precursor to the model and perform a gene deletion study for each demand reaction. First, the components of the biomass reaction can be identified and demand reactions can be added by using the `addDemandReaction` function:

```
model = modelOri;
bool=model.S(:,strcmp(model.rxns,'Biomass_Ecoli_core_N(w/GAM)-Nmet2'))~=0;
biomassComponents=model.mets(bool);
[modelBiomass,rxnNames] = addDemandReaction(model,biomassComponents);
```

Adding the following reactions to the model:

```
DM_3pg[c]    3pg[c]    ->
DM_accoa[c]   accoa[c]   ->
DM_adp[c]    adp[c]    ->
DM_akg[c]    akg[c]    ->
DM_atp[c]    atp[c]    ->
DM_coa[c]    coa[c]    ->
DM_e4p[c]    e4p[c]    ->
DM_f6p[c]    f6p[c]    ->
DM_g3p[c]    g3p[c]    ->
DM_g6p[c]    g6p[c]    ->
DM_gln-L[c]   gln-L[c]   ->
DM_glu-L[c]   glu-L[c]   ->
DM_h2o[c]    h2o[c]    ->
DM_h[c]      h[c]      ->
DM_nad[c]    nad[c]    ->
DM_nadh[c]   nadh[c]   ->
DM_nadp[c]   nadp[c]   ->
DM_nadph[c]  nadph[c]  ->
DM_oaa[c]    oaa[c]    ->
DM_pep[c]    pep[c]    ->
DM_pi[c]     pi[c]     ->
DM_pyr[c]    pyr[c]    ->
DM_r5p[c]    r5p[c]    ->
```

Next, gene knockout screens can be performed with each of these demand reactions set as the objective, one at a time:

```
for i = 1:length(rxnNames)
    modelBiomass = changeObjective(modelBiomass,rxnNames{i});
    [grRatio,grRateKO,grRateWT,hasEffect,delRxns,fluxSolution] =
singleGeneDeletion(modelBiomass, 'FBA',model.genes,0);
    biomassPrecursorGeneEss(:,i) = grRateKO;
    biomassPrecursorGeneEssRatio(:,i) = grRatio;
end
```

```
11111111111111111111111111111111100%    [.....]
```

```
%biomassPrecursorGeneEssRatio(~isfinite(biomassPrecursorGeneEssRatio))=0;
```

The resulting matrix `biomassPrecursorGeneEssRatio` is plotted with the `imagesc` function in Figure 9, and indicate which biomass precursors become blocked by certain gene knockouts.

```
figure;
imagesc(biomassPrecursorGeneEssRatio);
colormap('hsv')
yticks(1:length(model.genes));
yticklabels(model.genes)
```

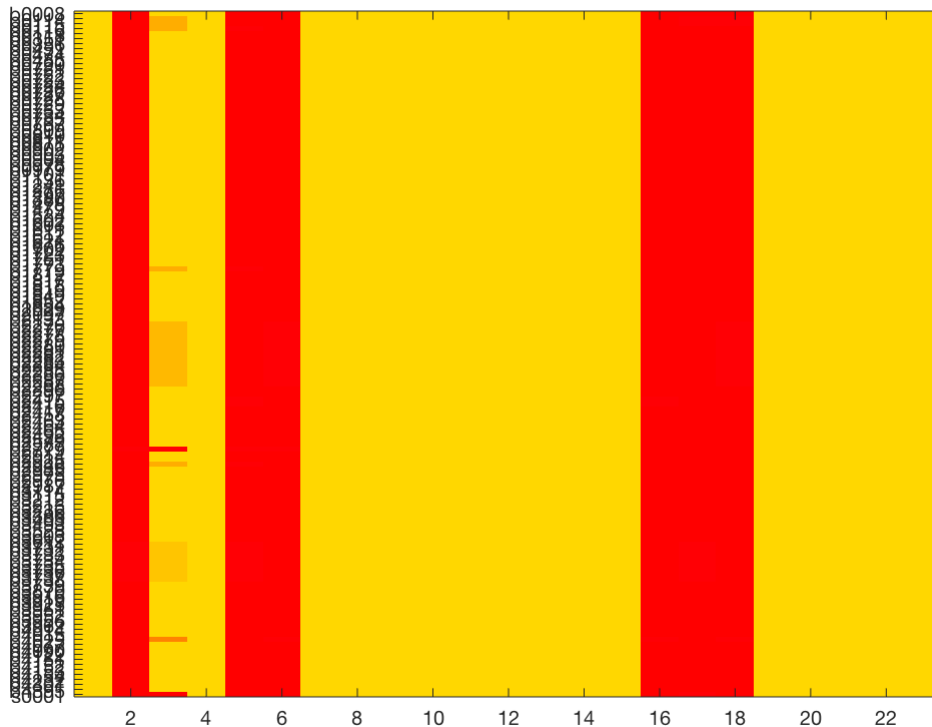


Figure 9 Gene essentiality for biomass precursor synthesis. Heat map shows the relative biomass precursor synthesis rate of mutant compared to wild-type. The 23 biomass precursors are 3pg, accoa, adp, akg, atp, coa, e4p, f6p, g3p, g6p, gln-L, glu-L, h2o, h, nad, nadh, nadp, nadph, oaa, pep, pi, pyr, r5p.

**Which biomass precursors cannot be net synthesised by the E. coli core model? What does this say about the E. coli core model itself?**

**Hint: think about steady state versus mass balance**

Some precursors (like atp[c]) cannot be net produced by any of the gene knockout strains because of the demand reactions that consume them and there are no set of reactions that de novo synthesise that precursor. The addDemandReaction function produces a demand reaction that does not regenerate ADP when ATP is consumed or NAD<sup>+</sup> when NADH is consumed, so these reactions cannot carry flux at steady-state. The genome-scale E. coli model contains all the reactions for de novo synthesis of ATP.

### Example 8. Which non-essential gene knockouts have the greatest effect on the network flexibility?

Another question one might be interested in asking is what are the consequences for the network of deleting a non-essential gene? Given an objective, the flux span is the difference between the maximum and minimum alternate optimal flux values for a reaction. A reaction is said to become more flexible if its flux span increases and less flexible if its flux span decreases.

**What reactions are increased and decreased in flux span in response to deletion of these 6 genes? {'b0114', 'b0723', 'b0726', 'b0767', 'b1602', 'b1702'} with reference to a particular example, what is the biochemical interpretation of an increased flux span in response to a gene deletion?**

**Hint:** `fluxVariability`, `hist`

To address this question, first delete the network genes individually (a single gene deletion study) and then perform FVA (Example 3) on non-essential gene deletion strains. In many cases one would expect that the deletion of a gene has only minor consequences to the network, especially if the deletion does not alter the maximal growth rate. In some cases however, the deletion may reduce the overall network flexibility or maybe even increase the flux range through specific reactions. In fact, this latter property is used to design optimal production strains (e.g., using OptKnock [9]), by redirecting fluxes through the network.

The absolute flux span is a measure of flux range for each reaction. It is calculated as  $f_i = \text{abs}(v_{\text{max},i} - v_{\text{min},i})$ , where  $v_{\text{min},i}$  and  $v_{\text{max},i}$  are the minimal and maximal flux rate as determined using FVA. First, calculate wild-type flux variability:

```
[minFluxAll(:,1),maxFluxAll(:,1)] = fluxVariability(model);
```

Next, calculate the knockout strain flux variabilities and flux spans:

```
genes=model.genes([2,14,16,23,42,48]);  
for i = 1 : length(genes)  
    [modelDel] = deleteModelGenes(model,genes{i});  
    [minFluxAll(:,i+1),maxFluxAll(:,i+1)] = fluxVariability(modelDel);  
end  
  
fluxSpan = abs(maxFluxAll - minFluxAll);  
for i = 1 : size(fluxSpan,2)  
    fluxSpanRelative(:,i) = fluxSpan(:,i)./fluxSpan(:,1);  
end
```

This example with six mutant strains shows that the majority of the network reactions have reduced flux span compared to wild-type (Figure 10). However, some of these knockout strains have a few reactions which have much higher flux span than wildtype. For example, in one of the knockout strains ( `sucA`), the flux span is increased 14 times through the phosphoenolpyruvate carboxykinase reaction (PPCK).

Finally, histograms can be plotted to show how many reactions have increased or decreased flux spans for each knockout:

```
for i =2:7  
    subplot(2,3,i-1)  
    hist(fluxSpanRelative(:,i),20);  
    title(genes{i-1});  
end
```

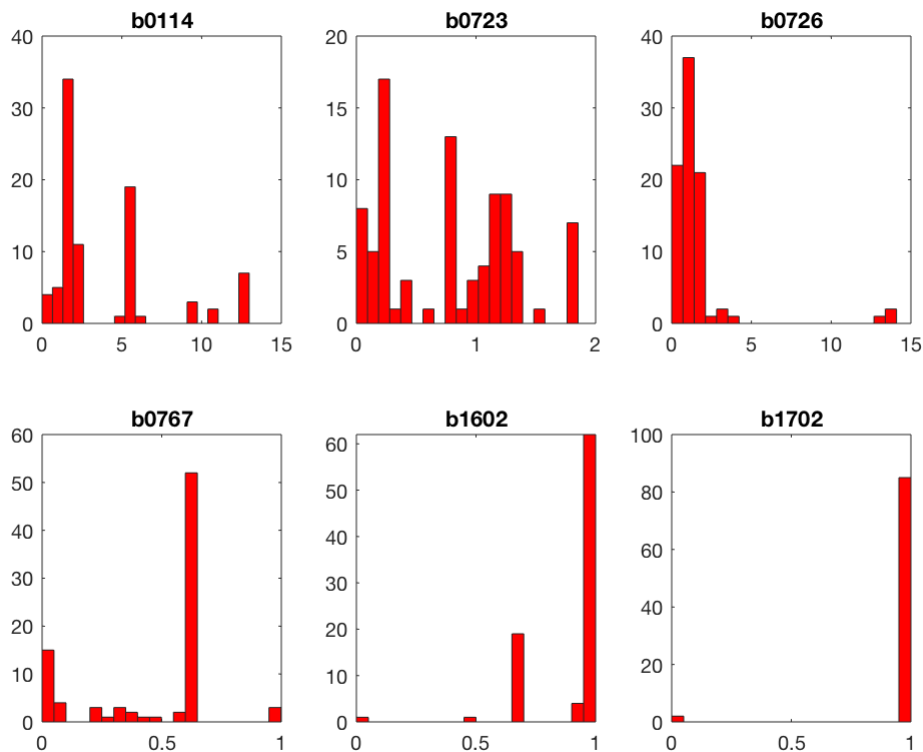


Figure 10 Histograms of relative flux spans for 6 gene knockout mutants.

## TIMING

2 hrs

## Acknowledgments

This tutorial was originally written by Jeff Orth and Ines Thiele for the publication "What is flux balance analysis?"

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