Benchmark solvers for solving whole body metabolic models

Author(s): Ronan M.T. Fleming, School of Medicine, University of Galway

Reviewer(s):

Introduction

Compare the time taken to solve different formulations of constraint-based modelling problems involving whole body metabolic models with different solvers and different methods for each solver with the option to repeat the analysis to compute mean and variance of solution times.

EQUIPMENT SETUP

Initialize the COBRA Toolbox.

Please ensure that The COBRA Toolbox has been properly installed, and initialized using the `initCobraToolbox` function.

```
if 0 %set to true if your toolbox has not been initialised
    initCobraToolbox(false) % false, as we don't want to update
end
```

PROCEDURE

Define the location to save your results

```
if 1
    resultsFolder = '~/drive/sbgCloud/projects/variationalKinetics/
results/WBM/';
else
    resultsFolder = pwd;
end
```

Load whole body metabolic model - change this to suit your own setup.

```
modelToUse ='Harvey';
%modelToUse ='Harvetta';
driver_loadBenchmarkWBMsolvers
```

Set parameters for benchmark

Model perturbation parameters

```
%model.ub(model.c~=0)=inf;
```

```
clear param T T0
param.replaceLargeBoundsWithInf=1;
param.relaxTightBounds=1;
param.relaxTightBounds_lowerExponent = 3; %the minimum difference between
ub_j and lb_j is 10^(param.relaxTightBounds_lowerExponent)
param.relaxTightBounds_higherExponent = 10;
param.setUpperBoundOnObjectiveToInf=1;
```

COBRA toolbox & solver parameters

```
param.printLevel = 0; %  {(0),1,2} 1 output from optimiseVKmode, 2 also
output from solver
param.feasTol = 1e-5;
param.optTol = 1e-5;
param.lifted = 1;
param.multiscale = 1;
param.debug = 0;
```

Set the maximum time limit allowed to solve a single instance. Useful for eliminating slow instances in a large batch of trials.

```
param.timelimit = 200;
```

Select whether to compare one or a set of solvers

```
compareSolvers = 1;
```

Select whether to compare one or a set of different formulations of constraint-based modelling problems involving whole body metabolic models.

```
compareSolveWBMmethods = 1;
```

Select whether to compare one or a set of available methods (algorithms) for each solver

```
compareSolverMethods = 1;
```

Define the number of times to replicate the same formulation, solver, method combination.

```
nReplicates = 2;
```

Display and (optionally) modify properties of the whole body model that may effect solve time

```
[nMet,nRxn]=size(model.S)
```

```
nMet =
58095
nRxn =
83395
```

Identify large bounds not at the maximum

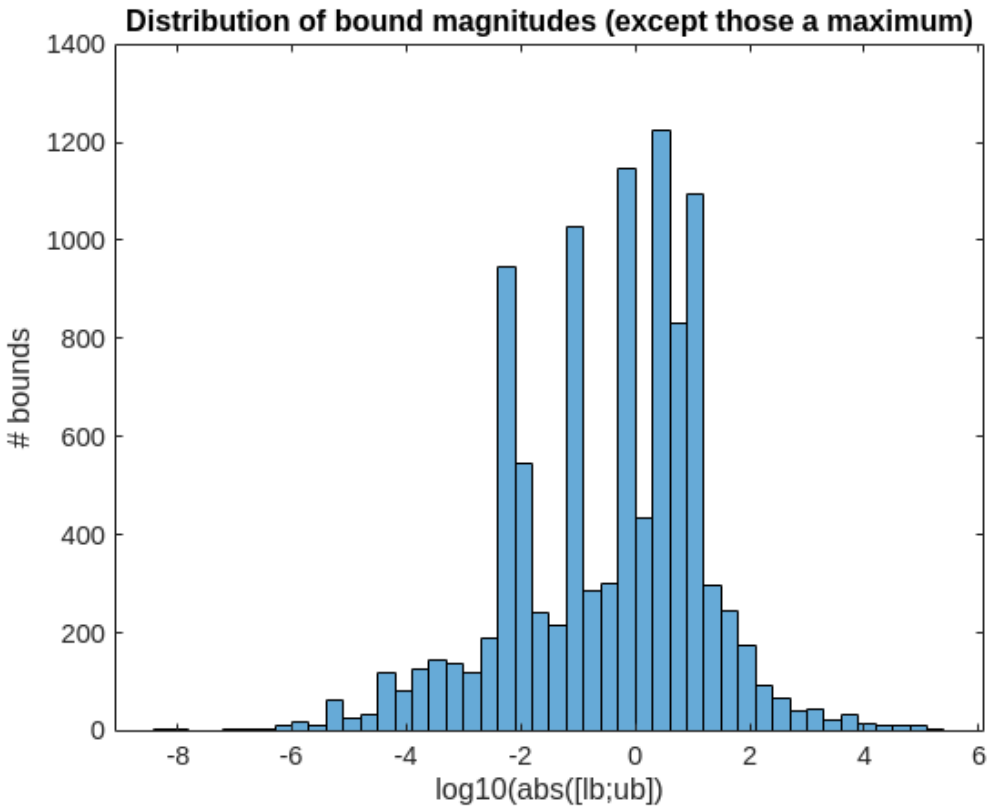
```
boundMagnitudes = [abs(model.lb);abs(model.ub)];
```

```
boundMagnitudes(~isfinite(boundMagnitudes))=0;
largestMagnitudeBound = max(boundMagnitudes);
fprintf('%g%s\n',
(nnz(largestMagnitudeBound==[abs(model.lb);abs(model.ub)])*100)/
(length(model.lb)*2),' = percent of bounds at maximum')
```

```
62.2112 = percent of bounds at maximum
```

Display bounds that are not at the maximum

```
if 1
    figure
    histogram(log10(boundMagnitudes(boundMagnitudes~=largestMagnitudeBound &
boundMagnitudes~=0)))
    xlabel('log10(abs([lb;ub])')
    ylabel('# bounds')
    title('Distribution of bound magnitudes (except those a maximum)')
end
```



Replace large bounds with inf or -inf. This is a good idea. Better to leave this option on.

```
if param.replaceLargeBoundsWithInf && largestMagnitudeBound>1e3
    model.lb(-largestMagnitudeBound==model.lb)=-inf;
    model.ub(largestMagnitudeBound==model.ub)= inf;
end
boolMagnitudes = boundMagnitudes~=largestMagnitudeBound & boundMagnitudes~=0
& boundMagnitudes<1e-4;
```

```
boolRxns = boolMagnitudes(1:nRxn) | boolMagnitudes(nRxn+1:2*nRxn);
```

Optionally, print the bounds for reactions with small magnitude

```
if 0
    printFluxBounds(model,model.rxns(boolRxns))
end
fprintf('%g%s\n',nnz(boolRxns)*100/length(boolRxns),' = percent of bounds
with magnutide less than 1e-4')
```

```
0.389712 = percent of bounds with magnutide less than 1e-4
```

Optionally, print the bounds for reactions with small difference

```
boundDifference = model.ub - model.lb;
bool = length(model.rxns);
Z = table(boundDifference,model.rxns,model.rxnNames,'VariableNames',
{'boundDifference','rxns','rxnNames'});
if any(boundDifference<0)
    error(['lb > ub for ' num2str(nnz(boundDifference)) ' reactions'])
end
boolDifference = boundDifference<1e-5 & boundDifference~=0;
Z = sortrows(Z(boolDifference,:),'boundDifference');
if 0
    printFluxBounds(model,Z.rxns,1)
end
fprintf('%g%s\n',nnz(boolRxns)*100/length(boolRxns), ' = percent of bounds
with difference (ub - lb) less than 1e-5')
```

```
0.389712 = percent of bounds with difference (ub - lb) less than 1e-5
```

```
forwardBoolDifference = boolDifference & model.lb>=0 & model.ub>0;
reverseBoolDifference = boolDifference & model.lb<0 & model.ub<=0;
reversibleBoolDifference = boolDifference & model.lb<0 & model.ub>0;

if any((forwardBoolDifference | reverseBoolDifference |
reversibleBoolDifference)~=boolDifference)
    error('missing bool difference')
end
```

Optionally relax bounds that are very tight

```
if param.relaxTightBounds
    modelOld=model;
    done=false(nRxn,1);
    for
x=param.relaxTightBounds_higherExponent:-1:param.relaxTightBounds_lowerExpone
nt
        %calulate the difference between the bounds each time
        boundDifference = model.ub - model.lb;

        %forward
```

```
        bool = forwardBoolDifference & (boundDifference <= 10^(-x));
        model.ub(bool & ~done) = model.ub(bool & ~done)*(10^(x-
param.relaxTightBounds_lowerExponent+1));
        done = done | bool;

        %reverse
        bool = reverseBoolDifference & (boundDifference <= 10^(-x));
        model.lb(bool & ~done) = model.lb(bool & ~done)*(10^(x-
param.relaxTightBounds_lowerExponent+1));
        done = done | bool;

        %reversible
        bool = reversibleBoolDifference & (boundDifference <= 10^(-x));
        model.lb(bool & ~done) = model.lb(bool & ~done)*(10^((x-
param.relaxTightBounds_lowerExponent+1)/2));
        model.ub(bool & ~done) = model.ub(bool & ~done)*(10^((x-
param.relaxTightBounds_lowerExponent+1)/2));
        done = done | bool;

        %reset
        %done=false(nRxn,1);
    end
    fprintf('%g%s\n',nnz(done), [' = reactions with tight bounds relaxed to
at least' num2str(param.relaxTightBounds_lowerExponent) ' for ub - lb'])
    if 1
        printFluxBounds(model,Z.rxns,1)
    end
end
```

```
69 = reactions with tight bounds relaxed to at least3 for ub - lb
               Reaction ID       Lower Bound       Upper Bound
     BBB_ESTRADIOL[CSF]exp         0.000e+00         5.544e-03    Estradiol Glucuronide Transport via Bica
 BBB_LEUKTRB4WCOOH[CSF]exp         0.000e+00         7.560e-03    Transport of W-Carboxy Leukotriene B4, A
   BBB_LEUKTRB4WOH[CSF]exp         0.000e+00         7.560e-03    Transport of W-Hydroxyl Leukotriene B4,
            EX_sphings[u]         0.000e+00         8.374e-03    Exchange of Sphingosine
           EX_leuktrB4[u]         0.000e+00         1.231e-03    Exchange of Leukotriene B4
      BBB_PRGNLONE[CSF]exp         0.000e+00         9.576e-03    Steroid SulfotransferaseTransport of Pre
      BBB_LEUKTRC4[CSF]exp         0.000e+00         1.018e-03    Transport of Leukotriene C4 via Bicarbon
          EX_prostgf2[u]         0.000e+00         1.167e-03    Exchange of Prostaglandin F2Alpha
        BBB_C14771[CSF]exp         0.000e+00         1.285e-03    Transport of 14, 15-EET, Active Transpor
           EX_thyox_L[u]         0.000e+00         1.315e-03    Exchange of L-Thyroxine
             EX_sphgn[u]         0.000e+00         1.337e-03    Exchange of Sphinganine
          BBB_MEPI[CSF]exp         0.000e+00         2.419e-03    Metanephrine Secretion via Secretory Ves
            EX_C05767[u]         0.000e+00         3.310e-03    Exchange of Uroporphyrin I
        BBB_C04805[CSF]exp         0.000e+00         3.578e-03    Transport of 5(S)-HETE, Active Transport
           EX_estriol[u]         0.000e+00         4.456e-03    Exchange of Estriol
     Kidney_EX_no(e)_[bc]        -4.484e-03         0.000e+00    Exchange of Nitric Oxide (frombloodto[e]
        EX_estradiol[u]         0.000e+00         4.922e-03    Exchange of Estradiol
          BBB_BTN[CSF]exp         0.000e+00         5.902e-03    Biotinidase (Biotin)Biotinidase (Biotin)
            EX_estrone[u]         0.000e+00         6.365e-03    Exchange of Estrone
        BBB_CE2047[CSF]exp         0.000e+00         6.905e-03    Transport (ATP-Dependent) into Extracell
        BBB_CE2049[CSF]exp         0.000e+00         7.560e-03    Transport (ATP-Dependent) into Extracell
    BBB_12HARACHD[CSF]exp         0.000e+00         7.711e-03    Transport of 12 Hydroxy Arachidonic Acid
         BBB_ANTH[CSF]exp         0.000e+00         7.762e-03    Transport of Anthranilate (BBB)
      EX_pcholn204_hs[u]         0.000e+00         8.062e-03    "Exchange of 1-Eicosatetraenoylglyceroph
           EX_C05298[u]         0.000e+00         8.699e-03    Exchange of 2-Hydroxyestrone
```

```
            EX_C05301[u]         0.000e+00         9.335e-03    Exchange of 2-Hydroxyestradiol-17Beta
           EX_tdchola[u]         0.000e+00         9.441e-03    Exchange of Taurochenodeoxycholate
     BBB_PRGSTRN[CSF]exp         0.000e+00         1.008e-03    Progesterone Transport (BBB)
  BBB_TSTSTERONE[CSF]exp         0.000e+00         1.008e-03    Glucuronidated Compound TransportGlucuro
   BBB_ANDRSTNDN[CSF]exp         0.000e+00         1.008e-03    Transport of Androst-4-Ene-3, 17-Dione,
        BBB_DHEA[CSF]exp         0.000e+00         1.008e-03    Dehydroepiandrosterone Sulfate Transport
     BBB_CE0955[CSF]exp         0.000e+00         1.124e-03    Transport of 6-Oxo-Prostaglandin F1Alpha
           EX_C05302[u]         0.000e+00         1.231e-03    Exchange of 2-Methoxyestradiol-17Beta
           EX_C05299[u]         0.000e+00         1.273e-03    Exchange of 2-Methoxyestrone
           EX_dgchol[u]         0.000e+00         1.379e-03    Exchange of Chenodeoxyglycocholate
           EX_argsuc[u]         0.000e+00         1.379e-03    Exchange of L-Arginosuccinic Acid
     BBB_XOL27OH[CSF]exp         0.000e+00         1.411e-03    27 Hydroxy Cholesterol TransportTranspor
      Diet_EX_adpcbl[d]        -4.332e-03        -2.888e-06    Diet_EX_adpcbl[d]
      BBB_IM4AC[CSF]exp         0.000e+00         1.562e-03    Assumed Passive Diffusion into Extracell
     BBB_35CGMP[CSF]exp         0.000e+00         1.714e-03    35CGMP Nuclear Transport (BBB)
     BBB_C14826[CSF]exp         0.000e+00         1.799e-03    Transport of 12 (13)-Epome, FATPTranspor
     BBB_XOL24OH[CSF]exp         0.000e+00         1.814e-03    Transport of (24S)-24-Hydroxycholesterol
      EX_pcholhep_hs[u]         0.000e+00         1.846e-03    Exchange of 1-Heptadecanoylglycerophosph
           EX_C05770[u]         0.000e+00         1.952e-03    Exchange of Coproporphyrin Iii
     BBB_C14825[CSF]exp         0.000e+00         1.981e-03    Formation of 9 (10)-EpomeTransport of 9
           EX_5htrp[u]         0.000e+00         1.986e-03    Exchange of 5-Hydroxy-L-Tryptophan
     BBB_PROSTGF2[CSF]exp         0.000e+00         2.016e-03    Prostaglandin Uniport (BBB)
           EX_adrnl[u]         0.000e+00         2.100e-03    Exchange of Adrenaline
      EX_pcholste_hs[u]         0.000e+00         2.122e-03    Exchange of 1-Stearoylglycerophosphochol
     BBB_3MOXTYR[CSF]exp         0.000e+00         2.280e-03    3-Methoxytyramine:Oxygen Oxidoreductase
     BBB_LEUKTRB4[CSF]exp         0.000e+00         2.313e-03    Transport of Leukotriene B4Transport of
            EX_fol[u]         0.000e+00         2.546e-03    Exchange of Folate
         EX_aldstrn[u]         0.000e+00         2.970e-03    Exchange of Aldosterone
     BBB_34DHPHA[CSF]exp         0.000e+00         3.024e-03    3, 4-Dihydroxyphenylacetate:Amet O-Methy
       EX_prostge2[u]         0.000e+00         4.031e-03    Exchange of Prostaglandin E2
          EX_crtsl[u]         0.000e+00         4.456e-03    Exchange of Cortisol
     BBB_34DHPHE[CSF]exp         0.000e+00         5.040e-03    3, 4-Dihydroxy-L-Phenylalanine Transport
       BBB_5HTRP[CSF]exp         0.000e+00         5.040e-03    5-Hydroxy-L-Tryptophan Secretion via Sec
        BBB_DOPA[CSF]exp         0.000e+00         5.040e-03    Dopamine Beta-MonooxygenaseL-Dopachrome
        BBB_SRTN[CSF]exp         0.000e+00         5.040e-03    Acetyl Coenzyme A:Aralkylamine N-Acetylt
       BBB_THMMP[CSF]exp         0.000e+00         5.040e-03    Thiamine Monophosphate Transport in via
  BBB_NORMETE_L[CSF]exp         0.000e+00         5.040e-03    Export ofnormete_L[csf] from CSF to bloo
      BBB_MHISTA[CSF]exp         0.000e+00         5.040e-03    Facilitated Diffusion Through Uniport Oc
      BBB_CE2705[CSF]exp         0.000e+00         5.040e-03    Transport by Ent1/Ent2 into Extracellula
      BBB_CE4890[CSF]exp         0.000e+00         5.040e-03    Facilitated Diffusion Through Uniport Oc
      BBB_C09642[CSF]exp         0.000e+00         5.040e-03    Facilitated Diffusion Through Uniport Oc
   Gall_EX_adrnl(e)_[bc]        -7.915e-03        -0.000e+00    Exchange of Adrenaline (frombloodto[e]
           EX_34dhphe[u]         0.000e+00         8.487e-03    "Exchange of 3, 4-Dihydroxy-L-Phenylalan
Kidney_EX_leuktrD4(e)_[bc]        -9.342e-03         0.000e+00    Exchange of Leukotriene D4 (frombloodto[
```

```
disp(min(boundDifference(boundDifference~=0)))
```

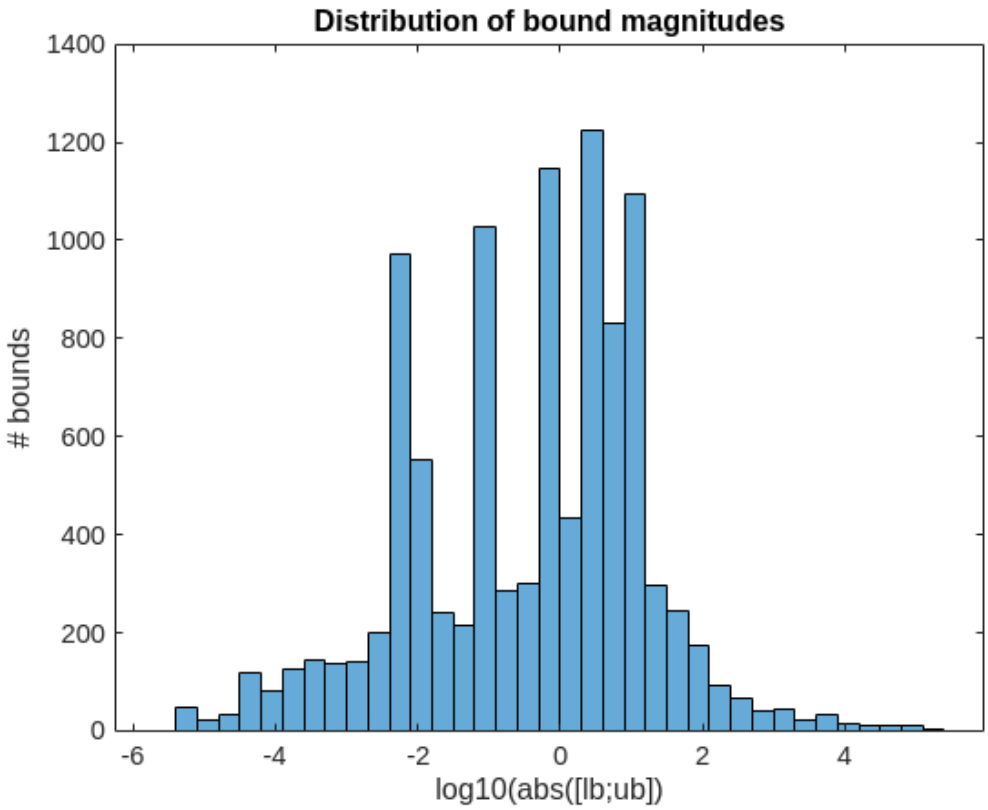
```
   1.0080e-05
```

Optionally, remove fixed upper bound on biomass reaction

```
% Optionally, relax bounds that are fixed for the objective
if param.setUpperBoundOnObjectiveToInf
    if any(contains(modelToUse,{'Harvey','Harvetta'})) && nnz(model.c)==1
        biomassRxnAbbr = model.rxns{model.c~=0};
        if
model.ub(ismember(model.rxns,biomassRxnAbbr))==model.ub(ismember(model.rxns,b
iomassRxnAbbr))
            if strcmp(model.osenseStr,'max')
                model.ub(ismember(model.rxns,biomassRxnAbbr))=inf;
```

```
            else
                model.lb(ismember(model.rxns,biomassRxnAbbr))=-inf;
            end
        end
    end
end
```

```
if 1
    boundMagnitudes = [abs(model.lb);abs(model.ub)];
    figure
    histogram(log10(boundMagnitudes))
    xlabel('log10(abs([lb;ub])')
    ylabel('# bounds')
    title('Distribution of bound magnitudes')
end
```



Prepare a benchmark table, choose the solver and solve

```
VariableNames={'interface','solver','method','problem','model','stat','origSt
at','time','obj','f1','f2','f0'};
% Define the corresponding variable types
VariableTypes = {'string', 'string',
'string','string','string','double','string','double','double','double','doub
le','double'};
```

```
T = table('Size', [0
length(VariableNames)],'VariableNames',VariableNames,'VariableTypes',
VariableTypes);
```

Select the solvers to compare

```
if compareSolvers
    solvers = {'mosek','ibm_cplex','gurobi'};
    %solvers = { 'ibm_cplex','mosek','gurobi'};
    %solvers = { 'mosek','ibm_cplex','gurobi'};
else
    % Choose the solver
    % solvers = {'gurobi'};
    %solvers = {'ibm_cplex'};
    solvers = {'mosek'};
end
```

Select the formulations to compare

```
if compareSolveWBMmethods
    %solveWBMmethods = {'LP','QP','QRLP','QRQP','zero','oneInternal'};
    % solveWBMmethods = {'LP','QP'};%,'zero','oneInternal'};
    %solveWBMmethods = {'LP','oneInternal'};
    solveWBMmethods = {'QP','LP'};
else
    % Choose type of problem to solve
    solveWBMmethods = {'LP'};
    %solveWBMmethods = {'QP'};
    %solveWBMmethods = {'QRLP'};
    %solveWBMmethods = {'QRQP'};
end
```

Define the methods (algorithms) available for different solvers

```
if compareSolverMethods
    % CPLEX
    % 0    CPX_ALG_AUTOMATIC     Automatic: let CPLEX choose; default
    % 1    CPX_ALG_PRIMAL     Primal simplex
    % 2    CPX_ALG_DUAL     Dual simplex
    % 3    CPX_ALG_NET     Network simplex
    % 4    CPX_ALG_BARRIER     Barrier
    % 5    CPX_ALG_SIFTING     Sifting
    % 6    CPX_ALG_CONCURRENT     Concurrent (Dual, Barrier, and Primal in
opportunistic parallel mode; Dual and Barrier in deterministic parallel mode)
    %https://www.ibm.com/docs/en/icos/12.10.0?topic=parameters-algorithm-
continuous-linear-problems
    cplexLPMethods
={'AUTOMATIC','PRIMAL','DUAL','NETWORK','BARRIER','SIFTING','CONCURRENT'};
    % 0    CPX_ALG_AUTOMATIC    Automatic: let CPLEX choose; default
```

```
    % 1    CPX_ALG_PRIMAL    Use the primal simplex optimizer.
    % 2    CPX_ALG_DUAL    Use the dual simplex optimizer.
    % 3    CPX_ALG_NET    Use the network optimizer.
    % 4    CPX_ALG_BARRIER    Use the barrier optimizer.
    % 6    CPX_ALG_CONCURRENT    Use the concurrent optimizer.
    % https://www.ibm.com/docs/en/icos/12.10.0?topic=parameters-algorithm-
continuous-quadratic-optimization
    cplexQPMethods
={'AUTOMATIC','PRIMAL','DUAL','NETWORK','BARRIER','CONCURRENT'};

    % Mosek

    % MSK_IPAR_OPTIMIZER
    %    The parameter controls which optimizer is used to optimize the task.
    %    Default "FREE"
    %    Accepted "FREE", "INTPNT", "CONIC", "PRIMAL_SIMPLEX",
"DUAL_SIMPLEX", "FREE_SIMPLEX", "MIXED_INT"
    %    Example param.MSK_IPAR_OPTIMIZER = 'MSK_OPTIMIZER_FREE'
    mosekLPMethods ={'FREE', 'INTPNT', 'CONIC', 'PRIMAL_SIMPLEX',
'DUAL_SIMPLEX', 'FREE_SIMPLEX'};
    mosekQPMethods ={'FREE', 'INTPNT'}; %CONIC not yet encoded in
solveCobraQP

    % Gurobi
    % https://www.gurobi.com/documentation/current/refman/method.html
    % Algorithm used to solve continuous models
    % Algorithm used to solve continuous models or the initial root
relaxation of a MIP model. Options are:
    gurobiLPMethods =
{'AUTOMATIC','PRIMAL','DUAL','BARRIER','CONCURRENT','DETERMINISTIC_CONCURRENT
'};
    gurobiQPMethods = {'AUTOMATIC','PRIMAL','DUAL','BARRIER'};
else
    if 0
        gurobiLPMethods={'BARRIER'};
        gurobiQPMethods={'BARRIER'};
        cplexLPMethods={'BARRIER'};
        cplexQPMethods={'BARRIER'};
        mosekLPMethods = {'CONIC'};
        mosekQPMethods = {'FREE'};
    else
        gurobiLPMethods={'AUTOMATIC'};
        gurobiQPMethods={'AUTOMATIC'};
        cplexLPMethods={'AUTOMATIC'};
        cplexQPMethods={'AUTOMATIC'};
        mosekLPMethods = {'FREE'};
        mosekQPMethods = {'FREE'};
    end
end
```

Set the min norm weight for QP problems

```
minNormWeight = 1e-4;
%model.c(:)=0;
```

Solve the ensemble of instances

```
for ind = 1:nReplicates
    for i = 1:length(solveWBMmethods)
        param.solveWBMmethod = solveWBMmethods{i};
        switch param.solveWBMmethod
            case 'LP'
                param.minNorm = [];
            case 'QP'
                param.minNorm = minNormWeight;
            case 'QRLP'
                param.minNorm = [];
            case 'QRQP'
                param.minNorm = minNormWeight;
            case 'zero'
                param.minNorm = 'zero';
            case 'oneInternal'
                if isfield(model,'SConsistentRxnBool')
                    param.minNorm = 'oneInternal';
                else
                    error('param.solveWBMmethod = oneInternal cannot be
implemented as model.SConsistentRxnBool is missing')
                end
        end

        for j = 1:length(solvers)

            param.solver=solvers{j};

            switch param.solver
                case 'gurobi'
                    if any(strcmp(param.solveWBMmethod,
{'LP','zero','oneInternal'}))
                        solverMethods = gurobiLPMethods;
                    else
                        solverMethods = gurobiQPMethods;
                    end
                case 'ibm_cplex'
                    if any(strcmp(param.solveWBMmethod,
{'LP','zero','oneInternal'}))
                        solverMethods = cplexLPMethods;
                    else
                        solverMethods = cplexQPMethods;
                    end
                case 'mosek'
```

```
                    if any(strcmp(param.solveWBMmethod,
{'LP','zero','oneInternal'}))
                        solverMethods = mosekLPMethods;
                    else
                        solverMethods = mosekQPMethods;
                    end
            end

            % Solve a problem with selected solver and each method available
to that solver
            for k=1:length(solverMethods)
                if any(strcmp(param.solveWBMmethod,
{'LP','zero','oneInternal'}))
                    param.lpmethod = solverMethods{k};
                    if isfield(param,'qpmethod')
                        param = rmfield(param,'qpmethod');
                    end
                else
                    param.qpmethod = solverMethods{k};
                    if isfield(param,'lpmethod')
                        param = rmfield(param,'lpmethod');
                    end
                end
                tic
                try
                    %return
                    solution = optimizeCbModel(model,'min',
param.minNorm,1,param);
                catch ME
                    disp('--------------------fail--------------')
                    disp('Error Message:')
                    disp(ME.message)
                    disp(param.solver)
                    disp(solverMethods{k})
                    disp(param.solveWBMmethod)
                    disp(param)
                    disp('--------------------fail--------------')
                end
                T = [T; {'optimizeCbModel', param.solver, solverMethods{k},
param.solveWBMmethod, modelToUse, solution.stat,{solution.origStat},toc,
{solution.obj},{solution.f1},{solution.f2},{solution.f0}}];
                %display(T(end,:));
            end
            display(T)
        end
    end
end
```

1×12 table

interface	solver	method	problem	model	stat	origStat	time	ob

```
    ________________    _______    ______    _______    ________    ____    _________    _____    _____

    "optimizeCbModel"    "mosek"    "FREE"     "QP"      "Harvey"     1      "OPTIMAL"    62.06    {[7.035

  1×12 table

       interface          solver      method      problem      model       stat     origStat       time

    ________________    _______    ________    _______    ________    ____    _________    ______    ____

    "optimizeCbModel"    "mosek"    "INTPNT"     "QP"      "Harvey"     1      "OPTIMAL"    62.152    {[7.
T = 2×12 table
                                                                                                    ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"

```
  1×12 table

       interface          solver        method       problem      model       stat      origStat          ti

    ________________    ___________    __________    _______    ________    ____    _____________    ____

    "optimizeCbModel"    "ibm_cplex"    "AUTOMATIC"     "QP"      "Harvey"     3      "non-optimal"    30.

  1×12 table

       interface          solver        method       problem      model       stat          origStat

    ________________    ___________    ________    _______    ________    ____    ____________________

    "optimizeCbModel"    "ibm_cplex"    "PRIMAL"     "QP"      "Harvey"     -1     "time limit exceeded"

  1×12 table

       interface          solver        method       problem      model       stat          origStat

    ________________    ___________    ______    _______    ________    ____    ____________________

    "optimizeCbModel"    "ibm_cplex"    "DUAL"     "QP"      "Harvey"     -1     "time limit exceeded"

  1×12 table

       interface          solver        method       problem      model       stat          origStat

    ________________    ___________    ________    _______    ________    ____    ____________________

    "optimizeCbModel"    "ibm_cplex"    "NETWORK"     "QP"      "Harvey"     -1     "time limit exceeded"

  1×12 table

       interface          solver        method       problem      model       stat      origStat          time

    ________________    ___________    ________    _______    ________    ____    _____________    _____

    "optimizeCbModel"    "ibm_cplex"    "BARRIER"     "QP"      "Harvey"     3      "non-optimal"    30.37

  1×12 table

       interface          solver          method         problem      model       stat          origStat

    ________________    ___________    ____________    _______    ________    ____    ____________________

    "optimizeCbModel"    "ibm_cplex"    "CONCURRENT"     "QP"      "Harvey"     -1     "time limit exceede
T = 8×12 table
                                                                                                      ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"

```
  1×12 table

      interface          solver       method        problem      model       stat      origStat          time
    _________________    ________    ___________    _______    ________    ____    _____________    _______

    "optimizeCbModel"    "gurobi"    "AUTOMATIC"     "QP"      "Harvey"     -1     "TIME_LIMIT"     201.78

  1×12 table

      interface          solver       method       problem      model       stat      origStat          time
    _________________    ________    ________    _________    ________    ____    _____________    _______

    "optimizeCbModel"    "gurobi"    "PRIMAL"      "QP"       "Harvey"     -1     "TIME_LIMIT"     200.18

  1×12 table

      interface          solver       method      problem      model       stat      origStat          time
    _________________    ________    ______    _________    ________    ____    _____________    _______

    "optimizeCbModel"    "gurobi"    "DUAL"      "QP"       "Harvey"     -1     "TIME_LIMIT"     200.18    {[

  1×12 table

      interface          solver       method       problem      model       stat      origStat          time
    _________________    ________    ________    _________    ________    ____    _____________    _______

    "optimizeCbModel"    "gurobi"    "BARRIER"     "QP"       "Harvey"     -1     "TIME_LIMIT"     200.2
T = 12×12 table
                                                                                                          ...
```


	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"

	interface	solver	method	problem	model	stat	origStat
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"

```
  1×12 table

      interface          solver     method     problem     model      stat     origStat       time          obj
    _________________    _______    _______    _______    ________    ____    __________    _______    _______

    "optimizeCbModel"    "mosek"    "FREE"      "LP"      "Harvey"     1      "OPTIMAL"     23.359    {[2.65

  1×12 table

      interface          solver      method      problem     model      stat     origStat       time          o
    _________________    _______    _________    _______    ________    ____    __________    _______    _____

    "optimizeCbModel"    "mosek"    "INTPNT"     "LP"       "Harvey"     1      "OPTIMAL"     23.298    {[2.
OPTIMAL
  1×12 table

      interface          solver     method     problem     model      stat     origStat       time          ob
    _________________    _______    _______    _______    ________    ____    __________    _______    ______

    "optimizeCbModel"    "mosek"    "CONIC"     "LP"       "Harvey"     1      "OPTIMAL"     16.838    {[2.7
Mosek returned an error or warning, open the following link in your browser:
https://docs.mosek.com/latest/toolbox/response-codes.html#mosek.rescode.trm_max_time
  1×12 table

      interface          solver          method          problem     model      stat                    orig
    _________________    _______    ________________    _______    ________    ____    ____________________

    "optimizeCbModel"    "mosek"    "PRIMAL_SIMPLEX"     "LP"      "Harvey"     -1     "DUAL_FEASIBLE & MS
Mosek returned an error or warning, open the following link in your browser:
https://docs.mosek.com/latest/toolbox/response-codes.html#mosek.rescode.trm_max_time
  1×12 table

      interface          solver          method          problem     model      stat                  origStat
    _________________    _______    ________________    _______    ________    ____    ____________________

    "optimizeCbModel"    "mosek"    "DUAL_SIMPLEX"      "LP"      "Harvey"     -1     "UNKNOWN & MSK_RES_TR
Mosek returned an error or warning, open the following link in your browser:
https://docs.mosek.com/latest/toolbox/response-codes.html#mosek.rescode.trm_max_time
  1×12 table

      interface          solver          method          problem     model      stat                  origStat
    _________________    _______    ________________    _______    ________    ____    ____________________

    "optimizeCbModel"    "mosek"    "FREE_SIMPLEX"      "LP"      "Harvey"     -1     "UNKNOWN & MSK_RES_TR
T = 18×12 table
                                                                                                          ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"

	interface	solver	method	problem	model	stat	origStat
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

⋮

```
1×12 table

    interface          solver          method         problem      model       stat     origStat       time
  _______________    ___________    ____________    ________    _________    ____    __________    _______

  "optimizeCbModel"    "ibm_cplex"    "AUTOMATIC"      "LP"       "Harvey"      1      "optimal"     17.237

1×12 table

    interface          solver          method       problem      model       stat          origStat
  _______________    ___________    ____________    ________    _________    ____    ____________________

  "optimizeCbModel"    "ibm_cplex"    "PRIMAL"       "LP"       "Harvey"      3      "time limit exceeded"

1×12 table

    interface          solver          method       problem      model       stat                  origStat
  _______________    ___________    ____________    ________    _________    ____    ____________________________

  "optimizeCbModel"    "ibm_cplex"    "DUAL"       "LP"       "Harvey"      3      "optimal with unscaled in

1×12 table

    interface          solver          method       problem      model       stat                  origSta
  _______________    ___________    ____________    ________    _________    ____    ____________________________

  "optimizeCbModel"    "ibm_cplex"    "NETWORK"      "LP"       "Harvey"      3      "optimal with unscaled

1×12 table

    interface          solver          method       problem      model       stat     origStat       time
  _______________    ___________    ____________    ________    _________    ____    __________    _______
```

```
    "optimizeCbModel"    "ibm_cplex"    "BARRIER"     "LP"      "Harvey"     1      "optimal"    16.065

  1×12 table

       interface          solver        method      problem     model       stat     origStat       time
    ________________    ___________    _________    _______    ________    ____    _________    ______

    "optimizeCbModel"    "ibm_cplex"    "SIFTING"     "LP"      "Harvey"     1      "optimal"    100.04

  1×12 table

       interface          solver          method        problem     model       stat     origStat       time
    ________________    ___________    _____________    _______    ________    ____    _________    _______

    "optimizeCbModel"    "ibm_cplex"    "CONCURRENT"     "LP"      "Harvey"     1      "optimal"    17.236
T = 25×12 table
                                                                                                          ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

```
    ⋮

  1×12 table

       interface          solver        method      problem     model       stat     origStat       time
    ________________    ___________    _________    _______    ________    ____    _________    ______

    "optimizeCbModel"    "gurobi"    "AUTOMATIC"     "LP"      "Harvey"     1      "OPTIMAL"    41.334

  1×12 table

       interface          solver        method      problem     model       stat     origStat       time
    ________________    ___________    _________    _______    ________    ____    _________    ______

    "optimizeCbModel"    "gurobi"    "PRIMAL"     "LP"      "Harvey"     -1      "TIME_LIMIT"    200.2    {
```

```
1×12 table

      interface          solver      method     problem      model       stat       origStat        time        __

    "optimizeCbModel"    "gurobi"    "DUAL"      "LP"       "Harvey"      -1      "TIME_LIMIT"     200.19     {[

1×12 table

      interface          solver       method       problem      model       stat      origStat        time        __

    "optimizeCbModel"    "gurobi"    "BARRIER"     "LP"       "Harvey"      1       "OPTIMAL"      31.246     {[

1×12 table

      interface          solver        method         problem      model       stat      origStat        time

    "optimizeCbModel"    "gurobi"    "CONCURRENT"     "LP"       "Harvey"      1       "OPTIMAL"      41.333

1×12 table

      interface          solver                  method                    problem      model       stat     origStat

    "optimizeCbModel"    "gurobi"    "DETERMINISTIC_CONCURRENT"     "LP"       "Harvey"      1       "OPTIMAL
T = 31×12 table
                                                                                                                ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

```
    ⋮
1×12 table
```

```
    interface          solver     method     problem     model       stat     origStat       time          o
    ________________    _______    ______    ________    ________    ____    _________    ______    ______

    "optimizeCbModel"    "mosek"    "FREE"     "QP"       "Harvey"     1      "OPTIMAL"    61.866    {[7.03

  1×12 table

    interface          solver      method     problem     model       stat     origStat       time

    ________________    _______    ________    ________    ________    ____    _________    ______    ____

    "optimizeCbModel"    "mosek"    "INTPNT"     "QP"       "Harvey"     1      "OPTIMAL"    61.835    {[7.
T = 33×12 table
                                                                                                      ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

```
    ⋮

  1×12 table

    interface          solver       method      problem     model       stat     origStat          ti

    ________________    _________    _________    _______    ________    ____    _____________    ____

    "optimizeCbModel"    "ibm_cplex"    "AUTOMATIC"     "QP"       "Harvey"     3      "non-optimal"    30.

  1×12 table

    interface          solver       method      problem     model       stat          origStat

    ________________    _________    ________    _______    ________    ____    ____________________

    "optimizeCbModel"    "ibm_cplex"    "PRIMAL"     "QP"       "Harvey"     -1     "time limit exceeded"

  1×12 table

    interface          solver       method     problem     model       stat          origStat

    ________________    _________    ______    _______    ________    ____    ____________________
```

```
    "optimizeCbModel"    "ibm_cplex"    "DUAL"     "QP"      "Harvey"     -1     "time limit exceeded"

  1×12 table

       interface          solver        method      problem     model       stat          origStat
    ________________    ___________    ________    _______    ________    ____    ____________________

    "optimizeCbModel"    "ibm_cplex"    "NETWORK"     "QP"      "Harvey"      -1     "time limit exceeded"

  1×12 table

       interface          solver        method      problem     model       stat       origStat        time
    ________________    ___________    ________    _______    ________    ____    _______________    _____

    "optimizeCbModel"    "ibm_cplex"    "BARRIER"     "QP"      "Harvey"      3      "non-optimal"    30.21

  1×12 table

       interface          solver          method        problem     model       stat          origStat
    ________________    ___________    ____________    _______    ________    ____    ____________________

    "optimizeCbModel"    "ibm_cplex"    "CONCURRENT"     "QP"      "Harvey"      -1     "time limit exceede
T = 39×12 table
                                                                                                      ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

```
    ⋮

  1×12 table

       interface          solver        method      problem     model       stat          origStat        time
    ________________    ___________    ________    _______    ________    ____    ____________________    ____

    "optimizeCbModel"    "gurobi"    "AUTOMATIC"     "QP"      "Harvey"      -1     "TIME_LIMIT"    201.39
```

```
1×12 table

      interface          solver      method      problem      model       stat       origStat        time
    _________________    ________    ________    _______    _________    ____    _____________    ______

    "optimizeCbModel"    "gurobi"    "PRIMAL"      "QP"       "Harvey"     -1     "TIME_LIMIT"    200.21

1×12 table

      interface          solver      method      problem      model       stat       origStat        time
    _________________    ________    ______    _______    _________    ____    _____________    ______    __

    "optimizeCbModel"    "gurobi"    "DUAL"      "QP"       "Harvey"     -1     "TIME_LIMIT"    200.18    {[

1×12 table

      interface          solver      method      problem      model       stat       origStat        time
    _________________    ________    _________    _______    _________    ____    _____________    ____    _

    "optimizeCbModel"    "gurobi"    "BARRIER"     "QP"       "Harvey"     -1     "TIME_LIMIT"    201     {
T = 43×12 table
                                                                                                        ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

```
    ⋮

1×12 table

      interface          solver      method      problem      model       stat       origStat        time          obj
    _________________    ________    ______    _______    _________    ____    _____________    ______    _______

    "optimizeCbModel"    "mosek"     "FREE"      "LP"       "Harvey"     1      "OPTIMAL"       23.077    {[2.65

1×12 table
```

```
    interface        solver      method      problem      model       stat     origStat       time          o
    _________________    _______    ________    _______    ________    ____    _________    ______    ____

    "optimizeCbModel"    "mosek"    "INTPNT"     "LP"      "Harvey"     1      "OPTIMAL"    23.255    {[2.
OPTIMAL
  1×12 table

    interface        solver     method     problem     model       stat     origStat       time          ob
    _________________    _______    _______    _______    ________    ____    _________    ______    ____

    "optimizeCbModel"    "mosek"    "CONIC"     "LP"      "Harvey"     1      "OPTIMAL"    16.841    {[2.7
Mosek returned an error or warning, open the following link in your browser:
https://docs.mosek.com/latest/toolbox/response-codes.html#mosek.rescode.trm_max_time
  1×12 table

    interface        solver         method          problem     model       stat                    orig
    _________________    _______    _____________________    _______    ________    ____    ____________________

    "optimizeCbModel"    "mosek"    "PRIMAL_SIMPLEX"     "LP"      "Harvey"     -1     "DUAL_FEASIBLE & MS
Mosek returned an error or warning, open the following link in your browser:
https://docs.mosek.com/latest/toolbox/response-codes.html#mosek.rescode.trm_max_time
  1×12 table

    interface        solver         method          problem     model       stat                  origStat
    _________________    _______    _____________________    _______    ________    ____    ____________________

    "optimizeCbModel"    "mosek"    "DUAL_SIMPLEX"     "LP"      "Harvey"     -1     "UNKNOWN & MSK_RES_TR
Mosek returned an error or warning, open the following link in your browser:
https://docs.mosek.com/latest/toolbox/response-codes.html#mosek.rescode.trm_max_time
  1×12 table

    interface        solver         method          problem     model       stat                  origStat
    _________________    _______    _____________________    _______    ________    ____    ____________________

    "optimizeCbModel"    "mosek"    "FREE_SIMPLEX"     "LP"      "Harvey"     -1     "UNKNOWN & MSK_RES_TR
T = 49×12 table
                                                                                                          ...
```

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"

	interface	solver	method	problem	model	stat	origStat
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

⋮

1×12 table

interface	solver	method	problem	model	stat	origStat	time
"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"LP"	"Harvey"	1	"optimal"	17.153

1×12 table

interface	solver	method	problem	model	stat	origStat
"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"LP"	"Harvey"	3	"time limit exceeded"

1×12 table

interface	solver	method	problem	model	stat	origStat
"optimizeCbModel"	"ibm_cplex"	"DUAL"	"LP"	"Harvey"	3	"optimal with unscaled in

1×12 table

interface	solver	method	problem	model	stat	origSta
"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"LP"	"Harvey"	3	"optimal with unscaled

1×12 table

interface	solver	method	problem	model	stat	origStat	time
"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"LP"	"Harvey"	1	"optimal"	16.145

1×12 table

interface	solver	method	problem	model	stat	origStat	time
"optimizeCbModel"	"ibm_cplex"	"SIFTING"	"LP"	"Harvey"	1	"optimal"	99.974

1×12 table

interface	solver	method	problem	model	stat	origStat	time
"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"LP"	"Harvey"	1	"optimal"	17.228

T = 56×12 table

...

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"

	interface	solver	method	problem	model	stat	origStat
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

⋮

```
1×12 table

      interface         solver        method       problem     model      stat     origStat     time
  _________________    ________    ____________    _______    ________    ____    __________    ______

  "optimizeCbModel"    "gurobi"    "AUTOMATIC"      "LP"      "Harvey"     1      "OPTIMAL"     41.581

1×12 table

      interface         solver       method      problem     model      stat      origStat       time
  _________________    ________    __________    _______    ________    ____    ____________    ______

  "optimizeCbModel"    "gurobi"    "PRIMAL"       "LP"      "Harvey"     -1     "TIME_LIMIT"    200.19

1×12 table

      interface         solver      method    problem     model      stat      origStat       time
  _________________    ________    ______    _______    ________    ____    ____________    ______    __

  "optimizeCbModel"    "gurobi"    "DUAL"     "LP"      "Harvey"     -1     "TIME_LIMIT"    200.19    {[

1×12 table

      interface         solver       method      problem     model      stat     origStat      time
  _________________    ________    ___________    _______    ________    ____    __________    ______    __

  "optimizeCbModel"    "gurobi"    "BARRIER"      "LP"      "Harvey"     1      "OPTIMAL"     30.577    {[

1×12 table

      interface         solver        method        problem     model      stat     origStat     time
  _________________    ________    _____________    _______    ________    ____    __________    ______
```

```
    "optimizeCbModel"    "gurobi"    "CONCURRENT"      "LP"       "Harvey"      1       "OPTIMAL"     42.292

  1×12 table

        interface          solver                 method                problem      model        stat     origStat
    _________________    ________    ___________________________    _______    _________    ____    ________

    "optimizeCbModel"    "gurobi"    "DETERMINISTIC_CONCURRENT"      "LP"       "Harvey"      1       "OPTIMAL
```

T = 62×12 table

...

	interface	solver	method	problem	model	stat	origStat
1	"optimizeCbModel"	"mosek"	"FREE"	"QP"	"Harvey"	1	"OPTIMAL"
2	"optimizeCbModel"	"mosek"	"INTPNT"	"QP"	"Harvey"	1	"OPTIMAL"
3	"optimizeCbModel"	"ibm_cplex"	"AUTOMATIC"	"QP"	"Harvey"	3	"non-optimal"
4	"optimizeCbModel"	"ibm_cplex"	"PRIMAL"	"QP"	"Harvey"	-1	"time limit exceeded"
5	"optimizeCbModel"	"ibm_cplex"	"DUAL"	"QP"	"Harvey"	-1	"time limit exceeded"
6	"optimizeCbModel"	"ibm_cplex"	"NETWORK"	"QP"	"Harvey"	-1	"time limit exceeded"
7	"optimizeCbModel"	"ibm_cplex"	"BARRIER"	"QP"	"Harvey"	3	"non-optimal"
8	"optimizeCbModel"	"ibm_cplex"	"CONCURRENT"	"QP"	"Harvey"	-1	"time limit exceeded"
9	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1	"TIME_LIMIT"
10	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
11	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1	"TIME_LIMIT"
12	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1	"TIME_LIMIT"
13	"optimizeCbModel"	"mosek"	"FREE"	"LP"	"Harvey"	1	"OPTIMAL"
14	"optimizeCbModel"	"mosek"	"INTPNT"	"LP"	"Harvey"	1	"OPTIMAL"

⋮

```
T.method = replace(T.method,'MSK_OPTIMIZER_','');
T.method = replace(T.method,'_',' ');
T.solver = replace(T.solver,'ibm_','');
T.approach = append(T.solver, ' ', T.method);
T =sortrows(T,{'stat','time'},{'ascend','ascend'});
display(T)
```

T = 62×13 table

...

	interface	solver	method	problem	model	stat
1	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1
2	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1
3	"optimizeCbModel"	"gurobi"	"DUAL"	"QP"	"Harvey"	-1
4	"optimizeCbModel"	"gurobi"	"DUAL"	"LP"	"Harvey"	-1

	interface	solver	method	problem	model	stat
5	"optimizeCbModel"	"gurobi"	"DUAL"	"LP"	"Harvey"	-1
6	"optimizeCbModel"	"gurobi"	"PRIMAL"	"LP"	"Harvey"	-1
7	"optimizeCbModel"	"gurobi"	"PRIMAL"	"LP"	"Harvey"	-1
8	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1
9	"optimizeCbModel"	"gurobi"	"PRIMAL"	"QP"	"Harvey"	-1
10	"optimizeCbModel"	"mosek"	"PRIMAL SIMPLEX"	"LP"	"Harvey"	-1
11	"optimizeCbModel"	"gurobi"	"BARRIER"	"QP"	"Harvey"	-1
12	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1
13	"optimizeCbModel"	"gurobi"	"AUTOMATIC"	"QP"	"Harvey"	-1
14	"optimizeCbModel"	"mosek"	"DUAL SIMPLEX"	"LP"	"Harvey"	-1

⋮

```
save([resultsFolder 'results_benchmarkWBMsolvers.mat'],'T')
```

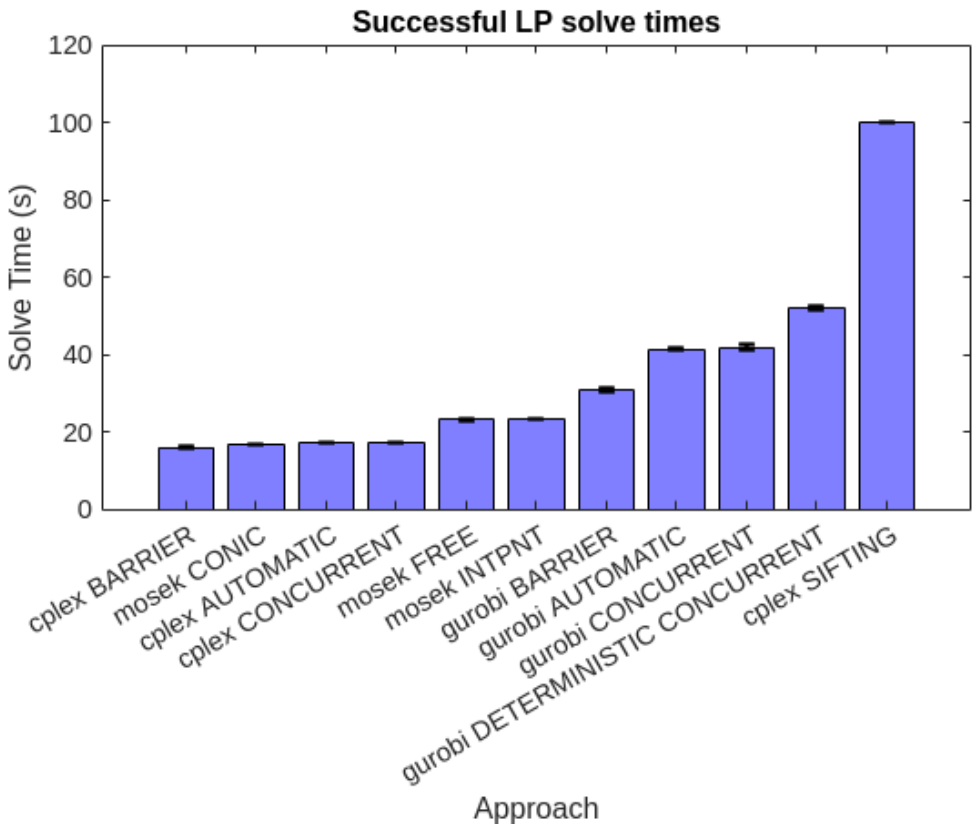
```
if 1
    if 0
        % Create the first histogram
        histogram(T.time(T.stat==1 &
strcmp(T.problem,'LP')),'NumBins',100,'FaceColor', 'r','FaceAlpha', 0.5);  %
'r' sets the color to red
        hold on;  % Keep the current plot so that the second histogram is
overlaid

        % Create the second histogram
        histogram(T.time(T.stat==1 &
strcmp(T.problem,'QP')),'NumBins',100,'FaceColor', 'b','FaceAlpha', 0.5);  %
'r' sets the color to red
        xlabel({'Whole body metabolic model LP solution time (seconds)',
[int2str(nMet) ' metabolites, ' int2str(nRxn) ' reactions.']})
        ylabel('Number of solutions')
        title('Solution time depends on solver, method and problem');
        legend('LP', 'QP');
        hold off;  % Release the hold for future plots

    else

        if ~exist('T0','var')
            T0 = T;
        else
            T = T0;
        end
```

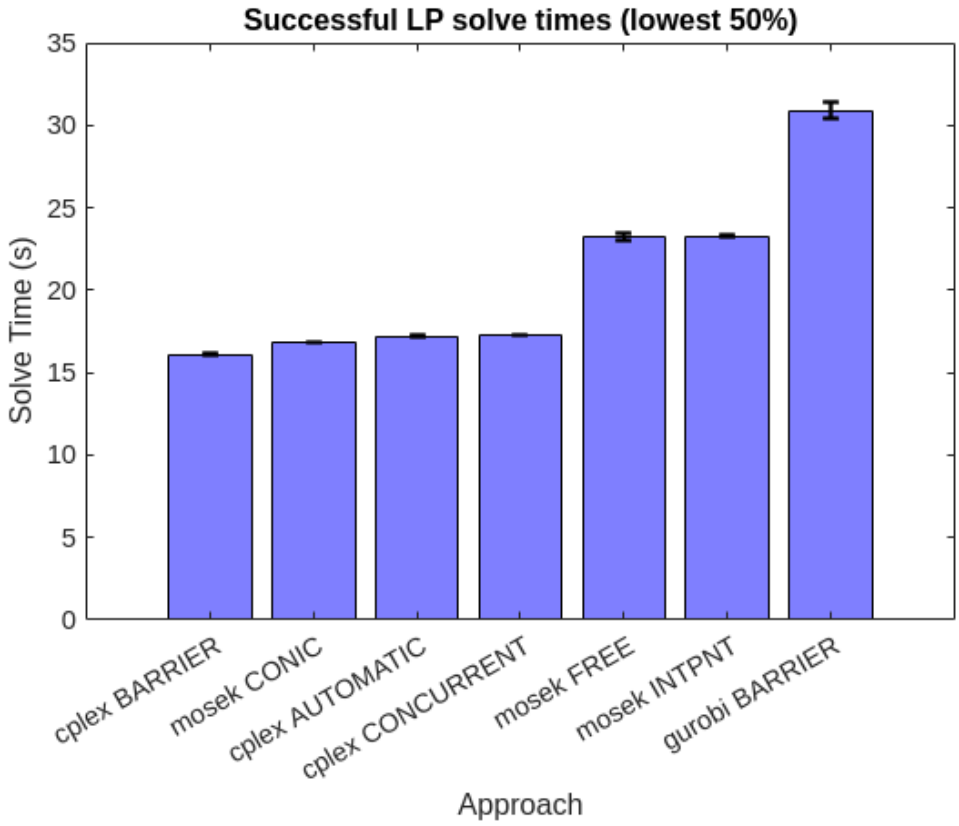
```
        % Concatenate solver and method into 'approach'
        T.approach = append(T.solver, ' ', T.method);
        T = T(strcmp(T.problem,'LP') & T.stat==1,:);
        % Calculate the mean solve time and standard deviation for each
approach
        avg_times = varfun(@mean, T, 'InputVariables', 'time',
'GroupingVariables', 'approach');
        std_times = varfun(@std, T, 'InputVariables', 'time',
'GroupingVariables', 'approach');

        times = avg_times;
        times.std_time = std_times.std_time;
        % Sort both the avg_times and std_times by the mean solve time
        [times, sort_idx] = sortrows(times, 'mean_time');

        figure
        % Create a bar plot with the sorted data
        b = bar(times.mean_time, 'FaceColor', 'b', 'FaceAlpha', 0.5);
        hold on;

        % Add error bars using the sorted standard deviations
        errorbar(times.mean_time, times.std_time, 'k', 'linestyle', 'none',
'LineWidth', 1.5);
        xticks(1:length(times.approach))
        xticklabels(times.approach)

        % Add labels and title
        xlabel('Approach', 'Interpreter', 'none');
        ylabel('Solve Time (s)');
        title('Successful LP solve times', 'Interpreter', 'none');

        if size(T,1)>1
            figure
            % fastest times
            times = times(times.mean_time<mean(times.mean_time),:);
            % Create a bar plot with the sorted data
            b = bar(times.mean_time, 'FaceColor', 'b', 'FaceAlpha', 0.5);
            hold on;

            % Add error bars using the sorted standard deviations
            errorbar(times.mean_time, times.std_time, 'k', 'linestyle',
'none', 'LineWidth', 1.5);
            xticks(1:length(times.approach))
            xticklabels(times.approach)

            % Add labels and title
            xlabel('Approach', 'Interpreter', 'none');
            ylabel('Solve Time (s)');
```

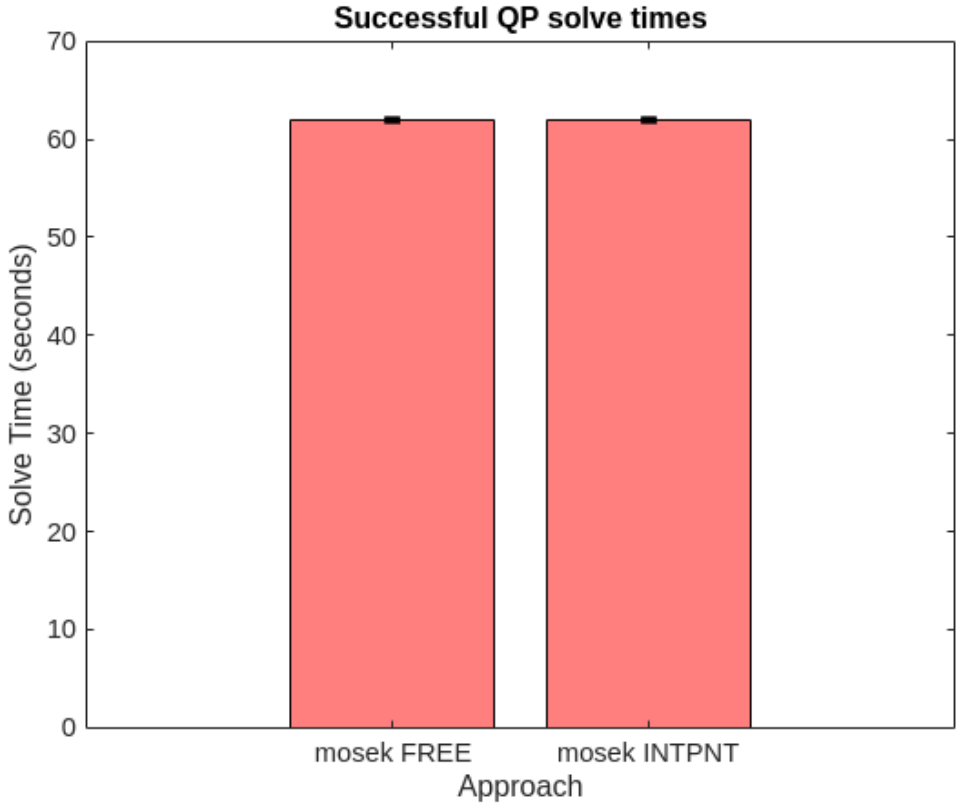
```
            title('Successful LP solve times (lowest 50%)', 'Interpreter',
'none');
        end


        T = T0;
        % Concatenate solver and method into 'approach'
        T.approach = append(T.solver, ' ', T.method);
        figure
        T = T(strcmp(T.problem,'QP') & T.stat==1,:);
        % Calculate the mean solve time and standard deviation for each
approach
        avg_times = varfun(@mean, T, 'InputVariables', 'time',
'GroupingVariables', 'approach');
        std_times = varfun(@std, T, 'InputVariables', 'time',
'GroupingVariables', 'approach');
        times = avg_times;
        times.std_time = std_times.std_time;
        % Sort both the avg_times and std_times by the mean solve time
        [times, sort_idx] = sortrows(times, 'mean_time');

        % Create a bar plot with the sorted data
        b = bar(times.mean_time, 'FaceColor', 'r', 'FaceAlpha', 0.5);
        hold on;

        % Add error bars using the sorted standard deviations
        errorbar(times.mean_time, times.std_time, 'k', 'linestyle', 'none',
'LineWidth', 1.5);
        xticks(1:length(times.approach))
        xticklabels(times.approach)

        % Add labels and title
        xlabel('Approach', 'Interpreter', 'none');
        ylabel('Solve Time (seconds)');
        title('Successful QP solve times', 'Interpreter', 'none');

        if size(T,1)>1
            figure
            % fastest times
            times = times(times.mean_time<mean(times.mean_time),:);
            % Create a bar plot with the sorted data
            b = bar(times.mean_time, 'FaceColor', 'r', 'FaceAlpha', 0.5);
            hold on;
            % Add error bars using the sorted standard deviations
            errorbar(times.mean_time, times.std_time, 'k', 'linestyle',
'none', 'LineWidth', 1.5);
            xticks(1:length(times.approach))
            xticklabels(times.approach)

            % Add labels and title
```

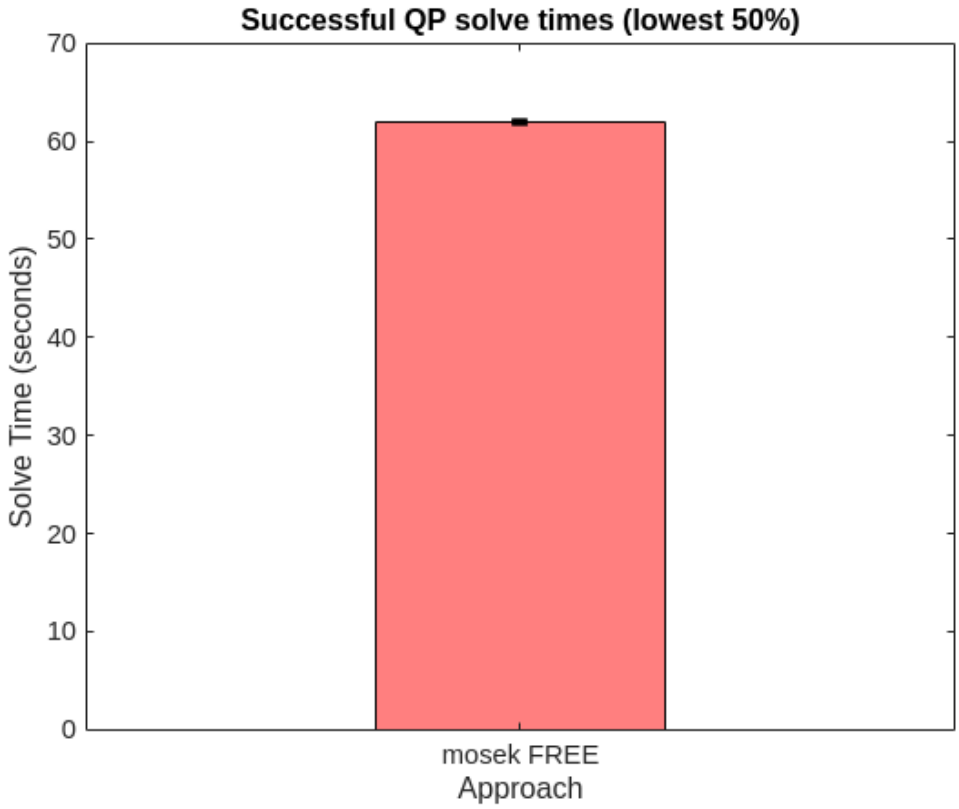
```
            xlabel('Approach', 'Interpreter', 'none');
            ylabel('Solve Time (seconds)');
            title('Successful QP solve times (lowest 50%)', 'Interpreter',
'none');
        end
    end
```

```
end
```




Successful LP solve times (lowest 50%)
Solve Time (s)
0
5
10
15
20
25
30
35
cplex BARRIER
mosek CONIC
cplex AUTOMATIC
cplex CONCURRENT
mosek FREE
mosek INTPNT
gurobi BARRIER
Approach


Successful QP solve times
Solve Time (seconds)
0
10
20
30
40
50
60
70
mosek FREE
mosek INTPNT
Approach



Acknowledgments

Please note an acknowledgments section can be included.

REFERENCES