# User Documentation for KINSOL v7.1.0 SUNDIALS v7.1.0

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#### **CONTRIBUTORS**

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# **Chapter 1**

# Introduction

KINSOL is part of a software family called SUNDIALS: SUite of Nonlinear and DIfferential/ALgebraic equation Solvers [33]. This suite consists of CVODE, ARKODE, KINSOL, and IDA, and variants of these with sensitivity analysis capabilities.

KINSOL is a general-purpose nonlinear system solver based on Newton-Krylov solver technology. A fixed point iteration is also included with the release of KINSOL v.2.8.0 and higher.

### 1.1 Historical Background

The first nonlinear solver packages based on Newton-Krylov methods were written in Fortran. In particular, the NKSOL package, written at LLNL, was the first Newton-Krylov solver package written for solution of systems arising in the solution of partial differential equations [17]. This Fortran code made use of Newton's method to solve the discrete nonlinear systems and applied a preconditioned Krylov linear solver for solution of the Jacobian system at each nonlinear iteration. The key to the Newton-Krylov method was that the matrix-vector multiplies required by the Krylov method could effectively be approximated by a finite difference of the nonlinear system-defining function, avoiding a requirement for the formation of the actual Jacobian matrix. Significantly less memory was required for the solver as a result.

In the late 1990s, there was a push at LLNL to rewrite the nonlinear solver in C and port it to distributed memory parallel machines. Both Newton and Krylov methods are easily implemented in parallel, and this effort gave rise to the KINSOL package. KINSOL is similar to NKSOL in functionality, except that it provides for more options in the choice of linear system methods and tolerances, and has a more modular design to provide flexibility for future enhancements.

At present, KINSOL may utilize a variety of Krylov methods provided in SUNDIALS. These methods include the GM-RES (Generalized Minimal RESidual) [46], FGMRES (Flexible Generalized Minimum RESidual) [45], Bi-CGStab (Bi-Conjugate Gradient Stabilized) [51], TFQMR (Transpose-Free Quasi-Minimal Residual) [28], and PCG (Preconditioned Conjugate Gradient) [32] linear iterative methods. As Krylov methods, these require little matrix storage for solving the Newton equations as compared to direct methods. However, the algorithms allow for a user-supplied preconditioner, and, for most problems, preconditioning is essential for an efficient solution. For very large nonlinear algebraic systems, the Krylov methods are preferable over direct linear solver methods, and are often the only feasible choice. Among the Krylov methods in SUNDIALS, we recommend GMRES as the best overall choice. However, users are encouraged to compare all options, especially if encountering convergence failures with GMRES. Bi-CGStab and TFQMR have an advantage in storage requirements, in that the number of workspace vectors they require is fixed, while that number for GMRES depends on the desired Krylov subspace size. FGMRES has an advantage in that it is designed to support preconditioners that vary between iterations (e.g., iterative methods). PCG exhibits rapid convergence and minimal workspace vectors, but only works for symmetric linear systems.

For the sake of completeness in functionality, direct linear system solvers are included in KINSOL. These include methods for both dense and banded linear systems, with Jacobians that are either user-supplied or generated internally by difference quotients. KINSOL also includes interfaces to sparse direct solvers, including KLU [4, 20] and the threaded sparse direct solver, SuperLU\_MT [9, 22, 40], among others (see Chapter §8 for further details).

In the process of translating NKSOL into C, the overall KINSOL organization has been changed considerably. One key feature of the KINSOL organization is that a separate module devoted to vector operations was created. This module facilitated extension to multiprosessor environments with minimal impact on the rest of the solver. The vector module design is shared across the SUNDIALS suite. This *N\_Vector* module is written in terms of abstract vector operations with the actual routines attached by a particular implementation (such as serial or parallel) of *N\_Vector*. This abstraction allows writing the SUNDIALS solvers in a manner independent of the actual *N\_Vector* implementation (which can be user-supplied), as well as allowing more than one *N\_Vector* module linked into an executable file. SUNDIALS (and thus KINSOL) is supplied with serial, MPI-parallel, OpenMP and Pthreads thread-parallel *N\_Vector* implementations, as well as multiple *N\_Vector* implementations designed to leverage GPU architectures (see Chapter §6 for further details).

There are several motivations for choosing the C language for KINSOL. First, a general movement away from Fortran and toward C in scientific computing was apparent. Second, the pointer, structure, and dynamic memory allocation features in C are extremely useful in software of this complexity, with the great variety of method options offered. Finally, we prefer C over C++ for KINSOL because of the wider availability of C compilers, the potentially greater efficiency of C, and the greater ease of interfacing the solver to applications written in Fortran.

### 1.2 Changes to SUNDIALS in release 7.1.0

#### **Major Features**

Created shared user interface functions for ARKODE to allow more uniform control over time-stepping algorithms, improved extensibility, and simplified code maintenance. The corresponding stepper-specific user-callable functions are now deprecated and will be removed in a future major release.

Added CMake infrastructure that enables externally maintained addons/plugins to be *optionally* built with SUNDIALS. See Contributing for details.

#### **New Features and Enhancements**

Added support for Kokkos Kernels v4.

Added the following Runge-Kutta Butcher tables

- ARKODE\_FORWARD\_EULER\_1\_1
- ARKODE\_RALSTON\_EULER\_2\_1\_2
- ARKODE\_EXPLICIT\_MIDPOINT\_EULER\_2\_1\_2
- ARKODE\_BACKWARD\_EULER\_1\_1
- ARKODE\_IMPLICIT\_MIDPOINT\_1\_2
- ARKODE\_IMPLICIT\_TRAPEZOIDAL\_2\_2

Added the following MRI coupling tables

- ARKODE\_MRI\_GARK\_FORWARD\_EULER
- ARKODE\_MRI\_GARK\_RALSTON2
- ARKODE\_MRI\_GARK\_RALSTON3
- ARKODE\_MRI\_GARK\_BACKWARD\_EULER
- ARKODE\_MRI\_GARK\_IMPLICIT\_MIDPOINT

- ARKODE\_IMEX\_MRI\_GARK\_EULER
- ARKODE\_IMEX\_MRI\_GARK\_TRAPEZOIDAL
- ARKODE\_IMEX\_MRI\_GARK\_MIDPOINT

Added ARKodeButcherTable\_ERKIDToName() and ARKodeButcherTable\_DIRKIDToName() to convert a Butcher table ID to a string representation.

Added the function ARKodeSetAutonomous() in ARKODE to indicate that the implicit right-hand side function does not explicitly depend on time. When using the trivial predictor, an autonomous problem may reuse implicit function evaluations across stage solves to reduce the total number of function evaluations.

Users may now disable interpolated output in ARKODE by passing ARK\_INTERP\_NONE to ARKodeSetInterpolant-Type(). When interpolation is disabled, rootfinding is not supported, implicit methods must use the trivial predictor (the default option), and interpolation at stop times cannot be used (interpolating at stop times is disabled by default). With interpolation disabled, calling ARKodeEvolve() in ARK\_NORMAL mode will return at or past the requested output time (setting a stop time may still be used to halt the integrator at a specific time). Disabling interpolation will reduce the memory footprint of an integrator by two or more state vectors (depending on the interpolant type and degree) which can be beneficial when interpolation is not needed e.g., when integrating to a final time without output in between or using an explicit fast time scale integrator with an MRI method.

Added "Resize" capability to ARKODE's SPRKStep time-stepping module.

Enabled the Fortran interfaces to build with 32-bit sunindextype.

#### **Bug Fixes**

Updated the CMake variable HIP\_PLATFORM default to amd as the previous default, hcc, is no longer recognized in ROCm 5.7.0 or newer. The new default is also valid in older version of ROCm (at least back to version 4.3.1).

Renamed the DPCPP value for the SUNDIALS\_GINKGO\_BACKENDS CMake option to SYCL to match Ginkgo's updated naming convention.

Changed the CMake version compatibility mode for SUNDIALS to AnyNewerVersion instead of SameMajorVersion. This fixes the issue seen here.

Fixed a CMake bug that caused an MPI linking error for our C++ examples in some instances. Fixes GitHub Issue #464.

Fixed the runtime library installation path for windows systems. This fix changes the default library installation path from CMAKE\_INSTALL\_PREFIX/CMAKE\_INSTALL\_BINDIR.

Fixed conflicting .1ib files between shared and static libs when using MSVC on Windows

Fixed invalid SUNDIALS\_EXPORT generated macro when building both shared and static libs.

Fixed a bug in some Fortran examples where c\_null\_ptr was passed as an argument to a function pointer instead of c\_null\_funptr. This caused compilation issues with the Cray Fortran compiler.

Fixed a bug in the HIP execution policies where WARP\_SIZE would not be set with ROCm 6.0.0 or newer.

Fixed a bug that caused error messages to be cut off in some cases. Fixes GitHub Issue #461.

Fixed a memory leak when an error handler was added to a SUNContext. Fixes GitHub Issue #466.

Fixed a bug where MRIStepEvolve() would not handle a recoverable error produced from evolving the inner stepper.

Added missing SetRootDirection and SetNoInactiveRootWarn functions to ARKODE's SPRKStep time-stepping module.

Fixed a bug in ARKodeSPRKTable\_Create() where the coefficient arrays were not allocated.

Fix bug on LLP64 platforms (like Windows 64-bit) where KLU\_INDEXTYPE could be 32 bits wide even if SUNDIALS\_-INT64\_T is defined.

Check if size of SuiteSparse\_long is 8 if the size of sunindextype is 8 when using KLU.

Fixed several build errors with the Fortran interfaces on Windows systems.

#### **Deprecation Notices**

Numerous ARKODE stepper-specific functions are now deprecated in favor of ARKODE-wide functions.

Deprecated the *ARKStepSetOptimalParams* function. Since this function does not have an ARKODE-wide equivalent, instructions have been added to the user guide for how to retain the current functionality using other user-callable functions.

The unsupported implementations of N\_VGetArrayPointer and N\_VSetArrayPointer for the *hypre* and PETSc vectors are now deprecated. Users should access the underlying wrapped external library vector objects instead with N\_VGetVector\_ParHyp and N\_VGetVector\_Petsc, respectively.

For changes in prior versions of SUNDIALS see §14.

### 1.3 Reading this User Guide

This user guide is a combination of general usage instructions and specific examples. We expect that some readers will want to concentrate on the general instructions, while others will refer mostly to the examples, and the organization is intended to accommodate both styles.

There are different possible levels of usage of KINSOL. The most casual user, with a small nonlinear system, can get by with reading all of Chapter §2, then Chapter :numref:KINSOL.Usage.CC through §5 only, and looking at examples in [19]. In a different direction, a more expert user with a nonlinear system may want to (a) use a package preconditioner (§5.5), (b) supply his/her own Jacobian or preconditioner routines (§5.4), (c) supply a new N\_Vector module (Chapter §6), or even (d) supply a different linear solver module (§5.3.2 and Chapter §8).

The structure of this document is as follows:

- In Chapter §2, we provide short descriptions of the numerical methods implemented by KINSOL for the solution of nonlinear systems.
- The following chapter describes the software organization of the KINSOL solver (§3).
- Chapter :numref:KINSOL.Usage.CC is the main usage document for KINSOL for C applications. It includes a complete description of the user interface for the solution of nonlinear algebraic systems.
- Chapter §6 gives a brief overview of the generic N\_Vector module shared among the various components of SUNDIALS, and details on the four N\_Vector implementations provided with SUNDIALS.
- Chapter §7 gives a brief overview of the generic SUNMatrix module shared among the various components of SUNDIALS, and details on the SUNMatrix implementations provided with SUNDIALS.
- Chapter §8 gives a brief overview of the generic SUNLinearSolver module shared among the various components of SUNDIALS. This chapter contains details on the SUNLinearSolver implementations provided with SUNDIALS. The chapter also contains details on the SUNLinearSolver implementations provided with SUNDIALS that interface with external linear solver libraries.
- Finally, in the appendices, we provide detailed instructions for the installation of KINSOL, within the structure of SUNDIALS (Appendix §10), as well as a list of all the constants used for input to and output from KINSOL functions (Appendix §12).

Finally, the reader should be aware of the following notational conventions in this user guide: program listings and identifiers (such as KINInit) within textual explanations appear in typewriter type style; fields in C structures (such as *content*) appear in italics; and packages or modules are written in all capitals. Usage and

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#### 1.4.3 SUNDIALS Release Numbers

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# 1.5 Acknowledgments

We wish to acknowledge the contributions to previous versions of the KINSOL code and user guide by Allan G. Taylor.

# **Chapter 2**

# **Mathematical Considerations**

KINSOL solves nonlinear algebraic systems in real N-space.

Using Newton's method, or the Picard iteration, one can solve

$$F(u) = 0, \quad F: \mathbb{R}^N \to \mathbb{R}^N, \tag{2.1}$$

given an initial guess  $u_0$ . Using a fixed-point iteration, the convergence of which can be improved with Anderson acceleration, one can solve

$$G(u) = u, \quad G: \mathbb{R}^N \to \mathbb{R}^N,$$
 (2.2)

given an initial guess  $u_0$ .

#### 2.1 Basic Newton iteration

Depending on the linear solver used, KINSOL can employ either an Inexact Newton method [15, 17, 21, 23, 36], or a Modified Newton method. At the highest level, KINSOL implements the following iteration scheme:

- 1. Set  $u_0 =$ an initial guess
- 2. For  $n = 0, 1, 2, \dots$  until convergence do:
  - a. Solve  $J(u_n)\delta_n = -F(u_n)$
  - b. Set  $u_{n+1} = u_n + \lambda \delta_n$ ,  $0 < \lambda \le 1$
  - c. Test for convergence

Here,  $u_n$  is the nth iterate to u, and J(u) = F'(u) is the system Jacobian. At each stage in the iteration process, a scalar multiple of the step  $\delta_n$ , is added to  $u_n$  to produce a new iterate,  $u_{n+1}$ . A test for convergence is made before the iteration continues.

#### 2.2 Newton method variants

For solving the linear system given in step (2a), KINSOL provides several choices, including the option of a user-supplied linear solver module. The linear solver modules distributed with SUNDIALS are organized in two families, a *direct* family comprising direct linear solvers for dense, banded, or sparse matrices and a *spils* family comprising scaled preconditioned iterative (Krylov) linear solvers. The methods offered through these modules are as follows:

- dense direct solvers, using either an internal implementation or a BLAS/LAPACK implementation (serial or threaded vector modules only),
- band direct solvers, using either an internal implementation or a BLAS/LAPACK implementation (serial or threaded vector modules only),
- sparse direct solver interfaces to various libraries, including KLU [4, 20], SuperLU\_MT [9, 22, 40], SuperLU\_Dist [8, 30, 41, 42], and cuSPARSE [7] [Note that users will need to download and install the relevant external packages independent of KINSOL],
- SPGMR, a scaled preconditioned GMRES (Generalized Minimal Residual method) solver,
- SPFGMR, a scaled preconditioned FGMRES (Flexible Generalized Minimal Residual method) solver,
- SPBCG, a scaled preconditioned Bi-CGStab (Bi-Conjugate Gradient Stable method) solver,
- SPTFQMR, a scaled preconditioned TFQMR (Transpose-Free Quasi-Minimal Residual method) solver, or
- PCG, a scaled preconditioned CG (Conjugate Gradient method) solver.

When using a direct linear solver, the linear system in 2a is solved exactly, thus resulting in a Modified Newton method (the Jacobian matrix is normally out of date; see below). Note that KINSOL allows the user to enforce a Jacobian evaluation at each iteration thus allowing for an Exact Newton iteration. Note that each direct linear solver is only compatible with a subset of vector representations (see §8.1.7 for details).

When using an iterative linear solver, the linear system in (2a) is solved only approximately, thus resulting in an Inexact Newton method. Here right preconditioning is available by way of the preconditioning setup and solve routines supplied by the user, in which case the iterative method is applied to the linear systems  $(JP^{-1})(P\delta) = -F$ , where P denotes the right preconditioning matrix.

Additionally, it is possible for users to supply a matrix-based iterative linear solver to KINSOL, resulting in a Modified Inexact Newton method. As with the direct linear solvers, the Jacobian matrix is updated infrequently; similarly as with iterative linear solvers the linear system is solved only approximately.

# 2.3 Jacobian information update strategy

In general, unless specified otherwise by the user, KINSOL strives to update Jacobian information (the actual system Jacobian J in the case of matrix-based linear solvers, and the preconditioner matrix P in the case of iterative linear solvers) as infrequently as possible to balance the high costs of matrix operations against other costs. Specifically, these updates occur when:

- the problem is initialized,
- $\|\lambda \delta_{n-1}\|_{D_u,\infty} > 1.5$  (Inexact Newton only),
- mbset= 10 nonlinear iterations have passed since the last update,
- the linear solver failed recoverably with outdated Jacobian information,
- the global strategy failed with outdated Jacobian information, or
- $\|\lambda \delta_n\|_{D_n,\infty}$  < steptol with outdated Jacobian or preconditioner information,

where the norm  $\|\cdot\|_{D_u,\infty}$  is defined below in (2.3).

KINSOL allows, through optional solver inputs, changes to the above strategy. Indeed, the user can disable the initial Jacobian information evaluation or change the default value of mbset, the number of nonlinear iterations after which a Jacobian information update is enforced.

### 2.4 Scaling

To address the case of ill-conditioned nonlinear systems, KINSOL allows users to prescribe scaling factors both for the solution vector and for the residual vector. For scaling to be used, the user should supply values  $D_u$ , which are diagonal elements of the scaling matrix such that  $D_u u_n$  has all components roughly the same magnitude when  $u_n$  is close to a solution, and  $D_F$ , which are diagonal scaling matrix elements such that  $D_F F$  has all components roughly the same magnitude when  $u_n$  is not too close to a solution. Based on these scaling matrices, we define the following scaled norms:

$$||z||_{D_u} = ||D_u z||_2, \ ||z||_{D_E} = ||D_F z||_2, \ ||z||_{D_u,\infty} = ||D_u z||_\infty, \ \text{and} \ ||z||_{D_E,\infty} = ||D_F z||_\infty$$
 (2.3)

where  $\|\cdot\|_{\infty}$  is the max norm. When scaling values are provided for the solution vector, these values are automatically incorporated into the calculation of the perturbations used for the default difference quotient approximations for Jacobian information; see (2.6) and (2.8) below.

## 2.5 Globalization strategy

Two methods of applying a computed step  $\delta_n$  to the previously computed solution vector are implemented. The first and simplest is the standard Newton strategy which applies step 2(b) as above with  $\lambda$  always set to 1. The other method is a global strategy, which attempts to use the direction implied by  $\delta_n$  in the most efficient way for furthering convergence of the nonlinear problem. This technique is implemented in the second strategy, called Linesearch. This option employs both the  $\alpha$  and  $\beta$  conditions of the Goldstein-Armijo linesearch given in [23] for step 2(b), where  $\lambda$  is chosen to guarantee a sufficient decrease in F relative to the step length as well as a minimum step length relative to the initial rate of decrease of F. One property of the algorithm is that the full Newton step tends to be taken close to the solution.

KINSOL implements a backtracking algorithm to first find a value  $\lambda$  such that  $u_n + \lambda \delta_n$  satisfies the sufficient decrease condition (or  $\alpha$ -condition)

$$F(u_n + \lambda \delta_n) \le F(u_n) + \alpha \nabla F(u_n)^T \lambda \delta_n$$
,

where  $\alpha = 10^{-4}$ . Although backtracking in itself guarantees that the step is not too small, KINSOL secondly relaxes  $\lambda$  to satisfy the so-called  $\beta$ -condition (equivalent to Wolfe's curvature condition):

$$F(u_n + \lambda \delta_n) \ge F(u_n) + \beta \nabla F(u_n)^T \lambda \delta_n$$

where  $\beta = 0.9$ . During this second phase,  $\lambda$  is allowed to vary in the interval  $[\lambda_{min}, \lambda_{max}]$  where

$$\lambda_{min} = \frac{steptol}{\|\bar{\delta}_n\|_{\infty}} \,, \quad \bar{\delta}_n^j = \frac{\delta_n^j}{1/D_u^j + |u^j|} \,,$$

and  $\lambda_{max}$  corresponds to the maximum feasible step size at the current iteration (typically  $\lambda_{max} = stepmax/\|\delta_n\|_{D_u}$ ). In the above expressions,  $v^j$  denotes the jth component of a vector v.

For more details, the reader is referred to [23].

2.4. Scaling 9

### 2.6 Nonlinear iteration stopping criteria

Stopping criteria for the Newton method are applied to both of the nonlinear residual and the step length. For the former, the Newton iteration must pass a stopping test

$$||F(u_n)||_{D_{F,\infty}} < \text{ftol}$$
,

where ftol is an input scalar tolerance with a default value of  $U^{1/3}$ . Here U is the machine unit roundoff. For the latter, the Newton method will terminate when the maximum scaled step is below a given tolerance

$$\|\lambda \delta_n\|_{D_{\infty,\infty}} < \text{steptol}$$
,

where steptol is an input scalar tolerance with a default value of  $U^{2/3}$ . Only the first condition (small residual) is considered a successful completion of KINSOL. The second condition (small step) may indicate that the iteration is stalled near a point for which the residual is still unacceptable.

#### 2.7 Additional constraints

As a user option, KINSOL permits the application of inequality constraints,  $u^i > 0$  and  $u^i < 0$ , as well as  $u^i \geq 0$  and  $u^i \leq 0$ , where  $u^i$  is the ith component of u. Any such constraint, or no constraint, may be imposed on each component. KINSOL will reduce step lengths in order to ensure that no constraint is violated. Specifically, if a new Newton iterate will violate a constraint, the maximum step length along the Newton direction that will satisfy all constraints is found, and  $\delta_n$  in Step 2(b) is scaled to take a step of that length.

# 2.8 Residual monitoring for Modified Newton method

When using a matrix-based linear solver, in addition to the strategy described above for the update of the Jacobian matrix, KINSOL also provides an optional nonlinear residual monitoring scheme to control when the system Jacobian is updated. Specifically, a Jacobian update will also occur when mbsetsub=5 nonlinear iterations have passed since the last update and

$$||F(u_n)||_{D_E} > \omega ||F(u_m)||_{D_E}$$

where  $u_n$  is the current iterate and  $u_m$  is the iterate at the last Jacobian update. The scalar  $\omega$  is given by

$$\omega = \min \left( \omega_{min} \, e^{\max(0, \rho - 1)}, \omega_{max} \right) \,, \tag{2.4}$$

with  $\rho$  defined as

$$\rho = \frac{\|F(u_n)\|_{D_F}}{\text{ftol}} \,,$$

where ftol is the input scalar tolerance discussed before. Optionally, a constant value  $\omega_{const}$  can be used for the parameter  $\omega$ .

The constants controlling the nonlinear residual monitoring algorithm can be changed from their default values through optional inputs to KINSOL. These include the parameters  $\omega_{min}$  and  $\omega_{max}$ , the constant value  $\omega_{const}$ , and the threshold mbsetsub.

# 2.9 Stopping criteria for iterative linear solvers

When using an Inexact Newton method (i.e. when an iterative linear solver is used), the convergence of the overall nonlinear solver is intimately coupled with the accuracy with which the linear solver in 2(a) above is solved. KINSOL provides three options for stopping criteria for the linear system solver, including the two algorithms of Eisenstat and Walker [26]. More precisely, the Krylov iteration must pass a stopping test

$$||J\delta_n + F||_{D_F} < (\eta_n + U)||F||_{D_F},$$

where  $\eta_n$  is one of:

#### Eisenstat and Walker Choice 1

$$\eta_n = \frac{| \|F(u_n)\|_{D_F} - \|F(u_{n-1}) + J(u_{n-1})\delta_n\|_{D_F} |}{\|F(u_{n-1})\|_{D_F}},$$

#### Eisenstat and Walker Choice 2

$$\eta_n = \gamma \left( \frac{\|F(u_n)\|_{D_F}}{\|F(u_{n-1})\|_{D_F}} \right)^{\alpha},$$

where default values of  $\gamma$  and  $\alpha$  are 0.9 and 2, respectively.

#### Constant $\eta$

$$\eta_n = \text{constant},$$

with 0.1 as the default.

The default strategy is "Eisenstat and Walker Choice 1". For both options 1 and 2, appropriate safeguards are incorporated to ensure that  $\eta$  does not decrease too quickly [26].

# 2.10 Difference quotient Jacobian approximations

With the SUNMATRIX\_DENSE and SUNMATRIX\_BAND matrix modules, the Jacobian may be supplied by a user routine, or approximated by difference quotients, at the user's option. In the latter case, we use the usual approximation

$$J^{ij} = [F^{i}(u + \sigma_{i}e^{j}) - F^{i}(u)]/\sigma_{i}.$$
(2.5)

The increments  $\sigma_j$  are given by

$$\sigma_j = \sqrt{U} \max\left\{ |u^j|, 1/D_u^j \right\}. \tag{2.6}$$

In the dense case, this scheme requires N evaluations of F, one for each column of J. In the band case, the columns of J are computed in groups, by the Curtis-Powell-Reid algorithm, with the number of F evaluations equal to the bandwidth. The parameter U above can (optionally) be replaced by a user-specified value, relfunc.

We note that with sparse and user-supplied matrix-based linear solvers, the Jacobian *must* be supplied by a user routine, i.e. it is not approximated internally within KINSOL.

In the case of a matrix-free iterative linear solver, Jacobian information is needed only as matrix-vector products Jv. If a routine for Jv is not supplied, these products are approximated by directional difference quotients as

$$J(u)v \approx [F(u+\sigma v) - F(u)]/\sigma, \qquad (2.7)$$

where u is the current approximation to a root of (2.1), and  $\sigma$  is a scalar. The choice of  $\sigma$  is taken from [17] and is given by

$$\sigma = \frac{\max\{|u^T v|, u_{typ}^T | v|\}}{\|v\|_2^2} \operatorname{sign}(u^T v) \sqrt{U}, \qquad (2.8)$$

where  $u_{typ}$  is a vector of typical values for the absolute values of the solution (and can be taken to be inverses of the scale factors given for u as described below). This formula is suitable for *scaled* vectors u and v, and so is applied to  $D_u u$  and  $D_u v$ . The parameter U above can (optionally) be replaced by a user-specified value, rel func. Convergence of the Newton method is maintained as long as the value of  $\sigma$  remains appropriately small, as shown in [15].

#### 2.11 Basic Fixed Point iteration

The basic fixed-point iteration scheme implemented in KINSOL is given by:

- 1. Set  $u_0 =$ an initial guess
- 2. For  $n = 0, 1, 2, \dots$  until convergence do:
  - Set  $u_{n+1} = (1 \beta)u_n + \beta G(u_n)$ .
  - Test for convergence.

Here,  $u_n$  is the n-th iterate to u. At each stage in the iteration process, the function G is applied to the current iterate with the damping parameter  $\beta$  to produce a new iterate,  $u_{n+1}$ . A test for convergence is made before the iteration continues.

For Picard iteration, as implemented in KINSOL, we consider a special form of the nonlinear function F, such that F(u) = Lu - N(u), where L is a constant nonsingular matrix and N is (in general) nonlinear. Then the fixed-point function G is defined as  $G(u) = u - L^{-1}F(u)$ . The Picard iteration is given by:

- 1. Set  $u_0 =$ an initial guess
- 2. For  $n = 0, 1, 2, \dots$  until convergence do:
  - Set  $u_{n+1} = (1 \beta)u_n + \beta G(u_n)$  where  $G(u_n) \equiv u_n L^{-1}F(u_n)$ .
  - Test  $F(u_{n+1})$  for convergence.

Here,  $u_n$  is the n-th iterate to u. Within each iteration, the Picard step is computed then added to  $u_n$  with the damping parameter  $\beta$  to produce the new iterate. Next, the nonlinear residual function is evaluated at the new iterate, and convergence is checked. Noting that  $L^{-1}N(u) = u - L^{-1}F(u)$ , the above iteration can be written in the same form as a Newton iteration except that here, L is in the role of the Jacobian. Within KINSOL, however, we leave this in a fixed-point form as above. For more information, see page 182 of [44].

#### 2.12 Anderson Acceleration

The Picard and fixed point methods can be significantly accelerated using Anderson's method [11, 27, 43, 52]. Anderson acceleration can be formulated as follows:

- 1. Set  $u_0 =$  an initial guess and  $m \ge 1$
- 2. Set  $u_1 = G(u_0)$
- 3. For  $n = 1, 2, \dots$  until convergence do:
  - a. Set  $m_n = \min\{m, n\}$
  - b. Set  $F_n = (f_{n-m_n}, \dots, f_n)$ , where  $f_i = G(u_i) u_i$

c. Determine 
$$\alpha^{(n)}=(\alpha_0^{(n)},\dots,\alpha_{m_n}^{(n)})$$
 that solves  $\min_{\alpha}\|F_n\alpha^T\|_2$  such that  $\sum_{i=0}^{m_n}\alpha_i=1$ 

d. Set 
$$u_{n+1} = \beta \sum_{i=0}^{m_n} \alpha_i^{(n)} G(u_{n-m_n+i}) + (1-\beta) \sum_{i=0}^{m_n} \alpha_i^{(n)} u_{n-m_n+i}$$

e. Test for convergence

It has been implemented in KINSOL by turning the constrained linear least-squares problem in step 3c into an unconstrained one leading to the algorithm given below:

- 1. Set  $u_0 =$  an initial guess and  $m \ge 1$
- 2. Set  $u_1 = G(u_0)$
- 3. For  $n = 1, 2, \dots$  until convergence do:
  - a. Set  $m_n = \min\{m, n\}$

b. Set 
$$\Delta F_n = (\Delta f_{n-m_n}, \dots, \Delta f_{n-1})$$
, where  $\Delta f_i = f_{i+1} - f_i$  and  $f_i = G(u_i) - u_i$ 

c. Determine 
$$\gamma^{(n)}=(\gamma_0^{(n)},\dots,\gamma_{m_n-1}^{(n)})$$
 that solves  $\min_{\gamma}\|f_n-\Delta F_n\gamma^T\|_2$ 

d. Set 
$$u_{n+1} = G(u_n) - \sum_{i=0}^{m_n-1} \gamma_i^{(n)} \Delta g_{n-m_n+i} - (1-\beta)(f(u_n) - \sum_{i=0}^{m_n-1} \gamma_i^{(n)} \Delta f_{n-m_n+i})$$
 with  $\Delta g_i = G(u_{i+1}) - G(u_i)$ 

e. Test for convergence

The least-squares problem in 3c is solved by applying a QR factorization to  $\Delta F_n = Q_n R_n$  and solving  $R_n \gamma = Q_n^T f_n$ . By default the damping is disabled i.e.,  $\beta = 1.0$ .

The Anderson acceleration implementation includes an option to delay the start of acceleration until after a given number of initial fixed-point or Picard iterations have been completed. This delay can be beneficial when the underlying method has strong global convergence properties as the initial iterations may help bring the iterates closer to a solution before starting the acceleration.

# 2.13 Anderson Acceleration QR Factorization

The default QR factorization routine used in Anderson acceleration is Modified Gram-Schmidt, a stable orthogonalization routine that requires an increasing number of synchronizations per iteration dependent upon the number of vectors being orthogonalized against. While practical use of Anderson acceleration only requires a small number of vectors to be used in the QR factorization, this linearly scaling number of synchronizations per iteration can yield poor performance when Anderson acceleration is performed in a parallel setting. To combat this poor performance, low synchronization QR routines are available to the user, in particular: Inverse Compact WY Modified Gram-Schmidt [10], along with variants of Classical Gram-Schmidt with Reorthogonalization [31]. While all of these QR factorization routines are mathematically equivalent, they do not exhibit the same stability when performed with floating point arithmetic or in a parallel setting.

Inverse Compact WY Modified Gram-Schmidt, which is based on triangular solve variants of Gram-Schmidt that were developed within the context of GMRES, is an option that only requires two synchronizations per iteration. Additionally, it adds a lower triangular solve at every iteration, but this generally does not affect performance due to the system solve being small i.e., the number of vectors being orthogonalized against.

The remaining orthogonalization options are based on and include Classical Gram-Schmidt with Reorthogonalization (CGS-2). CGS-2 only requires three synchronizations per iteration, but does not exhibit the same stability as Modified Gram-Schmidt. Classical Gram-Schmidt with Delayed Reorthogolonization has the same stability as CGS-2, but it reduces the number of synchronizations per iteration to two.

# 2.14 Fixed-point - Anderson Acceleration Stopping Criterion

The default stopping criterion is

$$||u_{n+1}-u_n||_{D_F,\infty}<\operatorname{gtol},$$

where  $D_F$  is a user-defined diagonal matrix that can be the identity or a scaling matrix chosen so that the components of  $D_F(G(u)-u)$  have roughly the same order of magnitude. Note that when using Anderson acceleration, convergence is checked after the acceleration is applied.

# 2.15 Picard - Anderson Acceleration Stopping Criterion

The default stopping criterion is

$$||F(u_{n+1})||_{D_F,\infty} < \text{ftol}$$
,

where  $D_F$  is a user-defined diagonal matrix that can be the identity or a scaling matrix chosen so that the components of  $D_F F(u)$  have roughly the same order of magnitude. Note that when using Anderson acceleration, convergence is checked after the acceleration is applied.

# **Chapter 3**

# **Code Organization**

The KINSOL package is written in ANSI C. The following summarizes the basic structure of the package, although knowledge of this structure is not necessary for its use.

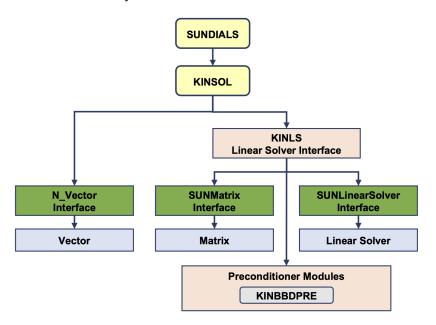


Fig. 3.1: Overall structure diagram of the KINSOL package. Components specific to KINSOL begin with "KINSOL" (KINLS and KINSOLBBDPRE), all other items correspond to generic SUNDIALS vector, matrix, and solver interfaces.

The overall organization of the KINSOL package is shown in Fig. 3.1. KINSOL utilizes generic linear solvers defined by the SUNLinearSolver (see §8). As such, KINSOL has no knowledge of the method being used to solve the linear and nonlinear systems that arise. For any given user problem, there exists a single nonlinear solver interface and, if necessary, one of the linear system solver interfaces is specified, and invoked as needed during the integration.

KINSOL has a single unified linear solver interface, KINSOLLS, supporting both direct and iterative linear solvers built using the generic SUNLinearSolver interface (see §8). These solvers may utilize a SUNMatrix object (see §7) for storing Jacobian information, or they may be matrix-free. Since KINSOL can operate on any valid SUNLinearSolver, the set of linear solver modules available to KINSOL will expand as new SUNLinearSolver implementations are developed.

For users employing SUNMATRIX\_DENSE or SUNMATRIX\_BAND Jacobian matrices, KINSOL includes algorithms

for their approximation through difference quotients, although the user also has the option of supplying a routine to compute the Jacobian (or an approximation to it) directly. This user-supplied routine is required when using sparse or user-supplied Jacobian matrices.

For users employing matrix-free iterative linear solvers, KINSOL includes an algorithm for the approximation by difference quotients of the product Jv. Again, the user has the option of providing routines for this operation, in two phases: setup (preprocessing of Jacobian data) and multiplication.

For preconditioned iterative methods, the preconditioning must be supplied by the user, again in two phases: setup and solve. While there is no default choice of preconditioner analogous to the difference-quotient approximation in the direct case, the references [16, 18], together with the example and demonstration programs included with KINSOL, offer considerable assistance in building preconditioners.

KINSOL's linear solver interface consists of four primary phases, devoted to (1) memory allocation and initialization, (2) setup of the matrix data involved, (3) solution of the system, and (4) freeing of memory. The setup and solution phases are separate because the evaluation of Jacobians and preconditioners is done only periodically during the integration, and only as required to achieve convergence. The call list within the central KINSOL module to each of the four associated functions is fixed, thus allowing the central module to be completely independent of the linear system method.

KINSOL also provides a preconditioner module, for use with any of the Krylov iterative linear solvers. It works in conjunction with the *NVECTOR\_PARALLEL* and generates a preconditioner that is a block-diagonal matrix with each block being a banded matrix.

All state information used by KINSOL to solve a given problem is stored in N\_Vector instances. There is no global data in the KINSOL package, and so, in this respect, it is reentrant. State information specific to the linear and nonlinear solver are saved in the SUNLinearSolver and SUNNonlinearSolver instances respectively. The reentrancy of KINSOL enables the setting where two or more problems are solved by intermixed or parallel calls to different instances of the package from within a single user program.

# **Chapter 4**

# **Getting Started**

The packages that make up SUNDIALS are built upon shared classes for vectors, matrices, and algebraic solvers. In addition, the packages all leverage some other common infrastructure, which we discuss in this section.

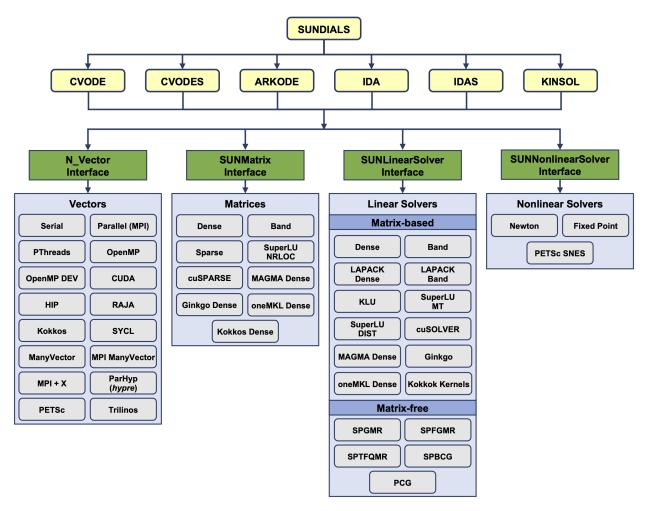


Fig. 4.1: High-level diagram of the SUNDIALS suite.

# 4.1 Data Types

SUNDIALS defines several data types in the header file sundials\_types.h. These types are used in the SUNDIALS API and internally in SUNDIALS. It is not necessary to use these types in your application, but the type must be compatible with the SUNDIALS types in the API when calling SUNDIALS functions. The types that are defined are:

- sunreal type the floating-point type used by the SUNDIALS packages
- sunindextype the integer type used for vector and matrix indices
- sunbooleantype the type used for logic operations within SUNDIALS
- SUNOutputFormat an enumerated type for SUNDIALS output formats
- SUNComm a simple typedef to an int when SUNDIALS is built without MPI, or a MPI\_Comm when built with MPI.

#### 4.1.1 Floating point types

#### type sunrealtype

The type sunrealtype can be float, double, or long double, with the default being double. The user can change the precision of the arithmetic used in the SUNDIALS solvers at the configuration stage (see *SUNDIALS\_-PRECISION*).

Additionally, based on the current precision, sundials\_types.h defines SUN\_BIG\_REAL to be the largest value representable as a sunrealtype, SUN\_SMALL\_REAL to be the smallest value representable as a sunrealtype, and SUN\_-UNIT\_ROUNDOFF to be the difference between 1.0 and the minimum sunrealtype greater than 1.0.

Within SUNDIALS, real constants are set by way of a macro called SUN\_RCONST. It is this macro that needs the ability to branch on the definition of sunrealtype. In ANSI C, a floating-point constant with no suffix is stored as a double. Placing the suffix "F" at the end of a floating point constant makes it a float, whereas using the suffix "L" makes it a long double. For example,

```
#define A 1.0
#define B 1.0F
#define C 1.0L
```

defines A to be a double constant equal to 1.0, B to be a float constant equal to 1.0, and C to be a long double constant equal to 1.0. The macro call SUN\_RCONST(1.0) automatically expands to 1.0 if sunrealtype is double, to 1.0F if sunrealtype is float, or to 1.0L if sunrealtype is long double. SUNDIALS uses the SUN\_RCONST macro internally to declare all of its floating-point constants.

Additionally, SUNDIALS defines several macros for common mathematical functions *e.g.*, fabs, sqrt, exp, etc. in sundials\_math.h. The macros are prefixed with SUNR and expand to the appropriate C function based on the sunrealtype. For example, the macro SUNRabs expands to the C function fabs when sunrealtype is double, fabsf when sunrealtype is float, and fabsl when sunrealtype is long double.

A user program which uses the type sunrealtype, the SUN\_RCONST macro, and the SUNR mathematical function macros is precision-independent except for any calls to precision-specific library functions. Our example programs use sunrealtype, SUN\_RCONST, and the SUNR macros. Users can, however, use the type double, float, or long double in their code (assuming that this usage is consistent with the typedef for sunrealtype) and call the appropriate math library functions directly. Thus, a previously existing piece of C or C++ code can use SUNDIALS without modifying the code to use sunrealtype, SUN\_RCONST, or the SUNR macros so long as the SUNDIALS libraries are built to use the corresponding precision (see §11.2).

### 4.1.2 Integer types used for indexing

#### type sunindextype

The type sunindextype is used for indexing array entries in SUNDIALS modules as well as for storing the total problem size (*e.g.*, vector lengths and matrix sizes). During configuration sunindextype may be selected to be either a 32- or 64-bit *signed* integer with the default being 64-bit (see *SUNDIALS\_INDEX\_SIZE*).

When using a 32-bit integer the total problem size is limited to  $2^{31} - 1$  and with 64-bit integers the limit is  $2^{63} - 1$ . For users with problem sizes that exceed the 64-bit limit an advanced configuration option is available to specify the type used for sunindextype (see SUNDIALS\_INDEX\_TYPE).

A user program which uses sunindextype to handle indices will work with both index storage types except for any calls to index storage-specific external libraries. Our C and C++ example programs use sunindextype. Users can, however, use any compatible type (e.g., int, long int, int32\_t, int64\_t, or long long int) in their code, assuming that this usage is consistent with the typedef for sunindextype on their architecture. Thus, a previously existing piece of C or C++ code can use SUNDIALS without modifying the code to use sunindextype, so long as the SUNDIALS libraries use the appropriate index storage type (for details see §11.2).

#### 4.1.3 Boolean type

#### type sunbooleantype

As ANSI C89 (ISO C90) does not have a built-in boolean data type, SUNDIALS defines the type sunboolean-type as an int.

The advantage of using the name sunbooleantype (instead of int) is an increase in code readability. It also allows the programmer to make a distinction between int and boolean data. Variables of type sunbooleantype are intended to have only the two values: *SUNFALSE* or *SUNTRUE*.

#### SUNFALSE

False (0)

#### **SUNTRUE**

True (1)

#### 4.1.4 Output formatting type

#### enum SUNOutputFormat

The enumerated type SUNOutputFormat defines the enumeration constants for SUNDIALS output formats

#### enumerator SUN\_OUTPUTFORMAT\_TABLE

The output will be a table of values

#### enumerator SUN\_OUTPUTFORMAT\_CSV

The output will be a comma-separated list of key and value pairs e.g., key1,value1,key2,value2,...

**Note:** The file scripts/sundials\_csv.py provides python utility functions to read and output the data from a SUNDIALS CSV output file using the key and value pair format.

4.1. Data Types

#### 4.1.5 MPI types

#### type SUNComm

A simple typedef to an *int* when SUNDIALS is built without MPI, or a MPI\_Comm when built with MPI. This type exists solely to ensure SUNDIALS can support MPI and non-MPI builds.

#### SUN\_COMM\_NULL

A macro defined as 0 when SUNDIALS is built without MPI, or as MPI\_COMM\_NULL when built with MPI.

# 4.2 The SUNContext Type

New in version 6.0.0.

All of the SUNDIALS objects (vectors, linear and nonlinear solvers, matrices, etc.) that collectively form a SUNDIALS simulation, hold a reference to a common simulation context object defined by the *SUNContext* class.

#### type SUNContext

An opaque pointer used by SUNDIALS objects for error handling, logging, profiling, etc.

Users should create a SUNContext object prior to any other calls to SUNDIALS library functions by calling:

```
SUNErrCode SUNContext_Create(SUNComm comm, SUNContext *sunctx)
```

Creates a *SUNContext* object associated with the thread of execution. The data of the *SUNContext* class is private.

#### **Parameters**

- comm the MPI communicator or SUN\_COMM\_NULL if not using MPI.
- **sunctx** [in,out] upon successful exit, a pointer to the newly created *SUNContext* object.

#### Returns

SUNErrCode indicating success or failure.

The created *SUNContext* object should be provided to the constructor routines for different SUNDIALS classes/modules e.g.,

```
SUNContext sunctx;
void* package_mem;
N_Vector x;

SUNContext_Create(SUN_COMM_NULL, &sunctx);

package_mem = CVodeCreate(..., sunctx);

package_mem = IDACreate(..., sunctx);

package_mem = KINCreate(..., sunctx);

package_mem = ARKStepCreate(..., sunctx);

x = N_VNew_<SomeVector>(..., sunctx);
```

After all other SUNDIALS code, the SUNContext object should be freed with a call to:

```
SUNErrCode SUNContext_Free(SUNContext *sunctx)
```

Frees the SUNContext object.

#### **Parameters**

• **sunctx** – pointer to a valid *SUNContext* object, NULL upon successful return.

#### Returns

SUNErrCode indicating success or failure.

Warning: When MPI is being used, the SUNContext\_Free() must be called prior to MPI\_Finalize.

The SUNContext API further consists of the following functions:

#### SUNErrCode SUNContext\_GetLastError(SUNContext sunctx)

Gets the last error code set by a SUNDIALS function call. The function then resets the last error code to SUN\_-SUCCESS.

#### **Parameters**

• sunctx – a valid SUNContext object.

#### Returns

the last SUNErrCode recorded.

#### SUNErrCode SUNContext\_PeekLastError(SUNContext sunctx)

Gets the last error code set by a SUNDIALS function call. The function *does not* reset the last error code to *SUN\_SUCCESS*.

#### **Parameters**

• sunctx – a valid SUNContext object.

#### Returns

the last SUNErrCode recorded.

# SUNErrCode SUNContext\_PushErrHandler(SUNContext sunctx, SUNErrHandlerFn err\_fn, void \*err\_user\_data)

Pushes a new SUNErrHandlerFn onto the error handler stack so that it is called when an error occurs inside of SUNDIALS.

#### **Parameters**

- **sunctx** a valid *SUNContext* object.
- err\_fn a callback function of type SUNErrHandlerFn to be pushed onto the error handler stack.
- err\_user\_data a pointer that will be passed back to the callback function when it is called.

#### Returns

SUNErrCode indicating success or failure.

#### SUNErrCode SUNContext\_PopErrHandler(SUNContext sunctx)

Pops the last SUNErrHandlerFn off of the error handler stack.

#### **Parameters**

• **sunctx** – a valid *SUNContext* object.

#### Returns

SUNErrCode indicating success or failure.

#### SUNErrCode SUNContext\_ClearErrHandlers(SUNContext sunctx)

Clears the entire error handler stack. After doing this it is important to push an error handler onto the stack with *SUNContext\_PushErrHandler* otherwise errors will be ignored.

#### **Parameters**

• **sunctx** – a valid *SUNContext* object.

#### Returns

SUNErrCode indicating success or failure.

#### SUNErrCode SUNContext\_GetProfiler(SUNContext sunctx, SUNProfiler \*profiler)

Gets the SUNProfiler object associated with the SUNContext object.

#### **Parameters**

- **sunctx** a valid *SUNContext* object.
- **profiler** [in,out] a pointer to the *SUNProfiler* object associated with this context; will be NULL if profiling is not enabled.

#### Returns

SUNErrCode indicating success or failure.

#### SUNErrCode SUNContext\_SetProfiler(SUNContext sunctx, SUNProfiler profiler)

Sets the SUNProfiler object associated with the SUNContext object.

#### **Parameters**

- **sunctx** a valid *SUNContext* object.
- profiler a SUNProfiler object to associate with this context; this is ignored if profiling
  is not enabled.

#### Returns

SUNErrCode indicating success or failure.

#### SUNErrCode SUNContext\_SetLogger(SUNContext sunctx, SUNLogger logger)

Sets the SUNLogger object associated with the SUNContext object.

#### **Parameters**

- sunctx a valid SUNContext object.
- logger a SUNLogger object to associate with this context; this is ignored if logging is not
  enabled.

#### Returns

SUNErrCode indicating success or failure.

New in version 6.2.0.

### SUNErrCode SUNContext\_GetLogger(SUNContext sunctx, SUNLogger \*logger)

Gets the SUNLogger object associated with the SUNContext object.

#### **Parameters**

- sunctx a valid SUNContext object.
- **logger** [in,out] a pointer to the *SUNLogger* object associated with this context; will be NULL if logging is not enabled.

#### Returns

SUNErrCode indicating success or failure.

New in version 6.2.0.

### 4.2.1 Implications for task-based programming and multi-threading

Applications that need to have *concurrently initialized* SUNDIALS simulations need to take care to understand the following:

- #. A SUNContext object must only be associated with one SUNDIALS simulation (a solver object and its associated vectors etc.) at a time.
  - Concurrently initialized is not the same as concurrently executing. Even if two SUNDIALS simulations execute sequentially, if both are initialized at the same time with the same *SUNContext*, behavior is undefined.
  - It is OK to reuse a SUNContext object with another SUNDIALS simulation after the first simulation has completed and all of the simulation's associated objects (vectors, matrices, algebraic solvers, etc.) have been destroyed.
- #. The creation and destruction of a *SUNContext* object is cheap, especially in comparison to the cost of creating/destroying a SUNDIALS solver object.

The following (incomplete) code examples demonstrate these points using CVODE as the example SUNDIALS package.

```
SUNContext sunctxs[num_threads];
int cvode_initialized[num_threads];
void* cvode_mem[num_threads];
// Create
for (int i = 0; i < num\_threads; i++) {
   sunctxs[i] = SUNContext_Create(...);
   cvode_mem[i] = CVodeCreate(..., sunctxs[i]);
   cvode_initialized[i] = 0; // not yet initialized
   // set optional cvode inputs...
}
// Solve
#pragma omp parallel for
for (int i = 0; i < num_problems; i++) {
   int retval = 0:
   int tid = omp_get_thread_num();
   if (!cvode_initialized[tid]) {
      retval = CVodeInit(cvode_mem[tid], ...);
      cvode_initialized[tid] = 1;
   } else {
      retval = CVodeReInit(cvode_mem[tid], ...);
   CVode(cvode_mem[i], ...);
}
// Destroy
for (int i = 0; i < num_threads; i++) {</pre>
   // get optional cvode outputs...
   CVodeFree(&cvode_mem[i]);
   SUNContext_Free(&sunctxs[i]);
}
```

Since each thread has its own unique CVODE and SUNContext object pair, there should be no thread-safety issues. Users should be sure that you apply the same idea to the other SUNDIALS objects needed as well (e.g. an N\_Vector).

The variation of the above code example demonstrates another possible approach:

```
// Create, Solve, Destroy
#pragma omp parallel for
for (int i = 0; i < num_problems; i++) {
    int retval = 0;
    void* cvode_mem;
    SUNContext sunctx;

    sunctx = SUNContext_Create(...);
    cvode_mem = CVodeCreate(..., sunctx);
    retval = CVodeInit(cvode_mem, ...);

// set optional cvode inputs...

CVode(cvode_mem, ...);

// get optional cvode outputs...

CVodeFree(&cvode_mem);
    SUNContext_Free(&sunctx);
}</pre>
```

So long as the overhead of creating/destroying the CVODE object is small compared to the cost of solving the ODE, this approach is a fine alternative to the first approach since <code>SUNContext\_Create()</code> and <code>SUNContext\_Free()</code> are much cheaper than the CVODE create/free routines.

#### 4.2.2 Convenience class for C++ Users

For C++ users a RAII safe class, sundials::Context, is provided:

```
namespace sundials {
class Context : public sundials::ConvertibleTo<SUNContext>
{
public:
explicit Context(SUNComm comm = SUN_COMM_NULL)
   sunctx_ = std::make_unique<SUNContext>();
   SUNContext_Create(comm, sunctx_.get());
}
/* disallow copy, but allow move construction */
Context(const Context&) = delete;
Context(Context&&)
                    = default;
/* disallow copy, but allow move operators */
Context& operator=(const Context&) = delete;
Context& operator=(Context&&) = default;
SUNContext Convert() override
{
   return *sunctx_.get();
```

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```
}
SUNContext Convert() const override
   return *sunctx_.get();
}
operator SUNContext() override
{
   return *sunctx_.get();
}
operator SUNContext() const override
   return *sunctx_.get();
}
~Context()
{
   if (sunctx_) SUNContext_Free(sunctx_.get());
}
private:
std::unique_ptr<SUNContext> sunctx_;
};
} // namespace sundials
```

# 4.3 Error Checking

New in version 7.0.0.

Until version 7.0.0, error reporting and handling was inconsistent throughout SUNDIALS. Starting with version 7.0.0 all of SUNDIALS (the core, implementations of core modules, and packages) reports error mesages through the SUNLogger API. Furthermore, functions in the SUNDIALS core API (i.e., SUN or N\_V functions only) either return a SUNErrCode, or (if they don't return a SUNErrCode) they internally record an error code (if an error occurs) within the SUNContext for the execution stream. This "last error" is accessible via the SUNContext\_GetLastError() or SUNContext\_PeekLastError() functions.

#### typedef int SUNErrCode

Thus, in user code, SUNDIALS core API functions can be checked for errors in one of two ways:

```
SUNContext sunctx;
SUNErrCode sunerr;
N_Vector v;
int length;
sunrealtype dotprod;

// Every code that uses SUNDIALS must create a SUNContext.
sunctx = SUNContext_Create(...);

// Create a SUNDIALS serial vector.
// Some functions do not return an error code.

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```

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```
// We have to check for errors in these functions using SUNContext_GetLastError.
length = 2;
v = N_VNew_Serial(length, sunctx);
sunerr = SUNContext_GetLastError(sunctx);
if (sunerr) { /* an error occured, do something */ }

// If the function returns a SUNErrCode, we can check it directly
sunerr = N_VLinearCombination(...);
if (sunerr) { /* an error occured, do something */ }

// Another function that does not return a SUNErrCode.
dotprod = N_VDotProd(...);
SUNContext_GetLastError(sunctx);
if (sunerr) {
    /* an error occured, do something */
} else {
    print("dotprod = %.2f\n", dotprod);
}
```

The function *SUNGetErrMsg()* can be used to get a message describing the error code.

const char \*SUNGetErrMsg(SUNErrCode code)

Returns a message describing the error code.

#### **Parameters**

• code – the error code

#### Returns

a message describing the error code.

**Note:** It is recommended in most cases that users check for an error after calling SUNDIALS functions. However, users concerned with getting the most performance might choose to exclude or limit these checks.

**Warning:** If a function returns a *SUNErrCode* then the return value is the only place the error is available i.e., these functions do not store their error code as the "last error" so it is invalid to use *SUNContext\_GetLastError()* to check these functions for errors.

#### 4.3.1 Error Handler Functions

When an error occurs in SUNDIALS, it calls error handler functions that have been pushed onto the error handler stack in last-in first-out order. Specific error handlers can be enabled by pushing them onto the error handler stack with the function <code>SUNContext\_PushErrHandler()</code>. They may disabled by calling <code>SUNContext\_PopErrHandler()</code> or <code>SUNContext\_ClearErrHandlers()</code>. A SUNDIALS error handler function has the type

typedef void (\*SUNErrHandlerFn)(int line, const char \*func, const char \*file, const char \*msg, SUNErrCode err\_code, void \*err\_user\_data, SUNContext sunctx)

SUNDIALS provides a few different error handlers that can be used, or a custom one defined by the user can be provided (useful for linking SUNDIALS errors to your application's error handling). The default error handler is <code>SUNLogErrHandlerFn()</code> which logs an error to a specified file or <code>stderr</code> if no file is specified.

The error handlers provided in SUNDIALS are:

void **SUNLogErrHandlerFn**(int line, const char \*func, const char \*file, const char \*msg, *SUNErrCode* err\_code, void \*err user data, *SUNContext* sunctx)

Logs the error that occurred using the SUNLogger from sunctx. This is the default error handler.

#### **Parameters**

- line the line number at which the error occured
- func the function in which the error occured
- file the file in which the error occured
- msg the message to log, if this is NULL then the default error message for the error code will be used
- err\_code the error code for the error that occured
- **err\_user\_data** the user pointer provided to *SUNContext\_PushErrHandler()*
- sunctx pointer to a valid SUNContext object

#### Returns

void

void **SUNAbortErrHandlerFn**(int line, const char \*func, const char \*file, const char \*msg, *SUNErrCode* err\_code, void \*err\_user\_data, *SUNContext* sunctx)

Logs the error and aborts the program if an error occured.

#### **Parameters**

- line the line number at which the error occured
- func the function in which the error occured
- file the file in which the error occured
- msg this parameter is ignored
- **err\_code** the error code for the error that occured
- **err\_user\_data** the user pointer provided to *SUNContext\_PushErrHandler()*
- sunctx pointer to a valid SUNContext object

#### Returns

void

void **SUNMPIAbortErrHandlerFn**(int line, const char \*func, const char \*file, const char \*msg, *SUNErrCode* err\_code, void \*err\_user\_data, *SUNContext* sunctx)

Logs the error and calls MPI\_Abort if an error occured.

#### **Parameters**

- line the line number at which the error occured
- func the function in which the error occured
- **file** the file in which the error occured
- msg this parameter is ignored
- **err\_code** the error code for the error that occured
- **err\_user\_data** the user pointer provided to *SUNContext\_PushErrHandler()*

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• **sunctx** – pointer to a valid *SUNContext* object

Returns

void

### 4.4 Status and Error Logging

New in version 6.2.0.

SUNDIALS includes a built-in logging functionality which can be used to direct error messages, warning messages, informational output, and debugging output to specified files. This capability requires enabling both build-time and run-time options to ensure the best possible performance is achieved.

#### 4.4.1 Enabling Logging

To enable logging, the CMake option *SUNDIALS\_LOGGING\_LEVEL* must be set to a value greater than **0** when configuring SUNDIALS. This option specifies the maximum desired output level. See the documentation entry for *SUNDIALS\_-LOGGING\_LEVEL* for the numeric values correspond to errors, warnings, info output, and debug output where errors < warnings < info output < debug output < extra debug output. More details in regards to configuring SUNDIALS with CMake can be found in §10.

**Note:** As of version 7.0.0, enabling MPI in SUNDIALS enables MPI-aware logging.

When SUNDIALS is built with logging enabled, then the default logger (stored in the *SUNContext* object) may be configured through environment variables without any changes to user code. The available environment variables are:

SUNLOGGER\_ERROR\_FILENAME SUNLOGGER\_WARNING\_FILENAME SUNLOGGER\_INFO\_FILENAME SUNLOGGER\_DEBUG\_FILENAME

These environment variables may be set to a filename string. There are two special filenames: stdout and stderr. These two filenames will result in output going to the standard output file and standard error file. The different variables may all be set to the same file, or to distinct files, or some combination there of. To disable output for one of the streams, then do not set the environment variable, or set it to an empty string.

**Warning:** A non-default logger should be created prior to any other SUNDIALS calls in order to capture all log events.

**Note:** If *SUNDIALS\_LOGGING\_LEVEL* was set to 1 (corresponding to error-level output) at build-time, then setting the environment variable SUNLOGGER\_INFO\_FILENAME will do nothing.

**Note:** Extra debugging output is turned on by setting  $SUNDIALS\_LOGGING\_LEVEL$  to 5. This extra output includes vector-values (so long as the  $N\_Vector$  used supports printing).

## 4.4.2 Logger API

The central piece of the Logger API is the SUNLogger type:

## type SUNLogger

An opaque pointer containing logging information.

When SUNDIALS is built with logging enabled, a default logging object is stored in the *SUNContext* object and can be accessed with a call to *SUNContext\_GetLogger()*.

The enumerated type SUNLogLevel is used by some of the logging functions to identify the output level or file.

## enum SUNLogLevel

The SUNDIALS logging level

## enumerator SUN\_LOGLEVEL\_ALL

Represents all output levels

## enumerator SUN\_LOGLEVEL\_NONE

Represents none of the output levels

## enumerator SUN\_LOGLEVEL\_ERROR

Represents error-level logging messages

## enumerator SUN\_LOGLEVEL\_WARNING

Represents warning-level logging messages

## enumerator SUN\_LOGLEVEL\_INFO

Represents info-level logging messages

## enumerator SUN\_LOGLEVEL\_DEBUG

Represents deubg-level logging messages

The SUNLogger class provides the following methods.

```
int SUNLogger_Create(SUNComm comm, int output_rank, SUNLogger *logger)
```

Creates a new SUNLogger object.

## **Arguments:**

- comm the MPI communicator to use, if MPI is enabled, otherwise can be SUN\_COMM\_NULL.
- output\_rank the MPI rank used for output (can be -1 to print to all ranks).
- logger [in,out] On input this is a pointer to a

SUNLogger, on output it will point to a new SUNLogger instance.

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

## int SUNLogger\_CreateFromEnv(SUNComm comm, SUNLogger \*logger)

Creates a new SUNLogger object and opens the output streams/files from the environment variables:

```
SUNLOGGER_ERROR_FILENAME
SUNLOGGER_WARNING_FILENAME
SUNLOGGER_INFO_FILENAME
SUNLOGGER_DEBUG_FILENAME
```

- comm the MPI communicator to use, if MPI is enabled, otherwise can be SUN\_COMM\_NULL.
- logger [in,out] On input this is a pointer to a SUNLogger, on output it will point to a new SUNLogger instance.

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

## int **SUNLogger\_SetErrorFilename**(SUNLogger logger, const char \*error\_filename)

Sets the filename for error output.

## **Arguments:**

- logger a SUNLogger object.
- error\_filename the name of the file to use for error output.

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

## int SUNLogger\_SetWarningFilename(SUNLogger logger, const char \*warning\_filename)

Sets the filename for warning output.

## **Arguments:**

- logger a SUNLogger object.
- warning\_filename the name of the file to use for warning output.

#### **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

## int **SUNLogger\_SetInfoFilename**(*SUNLogger* logger, const char \*info\_filename)

Sets the filename for info output.

## **Arguments:**

- logger a SUNLogger object.
- info\_filename the name of the file to use for info output.

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

## int **SUNLogger\_SetDebugFilename**(SUNLogger logger, const char \*debug filename)

Sets the filename for debug output.

## **Arguments:**

- logger a SUNLogger object.
- debug\_filename the name of the file to use for debug output.

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

# int **SUNLogger\_QueueMsg**(*SUNLogger* logger, *SUNLogLevel* lvl, const char \*scope, const char \*label, const char \*msg\_txt, ...)

Queues a message to the output log level.

- logger a SUNLogger object.
- lvl the message log level (i.e. error, warning, info, debug).
- scope the message scope (e.g. the function name).
- label the message label.
- msg\_txt the message text itself.
- ... the format string arguments

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

**Warning:** When compiling for ANSI C / C89 / C90 (and without compiler extensions), it is dangerous to pass any user input to this function because it falls back to using sprintf with a fixed buffer size.

It is **highly recommended** to compile with C99 or newer if your compiler does not support snprintf through extensions.

## int SUNLogger\_Flush(SUNLogger logger, SUNLogLevel lvl)

Flush the message queue(s).

## **Arguments:**

- logger a SUNLogger object.
- 1v1 the message log level (i.e. error, warning, info, debug or all).

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

## int **SUNLogger\_GetOutputRank**(SUNLogger logger, int \*output rank)

Get the output MPI rank for the logger.

## **Arguments:**

- logger a SUNLogger object.
- output\_rank [in,out] On input this is a pointer to an int, on output it points to the int holding the output rank.

## **Returns:**

• Returns zero if successful, or non-zero if an error occurred.

## int SUNLogger\_Destroy(SUNLogger \*logger)

Free the memory for the *SUNLogger* object.

## **Arguments:**

• logger – a pointer to the *SUNLogger* object.

## **Returns:**

• Returns zero if successful, or non-zero if an error occur.

## 4.4.3 Example Usage

As previously mentioned, if it is enabled at build time, there is a default *SUNLogger* attached to a *SUNContext* instance when it is created. This logger can be configured using the environment variables, e.g.,

```
SUNDIALS_INFO_FILENAME=stdout ./examples/cvode/serial/cvKrylovDemo_ls
```

SUNDIALS also includes several example codes that demonstrate how to use the logging interface via the C API.

```
examples/arkode/CXX_serial/ark_analytic_sys.cpp
examples/cvode/serial/cvAdvDiff_bnd.c
examples/cvode/parallel/cvAdvDiff_diag_p.c
examples/kinsol/CXX_parallel/kin_em_p.cpp
examples/kinsol/CUDA_mpi/kin_em_mpicuda.cpp
```

## 4.5 Performance Profiling

New in version 6.0.0.

SUNDIALS includes a lightweight performance profiling layer that can be enabled at compile-time. Optionally, this profiling layer can leverage Caliper [14] for more advanced instrumentation and profiling. By default, only SUNDIALS library code is profiled. However, a public profiling API can be utilized to leverage the SUNDIALS profiler to time user code regions as well (see §4.5.2).

## 4.5.1 Enabling Profiling

To enable profiling, SUNDIALS must be built with the CMake option SUNDIALS\_BUILD\_WITH\_PROFILING set to ON. To utilize Caliper support, the CMake option ENABLE\_CALIPER must also be set to ON. More details in regards to configuring SUNDIALS with CMake can be found in §10.

When SUNDIALS is built with profiling enabled and **without Caliper**, then the environment variable SUNPROFILER\_PRINT can be utilized to enable/disable the printing of profiler information. Setting SUNPROFILER\_PRINT=1 will cause the profiling information to be printed to stdout when the SUNDIALS simulation context is freed. Setting SUNPROFILER\_PRINT=0 will result in no profiling information being printed unless the *SUNProfiler\_Print()* function is called explicitly. By default, SUNPROFILER\_PRINT is assumed to be 0. SUNPROFILER\_PRINT can also be set to a file path where the output should be printed.

If Caliper is enabled, then users should refer to the Caliper documentation for information on getting profiler output. In most cases, this involves setting the CALI\_CONFIG environment variable.

**Note:** The SUNDIALS profiler requires POSIX timers or the Windows profileapi.h timers.

**Warning:** While the SUNDIALS profiling scheme is relatively lightweight, enabling profiling can still negatively impact performance. As such, it is recommended that profiling is enabled judiciously.

## 4.5.2 Profiler API

The primary way of interacting with the SUNDIALS profiler is through the following macros:

```
SUNDIALS_MARK_FUNCTION_BEGIN(profobj)
SUNDIALS_MARK_FUNCTION_END(profobj)
SUNDIALS_WRAP_STATEMENT(profobj, name, stmt)
SUNDIALS_MARK_BEGIN(profobj, name)
SUNDIALS_MARK_END(profobj, name)
```

Additionally, in C++ applications, the follow macro is available:

```
SUNDIALS_CXX_MARK_FUNCTION(profobj)
```

These macros can be used to time specific functions or code regions. When using the \*\_BEGIN macros, it is important that a matching \*\_END macro is placed at all exit points for the scope/function. The SUNDIALS\_CXX\_MARK\_FUNCTION macro only needs to be placed at the beginning of a function, and leverages RAII to implicitly end the region.

The profobj argument to the macro should be a SUNProfiler object, i.e.

## type SUNProfiler

An opaque pointer containing profiling information.

When SUNDIALS is built with profiling, a default profiling object is stored in the SUNContext object and can be accessed with a call to SUNContext\_GetProfiler().

The name argument should be a unique string indicating the name of the region/function. It is important that the name given to the \*\_BEGIN macros matches the name given to the \*\_END macros.

In addition to the macros, the following methods of the SUNProfiler class are available.

```
int SUNProfiler_Create(SUNComm comm, const char *title, SUNProfiler *p)
```

Creates a new SUNProfiler object.

## **Arguments:**

- comm the MPI communicator to use, if MPI is enabled, otherwise can be SUN\_COMM\_NULL.
- title a title or description of the profiler
- p [in,out] On input this is a pointer to a SUNProfiler, on output it will point to a new SUNProfiler instance

## **Returns:**

· Returns zero if successful, or non-zero if an error occurred

```
int SUNProfiler_Free(SUNProfiler *p)
```

Frees a SUNProfiler object.

## **Arguments:**

 $\bullet$  p – [in,out] On input this is a pointer to a SUNProfiler, on output it will be NULL

## **Returns:**

· Returns zero if successful, or non-zero if an error occurred

int **SUNProfiler\_Begin**(*SUNProfiler* p, const char \*name)

Starts timing the region indicated by the name.

- p a SUNProfiler object
- name a name for the profiling region

## **Returns:**

· Returns zero if successful, or non-zero if an error occurred

## int **SUNProfiler\_End**(SUNProfiler p, const char \*name)

Ends the timing of a region indicated by the name.

## **Arguments:**

- p a SUNProfiler object
- name a name for the profiling region

## **Returns:**

· Returns zero if successful, or non-zero if an error occurred

## int SUNProfiler\_GetElapsedTime(SUNProfiler p, const char \*name, double \*time)

Get the elapsed time for the timer "name" in seconds.

## **Arguments:**

- p a SUNProfiler object
- name the name for the profiling region of interest
- time upon return, the elapsed time for the timer

#### **Returns:**

· Returns zero if successful, or non-zero if an error occurred

## int SUNProfiler\_GetTimerResolution(SUNProfiler p, double \*resolution)

Get the timer resolution in seconds.

## **Arguments:**

- p a SUNProfiler object
- resolution upon return, the resolution for the timer

## **Returns:**

· Returns zero if successful, or non-zero if an error occurred

## int SUNProfiler\_Print(SUNProfiler p, FILE \*fp)

Prints out a profiling summary. When constructed with an MPI comm the summary will include the average and maximum time per rank (in seconds) spent in each marked up region.

## **Arguments:**

- p a SUNProfiler object
- fp the file handler to print to

## **Returns:**

· Returns zero if successful, or non-zero if an error occurred

## int SUNProfiler\_Reset(SUNProfiler p)

Resets the region timings and counters to zero.

• p – a SUNProfiler object

## **Returns:**

· Returns zero if successful, or non-zero if an error occurred

## 4.5.3 Example Usage

The following is an excerpt from the CVODE example code examples/cvode/serial/cvAdvDiff\_bnd.c. It is applicable to any of the SUNDIALS solver packages.

```
SUNContext ctx:
SUNProfiler profobj;
/* Create the SUNDIALS context */
retval = SUNContext_Create(SUN_COMM_NULL, &ctx);
/* Get a reference to the profiler */
retval = SUNContext_GetProfiler(ctx, &profobj);
/* ... */
SUNDIALS_MARK_BEGIN(profobj, "Integration loop");
umax = N_VMaxNorm(u);
PrintHeader(reltol, abstol, umax);
for(iout=1, tout=T1; iout <= NOUT; iout++, tout += DTOUT) {</pre>
   retval = CVode(cvode_mem, tout, u, &t, CV_NORMAL);
   umax = N_VMaxNorm(u);
   retval = CVodeGetNumSteps(cvode_mem, &nst);
   PrintOutput(t, umax, nst);
}
SUNDIALS_MARK_END(profobj, "Integration loop");
PrintFinalStats(cvode_mem); /* Print some final statistics
```

## 4.5.4 Other Considerations

If many regions are being timed, it may be necessary to increase the maximum number of profiler entries (the default is 2560). This can be done by setting the environment variable SUNPROFILER\_MAX\_ENTRIES.

## 4.6 Getting Version Information

SUNDIALS provides additional utilities to all packages, that may be used to retrieve SUNDIALS version information at runtime.

int SUNDIALSGetVersion(char \*version, int len)

This routine fills a string with SUNDIALS version information.

#### **Arguments:**

- *version* character array to hold the SUNDIALS version information.
- *len* allocated length of the *version* character array.

- · 0 if successful
- -1 if the input string is too short to store the SUNDIALS version

An array of 25 characters should be sufficient to hold the version information.

int **SUNDIALSGetVersionNumber**(int \*major, int \*minor, int \*patch, char \*label, int len)

This routine sets integers for the SUNDIALS major, minor, and patch release numbers and fills a string with the release label if applicable.

## **Arguments:**

- *major* SUNDIALS release major version number.
- minor SUNDIALS release minor version number.
- patch SUNDIALS release patch version number.
- label string to hold the SUNDIALS release label.
- len allocated length of the label character array.

## Return value:

- 0 if successful
- -1 if the input string is too short to store the SUNDIALS label

#### **Notes:**

An array of 10 characters should be sufficient to hold the label information. If a label is not used in the release version, no information is copied to *label*.

## 4.7 Fortran Interface

SUNDIALS provides modern, Fortran 2003 based, interfaces as Fortran modules to most of the C API including:

- The SUNDIALS core types, utilities, and data structures via the fsundials\_core\_mod module.
- All of the time-stepping modules in ARKODE:
  - The farkode\_arkstep\_mod, farkode\_erkstep\_mod, farkode\_mristep\_mod, and farkode\_sprk-step\_mod modules provide interfaces to the ARKStep, ERKStep, MRIStep, and SPRKStep integrators respectively.
  - The farkode\_mod module interfaces to the components of ARKODE which are shared by the timestepping modules.
- CVODE via the fcvode\_mod module.
- CVODES via the fcvodes\_mod module.
- IDA via the fida\_mod module.
- IDAS via the fidas\_mod module.
- KINSOL via the fkinsol\_mod module.

Additionally, all of the SUNDIALS base classes (*N\_Vector*, *SUNMatrix*, and *SUNLinearSolver*) include Fortran interface modules. A complete list of class implementations with Fortran 2003 interface modules is given in Table 4.1.

An interface module can be accessed with the use statement, e.g.

```
use fsundials_core_mod ! this is needed to access core SUNDIALS types, utilities, and data structures
use fcvode_mod ! this is needed to access CVODE functions and types
use fnvector_openmp_mod ! this is needed to access the OpenMP implementation of the N_Vector class
```

and by linking to the Fortran 2003 library in addition to the C library, e.g. libsundials\_fcore\_mod.<so|a>, libsundials\_core.<so|a>, libsundials\_fnvecpenmp\_mod.<so|a>, libsundials\_nvecopenmp.<so|a>, libsundials\_fcvode\_mod.<so|a> and libsundials\_cvode.<so|a>. The use statements mirror the #include statements needed when using the C API.

The Fortran 2003 interfaces leverage the <code>iso\_c\_binding</code> module and the <code>bind(C)</code> attribute to closely follow the SUNDIALS C API (modulo language differences). The SUNDIALS classes, e.g. <code>N\_Vector</code>, are interfaced as Fortran derived types, and function signatures are matched but with an F prepending the name, e.g. <code>FN\_VConst</code> instead of <code>N\_VConst()</code> or <code>FCVodeCreate</code> instead of <code>CVodeCreate</code>. Constants are named exactly as they are in the C API. Accordingly, using <code>SUNDIALS</code> via the Fortran 2003 interfaces looks just like using it in C. Some caveats stemming from the language differences are discussed in §4.7.2. A discussion on the topic of equivalent data types in C and Fortran 2003 is presented in §4.7.1.

Further information on the Fortran 2003 interfaces specific to the *N\_Vector*, *SUNMatrix*, *SUNLinearSolver*, and SUNNonlinearSolver classes is given alongside the C documentation. For details on where the Fortran 2003 module (.mod) files and libraries are installed see §10.

The Fortran 2003 interface modules were generated with SWIG Fortran [35], a fork of SWIG. Users who are interested in the SWIG code used in the generation process should contact the SUNDIALS development team.

Table 4.1. List of SONDIALS Fortial 2003 interface modules				
Class/Module	Fortran 2003 Module Name			
SUNDIALS core	fsundials_core_mode			
ARKODE	farkode_mod			
ARKODE::ARKSTEP	farkode_arkstep_mod			
ARKODE::ERKSTEP	farkode_erkstep_mod			
ARKODE::MRISTEP	farkode_mristep_mod			
ARKODE::SPRKSTEP	<pre>farkode_sprkstep_mod</pre>			
CVODE	fcvode_mod			
CVODES	fcvodes_mod			
IDA	fida_mod			
IDAS	fidas_mod			
KINSOL	fkinsol_mod			
NVECTOR_SERIAL	<pre>fnvector_serial_mod</pre>			
NVECTOR_OPENMP	<pre>fnvector_openmp_mod</pre>			
NVECTOR_PTHREADS	<pre>fnvector_pthreads_mod</pre>			
NVECTOR_PARALLEL	<pre>fnvector_parallel_mod</pre>			
NVECTOR_PARHYP	Not interfaced			
NVECTOR_PETSC	Not interfaced			
NVECTOR_CUDA	Not interfaced			
NVECTOR_RAJA	Not interfaced			
NVECTOR_SYCL	Not interfaced			
NVECTOR_MANVECTOR	<pre>fnvector_manyvector_mod</pre>			
NVECTOR_MPIMANVECTOR	<pre>fnvector_mpimanyvector_mod</pre>			
NVECTOR_MPIPLUSX	<pre>fnvector_mpiplusx_mod</pre>			
SUNMATRIX_BAND	fsunmatrix_band_mod			
SUNMATRIX_DENSE	fsunmatrix_dense_mod			
SUNMATRIX_MAGMADENSE	Not interfaced			
SUNMATRIX_ONEMKLDENSE	Not interfaced			

Table 4.1: List of SUNDIALS Fortran 2003 interface modules

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Table 4.1	l – continued	l from pro	evious	page

Class/Module	Fortran 2003 Module Name
SUNMATRIX_SPARSE	fsunmatrix_sparse_mod
SUNLINSOL_BAND	fsunlinsol_band_mod
SUNLINSOL_DENSE	fsunlinsol_dense_mod
SUNLINSOL_LAPACKBAND	Not interfaced
SUNLINSOL_LAPACKDENSE	Not interfaced
SUNLINSOL_MAGMADENSE	Not interfaced
SUNLINSOL_ONEMKLDENSE	Not interfaced
SUNLINSOL_KLU	fsunlinsol_klu_mod
SUNLINSOL_SLUMT	Not interfaced
SUNLINSOL_SLUDIST	Not interfaced
SUNLINSOL_SPGMR	fsunlinsol_spgmr_mod
SUNLINSOL_SPFGMR	fsunlinsol_spfgmr_mod
SUNLINSOL_SPBCGS	fsunlinsol_spbcgs_mod
SUNLINSOL_SPTFQMR	fsunlinsol_sptfqmr_mod
SUNLINSOL_PCG	fsunlinsol_pcg_mof
SUNNONLINSOL_NEWTON	fsunnonlinsol_newton_mod
SUNNONLINSOL_FIXEDPOINT	<pre>fsunnonlinsol_fixedpoint_mod</pre>
SUNNONLINSOL_PETSCSNES	Not interfaced

## 4.7.1 Data Types

Generally, the Fortran 2003 type that is equivalent to the C type is what one would expect. Primitive types map to the iso\_c\_binding type equivalent. SUNDIALS classes map to a Fortran derived type. However, the handling of pointer types is not always clear as they can depend on the parameter direction. Table 4.2 presents a summary of the type equivalencies with the parameter direction in mind.

**Warning:** Currently, the Fortran 2003 interfaces are only compatible with SUNDIALS builds where the sunrealtype is double-precision.

Changed in version 7.1.0: The Fortran interfaces can now be built with 32-bit sunindextype in addition to 64-bit sunindextype.

Table 4.2: C/Fortran-2003 Equivalent Types

C Type	Parameter Direction	Fortran 2003 type
SUNComm	in, inout, out, return	integer(c_int)
SUNErrCode	in, inout, out, return	<pre>integer(c_int)</pre>
double	in, inout, out, return	real(c_double)
int	in, inout, out, return	<pre>integer(c_int)</pre>
long	in, inout, out, return	<pre>integer(c_long)</pre>
sunbooleantype	in, inout, out, return	<pre>integer(c_int)</pre>
sunrealtype	in, inout, out, return	real(c_double)
sunindextype	in, inout, out, return	<pre>integer(c_long)</pre>
double*	in, inout, out	<pre>real(c_double), dimension(*)</pre>
double*	return	<pre>real(c_double), pointer, dimension(:)</pre>
int*	in, inout, out	<pre>real(c_int), dimension(*)</pre>
int*	return	<pre>real(c_int), pointer, dimension(:)</pre>
long*	in, inout, out	<pre>real(c_long), dimension(*)</pre>

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С Туре	Parameter Direction	Fortran 2003 type
long*	return	<pre>real(c_long), pointer, dimension(:)</pre>
sunrealtype*	in, inout, out	<pre>real(c_double), dimension(*)</pre>
sunrealtype*	return	<pre>real(c_double), pointer, dimension(:)</pre>
sunindextype*	in, inout, out	<pre>real(c_long), dimension(*)</pre>
sunindextype*	return	<pre>real(c_long), pointer, dimension(:)</pre>
sunrealtype[]	in, inout, out	<pre>real(c_double), dimension(*)</pre>
<pre>sunindextype[]</pre>	in, inout, out	<pre>integer(c_long), dimension(*)</pre>
N_Vector	in, inout, out	type(N_Vector)
N_Vector	return	<pre>type(N_Vector), pointer</pre>
SUNMatrix	in, inout, out	type(SUNMatrix)
SUNMatrix	return	type(SUNMatrix), pointer
SUNLinearSolver	in, inout, out	type(SUNLinearSolver)
SUNLinearSolver	return	type(SUNLinearSolver), pointer
SUNNonlinearSolver	in, inout, out	type(SUNNonlinearSolver)
SUNNonlinearSolver	return	type(SUNNonlinearSolver), pointer
FILE*	in, inout, out, return	type(c_ptr)
void*	in, inout, out, return	type(c_ptr)
T**	in, inout, out, return	type(c_ptr)
T***	in, inout, out, return	type(c_ptr)
T****	in, inout, out, return	type(c_ptr)

Table 4.2 – continued from previous page

## 4.7.2 Notable Fortran/C usage differences

While the Fortran 2003 interface to SUNDIALS closely follows the C API, some differences are inevitable due to the differences between Fortran and C. In this section, we note the most critical differences. Additionally, §4.7.1 discusses equivalencies of data types in the two languages.

## 4.7.2.1 Creating generic SUNDIALS objects

In the C API a SUNDIALS class, such as an *N\_Vector*, is actually a pointer to an underlying C struct. However, in the Fortran 2003 interface, the derived type is bound to the C struct, not the pointer to the struct. For example, type(N\_Vector) is bound to the C struct \_generic\_N\_Vector not the N\_Vector type. The consequence of this is that creating and declaring SUNDIALS objects in Fortran is nuanced. This is illustrated in the code snippets below:

C code:

```
N_Vector x;
x = N_VNew_Serial(N, sunctx);
```

Fortran code:

```
type(N_Vector), pointer :: x
x => FN_VNew_Serial(N, sunctx)
```

Note that in the Fortran declaration, the vector is a  $type(N_Vector)$ , pointer, and that the pointer assignment operator is then used.

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## 4.7.2.2 Arrays and pointers

Unlike in the C API, in the Fortran 2003 interface, arrays and pointers are treated differently when they are return values versus arguments to a function. Additionally, pointers which are meant to be out parameters, not arrays, in the C API must still be declared as a rank-1 array in Fortran. The reason for this is partially due to the Fortran 2003 standard for C bindings, and partially due to the tool used to generate the interfaces. Regardless, the code snippets below illustrate the differences.

C code:

```
N_Vector x;
sunrealtype* xdata;
long int leniw, lenrw;

/* create a new serial vector */
x = N_VNew_Serial(N, sunctx);

/* capturing a returned array/pointer */
xdata = N_VGetArrayPointer(x)

/* passing array/pointer to a function */
N_VSetArrayPointer(xdata, x)

/* pointers that are out-parameters */
N_VSpace(x, &leniw, &lenrw);
```

Fortran code:

## 4.7.2.3 Passing procedure pointers and user data

Since functions/subroutines passed to SUNDIALS will be called from within C code, the Fortran procedure must have the attribute bind(C). Additionally, when providing them as arguments to a Fortran 2003 interface routine, it is required to convert a procedure's Fortran address to C with the Fortran intrinsic c\_funloc.

Typically when passing user data to a SUNDIALS function, a user may simply cast some custom data structure as a void\*. When using the Fortran 2003 interfaces, the same thing can be achieved. Note, the custom data structure *does not* have to be bind(C) since it is never accessed on the C side.

C code:

```
MyUserData *udata;
void *cvode_mem;
ierr = CVodeSetUserData(cvode_mem, udata);
```

Fortran code:

```
type(MyUserData) :: udata
type(c_ptr) :: arkode_mem

ierr = FARKStepSetUserData(arkode_mem, c_loc(udata))
```

On the other hand, Fortran users may instead choose to store problem-specific data, e.g. problem parameters, within modules, and thus do not need the SUNDIALS-provided user\_data pointers to pass such data back to user-supplied functions. These users should supply the c\_null\_ptr input for user\_data arguments to the relevant SUNDIALS functions.

## 4.7.2.4 Passing NULL to optional parameters

In the SUNDIALS C API some functions have optional parameters that a caller can pass as NULL. If the optional parameter is of a type that is equivalent to a Fortran type(c\_ptr) (see §4.7.1), then a Fortran user can pass the intrinsic c\_null\_ptr. However, if the optional parameter is of a type that is not equivalent to type(c\_ptr), then a caller must provide a Fortran pointer that is dissociated. This is demonstrated in the code example below.

C code:

```
SUNLinearSolver LS;
N_Vector x, b;

/* SUNLinSolSolve expects a SUNMatrix or NULL as the second parameter. */
ierr = SUNLinSolSolve(LS, NULL, x, b);
```

Fortran code:

```
type(SUNLinearSolver), pointer :: LS
type(SUNMatrix), pointer :: A
type(N_Vector), pointer :: x, b

! Disassociate A
A => null()
! SUNLinSolSolve expects a type(SUNMatrix), pointer as the second parameter.
```

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```
! Therefore, we cannot pass a c_null_ptr, rather we pass a disassociated A. ierr = FSUNLinSolSolve(LS, A, x, b)
```

## 4.7.2.5 Working with N\_Vector arrays

Arrays of *N\_Vector* objects are interfaced to Fortran 2003 as an opaque type(c\_ptr). As such, it is not possible to directly index an array of *N\_Vector* objects returned by the *N\_Vector* "VectorArray" operations, or packages with sensitivity capabilities (CVODES and IDAS). Instead, SUNDIALS provides a utility function FN\_VGetVecAtIndexVectorArray wrapping *N\_VGetVecAtIndexVectorArray()*. The example below demonstrates accessing a vector in a vector array.

C code:

```
N_Vector x;
N_Vector* vecs;

/* Create an array of N_Vectors */
vecs = N_VCloneVectorArray(count, x);

/* Fill each array with ones */
for (int i = 0; i < count; ++i)
    N_VConst(vecs[i], 1.0);</pre>
```

Fortran code:

SUNDIALS also provides the functions  $N\_VSetVecAtIndexVectorArray()$  and  $N\_VNewVectorArray()$  for working with  $N\_Vector$  arrays, that have corresponding Fortran interfaces FN\_VSetVecAtIndexVectorArray and FN\_-VNewVectorArray, respectively. These functions are particularly useful for users of the Fortran interface to the  $NVEC-TOR\_MANYVECTOR$  or  $NVECTOR\_MPIMANYVECTOR$  when creating the subvector array. Both of these functions along with  $N\_VGetVecAtIndexVectorArray()$  (wrapped as FN\_VGetVecAtIndexVectorArray) are further described in §6.1.1.

## 4.7.2.6 Providing file pointers

There are a few functions in the SUNDIALS C API which take a FILE\* argument. Since there is no portable way to convert between a Fortran file descriptor and a C file pointer, SUNDIALS provides two utility functions for creating a FILE\* and destroying it. These functions are defined in the module fsundials\_core\_mod.

```
SUNErrCode SUNDIALSFileOpen (const char *filename, const char *mode, FILE **fp)
```

The function allocates a FILE\* by calling the C function fopen with the provided filename and I/O mode.

#### **Parameters**

- **filename** the path to the file, that should have Fortran type character(kind=C\_CHAR, len=\*). There are two special filenames: stdout and stderr these two filenames will result in output going to the standard output file and standard error file, respectively.
- **mode** the I/O mode to use for the file. This should have the Fortran type character(kind=C\_CHAR, len=\*). The string begins with one of the following characters:
  - r to open a text file for reading
  - r+ to open a text file for reading/writing
  - w to truncate a text file to zero length or create it for writing
  - w+ to open a text file for reading/writing or create it if it does not exist
  - a to open a text file for appending, see documentation of fopen for your system/compiler
  - a+ to open a text file for reading/appending, see documentation for fopen for your system/compiler
- **fp** The FILE\* that will be open when the function returns. This should be a *type*(*c\_ptr*) in the Fortran.

#### Returns

A SUNErrCode

Usage example:

```
type(c_ptr) :: fp

! Open up the file output.log for writing
ierr = FSUNDIALSFileOpen("output.log", "w+", fp)

! The C function ARKStepPrintMem takes void* arkode_mem and FILE* fp as arguments
call FARKStepPrintMem(arkode_mem, fp)

! Close the file
ierr = FSUNDIALSFileClose(fp)
```

Changed in version 7.0.0: The function signature was updated to return a *SUNErrCode* and take a *FILE*\*\* as the last input parameter rather then return a *FILE*\*.

```
SUNErrCode SUNDIALSFileClose(FILE **fp)
```

The function deallocates a C FILE\* by calling the C function fclose with the provided pointer.

## Parameters

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• fp - the C FILE\* that was previously obtained from fopen. This should have the Fortran type type(c\_ptr). Note that if either stdout or stderr were opened using SUNDIALS-FileOpen()

#### **Returns**

A SUNErrCode

Changed in version 7.0.0: The function signature was updated to return a *SUNErrCode* and the *fp* parameter was changed from *FILE*\* to *FILE*\*\*.

## 4.7.3 Important notes on portability

The SUNDIALS Fortran 2003 interface *should* be compatible with any compiler supporting the Fortran 2003 ISO standard.

Upon compilation of SUNDIALS, Fortran module (.mod) files are generated for each Fortran 2003 interface. These files are highly compiler specific, and thus it is almost always necessary to compile a consuming application with the same compiler that was used to generate the modules.

## 4.7.4 Common Issues

In this subsection, we list some common issues users run into when using the Fortran interfaces.

## **Strange Segmentation Fault in User-Supplied Functions**

One common issue we have seen trip up users (and even ourselves) has the symptom of segmentation fault in a user-supplied function (such as the RHS) when trying to use one of the callback arguments. For example, in the following RHS function, we will get a segfault on line 21:

```
integer(c_int) function ff(t, yvec, ydotvec, user_data) &
      result(ierr) bind(C)
2
      use, intrinsic :: iso_c_binding
      use fsundials_nvector_mod
      implicit none
      real(c_double) :: t ! <===== Missing value attribute</pre>
      type(N_Vector) :: yvec
      type(N_Vector) :: ydotvec
10
                      :: user_data
      type(c_ptr)
11
12
      real(c_double) :: e
13
      real(c_double) :: u, v
      real(c_double) :: tmp1, tmp2
15
      real(c_double), pointer :: yarr(:)
      real(c_double), pointer :: ydotarr(:)
17
      ! get N_Vector data arrays
19
      yarr => FN_VGetArrayPointer(yvec)
      ydotarr => FN_VGetArrayPointer(ydotvec) ! <==== SEGFAULTS HERE</pre>
21
      ! extract variables
23
      u = yarr(1)
24
      v = yarr(2)
25
```

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```
26
       ! fill in the RHS function:
27
         [0 \ 0]*[(-1+u^2-r(t))/(2*u)] + [
28
         [e -1] [(-2+v^2-s(t))/(2*v)]
                                           [sdot(t)/(2*vtrue(t))]
      tmp1 = (-ONE+u*u-r(t))/(TWO*u)
      tmp2 = (-TWO+v*v-s(t))/(TWO*v)
31
      ydotarr(1) = ZERO
32
      ydotarr(2) = e*tmp1 - tmp2 + sdot(t)/(TWO*vtrue(t))
33
      ! return success
35
      ierr = 0
      return
37
   end function
```

The subtle bug in the code causing the segfault is on line 8. It should read  $real(c\_double)$ , value :: t instead of  $real(c\_double)$  :: t (notice the value attribute). Fundamental types that are passed by value in C need the value attribute.

## 4.8 Features for GPU Accelerated Computing

In this section, we introduce the SUNDIALS GPU programming model and highlight SUNDIALS GPU features. The model leverages the fact that all of the SUNDIALS packages interact with simulation data either through the shared vector, matrix, and solver APIs or through user-supplied callback functions. Thus, under the model, the overall structure of the user's calling program, and the way users interact with the SUNDIALS packages is similar to using SUNDIALS in CPU-only environments.

## 4.8.1 SUNDIALS GPU Programming Model

As described in [13], within the SUNDIALS GPU programming model, all control logic executes on the CPU, and all simulation data resides wherever the vector or matrix object dictates as long as SUNDIALS is in control of the program. That is, SUNDIALS will not migrate data (explicitly) from one memory space to another. Except in the most advanced use cases, it is safe to assume that data is kept resident in the GPU-device memory space. The consequence of this is that, when control is passed from the user's calling program to SUNDIALS, simulation data in vector or matrix objects must be up-to-date in the device memory space. Similarly, when control is passed from SUNDIALS to the user's calling program, the user should assume that any simulation data in vector and matrix objects are up-to-date in the device memory space. To put it succinctly, it is the responsibility of the user's calling program to manage data coherency between the CPU and GPU-device memory spaces unless unified virtual memory (UVM), also known as managed memory, is being utilized. Typically, the GPU-enabled SUNDIALS modules provide functions to copy data from the host to the device and vice-versa as well as support for unmanaged memory or UVM. In practical terms, the way SUNDIALS handles distinct host and device memory spaces means that users need to ensure that the user-supplied functions, e.g. the right-hand side function, only operate on simulation data in the device memory space otherwise extra memory transfers will be required and performance will suffer. The exception to this rule is if some form of hybrid data partitioning (achievable with the NVECTOR\_MANYVECTOR, see §6.17) is utilized.

SUNDIALS provides many native shared features and modules that are GPU-enabled. Currently, these include the NVIDIA CUDA platform [5], AMD ROCm/HIP [2], and Intel oneAPI [3]. Table 4.3—Table 4.5 summarize the shared SUNDIALS modules that are GPU-enabled, what GPU programming environments they support, and what class of memory they support (unmanaged or UVM). Users may also supply their own GPU-enabled *N\_Vector*, *SUNMatrix*, or *SUNLinearSolver* implementation, and the capabilties will be leveraged since SUNDIALS operates on data through these APIs.

In addition, SUNDIALS provides a memory management helper module (see §9) to support applications which implement their own memory management or memory pooling.

Table 4.3: List of SUNDIALS GPU-enabled N\_Vector Modules

Module	CUDA	ROCm/HIP	oneAPI	Unmanaged Memory	UVM
NVECTOR_CUDA	X			X	X
NVECTOR_HIP	X	X		X	X
NVECTOR_SYCL	$X^3$	$X^3$	X	X	X
NVECTOR_RAJA	X	X	X	X	X
NVECTOR_KOKKOS	X	X	X	X	X
NVECTOR_OPENMPDEV	X	$X^2$	$X^2$	X	

Table 4.4: List of SUNDIALS GPU-enabled SUNMatrix Modules

Module	CUDA	ROCm/HIP	oneAPI	<b>Unmanaged Memory</b>	UVM
SUNMATRIX_CUSPARSE	X			X	X
SUNMATRIX_ONEMKLDENSE	$X^3$	$X^3$	X	X	X
SUNMATRIX_MAGMADENSE	X	X		X	X
SUNMATRIX_GINKGO	X	X		X	X
SUNMATRIX_KOKKOSDENSE	X	X		X	X

Table 4.5: List of SUNDIALS GPU-enabled SUNLinearSolver Modules

Module	CUDA	ROCm/HIP	oneAPI	Unmanaged Memory	UVM
SUNLINSOL_CUSOLVERSP	X			X	X
SUNLINSOL_ONEMKLDENSE	$X^3$	$X^3$	X	X	X
SUNLINSOL_MAGMADENSE	X			X	X
$SUNLINSOL\_GINKGO$	X	X		X	X
SUNLINSOL_KOKKOSDENSE	X	X		X	X
SUNLINSOL_SPGMR	$X^1$	$X^1$	$X^1$	$X^1$	$X^1$
SUNLINSOL_SPFGMR	$X^1$	$X^1$	$X^1$	$X^1$	$X^1$
SUNLINSOL_SPTFQMR	$X^1$	$X^1$	$X^1$	$X^1$	$X^1$
SUNLINSOL_SPBCGS	$X^1$	$X^1$	$X^1$	$X^1$	$X^1$
$SUNLINSOL\_PCG$	$X^1$	$X^1$	$X^1$	$X^1$	$X^1$

Notes regarding the above tables:

- 1. This module inherits support from the NVECTOR module used
- 2. Support for ROCm/HIP and oneAPI are currently untested.
- 3. Support for CUDA and ROCm/HIP are currently untested.

In addition, note that implicit UVM (i.e. malloc returning UVM) is not accounted for.

## 4.8.2 Steps for Using GPU Accelerated SUNDIALS

For any SUNDIALS package, the generalized steps a user needs to take to use GPU accelerated SUNDIALS are:

- 1. Utilize a GPU-enabled N\_Vector implementation. Initial data can be loaded on the host, but must be in the device memory space prior to handing control to SUNDIALS.
- 2. Utilize a GPU-enabled SUNLinearSolver linear solver (if applicable).
- 3. Utilize a GPU-enabled SUNMatrix implementation (if using a matrix-based linear solver).
- 4. Utilize a GPU-enabled SUNNonlinearSolver nonlinear solver (if applicable).
- 5. Write user-supplied functions so that they use data only in the device memory space (again, unless an atypical data partitioning is used). A few examples of these functions are the right-hand side evaluation function, the Jacobian evaluation function, or the preconditioner evaluation function. In the context of CUDA and the right-hand side function, one way a user might ensure data is accessed on the device is, for example, calling a CUDA kernel, which does all of the computation, from a CPU function which simply extracts the underlying device data array from the *N\_Vector* object that is passed from SUNDIALS to the user-supplied function.

Users should refer to the above tables for a complete list of GPU-enabled native SUNDIALS modules.

## **Chapter 5**

# Using KINSOL for the Solution of Nonlinear Systems

This section is concerned with the use of KINSOL for the solution of nonlinear systems.

The following sections treat the header files and the layout of the user's main program, and provide descriptions of the KINSOL user-callable functions and user-supplied functions. The sample programs described in the companion document [19] may also be helpful. Those codes may be used as templates (with the removal of some lines used in testing) and are included in the KINSOL package.

KINSOL uses various constants for both input and output. These are defined as needed in this chapter, but for convenience are also listed separately in §12.

The user should be aware that not all SUNLinearSolver and SUNMatrix objects are compatible with all N\_Vector implementations. Details on compatibility are given in the documentation for each SUNMatrix (Chapter §7) and SUNLinearSolver (Chapter §8) implementation. For example, NVECTOR\_PARALLEL is not compatible with the dense, banded, or sparse SUNMatrix types, or with the corresponding dense, banded, or sparse SUNLinearSolver objects. Please check Chapters §7 and §8 to verify compatibility between these objects. In addition to that documentation, we note that the KINBBDPRE preconditioner can only be used with NVECTOR\_PARALLEL. It is not recommended to use a threaded vector object with SuperLU\_MT unless it is the NVECTOR\_OPENMP module, and SuperLU\_MT is also compiled with OpenMP.

## 5.1 Access to library and header files

At this point, it is assumed that the installation of KINSOL, following the procedure described in §10, has been completed successfully. In the proceeding text, the directories libdir and incdir are the installation library and include directories, respectively. For a default installation, these are instdir/lib and instdir/include, respectively, where instdir is the directory where SUNDIALS was installed.

Regardless of where the user's application program resides, its associated compilation and load commands must make reference to the appropriate locations for the library and header files required by KINSOL. KINSOL symbols are found in libdir/libsundials\_kinsol.lib. Thus, in addition to linking to libdir/libsundials\_core.lib, KINSOL users need to link to the KINSOL library. Symbols for additional SUNDIALS modules, vectors and algebraic solvers, are found in

```
<libdir>/libsundials_nvec*.lib
<libdir>/libsundials_sunmat*.lib
<libdir>/libsundials_sunlinsol*.lib
```

(continues on next page)

(continued from previous page)

```
<libdir>/libsundials_sunnonlinsol*.lib
<libdir>/libsundials_sunmem*.lib
```

The file extension .lib is typically .so for shared libraries and .a for static libraries.

The relevant header files for KINSOL are located in the subdirectories incdir/include/kinsol. To use KINSOL the application needs to include the header file for KINSOL in addition to the SUNDIALS core header file:

```
#include <sundials_sundials_core.h> // Provides core SUNDIALS types
#include <kinsol/kinsol.h> // KINSOL provides methods for solving nonlinear systems
```

The calling program must also include an *N\_Vector* implementation header file, of the form nvector/nvector\_\*.h. See §6 for the appropriate name.

If using a Newton or Picard nonlinear solver that requires the solution of a linear system, the calling program must also include a SUNLinearSolver implementation header file, of the from sunlinsol/sunlinsol\_\*.h where \* is the name of the linear solver (see Chapter §8 for more information).

If the linear solver is matrix-based, the linear solver header will also include a header file of the from sunmatrix/sunmatrix\_\*.h where \* is the name of the matrix implementation compatible with the linear solver. (see Chapter §7 for more information).

Other headers may be needed, according to the choice of preconditioner, etc. For example, in the example kinFood-Web\_kry\_p (see [19]), preconditioning is done with a block-diagonal matrix. For this, even though the SUNLINSOL\_-SPGMR linear solver is used, the header sundials/sundials\_dense.h is included for access to the underlying generic dense matrix arithmetic routines.

## 5.2 A skeleton of the user's main program

The following is a skeleton of the user's main program (or calling program) for the solution of a nonlinear system problem. Most of the steps are independent of the N\_Vector, SUNMatrix, and SUNLinearSolver implementations used. For the steps that are not, refer to §6, §7, and §8 for the specific name of the function to be called or macro to be referenced.

1. **Initialize parallel or multi-threaded environment** (*if appropriate*)

For example, call MPI\_Init to initialize MPI if used.

2. Create the SUNDIALS context object

Call SUNContext\_Create() to allocate the SUNContext object.

3. Set the problem dimensions etc.

This generally includes the problem size N, and may include the local vector length Nlocal.

4. Create the vector with the initial guess

Construct an N\_Vector of initial guess values using the appropriate functions defined by the particular N\_Vector implementation (see §6 for details).

For native SUNDIALS vector implementations, use a call of the form  $y0 = N_vMake_***(..., ydata)$  if the array containing the initial values of y already exists. Otherwise, create a new vector by making a call of the form  $N_vVNew_***(...)$ , and then set its elements by accessing the underlying data with a call of the form  $ydata = N_vGetArrayPointer(y0)$ . Here, \*\*\* is the name of the vector implementation.

For *hypre*, PETSc, and Trilinos vector wrappers, first create and initialize the underlying vector, and then create an N\_Vector wrapper with a call of the form y0 = N\_VMake\_\*\*\*(yvec), where yvec is a *hypre*, PETSc, or

Trilinos vector. Note that calls like N\_VNew\_\*\*\*(...) and N\_VGetArrayPointer(...) are not available for these vector wrappers.

## 5. Create matrix object (if appropriate)

If a linear solver is required (e.g., when using the default Newton solver) and the linear solver will be a matrix-based linear solver, then a template Jacobian matrix must be created by calling the appropriate constructor defined by the particular SUNMatrix implementation.

For the native SUNDIALS SUNMatrix implementations, the matrix object may be created using a call of the form SUN\*\*\*Matrix(...) where \*\*\* is the name of the matrix (see §7 for details).

## 6. Create linear solver object (if appropriate)

If a linear solver is required (e.g., when using the default Newton solver), then the desired linear solver object must be created by calling the appropriate constructor defined by the particular SUNLinearSolver implementation.

For any of the native SUNDIALS SUNLinearSolver implementations, the linear solver object may be created using a call of the form SUNLinearSolver LS = SUNLinSol\_\*\*\*(...); where \*\*\* is the name of the linear solver (see §8 for details).

## 7. Create KINSOL object

Call KINCreate() to create the KINSOL solver object.

## 8. Initialize KINSOL solver

Call KINInit() to allocate internal memory.

## 9. Attach the linear solver (if appropriate)

If a linear solver was created above, initialize the KINLS linear solver interface by attaching the linear solver object (and matrix object, if applicable) with *KINSetLinearSolver()*.

## 10. Set linear solver optional inputs (if appropriate)

See Table 5.1 for KINLS optional inputs and Chapter §8 for linear solver specific optional inputs.

## 11. Set optional inputs

Call KINSet\*\*\* functions to change any optional inputs that control the behavior of KINSOL from their default values. See §5.3.4 for details.

## 12. Solve problem

Call ier = KINSol(...) to solve the nonlinear problem for a given initial guess.

See KINSol() for details.

## 13. Get optional outputs

Call KINGet\*\*\* functions to obtain optional output. See §5.3.5 for details.

## 14. Deallocate memory

Upon completion of the integration call the following, as necessary, to free any objects or memory allocated above:

- Call *N\_VDestroy()* to free vector objects.
- Call SUNMatDestroy() to free matrix objects.
- Call SUNLinSolFree() to free linear solvers objects.
- Call SUNNonlinSolFree() to free nonlinear solvers objects.
- Call KINFree() to free the memory allocated by KINSOL.

• Call SUNContext\_Free() to free the SUNContext object

## 15. Finalize MPI, if used

Call MPI\_Finalize to terminate MPI.

## **5.3** User-callable functions

This section describes the KINSOL functions that are called by the user to setup and then solve an IVP. Some of these are required. However, starting with §5.3.4, the functions listed involve optional inputs/outputs or restarting, and those paragraphs may be skipped for a casual use of KINSOL. In any case, refer to §5.2 for the correct order of these calls.

On an error, each user-callable function returns a negative value and sends an error message to the error handler routine, which prints the message on stderr by default. However, the user can set a file as error output or can provide his own error handler function (see §5.3.4).

## 5.3.1 KINSOL initialization and deallocation functions

void KINCreate(SUNContext sunctx)

The function *KINCreate()* instantiates a KINSOL solver object.

## **Arguments:**

• sunctx – the SUNContext object (see §4.2)

#### **Return value:**

• void

int KINInit(void \*kin\_mem, KINSysFn func, N\_Vector tmpl)

The function *KINInit()* specifies the problem-defining function, allocates internal memory, and initializes KIN-SOL.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block returned by KINCreate().
- func is the CC function which computes the system function F(u) (or G(u) for fixed-point iteration) in the nonlinear problem. This function has the form func (u, fval, user\_data). (For full details see §5.4.1).
- tmpl is any N\_Vector (e.g. the initial guess vector u) which is used as a template to create (by cloning) necessary vectors in kin\_mem.

## Return value:

- KIN\_SUCCESS The call to KINInit() was successful.
- KIN\_MEM\_NULL The KINSOL memory block was not initialized through a previous call to KINCreate().
- KIN\_MEM\_FAIL A memory allocation request has failed.
- KIN\_ILL\_INPUT An input argument to KINInit() has an illegal value.

#### **Notes:**

If an error occurred, KINInit() sends an error message to the error handler function.

#### void KINFree(void \*\*kin mem)

The function *KINFree()* frees the pointer allocated by a previous call to *KINCreate()*.

## **Arguments:**

• kin\_mem – pointer to the KINSOL solver object.

## Return value:

• void

## **5.3.2** Linear solver specification functions

As previously explained, Newton and Picard iterations require the solution of linear systems of the form  $J\delta=-F$ . Solution of these linear systems is handled using the KINLS linear solver interface. This interface supports all valid SUNLinearSolver modules. Here, matrix-based SUNLinearSolver modules utilize SUNMatrix objects to store the Jacobian matrix J=F'(u) and factorizations used throughout the solution process. Conversely, matrix-free SUNLinearSolver modules instead use iterative methods to solve the linear systems of equations, and only require the *action* of the Jacobian on a vector, Jv.

With most iterative linear solvers, preconditioning can be done on the left only, on the right only, on both the left and the right, or not at all. However, only right preconditioning is supported within KINLS. If preconditioning is done, user-supplied functions define the linear operator corresponding to a right preconditioner matrix P, which should approximate the system Jacobian matrix J. For the specification of a preconditioner, see the iterative linear solver sections in §5.3.4 and §5.4. A preconditioner matrix P must approximate the Jacobian J, at least crudely.

To specify a generic linear solver to KINSOL, after the call to <code>KINCreate()</code> but before any calls to <code>KINSOl()</code>, the user's program must create the appropriate <code>SUNLinearSolver</code> object and call the function <code>KINSetLinearSolver()</code>, as documented below. To create the <code>SUNLinearSolver</code> object, the user may call one of the <code>SUNDIALS-packaged SUNLinearSolver</code> module constructor routines via a call of the form

```
SUNLinearSolver LS = SUNLinSol_*(...);
```

For a current list of such constructor routines see §8.

Alternately, a user-supplied SUNLinearSolver module may be created and used instead. The use of each of the generic linear solvers involves certain constants, functions and possibly some macros, that are likely to be needed in the user code. These are available in the corresponding header file associated with the specific SUNMatrix or SUNLinearSolver module in question, as described in Chapters §7 and §8.

Once this solver object has been constructed, the user should attach it to KINSOL via a call to <code>KINSetLinear-Solver()</code>. The first argument passed to this function is the KINSOL memory pointer returned by <code>KINCreate()</code>; the second argument is the desired <code>SUNLinearSolver</code> object to use for solving Newton or Picard systems. The third argument is an optional <code>SUNMatrix</code> object to accompany matrix-based <code>SUNLinearSolver</code> inputs (for matrix-free linear solvers, the third argument should be <code>NULL</code>). A call to this function initializes the KINLS linear solver interface, linking it to the main KINSOL solver, and allows the user to specify additional parameters and routines pertinent to their choice of linear solver.

```
int KINSetLinearSolver(void *kin_mem, SUNLinearSolver LS, SUNMatrix J)
```

The function <code>KINSetLinearSolver()</code> attaches a generic SUNLinSol object LS and corresponding template Jacobian SUNMatrix object J (if applicable) to KINSOL, initializing the KINLS linear solver interface.

- kin\_mem pointer to the KINSOL memory block.
- LS SUNLINSOL object to use for solving Newton linear systems.
- J SUNMATRIX object for used as a template for the Jacobian (or NULL if not applicable).

## Return value:

- KINLS\_SUCCESS The KINLS initialization was successful.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_ILL\_INPUT The KINLS interface is not compatible with the LS or J input objects or is incompatible with the current NVECTOR module.
- KINLS\_SUNLS\_FAIL A call to the LS object failed.
- KINLS\_MEM\_FAIL A memory allocation request failed.

## **Notes:**

If LS is a matrix-based linear solver, then the template Jacobian matrix J will be used in the solve process, so if additional storage is required within the SUNMatrix object (e.g. for factorization of a banded matrix), ensure that the input object is allocated with sufficient size (see the documentation of the particular SUNMatrix type in Chapter §7 for further information).

New in version 4.0.0: Replaces the deprecated functions KINDlsSetLinearSolver and KINSpilsSetLinearSolver.

## **5.3.3 KINSOL solver function**

This is the central step in the solution process, the call to solve the nonlinear algebraic system.

int **KINSol** (void \*kin\_mem, N\_Vector u, int strategy, N\_Vector u\_scale, N\_Vector f\_scale)

The function KINSol() computes an approximate solution to the nonlinear system.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- u vector set to initial guess by user before calling KINSol(), but which upon return contains an approximate solution of the nonlinear system F(u) = 0.
- strategy strategy used to solve the nonlinear system. It must be of the following:
  - KIN\_NONE basic Newton iteration
  - KIN\_LINESEARCH Newton with globalization
  - KIN\_FP fixed-point iteration with Anderson Acceleration (no linear solver needed)
  - KIN\_PICARD Picard iteration with Anderson Acceleration (uses a linear solver)
- u\_scale vector containing diagonal elements of scaling matrix  $D_u$  for vector u chosen so that the components of  $D_u$  u (as a matrix multiplication) all have roughly the same magnitude when u is close to a root of F(u).
- f\_scale vector containing diagonal elements of scaling matrix  $D_F$  for F(u) chosen so that the components of  $D_F$  F(u) (as a matrix multiplication) all have roughly the same magnitude when u is not too near a root of F(u). In the case of a fixed-point iteration, consider F(u) = G(u) u.

- KIN\_SUCCESS KINSol() succeeded; the scaled norm of F(u) is less than fnormtol.
- KIN\_INITIAL\_GUESS\_OK The guess  $\mathbf{u}=u_0$  satisfied the system F(u)=0 within the tolerances specified (the scaled norm of  $F(u_0)$  is less than 0.01\*fnormtol).
- KIN\_STEP\_LT\_STPTOL KINSOL stopped based on scaled step length. This means that the current iterate may be an approximate solution of the given nonlinear system, but it is also quite possible that

the algorithm is "stalled" (making insufficient progress) near an invalid solution, or that the scalar scsteptol is too large (see *KINSetScaledStepTol(*) in §5.3.4 to change scsteptol from its default value).

- KIN\_MEM\_NULL The KINSOL memory block pointer was NULL.
- KIN\_ILL\_INPUT An input parameter was invalid.
- KIN\_NO\_MALLOC The KINSOL memory was not allocated by a call to KINCreate().
- KIN\_MEM\_FAIL A memory allocation failed.
- KIN\_LINESEARCH\_NONCONV The line search algorithm was unable to find an iterate sufficiently distinct from the current iterate, or could not find an iterate satisfying the sufficient decrease condition. Failure to satisfy the sufficient decrease condition could mean the current iterate is "close" to an approximate solution of the given nonlinear system, the difference approximation of the matrix-vector product J(u) v is inaccurate, or the real scalar scsteptol is too large.
- KIN\_MAXITER\_REACHED The maximum number of nonlinear iterations has been reached.
- KIN\_MXNEWT\_5X\_EXCEEDED Five consecutive steps have been taken that satisfy the inequality  $||D_u p||_{L2} > 0.99$  mxnewtstep, where p denotes the current step and mxnewtstep is a scalar upper bound on the scaled step length. Such a failure may mean that  $||D_F F(u)||_{L2}$  asymptotes from above to a positive value, or the real scalar mxnewtstep is too small.
- KIN\_LINESEARCH\_BCFAIL The line search algorithm was unable to satisfy the "beta-condition" for MXNBCF+1 nonlinear iterations (not necessarily consecutive), which may indicate the algorithm is making poor progress.
- KIN\_LINSOLV\_NO\_RECOVERY The user-supplied routine psolve encountered a recoverable error, but the preconditioner is already current.
- KIN\_LINIT\_FAIL The KINLS initialization routine (linit) encountered an error.
- KIN\_LSETUP\_FAIL The KINLS setup routine (1setup) encountered an error; e.g., the user-supplied routine pset (used to set up the preconditioner data) encountered an unrecoverable error.
- KIN\_LSOLVE\_FAIL The KINLS solve routine (lsolve) encountered an error; e.g., the user-supplied routine psolve (used to to solve the preconditioned linear system) encountered an unrecoverable error.
- KIN\_SYSFUNC\_FAIL The system function failed in an unrecoverable manner.
- KIN\_FIRST\_SYSFUNC\_ERR The system function failed recoverably at the first call.
- KIN\_REPTD\_SYSFUNC\_ERR The system function had repeated recoverable errors. No recovery is possible.

#### **Notes:**

The components of vectors u\_scale and f\_scale should be strictly positive. KIN\_SUCCESS=0, KIN\_INITIAL\_GUESS\_OK=1, and KIN\_STEP\_LT\_STPTOL=2. All remaining return values are negative and therefore a test flag < 0 will trap all KINSol() failures.

**5.3.** User-callable functions

## **5.3.4** Optional input functions

There are numerous optional input parameters that control the behavior of the KINSOL solver. KINSOL provides functions that can be used to change these from their default values. Table 5.1 lists all optional input functions in KINSOL which are then described in detail in the remainder of this section, beginning with those for the main KINSOL solver and continuing with those for the KINLS linear solver interface.

We note that, on error return, all of these functions also send an error message to the error handler function. We also note that all error return values are negative, so a test retval < 0 will catch any error.

**Optional** input **Function name Default** KINSOL main solver Data for problem-defining function KINSetUserData() NULL Max. number of nonlinear iterations KINSetNumMaxIters() 200 No initial matrix setup SUNFALSE KINSetNoInitSetup() No residual monitoring KINSetNoResMon() SUNFALSE. Max. iterations without matrix setup KINSetMaxSetupCalls() 10 Max. iterations without residual check KINSetMaxSubSetupCalls() 5 KIN\_ETACHOICE1 Form of  $\eta$  coefficient KINSetEtaForm() Constant value of  $\eta$ KINSetEtaConstValue() 0.1 Values of  $\gamma$  and  $\alpha$ 0.9 and 2.0 KINSetEtaParams() 0.00001 and 0.9 Values of  $\omega_{min}$  and  $\omega_{max}$ KINSetResMonParams() Constant value of  $\omega$ KINSetResMonConstValue() 0.9 Lower bound on  $\epsilon$ SUNFALSE KINSetNoMinEps() Max. scaled length of Newton step KINSetMaxNewtonStep()  $1000|D_u u_0|_2$ Max. number of  $\beta$ -condition failures KINSetMaxBetaFails() 10 √uround Rel. error for D.Q. JvKINSetRelErrFunc()  $uround^{1/3}$ Function-norm stopping tolerance KINSetFuncNormTol()  $uround^{2/3}$ Scaled-step stopping tolerance KINSetScaledStepTol() NULL Inequality constraints on solution KINSetConstraints() Nonlinear system function KINSetSvsFunc() none Return the newest fixed point iteration KINSetReturnNewest() SUNFALSE Fixed point/Picard damping parameter KINSetDamping() 1.0 0 Anderson Acceleration subspace size KINSetMAA() 1.0 Anderson Acceleration damping parameter KINSetDampingAA() Anderson Acceleration delay KINSetDelayAA() 0 Anderson Acceleration orthogonalization routine KINSetOrthAA() KIN\_ORTH\_MGS KINLS linear solver interface Jacobian function KINSetJacFn() DO NULL, NULL, NULL Preconditioner functions and data KINSetPreconditioner() Jacobian-times-vector function and data KINSetJacTimesVecFn() internal DO, NULL Jacobian-times-vector system function KINSetJacTimesVecSysFn() NULL

Table 5.1: Optional inputs for KINSOL and KINLS

## int KINSetUserData(void \*kin\_mem, void \*user\_data)

The function *KINSetUserData()* specifies the pointer to user-defined memory that is to be passed to all user-supplied functions.

- kin\_mem pointer to the KINSOL memory block.
- user\_data pointer to the user-defined memory.

## **Return value:**

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## **Notes:**

If specified, the pointer to user\_data is passed to all user-supplied functions that have it as an argument. Otherwise, a NULL pointer is passed.

**Warning:** If user\_data is needed in user linear solver or preconditioner functions, the call to *KINSe-tUserData()* must be made before the call to specify the linear solver module.

## int KINSetNumMaxIters(void \*kin\_mem, long int mxiter)

The function KINSetNumMaxIters() specifies the maximum number of nonlinear iterations allowed.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- mxiter maximum number of nonlinear iterations.

## Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The maximum number of iterations was non-positive.

## **Notes:**

The default value for mxiter is MXITER\_DEFAULT = 200.

## int **KINSetNoInitSetup**(void \*kin\_mem, *sunbooleantype* noInitSetup)

The function *KINSetNoInitSetup()* specifies whether an initial call to the preconditioner or Jacobian setup function should be made or not.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- noInitSetup flag controlling whether an initial call to the preconditioner or Jacobian setup function is made (pass SUNFALSE) or not made (pass SUNTRUE).

## Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

#### **Notes:**

The default value for noInitSetup is SUNFALSE, meaning that an initial call to the preconditioner or Jacobian setup function will be made. A call to this function is useful when solving a sequence of problems, in which the final preconditioner or Jacobian value from one problem is to be used initially for the next problem.

## int **KINSetNoResMon**(void \*kin mem, *sunbooleantype* noNNIResMon)

The function *KINSetNoResMon()* specifies whether or not the nonlinear residual monitoring scheme is used to control Jacobian updating

#### **Arguments:**

• kin\_mem – pointer to the KINSOL memory block.

 noNNIResMon – flag controlling whether residual monitoring is used (pass SUNFALSE) or not used (pass SUNTRUE).

#### Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

#### **Notes:**

When using a direct solver, the default value for noNNIResMon is SUNFALSE, meaning that the nonlinear residual will be monitored.

Warning: Residual monitoring is only available for use with matrix-based linear solver modules.

## int KINSetMaxSetupCalls(void \*kin\_mem, long int msbset)

The function KINSetMaxSetupCalls() specifies the maximum number of nonlinear iterations that can be performed between calls to the preconditioner or Jacobian setup function.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- msbset maximum number of nonlinear iterations without a call to the preconditioner or Jacobian setup function. Pass 0 to indicate the default.

#### **Return value:**

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument msbset was negative.

## Notes:

The default value for msbset is MSBSET\_DEFAULT=10. The value of msbset should be a multiple of msbsetsub (see KINSetMaxSubSetupCalls()).

## int KINSetMaxSubSetupCalls(void \*kin mem, long int msbsetsub)

The function *KINSetMaxSubSetupCalls()* specifies the maximum number of nonlinear iterations between checks by the residual monitoring algorithm.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- msbsetsub maximum number of nonlinear iterations without checking the nonlinear residual. Pass 0 to indicate the default.

#### Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument msbsetsub was negative.

#### **Notes:**

The default value for msbsetsub is MSBSET\_SUB\_DEFAULT = 5. The value of msbset (see KINSet-MaxSetupCalls()) should be a multiple of msbsetsub.

Warning: Residual monitoring is only available for use with matrix-based linear solver modules.

#### int **KINSetEtaForm**(void \*kin mem, int etachoice)

The function KINSetEtaForm() specifies the method for computing the value of the  $\eta$  coefficient used in the calculation of the linear solver convergence tolerance.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- etachoice flag indicating the method for computing  $\eta$ . The value must be one of KIN\_ETACHOICE1 , KIN\_ETACHOICE2 , or KIN\_ETACONSTANT (see Chapter §2 for details).

#### **Return value:**

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument etachoice had an illegal value.

#### **Notes:**

The default value for etachoice is KIN\_ETACHOICE1. When using either KIN\_ETACHOICE1 or KIN\_-ETACHOICE2 the safeguard

$$\eta_n = \max(\eta_n, \eta_{\text{safe}})$$

is applied when  $\eta_{\text{safe}} > 0.1$ . For KIN\_ETACHOICE1

$$\eta_{\mathrm{safe}} = \eta_{n-1}^{rac{1+\sqrt{5}}{2}}$$

and for KIN\_ETACHOICE2

$$\eta_{\text{safe}} = \gamma \eta_{n-1}^{\alpha}$$

where  $\gamma$  and  $\alpha$  can be set with KINSetEtaParams().

The following safeguards are always applied when using either KIN\_ETACHOICE1 or KIN\_ETACHOICE2 so that  $\eta_{\min} \leq \eta_n \leq \eta_{\max}$ :

$$\eta_n = \max(\eta_n, \eta_{\min})$$

$$\eta_n = \min(\eta_n, \eta_{\max})$$

where  $\eta_{\min} = 10^{-4}$  and  $\eta_{\max} = 0.9$ .

## int KINSetEtaConstValue(void \*kin\_mem, sunrealtype eta)

The function KINSetEtaConstValue() specifies the constant value for  $\eta$  in the case etachoice = KIN\_-ETACONSTANT.

#### **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- eta constant value for  $\eta$ . Pass 0.0 to indicate the default.

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument eta had an illegal value

The default value for eta is 0.1. The legal values are  $0.0 < \text{eta} \le 1.0$ .

## int **KINSetEtaParams** (void \*kin\_mem, sunrealtype egamma, sunrealtype ealpha)

The function  $\mathit{KINSetEtaParams}()$  specifies the parameters  $\gamma$  and  $\alpha$  in the formula for  $\eta$ , in the case etachoice =  $\mathit{KIN\_ETACHOICE2}$ .

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- egamma value of the  $\gamma$  parameter. Pass 0.0 to indicate the default.
- ealpha value of the  $\alpha$  parameter. Pass 0.0 to indicate the default.

## **Return value:**

- KIN\_SUCCESS The optional values have been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT One of the arguments egamma or ealpha had an illegal value.

## Notes:

The default values for egamma and ealpha are 0.9 and 2.0, respectively. The legal values are 0.0 < egamma  $\leq 1.0$  and 1.0 < ealpha  $\leq 2.0$ .

## int **KINSetResMonConstValue**(void \*kin\_mem, *sunrealtype* omegaconst)

The function KINSetResMonConstValue() specifies the constant value for  $\omega$  when using residual monitoring.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- omegaconst constant value for  $\omega$ . Passing 0.0 results in using Eqn. (2.4).

## Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument omegaconst had an illegal value

## Notes:

The default value for omegaconst is 0.9. The legal values are 0.0 < omegaconst < 1.0.

## int KINSetResMonParams (void \*kin\_mem, sunrealtype omegamin, sunrealtype omegamax)

The function KINSetResMonParams() specifies the parameters  $\omega_{min}$  and  $\omega_{max}$  in the formula (2.4) for  $\omega$ .

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- omegamin value of the  $\omega_{min}$  parameter. Pass 0.0 to indicate the default.
- omegamax value of the  $\omega_{max}$  parameter. Pass 0.0 to indicate the default.

- KIN\_SUCCESS The optional values have been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT One of the arguments omegamin or omegamax had an illegal value.

The default values for omegamin and omegamax are 0.00001 and 0.9, respectively. The legal values are 0.0 < omegamin < omegamax < 1.0.

Warning: Residual monitoring is only available for use with matrix-based linear solver modules.

## int KINSetNoMinEps(void \*kin\_mem, sunbooleantype noMinEps)

The function KINSetNoMinEps() specifies a flag that controls whether or not the value of  $\epsilon$ , the scaled linear residual tolerance, is bounded from below.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- noMinEps flag controlling the bound on  $\epsilon$ . If SUNFALSE is passed the value of  $\epsilon$  is constrained and if SUNTRUE is passed then  $\epsilon$  is not constrained.

#### **Return value:**

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## **Notes:**

The default value for noMinEps is SUNFALSE, meaning that a positive minimum value, equal to 0.01'\* "fnormtol', is applied to  $\epsilon$  (see KINSetFuncNormTol() below).

## int KINSetMaxNewtonStep(void \*kin\_mem, sunrealtype mxnewtstep)

The function KINSetMaxNewtonStep() specifies the maximum allowable scaled length of the Newton step.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- mxnewtstep maximum scaled step length ( $\geq 0.0$ ). Pass 0.0 to indicate the default.

## Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The input value was negative.

## Notes:

The default value of mxnewtstep is  $1000 \|u_0\|_{D_u}$ , where  $u_0$  is the initial guess.

## int KINSetMaxBetaFails(void \*kin\_mem, sunrealtype mxnbcf)

The function KINSetMaxBetaFails() specifies the maximum number of  $\beta$ -condition failures in the linesearch algorithm.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- mxnbcf maximum number of  $\beta$  -condition failures. Pass 0.0 to indicate the default.

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT mxnbcf was negative.

The default value of mxnbcf is MXNBCF DEFAULT = 10.

## int KINSetRelErrFunc(void \*kin\_mem, sunrealtype relfunc)

The function KINSetRelErrFunc() specifies the relative error in computing F(u), which is used in the difference quotient approximation to the Jacobian matrix [see Eq. (2.6)] or the Jacobian-vector product [see Eq. (2.8)]. The value stored is  $\sqrt{\text{relfunc}}$ .

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- relfunc relative error in F(u) (relfunc  $\geq 0.0$ ). Pass 0.0 to indicate the default.

## Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The relative error was negative.

## **Notes:**

The default value for relfunc is U = unit roundoff.

## int **KINSetFuncNormTol** (void \*kin mem, *sunrealtype* fnormtol)

The function KINSetFuncNormTol() specifies the scalar used as a stopping tolerance on the scaled maximum norm of the system function F(u).

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- fnormtol tolerance for stopping based on scaled function norm (≥ 0.0). Pass 0.0 to indicate the
  default.

#### Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The tolerance was negative.

## **Notes:**

The default value for fnormtol is (unit roundoff)  $^{1/3}$ .

## int KINSetScaledStepTol(void \*kin\_mem, sunrealtype scsteptol)

The function *KINSetScaledStepTol()* specifies the scalar used as a stopping tolerance on the minimum scaled step length.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- scsteptol tolerance for stopping based on scaled step length  $(\geq 0.0)$ . Pass 0.0 to indicate the default.

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The tolerance was non-positive.

The default value for scsteptol is (unit roundoff)  $^{2/3}$ .

## int KINSetConstraints(void \*kin\_mem, N\_Vector constraints)

The function KINSetConstraints() specifies a vector that defines inequality constraints for each component of the solution vector u.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- constraints vector of constraint flags. If constraints[i] is
  - 0.0 then no constraint is imposed on  $u_i$ .
  - 1.0 then  $u_i$  will be constrained to be  $u_i \geq 0.0$ .
  - -1.0 then  $u_i$  will be constrained to be  $u_i \leq 0.0$ .
  - 2.0 then  $u_i$  will be constrained to be  $u_i > 0.0$ .
  - -2.0 then  $u_i$  will be constrained to be  $u_i < 0.0$ .

#### Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The constraint vector contains illegal values.

#### Notes:

The presence of a non-NULL constraints vector that is not 0.0 in all components will cause constraint checking to be performed. If a NULL vector is supplied, constraint checking will be disabled. The function creates a private copy of the constraints vector. Consequently, the user-supplied vector can be freed after the function call, and the constraints can only be changed by calling this function.

## int **KINSetSysFunc**(void \*kin\_mem, *KINSysFn* func)

The function KINSetSysFunc() specifies the user-provided function that evaluates the nonlinear system function F(u) or G(u).

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- func user-supplied function that evaluates F(u) (or G(u) for fixed-point iteration).

## Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument func was NULL.

## Notes:

The nonlinear system function is initially specified through *KINInit()*. The option of changing the system function is provided for a user who wishes to solve several problems of the same size but with different functions.

## int KINSetReturnNewest(void \*kin\_mem, sunbooleantype ret\_newest)

The function *KINSetReturnNewest()* specifies if the fixed point iteration should return the newest iteration or the iteration consistent with the last function evaluation.

- kin\_mem pointer to the KINSOL memory block.
- ret\_newest SUNTRUE return the newest iteration. SUNFALSE return the iteration consistent with the last function evaluation.

#### **Return value:**

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

#### Notes:

The default value of ret\_newest is SUNFALSE.

## int **KINSetDamping**(void \*kin\_mem, sunrealtype beta)

The function <code>KINSetDamping()</code> specifies the value of the damping parameter in the fixed point or Picard iteration.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- beta the damping parameter value  $0 < beta \le 1.0$ .

## **Return value:**

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument beta was zero or negative.

#### **Notes:**

This function sets the damping parameter value, which needs to be greater than zero and less than one if damping is to be used. A value  $\geq 1$  disables damping. The default value of beta is 1.0, indicating no damping. To set the damping parameter used in Anderson acceleration see KINSetDampingAA(). With the fixed point iteration the difference between successive iterations is used to determine convergence. As such, when damping is enabled, the tolerance used to stop the fixed point iteration is scaled by beta to account for the effects of damping. If beta is extremely small (close to zero), this can lead to an excessively tight tolerance.

## int **KINSetMAA**(void \*kin\_mem, long int maa)

The function *KINSetMAA()* specifies the size of the subspace used with Anderson acceleration in conjunction with Picard or fixed-point iteration.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- maa subspace size for various methods. A value of 0 means no acceleration, while a positive value means acceleration will be done.

## Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument maa was negative.

## **Notes:**

This function sets the subspace size, which needs to be > 0 if Anderson Acceleration is to be used. It also allocates additional memory necessary for Anderson Acceleration. The default value of maa is 0, indicating no acceleration. The value of maa should always be less than mxiter. This function MUST be

called before calling *KINInit()*. If the user calls the function KINSetNumMaxIters, that call should be made before the call to KINSetMAA, as the latter uses the value of mxiter.

#### int **KINSetDampingAA** (void \*kin\_mem, *sunrealtype* beta)

The function KINSetDampingAA() specifies the value of the Anderson acceleration damping paramter.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- beta the damping parameter value  $0 < beta \le 1.0$ .

#### Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument beta was zero or negative.

#### Notes:

This function sets the damping parameter value, which needs to be greater than zero and less than one if damping is to be used. A value  $\geq 1$  disables damping. The default value of beta is 1.0, indicating no damping. When delaying the start of Anderson acceleration with KINSetDelayAA(), use KINSetDamping() to set the damping parameter in the fixed point or Picard iterations before Anderson acceleration begins. When using Anderson acceleration without delay, the value provided to KINSetDampingAA() is applied to all iterations and any value provided to KINSetDamping() is ignored.

## int **KINSetDelayAA** (void \*kin mem, long int delay)

The function KINSetDelayAA() specifies the number of iterations to delay the start of Anderson acceleration.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- delay the number of iterations to delay Anderson acceleration.

#### Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument delay was less than zero.

#### **Notes:**

The default value of delay is 0, indicating no delay.

## int KINSetOrthAA (void \*kin\_mem, int orthaa)

The function *KINSetOrthAA()* specifies the orthogonalization routine to be used in the QR factorization portion of Anderson acceleration.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- orthaa the orthogonalization routine parameter. Can be set to any of the following
  - KIN\_ORTH\_MGS Modified Gram Schmidt (default)
  - KIN\_ORTH\_ICWY Inverse Compact WY Modified Gram Schmidt
  - KIN\_ORTH\_CGS2 Classical Gram Schmidt with Reorthogonalization (CGS2)
  - KIN\_ORTH\_DCGS2 Classical Gram Schmidt with Delayed Reorthogonlization

#### Return value:

- KIN\_SUCCESS The optional value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT The argument orthaa was not one of the predefined orthogonalization routines defined in KINSOL.

**Note:** This function *must* be called before calling *KINInit()*.

An example of how to use this function can be found in examples/kinsol/serial/kinAnalytic\_fp.c

## **5.3.4.1** Linear solver interface optional input functions

For matrix-based linear solver modules, the KINLS solver interface needs a function to compute an approximation to the Jacobian matrix J(u). This function must be of type KINLsJacFn. The user can supply a Jacobian function, or if using the  $SUNMATRIX\_DENSE$  or  $SUNMATRIX\_BAND$  modules for J can use the default internal difference quotient approximation that comes with the KINLS solver. To specify a user-supplied Jacobian function jac, KINLS provides the function KINSetJacFn(). The KINLS interface passes the pointer user\_data to the Jacobian function. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied Jacobian function, without using global data in the program. The pointer user\_data may be specified through KINSetUserData().

## int KINSetJacFn(void \*kin\_mem, KINLsJacFn jac)

The function *KINSetJacFn()* specifies the Jacobian approximation function to be used for a matrix-based solver within the KINLS interface.

#### **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- jac user-defined Jacobian approximation function. See *KINLsJacFn* for more details.

#### Return value:

- KINLS\_SUCCESS The optional value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver interface has not been initialized.

## **Notes:**

This function must be called after the KINLS linear solver interface has been initialized through a call to <code>KINSetLinearSolver()</code>. By default, KINLS uses an internal difference quotient function for the <code>SUNMATRIX\_DENSE</code> and <code>SUNMATRIX\_BAND</code> modules. If <code>NULL</code> is passed to <code>jac</code>, this default function is used. An error will occur if no <code>jac</code> is supplied when using other matrix types.

New in version 4.0.0: Replaces the deprecated function KINDlsSetJacFn.

When using matrix-free linear solver modules, the KINLS linear solver interface requires a function to compute an approximation to the product between the Jacobian matrix J(u) and a vector v. The user can supply his/her own Jacobian-times-vector approximation function, or use the internal difference quotient approximation that comes with the KINLS solver interface.

A user-defined Jacobian-vector function must be of type <code>KINLsJacTimesVecFn</code> and can be specified through a call to <code>KINSetJacTimesVecFn()</code> (see §5.4.3 for specification details). The pointer user\_data received through <code>KINSetUserData()</code> (or a pointer to NULL if user\_data was not specified) is passed to the Jacobian-times-vector function jtimes each time it is called. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied functions without using global data in the program.

## int **KINSetJacTimesVecFn** (void \*kin\_mem, *KINLsJacTimesVecFn* jtimes)

The function *KINSetJacTimesVecFn()* specifies the Jacobian-vector product function.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- jtimes user-defined Jacobian-vector product function.

#### Return value:

- KINLS\_SUCCESS The optional value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.
- KINLS\_SUNLS\_FAIL An error occurred when setting up the system matrix-times-vector routines in the SUNLINSOL object used by the KINLS interface.

#### Notes:

The default is to use an internal difference quotient for jtimes. If NULL is passed as jtimes, this default is used. This function must be called after the KINLS linear solver interface has been initialized through a call to KINSetLinearSolver(). The function type KINLsJacTimesVecFn is described in §5.4.3.

New in version 4.0.0: Replaces the deprecated function KINSpilsSetJacTimesVecFn.

When using the internal difference quotient the user may optionally supply an alternative system function for use in the Jacobian-vector product approximation by calling <code>KINSetJacTimesVecSysFn()</code>. The alternative system function should compute a suitable (and differentiable) approximation of the system function provided to <code>KINInit()</code>. For example, as done in [24] when solving the nonlinear systems that arise in the implicit integration of ordinary differential equations, the alternative function may use lagged values when evaluating a nonlinearity to avoid differencing a potentially non-differentiable factor.

## int **KINSetJacTimesVecSysFn**(void \*kin\_mem, *KINSysFn* jtimesSysFn)

The function *KINSetJacTimesVecSysFn()* specifies an alternative system function for use in the internal Jacobian-vector product difference quotient approximation.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- jtimesSysFn—is the CC function which computes the alternative system function to use in Jacobian-vector product difference quotient approximations. This function has the form func(u, fval, user\_data). (For full details see §5.4.1.)

#### Return value:

- KINLS\_SUCCESS The optional value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.
- KINLS\_ILL\_INPUT The internal difference quotient approximation is disabled.

#### **Notes:**

The default is to use the system function provided to *KINInit()* in the internal difference quotient. If the input system function is NULL, the default is used. This function must be called after the KINLS linear solver interface has been initialized through a call to *KINSetLinearSolver()*.

When using an iterative linear solver, the user may supply a preconditioning operator to aid in solution of the system. This operator consists of two user-supplied functions, psetup and psolve, that are supplied to KINLS using the function *KINSetPreconditioner()*. The psetup function supplied to this routine should handle evaluation and

preprocessing of any Jacobian data needed by the user's preconditioner solve function, psolve. Both of these functions are fully specified in §5.4. The user data pointer received through *KINSetUserData()* (or a pointer to NULL if user data was not specified) is passed to the psetup and psolve functions. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied preconditioner functions without using global data in the program.

int KINSetPreconditioner(void \*kin\_mem, KINLsPrecSetupFn psetup, KINLsPrecSolveFn psolve)

The function KINSetPreconditioner() specifies the preconditioner setup and solve functions.

#### **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- psetup user-defined function to set up the preconditioner. See *KINLsPrecSetupFn* for more details. Pass NULL if no setup is necessary.
- psolve user-defined preconditioner solve function. See KINLsPrecSolveFn for more details.

#### **Return value:**

- KINLS\_SUCCESS The optional values have been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.
- KINLS\_SUNLS\_FAIL An error occurred when setting up preconditioning in the SUNLinearSolver object used by the KINLS interface.

#### Notes:

The default is NULL for both arguments (i.e., no preconditioning). This function must be called after the KINLS linear solver interface has been initialized through a call to KINSetLinearSolver().

New in version 4.0.0: Replaces the deprecated function KINSpilsSetPreconditioner.

## **5.3.5** Optional output functions

KINSOL provides an extensive list of functions that can be used to obtain solver performance information. Table 5.2 lists all optional output functions in KINSOL, which are then described in detail in the remainder of this section, beginning with those for the main KINSOL solver and continuing with those for the KINLS linear solver interface. Where the name of an output from a linear solver module would otherwise conflict with the name of an optional output from the main solver, a suffix LS (for Linear Solver) has been added here (e.g., lenrwLS).

Table 5.2: Optional outputs from KINSOL and KINLS

Optional output	Function name
KINSOL main solver	
Size of KINSOL real and integer workspaces	<pre>KINGetWorkSpace()</pre>
Number of function evaluations	<pre>KINGetNumFuncEvals()</pre>
Number of nonlinear iterations	<pre>KINGetNumNonlinSolvIters()</pre>
Number of $\beta$ -condition failures	<pre>KINGetNumBetaCondFails()</pre>
Number of backtrack operations	<pre>KINGetNumBacktrackOps()</pre>
Scaled norm of $F$	<pre>KINGetFuncNorm()</pre>
Scaled norm of the step	<pre>KINGetStepLength()</pre>
User data pointer	<pre>KINGetUserData()</pre>
Print all statistics	<pre>KINPrintAllStats()</pre>
Name of constant associated with a return flag	<pre>KINGetReturnFlagName()</pre>
KINLS linear solver interface	
Stored Jacobian of the nonlinear system	<pre>KINGetJac()</pre>
Nonlinear iteration number at which the Jacobian was evaluated	<pre>KINGetJacNumIters()</pre>
Size of real and integer workspaces	<pre>KINGetLinWorkSpace()</pre>
No. of Jacobian evaluations	<pre>KINGetNumJacEvals()</pre>
No. of $F$ calls for D.Q. Jacobian[-vector] evals.	<pre>KINGetNumLinFuncEvals()</pre>
No. of linear iterations	<pre>KINGetNumLinIters()</pre>
No. of linear convergence failures	<pre>KINGetNumLinConvFails()</pre>
No. of preconditioner evaluations	<pre>KINGetNumPrecEvals()</pre>
No. of preconditioner solves	<pre>KINGetNumPrecSolves()</pre>
No. of Jacobian-vector product evaluations	<pre>KINGetNumJtimesEvals()</pre>
Last return from a KINLS function	<pre>KINGetLastLinFlag()</pre>
Name of constant associated with a return flag	<pre>KINGetLinReturnFlagName()</pre>

## 5.3.5.1 Main solver optional output functions

KINSOL provides several user-callable functions that can be used to obtain different quantities that may be of interest to the user, such as solver workspace requirements and solver performance statistics. These optional output functions are described next.

int **KINGetWorkSpace**(void \*kin\_mem, long int lenrw, long int leniw)

The function KINGetWorkSpace() returns the KINSOL integer and real workspace sizes.

#### **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- lenrw the number of sunrealtype values in the KINSOL workspace.
- leniw the number of integer values in the KINSOL workspace.

#### Return value:

- KIN\_SUCCESS The optional output values have been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## **Notes:**

KINSOL solver In terms of the problem size N, the actual size of the real workspace is 17 + 5N sunrealtype words. The real workspace is increased by an additional N words if constraint checking is enabled (see KINSetConstraints()).

The actual size of the integer workspace (without distinction between int and long int) is 22 + 5N (increased by N if constraint checking is enabled).

#### int **KINGetNumFuncEvals**(void \*kin mem, long int nfevals)

The function KINGetNumFuncEvals() returns the number of evaluations of the system function.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- nfevals number of calls to the user-supplied function that evaluates F(u).

#### **Return value:**

- KIN\_SUCCESS The optional output value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## int KINGetNumNonlinSolvIters(void \*kin\_mem, long int nniters)

The function *KINGetNumNonlinSolvIters()* returns the number of nonlinear iterations.

#### **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- nniters number of nonlinear iterations.

#### Return value:

- KIN\_SUCCESS The optional output value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## int KINGetNumBetaCondFails(void \*kin\_mem, long int nbcfails)

The function *KINGetNumBetaCondFails()* returns the number of  $\beta$ -condition failures.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- nbcfails number of  $\beta$  -condition failures.

## Return value:

- KIN\_SUCCESS The optional output value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## int KINGetNumBacktrackOps (void \*kin\_mem, long int nbacktr)

The function *KINGetNumBacktrackOps()* returns the number of backtrack operations (step length adjustments) performed by the line search algorithm.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- nbacktr number of backtrack operations.

#### Return value:

- KIN\_SUCCESS The optional output value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## int KINGetFuncNorm(void \*kin\_mem, sunrealtype fnorm)

The function  $\mathit{KINGetFuncNorm}()$  returns the scaled Euclidean  $\ell_2$  norm of the nonlinear system function F(u) evaluated at the current iterate.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- fnorm current scaled norm of F(u).

#### Return value:

- KIN\_SUCCESS The optional output value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## int KINGetStepLength(void \*kin\_mem, sunrealtype steplength)

The function KINGetStepLength() returns the scaled Euclidean  $\ell_2$  norm of the step used during the previous iteration.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- steplength scaled norm of the Newton step.

## **Return value:**

- KIN\_SUCCESS The optional output value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

## int KINGetUserData(void \*kin\_mem, void \*\*user\_data)

The function *KINGetUserData()* returns the user data pointer provided to *KINSetUserData()*.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- user\_data memory reference to a user data pointer.

#### Return value:

- KIN\_SUCCESS The optional output value has been successfully set.
- KIN\_MEM\_NULL The kin\_mem pointer is NULL.

New in version 6.3.0.

## int **KINPrintAllStats**(void \*cvode\_mem, FILE \*outfile, *SUNOutputFormat* fmt)

The function KINPrintAllStats() outputs all of the nonlinear solver, linear solver, and other statistics.

#### **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- outfile pointer to output file.
- fmt the output format:
  - SUN\_OUTPUTFORMAT\_TABLE prints a table of values
  - SUN\_OUTPUTFORMAT\_CSV prints a comma-separated list of key and value pairs e.g., key1, value1, key2, value2,...

#### Return value:

• KIN\_SUCCESS – The output was successfully.

- KIN\_MEM\_NULL The kin\_mem pointer is NULL.
- KIN\_ILL\_INPUT An invalid formatting option was provided.

**Note:** The file scripts/sundials\_csv.py provides python utility functions to read and output the data from a SUNDIALS CSV output file using the key and value pair format.

New in version 6.2.0.

## char \*KINGetReturnFlagName(int flag)

The function KINGetReturnFlagName() returns the name of the KINSOL constant corresponding to flag.

## **Arguments:**

• flag – return flag from a KINSOL function.

#### **Return value:**

• A string containing the name of the corresponding constant

#### 5.3.5.2 KINLS linear solver interface optional output functions

The following optional outputs are available from the KINLS modules:

int KINGetJac(void \*kin\_mem, SUNMatrix \*J)

Returns the internally stored copy of the Jacobian matrix of the nonlinear system function.

#### **Parameters**

- kin\_mem the KINSOL solver object
- **J** the Jacobian matrix

#### Return values

- KINLS\_SUCCESS the output value has been successfully set
- KINLS\_MEM\_NULL kin\_mem was NULL
- KINLS\_LMEM\_NULL the linear solver interface has not been initialized

**Warning:** With linear solvers that overwrite the input Jacobian matrix as part of the linear solver setup (e.g., performing an in-place LU factorization) the matrix returned by *KINGetJac()* may differ from the matrix returned by the last Jacobian evaluation.

**Warning:** This function is provided for debugging purposes and the values in the returned matrix should not be altered.

int KINGetJacNumIters(void \*kin\_mem, sunrealtype \*nni\_J)

Returns the nonlinear iteration number at which the Jacobian was evaluated.

## **Parameters**

- kin\_mem the KINSOL memory structure
- **nni J** the nonlinear iteration number

#### **Return values**

- KINLS\_SUCCESS the output value has been successfully set
- KINLS\_MEM\_NULL kin\_mem was NULL
- KINLS\_LMEM\_NULL the linear solver interface has not been initialized

#### int **KINGetLinWorkSpace**(void \*kin mem, long int \*lenrwLS, long int \*leniwLS)

The function <code>KINGetLinWorkSpace()</code> returns the sizes of the real and integer workspaces used by the KINLS linear solver interface.

#### **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- lenrwLS the number of real values in the KINLS workspace.
- leniwLS the number of integer values in the KINLS workspace.

#### Return value:

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

#### **Notes:**

The workspace requirements reported by this routine correspond only to memory allocated within this interface and to memory allocated by the SUNLinearSolver object attached to it. The template Jacobian matrix allocated by the user outside of KINLS is not included in this report.

New in version 4.0.0: Replaces the deprecated function KINDlsGetWorkspace and KINSpilsGetWorkspace.

## int KINGetNumJacEvals(void \*kin\_mem, long int \*njevals)

The function <code>KINGetNumJacEvals()</code> returns the cumulative number of calls to the KINLS Jacobian approximation function.

#### **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- njevals the cumulative number of calls to the Jacobian function total so far.

#### Return value:

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

New in version 4.0.0: Replaces the deprecated function KINDlsGetNumJacEvals.

## int KINGetNumLinFuncEvals(void \*kin\_mem, long int \*nrevalsLS)

The function *KINGetNumLinFuncEvals()* returns the cumulative number of calls to the user residual function due to the finite difference Jacobian approximation or finite difference Jacobian-vector product approximation.

#### **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- nrevalsLS the cumulative number of calls to the user residual function.

#### Return value:

• KINLS\_SUCCESS – The optional output value has been successfully set.

**5.3.** User-callable functions

- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

#### **Notes:**

The value nrevalsLS is incremented only if one of the default internal difference quotient functions is used.

New in version 4.0.0: Replaces the deprecated functions KINDlsGetNumRhsEvals and KINSpilsGetNumRhsEvals.

## int KINGetNumLinIters(void \*kin\_mem, long int \*nliters)

The function KINGetNumLinIters() returns the cumulative number of linear iterations.

## **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- nliters the current number of linear iterations.

#### Return value:

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

New in version 4.0.0: Replaces the deprecated function KINSpilsGetNumLinIters.

## int KINGetNumLinConvFails(void \*kin\_mem, long int \*nlcfails)

The function KINGetNumLinConvFails() returns the cumulative number of linear convergence failures.

#### **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- nlcfails the current number of linear convergence failures.

#### Return value:

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

New in version 4.0.0: Replaces the deprecated function KINSpilsGetNumConvFails.

## int **KINGetNumPrecEvals**(void \*kin\_mem, long int \*npevals)

The function *KINGetNumPrecEvals()* returns the cumulative number of preconditioner evaluations, i.e., the number of calls made to psetup.

## **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- npevals the cumulative number of calls to psetup.

## Return value:

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

New in version 4.0.0: Replaces the deprecated function KINSpilsGetNumPrecEvals.

#### int **KINGetNumPrecSolves** (void \*kin\_mem, long int \*npsolves)

The function *KINGetNumPrecSolves()* returns the cumulative number of calls made to the preconditioner solve function, psolve.

#### **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- npsolves the cumulative number of calls to psolve.

#### Return value:

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

New in version 4.0.0: Replaces the deprecated function KINSpilsGetNumPrecSolves.

## int **KINGetNumJtimesEvals**(void \*kin\_mem, long int \*njvevals)

The function *KINGetNumJtimesEvals()* returns the cumulative number of calls made to the Jacobian-vector product function, jtimes.

## **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- njvevals the cumulative number of calls to jtimes.

#### Return value:

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

New in version 4.0.0: Replaces the deprecated function KINSpilsGetNumJtimesEvals.

## int KINGetLastLinFlag(void \*kin\_mem, long int \*lsflag)

The function *KINGetLastLinFlag()* returns the last return value from an KINLS routine.

## **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- 1sflag the value of the last return flag from an KINLS function.

#### **Return value:**

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer is NULL.
- KINLS\_LMEM\_NULL The KINLS linear solver has not been initialized.

## **Notes:**

If the KINLS setup function failed (i.e., KINSO1() returned KIN\_LSETUP\_FAIL) when using the SUN-LINSOL\_DENSE or SUNLINSOL\_BAND modules, then the value of 1sflag is equal to the column index (numbered from one) at which a zero diagonal element was encountered during the LU factorization of the (dense or banded) Jacobian matrix.

If the KINLS setup function failed when using another SUNLinearSolver object, then lsflag will be SUNLS\_PSET\_FAIL\_UNREC, SUNLS\_ASET\_FAIL\_UNREC, or SUN\_ERR\_EXT\_FAIL.

**5.3.** User-callable functions

If the KINLS solve function failed (KINSol()) returned KIN\_LSOLVE\_FAIL), 1sflag contains the error return flag from the SUNLinearSolver object, which will be one of: SUN\_ERR\_ARG\_CORRUPTRRUPT, indicating that the SUNLinearSolver memory is NULL; SUNLS\_ATIMES\_FAIL\_UNREC, indicating an unrecoverable failure in the J\*v function; SUNLS\_PSOLVE\_FAIL\_UNREC, indicating that the preconditioner solve function psolve failed unrecoverably; SUNLS\_GS\_FAIL, indicating a failure in the Gram-Schmidt procedure (generated only in SPGMR or SPFGMR); SUNLS\_QRSOL\_FAIL, indicating that the matrix R was found to be singular during the QR solve phase (SPGMR and SPFGMR only); or SUN\_ERR\_EXT\_FAIL, indicating an unrecoverable failure in an external iterative linear solver package.

New in version 4.0.0: Replaces the deprecated functions KINDlsGetLastFlag and KINSpilsGetLastFlag.

## char \*KINGetLinReturnFlagName(long int lsflag)

The function KINGetLinReturnFlagName() returns the name of the KINLS constant corresponding to lsflag.

## **Arguments:**

• flag – the flag returned by a call to an KINSOL function

#### **Return value:**

• char\* – the flag name string or if  $1 \le \mathtt{lsflag} \le N$  (LU factorization failed), this function returns "NONE".

New in version 4.0.0: Replaces the deprecated functions KINDlsGetReturnFlagName and KINSpilsGetReturnFlagName.

## **5.4** User-supplied functions

The user-supplied functions consist of one function defining the nonlinear system, (optionally) a function that handles error and warning messages, (optionally) a function that handles informational messages, (optionally) one or two functions that provides Jacobian-related information for the linear solver, and (optionally) one or two functions that define the preconditioner for use in any of the Krylov iterative algorithms.

## **5.4.1 Problem defining function**

The user must provide a function of type *KINSysFn* defined as follows:

typedef int (\***KINSysFn**)(*N\_Vector* u, *N\_Vector* fval, void \*user\_data)

This function computes the F(u) (or G(u) for fixed-point iteration and Anderson acceleration) for a given value of the vector u.

#### **Arguments:**

- $\mathbf{u}$  is the current value of the dependent variable vector, u
- fval is the output vector F(u)
- user\_data is a pointer to user data, the same as the user\_data pointer parameter passed to KIN-SetUserData()

#### Return value:

An *KINSysFn* function type should return a value of 0 if successful, a positive value if a recoverable error occurred (in which case KINSOL will attempt to correct), or a negative value if a nonrecoverable error occurred. In the last case, the integrator halts. If a recoverable error occurred, the integrator will attempt to correct and retry.

#### **Notes:**

Allocation of memory for fval is handled within KINSOL.

## **5.4.2** Jacobian construction (matrix-based linear solvers)

If a matrix-based linear solver module is used (i.e. a non-NULL SUNMatrix object was supplied to KINSetLinear-Solver()), the user may provide a function of type KINLsJacFn defined as follows:

typedef int (\***KINLsJacFn**)(*N\_Vector* u, *N\_Vector* fu, *SUNMatrix* J, void \*user\_data, *N\_Vector* tmp1, *N\_Vector* tmp2)

This function computes the Jacobian matrix J(u) (or an approximation to it).

## **Arguments:**

- u is the current (unscaled) iterate.
- $\mathbf{fu}$  is the current value of the vector, F(u).
- J is the output (approximate) Jacobian matrix (of type SUNMatrix), F'(u).
- user\_data is a pointer to user data, the same as the user\_data parameter passed to KINSetUser-Data().
- tmp1, tmp2, are pointers to memory allocated for variables of type N\_Vector which can be used by *KINLsJacFn* function as temporary storage or work space.

#### **Return value:**

An KINLsJacFn should return 0 if successful, or a non-zero value otherwise.

#### **Notes:**

Information regarding the structure of the specific SUNMatrix structure (e.g. number of rows, upper/lower bandwidth, sparsity type) may be obtained through using the implementation-specific SUNMatrix interface functions (see Chapter §7 for details).

With direct linear solvers (i.e., linear solvers with type SUNLINEARSOLVER\_DIRECT), the Jacobian matrix J(u) is zeroed out prior to calling the user-supplied Jacobian function so only nonzero elements need to be loaded into J.

If the user's *KINLsJacFn* function uses difference quotient approximations, it may need to access quantities not in the call list. These quantities may include the scale vectors and the unit roundoff. To obtain the scale vectors, the user will need to add to user\_data pointers to u\_scale and/or f\_scale as needed. The unit roundoff can be accessed as SUN\_UNIT\_ROUNDOFF defined in sundials\_types.h.

#### dense:

A user-supplied dense Jacobian function must load the N  $\times$  N dense matrix J with an approximation to the Jacobian matrix J(u) at the point (u). The accessor macros SM\_ELEMENT\_D and SM\_COLUMN\_D allow the user to read and write dense matrix elements without making explicit references to the underlying representation of the SUNMATRIX\_DENSE type. SM\_ELEMENT\_D(J, i, j) references the (i, j)-th element of the dense matrix J (with i, j = 0 ... N - 1). This macro is meant for small problems for which efficiency of access is not a major concern. Thus, in terms of the indices m and n ranging from 1 to N, the Jacobian element  $J_{m,n}$  can be set using the statement SM\_ELEMENT\_D(J, m-1, n-1) =  $J_{m,n}$ . Alternatively, SM\_-COLUMN\_D(J, j) returns a pointer to the first element of the j-th column of J (with j = 0 ... N - 1), and the elements of the j-th column can then be accessed using ordinary array indexing. Consequently,  $J_{m,n}$  can be loaded using the statements col\_n = SM\_COLUMN\_D(J, n-1); col\_n[m-1] =  $J_{m,n}$ . For large problems, it is more efficient to use SM\_COLUMN\_D than to use SM\_ELEMENT\_D. Note that both of these macros number rows and columns starting from 0. The SUNMATRIX\_DENSE type and accessor macros are documented in §7.3.

#### banded:

A user-supplied banded Jacobian function must load the N  $\times$  N banded matrix J with an approximation to the Jacobian matrix J(u) at the point (u). The accessor macros SM\_ELEMENT\_B, SM\_COLUMN\_B, and SM\_COLUMN\_ELEMENT\_B allow the user to read and write banded matrix elements without making specific

references to the underlying representation of the SUNMATRIX\_BAND type. SM\_ELEMENT\_B(J, i, j) references the (i, j)-th element of the banded matrix J, counting from 0. This macro is meant for use in small problems for which efficiency of access is not a major concern. Thus, in terms of the indices m and n ranging from 1 to N with (m, n) within the band defined by mupper and mlower, the Jacobian element  $J_{m,n}$  can be loaded using the statement SM\_ELEMENT\_B(J, m-1, n-1) =  $J_{m,n}$ . The elements within the band are those with -mupper  $\leq$  m-n  $\leq$  mlower. Alternatively, SM\_COLUMN\_B(J, j) returns a pointer to the diagonal element of the j-th column of J, and if we assign this address to sunreal type \*col\_j, then the i-th element of the j-th column is given by SM\_COLUMN\_ELEMENT\_B(col\_j, i, j), counting from 0. Thus, for (m,n) within the band,  $J_{m,n}$  can be loaded by setting col\_n = SM\_COLUMN\_B(J, n-1); and SM\_COLUMN\_ELEMENT\_B(col\_n, m-1, n-1) =  $J_{m,n}$ . The elements of the j-th column can also be accessed via ordinary array indexing, but this approach requires knowledge of the underlying storage for a band matrix of type SUNMATRIX\_BAND. The array col\_n can be indexed from -mupper to mlower. For large problems, it is more efficient to use SM\_COLUMN\_B and SM\_COLUMN\_ELEMENT\_B than to use the SM\_-ELEMENT\_B macro. As in the dense case, these macros all number rows and columns starting from 0. The SUNMATRIX\_BAND type and accessor macros are documented in §7.6.

#### sparse:

A user-supplied sparse Jacobian function must load the N  $\times$  N compressed-sparse-column or compressed-sparse-row matrix J with an approximation to the Jacobian matrix J(u) at the point (u). Storage for J already exists on entry to this function, although the user should ensure that sufficient space is allocated in J to hold the nonzero values to be set; if the existing space is insufficient the user may reallocate the data and index arrays as needed. The amount of allocated space in a SUNMATRIX\_SPARSE object may be accessed using the macro SM\_NNZ\_S or the routine SUNSparseMatrix\_NNZ. The SUNMATRIX\_SPARSE type and accessor macros are documented in §7.8.

New in version 4.0.0: Replaces the deprecated type KINDlsJacFn.

## **5.4.3** Jacobian-vector product (matrix-free linear solvers)

If a matrix-free linear solver is to be used (i.e., a NULL-valued SUNMatrix was supplied to KINSetLinearSolver()), the user may provide a function of type KINLsJacTimesVecFn in the following form, to compute matrix-vector products Jv. If such a function is not supplied, the default is a difference quotient approximation to these products.

typedef int (\*KINLsJacTimesVecFn)(*N\_Vector* v, *N\_Vector* Jv, *N\_Vector* u, *sunbooleantype* \*new\_u, void \*user data)

This function computes the product Jv (or an approximation to it).

## **Arguments:**

- v is the vector by which the Jacobian must be multiplied to the right.
- Jv is the computed output vector.
- $\mathbf{u}$  is the current value of the dependent variable vector.
- user\_data is a pointer to user data, the same as the user\_data parameter passed to *KINSetUser-Data()*.

#### Return value:

The value returned by the Jacobian-times-vector function should be 0 if successful. If a recoverable failure occurred, the return value should be positive. In this case, KINSOL will attempt to correct by calling the preconditioner setup function. If this information is current, KINSOL halts. If the Jacobian-times-vector function encounters an unrecoverable error, it should return a negative value, prompting KINSOL to halt.

#### **Notes:**

If a user-defined routine is not given, then an internal jtimes function, using a difference quotient approximation, is used.

This function must return a value of J \* v that uses the *current* value of J, i.e. as evaluated at the current u.

If the user's *KINLsJacTimesVecFn* function uses difference quotient approximations, it may need to access quantities not in the call list. These might include the scale vectors and the unit roundoff. To obtain the scale vectors, the user will need to add to user\_data pointers to u\_scale and/or f\_scale as needed. The unit roundoff can be accessed as SUN\_UNIT\_ROUNDOFF defined in sundials\_types.h.

New in version 4.0.0: Replaces the deprecated type KINSpilsJacTimesVecFn.

## **5.4.4** Preconditioner solve (iterative linear solvers)

If a user-supplied preconditioner is to be used with a SUNLinearSolver solver module, then the user must provide a function to solve the linear system Pz = r where P is the preconditioner matrix which approximates (at least crudely) the Jacobian matrix J = F'(u). This function must be of type KINLsPrecSolveFn, defined as follows:

typedef int (\*KINLsPrecSolveFn)(*N\_Vector* u, *N\_Vector* uscale, *N\_Vector* fval, *N\_Vector* fscale, *N\_Vector* v, void \*user data)

This function solves the preconditioning system Pz = r.

## **Arguments:**

- u is the current (unscaled) value of the iterate.
- uscale is a vector containing diagonal elements of the scaling matrix u
- fval is the vector F(u) evaluated at u
- fscale is a vector containing diagonal elements of the scaling matrix for fval
- v on input, v is set to the right-hand side vector of the linear system, r. On output, v must contain the solution z of the linear system Pz = r
- user\_data is a pointer to user data, the same as the user\_data parameter passed to *KINSetUser-Data()*.

## **Return value:**

The value returned by the preconditioner solve function should be 0 if successful, positive for a recoverable error, or negative for an unrecoverable error.

#### **Notes:**

If the preconditioner solve function fails recoverably and if the preconditioner information (set by the preconditioner setup function) is out of date, KINSOL attempts to correct by calling the setup function. If the preconditioner data is current, KINSOL halts.

## **5.4.5** Preconditioner setup (iterative linear solvers)

If the user's preconditioner requires that any Jacobian-related data be evaluated or preprocessed, then this needs to be done in a user-supplied function of type *KINLsPrecSetupFn*, defined as follows:

typedef int (\*KINLsPrecSetupFn)(N\_Vector u, N\_Vector uscale, N\_Vector fval, N\_Vector fscale, void \*user\_data)

This function evaluates and/or preprocesses Jacobian-related data needed by the preconditioner solve function.

#### **Arguments:**

- u is the current (unscaled) value of the iterate.
- uscale is a vector containing diagonal elements of the scaling matrix u
- fval is the vector F(u) evaluated at u

- fscale is a vector containing diagonal elements of the scaling matrix for fval
- user\_data is a pointer to user data, the same as the user\_data parameter passed to KINSetUser-Data().

#### Return value:

The value returned by the preconditioner setup function should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

#### **Notes:**

The user-supplied preconditioner setup subroutine should compute the right preconditioner matrix P (stored in the memory block referenced by the user\_data pointer) used to form the scaled preconditioned linear system

$$(D_F J(u) P^{-1} D_u^{-1})(D_u P x) = -D_F F(u)$$
,

where  $D_u$  and  $D_F$  denote the diagonal scaling matrices whose diagonal elements are stored in the vectors uscale and fscale, respectively.

The preconditioner setup routine will not be called prior to every call made to the preconditioner solve function, but will instead be called only as often as necessary to achieve convergence of the Newton iteration.

If the user's *KINLsPrecSetupFn* function uses difference quotient approximations, it may need to access quantities not in the call list. These might include the scale vectors and the unit roundoff. To obtain the scale vectors, the user will need to add to user\_data pointers to u\_scale and/or f\_scale as needed. The unit roundoff can be accessed as SUN\_UNIT\_ROUNDOFF defined in sundials\_types.h.

If the preconditioner solve routine requires no preparation, then a preconditioner setup function need not be given.

# 5.5 A parallel band-block-diagonal preconditioner module

The efficiency of Krylov iterative methods for the solution of linear systems can be greatly enhanced through preconditioning. For problems in which the user cannot define a more effective, problem-specific preconditioner, KINSOL provides a band-block-diagonal preconditioner module KINBBDPRE, to be used with the parallel N\_Vector module described in §6.5.

This module provides a preconditioner matrix for KINSOL that is block-diagonal with banded blocks. The blocking corresponds to the distribution of the dependent variable vector u amongst the processes. Each preconditioner block is generated from the Jacobian of the local part (associated with the current process) of a given function G(u) approximating F(u) (G=F is allowed). The blocks are generated by each process via a difference quotient scheme, utilizing a specified band structure. This structure is given by upper and lower half-bandwidths, mudq and mldq, defined as the number of non-zero diagonals above and below the main diagonal, respectively. However, from the resulting approximate Jacobain blocks, only a matrix of bandwidth mukeep + mlkeep +1 is retained.

Neither pair of parameters need be the true half-bandwidths of the Jacobian of the local block of G, if smaller values provide a more efficient preconditioner. Such an efficiency gain may occur if the couplings in the system outside a certain bandwidth are considerably weaker than those within the band. Reducing mukeep and mlkeep while keeping mudq and mldq at their true values, discards the elements outside the narrower band. Reducing both pairs has the additional effect of lumping the outer Jacobian elements into the computed elements within the band, and requires more caution and experimentation to see whether the lower cost of narrower band matrices offsets the loss of accuracy in the blocks.

The KINBBDPRE module calls two user-provided functions to construct P: a required function Gloc (of type KINBBD-LocalFn) which approximates the nonlinear system function  $G(u) \approx F(u)$  and which is computed locally, and an

optional function  $\operatorname{Gcomm}$  (of type  $\operatorname{KINBBDComm}Fn$ ) which performs all interprocess communication necessary to evaluate the approximate function G. These are in addition to the user-supplied nonlinear system function that evaluates F(u). Both functions take as input the same pointer  $\operatorname{user\_data}$  as that passed by the user to  $\operatorname{KINSetUserData}()$  and passed to the user's function func, and neither function has a return value. The user is responsible for providing space (presumably within  $\operatorname{user\_data}$ ) for components of  $\operatorname{u}$  that are communicated by  $\operatorname{Gcomm}$  from the other processes, and that are then used by  $\operatorname{Gloc}$ , which should not do any communication.

typedef int (\*KINBBDLocalFn)(sunindextype Nlocal, N\_Vector u, N\_Vector gval, void \*user\_data)

This Gloc function computes G(u), and outputs the resulting vector as gval.

#### **Arguments:**

- Nlocal is the local vector length.
- u is the current value of the iterate.
- gval is the output vector.
- user\_data is a pointer to user data, the same as the user\_data parameter passed to *KINSetUser-Data()*.

## Return value:

An KINBBDLocalFn function type should return 0 to indicate success, or non-zero if an error occured.

#### Notes:

This function must assume that all inter-processor communication of data needed to calculate gval has already been done, and this data is accessible within user\_data.

The case where G is mathematically identical to F is allowed.

typedef int (\*KINBBDCommFn)(sunindextype Nlocal, N\_Vector u, void \*user\_data)

This Gcomm function performs all inter-processor communications necessary for the execution of the Gloc function above, using the input vectors u.

## **Arguments:**

- Nlocal is the local vector length.
- u is the current value of the iterate.
- user\_data is a pointer to user data, the same as the user\_data parameter passed to *KINSetUser-Data()*.

#### Return value:

An KINBBDLocalFn function type should return 0 to indicate success, or non-zero if an error occured.

## Notes:

The Gcomm function is expected to save communicated data in space defined within the structure user\_data.

Each call to the Gcomm function is preceded by a call to the residual function func with the same u argument. Thus Gcomm can omit any communications done by func if relevant to the evaluation of Gloc. If all necessary communication was done in func, then Gcomm = NULL can be passed in the call to KINBB-DPrecInit().

Besides the header files required for the integration of the DAE problem (see §5.1), to use the KINBBDPRE module, the main program must include the header file kin\_bbdpre.h which declares the needed function prototypes.

The following is a summary of the usage of this module and describes the sequence of calls in the user main program. Steps that are unchanged from the user main program presented in §5.2 are not bold.

- 1. Initialize parallel or multi-threaded environment (*if appropriate*)
- 2. Create the SUNDIALS context object

- 3. Set the problem dimensions etc.
- 4. Create the vector with the initial guess
- 5. Create matrix object (if appropriate)
- 6. Create linear solver object (if appropriate)

When creating the iterative linear solver object, specify the use of right preconditioning (SUN\_PREC\_RIGHT) as KINSOL only supports right preconditioning.

- 7. Create nonlinear solver object (if appropriate)
- 8. Create KINSOL object
- 9. Initialize KINSOL solver
- 10. Attach the linear solver (if appropriate)
- 11. **Set linear solver optional inputs** (*if appropriate*)

Note that the user should not overwrite the preconditioner setup function or solve function through calls to <code>KINSetPreconditioner()</code> optional input function.

## 12. Initialize the KINBBDPRE preconditioner module

Call KINBBDPrecInit() to allocate memory and initialize the internal preconditioner data. The last two arguments of KINBBDPrecInit() are the two user-supplied functions described above.

- 13. Set optional inputs
- 14. Solve problem

## 15. Get optional outputs

Additional optional outputs associated with KINBBDPRE are available by way of two routines described below, <code>KINBBDPrecGetWorkSpace()</code> and <code>KINBBDPrecGetNumGfnEvals()</code>.

- 16. Deallocate memory
- 17. Finalize MPI, if used

The user-callable functions that initialize or re-initialize the KINBBDPRE preconditioner module are described next.

int **KINBBDPrecInit**(void \*kin\_mem, sunindextype Nlocal, sunindextype mudq, sunindextype mldq, sunindextype mukeep, sunindextype mlkeep, sunrealtype dq\_rel\_u, KINBBDLocalFn Gloc, KINBBDCommFn Gcomm)

The function KINBBDPrecInit() initializes and allocates memory for the KINBBDPRE preconditioner.

## **Arguments:**

- kin\_mem pointer to the KINSOL memory block.
- Nlocal local vector length.
- mudq upper half-bandwidth to be used in the difference-quotient Jacobian approximation.
- mldq lower half-bandwidth to be used in the difference-quotient Jacobian approximation.
- mukeep upper half-bandwidth of the retained banded approximate Jacobian block.
- mlkeep lower half-bandwidth of the retained banded approximate Jacobian block.
- $dq_rel_u$  the relative increment in components of u used in the difference quotient approximations. The default is  $dq_rel_u = \sqrt{unit \ roundoff}$ , which can be specified by passing  $dq_rel_u = 0.0$ .
- Gloc the CC function which computes the approximation  $G(u) \approx F(u)$ .

• Gcomm – the optional CC function which performs all interprocess communication required for the computation of G(u).

#### Return value:

- KINLS\_SUCCESS The call to KINBBDPrecInit() was successful.
- KINLS\_MEM\_NULL The kin\_mem pointer was NULL.
- KINLS\_MEM\_FAIL A memory allocation request has failed.
- KINLS\_LMEM\_NULL The KINLS linear solver interface has not been initialized.
- KINLS\_ILL\_INPUT The supplied vector implementation was not compatible with the block band preconditioner.

#### Notes:

If one of the half-bandwidths mudq or mldq to be used in the difference-quotient calculation of the approximate Jacobian is negative or exceeds the value Nlocal-1, it is replaced with 0 or Nlocal-1 accordingly.

The half-bandwidths mudq and mldq need not be the true half-bandwidths of the Jacobian of the local block of G, when smaller values may provide greater efficiency.

Also, the half-bandwidths mukeep and mlkeep of the retained banded approximate Jacobian block may be even smaller, to reduce storage and computation costs further.

For all four half-bandwidths, the values need not be the same for every process.

The following two optional output functions are available for use with the KINBBDPRE module:

#### int **KINBBDPrecGetWorkSpace**(void \*kin mem, long int \*lenrwBBDP, long int \*leniwBBDP)

The function KINBBDPrecGetWorkSpace() returns the local sizes of the KINBBDPRE real and integer workspaces.

## **Arguments:**

- kin\_mem pointer to the KINSOL solver object.
- lenrwBBDP local number of real values in the KINBBDPRE workspace.
- leniwBBDP local number of integer values in the KINBBDPRE workspace.

#### **Return value:**

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer was NULL.
- KINLS\_PMEM\_NULL The KINBBDPRE preconditioner has not been initialized.

#### **Notes:**

The workspace requirements reported by this routine correspond only to memory allocated within the KINBBDPRE module (the banded matrix approximation, banded SUNLinearSolver object, temporary vectors). These values are local to each process.

The workspaces referred to here exist in addition to those given by the corresponding <code>KINGetLin-WorkSpace()</code> function.

## int **KINBBDPrecGetNumGfnEvals**(void \*kin\_mem, long int \*ngevalsBBDP)

The function <code>KINBBDPrecGetNumGfnEvals()</code> returns the cumulative number of calls to the user <code>Gres</code> function due to the finite difference approximation of the Jacobian blocks used within KINBBDPRE's preconditioner setup function.

#### **Arguments:**

• kin\_mem – pointer to the KINSOL solver object.

• ngevalsBBDP – the cumulative number of calls to the user Gres function.

#### **Return value:**

- KINLS\_SUCCESS The optional output value has been successfully set.
- KINLS\_MEM\_NULL The kin\_mem pointer was NULL.
- KINLS\_PMEM\_NULL The KINBBDPRE preconditioner has not been initialized.

In addition to the ngevalsBBDP evaluations of Gres, the costs associated with KINBBDPRE also includes nlin-setups LU factorizations, nlinsetups calls to Gcomm, npsolves banded backsolve calls, and nrevalsLS residual function evaluations, where nlinsetups is an optional KINSOL output (see §5.3.5.1), and npsolves and nrevalsLS are linear solver optional outputs (see §5.3.5.2).

## 5.6 Alternative to KINSOL for difficult systems

A nonlinear system F(u) = 0 may be difficult to solve with KINSOL (or any other nonlinear system solver) for a variety of reasons. The possible reasons include high nonlinearity, small region of convergence, and lack of a good initial guess. For systems with such difficulties, there is an alternative approach that may be more successful. This is an old idea, but deserves some new attention.

If the nonlinear system is F(u) = 0, consider instead the ODE system  $du/dt = -M^{-1}F(u)$ , where M is a nonsingular matrix that is an approximation (even a crude approximation) to the system Jacobian  $F_u = dF/du$ . Whatever M is, if this ODE is solved until it reaches a steady state  $u^*$ , then  $u^*$  is a zero of the right-hand side of the ODE, and hence a solution to F(u) = 0. There is no issue of having a close enough initial guess.

A further basis for this choice of ODE is the following: If M approximates  $F_u$ , then the Jacobian of the ODE system,  $-M^{-1}F_u$ , is approximately equal to -I where I is the identity matrix. This means that (in a local approximation sense) the solution modes of the ODE behave like  $\exp(-t)$ , and that asymptotically the approach to the steady state goes as  $\exp(-t)$ . Of course, the closer M is to  $F_u$ , the better this basis applies.

Using (say) CVODE to solve the above ODE system requires, in addition to the objective function F(u), the calculation of a suitable matrix M and its inverse, or at least a routine that solves linear systems Mx=b. This is similar to the KINSOL requirement of supplying the system Jacobian J (or solutions to Jx=b), but differs in that M may be simpler than J and hence easier to deal with. Depending on the nature of M, this may be handled best with a direct solver, or with a preconditioned Krylov solver. The latter calls for the use of a preconditioner P that may be a crude approximation to M, hence even easier to solve. Note if using ARKODE, the ODE system may be posed in the linearly implicit from Mdu/dt=-F(u) where M is the "mass matrix" for the system. This use case requires supplying ARKODE with a function to evaluate M or to compute its action on a vector (Mv=w) and attaching a linear solver (direct or iterative) to solve the linear systems Mx=b.

The solution of the ODE may be made easier by solving instead the equivalent DAE, Mdu/dt + F(u) = 0. Applying IDA to this system requires solutions to linear systems whose matrix is the DAE system Jacobian,  $J = F_u + \alpha M$ , where  $\alpha$  is the scalar coefficient  $c_j$  supplied to the user's Jacobian or preconditioner routine. Selecting a preconditioned Krylov method requires an approximation to this Jacobian as preconditioner P. Given that M approximates  $F_u$  (possibly crudely), the appropriate approximation to J is  $P = M + \alpha M = (1 + \alpha)M$ . Again the user must supply a routine that solves linear systems Px = b, or  $Mx = b/(1 + \alpha)$ . If M is too difficult to solve, than an approximation M' that is easier can be substituted, as long as it achieves convergence. As always, there is a trade-off between the expense of solving M and the difficulty of achieving convergence in the linear solver.

For the solution of either the ODE or DAE system above, the chances for convergence can be improved with a piecewise constant choice for M. Specifically, starting from an initial guess  $u_0$ , an initial choice for M might be  $M_0 = F_u(u_0)$ , or some approximation to this Jacobian. Then one could integrate  $M_0 du/dt + F(u) = 0$  from t = 0 to t = T for some sizable value T, evaluate  $F_u(u(T))$ , and take  $M_1$  to be an approximation to that Jacobian. Then integrate using  $M_1$  from t = T to t = 2T, and repeat the process until it converges to a steady state.

# **Chapter 6**

# **Vector Data Structures**

The SUNDIALS library comes packaged with a variety of NVECTOR implementations, designed for simulations in serial, shared-memory parallel, and distributed-memory parallel environments, as well as interfaces to vector data structures used within external linear solver libraries. All native implementations assume that the process-local data is stored contiguously, and they in turn provide a variety of standard vector algebra operations that may be performed on the data.

In addition, SUNDIALS provides a simple interface for generic vectors (akin to a C++ abstract base class). All of the major SUNDIALS solvers (CVODE(s), IDA(s), KINSOL, ARKODE) in turn are constructed to only depend on these generic vector operations, making them immediately extensible to new user-defined vector objects. The only exceptions to this rule relate to the dense, banded and sparse-direct linear system solvers, since they rely on particular data storage and access patterns in the NVECTORS used.

# **6.1 Description of the NVECTOR Modules**

SUNDIALS solvers are written in a data-independent manner. They all operate on generic vectors (of type *N\_Vector*) through a set of operations defined by, and specific to, the particular vector implementation. Users can provide a custom vector implementation or use one provided with SUNDIALS. The generic operations are described below. In the sections following, the implementations provided with SUNDIALS are described.

```
An N_Vector is a pointer to the _generic_N_Vector structure:
```

typedef struct <u>\_generic\_N\_Vector</u> \*N\_Vector

#### struct **\_generic\_N\_Vector**

The structure defining the SUNDIALS vector class.

void \*content

Pointer to vector-specific member data.

N\_Vector\_Ops ops

A virtual table of vector operations provided by a specific implementation.

SUNContext sunctx

The SUNDIALS simulation context

The virtual table structure is defined as

typedef generic N Vector Ops \*N\_Vector\_Ops

```
struct _generic_N_Vector_Ops
     The structure defining N_{-}Vector operations.
     N Vector ID (*nvgetvectorid)(N Vector)
          The function implementing N_VGetVectorID()
     N Vector (*nvclone)(N Vector)
          The function implementing N_{VClone}()
     N Vector (*nvcloneempty)(N Vector)
          The function implementing N_VCloneEmpty()
     void (*nvdestroy)(N_Vector)
          The function implementing N_VDestroy()
     void (*nvspace)(N_Vector, sunindextype*, sunindextype*)
          The function implementing N_{VSpace}
     sunrealtype *(*nvgetarraypointer)(N_Vector)
          The function implementing N_VGetArrayPointer()
     sunrealtype *(*nvgetdevicearraypointer)(N_Vector)
          The function implementing N_VGetDeviceArrayPointer()
     void (*nvsetarraypointer)(sunrealtype*, N_Vector)
          The function implementing N_VSetArrayPointer()
     SUNComm (*nvgetcommunicator)(N Vector)
          The function implementing N_VGetCommunicator()
     sunindextype (*nvgetlength)(N_Vector)
          The function implementing N_VGetLength()
     sunindextype (*nvgetlocallength)(N_Vector)
          The function implementing N_VGetLocalLength()
     void (*nvlinearsum)(sunrealtype, N_Vector, sunrealtype, N_Vector, N_Vector)
          The function implementing N_VLinearSum()
     void (*nvconst)(sunrealtype, N_Vector)
          The function implementing N_VConst()
     void (*nvprod)(N_Vector, N_Vector, N_Vector)
          The function implementing N_{-}VProd()
     void (*nvdiv)(N_Vector, N_Vector, N_Vector)
          The function implementing N_{VDiv}
     void (*nvscale)(sunrealtype, N_Vector, N_Vector)
          The function implementing N_{VScale}
     void (*nvabs)(N_Vector, N_Vector)
          The function implementing N_{VAbs}
     void (*nvinv)(N_Vector, N_Vector)
          The function implementing N_{-}VInv()
```

```
void (*nvaddconst)(N Vector, sunrealtype, N Vector)
     The function implementing N_VAddConst()
sunrealtype (*nvdotprod)(N Vector, N Vector)
     The function implementing N_{VDotProd}()
sunrealtype (*nvmaxnorm)(N Vector)
     The function implementing N_{VMaxNorm}
sunrealtype (*nvwrmsnorm)(N Vector, N Vector)
     The function implementing N_{VWrmsNorm}()
sunrealtype (*nvwrmsnormmask)(N_Vector, N_Vector, N_Vector)
     The function implementing N_VWrmsNormMask()
sunrealtype (*nvmin)(N_Vector)
     The function implementing N_{VMin}
sunrealtype (*nvwl2norm)(N_Vector, N_Vector)
     The function implementing N_{VWL2Norm}()
sunrealtype (*nvl1norm)(N_Vector)
     The function implementing N_{VL1Norm}()
void (*nvcompare)(sunrealtype, N_Vector, N_Vector)
     The function implementing N_{VCompare}
sunbooleantype (*nvinvtest)(N_Vector, N_Vector)
     The function implementing N_VInvTest()
sunbooleantype (*nvconstrmask)(N_Vector, N_Vector, N_Vector)
     The function implementing N_VConstrMask()
sunrealtype (*nvminquotient)(N_Vector, N_Vector)
     The function implementing N_VMinQuotient()
SUNErrCode (*nvlinearcombination)(int, sunrealtype*, N_Vector*, N_Vector)
     The function implementing N_VLinearCombination()
SUNErrCode (*nvscaleaddmulti)(int, sunrealtype*, N_Vector, N_Vector*, N_Vector*)
     The function implementing N_VScaleAddMulti()
SUNErrCode (*nvdotprodmulti)(int, N_Vector, N_Vector*, sunrealtype*)
     The function implementing N_VDotProdMulti()
SUNErrCode (*nvlinearsumvectorarray)(int, sunrealtype, N_Vector*, sunrealtype, N_Vector*,
N Vector*)
     The function implementing N_VLinearSumVectorArray()
SUNErrCode (*nvscalevectorarray)(int, sunrealtype*, N Vector*, N Vector*)
     The function implementing N_VScaleVectorArray()
SUNErrCode (*nvconstvectorarray)(int, sunrealtype, N Vector*)
     The function implementing N_VConstVectorArray()
SUNErrCode (*nvwrmsnormvectorarray)(int, N Vector*, N Vector*, sunrealtype*)
     The function implementing N_VWrmsNormVectorArray()
```

```
SUNErrCode (*nvwrmsnormmaskvectorarray)(int, N Vector*, N Vector*, N Vector, sunrealtype*)
    The function implementing N_VWrmsNormMaskVectorArray()
SUNErrCode (*nvscaleaddmultivectorarray)(int, int, sunrealtype*, N Vector*, N Vector**,
N Vector**)
    The function implementing N_VScaleAddMultiVectorArray()
SUNErrCode (*nvlinearcombinationvectorarray)(int, int, sunrealtype*, N Vector**, N Vector*)
    The function implementing N_VLinearCombinationVectorArray()
sunrealtype (*nvdotprodlocal)(N Vector, N Vector)
    The function implementing N_VDotProdLocal()
sunrealtype (*nvmaxnormlocal)(N Vector)
    The function implementing N_VMaxNormLocal()
sunrealtype (*nvminlocal)(N Vector)
    The function implementing N_VMinLocal()
sunrealtype (*nvl1normlocal)(N_Vector)
    The function implementing N_VL1NormLocal()
sunbooleantype (*nvinvtestlocal)(N Vector, N Vector)
    The function implementing N_{VInvTestLocal} ()
sunbooleantype (*nvconstrmasklocal)(N Vector, N Vector, N Vector)
    The function implementing N_VConstrMaskLocal()
sunrealtype (*nvminquotientlocal)(N Vector, N Vector)
    The function implementing N_VMinQuotientLocal()
sunrealtype (*nvwsqrsumlocal)(N Vector, N Vector)
    The function implementing N_VWSqrSumLocal()
sunrealtype (*nvwsqrsummasklocal)(N_Vector, N_Vector, N_Vector)
    The function implementing N_VWSqrSumMaskLocal()
SUNErrCode (*nvdotprodmultilocal)(int, N Vector, N Vector*, sunrealtype*)
    The function implementing N_VDotProdMultiLocal()
SUNErrCode (*nvdotprodmultiallreduce)(int, N_Vector, sunrealtype*)
    The function implementing N_VDotProdMultiAllReduce()
SUNErrCode (*nvbufsize)(N Vector, sunindextype*)
    The function implementing N_VBufSize()
SUNErrCode (*nvbufpack)(N Vector, void*)
    The function implementing N_VBufPack()
SUNErrCode (*nvbufunpack)(N Vector, void*)
    The function implementing N_VBufUnpack()
void (*nvprint)(N Vector)
    The function implementing N_{-}VPrint()
```

```
void (*nvprintfile)(N_Vector, FILE*)
```

The function implementing *N\_VPrintFile()* 

The generic NVECTOR module defines and implements the vector operations acting on a N\_Vector. These routines are nothing but wrappers for the vector operations defined by a particular NVECTOR implementation, which are accessed through the ops field of the N\_Vector structure. To illustrate this point we show below the implementation of a typical vector operation from the generic NVECTOR module, namely N\_VScale, which performs the operation  $z \leftarrow cx$  for vectors x and z and a scalar c:

```
void N_VScale(sunrealtype c, N_Vector x, N_Vector z) {
  z->ops->nvscale(c, x, z);
}
```

§6.2 contains a complete list of all standard vector operations defined by the generic NVECTOR module. §6.2.2, §6.2.3, §6.2.4, §6.2.5, and §6.2.6 list *optional* fused, vector array, local reduction, single buffer reduction, and exchange operations, respectively.

Fused and vector array operations (see §6.2.2 and §6.2.3) are intended to increase data reuse, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. If a particular NVECTOR implementation defines a fused or vector array operation as NULL, the generic NVECTOR module will automatically call standard vector operations as necessary to complete the desired operation. In all SUNDIALS-provided NVECTOR implementations, all fused and vector array operations are disabled by default. However, these implementations provide additional user-callable functions to enable/disable any or all of the fused and vector array operations. See the following sections for the implementation specific functions to enable/disable operations.

Local reduction operations (see §6.2.4) are similarly intended to reduce parallel communication on distributed memory systems, particularly when NVECTOR objects are combined together within an NVECTOR\_MANYVECTOR object (see §6.17). If a particular NVECTOR implementation defines a local reduction operation as NULL, the NVECTOR\_MANYVECTOR module will automatically call standard vector reduction operations as necessary to complete the desired operation. All SUNDIALS-provided NVECTOR implementations include these local reduction operations, which may be used as templates for user-defined implementations.

The single buffer reduction operations (§6.2.5) are used in low-synchronization methods to combine separate reductions into one MPI\_Allreduce call.

The exchange operations (see §6.2.6) are intended only for use with the XBraid library for parallel-in-time integration (accessible from ARKODE) and are otherwise unused by SUNDIALS packages.

## **6.1.1 NVECTOR Utility Functions**

The generic NVECTOR module also defines several utility functions to aid in creation and management of arrays of N\_Vector objects – these functions are particularly useful for Fortran users to utilize the NVECTOR\_MANYVECTOR or SUNDIALS' sensitivity-enabled packages CVODES and IDAS.

The functions *N\_VCloneVectorArray()* and *N\_VCloneVectorArrayEmpty()* create (by cloning) an array of *count* variables of type *N\_Vector*, each of the same type as an existing *N\_Vector* input:

```
N_Vector *N_VCloneVectorArray(int count, N_Vector w)
```

Clones an array of count  $N_Vector$  objects, allocating their data arrays (similar to  $N_Vclone()$ ).

#### **Arguments:**

- count number of N\_Vector objects to create.
- w template *N\_Vector* to clone.

## **Return value:**

• pointer to a new N\_Vector array on success.

• NULL pointer on failure.

#### N Vector \*N\_VCloneVectorArrayEmpty(int count, N Vector w)

Clones an array of count  $N_{\text{vector}}$  objects, leaving their data arrays unallocated (similar to  $N_{\text{vector}}$  *VCloneEmpty()*).

#### **Arguments:**

- count number of N\_Vector objects to create.
- w template *N\_Vector* to clone.

#### Return value:

- pointer to a new N\_Vector array on success.
- NULL pointer on failure.

An array of variables of type *N\_Vector* can be destroyed by calling *N\_VDestroyVectorArray()*:

## void N\_VDestroyVectorArray(N\_Vector \*vs, int count)

Destroys an array of count N\_Vector objects.

#### **Arguments:**

- vs N\_Vector array to destroy.
- count number of N\_Vector objects in vs array.

#### Notes:

This routine will internally call the N\_Vector implementation-specific N\_VDestroy() operation.

If vs was allocated using  $N\_VCloneVectorArray()$  then the data arrays for each  $N\_Vector$  object will be freed; if vs was allocated using  $N\_VCloneVectorArrayEmpty()$  then it is the user's responsibility to free the data for each  $N\_Vector$  object.

Finally, we note that users of the Fortran 2003 interface may be interested in the additional utility functions *N\_VNewVectorArray()*, *N\_VGetVecAtIndexVectorArray()*, and *N\_VSetVecAtIndexVectorArray()*, that are wrapped as FN\_NewVectorArray, FN\_VGetVecAtIndexVectorArray, and FN\_VSetVecAtIndexVectorArray, respectively. These functions allow a Fortran 2003 user to create an empty vector array, access a vector from this array, and set a vector within this array:

## *N\_Vector* \***N\_VNewVectorArray**(int count, *SUNContext* sunctx)

Creates an array of count N\_Vector objects, the pointers to each are initialized as NULL.

## **Arguments:**

- $count length of desired N\_Vector array.$
- sunctx a SUNContext object

## **Return value:**

- pointer to a new N\_Vector array on success.
- NULL pointer on failure.

Changed in version 7.0.0: The function signature was updated to add the SUNContext argument.

## *N\_Vector* \*N\_VGetVecAtIndexVectorArray(*N\_Vector* \*vs, int index)

Accesses the N\_Vector at the location index within the N\_Vector array vs.

## **Arguments:**

• vs - N\_Vector array.

• index – desired N\_Vector to access from within vs.

#### **Return value:**

- pointer to the indexed N\_Vector on success.
- NULL pointer on failure (index < 0 or vs == NULL).

#### Notes:

This routine does not verify that index is within the extent of vs, since vs is a simple N\_Vector array that does not internally store its allocated length.

## void **N\_VSetVecAtIndexVectorArray**(*N\_Vector* \*vs, int index, *N\_Vector* w)

Sets a pointer to w at the location index within the vector array vs.

## **Arguments:**

- vs N\_Vector array.
- index desired location to place the pointer to w within vs.
- w N\_Vector to set within vs.

#### **Notes:**

This routine does not verify that index is within the extent of vs, since vs is a simple N\_Vector array that does not internally store its allocated length.

## 6.1.2 Implementing a custom NVECTOR

A particular implementation of the NVECTOR module must:

- Specify the *content* field of the N\_Vector structure.
- Define and implement the vector operations. Note that the names of these routines should be unique to that implementation in order to permit using more than one NVECTOR module (each with different N\_Vector internal data representations) in the same code.
- Define and implement user-callable constructor and destructor routines to create and free an N\_Vector with the new *content* field and with *ops* pointing to the new vector operations.
- Optionally, define and implement additional user-callable routines acting on the newly-defined N\_Vector (e.g., a routine to print the content for debugging purposes).
- Optionally, provide accessor macros as needed for that particular implementation to be used to access different parts in the *content* field of the newly-defined N\_Vector.

To aid in the creation of custom NVECTOR modules, the generic NVECTOR module provides two utility functions  $N_-$  VNewEmpty() and  $N_-VCopyOps()$ . When used in custom NVECTOR constructors and clone routines these functions will ease the introduction of any new optional vector operations to the NVECTOR API by ensuring that only required operations need to be set, and that all operations are copied when cloning a vector.

## *N\_Vector* **N\_VNewEmpty**(*SUNContext* sunctx)

This allocates a new generic N\_Vector object and initializes its content pointer and the function pointers in the operations structure to NULL.

**Return value:** If successful, this function returns an N\_Vector object. If an error occurs when allocating the object, then this routine will return NULL.

## void N\_VFreeEmpty(N\_Vector v)

This routine frees the generic N\_Vector object, under the assumption that any implementation-specific data that was allocated within the underlying content structure has already been freed. It will additionally test whether the ops pointer is NULL, and, if it is not, it will free it as well.

#### **Arguments:**

• v – an N\_Vector object

## SUNErrCode N\_VCopyOps(N\_Vector w, N\_Vector v)

This function copies the function pointers in the ops structure of w into the ops structure of v.

## **Arguments:**

- w the vector to copy operations from
- v the vector to copy operations to

Return value: Returns a SUNErrCode.

## enum N\_Vector\_ID

Each *N\_Vector* implementation included in SUNDIALS has a unique identifier specified in enumeration and shown in Table 6.1. It is recommended that a user supplied NVECTOR implementation use the SUNDIALS\_-NVEC\_CUSTOM identifier.

Table 6.1: Vector Identifications associated with vector kernels supplied with SUNDIALS

Vector ID	Vector type	ID Value
SUNDIALS_NVEC_SERIAL	Serial	0
SUNDIALS_NVEC_PARALLEL	Distributed memory parallel (MPI)	1
SUNDIALS_NVEC_OPENMP	OpenMP shared memory parallel	2
SUNDIALS_NVEC_PTHREADS	PThreads shared memory parallel	3
SUNDIALS_NVEC_PARHYP	hypre ParHyp parallel vector	4
SUNDIALS_NVEC_PETSC	PETSc parallel vector	5
SUNDIALS_NVEC_CUDA	CUDA vector	6
SUNDIALS_NVEC_HIP	HIP vector	7
SUNDIALS_NVEC_SYCL	SYCL vector	8
SUNDIALS_NVEC_RAJA	RAJA vector	9
SUNDIALS_NVEC_OPENMPDEV	OpenMP vector with device offloading	10
SUNDIALS_NVEC_TRILINOS	Trilinos Tpetra vector	11
SUNDIALS_NVEC_MANYVECTOR	"Many Vector" vector	12
SUNDIALS_NVEC_MPIMANYVECTOR	MPI-enabled "Many Vector" vector	13
SUNDIALS_NVEC_MPIPLUSX	MPI+X vector	14
SUNDIALS_NVEC_CUSTOM	User-provided custom vector	15

## **6.1.3** Support for complex-valued vectors

While SUNDIALS itself is written under an assumption of real-valued data, it does provide limited support for complex-valued problems. However, since none of the built-in NVECTOR modules supports complex-valued data, users must provide a custom NVECTOR implementation for this task. Many of the NVECTOR routines described in the subsection §6.2 naturally extend to complex-valued vectors; however, some do not. To this end, we provide the following guidance:

- N\_VMin() and N\_VMinLocal() should return the minimum of all real components of the vector, i.e.,  $m = \min_{0 \le i < n} \operatorname{real}(x_i)$ .
- $N\_VConst()$  (and similarly  $N\_VConstVectorArray()$ ) should set the real components of the vector to the input constant, and set all imaginary components to zero, i.e.,  $z_i = c + 0j$  for  $0 \le i < n$ .
- N\_VAddConst() should only update the real components of the vector with the input constant, leaving all imaginary components unchanged.

- N\_VWrmsNorm(), N\_VWrmsNormMask(), N\_VWSqrSumLocal() and N\_VWSqrSumMaskLocal() should assume that all entries of the weight vector w and the mask vector id are real-valued.
- N\_VDotProd() should mathematically return a complex number for complex-valued vectors; as this is not possible with SUNDIALS' current sunrealtype, this routine should be set to NULL in the custom NVECTOR implementation.
- N\_VCompare(), N\_VConstrMask(), N\_VMinQuotient(), N\_VConstrMaskLocal() and N\_VMinQuotient-Local() are ill-defined due to the lack of a clear ordering in the complex plane. These routines should be set to NULL in the custom NVECTOR implementation.

While many SUNDIALS solver modules may be utilized on complex-valued data, others cannot. Specifically, although each package's linear solver interface (e.g., ARKLS or CVLS) may be used on complex-valued problems, none of the built-in SUNMatrix or SUNLinearSolver modules will work (all of the direct linear solvers must store complex-valued data, and all of the iterative linear solvers require  $N_{-}VDotProd()$ ). Hence a complex-valued user must provide custom linear solver modules for their problem. At a minimum this will consist of a custom SUNLinearSolver implementation (see §8.1.8), and optionally a custom SUNMatrix as well. The user should then attach these modules as normal to the package's linear solver interface.

Finally, constraint-handling features of each package cannot be used for complex-valued data, due to the issue of ordering in the complex plane discussed above with N\_VCompare(), N\_VConstrMask(), N\_VMinQuotient(), N\_VConstrMaskLocal() and N\_VMinQuotientLocal().

We provide a simple example of a complex-valued example problem, including a custom complex-valued Fortran 2003 NVECTOR module, in the files examples/arkode/F2003\_custom/ark\_analytic\_complex\_f2003.f90, examples/arkode/F2003\_custom/fnvector\_complex\_mod.f90, and examples/arkode/F2003\_custom/test\_fnvector\_complex\_mod.f90.

# **6.2 Description of the NVECTOR operations**

## **6.2.1** Standard vector operations

The standard vector operations defined by the generic N\_Vector module are defined as follows. For each of these operations, we give the name, usage of the function, and a description of its mathematical operations below.

```
N_Vector_ID N_VGetVectorID(N_Vector w)
```

Returns the vector type identifier for the vector  $\mathbf{w}$ . It is used to determine the vector implementation type (e.g. serial, parallel, ...) from the abstract  $\mathbf{N}$ \_Vector interface. Returned values are given in Table 6.1.

Usage:

```
id = N_VGetVectorID(w);
```

## *N\_Vector* **N\_VClone**(*N\_Vector* w)

Creates a new  $N_{vector}$  of the same type as an existing vector w and sets the ops field. It does not copy the vector, but rather allocates storage for the new vector.

Usage:

```
v = N_VClone(w);
```

## N\_Vector N\_VCloneEmpty(N\_Vector w)

Creates a new  $N_{vector}$  of the same type as an existing vector w and sets the ops field. It does not allocate storage for the new vector's data.

Usage:

```
v = N VCloneEmpty(w);
```

#### void N\_VDestroy(N Vector v)

Destroys the N\_Vector v and frees memory allocated for its internal data.

Usage:

```
N_VDestroy(v);
```

void **N\_VSpace**(*N\_Vector* v, *sunindextype* \*lrw, *sunindextype* \*liw)

Returns storage requirements for the N\_Vector *v*:

- *lrw* contains the number of sunrealtype words
- *liw* contains the number of integer words.

This function is advisory only, for use in determining a user's total space requirements; it could be a dummy function in a user-supplied NVECTOR module if that information is not of interest.

Usage:

```
N_VSpace(nvSpec, &lrw, &liw);
```

#### sunrealtype \*N\_VGetArrayPointer(N\_Vector v)

Returns a pointer to a sunrealtype array from the N\_Vector v. Note that this assumes that the internal data in the N\_Vector is a contiguous array of sunrealtype and is accesible from the CPU.

This routine is only used in the solver-specific interfaces to the dense and banded (serial) linear solvers, and in the interfaces to the banded (serial) and band-block-diagonal (parallel) preconditioner modules provided with SUNDIALS.

Usage:

```
vdata = N_VGetArrayPointer(v);
```

## sunrealtype \*N\_VGetDeviceArrayPointer(N\_Vector v)

Returns a device pointer to a sunrealtype array from the N\_Vector v. Note that this assumes that the internal data in N\_Vector is a contiguous array of sunrealtype and is accessible from the device (e.g., GPU).

This operation is optional except when using the GPU-enabled direct linear solvers.

Usage:

```
vdata = N_VGetArrayPointer(v);
```

```
void N_VSetArrayPointer(sunrealtype *vdata, N_Vector v)
```

Replaces the data array pointer in an N\_Vector with a given array of sunrealtype. Note that this assumes that the internal data in the N\_Vector is a contiguous array of sunrealtype. This routine is only used in the interfaces to the dense (serial) linear solver, hence need not exist in a user-supplied NVECTOR module.

Usage:

```
N_VSetArrayPointer(vdata,v);
```

## SUNComm N\_VGetCommunicator(N\_Vector v)

Returns the *SUNComm* (which is just an MPI\_Comm when SUNDIALS is built with MPI, otherwise it is an int) associated with the vector (if applicable). For MPI-unaware vector implementations, this should return SUN\_-COMM\_NULL.

Usage:

```
MPI_Comm comm = N_VGetCommunicator(v); // Works if MPI is enabled
int comm = N_VGetCommunicator(v); // Works if MPI is disabled
SUNComm comm = N_VGetCommunicator(v); // Works with or without MPI
```

## sunindextype N\_VGetLength(N\_Vector v)

Returns the global length (number of "active" entries) in the NVECTOR  $\nu$ . This value should be cumulative across all processes if the vector is used in a parallel environment. If  $\nu$  contains additional storage, e.g., for parallel communication, those entries should not be included.

Usage:

```
global_length = N_VGetLength(v);
```

## sunindextype N\_VGetLocalLength(N\_Vector v)

Returns the local length (number of "active" entries) in the NVECTOR v. This value should be the length of the array returned by  $N_VGetArrayPointer()$  or  $N_VGetDeviceArrayPointer()$ .

Usage:

```
local_length = N_VGetLocalLength(v);
```

void **N\_VLinearSum**(sunrealtype a, N\_Vector x, sunrealtype b, N\_Vector y, N\_Vector z)

Performs the operation z = ax + by, where a and b are sunrealtype scalars and x and y are of type N\_Vector:

$$z_i = ax_i + by_i$$
,  $i = 0, \ldots, n-1$ .

The output vector z can be the same as either of the input vectors (x or y).

Usage:

```
N_VLinearSum(a, x, b, y, z);
```

void N\_VConst(sunrealtype c, N\_Vector z)

Sets all components of the N\_Vector z to sunrealtype c:

$$z_i = c, \quad i = 0, \dots, n - 1.$$

Usage:

N\_VConst(c, z);

void N\_VProd(N\_Vector x, N\_Vector y, N\_Vector z)

Sets the  $N_Vector z$  to be the component-wise product of the  $N_Vector inputs x$  and y:

$$z_i = x_i y_i, \quad i = 0, \dots, n - 1.$$

Usage:

void **N\_VDiv**(*N\_Vector* x, *N\_Vector* y, *N\_Vector* z)

Sets the N\_Vector *z* to be the component-wise ratio of the N\_Vector inputs *x* and *y*:

$$z_i = \frac{x_i}{y_i}, \quad i = 0, \dots, n - 1.$$

The  $y_i$  may not be tested for 0 values. It should only be called with a y that is guaranteed to have all nonzero components.

Usage:

N\_VDiv(x, y, z);

void N\_VScale(sunrealtype c, N\_Vector x, N\_Vector z)

Scales the  $N_Vector x$  by the sunreal type scalar c and returns the result in z:

$$z_i = cx_i, \quad i = 0, \dots, n - 1.$$

Usage:

N\_VScale(c, x, z);

void **N\_VAbs**(*N\_Vector* x, *N\_Vector* z)

Sets the components of the  $N_{vector}$  to be the absolute values of the components of the  $N_{vector}$  x:

$$z_i = |x_i|, \quad i = 0, \dots, n-1.$$

Usage:

 $N_VAbs(x, z);$ 

void N\_VInv(N\_Vector x, N\_Vector z)

Sets the components of the  $N_V$ ector z to be the inverses of the components of the  $N_V$ ector x:

$$z_i = \frac{1}{x_i}, \quad i = 0, \dots, n - 1.$$

This routine may not check for division by 0. It should be called only with an x which is guaranteed to have all nonzero components.

Usage:

N\_VInv(x, z);

void N\_VAddConst(N\_Vector x, sunrealtype b, N\_Vector z)

Adds the sunrealtype scalar b to all components of x and returns the result in the N\_Vector z:

$$z_i = x_i + b, \quad i = 0, \dots, n - 1.$$

Usage:

N\_VAddConst(x, b, z);

sunrealtype N\_VDotProd(N\_Vector x, N\_Vector z)

Returns the value of the dot-product of the vectors *x* and *y*:

$$d = \sum_{i=0}^{n-1} x_i y_i.$$

Usage:

d = N\_VDotProd(x, y);

## sunrealtype N\_VMaxNorm(N\_Vector x)

Returns the value of the  $l_{\infty}$  norm of the N\_Vector x:

$$m = \max_{0 \le i \le n} |x_i|.$$

Usage:

m = N\_VMaxNorm(x);

## sunrealtype N\_VWrmsNorm(N\_Vector x, N\_Vector w)

Returns the weighted root-mean-square norm of the N\_Vector x with (positive) sunrealtype weight vector w:

$$m = \sqrt{\left(\sum_{i=0}^{n-1} (x_i w_i)^2\right)/n}$$

Usage:

 $m = N_VWrmsNorm(x, w);$ 

## sunrealtype N\_VWrmsNormMask(N\_Vector x, N\_Vector w, N\_Vector id)

Returns the weighted root mean square norm of the N\_Vector *x* with sunrealtype weight vector *w* built using only the elements of *x* corresponding to positive elements of the N\_Vector *id*:

$$m = \sqrt{\left(\sum_{i=0}^{n-1} (x_i w_i H(id_i))^2\right)/n},$$

where 
$$H(\alpha) = \begin{cases} 1 & \alpha > 0 \\ 0 & \alpha \le 0 \end{cases}$$
.

Usage:

m = N\_VWrmsNormMask(x, w, id);

## sunrealtype N\_VMin(N\_Vector x)

Returns the smallest element of the N\_Vector *x*:

$$m = \min_{0 \le i < n} x_i.$$

Usage:

 $m = N_VMin(x);$ 

## sunrealtype N\_VWL2Norm(N\_Vector x, N\_Vector w)

Returns the weighted Euclidean  $l_2$  norm of the N\_Vector x with sunreal type weight vector w:

$$m = \sqrt{\sum_{i=0}^{n-1} (x_i w_i)^2}.$$

Usage:

 $m = N_VWL2Norm(x, w);$ 

sunrealtype N\_VL1Norm(N\_Vector x)

Returns the  $l_1$  norm of the N\_Vector x:

$$m = \sum_{i=0}^{n-1} |x_i|.$$

Usage:

 $m = N_VL1Norm(x);$ 

void N\_VCompare(sunrealtype c, N\_Vector x, N\_Vector z)

Compares the components of the N\_Vector x to the sunrealtype scalar c and returns an N\_Vector z such that for all  $0 \le i < n$ ,

$$z_i = \begin{cases} 1.0 & \text{if } |x_i| \ge c, \\ 0.0 & \text{otherwise} \end{cases}.$$

Usage:

N\_VCompare(c, x, z);

sunbooleantype N\_VInvTest(N\_Vector x, N\_Vector z)

Sets the components of the  $N_{vector}$  to be the inverses of the components of the  $N_{vector}$ , with prior testing for zero values:

$$z_i = \frac{1}{x_i}, \quad i = 0, \dots, n - 1.$$

This routine returns a boolean assigned to SUNTRUE if all components of x are nonzero (successful inversion) and returns SUNFALSE otherwise.

Usage:

t = N\_VInvTest(x, z);

sunbooleantype N\_VConstrMask(N\_Vector c, N\_Vector x, N\_Vector m)

Performs the following constraint tests based on the values in  $c_i$ :

There is no constraint on  $x_i$  if  $c_i = 0$ . This routine returns a boolean assigned to SUNFALSE if any element failed the constraint test and assigned to SUNTRUE if all passed. It also sets a mask vector m, with elements equal to 1.0 where the constraint test failed, and 0.0 where the test passed. This routine is used only for constraint checking.

Usage:

t = N\_VConstrMask(c, x, m);

*sunrealtype* **N\_VMinQuotient**(*N\_Vector* num, *N\_Vector* denom)

This routine returns the minimum of the quotients obtained by termwise dividing the elements of n by the elements in d:

$$\min_{0 \le i < n} \frac{\mathsf{num}_i}{\mathsf{denom}_i}.$$

A zero element in *denom* will be skipped. If no such quotients are found, then the large value SUN\_BIG\_REAL (defined in the header file sundials\_types.h) is returned.

Usage:

minq = N\_VMinQuotient(num, denom);

## **6.2.2** Fused operations

The following fused vector operations are *optional*. These operations are intended to increase data reuse, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. If a particular NVECTOR implementation defines one of the fused vector operations as NULL, the NVECTOR interface will call one of the above standard vector operations as necessary. As above, for each operation, we give the name, usage of the function, and a description of its mathematical operations below.

SUNErrCode N\_VLinearCombination(int nv, sunrealtype \*c, N\_Vector \*X, N\_Vector z)

This routine computes the linear combination of nv vectors with n elements:

$$z_i = \sum_{j=0}^{nv-1} c_j x_{j,i}, \quad i = 0, \dots, n-1,$$

where c is an array of nv scalars,  $x_j$  is a vector in the vector array X, and z is the output vector. If the output vector z is one of the vectors in X, then it must be the first vector in the vector array. The operation returns a SUNErrCode.

Usage:

retval = N\_VLinearCombination(nv, c, X, z);

SUNErrCode N\_VScaleAddMulti(int nv, sunrealtype \*c, N\_Vector x, N\_Vector \*Y, N\_Vector \*Z)

This routine scales and adds one vector to nv vectors with n elements:

$$z_{j,i} = c_j x_i + y_{j,i}, \quad j = 0, \dots, nv - 1 \quad i = 0, \dots, n - 1,$$

where c is an array of scalars, x is a vector,  $y_j$  is a vector in the vector array Y, and  $z_j$  is an output vector in the vector array Z. The operation returns a SUNErrCode.

Usage:

retval = N\_VScaleAddMulti(nv, c, x, Y, Z);

SUNErrCode N\_VDotProdMulti(int nv, N\_Vector x, N\_Vector \*Y, sunrealtype \*d)

This routine computes the dot product of a vector with nv vectors having n elements:

$$d_j = \sum_{i=0}^{n-1} x_i y_{j,i}, \quad j = 0, \dots, nv - 1,$$

where d is an array of scalars containing the computed dot products, x is a vector, and  $y_j$  is a vector the vector array Y. The operation returns a SUNErrCode.

Usage:

retval = N\_VDotProdMulti(nv, x, Y, d);

## 6.2.3 Vector array operations

The following vector array operations are also *optional*. As with the fused vector operations, these are intended to increase data reuse, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. If a particular NVECTOR implementation defines one of the fused or vector array operations as NULL, the NVECTOR interface will call one of the above standard vector operations as necessary. As above, for each operation, we give the name, usage of the function, and a description of its mathematical operations below.

SUNErrCode N\_VLinearSumVectorArray(int nv, sunrealtype a, N\_Vector X, sunrealtype b, N\_Vector \*Y, N\_Vector \*Z)

This routine computes the linear sum of two vector arrays of nv vectors with n elements:

$$z_{i,i} = ax_{i,i} + by_{i,i}, \quad i = 0, \dots, n-1 \quad j = 0, \dots, nv-1,$$

where a and b are scalars,  $x_j$  and  $y_j$  are vectors in the vector arrays X and Y respectively, and  $z_j$  is a vector in the output vector array Z. The operation returns a SUNErrCode.

Usage:

retval = N\_VLinearSumVectorArray(nv, a, X, b, Y, Z);

SUNErrCode N\_VScaleVectorArray(int nv, sunrealtype \*c, N\_Vector \*X, N\_Vector \*Z)

This routine scales each element in a vector of n elements in a vector array of nv vectors by a potentially different constant:

$$z_{i,i} = c_i x_{i,i}, \quad i = 0, \dots, n-1 \quad j = 0, \dots, nv-1,$$

where c is an array of scalars,  $x_j$  is a vector in the vector array X, and  $z_j$  is a vector in the output vector array Z. The operation returns a SUNErrCode.

Usage:

retval = N\_VScaleVectorArray(nv, c, X, Z);

SUNErrCode N\_VConstVectorArray(int nv, sunrealtype c, N\_Vector \*Z)

This routine sets each element in a vector of n elements in a vector array of nv vectors to the same value:

$$z_{i,i} = c, \quad i = 0, \dots, n-1 \quad j = 0, \dots, nv-1,$$

where c is a scalar and  $z_i$  is a vector in the vector array Z. The operation returns a SUNErrCode.

Usage:

retval = N\_VConstVectorArray(nv, c, Z);

SUNErrCode N\_VWrmsNormVectorArray(int nv, N\_Vector \*X, N\_Vector \*W, sunrealtype \*m)

This routine computes the weighted root mean square norm of each vector in a vector array:

$$m_j = \left(\frac{1}{n}\sum_{i=0}^{n-1} (x_{j,i}w_{j,i})^2\right)^{1/2}, \quad j = 0, \dots, nv - 1,$$

where  $x_j$  is a vector in the vector array X,  $w_j$  is a weight vector in the vector array W, and m is the output array of scalars containing the computed norms. The operation returns a *SUNErrCode*.

Usage:

retval = N\_VWrmsNormVectorArray(nv, X, W, m);

SUNErrCode N\_VWrmsNormMaskVectorArray(int nv, N\_Vector \*X, N\_Vector \*W, N\_Vector id, sunrealtype \*m)

This routine computes the masked weighted root mean square norm of each vector in a vector array:

$$m_j = \left(\frac{1}{n}\sum_{i=0}^{n-1} (x_{j,i}w_{j,i}H(id_i))^2\right)^{1/2}, \quad j = 0, \dots, nv - 1,$$

where  $H(id_i) = 1$  if  $id_i > 0$  and is zero otherwise,  $x_j$  is a vector in the vector array X,  $w_j$  is a weight vector in the vector array W, id is the mask vector, and m is the output array of scalars containing the computed norms. The operation returns a SUNErrCode.

Usage:

retval = N\_VWrmsNormMaskVectorArray(nv, X, W, id, m);

SUNErrCode N\_VScaleAddMultiVectorArray(int nv, int nsum, sunrealtype \*c,  $N_V$ ector \*X,  $N_V$ ector \*\*YY,  $N_V$ ector \*\*ZZ)

This routine scales and adds a vector array of nv vectors to nsum other vector arrays:

$$z_{k,j,i} = c_k x_{j,i} + y_{k,j,i}, \quad i = 0, \dots, n-1 \quad j = 0, \dots, nv-1, \quad k = 0, \dots, nsum-1$$

where c is an array of scalars,  $x_j$  is a vector in the vector array X,  $y_{k,j}$  is a vector in the array of vector arrays YY, and  $z_{k,j}$  is an output vector in the array of vector arrays ZZ. The operation returns a SUNErrCode.

Usage:

retval = N\_VScaleAddMultiVectorArray(nv, nsum, c, x, YY, ZZ);

SUNErrCode N\_VLinearCombinationVectorArray(int nv, int nsum, sunrealtype \*c, N\_Vector \*\*XX, N\_Vector \*7)

This routine computes the linear combination of *nsum* vector arrays containing *nv* vectors:

$$z_{j,i} = \sum_{k=0}^{nsum-1} c_k x_{k,j,i}, \quad i = 0, \dots, n-1 \quad j = 0, \dots, nv-1,$$

where c is an array of scalars,  $x_{k,j}$  is a vector in array of vector arrays XX, and  $z_{j,i}$  is an output vector in the vector array Z. If the output vector array is one of the vector arrays in XX, it must be the first vector array in XX. The operation returns a SUNErrCode.

Usage:

retval = N\_VLinearCombinationVectorArray(nv, nsum, c, XX, Z);

# **6.2.4** Local reduction operations

The following local reduction operations are also *optional*. As with the fused and vector array operations, these are intended to reduce parallel communication on distributed memory systems. If a particular NVECTOR implementation defines one of the local reduction operations as NULL, the NVECTOR interface will call one of the above standard vector operations as necessary. As above, for each operation, we give the name, usage of the function, and a description of its mathematical operations below.

```
sunrealtype N_VDotProdLocal(N_Vector x, N_Vector y)
```

This routine computes the MPI task-local portion of the ordinary dot product of x and y:

$$d = \sum_{i=0}^{n_{local}-1} x_i y_i,$$

where  $n_{local}$  corresponds to the number of components in the vector on this MPI task (or  $n_{local} = n$  for MPI-unaware applications).

Usage:

d = N\_VDotProdLocal(x, y);

#### sunrealtype N\_VMaxNormLocal(N\_Vector x)

This routine computes the MPI task-local portion of the maximum norm of the NVECTOR x:

$$m = \max_{0 \le i < n_{local}} |x_i|,$$

where  $n_{local}$  corresponds to the number of components in the vector on this MPI task (or  $n_{local} = n$  for MPI-unaware applications).

Usage:

m = N\_VMaxNormLocal(x);

#### sunrealtype N\_VMinLocal(N Vector x)

This routine computes the smallest element of the MPI task-local portion of the NVECTOR x:

$$m = \min_{0 \le i \le n_{local}} x_i,$$

where  $n_{local}$  corresponds to the number of components in the vector on this MPI task (or  $n_{local} = n$  for MPI-unaware applications).

Usage:

m = N\_VMinLocal(x);

# sunrealtype N\_VL1NormLocal(N\_Vector x)

This routine computes the MPI task-local portion of the  $l_1$  norm of the N\_Vector x:

$$n = \sum_{i=0}^{n_{local}-1} |x_i|,$$

where  $n_{local}$  corresponds to the number of components in the vector on this MPI task (or  $n_{local} = n$  for MPI-unaware applications).

Usage:

n = N\_VL1NormLocal(x);

## sunrealtype N\_VWSqrSumLocal(N\_Vector x, N\_Vector w)

This routine computes the MPI task-local portion of the weighted squared sum of the NVECTOR x with weight vector w:

$$s = \sum_{i=0}^{n_{local}-1} (x_i w_i)^2,$$

where  $n_{local}$  corresponds to the number of components in the vector on this MPI task (or  $n_{local} = n$  for MPI-unaware applications).

Usage:

s = N\_VWSqrSumLocal(x, w);

#### sunrealtype N\_VWSqrSumMaskLocal(N\_Vector x, N\_Vector w, N\_Vector id)

This routine computes the MPI task-local portion of the weighted squared sum of the NVECTOR x with weight vector w built using only the elements of x corresponding to positive elements of the NVECTOR id:

$$m = \sum_{i=0}^{n_{local}-1} (x_i w_i H(id_i))^2,$$

where

$$H(\alpha) = \begin{cases} 1 & \alpha > 0 \\ 0 & \alpha \le 0 \end{cases}$$

and  $n_{local}$  corresponds to the number of components in the vector on this MPI task (or  $n_{local} = n$  for MPI-unaware applications).

Usage:

s = N\_VWSqrSumMaskLocal(x, w, id);

# sunbooleantype N\_VInvTestLocal(N\_Vector x)

This routine sets the MPI task-local components of the NVECTOR z to be the inverses of the components of the NVECTOR x, with prior testing for zero values:

$$z_i = \frac{1}{x_i}, \ i = 0, \dots, n_{local} - 1$$

where  $n_{local}$  corresponds to the number of components in the vector on this MPI task (or  $n_{local} = n$  for MPIunaware applications). This routine returns a boolean assigned to SUNTRUE if all task-local components of x are nonzero (successful inversion) and returns SUNFALSE otherwise.

Usage:

t = N\_VInvTestLocal(x);

#### sunbooleantype N\_VConstrMaskLocal(N\_Vector c, N\_Vector x, N\_Vector m)

Performs the following constraint tests based on the values in  $c_i$ :

$$\begin{array}{lllll} x_i &>& 0 & \mbox{if} & c_i = 2, \\ x_i &\geq & 0 & \mbox{if} & c_i = 1, \\ x_i &<& 0 & \mbox{if} & c_i = -2, \\ x_i &\leq & 0 & \mbox{if} & c_i = -1. \end{array}$$

for all MPI task-local components of the vectors. This routine returns a boolean assigned to SUNFALSE if any task-local element failed the constraint test and assigned to SUNTRUE if all passed. It also sets a mask vector m, with elements equal to 1.0 where the constraint test failed, and 0.0 where the test passed. This routine is used only for constraint checking.

Usage:

```
t = N_VConstrMaskLocal(c, x, m);
```

sunrealtype N\_VMinQuotientLocal(N Vector num, N Vector denom)

This routine returns the minimum of the quotients obtained by term-wise dividing  $num_i$  by  $denom_i$ , for all MPI task-local components of the vectors. A zero element in denom will be skipped. If no such quotients are found, then the large value SUN\_BIG\_REAL (defined in the header file sundials\_types.h) is returned.

Usage:

```
minq = N_VMinQuotientLocal(num, denom);
```

# **6.2.5** Single Buffer Reduction Operations

The following *optional* operations are used to combine separate reductions into a single MPI call by splitting the local computation and communication into separate functions. These operations are used in low-synchronization orthogonalization methods to reduce the number of MPI Allreduce calls. If a particular NVECTOR implementation does not define these operations additional communication will be required.

SUNErrCode N\_VDotProdMultiLocal(int nv, N\_Vector x, N\_Vector \*Y, sunrealtype \*d)

This routine computes the MPI task-local portion of the dot product of a vector x with nv vectors  $y_i$ :

$$d_j = \sum_{i=0}^{n_{local}-1} x_i y_{j,i}, \quad j = 0, \dots, nv - 1,$$

where d is an array of scalars containing the computed dot products, x is a vector,  $y_j$  is a vector in the vector array Y, and  $n_{local}$  corresponds to the number of components in the vector on this MPI task. The operation returns a SUNErrCode.

Usage:

```
retval = N_VDotProdMultiLocal(nv, x, Y, d);
```

SUNErrCode N\_VDotProdMultiAllReduce(int nv, N\_Vector x, sunrealtype \*d)

This routine combines the MPI task-local portions of the dot product of a vector x with nv vectors:

```
retval = MPI_Allreduce(MPI_IN_PLACE, d, nv, MPI_SUNREALTYPE, MPI_SUM, comm)
```

where d is an array of nv scalars containing the local contributions to the dot product and comm is the MPI communicator associated with the vector x. The operation returns a SUNErrCode.

Usage:

```
retval = N_VDotProdMultiAllReduce(nv, x, d);
```

# **6.2.6** Exchange operations

The following vector exchange operations are also *optional* and are intended only for use when interfacing with the XBraid library for parallel-in-time integration. In that setting these operations are required but are otherwise unused by SUNDIALS packages and may be set to NULL. For each operation, we give the function signature, a description of the expected behavior, and an example of the function usage.

```
SUNErrCode N_VBufSize(N_Vector x, sunindextype *size)
```

This routine returns the buffer size need to exchange in the data in the vector x between computational nodes.

Usage:

```
flag = N_VBufSize(x, &buf_size)
```

```
SUNErrCode N_VBufPack(N Vector x, void *buf)
```

This routine fills the exchange buffer buf with the vector data in x.

Usage:

```
flag = N_VBufPack(x, &buf)
```

```
SUNErrCode N_VBufUnpack(N_Vector x, void *buf)
```

This routine unpacks the data in the exchange buffer *buf* into the vector *x*.

Usage:

```
flag = N_VBufUnpack(x, buf)
```

# **6.2.7 Output operations**

The following optional vector operations are for writing vector data either to stdout or to a given file.

```
void N_VPrint(N_Vector x)
```

This routine prints vector data to stdout

Usage:

```
N_VPrint(x);
```

```
void N_VPrintFile(N_Vector x, FILE *file)
```

This routine writes vector data to the given file pointer.

Usage:

```
FILE* fp = fopen("vector_data.txt", "w");
N_VPrintFile(x, fp);
fclose(fp);
```

# **6.3 NVECTOR functions used by KINSOL**

In Table 6.2 below, we list the vector functions used in the N\_Vector module used by the KINSOL package. The table also shows, for each function, which of the code modules uses the function. The KINSOL column shows function usage within the main integrator module, while the remaining columns show function usage within the KINLS linear solvers interface, and the KINBBDPRE preconditioner module.

At this point, we should emphasize that the KINSOL user does not need to know anything about the usage of vector functions by the KINSOL code modules in order to use KINSOL. The information is presented as an implementation detail for the interested reader.

Function name	KINSOL	KINLS	KINBBDPRE
N_VGetVectorID()			
N_VGetLength()	4		
N_VClone()	X	X	
N_VCloneEmpty()			
N_VDestroy()	X	X	
N_VSpace()	X	2	
N_VGetArrayPointer()	1	X	
N_VSetArrayPointer()	1		
N_VLinearSum()	X	X	
N_VConst()	X		
N_VProd()	X	X	
N_VDiv()	X		
N_VScale()	X	X	X
N_VAbs()	X		
N_VInv()	X		
N_VDotProd()	X	X	
N_VMaxNorm()	X		
N_VMin()	X		
N_VWL2Norm()	X	X	
N_VL1Norm()	3		
N_VConstrMask()	X		
N_VMinQuotient()	X		
N_VLinearCombination()	X	X	
N_VDotProdMulti()	X		

Table 6.2: List of vector functions usage by KINSOL code modules

Special cases (numbers match markings in table):

- 1. These routines are only required if an internal difference-quotient routine for constructing *SUNMATRIX\_DENSE* or *SUNMATRIX\_BAND* Jacobian matrices is used.
- 2. This routine is optional, and is only used in estimating space requirements for IDA modules for user feedback.
- 3. These routines are only required if the internal difference-quotient routine for approximating the Jacobian-vector product is used.
- 4. This routine is only used when an iterative SUNLinearSolver module that does not support the SUNLin-SolSetScalingVectors() routine is supplied to KINSOL.

Each SUNLinearSolver object may require additional N\_Vector routines not listed in the table above. Please see the the relevant descriptions of these modules in §8 for additional detail on their N\_Vector requirements.

The vector functions listed in §6.2 that are *not* used by KINSOL are *N\_VAddConst()*, *N\_VWrmsNorm()*, *N\_VWrmsNorm()*, *N\_VWrmsNorm()*, *N\_VInvTest()*, and *N\_VGetCommunicator()*. Therefore a user-supplied *N\_Vector* module for KINSOL could omit these functions.

The optional function *N\_VLinearCombination()* is only used when Anderson acceleration is enabled or the SPBCG, SPTFQMR, SPGMR, or SPFGMR linear solvers are used. *N\_VDotProd()* is only used when Anderson acceleration is enabled or Classical Gram-Schmidt is used with SPGMR or SPFGMR. The remaining operations from §6.2.2 and §6.2.3 are unused and a user-supplied *N\_Vector* module for KINSOL could omit these operations.

# **6.4 The NVECTOR\_SERIAL Module**

The serial implementation of the NVECTOR module provided with SUNDIALS, NVECTOR\_SERIAL, defines the *content* field of an N\_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, and a boolean flag *own\_data* which specifies the ownership of data.

```
struct _N_VectorContent_Serial {
   sunindextype length;
   sunbooleantype own_data;
   sunrealtype *data;
};
```

The header file to be included when using this module is nvector\_serial.h. The installed module library to link to is libsundials\_nvecserial.lib where .lib is typically .so for shared libraries and .a for static libraries.

# **6.4.1 NVECTOR SERIAL accessor macros**

The following five macros are provided to access the content of an NVECTOR\_SERIAL vector. The suffix \_S in the names denotes the serial version.

#### NV\_CONTENT\_S(v)

This macro gives access to the contents of the serial vector N\_Vector v.

The assignment  $v_{cont} = NV_{cont} = NV$ 

Implementation:

```
#define NV_CONTENT_S(v) ( (N_VectorContent_Serial)(v->content) )
```

#### NV\_OWN\_DATA\_S(v)

Access the *own\_data* component of the serial N\_Vector *v*.

Implementation:

```
#define NV_OWN_DATA_S(v) ( NV_CONTENT_S(v)->own_data )
```

#### NV\_DATA\_S(v)

The assignment  $v_{data} = NV_DATA_S(v)$  sets  $v_{data}$  to be a pointer to the first component of the data for the  $N_Vector v$ .

Similarly, the assignment  $NV_DATA_S(v) = v_data$  sets the component array of v to be  $v_data$  by storing the pointer  $v_data$ .

Implementation:

```
#define NV_DATA_S(v) ( NV_CONTENT_S(v)->data )
```

#### NV\_LENGTH\_S(v)

Access the *length* component of the serial N\_Vector v.

The assignment  $v_{en} = NV_{en} =$ 

Implementation:

```
#define NV_LENGTH_S(v) ( NV_CONTENT_S(v)->length )
```

#### $NV_Ith_S(v, i)$

This macro gives access to the individual components of the *data* array of an N\_Vector, using standard 0-based C indexing.

The assignment  $r = NV_{in}(v,i)$  sets r to be the value of the i-th component of v.

The assignment  $NV_Ith_S(v,i) = r$  sets the value of the i-th component of v to be r.

Here i ranges from 0 to n-1 for a vector of length n.

Implementation:

```
#define NV_Ith_S(v,i) ( NV_DATA_S(v)[i] )
```

# **6.4.2 NVECTOR\_SERIAL functions**

The NVECTOR\_SERIAL module defines serial implementations of all vector operations listed in §6.2.1, §6.2.2, §6.2.3, and §6.2.4. Their names are obtained from those in those sections by appending the suffix \_Serial (e.g. N\_-VDestroy\_Serial). All the standard vector operations listed in §6.2.1 with the suffix \_Serial appended are callable via the Fortran 2003 interface by prepending an F (e.g. FN\_VDestroy\_Serial).

The module NVECTOR\_SERIAL provides the following additional user-callable routines:

```
N_Vector N_VNew_Serial(sunindextype vec_length, SUNContext sunctx)
```

This function creates and allocates memory for a serial N\_Vector. Its only argument is the vector length.

N Vector N\_VNewEmpty\_Serial(sunindextype vec length, SUNContext sunctx)

This function creates a new serial N\_Vector with an empty (NULL) data array.

```
N_Vector N_VMake_Serial(sunindextype vec_length, sunrealtype *v_data, SUNContext sunctx)
```

This function creates and allocates memory for a serial vector with user-provided data array, v\_data.

(This function does *not* allocate memory for v\_data itself.)

```
void N_VPrint_Serial(N_Vector v)
```

This function prints the content of a serial vector to stdout.

```
void N_VPrintFile_Serial(N_Vector v, FILE *outfile)
```

This function prints the content of a serial vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_SERIAL module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with *N\_VNew\_Serial()*, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using *N\_VClone()*. This guarantees that the new vectors will have the same operations enabled/disabled as cloned

vectors inherit the same enable/disable options as the vector they are cloned, from while vectors created with  $N_-$ - $VNew\_Serial()$  will have the default settings for the NVECTOR SERIAL module.

#### SUNErrCode N\_VEnableFusedOps\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the serial vector. The return value is a SUNErrCode.

## SUNErrCode N\_VEnableLinearCombination\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the serial vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the serial vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableDotProdMulti\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the serial vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the serial vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the serial vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableConstVectorArray\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the serial vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableWrmsNormVectorArray\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the serial vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the serial vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Serial(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the serial vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearCombinationVectorArray\_Serial(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the serial vector. The return value is a *SUNErrCode*.

#### **Notes**

- When looping over the components of an N\_Vector v, it is more efficient to first obtain the component array via v\_data = NV\_DATA\_S(v), or equivalently v\_data = N\_VGetArrayPointer(v), and then access v\_data[i] within the loop than it is to use NV\_Ith\_S(v,i) within the loop.
- N\_VNewEmpty\_Serial() and N\_VMake\_Serial() set the field own\_data to SUNFALSE. The implementation of N\_VDestroy() will not attempt to free the pointer data for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the data pointer.

 To maximize efficiency, vector operations in the NVECTOR\_SERIAL implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same length.

# **6.4.3 NVECTOR\_SERIAL Fortran Interface**

The NVECTOR\_SERIAL module provides a Fortran 2003 module for use from Fortran applications.

The fnvector\_serial\_mod Fortran module defines interfaces to all NVECTOR\_SERIAL C functions using the intrinsic iso\_c\_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading F. For example, the function N\_VNew\_Serial is interfaced as FN\_VNew\_Serial.

The Fortran 2003 NVECTOR\_SERIAL interface module can be accessed with the use statement, i.e. use fnvector\_serial\_mod, and linking to the library libsundials\_fnvectorserial\_mod.lib in addition to the C library. For details on where the library and module file fnvector\_serial\_mod.mod are installed see §10. We note that the module is accessible from the Fortran 2003 SUNDIALS integrators without separately linking to the libsundials\_fnvectorserial\_mod library.

# **6.5** The NVECTOR\_PARALLEL Module

The NVECTOR\_PARALLEL implementation of the NVECTOR module provided with SUNDIALS is based on MPI. It defines the *content* field of an N\_Vector to be a structure containing the global and local lengths of the vector, a pointer to the beginning of a contiguous local data array, an MPI communicator, an a boolean flag *own\_data* indicating ownership of the data array *data*.

```
struct _N_VectorContent_Parallel {
    sunindextype local_length;
    sunindextype global_length;
    sunbooleantype own_data;
    sunrealtype *data;
    MPI_Comm comm;
};
```

The header file to be included when using this module is nvector\_parallel.h. The installed module library to link to is libsundials\_nvecparallel.lib where .lib is typically .so for shared libraries and .a for static libraries.

#### **6.5.1 NVECTOR PARALLEL accessor macros**

The following seven macros are provided to access the content of a NVECTOR\_PARALLEL vector. The suffix \_P in the names denotes the distributed memory parallel version.

#### NV\_CONTENT\_P(v)

This macro gives access to the contents of the parallel N\_Vector *v*.

The assignment v\_cont = NV\_CONTENT\_P(v) sets v\_cont to be a pointer to the N\_Vector *content* structure of type struct N\_VectorContent\_Parallel.

Implementation:

```
#define NV_CONTENT_P(v) ( (N_VectorContent_Parallel)(v->content) )
```

#### NV\_OWN\_DATA\_P(v)

Access the *own\_data* component of the parallel N\_Vector *v*.

Implementation:

#### $NV_DATA_P(v)$

The assignment  $v_{data} = NV_DATA_P(v)$  sets  $v_{data}$  to be a pointer to the first component of the *local\_data* for the  $N_Vector v$ .

The assignment  $NV_DATA_P(v) = v_data$  sets the component array of v to be  $v_data$  by storing the pointer  $v_data$  into data.

Implementation:

```
#define NV_DATA_P(v) ( NV_CONTENT_P(v)->data )
```

#### NV\_LOCLENGTH\_P(v)

The assignment  $v_1len = NV_LOCLENGTH_P(v)$  sets  $v_1len$  to be the length of the local part of v.

The call NV\_LOCLENGTH\_P(v) = llen\_v sets the *local\_length* of v to be llen\_v.

Implementation:

```
#define NV_LOCLENGTH_P(v) ( NV_CONTENT_P(v)->local_length )
```

#### NV\_GLOBLENGTH\_P(v)

The assignment v\_glen = NV\_GLOBLENGTH\_P(v) sets v\_glen to be the global\_length of the vector v.

The call NV\_GLOBLENGTH\_P(v) =  $glen_v$  sets the  $global_length$  of v to be  $glen_v$ .

Implementation:

```
#define NV_GLOBLENGTH_P(v) ( NV_CONTENT_P(v)->global_length )
```

#### $NV_COMM_P(v)$

This macro provides access to the MPI communicator used by the parallel  $N_{v}$ .

Implementation:

```
#define NV_COMM_P(v) ( NV_CONTENT_P(v)->comm )
```

## $NV_Ith_P(v, i)$

This macro gives access to the individual components of the *local\_data* array of an N\_Vector.

The assignment  $r = NV_{int} P(v, i)$  sets r to be the value of the i-th component of the local part of v.

The assignment  $NV_Ith_P(v,i) = r$  sets the value of the i-th component of the local part of v to be r.

Here i ranges from 0 to n-1, where n is the *local\_length*.

Implementation:

```
#define NV_Ith_P(v,i) ( NV_DATA_P(v)[i] )
```

## **6.5.2 NVECTOR PARALLEL functions**

The NVECTOR\_PARALLEL module defines parallel implementations of all vector operations listed in §6.2. Their names are obtained from the generic names by appending the suffix \_Parallel (e.g. N\_VDestroy\_Parallel). The module NVECTOR\_PARALLEL provides the following additional user-callable routines:

N\_Vector N\_VNew\_Parallel (MPI\_Comm comm, sunindextype local\_length, sunindextype global\_length, SUNContext sunctx)

This function creates and allocates memory for a parallel vector having global length *global\_length*, having processor-local length *local\_length*, and using the MPI communicator *comm*.

N\_Vector N\_VNewEmpty\_Parallel(MPI\_Comm comm, sunindextype local\_length, sunindextype global\_length, SUNContext sunctx)

This function creates a new parallel N\_Vector with an empty (NULL) data array.

N\_Vector N\_VMake\_Parallel(MPI\_Comm comm, sunindextype local\_length, sunindextype global\_length, sunrealtype \*v\_data, SUNContext sunctx)

This function creates and allocates memory for a parallel vector with user-provided data array.

(This function does *not* allocate memory for v\_data itself.)

## sunindextype N\_VGetLocalLength\_Parallel(N\_Vector v)

This function returns the local vector length.

#### void N\_VPrint\_Parallel(N\_Vector v)

This function prints the local content of a parallel vector to stdout.

#### void N\_VPrintFile\_Parallel(N\_Vector v, FILE \*outfile)

This function prints the local content of a parallel vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_PARALLEL module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with  $N_{VNew_Parallel}()$ , enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using  $N_{VClone}()$ . This guarantees that the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from, while vectors created with  $N_{VNew_Parallel}()$  will have the default settings for the NVECTOR PARALLEL module.

## SUNErrCode N\_VEnableFusedOps\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the parallel vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearCombination\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the parallel vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableScaleAddMulti\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the parallel vector. The return value is a <code>SUNErrCode</code>.

#### SUNErrCode N\_VEnableDotProdMulti\_Parallel(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the parallel vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the parallel vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the parallel vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableConstVectorArray\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the parallel vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_Parallel(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the parallel vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the parallel vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the parallel vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearCombinationVectorArray\_Parallel(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the parallel vector. The return value is a *SUNErrCode*.

#### **Notes**

- When looping over the components of an N\_Vector v, it is more efficient to first obtain the local component array via v\_data = N\_VGetArrayPointer(v), or equivalently v\_data = NV\_DATA\_P(v), and then access v\_data[i] within the loop than it is to use NV\_Ith\_P(v,i) within the loop.
- N\_VNewEmpty\_Parallel() and N\_VMake\_Parallel() set the field own\_data to SUNFALSE. The implementation of N\_VDestroy() will not attempt to free the pointer data for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the data pointer.
- To maximize efficiency, vector operations in the NVECTOR\_PARALLEL implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

## 6.5.3 NVECTOR PARALLEL Fortran Interface

The NVECTOR\_PARALLEL module provides a Fortran 2003 module for use from Fortran applications.

The fnvector\_parallel\_mod Fortran module defines interfaces to all NVECTOR\_PARALLEL C functions using the intrinsic  $iso_cbinding$  module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading F. For example, the function N\_VNew\_Parallel is interfaced as FN\_VNew\_Parallel.

The Fortran 2003 NVECTOR\_PARALLEL interface module can be accessed with the use statement, i.e. use fn-vector\_parallel\_mod, and linking to the library libsundials\_fnvectorparallel\_mod.lib in addition to the C library. For details on where the library and module file fnvector\_parallel\_mod.mod are installed see §10. We note that the module is accessible from the Fortran 2003 SUNDIALS integrators without separately linking to the libsundials\_fnvectorparallel\_mod library.

# **6.6 The NVECTOR\_OPENMP Module**

In situations where a user has a multi-core processing unit capable of running multiple parallel threads with shared memory, SUNDIALS provides an implementation of NVECTOR using OpenMP, called NVECTOR\_OPENMP, and an implementation using Pthreads, called NVECTOR\_PTHREADS. Testing has shown that vectors should be of length at least 100,000 before the overhead associated with creating and using the threads is made up by the parallelism in the vector calculations.

The OpenMP NVECTOR implementation provided with SUNDIALS, NVECTOR\_OPENMP, defines the *content* field of N\_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, a boolean flag *own\_data* which specifies the ownership of *data*, and the number of threads. Operations on the vector are threaded using OpenMP, the number of threads used is based on the supplied argument in the vector constructor.

```
struct _N_VectorContent_OpenMP {
   sunindextype length;
   sunbooleantype own_data;
   sunrealtype *data;
   int num_threads;
};
```

The header file to be included when using this module is nvector\_openmp.h. The installed module library to link to is libsundials\_nvecopenmp.lib where .lib is typically .so for shared libraries and .a for static libraries. The Fortran module file to use when using the Fortran 2003 interface to this module is fnvector\_openmp\_mod.mod.

# 6.6.1 NVECTOR\_OPENMP accessor macros

The following six macros are provided to access the content of an NVECTOR\_OPENMP vector. The suffix \_OMP in the names denotes the OpenMP version.

#### NV\_CONTENT\_OMP(v)

This macro gives access to the contents of the OpenMP vector N\_Vector v.

The assignment v\_cont = NV\_CONTENT\_OMP(v) sets v\_cont to be a pointer to the OpenMP N\_Vector content structure.

Implementation:

```
#define NV_CONTENT_OMP(v) ( (N_VectorContent_OpenMP)(v->content) )
```

#### NV\_OWN\_DATA\_OMP(v)

Access the *own data* component of the OpenMP N\_Vector v.

Implementation:

```
#define NV_OWN_DATA_OMP(v) ( NV_CONTENT_OMP(v)->own_data )
```

# $NV_DATA_OMP(v)$

The assignment  $v_{data} = NV_DATA_OMP(v)$  sets  $v_{data}$  to be a pointer to the first component of the *data* for the  $N_Vector v$ .

Similarly, the assignment  $NV_DATA_OMP(v) = v_data$  sets the component array of v to be  $v_data$  by storing the pointer  $v_data$ .

Implementation:

```
#define NV_DATA_OMP(v) ( NV_CONTENT_OMP(v)->data )
```

#### NV\_LENGTH\_OMP(v)

Access the *length* component of the OpenMP N\_Vector v.

The assignment  $v_len = NV_LENGTH_OMP(v)$  sets  $v_len$  to be the *length* of v. On the other hand, the call  $NV_LENGTH_OMP(v) = len_v$  sets the *length* of v to be  $len_v$ .

Implementation:

```
#define NV_LENGTH_OMP(v) ( NV_CONTENT_OMP(v)->length )
```

#### NV\_NUM\_THREADS\_OMP(v)

Access the *num\_threads* component of the OpenMP N\_Vector *v*.

The assignment v\_threads = NV\_NUM\_THREADS\_OMP(v) sets v\_threads to be the  $num\_threads$  of v. On the other hand, the call NV\_NUM\_THREADS\_OMP(v) = num\_threads\_v sets the  $num\_threads$  of v to be num\_threads\_v.

Implementation:

```
#define NV_NUM_THREADS_OMP(v) ( NV_CONTENT_OMP(v)->num_threads )
```

#### $NV_Ith_OMP(v, i)$

This macro gives access to the individual components of the *data* array of an N\_Vector, using standard 0-based C indexing.

The assignment  $r = NV_{in}$  omp (v, i) sets r to be the value of the i-th component of v.

The assignment  $NV_Ith_OMP(v,i) = r$  sets the value of the i-th component of v to be r.

Here i ranges from 0 to n-1 for a vector of length n.

Implementation:

```
#define NV_Ith_OMP(v,i) ( NV_DATA_OMP(v)[i] )
```

# **6.6.2 NVECTOR\_OPENMP functions**

The NVECTOR\_OPENMP module defines OpenMP implementations of all vector operations listed in  $\S6.2$ ,  $\S6.2.2$ ,  $\S6.2.3$ , and  $\S6.2.4$ . Their names are obtained from those in those sections by appending the suffix \_OpenMP (e.g. N\_-VDestroy\_OpenMP). All the standard vector operations listed in  $\S6.2$  with the suffix \_OpenMP appended are callable via the Fortran 2003 interface by prepending an  $F'(e.g. ``FN_VDestroy_OpenMP')$ .

The module NVECTOR\_OPENMP provides the following additional user-callable routines:

N\_Vector N\_VNew\_OpenMP(sunindextype vec\_length, int num\_threads, SUNContext sunctx)

This function creates and allocates memory for a OpenMP N\_Vector. Arguments are the vector length and number of threads.

N\_Vector N\_VNewEmpty\_OpenMP(sunindextype vec\_length, int num\_threads, SUNContext sunctx)

This function creates a new OpenMP N\_Vector with an empty (NULL) data array.

N\_Vector N\_VMake\_OpenMP(sunindextype vec\_length, sunrealtype \*v\_data, int num\_threads, SUNContext sunctx)

This function creates and allocates memory for a OpenMP vector with user-provided data array, v data.

(This function does *not* allocate memory for v\_data itself.)

#### void N\_VPrint\_OpenMP(N Vector v)

This function prints the content of an OpenMP vector to stdout.

#### void N\_VPrintFile\_OpenMP(N Vector v, FILE \*outfile)

This function prints the content of an OpenMP vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_OPENMP module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with <code>N\_VNew\_OpenMP()</code>, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using <code>N\_VClone()</code>. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with <code>N\_VNew\_OpenMP()</code> will have the default settings for the NVECTOR\_OPENMP module.

#### SUNErrCode N\_VEnableFusedOps\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the OpenMP vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearCombination\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the OpenMP vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the OpenMP vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableDotProdMulti\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the OpenMP vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableLinearSumVectorArray\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the OpenMP vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the OpenMP vector. The return value is a SUNErrCode.

## SUNErrCode N\_VEnableConstVectorArray\_OpenMP(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the OpenMP vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the OpenMP vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the OpenMP vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMultiVectorArray\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the OpenMP vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableLinearCombinationVectorArray\_OpenMP(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the OpenMP vector. The return value is a <code>SUNErrCode</code>.

#### **Notes**

- When looping over the components of an N\_Vector v, it is more efficient to first obtain the component array via v\_data = N\_VGetArrayPointer(v), or equivalently v\_data = NV\_DATA\_OMP(v) and then access v\_data[i] within the loop than it is to use NV\_Ith\_OMP(v,i) within the loop.
- N\_VNewEmpty\_OpenMP() and N\_VMake\_OpenMP() set the field own\_data to SUNFALSE. The implementation of N\_VDestroy() will not attempt to free the pointer data for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the data pointer.
- To maximize efficiency, vector operations in the NVECTOR\_OPENMP implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

# 6.6.3 NVECTOR\_OPENMP Fortran Interface

The NVECTOR OPENMP module provides a Fortran 2003 module for use from Fortran applications.

The fnvector\_openmp\_mod Fortran module defines interfaces to all NVECTOR\_OPENMP C functions using the intrinsic iso\_c\_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading F. For example, the function N\_VNew\_OpenMP is interfaced as FN\_VNew\_OpenMP.

The Fortran 2003 NVECTOR\_OPENMP interface module can be accessed with the use statement, i.e. use fnvector\_openmp\_mod, and linking to the library libsundials\_fnvectoropenmp\_mod.lib in addition to the C library. For details on where the library and module file fnvector\_openmp\_mod.mod are installed see §10.

# **6.7** The NVECTOR\_PTHREADS Module

In situations where a user has a multi-core processing unit capable of running multiple parallel threads with shared memory, SUNDIALS provides an implementation of NVECTOR using OpenMP, called NVECTOR\_OPENMP, and an implementation using Pthreads, called NVECTOR\_PTHREADS. Testing has shown that vectors should be of length at least 100,000 before the overhead associated with creating and using the threads is made up by the parallelism in the vector calculations.

The Pthreads NVECTOR implementation provided with SUNDIALS, denoted NVECTOR\_PTHREADS, defines the *content* field of N\_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, a boolean flag *own\_data* which specifies the ownership of *data*, and the number of threads. Operations on the vector are threaded using POSIX threads (Pthreads).

```
struct _N_VectorContent_Pthreads {
   sunindextype length;
   sunbooleantype own_data;
   sunrealtype *data;
   int num_threads;
};
```

The header file to be included when using this module is nvector\_pthreads.h. The installed module library to link to is libsundials\_nvecpthreads.lib where .lib is typically .so for shared libraries and .a for static libraries.

## 6.7.1 NVECTOR PTHREADS accessor macros

The following six macros are provided to access the content of an NVECTOR\_PTHREADS vector. The suffix \_PT in the names denotes the Pthreads version.

#### NV\_CONTENT\_PT(v)

This macro gives access to the contents of the Pthreads vector N\_Vector v.

The assignment v\_cont = NV\_CONTENT\_PT(v) sets v\_cont to be a pointer to the Pthreads N\_Vector content structure.

Implementation:

```
#define NV_CONTENT_PT(v) ( (N_VectorContent_Pthreads)(v->content) )
```

#### NV\_OWN\_DATA\_PT(v)

Access the *own\_data* component of the Pthreads N\_Vector *v*.

Implementation:

```
#define NV_OWN_DATA_PT(v) ( NV_CONTENT_PT(v)->own_data )
```

## $NV_DATA_PT(v)$

The assignment  $v_{data} = NV_DATA_PT(v)$  sets  $v_{data}$  to be a pointer to the first component of the data for the  $N_Vector v$ .

Similarly, the assignment  $NV_DATA_PT(v) = v_data$  sets the component array of v to be  $v_data$  by storing the pointer  $v_data$ .

Implementation:

```
#define NV_DATA_PT(v) ( NV_CONTENT_PT(v)->data )
```

# NV\_LENGTH\_PT(v)

Access the *length* component of the Pthreads N\_Vector v.

The assignment  $v_len = NV_LENGTH_PT(v)$  sets  $v_len$  to be the *length* of v. On the other hand, the call  $NV_LENGTH_PT(v) = len_v$  sets the *length* of v to be  $len_v$ .

Implementation:

```
#define NV_LENGTH_PT(v) ( NV_CONTENT_PT(v)->length )
```

#### NV\_NUM\_THREADS\_PT(v)

Access the  $num\_threads$  component of the Pthreads  $N\_Vector v$ .

The assignment  $v_{threads} = NV_{NUM\_THREADS\_PT(v)}$  sets  $v_{threads}$  to be the  $num\_threads$  of v. On the other hand, the call  $NV_{NUM\_THREADS\_PT(v)} = num\_threads\_v$  sets the  $num\_threads$  of v to be  $num\_threads\_v$ .

Implementation:

```
#define NV_NUM_THREADS_PT(v) ( NV_CONTENT_PT(v)->num_threads )
```

#### $NV_Ith_PT(v, i)$

This macro gives access to the individual components of the *data* array of an N\_Vector, using standard 0-based C indexing.

The assignment  $r = NV_{int} PT(v, i)$  sets r to be the value of the i-th component of v.

The assignment  $NV_{int} = r$  sets the value of the i-th component of v to be r.

Here i ranges from 0 to n-1 for a vector of length n.

Implementation:

#define NV\_Ith\_PT(v,i) ( NV\_DATA\_PT(v)[i] )

# 6.7.2 NVECTOR\_PTHREADS functions

The NVECTOR\_PTHREADS module defines Pthreads implementations of all vector operations listed in §6.2, §6.2.2, §6.2.3, and §6.2.4. Their names are obtained from those in those sections by appending the suffix \_Pthreads (e.g. N\_VDestroy\_Pthreads). All the standard vector operations listed in §6.2 are callable via the Fortran 2003 interface by prepending an F' (e.g. ``FN\_VDestroy\_Pthreads``). The module NVECTOR\_PTHREADS provides the following additional user-callable routines:

N\_Vector N\_VNew\_Pthreads(sunindextype vec\_length, int num\_threads, SUNContext sunctx)

This function creates and allocates memory for a Pthreads N\_Vector. Arguments are the vector length and number of threads.

N\_Vector N\_VNewEmpty\_Pthreads(sunindextype vec\_length, int num\_threads, SUNContext sunctx)

This function creates a new Pthreads N\_Vector with an empty (NULL) data array.

*N\_Vector* **N\_VMake\_Pthreads**(*sunindextype* vec\_length, *sunrealtype* \*v\_data, int num\_threads, *SUNContext* sunctx)

This function creates and allocates memory for a Pthreads vector with user-provided data array, v\_data.

(This function does *not* allocate memory for v\_data itself.)

void N\_VPrint\_Pthreads(N Vector v)

This function prints the content of a Pthreads vector to stdout.

void N\_VPrintFile\_Pthreads(N\_Vector v, FILE \*outfile)

This function prints the content of a Pthreads vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_PTHREADS module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with  $N_{VNew_Pthreads}()$ , enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using  $N_{VClone}()$ . This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with  $N_{VNew_Pthreads}()$  will have the default settings for the NVECTOR\_PTHREADS module.

```
SUNErrCode N_VEnableFusedOps_Pthreads(N_Vector v, sunbooleantype tf)
```

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the Pthreads vector. The return value is a SUNExrCode.

SUNErrCode N\_VEnableLinearCombination\_Pthreads(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the Pthreads vector. The return value is a *SUNErrCode*.

SUNErrCode N\_VEnableScaleAddMulti\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the Pthreads vector. The return value is a <code>SUNErrCode</code>.

#### SUNErrCode N\_VEnableDotProdMulti\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the Pthreads vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the Pthreads vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the Pthreads vector. The return value is a SUNErrCode.

## SUNErrCode N\_VEnableConstVectorArray\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the Pthreads vector. The return value is a SUNErrCode.

## SUNErrCode N\_VEnableWrmsNormVectorArray\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the Pthreads vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the Pthreads vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the Pthreads vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableLinearCombinationVectorArray\_Pthreads(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the Pthreads vector. The return value is a *SUNErrCode*.

#### Notes

- When looping over the components of an N\_Vector v, it is more efficient to first obtain the component array via v\_data = N\_VGetArrayPointer(v), or equivalently v\_data = NV\_DATA\_PT(v) and then access v\_data[i] within the loop than it is to use NV\_Ith\_S(v,i) within the loop.
- N\_VNewEmpty\_Pthreads() and N\_VMake\_Pthreads() set the field own\_data to SUNFALSE. The implementation of N\_VDestroy() will not attempt to free the pointer data for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the data pointer.
- To maximize efficiency, vector operations in the NVECTOR\_PTHREADS implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

# **6.7.3 NVECTOR\_PTHREADS Fortran Interface**

The NVECTOR\_PTHREADS module provides a Fortran 2003 module for use from Fortran applications.

The fnvector\_pthreads\_mod Fortran module defines interfaces to all NVECTOR\_PTHREADS C functions using the intrinsic iso\_c\_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading F. For example, the function N\_VNew\_Pthreads is interfaced as FN\_VNew\_Pthreads.

The Fortran 2003 NVECTOR\_PTHREADS interface module can be accessed with the use statement, i.e. use fn-vector\_pthreads\_mod, and linking to the library libsundials\_fnvectorpthreads\_mod.lib in addition to the C library. For details on where the library and module file fnvector\_pthreads\_mod.mod are installed see §10.

# 6.8 The NVECTOR\_PARHYP Module

The NVECTOR\_PARHYP implementation of the NVECTOR module provided with SUNDIALS is a wrapper around HYPRE's ParVector class. Most of the vector kernels simply call HYPRE vector operations. The implementation defines the *content* field of N\_Vector to be a structure containing the global and local lengths of the vector, a pointer to an object of type hypre\_ParVector, an MPI communicator, and a boolean flag *own\_parvector* indicating ownership of the HYPRE parallel vector object *x*.

```
struct _N_VectorContent_ParHyp {
   sunindextype local_length;
   sunbooleantype own_data;
   sunbooleantype own_parvector;
   sunrealtype *data;
   MPI_Comm comm;
   hypre_ParVector *x;
};
```

The header file to be included when using this module is nvector\_parhyp.h. The installed module library to link to is libsundials\_nvecparhyp.lib where .lib is typically .so for shared libraries and .a for static libraries.

Unlike native SUNDIALS vector types, NVECTOR\_PARHYP does not provide macros to access its member variables. Note that NVECTOR\_PARHYP requires SUNDIALS to be built with MPI support.

## **6.8.1 NVECTOR\_PARHYP functions**

The NVECTOR\_PARHYP module defines implementations of all vector operations listed in §6.2 except for N\_VSe-tArrayPointer() and N\_VGetArrayPointer() because accessing raw vector data is handled by low-level HYPRE functions. As such, this vector is not available for use with SUNDIALS Fortran interfaces. When access to raw vector data is needed, one should extract the HYPRE vector first, and then use HYPRE methods to access the data. Usage examples of NVECTOR\_PARHYP are provided in the cvAdvDiff\_non\_ph.c example programs for CVODE and the ark\_diurnal\_kry\_ph.c example program for ARKODE.

The names of parhyp methods are obtained from those in §6.2, §6.2.2, §6.2.3, and §6.2.4 by appending the suffix \_ParHyp (e.g. N\_VDestroy\_ParHyp). The module NVECTOR\_PARHYP provides the following additional user-callable routines:

```
N_Vector N_VNewEmpty_ParHyp(MPI_Comm comm, sunindextype local_length, sunindextype global_length, SUNContext sunctx)
```

This function creates a new parhyp N\_Vector with the pointer to the HYPRE vector set to NULL.

```
N_Vector N_VMake_ParHyp(hypre_ParVector *x, SUNContext sunctx)
```

This function creates an  $N_{vector}$  wrapper around an existing HYPRE parallel vector. It does *not* allocate memory for x itself.

```
hypre_ParVector *N_VGetVector_ParHyp(N_Vector v)
```

This function returns a pointer to the underlying HYPRE vector.

#### void **N\_VPrint\_ParHyp**(*N\_Vector* v)

This function prints the local content of a parhyp vector to stdout.

# void N\_VPrintFile\_ParHyp(N\_Vector v, FILE \*outfile)

This function prints the local content of a parhyp vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_PARHYP module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with N\_VMake\_ParHyp(), enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using N\_VClone(). This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with N\_VMake\_ParHyp() will have the default settings for the NVECTOR\_PARHYP module.

#### SUNErrCode N\_VEnableFusedOps\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the parhyp vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearCombination\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the parhyp vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableScaleAddMulti\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the parhyp vector. The return value is a SUNErrCode.

# SUNErrCode N\_VEnableDotProdMulti\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the parhyp vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableLinearSumVectorArray\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the parhyp vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the parhyp vector. The return value is a SUNErrCode.

# SUNErrCode N\_VEnableConstVectorArray\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the parhyp vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the parhyp vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the parhyp vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMultiVectorArray\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the parhyp vector. The return value is a SUNErrCode.

# SUNErrCode N\_VEnableLinearCombinationVectorArray\_ParHyp(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the parhyp vector. The return value is a <code>SUNErrCode</code>.

#### **Notes**

- When there is a need to access components of an N\_Vector\_ParHyp v, it is recommended to extract the HYPRE vector via x\_vec = N\_VGetVector\_ParHyp(v) and then access components using appropriate HYPRE functions.
- N\_VNewEmpty\_ParHyp(), and N\_VMake\_ParHyp() set the field own\_parvector to SUNFALSE. The implementation of N\_VDestroy() will not attempt to delete an underlying HYPRE vector for any N\_Vector with own\_parvector set to SUNFALSE. In such a case, it is the user's responsibility to delete the underlying vector.
- To maximize efficiency, vector operations in the NVECTOR\_PARHYP implementation that have more than one N\_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

# **6.9 The NVECTOR\_PETSC Module**

The NVECTOR\_PETSC module is an NVECTOR wrapper around the PETSc vector. It defines the *content* field of a N\_Vector to be a structure containing the global and local lengths of the vector, a pointer to the PETSc vector, an MPI communicator, and a boolean flag *own\_data* indicating ownership of the wrapped PETSc vector.

```
struct _N_VectorContent_Petsc {
    sunindextype local_length;
    sunindextype global_length;
    sunbooleantype own_data;
    Vec *pvec;
    MPI_Comm comm;
};
```

The header file to be included when using this module is nvector\_petsc.h. The installed module library to link to is libsundials\_nvecpetsc.lib where .lib is typically .so for shared libraries and .a for static libraries.

Unlike native SUNDIALS vector types, NVECTOR\_PETSC does not provide macros to access its member variables. Note that NVECTOR\_PETSC requires SUNDIALS to be built with MPI support.

## **6.9.1 NVECTOR PETSC functions**

The NVECTOR\_PETSC module defines implementations of all vector operations listed in §6.2 except for *N\_VGe-tArrayPointer()* and *N\_VSetArrayPointer()*. As such, this vector cannot be used with SUNDIALS Fortran interfaces. When access to raw vector data is needed, it is recommended to extract the PETSc vector first, and then use PETSc methods to access the data. Usage examples of NVECTOR\_PETSC is provided in example programs for IDA.

The names of vector operations are obtained from those in §6.2, §6.2.2, §6.2.3, and §6.2.4 by appending the suffice \_Petsc (e.g. N\_VDestroy\_Petsc). The module NVECTOR\_PETSC provides the following additional user-callable routines:

```
N_Vector N_VNewEmpty_Petsc(MPI_Comm comm, sunindextype local_length, sunindextype global_length, SUNContext sunctx)
```

This function creates a new PETSC N\_Vector with the pointer to the wrapped PETSc vector set to NULL. It is used by the N\_VMake\_Petsc and N\_VClone\_Petsc implementations. It should be used only with great caution.

```
N_Vector N_VMake_Petsc(Vec *pvec, SUNContext sunctx)
```

This function creates and allocates memory for an NVECTOR\_PETSC wrapper with a user-provided PETSc vector. It does *not* allocate memory for the vector pvec itself.

#### Vec \*N\_VGetVector\_Petsc(N Vector v)

This function returns a pointer to the underlying PETSc vector.

#### void N\_VPrint\_Petsc(N\_Vector v)

This function prints the global content of a wrapped PETSc vector to stdout.

#### void N\_VPrintFile\_Petsc(N Vector v, const char fname[])

This function prints the global content of a wrapped PETSc vector to fname.

By default all fused and vector array operations are disabled in the NVECTOR\_PETSC module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with  $N_{VMake_{Petsc}}$ , enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using  $N_{VClone}$ . This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with  $N_{VMake_{Petsc}}$  will have the default settings for the NVECTOR\_PETSC module.

#### SUNErrCode N\_VEnableFusedOps\_Petsc(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the PETSc vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearCombination\_Petsc(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the PETSc vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_Petsc(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the PETSc vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableDotProdMulti\_Petsc(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the PETSc vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Petsc(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the PETSc vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableScaleVectorArray\_Petsc(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the PETSc vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableConstVectorArray\_Petsc(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the PETSc vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_Petsc(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the PETSc vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_Petsc(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the PETSc vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Petsc(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the PETSc vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearCombinationVectorArray\_Petsc(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the PETSc vector. The return value is a *SUNErrCode*.

#### Notes

- When there is a need to access components of an N\_Vector\_Petsc v, it is recommeded to extract the PETSc vector via x\_vec = N\_VGetVector\_Petsc(v); and then access components using appropriate PETSc functions.
- The functions N\_VNewEmpty\_Petsc() and N\_VMake\_Petsc(), set the field own\_data to SUNFALSE. The implementation of N\_VDestroy() will not attempt to free the pointer pvec for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the pvec pointer.
- To maximize efficiency, vector operations in the NVECTOR\_PETSC implementation that have more than one N\_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

# 6.10 The NVECTOR\_CUDA Module

The NVECTOR\_CUDA module is an NVECTOR implementation in the CUDA language. The module allows for SUNDIALS vector kernels to run on NVIDIA GPU devices. It is intended for users who are already familiar with CUDA and GPU programming. Building this vector module requires a CUDA compiler and, by extension, a C++ compiler. The vector content layout is as follows:

```
struct _N_VectorContent_Cuda
{
   sunindextype
                      length:
   sunbooleantype
                      own_helper;
   SUNMemory
                      host_data;
                      device_data;
   SUNMemory
   SUNCudaExecPolicy* stream_exec_policy;
   SUNCudaExecPolicy* reduce_exec_policy;
   SUNMemoryHelper
                      mem_helper:
   void*
                      priv; /* 'private' data */
};
typedef struct _N_VectorContent_Cuda *N_VectorContent_Cuda;
```

The content members are the vector length (size), boolean flags that indicate if the vector owns the execution policies and memory helper objects (i.e., it is in change of freeing the objects), <code>SUNMemory</code> objects for the vector data on the host and device, pointers to execution policies that control how streaming and reduction kernels are launched, a <code>SUNMemoryHelper</code> for performing memory operations, and a private data structure which holds additional members that should not be accessed directly.

When instantiated with *N\_VNew\_Cuda()*, the underlying data will be allocated on both the host and the device. Alternatively, a user can provide host and device data arrays by using the *N\_VMake\_Cuda()* constructor. To use CUDA managed memory, the constructors *N\_VNewManaged\_Cuda()* and *N\_VMakeManaged\_Cuda()* are provided. Additionally, a user-defined SUNMemoryHelper for allocating/freeing data can be provided with the constructor *N\_VNewWith-MemHelp\_Cuda()*. Details on each of these constructors are provided below.

To use the NVECTOR\_CUDA module, include nvector\_cuda.h and link to the library libsundials\_nveccuda. lib. The extension, .lib, is typically .so for shared libraries and .a for static libraries.

## **6.10.1 NVECTOR CUDA functions**

Unlike other native SUNDIALS vector types, the NVECTOR\_CUDA module does not provide macros to access its member variables. Instead, user should use the accessor functions:

```
sunrealtype *N_VGetHostArrayPointer_Cuda(N_Vector v)
```

This function returns pointer to the vector data on the host.

```
sunrealtype *N_VGetDeviceArrayPointer_Cuda(N_Vector v)
```

This function returns pointer to the vector data on the device.

```
sunbooleantype N_VIsManagedMemory_Cuda(N_Vector v)
```

This function returns a boolean flag indiciating if the vector data array is in managed memory or not.

The NVECTOR\_CUDA module defines implementations of all standard vector operations defined in §6.2, §6.2.2, §6.2.3, and §6.2.4, except for N\_VSetArrayPointer(), and, if using unmanaged memory, N\_VGetArrayPointer(). As such, this vector can only be used with SUNDIALS direct solvers and preconditioners when using managed memory. The NVECTOR\_CUDA module provides separate functions to access data on the host and on the device for the unmanaged memory use case. It also provides methods for copying from the host to the device and vice versa. Usage examples of NVECTOR\_CUDA are provided in example programs for CVODE [34].

The names of vector operations are obtained from those in §6.2, §6.2.2, §6.2.3, and §6.2.4 by appending the suffix \_Cuda (e.g. N\_VDestroy\_Cuda). The module NVECTOR\_CUDA provides the following additional user-callable routines:

*N\_Vector* **N\_VNew\_Cuda**(*sunindextype* length, *SUNContext* sunctx)

This function creates and allocates memory for a CUDA N\_Vector. The vector data array is allocated on both the host and device.

*N\_Vector* **N\_VNewManaged\_Cuda**(*sunindextype* vec\_length, *SUNContext* sunctx)

This function creates and allocates memory for a CUDA N\_Vector. The vector data array is allocated in managed memory.

*N\_Vector* **N\_VNewWithMemHelp\_Cuda**(*sunindextype* length, *sunbooleantype* use\_managed\_mem, *SUNMemoryHelper* helper, *SUNContext* sunctx)

This function creates a new CUDA N\_Vector with a user-supplied SUNMemoryHelper for allocating/freeing memory.

N Vector N\_VNewEmpty\_Cuda(sunindextype vec length, SUNContext sunctx)

This function creates a new CUDA N\_Vector where the members of the content structure have not been allocated. This utility function is used by the other constructors to create a new vector.

*N\_Vector* **N\_VMake\_Cuda**(*sunindextype* vec\_length, *sunrealtype* \*h\_vdata, *sunrealtype* \*d\_vdata, *SUNContext* sunctx)

This function creates a CUDA N\_Vector with user-supplied vector data arrays for the host and the device.

N\_Vector N\_VMakeManaged\_Cuda (sunindextype vec\_length, sunrealtype \*vdata, SUNContext sunctx)

This function creates a CUDA N\_Vector with a user-supplied managed memory data array.

*N\_Vector* **N\_VMakeWithManagedAllocator\_Cuda**(*sunindextype* length, void \*(\*allocfn)(size\_t size), void (\*freefn)(void \*ptr))

This function creates a CUDA N\_Vector with a user-supplied memory allocator. It requires the user to provide a corresponding free function as well. The memory allocated by the allocator function must behave like CUDA managed memory.

The module NVECTOR\_CUDA also provides the following user-callable routines:

# void **N\_VSetKernelExecPolicy\_Cuda**(*N\_Vector* v, SUNCudaExecPolicy \*stream\_exec\_policy, SUNCudaExecPolicy \*reduce exec policy)

This function sets the execution policies which control the kernel parameters utilized when launching the streaming and reduction CUDA kernels. By default the vector is setup to use the SUNCudaThreadDirectExecPolicy and SUNCudaBlockReduceAtomicExecPolicy. Any custom execution policy for reductions must ensure that the grid dimensions (number of thread blocks) is a multiple of the CUDA warp size (32). See §6.10.2 below for more information about the SUNCudaExecPolicy class. Providing NULL for an argument will result in the default policy being restored.

The input execution policies are cloned and, as such, may be freed after being attached to the desired vectors. A NULL input policy will reset the execution policy to the default setting.

**Note:** Note: All vectors used in a single instance of a SUNDIALS package must use the same execution policy. It is **strongly recommended** that this function is called immediately after constructing the vector, and any subsequent vector be created by cloning to ensure consistent execution policies across vectors

## sunrealtype \*N\_VCopyToDevice\_Cuda(N\_Vector v)

This function copies host vector data to the device.

# sunrealtype \*N\_VCopyFromDevice\_Cuda(N\_Vector v)

This function copies vector data from the device to the host.

#### void N\_VPrint\_Cuda(N\_Vector v)

This function prints the content of a CUDA vector to stdout.

# void N\_VPrintFile\_Cuda(N\_Vector v, FILE \*outfile)

This function prints the content of a CUDA vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_CUDA module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with N\_VNew\_Cuda(), enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using N\_VClone(). This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with N\_VNew\_Cuda() will have the default settings for the NVECTOR\_CUDA module.

#### SUNErrCode N\_VEnableFusedOps\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the CUDA vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableLinearCombination\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the CUDA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the CUDA vector. The return value is a SUNErrCode.

## SUNErrCode N\_VEnableDotProdMulti\_Cuda(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the CUDA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the CUDA vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableScaleVectorArray\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the CUDA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableConstVectorArray\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the CUDA vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_Cuda(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the CUDA vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the CUDA vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the CUDA vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearCombinationVectorArray\_Cuda(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the CUDA vector. The return value is a *SUNErrCode*.

#### Notes

- When there is a need to access components of an N\_Vector\_Cuda, v, it is recommeded to use functions N\_-VGetDeviceArrayPointer\_Cuda() or N\_VGetHostArrayPointer\_Cuda(). However, when using managed memory, the function N\_VGetArrayPointer() may also be used.
- To maximize efficiency, vector operations in the NVECTOR\_CUDA implementation that have more than one N\_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

## **6.10.2** The SUNCudaExecPolicy Class

In order to provide maximum flexibility to users, the CUDA kernel execution parameters used by kernels within SUN-DIALS are defined by objects of the sundials::cuda::ExecPolicy abstract class type (this class can be accessed in the global namespace as SUNCudaExecPolicy). Thus, users may provide custom execution policies that fit the needs of their problem. The SUNCudaExecPolicy class is defined as

```
typedef sundials::cuda::ExecPolicy SUNCudaExecPolicy
```

where the sundials::cuda::ExecPolicy class is defined in the header file sundials\_cuda\_policies.hpp, as follows:

```
class sundials::cuda::ExecPolicy
    ExecPolicy(cudaStream_t stream = 0)
    virtual size_t gridSize(size_t numWorkUnits = 0, size_t blockDim = 0)
    virtual size_t blockSize(size_t numWorkUnits = 0, size_t gridDim = 0)
    virtual const cudaStream t *stream() const
```

```
virtual ExecPolicy *clone() const
ExecPolicy *clone_new_stream(cudaStream_t stream) const
virtual bool atomic() const
virtual ~ExecPolicy()
```

To define a custom execution policy, a user simply needs to create a class that inherits from the abstract class and implements the methods. The SUNDIALS provided sundials::cuda::ThreadDirectExecPolicy (aka in the global namespace as SUNCudaThreadDirectExecPolicy) class is a good example of a what a custom execution policy may look like:

```
class ThreadDirectExecPolicy : public ExecPolicy
public:
   ThreadDirectExecPolicy(const size_t blockDim, cudaStream_t stream = 0)
      : blockDim_(blockDim), ExecPolicy(stream)
   {}
   ThreadDirectExecPolicy(const ThreadDirectExecPolicy& ex)
      : blockDim_(ex.blockDim_), ExecPolicy(ex.stream_)
   {}
   virtual size_t gridSize(size_t numWorkUnits = 0, size_t /*blockDim*/ = 0) const
      /* ceil(n/m) = floor((n + m - 1) / m) */
      return (numWorkUnits + blockSize() - 1) / blockSize();
   }
   virtual size_t blockSize(size_t /*numWorkUnits*/ = 0, size_t /*gridDim*/ = 0) const
      return blockDim_;
   }
   virtual ExecPolicy* clone() const
      return static_cast<ExecPolicy*>(new ThreadDirectExecPolicy(*this));
   }
private:
   const size_t blockDim_;
};
```

In total, SUNDIALS provides 3 execution policies:

**SUNCudaThreadDirectExecPolicy**(const size\_t blockDim, const cudaStream\_t stream = 0)

Maps each CUDA thread to a work unit. The number of threads per block (blockDim) can be set to anything. The grid size will be calculated so that there are enough threads for one thread per element. If a CUDA stream is provided, it will be used to execute the kernel.

**SUNCudaGridStrideExecPolicy**(const size\_t blockDim, const size\_t gridDim, const cudaStream\_t stream = 0)

Is for kernels that use grid stride loops. The number of threads per block (blockDim) can be set to anything. The number of blocks (gridDim) can be set to anything. If a CUDA stream is provided, it will be used to execute the kernel.

#### **SUNCudaBlockReduceExecPolicy**(const size\_t blockDim, const cudaStream\_t stream = 0)

Is for kernels performing a reduction across indvidual thread blocks. The number of threads per block (blockDim) can be set to any valid multiple of the CUDA warp size. The grid size (gridDim) can be set to any value greater than 0. If it is set to 0, then the grid size will be chosen so that there is enough threads for one thread per work unit. If a CUDA stream is provided, it will be used to execute the kernel.

# **SUNCudaBlockReduceAtomicExecPolicy**(const size\_t blockDim, const cudaStream\_t stream = 0)

Is for kernels performing a reduction across indvidual thread blocks using atomic operations. The number of threads per block (blockDim) can be set to any valid multiple of the CUDA warp size. The grid size (gridDim) can be set to any value greater than 0. If it is set to 0, then the grid size will be chosen so that there is enough threads for one thread per work unit. If a CUDA stream is provided, it will be used to execute the kernel.

For example, a policy that uses 128 threads per block and a user provided stream can be created like so:

```
cudaStream_t stream;
cudaStreamCreate(&stream);
SUNCudaThreadDirectExecPolicy thread_direct(128, stream);
```

These default policy objects can be reused for multiple SUNDIALS data structures (e.g. a SUNMatrix and an  $N_-$ -Vector) since they do not hold any modifiable state information.

# **6.11 The NVECTOR\_HIP Module**

The NVECTOR\_HIP module is an NVECTOR implementation using the AMD ROCm HIP library [2]. The module allows for SUNDIALS vector kernels to run on AMD or NVIDIA GPU devices. It is intended for users who are already familiar with HIP and GPU programming. Building this vector module requires the HIP-clang compiler. The vector content layout is as follows:

```
struct _N_VectorContent_Hip
   sunindextype
                      length;
   sunbooleantype
                      own_helper;
   SUNMemory
                      host_data;
   SUNMemory
                      device_data;
   SUNHipExecPolicy* stream_exec_policy;
   SUNHipExecPolicy* reduce_exec_policy;
   SUNMemoryHelper
                      mem_helper;
   void*
                      priv; /* 'private' data */
};
typedef struct _N_VectorContent_Hip *N_VectorContent_Hip;
```

The content members are the vector length (size), a boolean flag that signals if the vector owns the data (i.e. it is in charge of freeing the data), pointers to vector data on the host and the device, pointers to *SUNHipExecPolicy* implementations that control how the HIP kernels are launched for streaming and reduction vector kernels, and a private data structure which holds additional members that should not be accessed directly.

When instantiated with  $N\_VNew\_Hip()$ , the underlying data will be allocated on both the host and the device. Alternatively, a user can provide host and device data arrays by using the  $N\_VMake\_Hip()$  constructor. To use managed memory, the constructors  $N\_VNewManaged\_Hip()$  and  $N\_VMakeManaged\_Hip()$  are provided. Additionally, a user-defined SUNMemoryHelper for allocating/freeing data can be provided with the constructor  $N\_VNewWithMemHelp\_-Hip()$ . Details on each of these constructors are provided below.

To use the NVECTOR\_HIP module, include nvector\_hip.h and link to the library libsundials\_nvechip.lib. The extension, .lib, is typically .so for shared libraries and .a for static libraries.

## **6.11.1 NVECTOR\_HIP functions**

Unlike other native SUNDIALS vector types, the NVECTOR\_HIP module does not provide macros to access its member variables. Instead, user should use the accessor functions:

```
sunrealtype *N_VGetHostArrayPointer_Hip(N_Vector v)
```

This function returns pointer to the vector data on the host.

```
sunrealtype *N_VGetDeviceArrayPointer_Hip(N_Vector v)
```

This function returns pointer to the vector data on the device.

```
sunbooleantype N_VIsManagedMemory_Hip(N_Vector v)
```

This function returns a boolean flag indiciating if the vector data array is in managed memory or not.

The NVECTOR\_HIP module defines implementations of all standard vector operations defined in §6.2, §6.2.2, §6.2.3, and §6.2.4, except for *N\_VSetArrayPointer()*. The names of vector operations are obtained from those in §6.2, §6.2.2, §6.2.3, and §6.2.4 by appending the suffix \_Hip (e.g. N\_VDestroy\_Hip). The module NVECTOR\_HIP provides the following additional user-callable routines:

```
N_Vector N_VNew_Hip(sunindextype length, SUNContext sunctx)
```

This function creates and allocates memory for a HIP N\_Vector. The vector data array is allocated on both the host and device.

```
N_Vector N_VNewManaged_Hip(sunindextype vec_length, SUNContext sunctx)
```

This function creates and allocates memory for a HIP N\_Vector. The vector data array is allocated in managed memory.

```
N_Vector N_VNewWithMemHelp_Hip(sunindextype length, sunbooleantype use_managed_mem, 
SUNMemoryHelper helper, SUNContext sunctx)
```

This function creates a new HIP N\_Vector with a user-supplied SUNMemoryHelper for allocating/freeing memory.

```
N_Vector N_VNewEmpty_Hip(sunindextype vec_length, SUNContext sunctx)
```

This function creates a new HIP N\_Vector where the members of the content structure have not been allocated. This utility function is used by the other constructors to create a new vector.

```
N_Vector N_VMake_Hip(sunindextype vec_length, sunrealtype *h_vdata, sunrealtype *d_vdata, SUNContext sunctx)
```

This function creates a HIP N\_Vector with user-supplied vector data arrays for the host and the device.

```
N_Vector N_VMakeManaged_Hip(sunindextype vec_length, sunrealtype *vdata, SUNContext sunctx)
```

This function creates a HIP N\_Vector with a user-supplied managed memory data array.

The module NVECTOR\_HIP also provides the following user-callable routines:

```
void N_VSetKernelExecPolicy_Hip(N_Vector v, SUNHipExecPolicy *stream_exec_policy, SUNHipExecPolicy *reduce_exec_policy)
```

This function sets the execution policies which control the kernel parameters utilized when launching the streaming and reduction HIP kernels. By default the vector is setup to use the <code>SUNHipThreadDirectExecPolicy()</code> and <code>SUNHipBlockReduceExecPolicy()</code>. Any custom execution policy for reductions must ensure that the grid dimensions (number of thread blocks) is a multiple of the HIP warp size (32 for NVIDIA GPUs, 64 for AMD GPUs). See §6.11.2 below for more information about the <code>SUNHipExecPolicy</code> class. Providing <code>NULL</code> for an argument will result in the default policy being restored.

The input execution policies are cloned and, as such, may be freed after being attached to the desired vectors. A NULL input policy will reset the execution policy to the default setting.

**Note:** Note: All vectors used in a single instance of a SUNDIALS package must use the same execution policy. It is **strongly recommended** that this function is called immediately after constructing the vector, and any subsequent vector be created by cloning to ensure consistent execution policies across vectors\*

#### sunrealtype \*N\_VCopyToDevice\_Hip(N Vector v)

This function copies host vector data to the device.

## sunrealtype \*N\_VCopyFromDevice\_Hip(N\_Vector v)

This function copies vector data from the device to the host.

#### void **N\_VPrint\_Hip**(*N\_Vector* v)

This function prints the content of a HIP vector to stdout.

#### void **N\_VPrintFile\_Hip**(*N\_Vector* v, FILE \*outfile)

This function prints the content of a HIP vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_HIP module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with  $N_{VNew_{Hip}}()$ , enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using  $N_{Vlone}()$ . This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with  $N_{VNew_{Hip}}()$  will have the default settings for the NVECTOR\_HIP module.

#### SUNErrCode N\_VEnableFusedOps\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the HIP vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearCombination\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the HIP vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the HIP vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableDotProdMulti\_Hip(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the HIP vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the HIP vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the HIP vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableConstVectorArray\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the HIP vector. The return value is a *SUNExyCode*.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the HIP vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the HIP vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Hip(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the HIP vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableLinearCombinationVectorArray\_Hip(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the HIP vector. The return value is a *SUNExrCode*.

#### **Notes**

- When there is a need to access components of an N\_Vector\_Hip, v, it is recommeded to use functions N\_-VGetDeviceArrayPointer\_Hip() or N\_VGetHostArrayPointer\_Hip(). However, when using managed memory, the function N\_VGetArrayPointer() may also be used.
- To maximize efficiency, vector operations in the NVECTOR\_HIP implementation that have more than one N\_-Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

# 6.11.2 The SUNHipExecPolicy Class

In order to provide maximum flexibility to users, the HIP kernel execution parameters used by kernels within SUN-DIALS are defined by objects of the sundials::hip::ExecPolicy abstract class type (this class can be accessed in the global namespace as SUNHipExecPolicy). Thus, users may provide custom execution policies that fit the needs of their problem. The SUNHipExecPolicy class is defined as

```
typedef sundials::hip::ExecPolicy SUNHipExecPolicy
```

where the sundials::hip::ExecPolicy class is defined in the header file sundials\_hip\_policies.hpp, as follows:

```
class sundials::hip::ExecPolicy
    ExecPolicy(hipStream_t stream = 0)
    virtual size_t gridSize(size_t numWorkUnits = 0, size_t blockDim = 0)
    virtual size_t blockSize(size_t numWorkUnits = 0, size_t gridDim = 0)
    virtual const hipStream_t *stream() const
    virtual ExecPolicy *clone() const
    ExecPolicy *clone_new_stream(hipStream_t stream) const
    virtual bool atomic() const
```

#### virtual ~ExecPolicy()

To define a custom execution policy, a user simply needs to create a class that inherits from the abstract class and implements the methods. The SUNDIALS provided sundials::hip::ThreadDirectExecPolicy (aka in the global namespace as SUNHipThreadDirectExecPolicy) class is a good example of a what a custom execution policy may look like:

```
class ThreadDirectExecPolicy : public ExecPolicy
public:
   ThreadDirectExecPolicy(const size_t blockDim, hipStream_t stream = 0)
      : blockDim_(blockDim), ExecPolicy(stream)
   {}
   ThreadDirectExecPolicy(const ThreadDirectExecPolicy& ex)
      : blockDim_(ex.blockDim_), ExecPolicy(ex.stream_)
   {}
  virtual size_t gridSize(size_t numWorkUnits = 0, size_t /*blockDim*/ = 0) const
      /* ceil(n/m) = floor((n + m - 1) / m) */
      return (numWorkUnits + blockSize() - 1) / blockSize();
   }
  virtual size_t blockSize(size_t /*numWorkUnits*/ = 0, size_t /*gridDim*/ = 0) const
      return blockDim_;
   }
  virtual ExecPolicy* clone() const
      return static_cast<ExecPolicy*>(new ThreadDirectExecPolicy(*this));
   }
private:
   const size_t blockDim_;
};
```

In total, SUNDIALS provides 4 execution policies:

#### **SUNHipThreadDirectExecPolicy**(const size\_t blockDim, const hipStream\_t stream = 0)

Maps each HIP thread to a work unit. The number of threads per block (blockDim) can be set to anything. The grid size will be calculated so that there are enough threads for one thread per element. If a HIP stream is provided, it will be used to execute the kernel.

```
SUNHipGridStrideExecPolicy(const size_t blockDim, const size_t gridDim, const hipStream_t stream = 0)
```

Is for kernels that use grid stride loops. The number of threads per block (blockDim) can be set to anything. The number of blocks (gridDim) can be set to anything. If a HIP stream is provided, it will be used to execute the kernel.

```
SUNHipBlockReduceExecPolicy(const size_t blockDim, const hipStream_t stream = 0)
```

Is for kernels performing a reduction across indvidual thread blocks. The number of threads per block (blockDim) can be set to any valid multiple of the HIP warp size. The grid size (gridDim) can be set to any value greater than 0. If it is set to 0, then the grid size will be chosen so that there is

enough threads for one thread per work unit. If a HIP stream is provided, it will be used to execute the kernel.

```
SUNHipBlockReduceAtomicExecPolicy (const size_t blockDim, const hipStream_t stream = 0)
```

Is for kernels performing a reduction across indvidual thread blocks using atomic operations. The number of threads per block (blockDim) can be set to any valid multiple of the HIP warp size. The grid size (gridDim) can be set to any value greater than 0. If it is set to 0, then the grid size will be chosen so that there is enough threads for one thread per work unit. If a HIP stream is provided, it will be used to execute the kernel.

For example, a policy that uses 128 threads per block and a user provided stream can be created like so:

```
hipStream_t stream;
hipStreamCreate(&stream);
SUNHipThreadDirectExecPolicy thread_direct(128, stream);
```

These default policy objects can be reused for multiple SUNDIALS data structures (e.g. a SUNMatrix and an  $N_-$ -Vector) since they do not hold any modifiable state information.

# **6.12 The NVECTOR\_SYCL Module**

The NVECTOR\_SYCL module is an experimental NVECTOR implementation using the SYCL abstraction layer. At present the only supported SYCL compiler is the DPC++ (Intel oneAPI) compiler. This module allows for SUNDIALS vector kernels to run on Intel GPU devices. The module is intended for users who are already familiar with SYCL and GPU programming.

The vector content layout is as follows:

```
struct _N_VectorContent_Syc1
{
   sunindextype
                      length;
   sunbooleantype
                      own_helper;
   SUNMemory
                      host_data:
   SUNMemory
                      device_data;
   SUNSyclExecPolicy* stream_exec_policy;
   SUNSyclExecPolicy* reduce_exec_policy;
   SUNMemoryHelper
                      mem_helper;
   sycl::queue*
                      queue;
   void*
                      priv; /* 'private' data */
};
typedef struct _N_VectorContent_Sycl *N_VectorContent_Sycl;
```

The content members are the vector length (size), boolean flags that indicate if the vector owns the execution policies and memory helper objects (i.e., it is in charge of freeing the objects), <code>SUNMemory</code> objects for the vector data on the host and device, pointers to execution policies that control how streaming and reduction kernels are launched, a <code>SUNMemoryHelper</code> for performing memory operations, the SYCL queue, and a private data structure which holds additional members that should not be accessed directly.

When instantiated with *N\_VNew\_Syc1()*, the underlying data will be allocated on both the host and the device. Alternatively, a user can provide host and device data arrays by using the *N\_VMake\_Syc1()* constructor. To use managed (shared) memory, the constructors *N\_VNewManaged\_Syc1()* and *N\_VMakeManaged\_Syc1()* are provided. Additionally, a user-defined SUNMemoryHelper for allocating/freeing data can be provided with the constructor *N\_VNewWith\_MemHelp\_Syc1()*. Details on each of these constructors are provided below.

The header file to include when using this is nvector\_sycl.h. The installed module library to link to is libsundials\_nvecsycl.lib. The extension .lib is typically .so for shared libraries .a for static libraries.

## 6.12.1 NVECTOR\_SYCL functions

The NVECTOR\_SYCL module implementations of all vector operations listed in §6.2, §6.2.2, §6.2.3, and §6.2.4, except for N\_VDotProdMulti(), N\_VWrmsNormVectorArray(), N\_VWrmsNormMaskVectorArray() as support for arrays of reduction vectors is not yet supported. These functions will be added to the NVECTOR\_SYCL implementation in the future. The names of vector operations are obtained from those in the aforementioned sections by appending the suffix \_Sycl (e.g., N\_VDestroy\_Sycl).

Additionally, the NVECTOR\_SYCL module provides the following user-callable constructors for creating a new NVECTOR\_SYCL:

N\_Vector N\_VNew\_Sycl (sunindextype vec\_length, sycl::queue \*Q, SUNContext sunctx)

This function creates and allocates memory for an NVECTOR\_SYCL. Vector data arrays are allocated on both the host and the device associated with the input queue. All operation are launched in the provided queue.

N\_Vector N\_VNewManaged\_Syc1(sunindextype vec\_length, syc1::queue \*Q, SUNContext sunctx)

This function creates and allocates memory for a NVECTOR\_SYCL. The vector data array is allocated in managed (shared) memory using the input queue. All operation are launched in the provided queue.

N\_Vector N\_VMake\_Sycl(sunindextype length, sunrealtype \*h\_vdata, sunrealtype \*d\_vdata, sycl::queue \*Q, SUNContext sunctx)

This function creates an NVECTOR\_SYCL with user-supplied host and device data arrays. This function does not allocate memory for data itself. All operation are launched in the provided queue.

N\_Vector N\_VMakeManaged\_Sycl (sunindextype length, sunrealtype \*vdata, sycl::queue \*Q, SUNContext sunctx)

This function creates an NVECTOR\_SYCL with a user-supplied managed (shared) data array. This function does not allocate memory for data itself. All operation are launched in the provided queue.

N\_Vector **N\_VNewWithMemHelp\_Sycl**(sunindextype length, sunbooleantype use\_managed\_mem, SUNMemoryHelper helper, sycl::queue \*Q, SUNContext sunctx)

This function creates an NVECTOR\_SYCL with a user-supplied SUNMemoryHelper for allocating/freeing memory. All operation are launched in the provided queue.

# N\_Vector N\_VNewEmpty\_Sycl()

This function creates a new N\_Vector where the members of the content structure have not been allocated. This utility function is used by the other constructors to create a new vector.

The following user-callable functions are provided for accessing the vector data arrays on the host and device and copying data between the two memory spaces. Note the generic NVECTOR operations  $N_VGetArrayPointer()$  and  $N_VSetArrayPointer()$  are mapped to the corresponding HostArray functions given below. To ensure memory coherency, a user will need to call the CopyTo or CopyFrom functions as necessary to transfer data between the host and device, unless managed (shared) memory is used.

# sunrealtype \*N\_VGetHostArrayPointer\_Sycl(N\_Vector v)

This function returns a pointer to the vector host data array.

sunrealtype \*N\_VGetDeviceArrayPointer\_Sycl(N\_Vector v)

This function returns a pointer to the vector device data array.

void N\_VSetHostArrayPointer\_Sycl(sunrealtype \*h\_vdata, N\_Vector v)

This function sets the host array pointer in the vector v.

#### void **N\_VSetDeviceArrayPointer\_Sycl** (sunrealtype \*d\_vdata, N\_Vector v)

This function sets the device array pointer in the vector v.

## void N\_VCopyToDevice\_Sycl(N\_Vector v)

This function copies host vector data to the device.

#### void N\_VCopyFromDevice\_Sycl(N\_Vector v)

This function copies vector data from the device to the host.

```
sunbooleantype N_VIsManagedMemory_Sycl(N Vector v)
```

This function returns SUNTRUE if the vector data is allocated as managed (shared) memory otherwise it returns SUNFALSE.

The following user-callable function is provided to set the execution policies for how SYCL kernels are launched on a device.

```
SUNErrCode N_VSetKernelExecPolicy_Sycl(N_Vector v, SUNSyclExecPolicy *stream_exec_policy, SUNSyclExecPolicy *reduce_exec_policy)
```

This function sets the execution policies which control the kernel parameters utilized when launching the streaming and reduction kernels. By default the vector is setup to use the <code>SUNSyclThreadDirectExecPolicy()</code> and <code>SUNSyclBlockReduceExecPolicy()</code>. See §6.12.2 below for more information about the <code>SUNSyclExecPolicy</code> class.

The input execution policies are cloned and, as such, may be freed after being attached to the desired vectors. A NULL input policy will reset the execution policy to the default setting.

**Note:** All vectors used in a single instance of a SUNDIALS package must use the same execution policy. It is **strongly recommended** that this function is called immediately after constructing the vector, and any subsequent vector be created by cloning to ensure consistent execution policies across vectors.

The following user-callable functions are provided to print the host vector data array. Unless managed memory is used, a user may need to call <code>N\_VCopyFromDevice\_Sycl()</code> to ensure consistency between the host and device array.

```
void N_VPrint_Sycl(N Vector v)
```

This function prints the host data array to stdout.

```
void N_VPrintFile_Sycl(N_Vector v, FILE *outfile)
```

This function prints the host data array to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_SYCL module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with one of the above constructors, enable/disable the desired operations on that vector with the functions below, and then use this vector in conjunction with *N\_VClone()* to create any additional vectors. This guarantees the new vectors will have the same operations enable/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created by any of the constructors above will have the default settings for the NVECTOR SYCL module.

#### SUNErrCode N\_VEnableFusedOps\_Sycl(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the SYCL vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearCombination\_Sycl(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the SYCL vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableScaleAddMulti\_Sycl(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the SYCL vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Sycl(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the SYCL vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_Syc1(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the SYCL vector. The return value is a SUNErrCode.

## SUNErrCode N\_VEnableConstVectorArray\_Sycl(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the SYCL vector. The return value is a SUNErrCode.

# SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Sycl(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the SYCL vector. The return value is a *SUNErrCode*.

## SUNErrCode N\_VEnableLinearCombinationVectorArray\_Sycl(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the SYCL vector. The return value is a *SUNErrCode*.

#### Notes

- When there is a need to access components of an NVECTOR\_SYCL, v, it is recommended to use N\_VGetDe-viceArrayPointer() to access the device array or N\_VGetArrayPointer() for the host array. When using managed (shared) memory, either function may be used. To ensure memory coherency, a user may need to call the CopyTo or CopyFrom functions as necessary to transfer data between the host and device, unless managed (shared) memory is used.
- To maximize efficiency, vector operations in the NVECTOR\_SYCL implementation that have more than one N\_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

# **6.12.2** The SUNSyclExecPolicy Class

In order to provide maximum flexibility to users, the SYCL kernel execution parameters used by kernels within SUN-DIALS are defined by objects of the sundials::sycl::ExecPolicy abstract class type (this class can be accessed in the global namespace as SUNSyclExecPolicy). Thus, users may provide custom execution policies that fit the needs of their problem. The SUNSyclExecPolicy class is defined as

```
typedef sundials::sycl::ExecPolicy SUNSyclExecPolicy
```

where the sundials::sycl::ExecPolicy class is defined in the header file sundials\_sycl\_policies.hpp, as follows:

```
class sundials::sycl::ExecPolicy
  virtual size_t gridSize(size_t numWorkUnits = 0, size_t blockDim = 0)
  virtual size_t blockSize(size_t numWorkUnits = 0, size_t gridDim = 0)
  virtual ExecPolicy *clone() const
```

#### virtual ~ExecPolicy()

For consistency the function names and behavior mirror the execution policies for the CUDA and HIP vectors. In the SYCL case the blockSize is the local work-group range in a one-dimensional nd\_range (threads per group). The gridSize is the number of local work groups so the global work-group range in a one-dimensional nd\_range is blockSize \* gridSize (total number of threads). All vector kernels are written with a many-to-one mapping where work units (vector elements) are mapped in a round-robin manner across the global range. As such, the blockSize and gridSize can be set to any positive value.

To define a custom execution policy, a user simply needs to create a class that inherits from the abstract class and implements the methods. The SUNDIALS provided sundials::sycl::ThreadDirectExecPolicy (aka in the global namespace as SUNSyclThreadDirectExecPolicy) class is a good example of a what a custom execution policy may look like:

```
class ThreadDirectExecPolicy : public ExecPolicy
public:
   ThreadDirectExecPolicy(const size_t blockDim)
      : blockDim_(blockDim)
   {}
   ThreadDirectExecPolicy(const ThreadDirectExecPolicy& ex)
      : blockDim_(ex.blockDim_)
   {}
  virtual size_t gridSize(size_t numWorkUnits = 0, size_t blockDim = 0) const
   {
      return (numWorkUnits + blockSize() - 1) / blockSize();
   }
  virtual size_t blockSize(size_t numWorkUnits = 0, size_t gridDim = 0) const
   {
      return blockDim_;
   }
  virtual ExecPolicy* clone() const
      return static_cast<ExecPolicy*>(new ThreadDirectExecPolicy(*this));
   }
private:
   const size_t blockDim_;
};
```

SUNDIALS provides the following execution policies:

#### **SUNSyclThreadDirectExecPolicy**(const size\_t blockDim)

Is for kernels performing streaming operations and maps each work unit (vector element) to a work-item (thread). Based on the local work-group range (number of threads per group, blockSize) the number of local work-groups (gridSize) is computed so there are enough work-items in the global work-group range (total number of threads, blockSize \* gridSize) for one work unit per work-item (thread).

#### **SUNSyclGridStrideExecPolicy**(const size\_t blockDim, const size\_t gridDim)

Is for kernels performing streaming operations and maps each work unit (vector element) to a workitem (thread) in a round-robin manner so the local work-group range (number of threads per group, blockSize) and the number of local work-groups (gridSize) can be set to any positive value. In this case the global work-group range (total number of threads, blockSize \* gridSize) may be less than the number of work units (vector elements).

# SUNSyclBlockReduceExecPolicy(const size\_t blockDim)

Is for kernels performing a reduction, the local work-group range (number of threads per group, blockSize) and the number of local work-groups (gridSize) can be set to any positive value or the gridSize may be set to 0 in which case the global range is chosen so that there are enough threads for at most two work units per work-item.

By default the NVECTOR\_SYCL module uses the SUNSyclThreadDirectExecPolicy and SUNSyclBlockReduce-ExecPolicy where the default blockDim is determined by querying the device for the max\_work\_group\_size. User may specify different policies by constructing a new SyclExecPolicy and attaching it with N\_VSetKernelExecPolicy\_Sycl(). For example, a policy that uses 128 work-items (threads) per group can be created and attached like so:

```
N_Vector v = N_VNew_Sycl(length, SUNContext sunctx);
SUNSyclThreadDirectExecPolicy thread_direct(128);
SUNSyclBlockReduceExecPolicy block_reduce(128);
flag = N_VSetKernelExecPolicy_Sycl(v, &thread_direct, &block_reduce);
```

These default policy objects can be reused for multiple SUNDIALS data structures (e.g. a SUNMatrix and an  $N_-$ -Vector) since they do not hold any modifiable state information.

# 6.13 The NVECTOR\_RAJA Module

The NVECTOR\_RAJA module is an experimental NVECTOR implementation using the RAJA hardware abstraction layer. In this implementation, RAJA allows for SUNDIALS vector kernels to run on AMD, NVIDIA, or Intel GPU devices. The module is intended for users who are already familiar with RAJA and GPU programming. Building this vector module requires a C++11 compliant compiler and either the NVIDIA CUDA programming environment, the AMD ROCM HIP programming environment, or a compiler that supports the SYCL abstraction layer. When using the AMD ROCM HIP environment, the HIP-clang compiler must be utilized. Users can select which backend to compile with by setting the SUNDIALS\_RAJA\_BACKENDS CMake variable to either CUDA, HIP, or SYCL. Besides the CUDA, HIP, and SYCL backends, RAJA has other backends such as serial, OpenMP, and OpenACC. These backends are not used in this SUNDIALS release.

The vector content layout is as follows:

```
struct _N_VectorContent_Raja
{
   sunindextype length;
   sunbooleantype own_data;
   sunrealtype* host_data;
   sunrealtype* device_data;
   void* priv; /* 'private' data */
};
```

The content members are the vector length (size), a boolean flag that signals if the vector owns the data (i.e., it is in charge of freeing the data), pointers to vector data on the host and the device, and a private data structure which holds the memory management type, which should not be accessed directly.

When instantiated with N\_VNew\_Raja(), the underlying data will be allocated on both the host and the device. Alternatively, a user can provide host and device data arrays by using the N\_VMake\_Raja() constructor. To use managed memory, the constructors N\_VNewManaged\_Raja() and N\_VMakeManaged\_Raja() are provided. Details on each of these constructors are provided below.

The header file to include when using this is nvector\_raja.h. The installed module library to link to is libsundials\_nvectudaraja.lib when using the CUDA backend, libsundials\_nvechipraja.lib when using the HIP backend, and libsundials\_nvecsyclraja.lib when using the SYCL backend. The extension .lib is typically .so for shared libraries .a for static libraries.

# 6.13.1 NVECTOR\_RAJA functions

Unlike other native SUNDIALS vector types, the NVECTOR\_RAJA module does not provide macros to access its member variables. Instead, user should use the accessor functions:

```
sunrealtype *N_VGetHostArrayPointer_Raja(N_Vector v)
```

This function returns pointer to the vector data on the host.

```
sunrealtype *N_VGetDeviceArrayPointer_Raja(N_Vector v)
```

This function returns pointer to the vector data on the device.

```
sunbooleantype N_VIsManagedMemory_Raja(N_Vector v)
```

This function returns a boolean flag indicating if the vector data is allocated in managed memory or not.

The NVECTOR\_RAJA module defines the implementations of all vector operations listed in §6.2, §6.2.2, §6.2.3, and §6.2.4, except for N\_VDotProdMulti(), N\_VWrmsNormVectorArray(), and N\_VWrmsNormMaskVectorArray() as support for arrays of reduction vectors is not yet supported in RAJA. These functions will be added to the NVECTOR\_RAJA implementation in the future. Additionally, the operations N\_VGetArrayPointer() and N\_VSetArrayPointer() are not implemented by the RAJA vector. As such, this vector cannot be used with SUNDIALS direct solvers and preconditioners. The NVECTOR\_RAJA module provides separate functions to access data on the host and on the device. It also provides methods for copying from the host to the device and vice versa. Usage examples of NVECTOR\_RAJA are provided in some example programs for CVODE [34].

The names of vector operations are obtained from those in §6.2, §6.2.2, §6.2.3, and §6.2.4 by appending the suffix \_Raja (e.g. N\_VDestroy\_Raja). The module NVECTOR RAJA provides the following additional user-callable routines:

```
N_Vector N_VNew_Raja(sunindextype vec_length, SUNContext sunctx)
```

This function creates and allocates memory for a RAJA N\_Vector. The memory is allocated on both the host and the device. Its only argument is the vector length.

# N\_Vector N\_VNewManaged\_Raja(sunindextype vec\_length, SUNContext sunctx)

This function creates and allocates memory for a RAJA N\_Vector. The vector data array is allocated in managed memory.

N\_Vector N\_VMake\_Raja(sunindextype length, sunrealtype \*h\_data, sunrealtype \*v\_data, SUNContext sunctx)

This function creates an NVECTOR\_RAJA with user-supplied host and device data arrays. This function does not allocate memory for data itself.

N\_Vector N\_VMakeManaged\_Raja(sunindextype length, sunrealtype \*vdata, SUNContext sunctx)

This function creates an NVECTOR\_RAJA with a user-supplied managed memory data array. This function does not allocate memory for data itself.

*N\_Vector* **N\_VNewWithMemHelp\_Raja**(*sunindextype* length, *sunbooleantype* use\_managed\_mem, *SUNMemoryHelper* helper, *SUNContext* sunctx)

This function creates an NVECTOR\_RAJA with a user-supplied SUNMemoryHelper for allocating/freeing memory.

### N\_Vector N\_VNewEmpty\_Raja()

This function creates a new N\_Vector where the members of the content structure have not been allocated. This utility function is used by the other constructors to create a new vector.

#### void N\_VCopyToDevice\_Raja(N\_Vector v)

This function copies host vector data to the device.

# void N\_VCopyFromDevice\_Raja(N\_Vector v)

This function copies vector data from the device to the host.

#### void N\_VPrint\_Raja(N\_Vector v)

This function prints the content of a RAJA vector to stdout.

#### void N\_VPrintFile\_Raja(N Vector v, FILE \*outfile)

This function prints the content of a RAJA vector to outfile.

By default all fused and vector array operations are disabled in the NVECTOR\_RAJA module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with  $N_{VNew_{Raja}}$ , enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using  $N_{Vlone}$ . This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with  $N_{VNew_{Raja}}$  will have the default settings for the NVECTOR\_RAJA module.

#### SUNErrCode N\_VEnableFusedOps\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the RAJA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearCombination\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the RAJA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the RAJA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the RAJA vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableScaleVectorArray\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the RAJA vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableConstVectorArray\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the RAJA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMultiVectorArray\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the RAJA vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearCombinationVectorArray\_Raja(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the RAJA vector. The return value is a *SUNErrCode*.

#### Notes

• When there is a need to access components of an NVECTOR\_RAJA vector, it is recommended to use functions N\_VGetDeviceArrayPointer\_Raja() or N\_VGetHostArrayPointer\_Raja(). However, when using managed memory, the function N\_VGetArrayPointer() may also be used.

• To maximize efficiency, vector operations in the NVECTOR\_RAJA implementation that have more than one N\_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same internal representations.

# **6.14 The NVECTOR\_KOKKOS Module**

New in version 6.4.0.

The NVECTOR\_KOKKOS N\_Vector implementation provides a vector data structure using Kokkos [25, 50] to support a variety of backends including serial, OpenMP, CUDA, HIP, and SYCL. Since Kokkos is a modern C++ library, the module is also written in modern C++ (it requires C++14) as a header only library. To utilize this N\_Vector users will need to include nvector/nvector\_kokkos.hpp. More instructions on building SUNDIALS with Kokkos enabled are given in §11.4. For instructions on building and using Kokkos, refer to the Kokkos documentation.

# 6.14.1 Using NVECTOR\_KOKKOS

The NVECTOR\_KOKKOS module is defined by the Vector templated class in the sundials::kokkos namespace:

To use the NVECTOR\_KOKKOS module, we construct an instance of the Vector class e.g.,

```
// Vector with extent length using the default execution space
sundials::kokkos::Vector<> x{length, sunctx};

// Vector with extent length using the Cuda execution space
sundials::kokkos::Vector<Kokkos::Cuda> x{length, sunctx};

// Vector based on an existing Kokkos::View
Kokkos::View<> view{"a view", length};
sundials::kokkos::Vector<> x{view, sunctx};

// Vector based on an existing Kokkos::View for device and host
Kokkos::View<Kokkos::Cuda> device_view{"a view", length};
Kokkos::View<Kokkos::HostMirror> host_view{Kokkos::create_mirror_view(device_view)};
sundials::kokkos::Vector<> x{device_view, host_view, sunctx};
```

Instances of the Vector class are implicitly or explicitly (using the Convert() method) convertible to a  $N\_Vector$  e.g.,

No further interaction with a Vector is required from this point, and it is possible to use the *N\_Vector* API to operate on x2 or x3.

**Warning:** N\_VDestroy() should never be called on a N\_Vector that was created via conversion from a sundials::kokkos::Vector. Doing so may result in a double free.

The underlying Vector can be extracted from a N\_Vector using GetVec() e.g.,

```
auto x_vec = GetVec<>(x3);
```

# 6.14.2 NVECTOR\_KOKKOS API

In this section we list the public API of the sundials::kokkos::Vector class.

template < class **ExecutionSpace** = Kokkos::DefaultExecutionSpace, class **MemorySpace** = class *ExecutionSpace*::memory\_space>

class **Vector**: public sundials::impl::BaseNVector, public sundials::ConvertibleTo<N\_Vector>

```
using view_type = Kokkos::View<sunrealtype*, MemorySpace>;
using size_type = typename view_type::size_type;
using host_view_type = typename view_type::HostMirror;
using memory_space = MemorySpace;
using exec_space = typename MemorySpace::execution_space;
using range_policy = Kokkos::RangePolicy<exec_space>;
Vector() = default
```

Default constructor – the vector must be copied or moved to.

Vector(size\_type length, SUNContext sunctx)

Constructs a single Vector which is based on a 1D Kokkos::View with the ExecutionSpace and MemorySpace provided as template arguments.

#### **Parameters**

- **length** length of the vector (i.e., the extent of the View)
- **sunctx** the SUNDIALS simulation context object (*SUNContext*)

**Vector**(*view type* view, SUNContext sunctx)

Constructs a single Vector from an existing Kokkos::View. The View ExecutionSpace and MemorySpace must match the ExecutionSpace and MemorySpace provided as template arguments.

#### **Parameters**

- view A 1D Kokkos::View
- **sunctx** the SUNDIALS simulation context object (*SUNContext*)

**Vector**(*view\_type* view, *host\_view\_type* host\_view, SUNContext sunctx)

Constructs a single Vector from an existing Kokkos::View for the device and the host. The Execution-Space and MemorySpace of the device View must match the ExecutionSpace and MemorySpace provided as template arguments.

# **Parameters**

• view - A 1D Kokkos:: View for the device

```
• host_view - A 1D Kokkos::View that is a Kokkos::HostMirrror for the device view
                   • sunctx – the SUNDIALS simulation context object (SUNContext)
     Vector (Vector &&that_vector) noexcept
           Move constructor.
     Vector (const Vector &that vector)
           Copy constructor. This creates a clone of the Vector, i.e., it creates a new Vector with the same properties,
           such as length, but it does not copy the data.
      Vector & operator=(Vector & & rhs) noexcept
           Move assignment.
      Vector & operator=(const Vector & rhs)
           Copy assignment. This creates a clone of the Vector, i.e., it creates a new Vector with the same properties,
           such as length, but it does not copy the data.
     virtual ~Vector() = default;
           Default destructor.
     size_type Length()
           Get the vector length i.e., extent(0).
     view_type View()
           Get the underlying Kokkos: View for the device.
     host_view_type HostView()
           Get the underlying Kokkos: View for the host.
     operator N_Vector() override
           Implicit conversion to a N_Vector.
     operator N_Vector() const override
           Implicit conversion to a N_Vector.
     N Vector Convert() override
           Explicit conversion to a N_Vector.
     N_Vector Convert() const override
           Explicit conversion to a N_Vector.
template<class VectorType>
inline VectorType *GetVec(N_Vector v)
     Get the Vector wrapped by a N_Vector.
void CopyToDevice(N_Vector v)
     Copy the data from the host view to the device view with Kokkos::deep_copy.
void CopyFromDevice(N_Vector v)
     Copy the data to the host view from the device view with Kokkos::deep_copy.
template<class VectorType>
void CopyToDevice(VectorType &v)
     Copy the data from the host view to the device view with Kokkos::deep_copy.
```

Copy the data to the host view from the device view with Kokkos::deep\_copy.

void CopyFromDevice(VectorType &v)

template<class VectorType>

# 6.15 The NVECTOR\_OPENMPDEV Module

In situations where a user has access to a device such as a GPU for offloading computation, SUNDIALS provides an NVECTOR implementation using OpenMP device offloading, called NVECTOR\_OPENMPDEV.

The NVECTOR\_OPENMPDEV implementation defines the *content* field of the N\_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguousdata array on the host, a pointer to the beginning of a contiguous data array on the device, and a boolean flag own\_data which specifies the ownership of host and device data arrays.

```
struct _N_VectorContent_OpenMPDEV
{
   sunindextype length;
   sunbooleantype own_data;
   sunrealtype *host_data;
   sunrealtype *dev_data;
};
```

The header file to include when using this module is nvector\_openmpdev.h. The installed module library to link to is libsundials\_nvecopenmpdev.lib where .lib is typically .so for shared libraries and .a for static libraries.

# 6.15.1 NVECTOR\_OPENMPDEV accessor macros

The following macros are provided to access the content of an NVECTOR\_OPENMPDEV vector.

#### NV\_CONTENT\_OMPDEV(v)

This macro gives access to the contents of the NVECTOR\_OPENMPDEV N\_vector v.

The assignment v\_cont = NV\_CONTENT\_S(v) sets v\_cont to be a pointer to the NVECTOR\_OPENMPDEV content structure.

Implementation:

```
{\it \#define NV\_CONTENT\_OMPDEV(v) ( (N\_VectorContent\_OpenMPDEV)(v->content) )}
```

# NV\_OWN\_DATA\_OMPDEV(v)

Access the *own\_data* component of the OpenMPDEV N\_Vector v.

The assignment  $v_{data} = NV_DATA_HOST_OMPDEV(v)$  sets  $v_{data}$  to be a pointer to the first component of the data on the host for the  $N_Vector\ v$ .

Implementation:

```
#define NV_OWN_DATA_OMPDEV(v) ( NV_CONTENT_OMPDEV(v)->own_data )
```

## NV\_DATA\_HOST\_OMPDEV(v)

The assignment  $NV_DATA_HOST_OMPDEV(v) = v_data$  sets the host component array of v to be  $v_data$  by storing the pointer  $v_data$ .

Implementation:

```
#define NV_DATA_HOST_OMPDEV(v) ( NV_CONTENT_OMPDEV(v)->host_data )
```

#### NV\_DATA\_DEV\_OMPDEV(v)

The assignment  $v_dev_data = NV_DATA_DEV_OMPDEV(v)$  sets  $v_dev_data$  to be a pointer to the first component of the data on the device for the  $N_Vector v$ . The assignment  $NV_DATA_DEV_OMPDEV(v) = v_dev_data$  sets the device component array of v to be  $v_dev_data$  by storing the pointer  $v_dev_data$ .

Implementation:

```
#define NV_DATA_DEV_OMPDEV(v) ( NV_CONTENT_OMPDEV(v)->dev_data )
```

#### NV\_LENGTH\_OMPDEV(V)

Access the *length* component of the OpenMPDEV N\_Vector v.

The assignment  $v_{len} = NV_{LENGTH_OMPDEV(v)}$  sets  $v_{len}$  to be the length of v. On the other hand, the call  $NV_{LENGTH_OMPDEV(v)} = len_v$  sets the length of v to be  $len_v$ .

#define NV\_LENGTH\_OMPDEV(v) ( NV\_CONTENT\_OMPDEV(v)->length )

# **6.15.2 NVECTOR\_OPENMPDEV functions**

The NVECTOR\_OPENMPDEV module defines OpenMP device offloading implementations of all vector operations listed in §6.2, §6.2.2, §6.2.3, and §6.2.4, except for *N\_VSetArrayPointer()*. As such, this vector cannot be used with the SUNDIALS direct solvers and preconditioners. It also provides methods for copying from the host to the device and vice versa.

The names of the vector operations are obtained from those in §6.2, §6.2.2, §6.2.3, and §6.2.4 by appending the suffix \_OpenMPDEV (e.g. N\_VDestroy\_OpenMPDEV). The module NVECTOR\_OPENMPDEV provides the following additional user-callable routines:

*N\_Vector* **N\_VNew\_OpenMPDEV**(*sunindextype* vec\_length, *SUNContext* sunctx)

This function creates and allocates memory for an NVECTOR\_OPENMPDEV N\_Vector.

*N\_Vector* **N\_VNewEmpty\_OpenMPDEV**(*sunindextype* vec\_length, *SUNContext* sunctx)

This function creates a new NVECTOR\_OPENMPDEV N\_Vector with an empty (NULL) data array.

N\_Vector N\_VMake\_OpenMPDEV(sunindextype vec\_length, sunrealtype \*h\_vdata, sunrealtype \*d\_vdata, SUNContext sunctx)

This function creates an NVECTOR\_OPENMPDEV vector with user-supplied vector data arrays h\_vdata and d\_vdata. This function does not allocate memory for data itself.

```
sunrealtype *N_VGetHostArrayPointer_OpenMPDEV(N_Vector v)
```

This function returns a pointer to the host data array.

```
sunrealtype *N_VGetDeviceArrayPointer_OpenMPDEV(N_Vector v)
```

This function returns a pointer to the device data array.

```
void N_VPrint_OpenMPDEV(N_Vector v)
```

This function prints the content of an NVECTOR\_OPENMPDEV vector to stdout.

```
void N_VPrintFile_OpenMPDEV(N Vector v, FILE *outfile)
```

This function prints the content of an NVECTOR\_OPENMPDEV vector to outfile.

```
void N_VCopyToDevice_OpenMPDEV(N_Vector v)
```

This function copies the content of an NVECTOR\_OPENMPDEV vector's host data array to the device data array.

#### void N\_VCopyFromDevice\_OpenMPDEV(N Vector v)

This function copies the content of an NVECTOR\_OPENMPDEV vector's device data array to the host data array.

By default all fused and vector array operations are disabled in the NVECTOR\_OPENMPDEV module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with N\_VNew\_OpenMPDEV, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using N\_VClone. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with N\_VNew\_OpenMPDEV will have the default settings for the NVECTOR\_OPENMPDEV module.

## SUNErrCode N\_VEnableFusedOps\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the NVECTOR OPENMPDEV vector. The return value is a *SUNErrCode*.

### SUNErrCode N\_VEnableLinearCombination\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the NVECTOR OPENMPDEV vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the NVECTOR\_OPENMPDEV vector. The return value is a <code>SUNErrCode</code>.

#### SUNErrCode N\_VEnableDotProdMulti\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the NVECTOR OPENMPDEV vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableLinearSumVectorArray\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the NVECTOR\_OPENMPDEV vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleVectorArray\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the NVECTOR\_-OPENMPDEV vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableConstVectorArray\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the NVECTOR\_OPENMPDEV vector. The return value is a *SUNErrCode*.

# $SUNErrCode \ {\tt N\_VEnableWrmsNormVectorArray\_OpenMPDEV} (N\_Vector\ v, sunboolean type\ tf)$

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the NVECTOR OPENMPDEV vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the NVECTOR\_OPENMPDEV vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMultiVectorArray\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the NVECTOR\_OPENMPDEV vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearCombinationVectorArray\_OpenMPDEV(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the NVECTOR\_OPENMPDEV vector. The return value is a <code>SUNErrCode</code>.

#### **Notes**

- When looping over the components of an N\_Vector v, it is most efficient to first obtain the component array via h\_data = N\_VGetArrayPointer(v) for the host array or v\_data = N\_VGetDeviceArrayPointer(v) for the device array, or equivalently to use the macros h\_data = NV\_DATA\_HOST\_OMPDEV(v) for the host array or v\_data = NV\_DATA\_DEV\_OMPDEV(v) for the device array, and then access h\_data[i] or v\_data[i] within the loop.
- When accessing individual components of an N\_Vector v on the host remember to first copy the array back from the device with N\_VCopyFromDevice\_OpenMPDEV(v) to ensure the array is up to date.
- N\_VNewEmpty\_OpenMPDEV() and N\_VMake\_OpenMPDEV() set the field own\_data to SUNFALSE. The implementation of N\_VDestroy() will not attempt to free the pointer data for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the data pointers.
- To maximize efficiency, vector operations in the NVECTOR\_OPENMPDEV implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same length.

# **6.16 The NVECTOR\_TRILINOS Module**

The NVECTOR\_TRILINOS module is an NVECTOR wrapper around the Trilinos Tpetra vector. The interface to Tpetra is implemented in the sundials::trilinos::nvector\_tpetra::TpetraVectorInterface class. This class simply stores a reference counting pointer to a Tpetra vector and inherits from an empty structure

```
struct _N_VectorContent_Trilinos {};
```

to interface the C++ class with the NVECTOR C code. A pointer to an instance of this class is kept in the *content* field of the N\_Vector object, to ensure that the Tpetra vector is not deleted for as long as the N\_Vector object exists.

The Tpetra vector type in the sundials::trilinos::nvector\_tpetra::TpetraVectorInterface class is defined as:

typedef Tpetra::Vector<sunrealtype, int, sunindextype> vector\_type;

The Tpetra vector will use the SUNDIALS-specified sunrealtype as its scalar type, int as the local ordinal type, and sunindextype as the global ordinal type. This type definition will use Tpetra's default node type. Available Kokkos node types as of the Trilinos 12.14 release are serial (single thread), OpenMP, Pthread, and CUDA. The default node type is selected when building the Kokkos package. For example, the Tpetra vector will use a CUDA node if Tpetra was built with CUDA support and the CUDA node was selected as the default when Tpetra was built.

The header file to include when using this module is nvector\_trilinos.h. The installed module library to link to is libsundials\_nvectrilinos.lib where .lib is typically .so for shared libraries and .a for static libraries.

# **6.16.1 NVECTOR TRILINOS functions**

The NVECTOR\_TRILINOS module defines implementations of all vector operations listed in §6.2, §6.2.2, §6.2.3, and §6.2.4, except for *N\_VGetArrayPointer()* and *N\_VSetArrayPointer()*. As such, this vector cannot be used with the SUNDIALS direct solvers and preconditioners. When access to raw vector data is needed, it is recommended to extract the Trilinos Tpetra vector first, and then use Tpetra vector methods to access the data. Usage examples of NVECTOR\_TRILINOS are provided in example programs for IDA.

The names of vector operations are obtained from those in §6.2 by appending the suffice \_Trilinos (e.g. N\_VDestroy\_Trilinos). Vector operations call existing Tpetra::Vector methods when available. Vector operations specific to SUNDIALS are implemented as standalone functions in the namespace sundials::trilinos::nvector\_tpetra::TpetraVector, located in the file SundialsTpetraVectorKernels.hpp. The module NVECTOR\_TRILINOS provides the following additional user-callable routines:

Teuchos::RCP<*vector\_type*> **N\_VGetVector\_Trilinos**(N\_Vector v)

This C++ function takes an N\_Vector as the argument and returns a reference counting pointer to the underlying Tpetra vector. This is a standalone function defined in the global namespace.

N\_Vector N\_VMake\_Trilinos (Teuchos::RCP<vector\_type> v)

This C++ function creates and allocates memory for an NVECTOR\_TRILINOS wrapper around a user-provided Tpetra vector. This is a standalone function defined in the global namespace.

#### **Notes**

• The template parameter vector\_type should be set as:

```
typedef sundials::trilinos::nvector_tpetra::TpetraVectorInterface::vector_type vector_type
```

This will ensure that data types used in Tpetra vector match those in SUNDIALS.

- When there is a need to access components of an N\_Vector\_Trilinos v, it is recommeded to extract the
  Trilinos vector object via x\_vec = N\_VGetVector\_Trilinos(v) and then access components using the appropriate Trilinos functions.
- The function N\_VDestroy\_Trilinos only deletes the N\_Vector wrapper. The underlying Tpetra vector object will exist for as long as there is at least one reference to it.

# 6.17 The NVECTOR\_MANYVECTOR Module

The NVECTOR\_MANYVECTOR module is designed to facilitate problems with an inherent data partitioning within a computational node for the solution vector. These data partitions are entirely user-defined, through construction of distinct NVECTOR modules for each component, that are then combined together to form the NVECTOR\_MANYVECTOR. Two potential use cases for this flexibility include:

- A. *Heterogenous computational architectures*: for data partitioning between different computing resources on a node, architecture-specific subvectors may be created for each partition. For example, a user could create one GPU-accelerated component based on *NVECTOR\_CUDA*, and another CPU threaded component based on *NVECTOR\_OPENMP*.
- B. Structure of arrays (SOA) data layouts: for problems that require separate subvectors for each solution component. For example, in an incompressible Navier-Stokes simulation, separate subvectors may be used for velocities and pressure, which are combined together into a single NVECTOR MANYVECTOR for the overall "solution".

The above use cases are neither exhaustive nor mutually exclusive, and the NVECTOR\_MANYVECTOR implementation should support arbitrary combinations of these cases.

The NVECTOR\_MANYVECTOR implementation is designed to work with any NVECTOR subvectors that implement the minimum "standard" set of operations in §6.2.1. Additionally, NVECTOR\_MANYVECTOR sets no limit on the number of subvectors that may be attached (aside from the limitations of using sunindextype for indexing, and standard per-node memory limitations). However, while this ostensibly supports subvectors with one entry each (i.e., one subvector for each solution entry), we anticipate that this extreme situation will hinder performance due to non-stride-one memory accesses and increased function call overhead. We therefore recommend a relatively coarse partitioning of the problem, although actual performance will likely be problem-dependent.

As a final note, in the coming years we plan to introduce additional algebraic solvers and time integration modules that will leverage the problem partitioning enabled by NVECTOR\_MANYVECTOR. However, even at present we anticipate that users will be able to leverage such data partitioning in their problem-defining ODE right-hand side function, DAE or nonlinear solver residual function, preconditioners, or custom <code>SUNLinearSolver</code> or <code>SUNNonlinearSolver</code> modules.

### **6.17.1 NVECTOR MANYVECTOR structure**

The NVECTOR\_MANYVECTOR implementation defines the *content* field of N\_Vector to be a structure containing the number of subvectors comprising the ManyVector, the global length of the ManyVector (including all subvectors), a pointer to the beginning of the array of subvectors, and a boolean flag own\_data indicating ownership of the subvectors that populate subvec\_array.

The header file to include when using this module is nvector\_manyvector.h. The installed module library to link against is libsundials\_nvecmanyvector.lib where .lib is typically .so for shared libraries and .a for static libraries.

### **6.17.2 NVECTOR MANYVECTOR functions**

The NVECTOR\_MANYVECTOR module implements all vector operations listed in §6.2 except for N\_VGetArray-Pointer(), N\_VSetArrayPointer(), N\_VScaleAddMultiVectorArray(), and N\_VLinearCombinationVectorArray(). As such, this vector cannot be used with the SUNDIALS direct solvers and preconditioners. Instead, the NVECTOR\_MANYVECTOR module provides functions to access subvectors, whose data may in turn be accessed according to their NVECTOR implementations.

The names of vector operations are obtained from those in §6.2 by appending the suffix \_ManyVector (e.g. N\_-VDestroy\_ManyVector). The module NVECTOR\_MANYVECTOR provides the following additional user-callable routines:

```
N_Vector N_VNew_ManyVector(sunindextype num_subvectors, N_Vector *vec_array, SUNContext sunctx)
```

This function creates a Many Vector from a set of existing NVECTOR objects.

This routine will copy all N\_Vector pointers from the input vec\_array, so the user may modify/free that pointer array after calling this function. However, this routine does *not* allocate any new subvectors, so the underlying NVECTOR objects themselves should not be destroyed before the ManyVector that contains them.

Upon successful completion, the new ManyVector is returned; otherwise this routine returns NULL (e.g., a memory allocation failure occurred).

Users of the Fortran 2003 interface to this function will first need to use the generic  $N_Vector$  utility functions  $N_VectorArray()$ , and  $N_VectorArray()$  to create the  $N_Vector*$  argument. This is further explained in §4.7.2.5, and the functions are documented in §6.1.1.

```
N_Vector N_VGetSubvector_ManyVector(N_Vector v, sunindextype vec_num)
```

This function returns the *vec\_num* subvector from the NVECTOR array.

```
sunindextype N_VGetSubvectorLocalLength_ManyVector(N Vector v, sunindextype vec num)
```

This function returns the local length of the *vec\_num* subvector from the NVECTOR array.

Usage:

```
local_length = N_VGetSubvectorLocalLength_ManyVector(v, 0);
```

```
sunrealtype *N_VGetSubvectorArrayPointer_ManyVector(N_Vector v, sunindextype vec_num)
```

This function returns the data array pointer for the vec\_num subvector from the NVECTOR array.

If the input  $vec\_num$  is invalid, or if the subvector does not support the N\_VGetArrayPointer operation, then NULL is returned.

# SUNErrCode N\_VSetSubvectorArrayPointer\_ManyVector(sunrealtype \*v\_data, N\_Vector v, sunindextype vec\_num)

This function sets the data array pointer for the *vec\_num* subvector from the NVECTOR array.

The function returns a SUNErrCode.

# sunindextype N\_VGetNumSubvectors\_ManyVector(N\_Vector v)

This function returns the overall number of subvectors in the ManyVector object.

By default all fused and vector array operations are disabled in the NVECTOR\_MANYVECTOR module, except for N\_VWrmsNormVectorArray() and N\_VWrmsNormMaskVectorArray(), that are enabled by default. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with N\_VNew\_ManyVector(), enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using N\_VClone(). This guarantees that the new vectors will have the same operations enabled/disabled, since cloned vectors inherit those configuration options from the vector they are cloned from, while vectors created with N\_VNew\_ManyVector() will have the default settings for the NVECTOR\_MANYVECTOR module. We note that these routines do not call the corresponding routines on subvectors, so those should be set up as desired before attaching them to the ManyVector in N\_VNew\_ManyVector().

### SUNErrCode N\_VEnableFusedOps\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the manyvector vector. The return value is a SUNErrCode.

# SUNErrCode N\_VEnableLinearCombination\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the manyvector vector. The return value is a *SUNErrCode*.

### SUNErrCode N\_VEnableScaleAddMulti\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the manyvector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableDotProdMulti\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the manyvector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableLinearSumVectorArray\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the manyvector vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableScaleVectorArray\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the manyvector vector. The return value is a *SUNErrCode*.

# SUNErrCode N\_VEnableConstVectorArray\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the manyvector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_ManyVector(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the manyvector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_ManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the manyvector vector. The return value is a *SUNErrCode*.

#### **Notes**

- N\_VNew\_ManyVector() sets the field own\_data = SUNFALSE. The ManyVector implementation of N\_VDe-stroy() will not attempt to call N\_VDestroy() on any subvectors contained in the subvector array for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the subvectors.
- To maximize efficiency, arithmetic vector operations in the NVECTOR\_MANYVECTOR implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same subvector representations.

# 6.18 The NVECTOR\_MPIMANYVECTOR Module

The NVECTOR\_MPIMANYVECTOR module is designed to facilitate problems with an inherent data partitioning for the solution vector, and when using distributed-memory parallel architectures. As such, this implementation supports all use cases allowed by the MPI-unaware NVECTOR\_MANYVECTOR implementation, as well as partitioning data between nodes in a parallel environment. These data partitions are entirely user-defined, through construction of distinct NVECTOR modules for each component, that are then combined together to form the NVECTOR\_MPI-MANYVECTOR. Three potential use cases for this module include:

- A. Heterogenous computational architectures (single-node or multi-node): for data partitioning between different computing resources on a node, architecture-specific subvectors may be created for each partition. For example, a user could create one MPI-parallel component based on NVECTOR\_PARALLEL, another GPU-accelerated component based on NVECTOR\_CUDA.
- B. *Process-based multiphysics decompositions (multi-node)*: for computations that combine separate MPI-based simulations together, each subvector may reside on a different MPI communicator, and the MPIManyVector combines these via an MPI *intercommunicator* that connects these distinct simulations together.
- C. Structure of arrays (SOA) data layouts (single-node or multi-node): for problems that require separate subvectors for each solution component. For example, in an incompressible Navier-Stokes simulation, separate subvectors may be used for velocities and pressure, which are combined together into a single MPIMany Vector for the overall "solution".

The above use cases are neither exhaustive nor mutually exclusive, and the NVECTOR\_MANYVECTOR implementation should support arbitrary combinations of these cases.

The NVECTOR\_MPIMANYVECTOR implementation is designed to work with any NVECTOR subvectors that implement the minimum "standard" set of operations in §6.2.1, however significant performance benefits may be obtained when subvectors additionally implement the optional local reduction operations listed in §6.2.4.

Additionally, NVECTOR\_MPIMANYVECTOR sets no limit on the number of subvectors that may be attached (aside from the limitations of using sunindextype for indexing, and standard per-node memory limitations). However, while this ostensibly supports subvectors with one entry each (i.e., one subvector for each solution entry), we anticipate that this extreme situation will hinder performance due to non-stride-one memory accesses and increased function call overhead. We therefore recommend a relatively coarse partitioning of the problem, although actual performance will likely be problem-dependent.

As a final note, in the coming years we plan to introduce additional algebraic solvers and time integration modules that will leverage the problem partitioning enabled by NVECTOR\_MPIMANYVECTOR. However, even at present we anticipate that users will be able to leverage such data partitioning in their problem-defining ODE right-hand side function, DAE or nonlinear solver residual function, preconditioners, or custom <code>SUNLinearSolver</code> or <code>SUNNonlinearSolver</code> modules.

### **6.18.1 NVECTOR MPIMANYVECTOR structure**

The NVECTOR\_MPIMANYVECTOR implementation defines the *content* field of N\_Vector to be a structure containing the MPI communicator (or MPI\_COMM\_NULL if running on a single-node), the number of subvectors comprising the MPIManyVector, the global length of the MPIManyVector (including all subvectors on all MPI ranks), a pointer to the beginning of the array of subvectors, and a boolean flag own\_data indicating ownership of the subvectors that populate subvec\_array.

The header file to include when using this module is nvector\_mpimanyvector.h. The installed module library to link against is libsundials\_nvecmpimanyvector.lib where .lib is typically .so for shared libraries and .a for static libraries.

**Note:** If SUNDIALS is configured with MPI disabled, then the MPIManyVector library will not be built. Furthermore, any user codes that include nvector\_mpimanyvector.h *must* be compiled using an MPI-aware compiler (whether the specific user code utilizes MPI or not). We note that the NVECTOR\_MANYVECTOR implementation is designed for ManyVector use cases in an MPI-unaware environment.

# **6.18.2 NVECTOR\_MPIMANYVECTOR functions**

The NVECTOR\_MPIMANYVECTOR module implements all vector operations listed in §6.2, except for N\_VGetAr-rayPointer(), N\_VScaleAddMultiVectorArray(), and N\_VLinearCombination-VectorArray(). As such, this vector cannot be used with the SUNDIALS direct solvers and preconditioners. Instead, the NVECTOR\_MPIMANYVECTOR module provides functions to access subvectors, whose data may in turn be accessed according to their NVECTOR implementations.

The names of vector operations are obtained from those in §6.2 by appending the suffix \_MPIManyVector (e.g. N\_-VDestroy\_MPIManyVector). The module NVECTOR\_MPIMANYVECTOR provides the following additional user-callable routines:

```
N_Vector N_VNew_MPIManyVector(sunindextype num_subvectors, N_Vector *vec_array, SUNContext sunctx)
```

This function creates a MPIManyVector from a set of existing NVECTOR objects, under the requirement that all MPI-aware subvectors use the same MPI communicator (this is checked internally). If none of the subvectors are MPI-aware, then this may equivalently be used to describe data partitioning within a single node. We note that this routine is designed to support use cases A and C above.

This routine will copy all N\_Vector pointers from the input vec\_array, so the user may modify/free that pointer array after calling this function. However, this routine does *not* allocate any new subvectors, so the underlying NVECTOR objects themselves should not be destroyed before the MPIManyVector that contains them.

Upon successful completion, the new MPIManyVector is returned; otherwise this routine returns NULL (e.g., if two MPI-aware subvectors use different MPI communicators).

Users of the Fortran 2003 interface to this function will first need to use the generic  $N_Vector$  utility functions  $N_VectorArray()$ , and  $N_VectorArray()$  to create the  $N_Vector*$  argument. This is further explained in §4.7.2.5, and the functions are documented in §6.1.1.

# N\_Vector N\_VMake\_MPIManyVector (MPI\_Comm comm, sunindextype num\_subvectors, N\_Vector \*vec\_array, SUNContext sunctx)

This function creates a MPIManyVector from a set of existing NVECTOR objects, and a user-created MPI communicator that "connects" these subvectors. Any MPI-aware subvectors may use different MPI communicators than the input *comm*. We note that this routine is designed to support any combination of the use cases above.

The input *comm* should be this user-created MPI communicator. This routine will internally call MPI\_Comm\_dup to create a copy of the input comm, so the user-supplied comm argument need not be retained after the call to N\_VMake\_MPIManyVector().

If all subvectors are MPI-unaware, then the input *comm* argument should be MPI\_COMM\_NULL, although in this case, it would be simpler to call N\_VNew\_MPIManyVector() instead, or to just use the NVECTOR\_MANYVECTOR module.

This routine will copy all N\_Vector pointers from the input  $vec\_array$ , so the user may modify/free that pointer array after calling this function. However, this routine does not allocate any new subvectors, so the underlying NVECTOR objects themselves should not be destroyed before the MPIManyVector that contains them.

Upon successful completion, the new MPIManyVector is returned; otherwise this routine returns NULL (e.g., if the input *vec\_array* is NULL).

#### N Vector N\_VGetSubvector\_MPIManyVector(N Vector v, sunindextype vec num)

This function returns the *vec\_num* subvector from the NVECTOR array.

# sunindextype N\_VGetSubvectorLocalLength\_MPIManyVector(N\_Vector v, sunindextype vec\_num)

This function returns the local length of the *vec\_num* subvector from the NVECTOR array.

Usage:

```
local_length = N_VGetSubvectorLocalLength_MPIManyVector(v, 0);
```

#### sunrealtype \*N\_VGetSubvectorArrayPointer\_MPIManyVector(N Vector v, sunindextype vec num)

This function returns the data array pointer for the *vec\_num* subvector from the NVECTOR array.

If the input *vec\_num* is invalid, or if the subvector does not support the N\_VGetArrayPointer operation, then NULL is returned.

# SUNErrCode N\_VSetSubvectorArrayPointer\_MPIManyVector(sunrealtype \*v\_data, N\_Vector v, sunindextype vec num)

This function sets the data array pointer for the *vec\_num* subvector from the NVECTOR array.

The function returns a SUNErrCode.

#### sunindextype N\_VGetNumSubvectors\_MPIManyVector(N\_Vector v)

This function returns the overall number of subvectors in the MPIMany Vector object.

By default all fused and vector array operations are disabled in the NVECTOR\_MPIMANYVECTOR module, except for N\_VWrmsNormVectorArray() and N\_VWrmsNormMaskVectorArray(), that are enabled by default. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with N\_VNew\_MPIManyVector() or N\_VMake\_MPIManyVector(), enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using N\_VClone(). This guarantees that the new vectors will have the same operations enabled/disabled, since cloned vectors inherit those configuration options from the vector they are cloned from, while vectors created with N\_VNew\_MPIManyVector() and N\_VMake\_MPIManyVector() will have the default settings for the NVECTOR\_MPIMANYVECTOR module. We note that these routines do not call the corresponding routines on subvectors, so those should be set up as desired before attaching them to the MPIManyVector in N\_VNew\_MPIManyVector() or N\_VMake\_MPIManyVector().

#### SUNErrCode N\_VEnableFusedOps\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the MPI-ManyVector vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableLinearCombination\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the MPI-Many Vector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableScaleAddMulti\_MPIManyVector(N Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the MPIMany Vector vector. The return value is a SUNErrCode.

## SUNErrCode N\_VEnableDotProdMulti\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the MPI-Many Vector vector. The return value is a SUNErrCode.

# SUNErrCode N\_VEnableLinearSumVectorArray\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the MPI-Many Vector vector. The return value is a SUNErrCode.

#### SUNErrCode N\_VEnableScaleVectorArray\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the MPI-ManyVector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableConstVectorArray\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the MPI-ManyVector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormVectorArray\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the MPIManyVector vector. The return value is a *SUNErrCode*.

#### SUNErrCode N\_VEnableWrmsNormMaskVectorArray\_MPIManyVector(N\_Vector v, sunbooleantype tf)

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the MPIManyVector vector. The return value is a *SUNErrCode*.

# Notes

- N\_VNew\_MPIManyVector() and N\_VMake\_MPIManyVector() set the field own\_data = SUNFALSE. The MPI-ManyVector implementation of N\_VDestroy() will not attempt to call N\_VDestroy() on any subvectors contained in the subvector array for any N\_Vector with own\_data set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the subvectors.
- To maximize efficiency, arithmetic vector operations in the NVECTOR\_MPIMANYVECTOR implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same subvector representations.

# 6.19 The NVECTOR MPIPLUSX Module

The NVECTOR\_MPIPLUSX module is designed to facilitate the MPI+X paradigm, where X is some form of on-node (local) parallelism (e.g. OpenMP, CUDA). This paradigm is becoming increasingly popular with the rise of heterogeneous computing architectures.

The NVECTOR\_MPIPLUSX implementation is designed to work with any NVECTOR that implements the minimum "standard" set of operations in §6.2.1. However, it is not recommended to use the NVECTOR\_PARALLEL, NVECTOR\_PARHYP, NVECTOR\_PETSC, or NVECTOR\_TRILINOS implementations underneath the NVECTOR\_MPIPLUSX module since they already provide MPI capabilities.

## **6.19.1 NVECTOR MPIPLUSX structure**

The NVECTOR\_MPIPLUSX implementation is a thin wrapper around the NVECTOR\_MPIMANYVECTOR. Accordingly, it adopts the same content structure as defined in §6.18.1.

The header file to include when using this module is nvector\_mpiplusx.h. The installed module library to link against is libsundials\_nvecmpiplusx.lib where .lib is typically .so for shared libraries and .a for static libraries.

**Note:** If SUNDIALS is configured with MPI disabled, then the mpiplusx library will not be built. Furthermore, any user codes that include nvector\_mpiplusx.h *must* be compiled using an MPI-aware compiler.

#### **6.19.2 NVECTOR MPIPLUSX functions**

The NVECTOR\_MPIPLUSX module adopts all vector operations listed in §6.2, from the NVECTOR\_MPI-MANYVECTOR (see §6.18) except for N\_VGetArrayPointer(), and N\_VSetArrayPointer(); the module provides its own implementation of these functions that call the local vector implementations. Therefore, the NVECTOR\_MPIPLUSX module implements all of the operations listed in the referenced sections except for N\_VScaleAddMul-tiVectorArray(), and N\_VLinearCombinationVectorArray(). Accordingly, it's compatibility with the SUNDIALS direct solvers and preconditioners depends on the local vector implementation.

The module NVECTOR MPIPLUSX provides the following additional user-callable routines:

N\_Vector N\_VMake\_MPIPlusX(MPI\_Comm comm, N\_Vector \*local\_vector, SUNContext sunctx)

This function creates a MPIPlusX vector from an exisiting local (i.e. on node) NVECTOR object, and a user-created MPI communicator.

The input comm should be this user-created MPI communicator. This routine will internally call MPI\_Comm\_dup to create a copy of the input comm, so the user-supplied comm argument need not be retained after the call to  $N_{VMake\_MPIP1usX}()$ .

This routine will copy the NVECTOR pointer to the input local\_vector, so the underlying local NVECTOR object should not be destroyed before the mpiplusx that contains it.

Upon successful completion, the new MPIPlusX is returned; otherwise this routine returns NULL (e.g., if the input *local\_vector* is NULL).

#### N\_Vector N\_VGetLocalVector\_MPIPlusX(N\_Vector v)

This function returns the local vector underneath the MPIPlusX NVECTOR.

#### sunindextype N\_VGetLocalLength\_MPIPlusX(N Vector v)

This function returns the local length of the vector underneath the MPIPlusX NVECTOR.

Usage:

```
local_length = N_VGetLocalLength_MPIPlusX(v);
```

```
sunrealtype *N_VGetArrayPointer_MPIPlusX(N_Vector v)
```

This function returns the data array pointer for the local vector.

If the local vector does not support the N\_VGetArrayPointer() operation, then NULL is returned.

```
void N_VSetArrayPointer_MPIPlusX(sunrealtype *v_data, N_Vector v)
```

This function sets the data array pointer for the local vector if the local vector implements the  $N\_VSetArray$ -Pointer() operation.

The NVECTOR\_MPIPLUSX module does not implement any fused or vector array operations. Instead users should enable/disable fused operations on the local vector.

#### **Notes**

- N\_VMake\_MPIPlusX() sets the field own\_data = SUNFALSE and the MPIPlusX implementation of N\_VDe-stroy() will not call N\_VDestroy() on the local vector. In this a case, it is the user's responsibility to deallocate the local vector.
- To maximize efficiency, arithmetic vector operations in the NVECTOR\_MPIPLUSX implementation that have more than one N\_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N\_Vector arguments that were all created with the same subvector representations.

# **6.20 NVECTOR Examples**

There are NVECTOR examples that may be installed for eac himplementation. Each implementation makes use of the functions in test\_nvector.c. These example functions show simple usage of the NVECTOR family of functions. The input to the examples are the vector length, number of threads (if threaded implementation), and a print timing flag.

The following is a list of the example functions in test\_nvector.c:

- Test\_N\_VClone: Creates clone of vector and checks validity of clone.
- Test\_N\_VCloneEmpty: Creates clone of empty vector and checks validity of clone.
- Test\_N\_VCloneVectorArray: Creates clone of vector array and checks validity of cloned array.
- Test\_N\_VCloneVectorArray: Creates clone of empty vector array and checks validity of cloned array.
- Test\_N\_VGetArrayPointer: Get array pointer.
- Test\_N\_VSetArrayPointer: Allocate new vector, set pointer to new vector array, and check values.
- Test\_N\_VGetLength: Compares self-reported length to calculated length.
- Test\_N\_VGetCommunicator: Compares self-reported communicator to the one used in constructor; or for MPI-unaware vectors it ensures that NULL is reported.
- Test\_N\_VLinearSum Case 1a: Test y = x + y
- Test\_N\_VLinearSum Case 1b: Test y = -x + y
- Test\_N\_VLinearSum Case 1c: Test y = ax + y
- Test\_N\_VLinearSum Case 2a: Test x = x + y
- Test\_N\_VLinearSum Case 2b: Test x = x y
- Test\_N\_VLinearSum Case 2c: Test x = x + by

- Test\_N\_VLinearSum Case 3: Test z = x + y
- Test\_N\_VLinearSum Case 4a: Test z = x y
- Test\_N\_VLinearSum Case 4b: Test z = -x + y
- Test\_N\_VLinearSum Case 5a: Test z = x + by
- Test\_N\_VLinearSum Case 5b: Test z = ax + y
- Test\_N\_VLinearSum Case 6a: Test z = -x + by
- Test\_N\_VLinearSum Case 6b: Test z = ax y
- Test\_N\_VLinearSum Case 7: Test z = a(x + y)
- Test\_N\_VLinearSum Case 8: Test z = a(x y)
- Test\_N\_VLinearSum Case 9: Test z = ax + by
- Test\_N\_VConst: Fill vector with constant and check result.
- Test\_N\_VProd: Test vector multiply: z = x \* y
- Test\_N\_VDiv: Test vector division: z = x / y
- Test\_N\_VScale: Case 1: scale: x = cx
- Test\_N\_VScale: Case 2: copy: z = x
- Test\_N\_VScale: Case 3: negate: z = -x
- Test\_N\_VScale: Case 4: combination: z = cx
- Test\_N\_VAbs: Create absolute value of vector.
- Test\_N\_VInv: Compute z[i] = 1 / x[i]
- \*\* Test\_N\_VAddConst: add constant vector: z = c + x
  - Test\_N\_VDotProd: Calculate dot product of two vectors.
  - Test\_N\_VMaxNorm: Create vector with known values, find and validate the max norm.
  - Test\_N\_VWrmsNorm: Create vector of known values, find and validate the weighted root mean square.
  - Test\_N\_VWrmsNormMask: Create vector of known values, find and validate the weighted root mean square using all elements except one.
  - Test\_N\_VMin: Create vector, find and validate the min.
  - Test\_N\_VWL2Norm: Create vector, find and validate the weighted Euclidean L2 norm.
  - Test\_N\_VL1Norm: Create vector, find and validate the L1 norm.
  - Test\_N\_VCompare: Compare vector with constant returning and validating comparison vector.
  - Test\_N\_VInvTest: Test z[i] = 1 / x[i]
  - Test\_N\_VConstrMask: Test mask of vector x with vector c.
  - Test\_N\_VMinQuotient: Fill two vectors with known values. Calculate and validate minimum quotient.
  - Test\_N\_VLinearCombination: Case 1a: Test x = a x
  - Test\_N\_VLinearCombination: Case 1b: Test z = a x
  - Test\_N\_VLinearCombination: Case 2a: Test x = a x + b y
  - Test\_N\_VLinearCombination: Case 2b: Test z = a x + b y

- Test\_N\_VLinearCombination: Case 3a: Test x = x + a y + b z
- Test\_N\_VLinearCombination: Case 3b: Test x = a x + b y + c z
- Test\_N\_VLinearCombination: Case 3c: Test w = a x + b y + c z
- Test\_N\_VScaleAddMulti: Case 1a: y = a x + y
- Test\_N\_VScaleAddMulti: Case 1b: z = a x + y
- Test\_N\_VScaleAddMulti: Case 2a: Y[i] = c[i] x + Y[i], i = 1,2,3
- Test\_N\_VScaleAddMulti: Case 2b: Z[i] = c[i] x + Y[i], i = 1,2,3
- Test\_N\_VDotProdMulti: Case 1: Calculate the dot product of two vectors
- Test\_N\_VDotProdMulti: Case 2: Calculate the dot product of one vector with three other vectors in a vector array.
- Test\_N\_VLinearSumVectorArray: Case 1: z = a x + b y
- Test\_N\_VLinearSumVectorArray: Case 2a: Z[i] = a X[i] + b Y[i]
- Test\_N\_VLinearSumVectorArray: Case 2b: X[i] = a X[i] + b Y[i]
- Test\_N\_VLinearSumVectorArray: Case 2c: Y[i] = a X[i] + b Y[i]
- Test\_N\_VScaleVectorArray: Case 1a: y = c y
- Test\_N\_VScaleVectorArray: Case 1b: z = c y
- Test\_N\_VScaleVectorArray: Case 2a: Y[i] = c[i] Y[i]
- Test\_N\_VScaleVectorArray: Case 2b: Z[i] = c[i] Y[i]
- Test\_N\_VConstVectorArray: Case 1a: z = c
- Test\_N\_VConstVectorArray: Case 1b: Z[i] = c
- Test\_N\_VWrmsNormVectorArray: Case 1a: Create a vector of know values, find and validate the weighted root mean square norm.
- Test\_N\_VWrmsNormVectorArray: Case 1b: Create a vector array of three vectors of know values, find and validate the weighted root mean square norm of each.
- Test\_N\_VWrmsNormMaskVectorArray: Case 1a: Create a vector of know values, find and validate the weighted root mean square norm using all elements except one.
- Test\_N\_VWrmsNormMaskVectorArray: Case 1b: Create a vector array of three vectors of know values, find and validate the weighted root mean square norm of each using all elements except one.
- Test\_N\_VScaleAddMultiVectorArray: Case 1a: y = a x + y
- Test\_N\_VScaleAddMultiVectorArray: Case 1b: z = a x + y
- Test\_N\_VScaleAddMultiVectorArray: Case 2a: Y[j][0] = a[j] X[0] + Y[j][0]
- Test\_N\_VScaleAddMultiVectorArray: Case 2b: Z[j][0] = a[j] X[0] + Y[j][0]
- Test\_N\_VScaleAddMultiVectorArray: Case 3a: Y[0][i] = a[0] X[i] + Y[0][i]
- Test\_N\_VScaleAddMultiVectorArray: Case 3b: Z[0][i] = a[0] X[i] + Y[0][i]
- Test\_N\_VScaleAddMultiVectorArray: Case 4a: Y[j][i] = a[j] X[i] + Y[j][i]
- Test\_N\_VScaleAddMultiVectorArray: Case 4b: Z[j][i] = a[j] X[i] + Y[j][i]
- Test\_N\_VLinearCombinationVectorArray: Case 1a: x = a x
- Test\_N\_VLinearCombinationVectorArray: Case 1b: z = a x

- Test\_N\_VLinearCombinationVectorArray: Case 2a: x = a x + b y
- Test\_N\_VLinearCombinationVectorArray: Case 2b: z = a x + b y
- Test\_N\_VLinearCombinationVectorArray: Case 3a: x = a x + b y + c z
- Test\_N\_VLinearCombinationVectorArray: Case 3b: w = a x + b y + c z
- Test\_N\_VLinearCombinationVectorArray: Case 4a: X[0][i] = c[0] X[0][i]
- Test\_N\_VLinearCombinationVectorArray: Case 4b: Z[i] = c[0] X[0][i]
- Test\_N\_VLinearCombinationVectorArray: Case 5a: X[0][i] = c[0] X[0][i] + c[1] X[1][i]
- Test\_N\_VLinearCombinationVectorArray: Case 5b: Z[i] = c[0] X[0][i] + c[1] X[1][i]
- Test\_N\_VLinearCombinationVectorArray: Case 6a: X[0][i] = X[0][i] + c[1] X[1][i] + c[2] X[2][i]
- Test\_N\_VLinearCombinationVectorArray: Case 6b: X[0][i] = c[0] X[0][i] + c[1] X[1][i] + c[2] X[2][i]
- Test\_N\_VLinearCombinationVectorArray: Case 6c: Z[i] = c[0] X[0][i] + c[1] X[1][i] + c[2] X[2][i]
- Test\_N\_VDotProdLocal: Calculate MPI task-local portion of the dot product of two vectors.
- Test\_N\_VMaxNormLocal: Create vector with known values, find and validate the MPI task-local portion of the max norm.
- Test\_N\_VMinLocal: Create vector, find and validate the MPI task-local min.
- Test\_N\_VL1NormLocal: Create vector, find and validate the MPI task-local portion of the L1 norm.
- Test\_N\_VWSqrSumLocal: Create vector of known values, find and validate the MPI task-local portion of the weighted squared sum of two vectors.
- Test\_N\_VWSqrSumMaskLocal: Create vector of known values, find and validate the MPI task-local portion of the weighted squared sum of two vectors, using all elements except one.
- Test\_N\_VInvTestLocal: Test the MPI task-local portion of z[i] = 1 / x[i]
- Test\_N\_VConstrMaskLocal: Test the MPI task-local portion of the mask of vector x with vector c.
- Test\_N\_VMinQuotientLocal: Fill two vectors with known values. Calculate and validate the MPI task-local minimum quotient.
- Test\_N\_VMBufSize: Tests for accuracy in the reported buffer size.
- Test\_N\_VMBufPack: Tests for accuracy in the buffer packing routine.
- Test\_N\_VMBufUnpack: Tests for accuracy in the buffer unpacking routine.

# **Chapter 7**

# **Matrix Data Structures**

The SUNDIALS library comes packaged with a variety of SUNMatrix implementations, designed for simulations requiring direct linear solvers for problems in serial or shared-memory parallel environments. SUNDIALS additionally provides a simple interface for generic matrices (akin to a C++ abstract base class). All of the major SUNDIALS packages (CVODE(s), IDA(s), KINSOL, ARKODE), are constructed to only depend on these generic matrix operations, making them immediately extensible to new user-defined matrix objects. For each of the SUNDIALS-provided matrix types, SUNDIALS also provides at least two SUNLinearSolver implementations that factor these matrix objects and use them in the solution of linear systems.

# 7.1 Description of the SUNMATRIX Modules

For problems that involve direct methods for solving linear systems, the SUNDIALS packages not only operate on generic vectors, but also on generic matrices (of type SUNMatrix), through a set of operations defined by the particular SUNMATRIX implementation. Users can provide their own specific implementation of the SUNMATRIX module, particularly in cases where they provide their own N\_Vector and/or linear solver modules, and require matrices that are compatible with those implementations. The generic SUNMatrix operations are described below, and descriptions of the SUNMATRIX implementations provided with SUNDIALS follow.

The generic SUNMatrix type has been modeled after the object-oriented style of the generic *N\_Vector* type. Specifically, a generic SUNMatrix is a pointer to a structure that has an implementation-dependent *content* field containing the description and actual data of the matrix, and an *ops* field pointing to a structure with generic matrix operations.

A *SUNMatrix* is a pointer to the *\_generic\_SUNMatrix* structure:

typedef struct \_generic\_SUNMatrix \*SUNMatrix

struct **\_generic\_SUNMatrix** 

The structure defining the SUNDIALS matrix class.

void \*content

Pointer to matrix-specific member data

struct \_generic\_SUNMatrix\_Ops \*ops

A virtual table of matrix operations provided by a specific implementation

SUNContext sunctx

The SUNDIALS simulation context

The virtual table structure is defined as

```
struct _generic_SUNMatrix_Ops
     The structure defining SUNMatrix operations.
     SUNMatrix ID (*getid)(SUNMatrix)
          The function implementing SUNMatGetID()
     SUNMatrix (*clone)(SUNMatrix)
          The function implementing SUNMatClone()
     void (*destroy)(SUNMatrix)
          The function implementing SUNMatDestroy()
     SUNErrCode (*zero)(SUNMatrix)
          The function implementing SUNMatZero()
     SUNErrCode (*copy)(SUNMatrix, SUNMatrix)
          The function implementing SUNMatCopy()
     SUNErrCode (*scaleadd)(sunrealtype, SUNMatrix, SUNMatrix)
          The function implementing SUNMatScaleAdd()
     SUNErrCode (*scaleaddi)(sunrealtype, SUNMatrix)
          The function implementing SUNMatScaleAddI()
     SUNErrCode (*matvecsetup)(SUNMatrix)
          The function implementing SUNMatMatvecSetup()
     SUNErrCode (*matvec)(SUNMatrix, N_Vector, N_Vector)
          The function implementing SUNMatMatvec()
     SUNErrCode (*space)(SUNMatrix, long int*, long int*)
          The function implementing SUNMatSpace()
```

The generic SUNMATRIX module defines and implements the matrix operations acting on a SUNMatrix. These routines are nothing but wrappers for the matrix operations defined by a particular SUNMATRIX implementation, which are accessed through the *ops* field of the SUNMatrix structure. To illustrate this point we show below the implementation of a typical matrix operation from the generic SUNMATRIX module, namely SUNMatZero, which sets all values of a matrix A to zero, returning a flag denoting a successful/failed operation:

```
SUNErrCode SUNMatZero(SUNMatrix A)
{
  return(A->ops->zero(A));
}
```

§7.2 contains a complete list of all matrix operations defined by the generic SUNMATRIX module. A particular implementation of the SUNMATRIX module must:

- Specify the *content* field of the SUNMatrix object.
- Define and implement a minimal subset of the matrix operations. See the documentation for each SUNDIALS package and/or linear solver to determine which SUNMATRIX operations they require.

Note that the names of these routines should be unique to that implementation in order to permit using more than one SUNMATRIX module (each with different SUNMatrix internal data representations) in the same code.

- Define and implement user-callable constructor and destructor routines to create and free a SUNMatrix with the new *content* field and with *ops* pointing to the new matrix operations.
- Optionally, define and implement additional user-callable routines acting on the newly defined SUNMatrix (e.g., a routine to print the *content* for debugging purposes).

• Optionally, provide accessor macros as needed for that particular implementation to be used to access different parts in the content field of the newly defined SUNMatrix.

To aid in the creation of custom SUNMATRIX modules the generic SUNMATRIX module provides three utility functions SUNMatNewEmpty(), SUNMatCopyOps(), and SUNMatFreeEmpty(). When used in custom SUNMATRIX constructors and clone routines these functions will ease the introduction of any new optional matrix operations to the SUNMATRIX API by ensuring only required operations need to be set and all operations are copied when cloning a matrix.

#### SUNMatrix SUNMatNewEmpty(SUNContext sunctx)

This function allocates a new generic SUNMatrix object and initializes its content pointer and the function pointers in the operations structure to NULL.

#### Return value:

If successful, this function returns a SUNMatrix object. If an error occurs when allocating the object, then this routine will return NULL.

#### SUNErrCode SUNMatCopyOps (SUNMatrix A, SUNMatrix B)

This function copies the function pointers in the ops structure of A into the ops structure of B.

#### **Arguments:**

- A the matrix to copy operations from.
- B the matrix to copy operations to.

#### **Return value:**

A SUNErrCode

# void SUNMatFreeEmpty(SUNMatrix A)

This routine frees the generic SUNMatrix object, under the assumption that any implementation-specific data that was allocated within the underlying content structure has already been freed. It will additionally test whether the ops pointer is NULL, and, if it is not, it will free it as well.

# **Arguments:**

• A – the SUNMatrix object to free

#### type **SUNMatrix\_ID**

Each SUNMATRIX implementation included in SUNDIALS has a unique identifier specified in enumeration and shown in Table 7.1. It is recommended that a user-supplied SUNMATRIX implementation use the SUNMATRIX\_-CUSTOM identifier.

Table 7.1: Identifiers associated with matrix kernels supplied with SUN-DIALS

Matrix ID	Matrix type
SUNMATRIX_BAND	Band $M \times M$ matrix
SUNMATRIX_CUSPARSE	CUDA sparse CSR matrix
SUNMATRIX_CUSTOM	User-provided custom matrix
SUNMATRIX_DENSE	Dense $M \times N$ matrix
SUNMATRIX_GINKGO	SUNMatrix wraper for Ginkgo matrices
SUNMATRIX_MAGMADENSE	Dense $M \times N$ matrix
SUNMATRIX_ONEMKLDENSE	oneMKL dense $M \times N$ matrix
SUNMATRIX_SLUNRLOC	SUNMatrix wrapper for SuperLU_DIST SuperMatrix
SUNMATRIX_SPARSE	Sparse (CSR or CSC) $M \times N$ matrix

# 7.2 Description of the SUNMATRIX operations

For each of the SUNMatrix operations, we give the name, usage of the function, and a description of its mathematical operations below.

#### SUNMatrix ID SUNMatGetID(SUNMatrix A)

Returns the type identifier for the matrix A. It is used to determine the matrix implementation type (e.g. dense, banded, sparse,...) from the abstract SUNMatrix interface. This is used to assess compatibility with SUNDIALS-provided linear solver implementations. Returned values are given in Table 7.1

Usage:

```
id = SUNMatGetID(A);
```

#### SUNMatrix SUNMatClone(SUNMatrix A)

Creates a new SUNMatrix of the same type as an existing matrix *A* and sets the *ops* field. It does not copy the matrix values, but rather allocates storage for the new matrix.

Usage:

```
B = SUNMatClone(A);
```

#### void SUNMatDestroy(SUNMatrix A)

Destroys the SUNMatrix A and frees memory allocated for its internal data.

Usage:

SUNMatDestroy(A);

#### SUNErrCode SUNMatSpace(SUNMatrix A, long int \*lrw, long int \*liw)

Returns the storage requirements for the matrix *A. lrw* contains the number of sunrealtype words and *liw* contains the number of integer words. The return value denotes success/failure of the operation.

This function is advisory only, for use in determining a user's total space requirements; it could be a dummy function in a user-supplied SUNMatrix module if that information is not of interest.

Usage:

```
retval = SUNMatSpace(A, &lrw, &liw);
```

# SUNErrCode SUNMatzero(SUNMatrix A)

Zeros all entries of the SUNMatrix A. The return value denotes the success/failure of the operation:

$$A_{i,j} = 0, \quad i = 1, \dots, m, \ j = 1, \dots, n.$$

Usage:

```
retval = SUNMatZero(A);
```

#### SUNErrCode SUNMatCopy(SUNMatrix A, SUNMatrix B)

Performs the operation *B gets A* for all entries of the matrices *A* and *B*. The return value denotes the success/failure of the operation:

$$B_{i,j} = A_{i,j}, \quad i = 1, \dots, m, \ j = 1, \dots, n.$$

Usage:

```
retval = SUNMatCopy(A,B);
```

#### SUNErrCode SUNMatScaleAdd(sunrealtype c, SUNMatrix A, SUNMatrix B)

Performs the operation A gets cA + B. The return value denotes the success/failure of the operation:

$$A_{i,j} = cA_{i,j} + B_{i,j}, \quad i = 1, \dots, m, \ j = 1, \dots, n.$$

Usage:

```
retval = SUNMatScaleAdd(c, A, B);
```

## SUNErrCode SUNMatScaleAddI(sunrealtype c, SUNMatrix A)

Performs the operation A gets cA + I. The return value denotes the success/failure of the operation:

$$A_{i,j} = cA_{i,j} + \delta_{i,j}, \quad i, j = 1, \dots, n.$$

Usage:

```
retval = SUNMatScaleAddI(c, A);
```

#### SUNErrCode SUNMatMatvecSetup(SUNMatrix A)

Performs any setup necessary to perform a matrix-vector product. The return value denotes the success/failure of the operation. It is useful for SUNMatrix implementations which need to prepare the matrix itself, or communication structures before performing the matrix-vector product.

Usage:

```
retval = SUNMatMatvecSetup(A);
```

#### SUNErrCode SUNMatMatvec(SUNMatrix A, N\_Vector x, N\_Vector y)

Performs the matrix-vector product y gets Ax. It should only be called with vectors x and y that are compatible with the matrix A – both in storage type and dimensions. The return value denotes the success/failure of the operation:

$$y_i = \sum_{j=1}^{n} A_{i,j} x_j, \quad i = 1, \dots, m.$$

Usage:

```
retval = SUNMatMatvec(A, x, y);
```

# 7.3 The SUNMATRIX\_DENSE Module

The dense implementation of the SUNMatrix module, SUNMATRIX\_DENSE, defines the *content* field of SUNMatrix to be the following structure:

```
struct _SUNMatrixContent_Dense {
   sunindextype M;
   sunindextype N;
   sunrealtype *data;
   sunindextype ldata;
   sunrealtype **cols;
};
```

These entries of the *content* field contain the following information:

- M number of rows
- N number of columns
- data pointer to a contiguous block of sunrealtype variables. The elements of the dense matrix are stored columnwise, i.e. the (i,j) element of a dense SUNMatrix object (with  $0 \le i < M$  and  $0 \le j < N$ ) may be accessed via data[j\*M+i].
- ldata length of the data array (= M N).
- cols array of pointers. cols[j] points to the first element of the j-th column of the matrix in the array data. The (i,j) element of a dense SUNMatrix (with  $0 \le i < M$  and  $0 \le j < N$ ) may be accessed may be accessed via cols[j][i].

The header file to be included when using this module is sunmatrix/sunmatrix\_dense.h.

The following macros are provided to access the content of a SUNMATRIX\_DENSE matrix. The prefix SM\_ in the names denotes that these macros are for *SUNMatrix* implementations, and the suffix \_D denotes that these are specific to the *dense* version.

#### SM\_CONTENT\_D(A)

This macro gives access to the contents of the dense SUNMatrix A.

The assignment A\_cont = SM\_CONTENT\_D(A) sets A\_cont to be a pointer to the dense SUNMatrix content structure.

Implementation:

```
#define SM_CONTENT_D(A) ( (SUNMatrixContent_Dense)(A->content) )
```

## $SM_ROWS_D(A)$

Access the number of rows in the dense SUNMatrix A.

This may be used either to retrieve or to set the value. For example, the assignment A\_rows = SM\_ROWS\_D(A) sets A\_rows to be the number of rows in the matrix A. Similarly, the assignment SM\_ROWS\_D(A) = A\_rows sets the number of columns in A to equal A\_rows.

Implementation:

```
#define SM_ROWS_D(A) ( SM_CONTENT_D(A)->M )
```

# $SM_COLUMNS_D(A)$

Access the number of columns in the dense SUNMatrix A.

This may be used either to retrieve or to set the value. For example, the assignment A\_columns = SM\_-COLUMNS\_D(A) sets A\_columns to be the number of columns in the matrix A. Similarly, the assignment SM\_-COLUMNS\_D(A) = A\_columns sets the number of columns in A to equal A\_columns

Implementation:

```
#define SM\_COLUMNS\_D(A) ( SM\_CONTENT\_D(A) -> N )
```

#### $SM_LDATA_D(A)$

Access the total data length in the dense SUNMatrix A.

This may be used either to retrieve or to set the value. For example, the assignment A\_ldata = SM\_LDATA\_D(A) sets A\_ldata to be the length of the data array in the matrix A. Similarly, the assignment SM\_LDATA\_D(A) = A\_ldata sets the parameter for the length of the data array in A to equal A\_ldata.

Implementation:

```
#define SM_LDATA_D(A) ( SM_CONTENT_D(A)->ldata )
```

#### $SM_DATA_D(A)$

This macro gives access to the data pointer for the matrix entries.

The assignment A\_data = SM\_DATA\_D(A) sets A\_data to be a pointer to the first component of the data array for the dense SUNMatrix A. The assignment SM\_DATA\_D(A) = A\_data sets the data array of A to be A\_data by storing the pointer A\_data.

Implementation:

```
#define SM_DATA_D(A) ( SM_CONTENT_D(A)->data )
```

#### SM\_COLS\_D(A)

This macro gives access to the cols pointer for the matrix entries.

The assignment  $A\_cols = SM\_COLS\_D(A)$  sets  $A\_cols$  to be a pointer to the array of column pointers for the dense SUNMatrix A. The assignment  $SM\_COLS\_D(A) = A\_cols$  sets the column pointer array of A to be  $A\_cols$  by storing the pointer  $A\_cols$ .

Implementation:

```
#define SM_COLS_D(A) ( SM_CONTENT_D(A)->cols )
```

#### SM\_COLUMN\_D(A)

This macros gives access to the individual columns of the data array of a dense SUNMatrix.

The assignment col\_j = SM\_COLUMN\_D(A, j) sets col\_j to be a pointer to the first entry of the j-th column of the  $M \times N$  dense matrix A (with  $0 \le j < N$ ). The type of the expression SM\_COLUMN\_D(A, j) is sunreal type \*. The pointer returned by the call SM\_COLUMN\_D(A, j) can be treated as an array which is indexed from 0 to M-1.

Implementation:

```
#define SM_COLUMN_D(A, j) ( (SM_CONTENT_D(A) -> cols)[j] )
```

#### SM\_ELEMENT\_D(A)

This macro gives access to the individual entries of the data array of a dense SUNMatrix.

The assignments SM\_ELEMENT\_D(A,i,j) = a\_ij and a\_ij = SM\_ELEMENT\_D(A,i,j) reference the  $A_{i,j}$  element of the  $M \times N$  dense matrix A (with  $0 \le i < M$  and  $0 \le j < N$ ).

Implementation:

The SUNMATRIX\_DENSE module defines dense implementations of all matrix operations listed in §7.2. Their names are obtained from those in that section by appending the suffix \_Dense (e.g. SUNMatCopy\_Dense). The module SUNMATRIX\_DENSE provides the following additional user-callable routines:

```
SUNMatrix SUNDenseMatrix(sunindextype M, sunindextype N, SUNContext sunctx)
```

This constructor function creates and allocates memory for a dense SUNMatrix. Its arguments are the number of rows, M, and columns, N, for the dense matrix.

```
void SUNDenseMatrix_Print(SUNMatrix A, FILE *outfile)
```

This function prints the content of a dense SUNMatrix to the output stream specified by outfile. Note: std-out or stderr may be used as arguments for outfile to print directly to standard output or standard error, respectively.

#### sunindextype SUNDenseMatrix\_Rows(SUNMatrix A)

This function returns the number of rows in the dense SUNMatrix.

#### sunindextype SUNDenseMatrix\_Columns(SUNMatrix A)

This function returns the number of columns in the dense SUNMatrix.

#### sunindextype SUNDenseMatrix\_LData(SUNMatrix A)

This function returns the length of the data array for the dense SUNMatrix.

```
sunrealtype *SUNDenseMatrix_Data(SUNMatrix A)
```

This function returns a pointer to the data array for the dense SUNMatrix.

```
sunrealtype **SUNDenseMatrix_Cols(SUNMatrix A)
```

This function returns a pointer to the cols array for the dense SUNMatrix.

```
sunrealtype *SUNDenseMatrix_Column(SUNMatrix A, sunindextype j)
```

This function returns a pointer to the first entry of the jth column of the dense SUNMatrix. The resulting pointer should be indexed over the range 0 to M-1.

#### **Notes**

- When looping over the components of a dense SUNMatrix A, the most efficient approaches are to:
  - First obtain the component array via A\_data = SUNDenseMatrix\_Data(A), or equivalently A\_data = SM\_DATA\_D(A), and then access A\_data[i] within the loop.
  - First obtain the array of column pointers via A\_cols = SUNDenseMatrix\_Cols(A), or equivalently A\_cols = SM\_COLS\_D(A), and then access A\_cols[j][i] within the loop.
  - Within a loop over the columns, access the column pointer via A\_colj = SUNDenseMatrix\_Column(A, j) and then to access the entries within that column using A\_colj[i] within the loop.

All three of these are more efficient than using SM\_ELEMENT\_D(A,i,j) within a double loop.

• Within the SUNMatMatvec\_Dense routine, internal consistency checks are performed to ensure that the matrix is called with consistent N\_Vector implementations. These are currently limited to: NVECTOR\_SERIAL, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS. As additional compatible vector implementations are added to SUNDIALS, these will be included within this compatibility check.

# 7.4 The SUNMATRIX\_MAGMADENSE Module

The SUNMATRIX\_MAGMADENSE module interfaces to the MAGMA linear algebra library and can target NVIDIA's CUDA programming model or AMD's HIP programming model [48]. All data stored by this matrix implementation resides on the GPU at all times. The implementation currently supports a standard LAPACK column-major storage format as well as a low-storage format for block-diagonal matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_0} & 0 & \cdots & 0 \\ 0 & \mathbf{A_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A_{n-1}} \end{bmatrix}$$

This matrix implementation is best paired with the SUNLinearSolver\_MagmaDense SUNLinearSolver.

The header file to include when using this module is sunmatrix/sunmatrix\_magmadense.h. The installed library to link to is libsundials\_sunmatrixmagmadense.lib where lib is typically .so for shared libraries and .a for static libraries.

**Warning:** The SUNMATRIX\_MAGMADENSE module is experimental and subject to change.

# 7.4.1 SUNMATRIX\_MAGMADENSE Functions

The SUNMATRIX\_MAGMADENSE module defines GPU-enabled implementations of all matrix operations listed in §7.2.

- SUNMatGetID\_MagmaDense returns SUNMATRIX\_MAGMADENSE
- SUNMatClone\_MagmaDense
- SUNMatDestroy\_MagmaDense
- SUNMatZero\_MagmaDense
- SUNMatCopy\_MagmaDense
- SUNMatScaleAdd\_MagmaDense
- SUNMatScaleAddI\_MagmaDense
- SUNMatMatvecSetup\_MagmaDense
- SUNMatMatvec\_MagmaDense
- SUNMatSpace\_MagmaDense

In addition, the SUNMATRIX\_MAGMADENSE module defines the following implementation specific functions:

SUNMatrix SUNMatrix\_MagmaDense(sunindextype M, sunindextype N, SUNMemoryType memtype, SUNMemoryHelper memhelper, void \*queue, SUNContext sunctx)

This constructor function creates and allocates memory for an  $M \times N$  SUNMATRIX\_MAGMADENSE SUNMatrix.

#### **Arguments:**

- M the number of matrix rows.
- N the number of matrix columns.
- memtype the type of memory to use for the matrix data; can be SUNMEMTYPE\_UVM or SUNMEMTYPE\_-DEVICE.
- memhelper the memory helper used for allocating data.
- queue a cudaStream\_t when using CUDA or a hipStream\_t when using HIP.
- *sunctx* the *SUNContext* object (see §4.2)

#### **Return value:**

If successful, a SUNMatrix object otherwise NULL.

SUNMatrix SUNMatrix\_MagmaDenseBlock(sunindextype nblocks, sunindextype M\_block, sunindextype N\_block, SUNMemoryType memtype, SUNMemoryHelper memhelper, void \*queue, SUNContext sunctx)

This constructor function creates and allocates memory for a block diagonal SUNMATRIX\_MAGMADENSE SUNMatrix with nblocks of size  $M \times N$ .

# **Arguments:**

- *nblocks* the number of matrix rows.
- *M* block the number of matrix rows in each block.

- N block the number of matrix columns in each block.
- memtype the type of memory to use for the matrix data; can be SUNMEMTYPE\_UVM or SUNMEMTYPE\_DEVICE.
- memhelper the memory helper used for allocating data.
- queue a cudaStream\_t when using CUDA or a hipStream\_t when using HIP.
- sunctx the SUNContext object (see §4.2)

#### Return value:

If successful, a SUNMatrix object otherwise NULL.

#### sunindextype SUNMatrix\_MagmaDense\_Rows(SUNMatrix A)

This function returns the number of rows in the SUNMatrix object. For block diagonal matrices, the number of rows is computed as  $M_{\rm block} \times {\rm nblocks}$ .

#### **Arguments:**

• A - a SUNMatrix object.

#### Return value:

If successful, the number of rows in the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

#### sunindextype SUNMatrix\_MagmaDense\_Columns(SUNMatrix A)

This function returns the number of columns in the SUNMatrix object. For block diagonal matrices, the number of columns is computed as  $N_{\text{block}} \times \text{nblocks}$ .

#### **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the number of columns in the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

#### sunindextype SUNMatrix\_MagmaDense\_BlockRows(SUNMatrix A)

This function returns the number of rows in a block of the SUNMatrix object.

#### **Arguments:**

• A - a SUNMatrix object.

#### Return value:

If successful, the number of rows in a block of the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

#### sunindextype SUNMatrix\_MagmaDense\_BlockColumns(SUNMatrix A)

This function returns the number of columns in a block of the SUNMatrix object.

#### **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the number of columns in a block of the SUNMatrix object otherwise SUNMATRIX\_ILL\_-TNPUT.

#### sunindextype SUNMatrix\_MagmaDense\_LData(SUNMatrix A)

This function returns the length of the SUNMatrix data array.

#### **Arguments:**

• A - a SUNMatrix object.

#### Return value:

If successful, the length of the SUNMatrix data array otherwise SUNMATRIX\_ILL\_INPUT.

#### sunindextype SUNMatrix\_MagmaDense\_NumBlocks(SUNMatrix A)

This function returns the number of blocks in the SUNMatrix object.

## **Arguments:**

• A - a SUNMatrix object.

#### Return value:

If successful, the number of blocks in the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

## sunrealtype \*SUNMatrix\_MagmaDense\_Data(SUNMatrix A)

This function returns the SUNMatrix data array.

#### **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the SUNMatrix data array otherwise NULL.

## sunrealtype \*\*SUNMatrix\_MagmaDense\_BlockData(SUNMatrix A)

This function returns an array of pointers that point to the start of the data array for each block in the SUNMatrix.

### **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, an array of data pointers to each of the SUNMatrix blocks otherwise NULL.

## sunrealtype \*SUNMatrix\_MagmaDense\_Block(SUNMatrix A, sunindextype k)

This function returns a pointer to the data array for block k in the SUNMatrix.

#### **Arguments:**

- A a SUNMatrix object.
- k the block index.

## Return value:

If successful, a pointer to the data array for the SUNMatrix block otherwise NULL.

**Note:** No bounds-checking is performed by this function, *j* should be strictly less than *nblocks*.

## sunrealtype \*SUNMatrix\_MagmaDense\_Column(SUNMatrix A, sunindextype j)

This function returns a pointer to the data array for column j in the SUNMatrix.

## **Arguments:**

- A a SUNMatrix object.
- j the column index.

## **Return value:**

If successful, a pointer to the data array for the SUNMatrix column otherwise NULL.

**Note:** No bounds-checking is performed by this function, j should be strictly less than  $nblocks * N_{block}$ .

## sunrealtype \*SUNMatrix\_MagmaDense\_BlockColumn(SUNMatrix A, sunindextype k, sunindextype j)

This function returns a pointer to the data array for column j of block k in the SUNMatrix.

## **Arguments:**

- A a SUNMatrix object.
- k the block index.
- j the column index.

#### Return value:

If successful, a pointer to the data array for the SUNMatrix column otherwise NULL.

**Note:** No bounds-checking is performed by this function, k should be strictly less than nblocks and k should be strictly less than  $N_{block}$ .

## SUNErrCode SUNMatrix\_MagmaDense\_CopyToDevice(SUNMatrix A, sunrealtype \*h\_data)

This function copies the matrix data to the GPU device from the provided host array.

#### **Arguments:**

- *A* − a SUNMatrix object
- $h_{data}$  a host array pointer to copy data from.

#### Return value:

- SUN\_SUCCESS if the copy is successful.
- SUN\_ERR\_ARG\_INCOMPATIBLE if the SUNMatrix is not a SUNMATRIX\_MAGMADENSE matrix.
- SUN\_ERR\_MEM\_FAIL if the copy fails.

## SUNErrCode SUNMatrix\_MagmaDense\_CopyFromDevice(SUNMatrix A, sunrealtype \*h\_data)

This function copies the matrix data from the GPU device to the provided host array.

#### **Arguments:**

- A a SUNMatrix object
- $h_{data}$  a host array pointer to copy data to.

#### Return value:

- SUN\_SUCCESS if the copy is successful.
- SUN\_ERR\_ARG\_INCOMPATIBLE if the SUNMatrix is not a SUNMATRIX\_MAGMADENSE matrix.
- SUN\_ERR\_MEM\_FAIL if the copy fails.

## 7.4.2 SUNMATRIX\_MAGMADENSE Usage Notes

**Warning:** When using the SUNMATRIX\_MAGMADENSE module with a SUNDIALS package (e.g. CVODE), the stream given to matrix should be the same stream used for the NVECTOR object that is provided to the package, and the NVECTOR object given to the SUNMatvec operation. If different streams are utilized, synchronization issues may occur.

# 7.5 The SUNMATRIX\_ONEMKLDENSE Module

The SUNMATRIX\_ONEMKLDENSE module is intended for interfacing with direct linear solvers from the Intel oneAPI Math Kernel Library (oneMKL) using the SYCL (DPC++) programming model. The implementation currently supports a standard LAPACK column-major storage format as well as a low-storage format for block-diagonal matrices,

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_0} & 0 & \cdots & 0 \\ 0 & \mathbf{A_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A_{n-1}} \end{bmatrix}$$

This matrix implementation is best paired with the SUNLinearSolver\_OneMklDense linear solver.

The header file to include when using this class is sunmatrix/sunmatrix\_onemkldense.h. The installed library to link to is libsundials\_sunmatrixonemkldense.lib where lib is typically .so for shared libraries and .a for static libraries.

Warning: The SUNMATRIX\_ONEMKLDENSE class is experimental and subject to change.

## 7.5.1 SUNMATRIX\_ONEMKLDENSE Functions

The SUNMATRIX\_ONEMKLDENSE class defines implementations of the following matrix operations listed in §7.2.

- SUNMatGetID\_OneMklDense returns SUNMATRIX\_ONEMKLDENSE
- SUNMatClone\_OneMklDense
- SUNMatDestroy\_OneMklDense
- SUNMatZero\_OneMklDense
- SUNMatCopy\_OneMklDense
- SUNMatScaleAdd\_OneMklDense
- SUNMatScaleAddI\_OneMklDense
- SUNMatMatvec\_OneMklDense
- SUNMatSpace\_OneMklDense

In addition, the SUNMATRIX\_ONEMKLDENSE class defines the following implementation specific functions.

## 7.5.1.1 Constructors

SUNMatrix SUNMatrix\_OneMklDense(sunindextype M, sunindextype N, SUNMemoryType memtype, SUNMemoryHelper memhelper, sycl::queue \*queue, SUNContext sunctx)

This constructor function creates and allocates memory for an  $M \times N$  SUNMATRIX\_ONEMKLDENSE SUNMatrix.

## **Arguments:**

- M the number of matrix rows.
- N the number of matrix columns.
- *memtype* the type of memory to use for the matrix data; can be SUNMEMTYPE\_UVM or SUNMEMTYPE\_- DEVICE.

- memhelper the memory helper used for allocating data.
- queue the SYCL queue to which operations will be submitted.
- *sunctx* the *SUNContext* object (see §4.2)

## **Return value:**

If successful, a SUNMatrix object otherwise NULL.

SUNMatrix SUNMatrix\_OneMklDenseBlock(sunindextype nblocks, sunindextype M\_block, sunindextype N\_block, SUNMemoryType memtype, SUNMemoryHelper memhelper, sycl::queue \*queue, SUNContext sunctx)

This constructor function creates and allocates memory for a block diagonal SUNMATRIX\_ONEMKLDENSE SUNMatrix with nblocks of size  $M_{block} \times N_{block}$ .

## **Arguments:**

- *nblocks* the number of matrix rows.
- *M\_block* the number of matrix rows in each block.
- *N\_block* the number of matrix columns in each block.
- memtype the type of memory to use for the matrix data; can be SUNMEMTYPE\_UVM or SUNMEMTYPE\_-DEVICE.
- *memhelper* the memory helper used for allocating data.
- queue the SYCL queue to which operations will be submitted.
- *sunctx* the *SUNContext* object (see §4.2)

#### Return value:

If successful, a SUNMatrix object otherwise NULL.

#### 7.5.1.2 Access Matrix Dimensions

## sunindextype SUNMatrix\_OneMklDense\_Rows(SUNMatrix A)

This function returns the number of rows in the SUNMatrix object. For block diagonal matrices, the number of rows is computed as  $M_{\rm block} \times {\rm nblocks}$ .

## **Arguments:**

• A - a SUNMatrix object.

#### Return value:

If successful, the number of rows in the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

## sunindextype SUNMatrix\_OneMklDense\_Columns(SUNMatrix A)

This function returns the number of columns in the SUNMatrix object. For block diagonal matrices, the number of columns is computed as  $N_{\text{block}} \times \text{nblocks}$ .

## **Arguments:**

• A - a SUNMatrix object.

#### Return value:

If successful, the number of columns in the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

#### 7.5.1.3 Access Matrix Block Dimensions

### sunindextype SUNMatrix\_OneMklDense\_NumBlocks(SUNMatrix A)

This function returns the number of blocks in the SUNMatrix object.

## **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the number of blocks in the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

### sunindextype SUNMatrix\_OneMklDense\_BlockRows(SUNMatrix A)

This function returns the number of rows in a block of the SUNMatrix object.

## **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the number of rows in a block of the SUNMatrix object otherwise SUNMATRIX\_ILL\_INPUT.

## sunindextype SUNMatrix\_OneMklDense\_BlockColumns(SUNMatrix A)

This function returns the number of columns in a block of the SUNMatrix object.

## **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the number of columns in a block of the SUNMatrix object otherwise SUNMATRIX\_ILL\_-INPUT.

## 7.5.1.4 Access Matrix Data

#### sunindextype SUNMatrix\_OneMklDense\_LData(SUNMatrix A)

This function returns the length of the SUNMatrix data array.

## **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the length of the SUNMatrix data array otherwise SUNMATRIX\_ILL\_INPUT.

## sunrealtype \*SUNMatrix\_OneMklDense\_Data(SUNMatrix A)

This function returns the SUNMatrix data array.

## **Arguments:**

• A - a SUNMatrix object.

## **Return value:**

If successful, the SUNMatrix data array otherwise NULL.

## sunrealtype \*SUNMatrix\_OneMklDense\_Column(SUNMatrix A, sunindextype j)

This function returns a pointer to the data array for column j in the SUNMatrix.

## **Arguments:**

- A a SUNMatrix object.
- j the column index.

#### **Return value:**

If successful, a pointer to the data array for the SUNMatrix column otherwise NULL.

**Note:** No bounds-checking is performed by this function, j should be strictly less than  $nblocks * N_{block}$ .

## 7.5.1.5 Access Matrix Block Data

## sunindextype SUNMatrix\_OneMklDense\_BlockLData(SUNMatrix A)

This function returns the length of the SUNMatrix data array for each block of the SUNMatrix object.

### **Arguments:**

• A - a SUNMatrix object.

#### **Return value:**

If successful, the length of the SUNMatrix data array for each block otherwise SUNMATRIX\_ILL\_INPUT.

## sunrealtype \*\*SUNMatrix\_OneMklDense\_BlockData(SUNMatrix A)

This function returns an array of pointers that point to the start of the data array for each block in the SUNMatrix.

#### **Arguments:**

• A - a SUNMatrix object.

#### Return value:

If successful, an array of data pointers to each of the SUNMatrix blocks otherwise NULL.

## sunrealtype \*SUNMatrix\_OneMklDense\_Block(SUNMatrix A, sunindextype k)

This function returns a pointer to the data array for block k in the SUNMatrix.

## **Arguments:**

- A a SUNMatrix object.
- k the block index.

## **Return value:**

If successful, a pointer to the data array for the SUNMatrix block otherwise NULL.

**Note:** No bounds-checking is performed by this function, *j* should be strictly less than *nblocks*.

## sunrealtype \*SUNMatrix\_OneMklDense\_BlockColumn(SUNMatrix A, sunindextype k, sunindextype j)

This function returns a pointer to the data array for column j of block k in the SUNMatrix.

## **Arguments:**

- A a SUNMatrix object.
- k the block index.
- j the column index.

## Return value:

If successful, a pointer to the data array for the SUNMatrix column otherwise NULL.

**Note:** No bounds-checking is performed by this function, k should be strictly less than nblocks and k should be strictly less than  $N_{block}$ .

## 7.5.1.6 Copy Data

## SUNErrCode SUNMatrix\_OneMklDense\_CopyToDevice(SUNMatrix A, sunrealtype \*h\_data)

This function copies the matrix data to the GPU device from the provided host array.

## **Arguments:**

- A a SUNMatrix object
- $h_{data}$  a host array pointer to copy data from.

#### **Return value:**

- SUN\_SUCCESS if the copy is successful.
- SUN\_ERR\_ARG\_INCOMPATIBLE if the SUNMatrix is not a SUNMATRIX\_ONEMKLDENSE matrix.
- SUN\_ERR\_MEM\_FAIL if the copy fails.

## SUNErrCode SUNMatrix\_OneMklDense\_CopyFromDevice(SUNMatrix A, sunrealtype \*h\_data)

This function copies the matrix data from the GPU device to the provided host array.

### **Arguments:**

- A a SUNMatrix object
- $h_{data}$  a host array pointer to copy data to.

#### Return value:

- SUN\_SUCCESS if the copy is successful.
- SUN\_ERR\_ARG\_INCOMPATIBLE if the SUNMatrix is not a SUNMATRIX\_ONEMKLDENSE matrix.
- SUN\_ERR\_MEM\_FAIL if the copy fails.

## 7.5.2 SUNMATRIX\_ONEMKLDENSE Usage Notes

**Warning:** The SUNMATRIX\_ONEMKLDENSE class only supports 64-bit indexing, thus SUNDIALS must be built for 64-bit indexing to use this class.

When using the SUNMATRIX\_ONEMKLDENSE class with a SUNDIALS package (e.g. CVODE), the queue given to matrix should be the same stream used for the NVECTOR object that is provided to the package, and the NVECTOR object given to the <code>SUNMatMatvec()</code> operation. If different streams are utilized, synchronization issues may occur.

# 7.6 The SUNMATRIX\_BAND Module

The banded implementation of the SUNMatrix module, SUNMATRIX\_BAND, defines the *content* field of SUNMatrix to be the following structure:

```
struct _SUNMatrixContent_Band {
   sunindextype M;
   sunindextype N;
   sunindextype mu;
   sunindextype ml;
   sunindextype smu;
```

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```
sunindextype ldim;
sunrealtype *data;
sunindextype ldata;
sunrealtype **cols;
};
```

A diagram of the underlying data representation in a banded matrix is shown in Fig. 7.1. A more complete description of the parts of this *content* field is given below:

- M number of rows
- N number of columns (N = M)
- $\operatorname{mu}$   $\operatorname{upper}$  half-bandwidth,  $0 \leq \operatorname{mu} < N$
- ml lower half-bandwidth,  $0 \le ml < N$
- smu storage upper bandwidth, mu  $\leq$  smu < N. The LU decomposition routines in the associated  $SUN-LINSOL\_BAND$  and  $SUNLINSOL\_LAPACKBAND$  modules write the LU factors into the existing storage for the band matrix. The upper triangular factor U, however, may have an upper bandwidth as big as min(N-1, mu+ml) because of partial pivoting. The smu field holds the upper half-bandwidth allocated for the band matrix.
- 1dim leading dimension (ldim  $\geq smu + ml + 1$ )
- data pointer to a contiguous block of sunrealtype variables. The elements of the banded matrix are stored columnwise (i.e. columns are stored one on top of the other in memory). Only elements within the specified half-bandwidths are stored. data is a pointer to ldata contiguous locations which hold the elements within the banded matrix.
- ldata length of the data array (=  $\operatorname{ldim} N$ )
- cols array of pointers. cols[j] is a pointer to the uppermost element within the band in the j-th column. This pointer may be treated as an array indexed from smu-mu (to access the uppermost element within the band in the j-th column) to smu+ml (to access the lowest element within the band in the j-th column). Indices from 0 to smu-mu-1 give access to extra storage elements required by the LU decomposition function. Finally, cols[j][i-j+smu] is the (i, j)-th element with  $j \text{mu} \le i \le j + \text{ml}$ .

The header file to be included when using this module is sunmatrix/sunmatrix\_band.h.

The following macros are provided to access the content of a SUNMATRIX\_BAND matrix. The prefix SM\_ in the names denotes that these macros are for *SUNMatrix* implementations, and the suffix \_B denotes that these are specific to the *banded* version.

#### SM\_CONTENT\_B(A)

This macro gives access to the contents of the banded SUNMatrix A.

The assignment A\_cont = SM\_CONTENT\_B(A) sets A\_cont to be a pointer to the banded SUNMatrix content structure.

Implementation:

```
#define SM_CONTENT_B(A) ( (SUNMatrixContent_Band) (A->content) )
```

## SM\_ROWS\_B(A)

Access the number of rows in the banded SUNMatrix A.

This may be used either to retrieve or to set the value. For example, the assignment A\_rows = SM\_ROWS\_B(A) sets A\_rows to be the number of rows in the matrix A. Similarly, the assignment SM\_ROWS\_B(A) = A\_rows sets the number of columns in A to equal A\_rows.

Implementation:

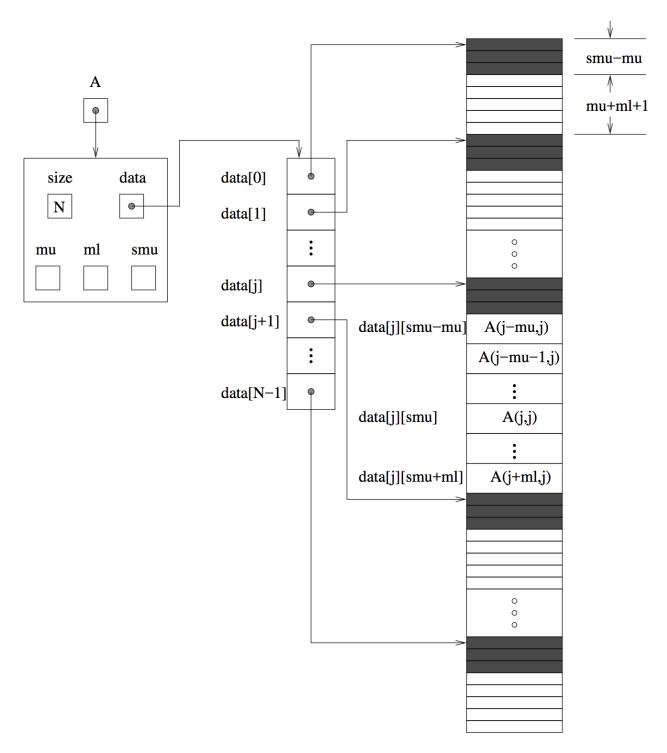


Fig. 7.1: Diagram of the storage for the SUNMATRIX\_BAND module. Here A is an  $N \times N$  band matrix with upper and lower half-bandwidths mu and ml, respectively. The rows and columns of A are numbered from 0 to N-1 and the (i,j)-th element of A is denoted A(i,j). The greyed out areas of the underlying component storage are used by the associated SUNLINSOL\_BAND or SUNLINSOL\_LAPACKBAND linear solver.

```
#define SM_ROWS_B(A) ( SM_CONTENT_B(A)->M )
```

## $SM_COLUMNS_B(A)$

Access the number of columns in the banded SUNMatrix A. As with  $SM_ROWS_B$ , this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_COLUMNS_B(A) ( SM_CONTENT_B(A)->N )
```

## $SM\_UBAND\_B(A)$

Access the mu parameter in the banded SUNMatrix A. As with SM\_ROWS\_B, this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_UBAND_B(A) ( SM_CONTENT_B(A)->mu )
```

## SM\_LBAND\_B(A)

Access the ml parameter in the banded SUNMatrix A. As with SM\_ROWS\_B, this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_LBAND_B(A) ( SM_CONTENT_B(A) -> m1 )
```

## SM\_SUBAND\_B(A)

Access the smu parameter in the banded SUNMatrix A. As with SM\_ROWS\_B, this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_SUBAND_B(A) ( SM_CONTENT_B(A) -> smu )
```

#### $SM_LDIM_B(A)$

Access the ldim parameter in the banded SUNMatrix A. As with SM\_ROWS\_B, this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_LDIM_B(A) ( SM_CONTENT_B(A)->ldim )
```

#### $SM_LDATA_B(A)$

Access the ldata parameter in the banded SUNMatrix A. As with  $SM_ROWS_B$ , this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_LDATA_B(A) ( SM_CONTENT_B(A)->ldata )
```

## SM\_DATA\_B(A)

This macro gives access to the data pointer for the matrix entries.

The assignment A\_data = SM\_DATA\_B(A) sets A\_data to be a pointer to the first component of the data array for the banded SUNMatrix A. The assignment SM\_DATA\_B(A) = A\_data sets the data array of A to be A\_data by storing the pointer A\_data.

Implementation:

```
#define SM_DATA_B(A) ( SM_CONTENT_B(A)->data )
```

#### $SM_COLS_B(A)$

This macro gives access to the cols pointer for the matrix entries.

The assignment  $A\_cols = SM\_COLS\_B(A)$  sets  $A\_cols$  to be a pointer to the array of column pointers for the banded SUNMatrix A. The assignment  $SM\_COLS\_B(A) = A\_cols$  sets the column pointer array of A to be  $A\_cols$  by storing the pointer  $A\_cols$ .

Implementation:

```
#define SM_COLS_B(A) ( SM_CONTENT_B(A)->cols )
```

## SM\_COLUMN\_B(A)

This macros gives access to the individual columns of the data array of a banded SUNMatrix.

The assignment col\_j = SM\_COLUMN\_B(A,j) sets col\_j to be a pointer to the diagonal element of the j-th column of the  $N \times N$  band matrix A,  $0 \le j \le N-1$ . The type of the expression SM\_COLUMN\_B(A,j) is sunrealtype \*. The pointer returned by the call SM\_COLUMN\_B(A,j) can be treated as an array which is indexed from -mu to ml.

Implementation:

```
      \#define SM\_COLUMN\_B(A,j) \quad ( \ ((SM\_CONTENT\_B(A)->cols)[j]) + SM\_SUBAND\_B(A) \ )
```

#### SM\_ELEMENT\_B(A)

This macro gives access to the individual entries of the data array of a banded SUNMatrix.

The assignments SM\_ELEMENT\_B(A,i,j) = a\_ij and a\_ij = SM\_ELEMENT\_B(A,i,j) reference the (i,j)-th element of the  $N \times N$  band matrix A, where  $0 \le i, j \le N-1$ . The location (i,j) should further satisfy  $j - \text{mu} \le i \le j + \text{ml}$ .

Implementation:

```
      \# define \  \, SM\_ELEMENT\_B(A,i,j) \quad ( \ (SM\_CONTENT\_B(A)->cols)[j][(i)-(j)+SM\_SUBAND\_B(A)] \ )
```

## SM\_COLUMN\_ELEMENT\_B(A)

This macro gives access to the individual entries of the data array of a banded SUNMatrix.

The assignments  $SM\_COLUMN\_ELEMENT\_B(col\_j,i,j) = a\_ij$  and  $a\_ij = SM\_COLUMN\_ELEMENT\_B(col\_j,i,j)$  reference the (i,j)-th entry of the band matrix A when used in conjunction with  $SM\_COLUMN\_B$  to reference the j-th column through  $col\_j$ . The index (i,j) should satisfy  $j - mu \le i \le j + ml$ .

Implementation:

```
#define SM_COLUMN_ELEMENT_B(col_j,i,j) (col_j[(i)-(j)])
```

The SUNMATRIX\_BAND module defines banded implementations of all matrix operations listed in §7.2. Their names are obtained from those in that section by appending the suffix \_Band (e.g. SUNMatCopy\_Band). The module SUNMATRIX\_BAND provides the following additional user-callable routines:

```
SUNMatrix SUNBandMatrix(sunindextype N, sunindextype mu, sunindextype ml, SUNContext sunctx)
```

This constructor function creates and allocates memory for a banded SUNMatrix. Its arguments are the matrix size, N, and the upper and lower half-bandwidths of the matrix, mu and ml. The stored upper bandwidth is set to mu+ml to accommodate subsequent factorization in the SUNLINSOL\_BAND and SUNLINSOL\_LAPACK-BAND modules.

# SUNMatrix SUNBandMatrixStorage(sunindextype N, sunindextype mu, sunindextype ml, sunindextype smu, SUNContext sunctx)

This constructor function creates and allocates memory for a banded SUNMatrix. Its arguments are the matrix size, N, the upper and lower half-bandwidths of the matrix, mu and ml, and the stored upper bandwidth, smu. When creating a band SUNMatrix, this value should be

- at least min(N-1,mu+ml) if the matrix will be used by the SUNLinSol\_Band module;
- exactly equal to mu+ml if the matrix will be used by the SUNLinSol LapackBand module;
- at least mu if used in some other manner.

**Note:** It is strongly recommended that users call the default constructor, <code>SUNBandMatrix()</code>, in all standard use cases. This advanced constructor is used internally within SUNDIALS solvers, and is provided to users who require banded matrices for non-default purposes.

#### void **SUNBandMatrix\_Print**(SUNMatrix A, FILE \*outfile)

This function prints the content of a banded SUNMatrix to the output stream specified by outfile. Note: stdout or stderr may be used as arguments for outfile to print directly to standard output or standard error, respectively.

## sunindextype SUNBandMatrix\_Rows(SUNMatrix A)

This function returns the number of rows in the banded SUNMatrix.

## sunindextype SUNBandMatrix\_Columns(SUNMatrix A)

This function returns the number of columns in the banded SUNMatrix.

#### sunindextype SUNBandMatrix\_LowerBandwidth(SUNMatrix A)

This function returns the lower half-bandwidth for the banded SUNMatrix.

## sunindextype SUNBandMatrix\_UpperBandwidth(SUNMatrix A)

This function returns the upper half-bandwidth of the banded SUNMatrix.

## $sunindex type \ {\tt SUNBandMatrix\_StoredUpperBandwidth} (SUNMatrix \ A)$

This function returns the stored upper half-bandwidth of the banded SUNMatrix.

## sunindextype SUNBandMatrix\_LDim(SUNMatrix A)

This function returns the length of the leading dimension of the banded SUNMatrix.

## sunindextype SUNBandMatrix\_LData(SUNMatrix A)

This function returns the length of the data array for the banded SUNMatrix.

## sunrealtype \*SUNBandMatrix\_Data(SUNMatrix A)

This function returns a pointer to the data array for the banded SUNMatrix.

```
sunrealtype **SUNBandMatrix_Cols(SUNMatrix A)
```

This function returns a pointer to the cols array for the band SUNMatrix.

```
sunrealtype *SUNBandMatrix_Column(SUNMatrix A, sunindextype j)
```

This function returns a pointer to the diagonal entry of the j-th column of the banded SUNMatrix. The resulting pointer should be indexed over the range -mu to ml.

**Warning:** When calling this function from the Fortran interfaces the shape of the array that is returned is [1], and the only element you can (legally) access is the diagonal element. Fortran users should instead work with the data array returned by <code>SUNBandMatrix\_Data()</code> directly.

#### **Notes**

- When looping over the components of a banded SUNMatrix A, the most efficient approaches are to:
  - First obtain the component array via A\_data = SUNBandMatrix\_Data(A), or equivalently A\_data = SM\_DATA\_B(A), and then access A\_data[i] within the loop.
  - First obtain the array of column pointers via A\_cols = SUNBandMatrix\_Cols(A), or equivalently A\_cols = SM\_COLS\_B(A), and then access A\_cols[j][i] within the loop.
  - Within a loop over the columns, access the column pointer via A\_colj = SUNBandMatrix\_Column(A,j) and then to access the entries within that column using SM\_COLUMN\_ELEMENT\_B(A\_colj,i,j).

All three of these are more efficient than using SM\_ELEMENT\_B(A,i,j) within a double loop.

Within the SUNMatMatvec\_Band routine, internal consistency checks are performed to ensure that the matrix
is called with consistent N\_Vector implementations. These are currently limited to: NVECTOR\_SERIAL,
NVECTOR\_OPENMP, and NVECTOR\_PTHREADS. As additional compatible vector implementations are
added to SUNDIALS, these will be included within this compatibility check.

## 7.7 The SUNMATRIX\_CUSPARSE Module

The SUNMATRIX\_CUSPARSE module is an interface to the NVIDIA cuSPARSE matrix for use on NVIDIA GPUs [7]. All data stored by this matrix implementation resides on the GPU at all times.

The header file to be included when using this module is sunmatrix/sunmatrix\_cusparse.h. The installed library to link to is libsundials\_sunmatrixcusparse.lib where .lib is typically .so for shared libraries and .a for static libraries.

# 7.7.1 SUNMATRIX\_CUSPARSE Description

The implementation currently supports the cuSPARSE CSR matrix format described in the cuSPARSE documentation, as well as a unique low-storage format for block-diagonal matrices of the form

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_0} & 0 & \cdots & 0 \\ 0 & \mathbf{A_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A_{n-1}} \end{bmatrix},$$

where all the block matrices  $\mathbf{A_j}$  share the same sparsity pattern. We will refer to this format as BCSR (not to be confused with the canonical BSR format where each block is stored as dense). In this format, the CSR column indices and row pointers are only stored for the first block and are computed only as necessary for other blocks. This can drastically reduce the amount of storage required compared to the regular CSR format when the number of blocks is large. This format is well-suited for, and intended to be used with, the SUNLinearSolver\_cuSolverSp\_batchQR linear solver (see §8.17).

The SUNMATRIX\_CUSPARSE module is experimental and subject to change.

## 7.7.2 SUNMATRIX\_CUSPARSE Functions

The SUNMATRIX\_CUSPARSE module defines GPU-enabled sparse implementations of all matrix operations listed in §7.2 except for the SUNMatSpace() and SUNMatMatvecSetup() operations:

- SUNMatGetID\_cuSparse returns SUNMATRIX\_CUSPARSE
- SUNMatClone\_cuSparse
- SUNMatDestroy\_cuSparse
- SUNMatZero\_cuSparse
- SUNMatCopy\_cuSparse
- SUNMatScaleAdd\_cuSparse performs A = cA + B, where A and B must have the same sparsity pattern
- SUNMatScaleAddI\_cuSparse performs A = cA + I, where the diagonal of A must be present
- SUNMatMatvec\_cuSparse

In addition, the SUNMATRIX\_CUSPARSE module defines the following implementation specific functions:

SUNMatrix SUNMatrix\_cuSparse\_NewCSR(int M, int N, int NNZ, cusparseHandle t cusp, SUNContext sunctx)

This constructor function creates and allocates memory for a SUNMATRIX\_CUSPARSE SUNMatrix that uses the CSR storage format. Its arguments are the number of rows and columns of the matrix, M and N, the number of nonzeros to be stored in the matrix, NNZ, and a valid cusparseHandle\_t.

SUNMatrix SUNMatrix\_cuSparse\_NewBlockCSR(int nblocks, int blockrows, int blockcols, int blocknnz, cusparseHandle\_t cusp, SUNContext sunctx)

This constructor function creates and allocates memory for a SUNMATRIX\_CUSPARSE SUNMatrix object that leverages the SUNMAT\_CUSPARSE\_BCSR storage format to store a block diagonal matrix where each block shares the same sparsity pattern. The blocks must be square. The function arguments are the number of blocks, nblocks, the number of rows, blockrows, the number of columns, blockcols, the number of nonzeros in each each block, blocknnz, and a valid cusparseHandle\_t.

**Warning:** The SUNMAT\_CUSPARSE\_BCSR format currently only supports square matrices, i.e., blockrows == blockcols.

SUNMatrix SUNMatrix\_cuSparse\_MakeCSR(cusparseMatDescr\_t mat\_descr, int M, int N, int NNZ, int \*rowptrs, int \*colind, sunrealtype \*data, cusparseHandle\_t cusp, SUNContext sunctx)

This constructor function creates a SUNMATRIX\_CUSPARSE SUNMatrix object from user provided pointers. Its arguments are a cusparseMatDescr\_t that must have index base CUSPARSE\_INDEX\_BASE\_ZERO, the number of rows and columns of the matrix, M and N, the number of nonzeros to be stored in the matrix, NNZ, and a valid cusparseHandle\_t.

## int SUNMatrix\_cuSparse\_Rows(SUNMatrix A)

This function returns the number of rows in the sparse SUNMatrix.

## int SUNMatrix\_cuSparse\_Columns(SUNMatrix A)

This function returns the number of columns in the sparse SUNMatrix.

## int SUNMatrix\_cuSparse\_NNZ(SUNMatrix A)

This function returns the number of entries allocated for nonzero storage for the sparse SUNMatrix.

## int SUNMatrix\_cuSparse\_SparseType(SUNMatrix A)

This function returns the storage type (SUNMAT\_CUSPARSE\_CSR or SUNMAT\_CUSPARSE\_BCSR) for the sparse SUNMatrix.

## sunrealtype \*SUNMatrix\_cuSparse\_Data(SUNMatrix A)

This function returns a pointer to the data array for the sparse SUNMatrix.

## int \*SUNMatrix\_cuSparse\_IndexValues(SUNMatrix A)

This function returns a pointer to the index value array for the sparse SUNMatrix – for the CSR format this is an array of column indices for each nonzero entry. For the BCSR format this is an array of the column indices for each nonzero entry in the first block only.

## int \*SUNMatrix\_cuSparse\_IndexPointers(SUNMatrix A)

This function returns a pointer to the index pointer array for the sparse SUNMatrix – for the CSR format this is an array of the locations of the first entry of each row in the data and indexvalues arrays, for the BCSR format this is an array of the locations of each row in the data and indexvalues arrays in the first block only.

## int SUNMatrix\_cuSparse\_NumBlocks(SUNMatrix A)

This function returns the number of matrix blocks.

## int SUNMatrix\_cuSparse\_BlockRows(SUNMatrix A)

This function returns the number of rows in a matrix block.

## int SUNMatrix\_cuSparse\_BlockColumns(SUNMatrix A)

This function returns the number of columns in a matrix block.

### int SUNMatrix\_cuSparse\_BlockNNZ(SUNMatrix A)

This function returns the number of nonzeros in each matrix block.

## sunrealtype \*SUNMatrix\_cuSparse\_BlockData(SUNMatrix A, int blockidx)

This function returns a pointer to the location in the data array where the data for the block, blockidx, begins. Thus, blockidx must be less than SUNMatrix\_cuSparse\_NumBlocks(A). The first block in the SUNMatrix is index 0, the second block is index 1, and so on.

#### cusparseMatDescr\_t SUNMatrix\_cuSparse\_MatDescr(SUNMatrix A)

This function returns the cusparseMatDescr\_t object associated with the matrix.

# SUNErrCode SUNMatrix\_cuSparse\_CopyToDevice(SUNMatrix A, sunrealtype \*h\_data, int \*h\_idxptrs, int \*h\_idxvals)

This functions copies the matrix information to the GPU device from the provided host arrays. A user may provide NULL for any of h\_data, h\_idxptrs, or h\_idxvals to avoid copying that information.

The function returns SUN\_SUCCESS if the copy operation(s) were successful, or a nonzero error code otherwise.

```
SUNErrCode SUNMatrix_cuSparse_CopyFromDevice(SUNMatrix A, sunrealtype *h_data, int *h_idxptrs, int *h_idxvals)
```

This functions copies the matrix information from the GPU device to the provided host arrays. A user may provide NULL for any of h\_data, h\_idxptrs, or h\_idxvals to avoid copying that information. Otherwise:

- The h\_data array must be at least SUNMatrix\_cuSparse\_NNZ(A)\*sizeof(sunrealtype) bytes.
- The h\_idxptrs array must be at least (SUNMatrix\_cuSparse\_BlockDim(A)+1)\*sizeof(int) bytes.
- The h\_idxvals array must be at least (SUNMatrix\_cuSparse\_BlockNNZ(A))\*sizeof(int) bytes.

The function returns SUN\_SUCCESS if the copy operation(s) were successful, or a nonzero error code otherwise.

### SUNErrCode SUNMatrix\_cuSparse\_SetFixedPattern(SUNMatrix A, sunbooleantype yesno)

This function changes the behavior of the the SUNMatZero operation on the object A. By default the matrix sparsity pattern is not considered to be fixed, thus, the SUNMatZero operation zeros out all data array as well as the indexvalues and indexpointers arrays. Providing a value of 1 or SUNTRUE for the yesno argument changes the behavior of SUNMatZero on A so that only the data is zeroed out, but not the indexvalues or indexpointers arrays. Providing a value of 0 or SUNFALSE for the yesno argument is equivalent to the default behavior.

### SUNErrCode SUNMatrix\_cuSparse\_SetKernelExecPolicy(SUNMatrix A, SUNCudaExecPolicy)\* exec policy)

This function sets the execution policies which control the kernel parameters utilized when launching the CUDA kernels. By default the matrix is setup to use a policy which tries to leverage the structure of the matrix. See §6.10.2 for more information about the SUNCudaExecPolicy class.

## 7.7.3 SUNMATRIX CUSPARSE Usage Notes

The SUNMATRIX\_CUSPARSE module only supports 32-bit indexing, thus SUNDIALS must be built for 32-bit indexing to use this module.

The SUNMATRIX\_CUSPARSE module can be used with CUDA streams by calling the cuSPARSE function cusparseSetStream on the cusparseHandle\_t that is provided to the SUNMATRIX\_CUSPARSE constructor.

**Warning:** When using the SUNMATRIX\_CUSPARSE module with a SUNDIALS package (e.g. ARKODE), the stream given to cuSPARSE should be the same stream used for the NVECTOR object that is provided to the package, and the NVECTOR object given to the SUNMatvec operation. If different streams are utilized, synchronization issues may occur.

# 7.8 The SUNMATRIX SPARSE Module

The sparse implementation of the SUNMatrix module, SUNMATRIX\_SPARSE, is designed to work with either *compressed-sparse-column* (CSC) or *compressed-sparse-row* (CSR) sparse matrix formats. To this end, it defines the *content* field of SUNMatrix to be the following structure:

```
struct _SUNMatrixContent_Sparse {
  sunindextype M;
  sunindextype N;
  sunindextype NNZ;
  sunindextype NP;
  sunrealtype *data;
  int sparsetype;
  sunindextype *indexvals;
  sunindextype *indexptrs;
  /* CSC indices */
  sunindextype **rowvals;
  sunindextype **colptrs;
  /* CSR indices */
  sunindextype **colvals;
  sunindextype **rowptrs;
};
```

A diagram of the underlying data representation in a sparse matrix is shown in Fig. 7.2. A more complete description of the parts of this *content* field is given below:

- M number of rows
- N number of columns
- NNZ maximum number of nonzero entries in the matrix (allocated length of data and indexvals arrays)
- NP number of index pointers (e.g. number of column pointers for CSC matrix). For CSC matrices NP=N, and for CSR matrices NP=M. This value is set automatically at construction based the input choice for sparsetype.
- data pointer to a contiguous block of sunrealtype variables (of length NNZ), containing the values of the nonzero entries in the matrix
- sparsetype type of the sparse matrix (CSC\_MAT or CSR\_MAT)
- indexvals pointer to a contiguous block of int variables (of length NNZ), containing the row indices (if CSC) or column indices (if CSR) of each nonzero matrix entry held in data
- indexptrs pointer to a contiguous block of int variables (of length NP+1). For CSC matrices each entry provides the index of the first column entry into the data and indexvals arrays, e.g. if indexptr[3]=7, then the first nonzero entry in the fourth column of the matrix is located in data[7], and is located in row indexvals[7] of the matrix. The last entry contains the total number of nonzero values in the matrix and hence points one past the end of the active data in the data and indexvals arrays. For CSR matrices, each entry provides the index of the first row entry into the data and indexvals arrays.

The following pointers are added to the SUNMATRIX\_SPARSE content structure for user convenience, to provide a more intuitive interface to the CSC and CSR sparse matrix data structures. They are set automatically when creating a sparse SUNMatrix, based on the sparse matrix storage type.

- rowvals pointer to indexvals when sparsetype is CSC\_MAT, otherwise set to NULL.
- colptrs pointer to indexptrs when sparsetype is CSC\_MAT, otherwise set to NULL.
- colvals pointer to indexvals when sparsetype is CSR\_MAT, otherwise set to NULL.
- rowptrs pointer to indexptrs when sparsetype is CSR\_MAT, otherwise set to NULL.

For example, the  $5 \times 4$  matrix

$$\left[\begin{array}{cccc} 0 & 3 & 1 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 7 & 0 & 0 \\ 1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

could be stored as a CSC matrix in this structure as either

```
M = 5;
N = 4;
NNZ = 8;
NP = N;
data = {3.0, 1.0, 3.0, 7.0, 1.0, 2.0, 9.0, 5.0};
sparsetype = CSC_MAT;
indexvals = {1, 3, 0, 2, 0, 1, 3, 4};
indexptrs = {0, 2, 4, 5, 8};
```

or

```
M = 5;

N = 4;

NNZ = 10;

NP = N;
```

(continues on next page)

(continued from previous page)

```
data = {3.0, 1.0, 3.0, 7.0, 1.0, 2.0, 9.0, 5.0, *, *};
sparsetype = CSC_MAT;
indexvals = {1, 3, 0, 2, 0, 1, 3, 4, *, *};
indexptrs = {0, 2, 4, 5, 8};
```

where the first has no unused space, and the second has additional storage (the entries marked with \* may contain any values). Note in both cases that the final value in indexptrs is 8, indicating the total number of nonzero entries in the matrix.

Similarly, in CSR format, the same matrix could be stored as

```
M = 5;
N = 4;
NNZ = 8;
NP = M;
data = {3.0, 1.0, 3.0, 2.0, 7.0, 1.0, 9.0, 5.0};
sparsetype = CSR_MAT;
indexvals = {1, 2, 0, 3, 1, 0, 3, 3};
indexptrs = {0, 2, 4, 5, 7, 8};
```

The header file to be included when using this module is sunmatrix/sunmatrix\_sparse.h.

The following macros are provided to access the content of a SUNMATRIX\_SPARSE matrix. The prefix SM\_ in the names denotes that these macros are for *SUNMatrix* implementations, and the suffix \_S denotes that these are specific to the *sparse* version.

#### $SM_CONTENT_S(A)$

This macro gives access to the contents of the sparse SUNMatrix A.

The assignment A\_cont = SM\_CONTENT\_S(A) sets A\_cont to be a pointer to the sparse SUNMatrix content structure.

Implementation:

```
#define SM_CONTENT_S(A) ( (SUNMatrixContent_Sparse)(A->content) )
```

## $SM_ROWS_S(A)$

Access the number of rows in the sparse SUNMatrix A.

This may be used either to retrieve or to set the value. For example, the assignment  $A_rows = SM_ROWS_S(A)$  sets  $A_rows$  to be the number of rows in the matrix A. Similarly, the assignment  $SM_ROWS_S(A) = A_rows$  sets the number of columns in A to equal  $A_rows$ .

Implementation:

```
#define SM_ROWS_S(A) ( SM_CONTENT_S(A)->M )
```

## SM\_COLUMNS\_S(A)

Access the number of columns in the sparse SUNMatrix A. As with SM\_ROWS\_S, this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_COLUMNS_S(A) ( SM_CONTENT_S(A) -> N )
```

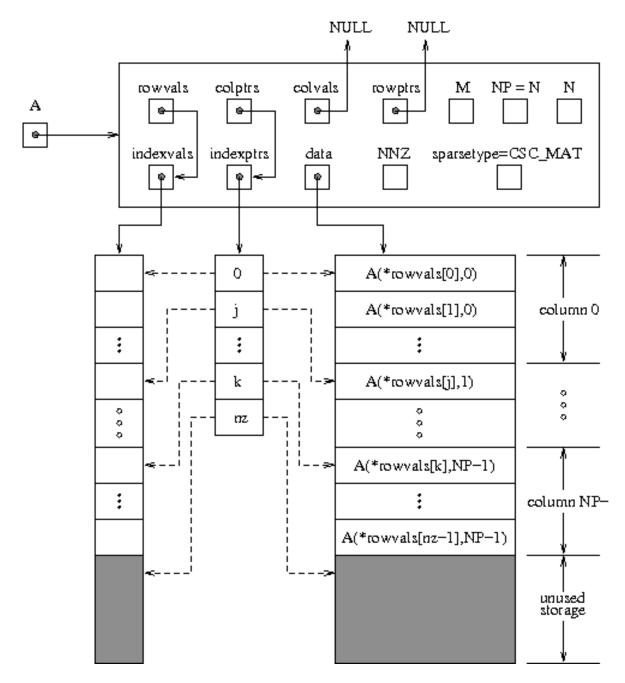


Fig. 7.2: Diagram of the storage for a compressed-sparse-column matrix of type SUNMATRIX\_SPARSE: Here A is an  $M \times N$  sparse CSC matrix with storage for up to NNZ nonzero entries (the allocated length of both data and indexvals). The entries in indexvals may assume values from 0 to M-1, corresponding to the row index (zero-based) of each nonzero value. The entries in data contain the values of the nonzero entries, with the row i, column j entry of A (again, zero-based) denoted as A(i,j). The indexptrs array contains N+1 entries; the first N denote the starting index of each column within the indexvals and data arrays, while the final entry points one past the final nonzero entry. Here, although NNZ values are allocated, only nz are actually filled in; the greyed-out portions of data and indexvals indicate extra allocated space.

#### $SM_NNZ_S(A)$

Access the allocated number of nonzeros in the sparse SUNMatrix A. As with  $SM_ROWS_S$ , this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_NNZ_S(A) ( SM_CONTENT_S(A)->NNZ )
```

## $SM_NP_S(A)$

Access the number of index pointers NP in the sparse SUNMatrix A. As with SM\_ROWS\_S, this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_NP_S(A) ( SM_CONTENT_S(A)->NP )
```

## SM\_SPARSETYPE\_S(A)

Access the sparsity type parameter in the sparse SUNMatrix A. As with SM\_ROWS\_S, this may be used either to retrieve or to set the value.

Implementation:

```
#define SM_SPARSETYPE_S(A) ( SM_CONTENT_S(A)->sparsetype )
```

## SM\_DATA\_S(A)

This macro gives access to the data pointer for the matrix entries.

The assignment  $A_{data} = SM_DATA_S(A)$  sets  $A_{data}$  to be a pointer to the first component of the data array for the sparse SUNMatrix A. The assignment  $SM_DATA_S(A) = A_{data}$  sets the data array of A to be  $A_{data}$  by storing the pointer  $A_{data}$ .

Implementation:

```
#define SM_DATA_S(A) ( SM_CONTENT_S(A)->data )
```

## SM\_INDEXVALS\_S(A)

This macro gives access to the indexvals pointer for the matrix entries.

The assignment A\_indexvals = SM\_INDEXVALS\_S(A) sets A\_indexvals to be a pointer to the array of index values (i.e. row indices for a CSC matrix, or column indices for a CSR matrix) for the sparse SUNMatrix A.

Implementation:

```
#define SM_INDEXVALS_S(A) ( SM_CONTENT_S(A)->indexvals )
```

## SM\_INDEXPTRS\_S(A)

This macro gives access to the indexptrs pointer for the matrix entries.

The assignment A\_indexptrs = SM\_INDEXPTRS\_S(A) sets A\_indexptrs to be a pointer to the array of index pointers (i.e. the starting indices in the data/indexvals arrays for each row or column in CSR or CSC formats, respectively).

Implementation:

```
#define SM_INDEXPTRS_S(A) ( SM_CONTENT_S(A)->indexptrs )
```

The SUNMATRIX\_SPARSE module defines sparse implementations of all matrix operations listed in §7.2. Their names are obtained from those in that section by appending the suffix \_Sparse (e.g. SUNMatCopy\_Sparse). The module SUNMATRIX\_SPARSE provides the following additional user-callable routines:

# SUNMatrix SUNSparseMatrix (sunindextype M, sunindextype N, sunindextype NNZ, int sparsetype, SUNContext sunctx)

This constructor function creates and allocates memory for a sparse SUNMatrix. Its arguments are the number of rows and columns of the matrix, M and N, the maximum number of nonzeros to be stored in the matrix, NNZ, and a flag *sparsetype* indicating whether to use CSR or CSC format (valid choices are CSR\_MAT or CSC\_MAT).

## SUNMatrix SUNSparseFromDenseMatrix(SUNMatrix A, sunrealtype droptol, int sparsetype)

This constructor function creates a new sparse matrix from an existing SUNMATRIX\_DENSE object by copying all values with magnitude larger than *droptol* into the sparse matrix structure.

### Requirements:

- A must have type SUNMATRIX\_DENSE
- droptol must be non-negative
- sparsetype must be either CSC\_MAT or CSR\_MAT

The function returns NULL if any requirements are violated, or if the matrix storage request cannot be satisfied.

## SUNMatrix SUNSparseFromBandMatrix (SUNMatrix A, sunrealtype droptol, int sparsetype)

This constructor function creates a new sparse matrix from an existing SUNMATRIX\_BAND object by copying all values with magnitude larger than *droptol* into the sparse matrix structure.

## Requirements:

- A must have type SUNMATRIX\_BAND
- droptol must be non-negative
- *sparsetype* must be either CSC\_MAT or CSR\_MAT.

The function returns NULL if any requirements are violated, or if the matrix storage request cannot be satisfied.

## SUNErrCode SUNSparseMatrix\_Realloc(SUNMatrix A)

This function reallocates internal storage arrays in a sparse matrix so that the resulting sparse matrix has no wasted space (i.e. the space allocated for nonzero entries equals the actual number of nonzeros, indexptrs[NP]). Returns a SUNErrCode.

## SUNErrCode SUNSparseMatrix\_Reallocate(SUNMatrix A, sunindextype NNZ)

Function to reallocate internal sparse matrix storage arrays so that the resulting sparse matrix has storage for a specified number of nonzeros. Returns a *SUNErrCode*.

## void SUNSparseMatrix\_Print(SUNMatrix A, FILE \*outfile)

This function prints the content of a sparse SUNMatrix to the output stream specified by outfile. Note: std-out or stderr may be used as arguments for outfile to print directly to standard output or standard error, respectively.

#### sunindextype SUNSparseMatrix\_Rows(SUNMatrix A)

This function returns the number of rows in the sparse SUNMatrix.

## sunindextype SUNSparseMatrix\_Columns(SUNMatrix A)

This function returns the number of columns in the sparse SUNMatrix.

## sunindextype SUNSparseMatrix\_NNZ(SUNMatrix A)

This function returns the number of entries allocated for nonzero storage for the sparse SUNMatrix.

## sunindextype SUNSparseMatrix\_NP(SUNMatrix A)

This function returns the number of index pointers for the sparse SUNMatrix (the indexptrs array has NP+1 entries).

### int SUNSparseMatrix\_SparseType(SUNMatrix A)

This function returns the storage type (CSR\_MAT or CSC\_MAT) for the sparse SUNMatrix.

```
sunrealtype *SUNSparseMatrix_Data(SUNMatrix A)
```

This function returns a pointer to the data array for the sparse SUNMatrix.

```
sunindextype *SUNSparseMatrix_IndexValues(SUNMatrix A)
```

This function returns a pointer to index value array for the sparse SUNMatrix – for CSR format this is the column index for each nonzero entry, for CSC format this is the row index for each nonzero entry.

```
sunindextype *SUNSparseMatrix_IndexPointers(SUNMatrix A)
```

This function returns a pointer to the index pointer array for the sparse SUNMatrix – for CSR format this is the location of the first entry of each row in the data and indexvalues arrays, for CSC format this is the location of the first entry of each column.

**Note:** Within the SUNMatMatvec\_Sparse routine, internal consistency checks are performed to ensure that the matrix is called with consistent N\_Vector implementations. These are currently limited to: NVECTOR\_SERIAL, NVECTOR\_OPENMP, NVECTOR\_PTHREADS, and NVECTOR\_CUDA when using managed memory. As additional compatible vector implementations are added to SUNDIALS, these will be included within this compatibility check.

# 7.9 The SUNMATRIX\_SLUNRLOC Module

The SUNMATRIX\_SLUNRLOC module is an interface to the SuperMatrix structure provided by the SuperLU\_DIST sparse matrix factorization and solver library written by X. Sherry Li and collaborators [8, 30, 41, 42]. It is designed to be used with the SuperLU\_DIST SUNLinearSolver module discussed in §8.15. To this end, it defines the content field of SUNMatrix to be the following structure:

A more complete description of the this content field is given below:

- own\_data a flag which indicates if the SUNMatrix is responsible for freeing A\_super
- grid pointer to the SuperLU\_DIST structure that stores the 2D process grid
- row\_to\_proc a mapping between the rows in the matrix and the process it resides on; will be NULL until the SUNMatMatvecSetup routine is called
- gsmv\_comm pointer to the SuperLU\_DIST structure that stores the communication information needed for matrix-vector multiplication; will be NULL until the SUNMatMatvecSetup routine is called
- A\_super pointer to the underlying SuperLU\_DIST SuperMatrix with Stype = SLU\_NR\_loc, Dtype = SLU\_D, Mtype = SLU\_GE; must have the full diagonal present to be used with SUNMatScaleAddI routine
- ACS\_super a column-sorted version of the matrix needed to perform matrix-vector multiplication; will be NULL until the routine SUNMatMatvecSetup routine is called

The header file to include when using this module is sunmatrix/sunmatrix\_slunrloc.h. The installed module library to link to is libsundials\_sunmatrixslunrloc.lib where .lib is typically .so for shared libraries and .a for static libraries.

## 7.9.1 SUNMATRIX\_SLUNRLOC Functions

The SUNMATRIX SLUNRLOC module provides the following user-callable routines:

SUNMatrix SUNMatrix\_SLUNRloc(SuperMatrix \*Asuper, gridinfo\_t \*grid, SUNContext sunctx)

This constructor function creates and allocates memory for a SUNMATRIX\_SLUNRLOC object. Its arguments are a fully-allocated SuperLU\_DIST SuperMatrix with Stype = SLU\_NR\_loc, Dtype = SLU\_D, Mtype = SLU\_GE and an initialized SuperLU\_DIST 2D process grid structure. It returns a SUNMatrix object if Asuper is compatible else it returns NULL.

## void SUNMatrix\_SLUNRloc\_Print(SUNMatrix A, FILE \*fp)

This function prints the underlying SuperMatrix content. It is useful for debugging. Its arguments are the SUNMatrix object and a FILE pointer to print to. It returns void.

## SuperMatrix \*SUNMatrix\_SLUNRloc\_SuperMatrix(SUNMatrix A)

This function returns the underlying SuperMatrix of A. Its only argument is the SUNMatrix object to access.

## gridinfo\_t \*SUNMatrix\_SLUNRloc\_ProcessGrid(SUNMatrix A)

This function returns the SuperLU\_DIST 2D process grid associated with A. Its only argument is the SUNMatrix object to access.

#### sunbooleantype SUNMatrix\_SLUNRloc\_OwnData(SUNMatrix A)

This function returns true if the SUNMatrix object is responsible for freeing the underlying SuperMatrix, otherwise it returns false. Its only argument is the SUNMatrix object to access.

The SUNMATRIX\_SLUNRLOC module also defines implementations of all generic SUNMatrix operations listed in §7.2:

- SUNMatGetID\_SLUNRloc returns SUNMATRIX\_SLUNRLOC
- SUNMatClone\_SLUNRloc
- SUNMatDestroy\_SLUNRloc
- SUNMatSpace\_SLUNRloc this only returns information for the storage within the matrix interface, i.e. storage for row\_to\_proc
- SUNMatZero\_SLUNRloc
- SUNMatCopy\_SLUNRloc
- SUNMatScaleAdd\_SLUNRloc performs A = cA + B, where A and B must have the same sparsity pattern
- SUNMatScaleAddI\_SLUNRloc performs A = cA + I, where the diagonal of A must be present
- SUNMatMatvecSetup\_SLUNRloc initializes the SuperLU\_DIST parallel communication structures needed to perform a matrix-vector product; only needs to be called before the first call to SUNMatMatvec() or if the matrix changed since the last setup
- SUNMatMatvec\_SLUNRloc

# 7.10 The SUNMATRIX\_GINKGO Module

New in version 6.4.0.

The SUNMATRIX\_GINKGO implementation of the SUNMatrix API provides an interface to the matrix data structure for the Ginkgo linear algebra library [12]. Ginkgo provides several different matrix formats and linear solvers which can run on a variety of hardware, such as NVIDIA, AMD, and Intel GPUs as well as multicore CPUs. Since Ginkgo is a modern C++ library, SUNMATRIX\_GINKGO is also written in modern C++ (it requires C++14). Unlike most other SUNDIALS modules, it is a header only library. To use the SUNMATRIX\_GINKGO SUNMatrix, users will need to include sunmatrix/sunmatrix\_ginkgo.hpp. More instructions on building SUNDIALS with Ginkgo enabled are given in §11.4. For instructions on building and using Ginkgo itself, refer to the Ginkgo website and documentation.

**Note:** It is assumed that users of this module are aware of how to use Ginkgo. This module does not try to encapsulate Ginkgo matrices, rather it provides a lightweight iteroperability layer between Ginkgo and SUNDIALS.

The SUNMATRIX\_GINKGO module is defined by the sundials::ginkgo::Matrix templated class:

```
template<typename GkoMatType>
class Matrix : public sundials::impl::BaseMatrix, public sundials::ConvertibleTo<SUNMatrix>;
```

## 7.10.1 Compatible Vectors

The N\_Vector to use with the SUNLINEARSOLVER\_GINKGO module depends on the gko::Executor utilized. That is, when using the gko::CudaExecutor you should use a CUDA capable N\_Vector (e.g., §6.10), gko::HipExecutor goes with a HIP capable N\_Vector (e.g., §6.11), gko::DpcppExecutor goes with a DPC++/SYCL capable N\_Vector (e.g., §6.12), and gko::OmpExecutor goes with a CPU based N\_Vector (e.g., §6.6). Specifically, what makes a N\_Vector compatible with different Ginkgo executors is where they store the data. The GPU enabled Ginkgo executors need the data to reside on the GPU, so the N\_Vector must implement N\_VGetDeviceArray-Pointer() and keep the data in GPU memory. The CPU-only enabled Ginkgo executors (e.g., gko::OmpExecutor and gko::ReferenceExecutor) need data to reside on the CPU and will use N\_VGetArrayPointer() to access the N\_Vector data.

## 7.10.2 Using SUNMATRIX\_GINKGO

To use the SUNMATRIX\_GINKGO module, we begin by creating an instance of a Ginkgo matrix using Ginkgo's API. For example, below we create a Ginkgo sparse matrix that uses the CSR storage format and then fill the diagonal of the matrix with ones to make an identity matrix:

```
auto gko_matrix{gko::matrix::Csr<sunrealtype, sunindextype>::create(gko_exec, matrix_dim)};
gko_matrix->read(gko::matrix_data<sunrealtype, sunindextype>::diag(matrix_dim, 1.0));
```

After we have a Ginkgo matrix object, we wrap it in an instance of the sundials::ginkgo::Matrix class. This object can be provided to other SUNDIALS functions that expect a SUNMatrix object via implicit conversion, or the Convert() method:

```
sundials::ginkgo::Matrix<gko::matrix::Csr> matrix{gko_matrix, sunctx};
SUNMatrix I1 = matrix.Convert(); // explicit conversion to SUNMatrix
SUNMatrix I2 = matrix; // implicit conversion to SUNMatrix
```

No further interaction with matrix is required from this point, and it is possible to to use the SUNMatrix API operating on I1 or I2 (or if needed, via Ginkgo operations on gko\_matrix).

**Warning:** SUNMatDestroy() should never be called on a SUNMatrix that was created via conversion from a sundials::ginkgo::Matrix. Doing so may result in a double free.

## 7.10.3 SUNMATRIX\_GINKGO API

```
In this section we list the public API of the sundials::ginkgo::Matrix class.
```

#### template<typename **GkoMatType**>

class Matrix: public sundials::impl::BaseMatrix, public sundials::ConvertibleTo<SUNMatrix>

Matrix() = default

Default constructor - means the matrix must be copied or moved to.

Matrix(std::shared\_ptr<GkoMatType> gko\_mat, SUNContext sunctx)

Constructs a Matrix from an existing Ginkgo matrix object.

## **Parameters**

- **gko\_mat** A Ginkgo matrix object
- **sunctx** The SUNDIALS simulation context object (SUNContext)

**Matrix** (*Matrix* &&that\_matrix) noexcept

Move constructor.

Matrix (const *Matrix* &that\_matrix)

Copy constructor (performs a deep copy).

*Matrix* & operator=(*Matrix* & & rhs) noexcept

Move assignment.

*Matrix* & operator=(const *Matrix* & rhs)

Copy assignment clones the gko::matrix and SUNMatrix. This is a deep copy (i.e. a new data array is created).

virtual ~Matrix() = default;

Default destructor.

std::shared\_ptr<*GkoMatType*> **GkoMtx**() const

Get the underlying Ginkgo matrix object.

std::shared\_ptr<const gko::Executor> **GkoExec()** const

Get the gko::Executor associated with the Ginkgo matrix.

const gko::dim<2> &GkoSize() const

Get the size, i.e. gko::dim, for the Ginkgo matrix.

operator SUNMatrix() override

Implicit conversion to a SUNMatrix.

operator SUNMatrix() const override

Implicit conversion to a SUNMatrix.

SUNMatrix Convert() override

Explicit conversion to a SUNMatrix.

SUNMatrix Convert() const override

Explicit conversion to a SUNMatrix.

# 7.11 The SUNMATRIX\_KOKKOSDENSE Module

New in version 6.4.0.

The SUNMATRIX\_KOKKOSDENSE *SUNMatrix* implementation provides a data structure for dense and dense batched (block-diagonal) matrices using Kokkos [25, 50] and KokkosKernels [49] to support a variety of backends including serial, OpenMP, CUDA, HIP, and SYCL. Since Kokkos is a modern C++ library, the module is also written in modern C++ (it requires C++14) as a header only library. To utilize this SUNMatrix users will need to include sunmatrix/sunmatrix\_kokkosdense.hpp. More instructions on building SUNDIALS with Kokkos and KokkosKernels enabled are given in §11.4. For instructions on building and using Kokkos and KokkosKernels, refer to the Kokkos and KokkosKernels, documentation.

## 7.11.1 Using SUNMATRIX\_KOKKOSDENSE

The SUNMATRIX\_KOKKOSDENSE module is defined by the DenseMatrix templated class in the sundials::kokkos namespace:

To use the SUNMATRIX\_KOKKOSDENSE module, we begin by constructing an instance of the Kokkos dense matrix e.g.,

Instances of the DenseMatrix class are implicitly or explicitly (using the *Convert()* method) convertible to a *SUN-Matrix* e.g.,

No further interaction with a DenseMatrix is required from this point, and it is possible to use the *SUNMatrix* API to operate on B or C.

**Warning:** SUNMatDestroy() should never be called on a SUNMatrix that was created via conversion from a sundials::kokkos::DenseMatrix. Doing so may result in a double free.

The underlying DenseMatrix can be extracted from a SUNMatrix using GetDenseMat() e.g.,

```
auto A_dense_mat = GetDenseMat<>(A_sunmat);
```

The SUNMATRIX\_KOKKOSDENSE module is compatible with the NVECTOR\_KOKKOS vector module (see §6.14) and SUNLINEARSOLVER\_KOKKOSDENSE linear solver module (see §8.19).

## 7.11.2 SUNMATRIX KOKKOSDENSE API

In this section we list the public API of the sundials::kokkos::DenseMatrix class.

template<class **ExeccutionSpace** = Kokkos::DefaultExecutionSpace, class **MemorySpace** = typename ExecutionSpace::memory\_space>

class **DenseMatrix**: public sundials::impl::BaseMatrix, public sundials::ConvertibleTo<SUNMatrix>

```
using exec_space = ExecutionSpace;
using memory_space = MemorySpace;
using view_type = Kokkos::View<sunrealtype***, memory_space>;
using size_type = typename view_type::size_type;
using range_policy = Kokkos::MDRangePolicy<exec_space, Kokkos::Rank<3>>;
using team_policy = typename Kokkos::TeamPolicy<exec_space>;
using member_type = typename Kokkos::TeamPolicy<exec_space>::member_type;
DenseMatrix() = default
```

Default constructor – the matrix must be copied or moved to.

DenseMatrix(size\_type rows, size\_type cols, SUNContext sunctx)

Constructs a single DenseMatrix using the default execution space instance.

## **Parameters**

- rows number of matrix rows
- cols number of matrix columns
- **sunctx** the SUNDIALS simulation context object (SUNContext)

**DenseMatrix**(*size\_type* rows, *size\_type* cols, *exec\_space* ex, SUNContext sunctx)

Constructs a single DenseMatrix using the provided execution space instance.

## **Parameters**

- rows number of matrix rows
- cols number of matrix columns
- **ex** an execuation space
- **sunctx** the SUNDIALS simulation context object (*SUNContext*)

**DenseMatrix**(size\_type blocks, size\_type block\_rows, size\_type block\_cols, SUNContext sunctx)

Constructs a batched (block-diagonal) DenseMatrix using the default execution space instance.

#### **Parameters**

• blocks – number of matrix blocks

- block\_rows number of rows in a block
- block\_cols number of columns in a block
- **sunctx** the SUNDIALS simulation context object (*SUNContext*)

**DenseMatrix**(*size\_type* blocks, *size\_type* block\_rows, *size\_type* block\_cols, *exec\_space* ex, SUNContext sunctx)

Constructs a batched (block-diagonal) DenseMatrix using the provided execution space instance.

#### **Parameters**

- blocks number of matrix blocks
- block rows number of rows in a block
- block\_cols number of columns in a block
- **ex** an execuation space
- **sunctx** the SUNDIALS simulation context object (SUNContext)

## DenseMatrix(DenseMatrix &&that\_matrix) noexcept

Move constructor.

## **DenseMatrix**(const *DenseMatrix* &that\_matrix)

Copy constructor. This creates a shallow clone of the Matrix, i.e., it creates a new Matrix with the same properties, such as size, but it does not copy the data.

## DenseMatrix & operator=(DenseMatrix & & rhs) noexcept

Move assignment.

## DenseMatrix & operator=(const DenseMatrix & rhs)

Copy assignment. This creates a shallow clone of the Matrix, i.e., it creates a new Matrix with the same properties, such as size, but it does not copy the data.

## virtual ~DenseMatrix() = default;

Default destructor.

## exec\_space ExecSpace()

Get the execution space instance used by the matrix.

```
view_type View()
```

Get the underlying Kokkos view with extents {blocks, block\_rows, block\_cols}.

## size\_type Blocks()

Get the number of blocks i.e., extent(0).

## size\_type BlockRows()

Get the number of rows in a block i.e., extent(1).

## size\_type BlockCols()

Get the number of columns in a block i.e., extent(2).

## size\_type Rows()

Get the number of rows in the block-diagonal matrix i.e., extent(0) \* extent(1).

## size\_type Cols()

Get the number of columns in the block-diagonal matrix i.e., extent(0) \* extent(2).

# 7.12 SUNMATRIX Examples

There are SUNMatrix examples that may be installed for each implementation, that make use of the functions in test\_sunmatrix.c. These example functions show simple usage of the SUNMatrix family of functions. The inputs to the examples depend on the matrix type, and are output to stdout if the example is run without the appropriate number of command-line arguments.

The following is a list of the example functions in test\_sunmatrix.c:

- Test\_SUNMatGetID: Verifies the returned matrix ID against the value that should be returned.
- Test\_SUNMatClone: Creates clone of an existing matrix, copies the data, and checks that their values match.
- Test\_SUNMatZero: Zeros out an existing matrix and checks that each entry equals 0.0.
- Test\_SUNMatCopy: Clones an input matrix, copies its data to a clone, and verifies that all values match.
- Test\_SUNMatScaleAdd: Given an input matrix A and an input identity matrix I, this test clones and copies A to a new matrix B, computes B = -B + B, and verifies that the resulting matrix entries equal 0. Additionally, if the matrix is square, this test clones and copies A to a new matrix D, clones and copies I to a new matrix C, computes D = D + I and C = C + A using SUNMatScaleAdd(), and then verifies that C = D.
- Test\_SUNMatScaleAddI: Given an input matrix A and an input identity matrix I, this clones and copies I to a new matrix B, computes B = -B + I using SUNMatScaleAddI(), and verifies that the resulting matrix entries equal 0.
- Test\_SUNMatMatvecSetup: verifies that SUNMatMatvecSetup() can be called.
- Test\_SUNMatMatvec Given an input matrix A and input vectors x and y such that y = Ax, this test has different behavior depending on whether A is square. If it is square, it clones and copies A to a new matrix B, computes B = 3B + I using SUNMatScaleAddI(), clones y to new vectors w and z, computes z = Bx using SUNMatMatvec(), computes z = Bx using N\_VLinearSum, and verifies that z = z. If z = Ax is not square, it just clones z = Ax using SUNMatMatvec(), and verifies that z = Ax using SUNMatMatvec(), and verifies that z = Ax
- Test\_SUNMatSpace: verifies that SUNMatSpace() can be called, and outputs the results to stdout.

# 7.13 SUNMatrix functions used by KINSOL

In Table 7.2, we list the matrix functions in the SUNMatrix module used within the KINSOL package. The table also shows, for each function, which of the code modules uses the function. The main KINSOL integrator does not call any SUNMatrix functions directly, so the table columns are specific to the KINLS and KINBBDPRE preconditioner modules. We further note that the KINLS interface only utilizes these routines when supplied with a *matrix-based* linear solver, i.e., the SUNMatrix object passed to *KINSetLinearSolver()* was not NULL.

At this point, we should emphasize that the KINSOL user does not need to know anything about the usage of matrix functions by the KINSOL code modules in order to use KINSOL. The information is presented as an implementation detail for the interested reader.

Table 7.2: List of matrix functions usage by KINSOL code modules

	KINLS	KINBBDPRE
<pre>SUNMatGetID()</pre>	X	
<pre>SUNMatDestroy()</pre>		X
SUNMatZero()	X	X
<pre>SUNMatSpace()</pre>		†

The matrix functions listed with a † symbol are optionally used, in that these are only called if they are implemented in the SUNMatrix module that is being used (i.e. their function pointers are non-NULL). The matrix functions listed in §7.1 that are *not* used by KINSOL are: SUNMatCopy(), SUNMatClone(), SUNMatScaleAdd(), SUNMatScaleAdd() and SUNMatMatvec(). Therefore a user-supplied SUNMatrix module for KINSOL could omit these functions.

We note that the KINBBDPRE preconditioner module is hard-coded to use the SUNDIALS-supplied band SUNMatrix type, so the most useful information above for user-supplied SUNMatrix implementations is the column relating the KINLS requirements.

# **Chapter 8**

# **Linear Algebraic Solvers**

For problems that require the solution of linear systems of equations, the SUNDIALS packages operate using generic linear solver modules defined through the <code>SUNLinearSolver</code>, or "SUNLinSol", API. This allows SUNDIALS packages to utilize any valid SUNLinSol implementation that provides a set of required functions. These functions can be divided into three categories. The first are the core linear solver functions. The second group consists of "set" routines to supply the linear solver object with functions provided by the SUNDIALS package, or for modification of solver parameters. The last group consists of "get" routines for retrieving artifacts (statistics, residual vectors, etc.) from the linear solver. All of these functions are defined in the header file <code>sundials/sundials\_linearsolver.h</code>.

The implementations provided with SUNDIALS work in coordination with the SUNDIALS *N\_Vector*, and optionally *SUNMatrix*, modules to provide a set of compatible data structures and solvers for the solution of linear systems using direct or iterative (matrix-based or matrix-free) methods. Moreover, advanced users can provide a customized SUNLinearSolver implementation to any SUNDIALS package, particularly in cases where they provide their own *N\_Vector* and/or SUNMatrix modules.

Historically, the SUNDIALS packages have been designed to specifically leverage the use of either *direct linear solvers* or matrix-free, *scaled, preconditioned, iterative linear solvers*. However, matrix-based iterative linear solvers are also supported.

The iterative linear solvers packaged with SUNDIALS leverage scaling and preconditioning, as applicable, to balance error between solution components and to accelerate convergence of the linear solver. To this end, instead of solving the linear system Ax = b directly, these apply the underlying iterative algorithm to the transformed system

$$\tilde{A}\tilde{x} = \tilde{b} \tag{8.1}$$

where

$$\tilde{A} = S_1 P_1^{-1} A P_2^{-1} S_2^{-1},$$

$$\tilde{b} = S_1 P_1^{-1} b,$$

$$\tilde{x} = S_2 P_2 x,$$
(8.2)

and where

- $P_1$  is the left preconditioner,
- $P_2$  is the right preconditioner,
- $S_1$  is a diagonal matrix of scale factors for  $P_1^{-1}b$ ,
- $S_2$  is a diagonal matrix of scale factors for  $P_2x$ .

SUNDIALS solvers request that iterative linear solvers stop based on the 2-norm of the scaled preconditioned residual meeting a prescribed tolerance, i.e.,

$$\left\|\tilde{b} - \tilde{A}\tilde{x}\right\|_2 < \text{tol.}$$

When provided an iterative SUNLinSol implementation that does not support the scaling matrices  $S_1$  and  $S_2$ , the SUNDIALS packages will adjust the value of tol accordingly (see the iterative linear tolerance section that follows for more details). In this case, they instead request that iterative linear solvers stop based on the criterion

$$||P_1^{-1}b - P_1^{-1}Ax||_2 < \text{tol.}$$

We note that the corresponding adjustments to tol in this case may not be optimal, in that they cannot balance error between specific entries of the solution x, only the aggregate error in the overall solution vector.

We further note that not all of the SUNDIALS-provided iterative linear solvers support the full range of the above options (e.g., separate left/right preconditioning), and that some of the SUNDIALS packages only utilize a subset of these options. Further details on these exceptions are described in the documentation for each SUNLinearSolver implementation, or for each SUNDIALS package.

For users interested in providing their own SUNLinSol module, the following section presents the SUNLinSol API and its implementation beginning with the definition of SUNLinSol functions in §8.1.1 – §8.1.3. This is followed by the definition of functions supplied to a linear solver implementation in §8.1.4. The linear solver return codes are described in Table 8.1. The SUNLinearSolver type and the generic SUNLinSol module are defined in §8.1.6. §8.1.8 lists the requirements for supplying a custom SUNLinSol module and discusses some intended use cases. Users wishing to supply their own SUNLinSol module are encouraged to use the SUNLinSol implementations provided with SUNDIALS as a template for supplying custom linear solver modules. The section that then follows describes the SUNLinSol functions required by this SUNDIALS package, and provides additional package specific details. Then the remaining sections of this chapter present the SUNLinSol modules provided with SUNDIALS.

## 8.1 The SUNLinear Solver API

The SUNLinSol API defines several linear solver operations that enable SUNDIALS packages to utilize this API. These functions can be divided into three categories. The first are the core linear solver functions. The second consist of "set" routines to supply the linear solver with functions provided by the SUNDIALS packages and to modify solver parameters. The final group consists of "get" routines for retrieving linear solver statistics. All of these functions are defined in the header file sundials/sundials\_linearsolver.h.

## **8.1.1 SUNLinearSolver core functions**

The core linear solver functions consist of two **required** functions: SUNLinSolGetType() returns the linear solver type, and SUNLinSolSolve() solves the linear system Ax = b.

The remaining **optional** functions return the solver ID (SUNLinSolGetID()), initialize the linear solver object once all solver-specific options have been set (SUNLinSolInitialize()), set up the linear solver object to utilize an updated matrix A(SUNLinSolSetup()), and destroy a linear solver object (SUNLinSolFree()).

## enum SUNLinearSolver\_Type

An identifier indicating the type of linear solver.

**Note:** See §8.1.8.1 for more information on intended use cases corresponding to the linear solver type.

#### enumerator SUNLINEARSOLVER\_DIRECT

The linear solver requires a matrix, and computes an "exact" solution to the linear system defined by that matrix.

#### enumerator SUNLINEARSOLVER\_ITERATIVE

The linear solver does not require a matrix (though one may be provided), and computes an inexact solution to the linear system using a matrix-free iterative algorithm. That is it solves the linear system defined by the package-supplied ATimes routine (see *SUNLinSolSetATimes()* below), even if that linear system differs from the one encoded in the matrix object (if one is provided). As the solver computes the solution only inexactly (or may diverge), the linear solver should check for solution convergence/accuracy as appropriate.

## enumerator SUNLINEARSOLVER\_MATRIX\_ITERATIVE

The linear solver module requires a matrix, and computes an inexact solution to the linear system defined by that matrix using an iterative algorithm. That is it solves the linear system defined by the matrix object even if that linear system differs from that encoded by the package-supplied ATimes routine. As the solver computes the solution only inexactly (or may diverge), the linear solver should check for solution convergence/accuracy as appropriate.

#### enumerator SUNLINEARSOLVER\_MATRIX\_EMBEDDED

The linear solver sets up and solves the specified linear system at each linear solve call. Any matrix-related data structures are held internally to the linear solver itself, and are not provided by the SUNDIALS package.

## SUNLinearSolver\_Type SUNLinSolGetType(SUNLinearSolver LS)

Returns the *SUNLinearSolver\_Type* type identifier for the linear solver.

## Usage:

```
type = SUNLinSolGetType(LS);
```

## SUNLinearSolver\_ID SUNLinSolGetID(SUNLinearSolver LS)

Returns a non-negative linear solver identifier (of type int) for the linear solver LS.

#### **Return value:**

Non-negative linear solver identifier (of type int), defined by the enumeration SUNLinearSolver\_- ID, with values shown in Table 8.2 and defined in the sundials\_linearsolver.h header file.

## **Usage:**

```
id = SUNLinSolGetID(LS);
```

**Note:** It is recommended that a user-supplied SUNLinearSolver return the SUNLINEARSOLVER\_CUSTOM identifier.

## SUNErrCode SUNLinSolInitialize(SUNLinearSolver LS)

Performs linear solver initialization (assuming that all solver-specific options have been set).

#### **Return value:**

A SUNErrCode.

## Usage:

```
retval = SUNLinSolInitialize(LS);
```

### int SUNLinSolSetup(SUNLinearSolver LS, SUNMatrix A)

Performs any linear solver setup needed, based on an updated system SUNMatrix A. This may be called frequently (e.g., with a full Newton method) or infrequently (for a modified Newton method), based on the type of integrator and/or nonlinear solver requesting the solves.

#### Return value:

Zero for a successful call, a positive value for a recoverable failure, and a negative value for an unrecoverable failure. Ideally this should return one of the generic error codes listed in Table 8.1.

### Usage:

```
retval = SUNLinSolSetup(LS, A);
```

int SUNLinSolSolve(SUNLinearSolver LS, SUNMatrix A, N\_Vector x, N\_Vector b, sunrealtype tol)

This required function solves a linear system Ax = b.

## **Arguments:**

- *LS* a SUNLinSol object.
- A a SUNMatrix object.
- x an N\_Vector object containing the initial guess for the solution of the linear system on input, and the solution to the linear system upon return.
- *b* an N\_Vector object containing the linear system right-hand side.
- *tol* the desired linear solver tolerance.

## Return value:

Zero for a successful call, a positive value for a recoverable failure, and a negative value for an unrecoverable failure. Ideally this should return one of the generic error codes listed in Table 8.1.

#### Notes:

**Direct solvers:** can ignore the *tol* argument.

**Matrix-free solvers:** (those that identify as SUNLINEARSOLVER\_ITERATIVE) can ignore the SUN-Matrix input A, and should rely on the matrix-vector product function supplied through the routine SUNLinSolSetATimes().

**Iterative solvers:** (those that identify as SUNLINEARSOLVER\_ITERATIVE or SUNLINEARSOLVER\_-MATRIX\_ITERATIVE) should attempt to solve to the specified tolerance *tol* in a weighted 2-norm. If the solver does not support scaling then it should just use a 2-norm.

**Matrix-embedded solvers:** should ignore the SUNMatrix input A as this will be NULL. It is assumed that within this function, the solver will call interface routines from the relevant SUNDIALS package to directly form the linear system matrix A, and then solve Ax = b before returning with the solution x.

## Usage:

```
retval = SUNLinSolSolve(LS, A, x, b, tol);
```

#### SUNErrCode SUNLinSolFree(SUNLinearSolver LS)

Frees memory allocated by the linear solver.

## Return value:

A SUNErrCode.

## **Usage:**

```
retval = SUNLinSolFree(LS);
```

#### **8.1.2** SUNLinearSolver "set" functions

The following functions supply linear solver modules with functions defined by the SUNDIALS packages and modify solver parameters. Only the routine for setting the matrix-vector product routine is required, and even then is only required for matrix-free linear solver modules. Otherwise, all other set functions are optional. SUNLinSol implementations that do not provide the functionality for any optional routine should leave the corresponding function pointer NULL instead of supplying a dummy routine.

```
SUNErrCode SUNLinSolSetATimes (SUNLinearSolver LS, void *A_data, SUNATimesFn ATimes)
```

Required for matrix-free linear solvers (otherwise optional).

Provides a *SUNATimesFn* function pointer, as well as a **void\*** pointer to a data structure used by this routine, to the linear solver object *LS*. SUNDIALS packages call this function to set the matrix-vector product function to either a solver-provided difference-quotient via vector operations or a user-supplied solver-specific routine.

#### Return value:

A SUNErrCode.

#### Usage:

```
retval = SUNLinSolSetATimes(LS, A_data, ATimes);
```

SUNErrCode SUNLinSolSetPreconditioner(SUNLinearSolver LS, void \*P\_data, SUNPSetupFn Pset, SUNPSolveFn Psol)

This *optional* routine provides SUNPSetupFn and SUNPSolveFn function pointers that implement the preconditioner solves  $P_1^{-1}$  and  $P_2^{-1}$  from (8.2). This routine is called by a SUNDIALS package, which provides translation between the generic Pset and Psol calls and the package- or user-supplied routines.

#### **Return value:**

A SUNErrCode.

## **Usage:**

```
retval = SUNLinSolSetPreconditioner(LS, Pdata, Pset, Psol);
```

## SUNErrCode SUNLinSolSetScalingVectors (SUNLinearSolver LS, N\_Vector s1, N\_Vector s2)

This optional routine provides left/right scaling vectors for the linear system solve. Here, s1 and s2 are vectors of positive scale factors containing the diagonal of the matrices  $S_1$  and  $S_2$  from (8.2), respectively. Neither vector needs to be tested for positivity, and a NULL argument for either indicates that the corresponding scaling matrix is the identity.

#### Return value:

A SUNErrCode.

#### **Usage:**

```
retval = SUNLinSolSetScalingVectors(LS, s1, s2);
```

## SUNErrCode SUNLinSolSetZeroGuess(SUNLinearSolver LS, sunbooleantype onoff)

This *optional* routine indicates if the upcoming *SUNLinSolSolve()* call will be made with a zero initial guess (SUNTRUE) or a non-zero initial guess (SUNFALSE).

#### **Return value:**

A SUNErrCode.

#### **Usage:**

```
retval = SUNLinSolSetZeroGuess(LS, onoff);
```

#### **Notes:**

It is assumed that the initial guess status is not retained across calls to <code>SUNLinSolSolve()</code>. As such, the linear solver interfaces in each of the <code>SUNDIALS</code> packages call <code>SUNLinSolSetZeroGuess()</code> prior to each call to <code>SUNLinSolSolve()</code>.

## 8.1.3 SUNLinearSolver "get" functions

The following functions allow SUNDIALS packages to retrieve results from a linear solve. *All routines are optional.* int SUNLinSolNumIters(SUNLinearSolver LS)

This optional routine should return the number of linear iterations performed in the most-recent "solve" call.

## **Usage:**

```
its = SUNLinSolNumIters(LS);
```

## sunrealtype SUNLinSolResNorm(SUNLinearSolver LS)

This optional routine should return the final residual norm from the most-recent "solve" call.

## Usage:

```
rnorm = SUNLinSolResNorm(LS);
```

## N Vector SUNLinSolResid(SUNLinearSolver LS)

If an iterative method computes the preconditioned initial residual and returns with a successful solve without performing any iterations (i.e., either the initial guess or the preconditioner is sufficiently accurate), then this *optional* routine may be called by the SUNDIALS package. This routine should return the N\_Vector containing the preconditioned initial residual vector.

#### Usage:

```
rvec = SUNLinSolResid(LS);
```

## **Notes:**

Since N\_Vector is actually a pointer, and the results are not modified, this routine should *not* require additional memory allocation. If the SUNLinSol object does not retain a vector for this purpose, then this function pointer should be set to NULL in the implementation.

#### sunindextype SUNLinSolLastFlag(SUNLinearSolver LS)

This *optional* routine should return the last error flag encountered within the linear solver. Although not called by the SUNDIALS packages directly, this may be called by the user to investigate linear solver issues after a failed solve.

## **Usage:**

```
lflag = SUNLinLastFlag(LS);
```

SUNErrCode SUNLinSolSpace(SUNLinearSolver LS, long int \*lenrwLS, long int \*lenrwLS)

This *optional* routine should return the storage requirements for the linear solver *LS*:

- lrw is a long int containing the number of sunrealtype words
- *liw* is a long int containing the number of integer words.

This function is advisory only, for use by users to help determine their total space requirements.

### Return value:

A SUNErrCode.

### **Usage:**

retval = SUNLinSolSpace(LS, &lrw, &liw);

# 8.1.4 Functions provided by SUNDIALS packages

To interface with SUNLinSol modules, the SUNDIALS packages supply a variety of routines for evaluating the matrix-vector product, and setting up and applying the preconditioner. These package-provided routines translate between the user-supplied ODE, DAE, or nonlinear systems and the generic linear solver API. The function types for these routines are defined in the header file sundials/sundials\_iterative.h, and are described below.

```
typedef int (*SUNATimesFn)(void *A_data, N_Vector v, N_Vector z)
```

Computes the action of a matrix on a vector, performing the operation  $z \leftarrow Av$ . Memory for z will already be allocated prior to calling this function. The parameter  $A\_data$  is a pointer to any information about A which the function needs in order to do its job. The vector v should be left unchanged.

#### **Return value:**

Zero for a successful call, and non-zero upon failure.

typedef int (\*SUNPSetupFn)(void \*P\_data)

Sets up any requisite problem data in preparation for calls to the corresponding SUNPSolveFn.

#### **Return value:**

Zero for a successful call, and non-zero upon failure.

Solves the preconditioner equation Pz=r for the vector z. Memory for z will already be allocated prior to calling this function. The parameter  $P\_data$  is a pointer to any information about P which the function needs in order to do its job (set up by the corresponding SUNPSetupFn). The parameter lr is input, and indicates whether P is to be taken as the left or right preconditioner: lr=1 for left and lr=2 for right. If preconditioning is on one side only, lr can be ignored. If the preconditioner is iterative, then it should strive to solve the preconditioner equation so that

$$||Pz - r||_{\text{wrms}} < tol$$

where the error weight vector for the WRMS norm may be accessed from the main package memory structure. The vector r should not be modified by the SUNPSolveFn.

#### Return value:

Zero for a successful call, a negative value for an unrecoverable failure condition, or a positive value for a recoverable failure condition (thus the calling routine may reattempt the solution after updating preconditioner data).

# 8.1.5 SUNLinearSolver return codes

The functions provided to SUNLinSol modules by each SUNDIALS package, and functions within the SUNDIALS-provided SUNLinSol implementations, utilize a common set of return codes, listed in Table 8.1. These adhere to a common pattern:

- 0 indicates success
- a positive value corresponds to a recoverable failure, and
- a negative value indicates a non-recoverable failure.

Aside from this pattern, the actual values of each error code provide additional information to the user in case of a linear solver failure.

Table 8.1: SUNLinSol error codes

P. 1	<b>T</b> 7 1	No. 1
Error code	Value	Meaning
SUN_SUCCESS	0	successful call or converged solve
SUNLS_ATIMES_NULL	-804	the Atimes function is NULL
SUNLS_ATIMES_FAIL UNREC	-805	an unrecoverable failure occurred in the ATimes routine
SUNLS_PSET_FAIL_UN- REC	-806	an unrecoverable failure occurred in the Pset routine
SUNLS_PSOLVE_NULL	-807	the preconditioner solve function is NULL
SUNLS_PSOLVE_FAIL UNREC	-808	an unrecoverable failure occurred in the Psolve routine
SUNLS_GS_FAIL	-810	a failure occurred during Gram-Schmidt orthogonalization (SPGMR/SPFGMR)
SUNLS_QRSOL_FAIL	-811	a singular \$R\$ matrix was encountered in a QR factorization (SPGMR/SPFGMR)
SUNLS_RES_REDUCED	801	an iterative solver reduced the residual, but did not converge to the desired tolerance
SUNLS_CONV_FAIL	802	an iterative solver did not converge (and the residual was not reduced)
SUNLS_ATIMES_FAIL REC	803	a recoverable failure occurred in the ATimes routine
SUNLS_PSET_FAIL_REC	804	a recoverable failure occurred in the Pset routine
SUNLS_PSOLVE_FAIL REC	805	a recoverable failure occurred in the Psolve routine
SUNLS_PACKAGE_FAIL REC	806	a recoverable failure occurred in an external linear solver package
SUNLS_QRFACT_FAIL	807	a singular matrix was encountered during a QR factorization (SPGMR/SPFGMR)
SUNLS_LUFACT_FAIL	808	a singular matrix was encountered during a LU factorization

# 8.1.6 The generic SUNLinearSolver module

SUNDIALS packages interact with linear solver implementations through the SUNLinearSolver class. A SUNLinearSolver is a pointer to the \_generic\_SUNLinearSolver structure: typedef struct \_generic\_SUNLinearSolver \*SUNLinearSolver struct \_generic\_SUNLinearSolver The structure defining the SUNDIALS linear solver class. void \*content Pointer to the linear solver-specific member data SUNLinearSolver\_Ops ops A virtual table of linear solver operations provided by a specific implementation SUNContext sunctx The SUNDIALS simulation context The virtual table structure is defined as typedef struct generic SUNLinearSolver Ops \*SUNLinearSolver\_Ops struct \_generic\_SUNLinearSolver\_Ops The structure defining *SUNLinearSolver* operations. SUNLinearSolver\_Type (\*gettype)(SUNLinearSolver) The function implementing SUNLinSolGetType() SUNLinearSolver\_ID (\*getid)(SUNLinearSolver) The function implementing SUNLinSolGetID() SUNErrCode (\*setatimes)(SUNLinearSolver, void\*, SUNATimesFn) The function implementing SUNLinSolSetATimes() SUNErrCode (\*setpreconditioner)(SUNLinearSolver, void\*, SUNPSetupFn, SUNPSolveFn) The function implementing SUNLinSolSetPreconditioner() SUNErrCode (\*setscalingvectors)(SUNLinearSolver, N Vector, N Vector) The function implementing SUNLinSolSetScalingVectors() SUNErrCode (\*setzeroguess)(SUNLinearSolver, sunbooleantype) The function implementing SUNLinSolSetZeroGuess() SUNErrCode (\*initialize)(SUNLinearSolver) The function implementing SUNLinSolInitialize() int (\*setup)(SUNLinearSolver, SUNMatrix) The function implementing SUNLinSolSetup() int (\*solve)(SUNLinearSolver, SUNMatrix, N Vector, N Vector, sunrealtype) The function implementing SUNLinSolSolve() int (\*numiters)(SUNLinearSolver) The function implementing SUNLinSolNumIters() sunrealtype (\*resnorm)(SUNLinearSolver)

The function implementing SUNLinSolResNorm()

```
sunindextype (*lastflag)(SUNLinearSolver)
    The function implementing SUNLinSolLastFlag()
SUNErrCode (*space)(SUNLinearSolver, long int*, long int*)
    The function implementing SUNLinSolSpace()
N_Vector (*resid)(SUNLinearSolver)
    The function implementing SUNLinSolResid()
SUNErrCode (*free)(SUNLinearSolver)
    The function implementing SUNLinSolFree()
```

The generic SUNLinSol class defines and implements the linear solver operations defined in §8.1.1 – §8.1.3. These routines are in fact only wrappers to the linear solver operations defined by a particular SUNLinSol implementation, which are accessed through the *ops* field of the SUNLinearSolver structure. To illustrate this point we show below the implementation of a typical linear solver operation from the SUNLinearSolver base class, namely *SUNLinSolIni-tialize()*, that initializes a SUNLinearSolver object for use after it has been created and configured, and returns a flag denoting a successful or failed operation:

```
int SUNLinSolInitialize(SUNLinearSolver S)
{
   return ((int) S->ops->initialize(S));
}
```

# 8.1.7 Compatibility of SUNLinear Solver modules

Not all SUNLinearSolver implementations are compatible with all SUNMatrix and N\_Vector implementations provided in SUNDIALS. More specifically, all of the SUNDIALS iterative linear solvers (SPGMR, SPFGMR, SPBCGS, SPTFQMR, and PCG) are compatible with all of the SUNDIALS N\_Vector modules, but the matrix-based direct SUNLinSol modules are specifically designed to work with distinct SUNMatrix and N\_Vector modules. In the list below, we summarize the compatibility of each matrix-based SUNLinearSolver module with the various SUNMatrix and N\_Vector modules. For a more thorough discussion of these compatibilities, we defer to the documentation for each individual SUNLinSol module in the sections that follow.

- Dense
  - SUNMatrix: *Dense* or user-supplied
  - N\_Vector: Serial, OpenMP, Pthreads, or user-supplied
- · LapackDense
  - SUNMatrix: Dense or user-supplied
  - N\_Vector: Serial, OpenMP, Pthreads, or user-supplied
- Band
  - SUNMatrix: *Band* or user-supplied
  - N\_Vector: Serial, OpenMP, Pthreads, or user-supplied
- LapackBand
  - SUNMatrix: Band or user-supplied
  - N\_Vector: Serial, OpenMP, Pthreads, or user-supplied
- KLU
  - SUNMatrix: Sparse or user-supplied

- N\_Vector: Serial, OpenMP, Pthreads, or user-supplied
- SuperLU MT
  - SUNMatrix: Sparse or user-supplied
  - N\_Vector: Serial, OpenMP, Pthreads, or user-supplied
- SuperLU Dist
  - SUNMatrix: SLUNRLOC or user-supplied
  - N\_Vector: Serial, OpenMP, Pthreads, Parallel, \*hypre\*, PETSc, or user-supplied
- Magma Dense
  - SUNMatrix: Magma Dense or user-supplied
  - N\_Vector: HIP, RAJA, or user-supplied
- · OneMKL Dense
  - SUNMatrix: One MKL Dense or user-supplied
  - N\_Vector: SYCL, RAJA, or user-supplied
- cuSolverSp batchQR
  - SUNMatrix: cuSparse or user-supplied
  - N\_Vector: CUDA, RAJA, or user-supplied

# 8.1.8 Implementing a custom SUNLinearSolver module

A particular implementation of the SUNLinearSolver module must:

- Specify the *content* field of the SUNLinSol module.
- Define and implement the required linear solver operations.

**Note:** The names of these routines should be unique to that implementation in order to permit using more than one SUNLinSol module (each with different SUNLinearSolver internal data representations) in the same code.

• Define and implement user-callable constructor and destructor routines to create and free a SUNLinearSolver with the new *content* field and with *ops* pointing to the new linear solver operations.

We note that the function pointers for all unsupported optional routines should be set to NULL in the *ops* structure. This allows the SUNDIALS package that is using the SUNLinSol object to know whether the associated functionality is supported.

To aid in the creation of custom SUNLinearSolver modules the generic SUNLinearSolver module provides the utility function <code>SUNLinSolNewEmpty()</code>. When used in custom SUNLinearSolver constructors this function will ease the introduction of any new optional linear solver operations to the SUNLinearSolver API by ensuring that only required operations need to be set.

# SUNLinearSolver SUNLinSolNewEmpty(SUNContext sunctx)

This function allocates a new generic SUNLinearSolver object and initializes its content pointer and the function pointers in the operations structure to NULL.

#### **Return value:**

If successful, this function returns a SUNLinearSolver object. If an error occurs when allocating the object, then this routine will return NULL.

### void SUNLinSolFreeEmpty(SUNLinearSolver LS)

This routine frees the generic SUNLinearSolver object, under the assumption that any implementation-specific data that was allocated within the underlying content structure has already been freed. It will additionally test whether the ops pointer is NULL, and, if it is not, it will free it as well.

## **Arguments:**

• LS – a SUNLinearSolver object

Additionally, a SUNLinearSolver implementation *may* do the following:

- Define and implement additional user-callable "set" routines acting on the SUNLinearSolver, e.g., for setting various configuration options to tune the linear solver for a particular problem.
- Provide additional user-callable "get" routines acting on the SUNLinearSolver object, e.g., for returning various solve statistics.

### enum SUNLinearSolver\_ID

Each SUNLinSol implementation included in SUNDIALS has a unique identifier specified in enumeration and shown in Table 8.2. It is recommended that a user-supplied SUNLinSol implementation use the SUNLINEAR-SOLVER\_CUSTOM identifier.

Table 8.2: Identifiers associated with SUNLinearSolver modules supplied with SUNDIALS

SUNLinSol ID	Linear solver type	ID Value
SUNLINEARSOLVER_BAND	Banded direct linear solver (internal)	0
SUNLINEARSOLVER_DENSE	Dense direct linear solver (internal)	1
SUNLINEARSOLVER_KLU	Sparse direct linear solver (KLU)	2
SUNLINEARSOLVER_LAPACKBAND	Banded direct linear solver (LAPACK)	3
SUNLINEARSOLVER_LAPACKDENSE	Dense direct linear solver (LAPACK)	4
SUNLINEARSOLVER_PCG	Preconditioned conjugate gradient iterative solver	5
SUNLINEARSOLVER_SPBCGS	Scaled-preconditioned BiCGStab iterative solver	6
SUNLINEARSOLVER_SPFGMR	Scaled-preconditioned FGMRES iterative solver	7
SUNLINEARSOLVER_SPGMR	Scaled-preconditioned GMRES iterative solver	8
SUNLINEARSOLVER_SPTFQMR	Scaled-preconditioned TFQMR iterative solver	9
SUNLINEARSOLVER_SUPERLUDIST	Parallel sparse direct linear solver (SuperLU_Dist)	10
SUNLINEARSOLVER_SUPERLUMT	Threaded sparse direct linear solver (SuperLUMT)	11
SUNLINEARSOLVER_CUSOLVERSP BATCHQR	Sparse direct linear solver (CUDA)	12
SUNLINEARSOLVER_MAGMADENSE	Dense or block-dense direct linear solver (MAGMA)	13
SUNLINEARSOLVER_ONEMKLDENSE	Dense or block-dense direct linear solver (OneMKL)	14
SUNLINEARSOLVER_CUSTOM	User-provided custom linear solver	15

### 8.1.8.1 Intended use cases

The SUNLinSol and SUNMATRIX APIs are designed to require a minimal set of routines to ease interfacing with custom or third-party linear solver libraries. Many external solvers provide routines with similar functionality and thus may require minimal effort to wrap within custom SUNMATRIX and SUNLinSol implementations. As SUNDIALS packages utilize generic SUNLinSol modules they may naturally leverage user-supplied SUNLinearSolver implementations, thus there exist a wide range of possible linear solver combinations. Some intended use cases for both the SUNDIALS-provided and user-supplied SUNLinSol modules are discussed in the sections below.

#### **Direct linear solvers**

Direct linear solver modules require a matrix and compute an "exact" solution to the linear system *defined by the matrix*. SUNDIALS packages strive to amortize the high cost of matrix construction by reusing matrix information for multiple nonlinear iterations or time steps. As a result, each package's linear solver interface recomputes matrix information as infrequently as possible.

Alternative matrix storage formats and compatible linear solvers that are not currently provided by, or interfaced with, SUNDIALS can leverage this infrastructure with minimal effort. To do so, a user must implement custom SUNMATRIX and SUNLinSol wrappers for the desired matrix format and/or linear solver following the APIs described in §7 and §8. *This user-supplied SUNLinSol module must then self-identify as having SUNLINEARSOLVER\_DIRECT type*.

#### Matrix-free iterative linear solvers

Matrix-free iterative linear solver modules do not require a matrix, and instead compute an inexact solution to the linear system *defined by the package-supplied* ATimes *routine*. SUNDIALS supplies multiple scaled, preconditioned iterative SUNLinSol modules that support scaling, allowing packages to handle non-dimensionalization, and users to define variables and equations as natural in their applications. However, for linear solvers that do not support left/right scaling, SUNDIALS packages must instead adjust the tolerance supplied to the linear solver to compensate (see the iterative linear tolerance section that follows for more details) – this strategy may be non-optimal since it cannot handle situations where the magnitudes of different solution components or equations vary dramatically within a single application.

To utilize alternative linear solvers that are not currently provided by, or interfaced with, SUNDIALS a user must implement a custom SUNLinSol wrapper for the linear solver following the API described in §8. *This user-supplied SUNLinSol module must then self-identify as having SUNLINEARSOLVER\_ITERATIVE type*.

### Matrix-based iterative linear solvers (reusing A)

Matrix-based iterative linear solver modules require a matrix and compute an inexact solution to the linear system *defined by the matrix*. This matrix will be updated infrequently and resued across multiple solves to amortize the cost of matrix construction. As in the direct linear solver case, only thin SUNMATRIX and SUNLinSol wrappers for the underlying matrix and linear solver structures need to be created to utilize such a linear solver. *This user-supplied SUNLinSol module must then self-identify as having SUNLINEARSOLVER\_MATRIX\_ITERATIVE type*.

At present, SUNDIALS has one example problem that uses this approach for wrapping a structured-grid matrix, linear solver, and preconditioner from the *hypre* library; this may be used as a template for other customized implementations (see examples/arkode/CXX\_parhyp/ark\_heat2D\_hypre.cpp).

#### Matrix-based iterative linear solvers (current A)

For users who wish to utilize a matrix-based iterative linear solver where the matrix is *purely for preconditioning* and the linear system is *defined by the package-supplied* ATimes *routine*, we envision two current possibilities.

The preferred approach is for users to employ one of the SUNDIALS scaled, preconditioned iterative linear solver implementations (SUNLinSol\_SPGMR(), SUNLinSol\_SPFGMR(), SUNLinSol\_SPFGGS(), SUNLinSol\_SPTFQMR(), or SUNLinSol\_PCG()) as the outer solver. The creation and storage of the preconditioner matrix, and interfacing with the corresponding matrix-based linear solver, can be handled through a package's preconditioner "setup" and "solve" functionality without creating SUNMATRIX and SUNLinSol implementations. This usage mode is recommended primarily because the SUNDIALS-provided modules support variable and equation scaling as described above.

A second approach supported by the linear solver APIs is as follows. If the SUNLinSol implementation is matrix-based, <code>self-identifies</code> as <code>having SUNLINEARSOLVER\_ITERATIVE</code> type, and also provides a non-NULL SUNLinSolSe-tATimes() routine, then each SUNDIALS package will call that routine to attach its package-specific matrix-vector product routine to the SUNLinSol object. The SUNDIALS package will then call the SUNLinSol-provided <code>SUNLinSolSetup()</code> routine (infrequently) to update matrix information, but will provide current matrix-vector products to the SUNLinSol implementation through the package-supplied <code>SUNATimesFn</code> routine.

### Application-specific linear solvers with embedded matrix structure

Many applications can exploit additional linear system structure arising from to the implicit couplings in their model equations. In certain circumstances, the linear solve Ax = b may be performed without the need for a global system matrix A, as the unformed A may be block diagonal or block triangular, and thus the overall linear solve may be performed through a sequence of smaller linear solves. In other circumstances, a linear system solve may be accomplished via specialized fast solvers, such as the fast Fourier transform, fast multipole method, or treecode, in which case no matrix structure may be explicitly necessary. In many of the above situations, construction and preprocessing of the linear system matrix A may be inexpensive, and thus increased performance may be possible if the current linear system information is used within every solve (instead of being lagged, as occurs with matrix-based solvers that reuse A).

To support such application-specific situations, SUNDIALS supports user-provided linear solvers with the SUNLINEAR-SOLVER\_MATRIX\_EMBEDDED type. For an application to leverage this support, it should define a custom SUNLinSol implementation having this type, that only needs to implement the required SUNLinSolGetType() and SUNLinSolSolve() operations. Within SUNLinSolSolve(), the linear solver implementation should call package-specific interface routines (e.g., ARKStepGetNonlinearSystemData, CVodeGetNonlinearSystemData, IDAGetNonlinearSystemData, ARKStepGetCurrentGamma, CVodeGetCurrentGamma, IDAGetCurrentCj, or MRIStepGetCurrentGamma) to construct the relevant system matrix A (or portions thereof), solve the linear system Ax = b, and return the solution vector x.

We note that when attaching this custom SUNLinearSolver object with the relevant SUNDIALS package SetLinear-Solver routine, the input SUNMatrix A should be set to NULL.

For templates of such user-provided "matrix-embedded" SUNLinSol implementations, see the SUNDIALS examples ark\_analytic\_mels.c, cvAnalytic\_mels.c, cvAnalytic\_mels.c, idaAnalytic\_mels.c, and idasAnalytic\_mels.c.

# 8.2 KINSOL SUNLinearSolver interface

Table 8.3 below lists the SUNLinearSolver module linear solver functions used within the KINLS interface. As with the SUNMatrix module, we emphasize that the KINSOL user does not need to know detailed usage of linear solver functions by the KINSOL code modules in order to use KINSOL. The information is presented as an implementation detail for the interested reader.

The linear solver functions listed below are marked with "x" to indicate that they are required, or with "†" to indicate that they are only called if they are non-NULL in the SUNLinearSolver implementation that is being used. Note:

- 1. *SUNLinSolNumIters()* is only used to accumulate overall iterative linear solver statistics. If it is not implemented by the SUNLinearSolver module, then KINLS will consider all solves as requiring zero iterations.
- 2. Although *SUNLinSolResNorm()* is optional, if it is not implemented by the SUNLinearSolver then KINLS will consider all solves a being *exact*.
- 3. Although KINLS does not call *SUNLinSolLastFlag()* directly, this routine is available for users to query linear solver issues directly.
- 4. Although KINLS does not call *SUNLinSolFree()* directly, this routine should be available for users to call when cleaning up from a simulation.

SUNLinSolGetType	X	X	X
SUNLinSolSetATimes	†	X	†
SUNLinSolSetPreconditioner	†	†	†
SUNLinSolSetScalingVectors	†	†	†
SUNLinSolInitialize	X	X	X
SUNLinSolSetup	X	X	X
SUNLinSolSolve	X	X	X
$^1$ SUNLinSolNumIters		†	†
$^2$ SUNLinSolResNorm		†	†
$^3$ SUNLinSolLastFlag			
<sup>4</sup> SUNLinSolFree			
SUNLinSolSpace	†	†	†

Table 8.3: List of linear solver function usage in the KINLS interface

Since there are a wide range of potential SUNLinearSolver use cases, the following subsections describe some details of the KINLS interface, in the case that interested users wish to develop custom SUNLinearSolver modules.

# 8.2.1 Lagged matrix information

If the SUNLinearSolver object self-identifies as having type SUNLINEARSOLVER\_DIRECT or SUNLINEARSOLVER\_-MATRIX\_ITERATIVE, then the SUNLinearSolver object solves a linear system *defined* by a SUNMatrix object. As a result, KINSOL can perform its optional residual monitoring scheme, described in §2.8.

### 8.2.2 Iterative linear solver tolerance

If the SUNLinearSolver object self-identifies as having type SUNLINEARSOLVER\_ITERATIVE or SUNLINEAR-SOLVER\_MATRIX\_ITERATIVE then KINLS will adjust the linear solver tolerance delta as described in §2.9 during the course of the nonlinear solve process. However, if the iterative linear solver does not support scaling matrices (i.e., the SUNLinSolSetScalingVectors routine is NULL), then KINLS will be unable to fully handle ill-conditioning in the nonlinear solve process through the solution and residual scaling operators described in §2.4. In this case, KINLS will attempt to adjust the linear solver tolerance to account for this lack of functionality. To this end, the following assumptions are made:

1. All residual components have similar magnitude; hence the scaling matrix  $D_F$  used in computing the linear residual norm (see §2.4) should satisfy the assumption

$$(D_F)_{i,i} \approx D_{F,mean}, \quad \text{for} \quad i = 0, \dots, n-1.$$

2. The SUNLinearSolver object uses a standard 2-norm to measure convergence.

Since KINSOL uses  $D_F$  as the left-scaling matrix,  $S_1 = D_F$ , then the linear solver convergence requirement is converted as follows (using the notation from equations (8.1) – (8.2):

$$\begin{split} &\|\tilde{b} - \tilde{A}\tilde{x}\|_{2} < \text{tol} \\ \Leftrightarrow &\|D_{F}P_{1}^{-1}b - D_{F}P_{1}^{-1}Ax\|_{2} < \text{tol} \\ \Leftrightarrow &\sum_{i=0}^{n-1} \left[ (D_{F})_{i,i} \left( P_{1}^{-1}(b - Ax) \right)_{i} \right]^{2} < \text{tol}^{2} \\ \Leftrightarrow &D_{F,mean}^{2} \sum_{i=0}^{n-1} \left[ \left( P_{1}^{-1}(b - Ax) \right)_{i} \right]^{2} < \text{tol}^{2} \\ \Leftrightarrow &\sum_{i=0}^{n-1} \left[ \left( P_{1}^{-1}(b - Ax) \right)_{i} \right]^{2} < \left( \frac{\text{tol}}{D_{F,mean}} \right)^{2} \\ \Leftrightarrow &\|P_{1}^{-1}(b - Ax)\|_{2} < \frac{\text{tol}}{D_{F,mean}} \end{split}$$

Therefore the tolerance scaling factor

$$D_{F,mean} = \frac{1}{\sqrt{n}} \left( \sum_{i=0}^{n-1} (D_F)_{i,i}^2 \right)^{1/2}$$

is computed and the scaled tolerance  $delta = tol/D_{F,mean}$  is supplied to the SUNLinearSolver object.

# 8.2.3 Matrix-embedded solver incompatibility

At present, KINLS is incompatible with SUNLinearSolver objects that self-identify as having type SUNLINEAR-SOLVER\_MATRIX\_EMBEDDED. Support for such user-supplied linear solvers may be added in a future release. Users interested in such support are recommended to contact the SUNDIALS development team.

# 8.3 The SUNLinSol\_Band Module

The SUNLinSol\_Band implementation of the SUNLinearSolver class is designed to be used with the corresponding *SUNMATRIX\_BAND* matrix type, and one of the serial or shared-memory N\_Vector implementations (NVECTOR\_-SERIAL, NVECTOR\_OPENMP or NVECTOR\_PTHREADS).

# 8.3.1 SUNLinSol\_Band Usage

The header file to be included when using this module is sunlinsol\_band.h. The SUNLinSol\_Band module is accessible from all SUNDIALS packages *without* linking to the libsundials\_sunlinsolband module library.

The SUNLinSol\_Band module provides the following user-callable constructor routine:

```
SUNLinearSolver SUNLinSol_Band(N_Vector y, SUNMatrix A, SUNContext sunctx)
```

This function creates and allocates memory for a band SUNLinearSolver.

### **Arguments:**

- y vector used to determine the linear system size
- A matrix used to assess compatibility
- sunctx the SUNContext object (see §4.2)

#### Return value:

New SUNLinSol\_Band object, or NULL if either A or y are incompatible.

#### **Notes:**

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUNMatrix implementations. These are currently limited to the SUNMATRIX\_BAND matrix type and the NVECTOR\_SERIAL, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

Additionally, this routine will verify that the input matrix  $\bf A$  is allocated with appropriate upper bandwidth storage for the LU factorization.

## 8.3.2 SUNLinSol\_Band Description

The SUNLinSol\_Band module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_Band {
  sunindextype N;
  sunindextype *pivots;
  sunindextype last_flag;
};
```

These entries of the *content* field contain the following information:

- N size of the linear system,
- pivots index array for partial pivoting in LU factorization,
- last\_flag last error return flag from internal function evaluations.

This solver is constructed to perform the following operations:

- The "setup" call performs an LU factorization with partial (row) pivoting, PA = LU, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX\_BAND object A, with pivoting information encoding P stored in the pivots array.
- The "solve" call performs pivoting and forward and backward substitution using the stored pivots array and the LU factors held in the SUNMATRIX\_BAND object.
- A must be allocated to accommodate the increase in upper bandwidth that occurs during factorization. More
  precisely, if A is a band matrix with upper bandwidth mu and lower bandwidth ml, then the upper triangular
  factor U can have upper bandwidth as big as smu = MIN(N-1,mu+ml). The lower triangular factor L has lower
  bandwidth ml.

The SUNLinSol\_Band module defines band implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_Band
- SUNLinSolInitialize\_Band this does nothing, since all consistency checks are performed at solver creation.
- ullet SUNLinSolSetup\_Band this performs the LU factorization.
- SUNLinSolSolve\_Band this uses the LU factors and pivots array to perform the solve.
- SUNLinSolLastFlag\_Band
- SUNLinSolSpace\_Band this only returns information for the storage *within* the solver object, i.e. storage for N, last\_flag, and pivots.
- SUNLinSolFree Band

# 8.4 The SUNLinSol\_Dense Module

The SUNLinSol\_Dense implementation of the SUNLinearSolver class is designed to be used with the corresponding SUNMATRIX\_DENSE matrix type, and one of the serial or shared-memory N\_Vector implementations (NVECTOR\_SERIAL, NVECTOR\_OPENMP or NVECTOR\_PTHREADS).

# 8.4.1 SUNLinSol\_Dense Usage

The header file to be included when using this module is sunlinsol\_dense.h. The SUNLinSol\_Dense module is accessible from all SUNDIALS solvers *without* linking to the libsundials\_sunlinsoldense module library.

The module SUNLinSol\_Dense provides the following user-callable constructor routine:

SUNLinearSolver SUNLinSol\_Dense(N\_Vector y, SUNMatrix A, SUNContext sunctx)

This function creates and allocates memory for a dense SUNLinearSolver.

### **Arguments:**

- y vector used to determine the linear system size.
- A matrix used to assess compatibility.
- sunctx the SUNContext object (see §4.2)

#### Return value:

New SUNLinSol\_Dense object, or NULL if either A or y are incompatible.

#### **Notes:**

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUN-Matrix implementations. These are currently limited to the SUNMATRIX\_DENSE matrix type and the NVECTOR\_SERIAL, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

# 8.4.2 SUNLinSol Dense Description

The SUNLinSol\_Dense module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_Dense {
  sunindextype N;
  sunindextype *pivots;
  sunindextype last_flag;
};
```

These entries of the *content* field contain the following information:

- N size of the linear system,
- pivots index array for partial pivoting in LU factorization,
- last\_flag last error return flag from internal function evaluations.

This solver is constructed to perform the following operations:

- The "setup" call performs an LU factorization with partial (row) pivoting ( $\mathcal{O}(N^3)$  cost), PA = LU, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX\_DENSE object A, with pivoting information encoding P stored in the pivots array.
- The "solve" call performs pivoting and forward and backward substitution using the stored pivots array and the LU factors held in the SUNMATRIX\_DENSE object ( $\mathcal{O}(N^2)$  cost).

The SUNLinSol\_Dense module defines dense implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_Dense
- SUNLinSolInitialize\_Dense this does nothing, since all consistency checks are performed at solver creation.
- SUNLinSolSetup\_Dense this performs the LU factorization.
- SUNLinSolSolve\_Dense this uses the *LU* factors and pivots array to perform the solve.
- SUNLinSolLastFlag\_Dense
- SUNLinSolSpace\_Dense this only returns information for the storage *within* the solver object, i.e. storage for N, last\_flag, and pivots.
- SUNLinSolFree\_Dense

# 8.5 The SUNLinSol\_KLU Module

The SUNLinSol\_KLU implementation of the SUNLinearSolver class is designed to be used with the corresponding SUNMATRIX\_SPARSE matrix type, and one of the serial or shared-memory N\_Vector implementations (NVECTOR\_SERIAL, NVECTOR\_OPENMP, or NVECTOR\_PTHREADS).

# 8.5.1 SUNLinSol\_KLU Usage

The header file to be included when using this module is sunlinsol/sunlinsol\_klu.h. The installed module library to link to is libsundials\_sunlinsolklu.lib where .lib is typically .so for shared libraries and .a for static libraries.

The module SUNLinSol\_KLU provides the following additional user-callable routines:

SUNLinearSolver SUNLinSol\_KLU(N\_Vector y, SUNMatrix A, SUNContext sunctx)

This constructor function creates and allocates memory for a SUNLinSol\_KLU object.

### **Arguments:**

- y vector used to determine the linear system size.
- *A* matrix used to assess compatibility.
- sunctx the SUNContext object (see §4.2)

#### Return value:

New SUNLinSol\_KLU object, or NULL if either A or y are incompatible.

#### Notes

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUN-Matrix implementations. These are currently limited to the SUNMATRIX\_SPARSE matrix type (using either CSR or CSC storage formats) and the NVECTOR\_SERIAL, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

### SUNErrCode SUNLinSol\_KLUReInit(SUNLinearSolver S, SUNMatrix A, sunindextype nnz, int reinit\_type)

This function reinitializes memory and flags for a new factorization (symbolic and numeric) to be conducted at the next solver setup call. This routine is useful in the cases where the number of nonzeroes has changed or if the structure of the linear system has changed which would require a new symbolic (and numeric factorization).

### **Arguments:**

- *S* existing SUNLinSol KLU object to reinitialize.
- A sparse SUNMatrix matrix (with updated structure) to use for reinitialization.
- nnz maximum number of nonzeros expected for Jacobian matrix.
- reinit\_type governs the level of reinitialization. The allowed values are:
  - The Jacobian matrix will be destroyed and a new one will be allocated based on the nnz value passed to this call. New symbolic and numeric factorizations will be completed at the next solver setup.
  - Only symbolic and numeric factorizations will be completed. It is assumed that the Jacobian size has not exceeded the size of nnz given in the sparse matrix provided to the original constructor routine (or the previous SUNKLUReInit call).

# Return value:

A SUNErrCode

#### **Notes:**

This routine assumes no other changes to solver use are necessary.

### SUNErrCode SUNLinSol\_KLUSetOrdering(SUNLinearSolver S, int ordering\_choice)

This function sets the ordering used by KLU for reducing fill in the linear solve.

### **Arguments:**

- *S* existing SUNLinSol\_KLU object to update.
- ordering choice type of ordering to use, options are:
  - 0. AMD,
  - 1. COLAMD, and
  - 2. the natural ordering.

The default is 1 for COLAMD.

#### Return value:

A SUNErrCode

# sun\_klu\_symbolic \*SUNLinSol\_KLUGetSymbolic(SUNLinearSolver S)

This function returns a pointer to the KLU symbolic factorization stored in the SUNLinSol\_KLU content structure.

### type sun\_klu\_symbolic

This type is an alias that depends on the SUNDIALS index size, see *sunindextype* and *SUNDIALS\_-INDEX\_SIZE*. It is equivalent to:

- klu\_symbolic when SUNDIALS is compiled with 32-bit indices
- klu\_l\_symbolic when SUNDIALS is compiled with 64-bit indices

# sun\_klu\_numeric \*SUNLinSol\_KLUGetNumeric(SUNLinearSolver S)

This function returns a pointer to the KLU numeric factorization stored in the SUNLinSol\_KLU content structure.

# type sun\_klu\_numeric

This type is an alias that depends on the SUNDIALS index size, see *sunindextype* and *SUNDIALS\_-INDEX\_SIZE*. It is equivalent to:

- klu\_numeric when SUNDIALS is compiled with 32-bit indices
- klu\_l\_numeric when SUNDIALS is compiled with 64-bit indices

#### sun klu common \*SUNLinSol\_KLUGetCommon(SUNLinearSolver S)

This function returns a pointer to the KLU common structure stored in the SUNLinSol\_KLU content structure.

# type sun\_klu\_common

This type is an alias that depends on the SUNDIALS index size, see *sunindextype* and *SUNDIALS\_-INDEX\_SIZE*. It is equivalent to:

- klu\_common when SUNDIALS is compiled with 32-bit indices
- klu\_l\_common when SUNDIALS is compiled with 64-bit indices

### 8.5.2 SUNLinSol KLU Description

The SUNLinSol KLU module defines the *content* field of a SUNLinearSolver to be the following structure:

These entries of the *content* field contain the following information:

- last\_flag last error return flag from internal function evaluations,
- first\_factorize flag indicating whether the factorization has ever been performed,
- symbolic KLU storage structure for symbolic factorization components, with underlying type klu\_symbolic
  or klu\_l\_symbolic, depending on whether SUNDIALS was installed with 32-bit versus 64-bit indices, respectively,
- numeric KLU storage structure for numeric factorization components, with underlying type klu\_numeric or klu\_l\_numeric, depending on whether SUNDIALS was installed with 32-bit versus 64-bit indices, respectively,
- common storage structure for common KLU solver components, with underlying type klu\_common or klu\_l\_common, depending on whether SUNDIALS was installed with 32-bit versus 64-bit indices, respectively,
- klu\_solver pointer to the appropriate KLU solver function (depending on whether it is using a CSR or CSC sparse matrix, and on whether SUNDIALS was installed with 32-bit or 64-bit indices).

The SUNLinSol\_KLU module is a SUNLinearSolver wrapper for the KLU sparse matrix factorization and solver library written by Tim Davis and collaborators ([4, 20]). In order to use the SUNLinSol\_KLU interface to KLU, it is assumed that KLU has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with KLU (see §11.4 for details). Additionally, this wrapper only supports double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to have *sunrealtype* set to either extended or single (see §4.1 for details). Since the KLU library supports both 32-bit and 64-bit integers, this interface will be compiled for either of the available *sunindextype* options.

The KLU library has a symbolic factorization routine that computes the permutation of the linear system matrix to block triangular form and the permutations that will pre-order the diagonal blocks (the only ones that need to be factored) to reduce fill-in (using AMD, COLAMD, CHOLAMD, natural, or an ordering given by the user). Of these ordering choices, the default value in the SUNLinSol\_KLU module is the COLAMD ordering.

KLU breaks the factorization into two separate parts. The first is a symbolic factorization and the second is a numeric factorization that returns the factored matrix along with final pivot information. KLU also has a refactor routine that can be called instead of the numeric factorization. This routine will reuse the pivot information. This routine also returns diagnostic information that a user can examine to determine if numerical stability is being lost and a full numerical factorization should be done instead of the refactor.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNLinSol\_KLU module is constructed to perform the following operations:

• The first time that the "setup" routine is called, it performs the symbolic factorization, followed by an initial numerical factorization.

- On subsequent calls to the "setup" routine, it calls the appropriate KLU "refactor" routine, followed by estimates of the numerical conditioning using the relevant "rcond", and if necessary "condest", routine(s). If these estimates of the condition number are larger than  $\varepsilon^{-2/3}$  (where  $\varepsilon$  is the double-precision unit roundoff), then a new factorization is performed.
- The module includes the routine SUNKLUReInit, that can be called by the user to force a full refactorization at the next "setup" call.
- The "solve" call performs pivoting and forward and backward substitution using the stored KLU data structures. We note that in this solve KLU operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

The SUNLinSol\_KLU module defines implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_KLU
- SUNLinSolInitialize\_KLU this sets the first\_factorize flag to 1, forcing both symbolic and numerical factorizations on the subsequent "setup" call.
- SUNLinSolSetup\_KLU this performs either a LU factorization or refactorization of the input matrix.
- SUNLinSolSolve\_KLU this calls the appropriate KLU solve routine to utilize the LU factors to solve the linear system.
- SUNLinSolLastFlag\_KLU
- SUNLinSolSpace\_KLU this only returns information for the storage within the solver *interface*, i.e. storage for the integers last\_flag and first\_factorize. For additional space requirements, see the KLU documentation.
- SUNLinSolFree KLU

# 8.6 The SUNLinSol\_LapackBand Module

The SUNLinSol\_LapackBand implementation of the SUNLinearSolver class is designed to be used with the corresponding SUNMATRIX\_BAND matrix type, and one of the serial or shared-memory N\_Vector implementations (NVECTOR\_SERIAL, NVECTOR\_OPENMP, or NVECTOR\_PTHREADS). The

# 8.6.1 SUNLinSol\_LapackBand Usage

The header file to be included when using this module is sunlinsol/sunlinsol\_lapackband.h. The installed module library to link to is libsundials\_sunlinsollapackband.lib where .lib is typically .so for shared libraries and .a for static libraries.

The module SUNLinSol\_LapackBand provides the following user-callable routine:

SUNLinearSolver SUNLinSol\_LapackBand(N\_Vector y, SUNMatrix A, SUNContext sunctx)

This function creates and allocates memory for a LAPACK band SUNLinearSolver.

### **Arguments:**

- y vector used to determine the linear system size.
- A matrix used to assess compatibility.
- *sunctx* the *SUNContext* object (see §4.2)

#### Return value:

New SUNLinSol\_LapackBand object, or NULL if either A or y are incompatible.

#### **Notes:**

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUNMatrix implementations. These are currently limited to the SUNMATRIX\_BAND matrix type and the NVECTOR\_SERIAL, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

Additionally, this routine will verify that the input matrix  ${\bf A}$  is allocated with appropriate upper bandwidth storage for the LU factorization.

# 8.6.2 SUNLinSol\_LapackBand Description

SUNLinSol\_LapackBand module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_Band {
  sunindextype N;
  sunindextype *pivots;
  sunindextype last_flag;
};
```

These entries of the *content* field contain the following information:

- N size of the linear system,
- pivots index array for partial pivoting in LU factorization,
- last\_flag last error return flag from internal function evaluations.

The SUNLinSol\_LapackBand module is a SUNLinearSolver wrapper for the LAPACK band matrix factorization and solve routines, \*GBTRF and \*GBTRS, where \* is either D or S, depending on whether SUNDIALS was configured to have *sunrealtype* set to double or single, respectively (see §4.1 for details). In order to use the SUNLinSol\_LapackBand module it is assumed that LAPACK has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with LAPACK (see §11.4 for details). We note that since there do not exist 128-bit floating-point factorization and solve routines in LAPACK, this interface cannot be compiled when using extended precision for *sunrealtype*. Similarly, since there do not exist 64-bit integer LAPACK routines, the SUNLinSol\_LapackBand module also cannot be compiled when using int64\_t for the *sunindextype*.

This solver is constructed to perform the following operations:

- The "setup" call performs an LU factorization with partial (row) pivoting, PA = LU, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX\_BAND object A, with pivoting information encoding P stored in the pivots array.
- ullet The "solve" call performs pivoting and forward and backward substitution using the stored pivots array and the LU factors held in the SUNMATRIX\_BAND object.
- A must be allocated to accommodate the increase in upper bandwidth that occurs during factorization. More
  precisely, if A is a band matrix with upper bandwidth mu and lower bandwidth ml, then the upper triangular
  factor U can have upper bandwidth as big as smu = MIN(N-1,mu+ml). The lower triangular factor L has lower
  bandwidth ml.

The SUNLinSol\_LapackBand module defines band implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_LapackBand
- SUNLinSolInitialize\_LapackBand this does nothing, since all consistency checks are performed at solver creation.

- SUNLinSolSetup\_LapackBand this calls either DGBTRF or SGBTRF to perform the LU factorization.
- SUNLinSolSolve\_LapackBand this calls either DGBTRS or SGBTRS to use the LU factors and pivots array
  to perform the solve.
- SUNLinSolLastFlag\_LapackBand
- SUNLinSolSpace\_LapackBand this only returns information for the storage *within* the solver object, i.e. storage for N, last\_flag, and pivots.
- SUNLinSolFree\_LapackBand

# 8.7 The SUNLinSol\_LapackDense Module

The SUNLinSol\_LapackDense implementation of the SUNLinearSolver class is designed to be used with the corresponding SUNMATRIX\_DENSE matrix type, and one of the serial or shared-memory N\_Vector implementations (NVECTOR\_SERIAL, NVECTOR\_OPENMP, or NVECTOR\_PTHREADS).

# 8.7.1 SUNLinSol\_LapackDense Usage

The header file to be included when using this module is sunlinsol\_lapackdense.h. The installed module library to link to is libsundials\_sunlinsollapackdense.lib where .lib is typically .so for shared libraries and .a for static libraries.

The module SUNLinSol\_LapackDense provides the following additional user-callable constructor routine:

SUNLinearSolver SUNLinSol\_LapackDense(N\_Vector y, SUNMatrix A, SUNContext sunctx)

This function creates and allocates memory for a LAPACK dense SUNLinearSolver.

### **Arguments:**

- y vector used to determine the linear system size.
- A matrix used to assess compatibility.
- *sunctx* the *SUNContext* object (see §4.2)

### **Return value:**

New SUNLinSol\_LapackDense object, or NULL if either A or y are incompatible.

### Notes:

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUN-Matrix implementations. These are currently limited to the SUNMATRIX\_DENSE matrix type and the NVECTOR\_SERIAL, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

# 8.7.2 SUNLinSol LapackDense Description

The SUNLinSol LapackDense module defines the content field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_Dense {
   sunindextype N;
   sunindextype *pivots;
   sunindextype last_flag;
};
```

These entries of the *content* field contain the following information:

- N size of the linear system,
- pivots index array for partial pivoting in LU factorization,
- last\_flag last error return flag from internal function evaluations.

The SUNLinSol\_LapackDense module is a SUNLinearSolver wrapper for the LAPACK dense matrix factorization and solve routines, \*GETRF and \*GETRS, where \* is either D or S, depending on whether SUNDIALS was configured to have *sunrealtype* set to double or single, respectively (see §4.1 for details). In order to use the SUNLinSol\_LapackDense module it is assumed that LAPACK has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with LAPACK (see §11.4 for details). We note that since there do not exist 128-bit floating-point factorization and solve routines in LAPACK, this interface cannot be compiled when using extended precision for *sunrealtype*. Similarly, since there do not exist 64-bit integer LAPACK routines, the SUNLinSol\_LapackDense module also cannot be compiled when using int64\_t for the *sunindextype*.

This solver is constructed to perform the following operations:

- The "setup" call performs an LU factorization with partial (row) pivoting  $(\mathcal{O}(N^3) \cot)$ , PA = LU, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX\_DENSE object A, with pivoting information encoding P stored in the pivots array.
- The "solve" call performs pivoting and forward and backward substitution using the stored pivots array and the LU factors held in the SUNMATRIX DENSE object  $(\mathcal{O}(N^2) \text{ cost})$ .

The SUNLinSol\_LapackDense module defines dense implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_LapackDense
- SUNLinSolInitialize\_LapackDense this does nothing, since all consistency checks are performed at solver creation.
- SUNLinSolSetup\_LapackDense this calls either DGETRF or SGETRF to perform the *LU* factorization.
- SUNLinSolSolve\_LapackDense this calls either DGETRS or SGETRS to use the LU factors and pivots array to perform the solve.
- SUNLinSolLastFlag\_LapackDense
- SUNLinSolSpace\_LapackDense this only returns information for the storage *within* the solver object, i.e. storage for N, last\_flag, and pivots.
- SUNLinSolFree\_LapackDense

# 8.8 The SUNLinSol\_MagmaDense Module

The SUNLinearSolver\_MagmaDense implementation of the SUNLinearSolver class is designed to be used with the SUNMATRIX\_MAGMADENSE matrix, and a GPU-enabled vector. The header file to include when using this module is sunlinsol/sunlinsol\_magmadense.h. The installed library to link to is libsundials\_sunlinsolmagmadense.lib where lib is typically .so for shared libraries and .a for static libraries.

Warning: The SUNLinearSolver\_MagmaDense module is experimental and subject to change.

# 8.8.1 SUNLinearSolver\_MagmaDense Description

The SUNLinearSolver\_MagmaDense implementation provides an interface to the dense LU and dense batched LU methods in the MAGMA linear algebra library [48]. The batched LU methods are leveraged when solving block diagonal linear systems of the form

$$\begin{bmatrix} \mathbf{A_0} & 0 & \cdots & 0 \\ 0 & \mathbf{A_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A_{n-1}} \end{bmatrix} x_j = b_j.$$

# 8.8.2 SUNLinearSolver\_MagmaDense Functions

The SUNLinearSolver\_MagmaDense module defines implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_MagmaDense
- SUNLinSolInitialize\_MagmaDense
- SUNLinSolSetup\_MagmaDense
- SUNLinSolSolve\_MagmaDense
- SUNLinSolLastFlag\_MagmaDense
- SUNLinSolFree\_MagmaDense

In addition, the module provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_MagmaDense(N\_Vector y, SUNMatrix A, SUNContext sunctx)

This constructor function creates and allocates memory for a SUNLinearSolver object.

### **Arguments:**

- y a vector for checking compatibility with the solver.
- A a SUNMATRIX\_MAGMADENSE matrix for checking compatibility with the solver.
- sunctx the SUNContext object (see §4.2)

### Return value:

If successful, a SUNLinearSolver object. If either *A* or *y* are incompatible then this routine will return NULL. This routine analyzes the input matrix and vector to determine the linear system size and to assess compatibility with the solver.

### SUNErrCode SUNLinSol\_MagmaDense\_SetAsync(SUNLinearSolver LS, sunbooleantype onoff)

This function can be used to toggle the linear solver between asynchronous and synchronous modes. In asynchronous mode (default), SUNLinearSolver operations are asynchronous with respect to the host. In synchronous mode, the host and GPU device are synchronized prior to the operation returning.

# **Arguments:**

- LS a SUNLinSol\_MagmaDense object
- onoff 0 for synchronous mode or 1 for asynchronous mode (default 1)

#### **Return value:**

• A SUNErrCode

# 8.8.3 SUNLinearSolver\_MagmaDense Content

The SUNLinearSolver\_MagmaDense module defines the object *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_MagmaDense {
                  last_flag;
  sunbooleantype async;
  sunindextype
                  N;
  SUNMemory
                  pivots;
  SUNMemory
                  pivotsarr;
  SUNMemory
                  dpivotsarr;
  SUNMemory
                  infoarr;
  SUNMemory
                  rhsarr;
  SUNMemoryHelper memhelp;
  magma_queue_t
                  q;
};
```

# 8.9 The SUNLinSol\_OneMklDense Module

The SUNLinearSolver\_OneMklDense implementation of the SUNLinearSolver class interfaces to the direct linear solvers from the Intel oneAPI Math Kernel Library (oneMKL) for solving dense systems or block-diagonal systems with dense blocks. This linear solver is best paired with the SUNMatrix\_OneMklDense matrix.

The header file to include when using this class is sunlinsol/sunlinsol\_onemkldense.h. The installed library to link to is libsundials\_sunlinsolonemkldense.lib where lib is typically .so for shared libraries and .a for static libraries.

Warning: The SUNLinearSolver OneMklDense class is experimental and subject to change.

# 8.9.1 SUNLinearSolver OneMklDense Functions

The SUNLinearSolver\_OneMklDense class defines implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_OneMklDense returns SUNLINEARSOLVER\_ONEMKLDENSE
- SUNLinSolInitialize\_OneMklDense
- SUNLinSolSetup\_OneMklDense
- SUNLinSolSolve\_OneMklDense
- SUNLinSolLastFlag\_OneMklDense
- SUNLinSolFree\_OneMklDense

In addition, the class provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_OneMklDense(N Vector y, SUNMatrix A, SUNContext sunctx)

This constructor function creates and allocates memory for a SUNLinearSolver object.

### **Arguments:**

- y a vector for checking compatibility with the solver.
- A a SUNMatrix OneMklDense matrix for checking compatibility with the solver.
- *sunctx* the *SUNContext* object (see §4.2)

#### Return value:

If successful, a SUNLinearSolver object. If either *A* or *y* are incompatible then this routine will return NULL. This routine analyzes the input matrix and vector to determine the linear system size and to assess compatibility with the solver.

# 8.9.2 SUNLinearSolver\_OneMklDense Usage Notes

**Warning:** The SUNLinearSolver\_OneMklDense class only supports 64-bit indexing, thus SUNDIALS must be built for 64-bit indexing to use this class.

When using the SUNLinearSolver\_OneMklDense class with a SUNDIALS package (e.g. CVODE), the queue given to the matrix is also used for the linear solver.

# 8.10 The SUNLinSol\_PCG Module

The SUNLinSol\_PCG implementation of the SUNLinearSolver class performs the PCG (Preconditioned Conjugate Gradient [32]) method; this is an iterative linear solver that is designed to be compatible with any N\_Vector implementation that supports a minimal subset of operations (N\_VClone(), N\_VDotProd(), N\_VScale(), N\_VLinearSum(), N\_VProd(), and N\_VDestroy()). Unlike the SPGMR and SPFGMR algorithms, PCG requires a fixed amount of memory that does not increase with the number of allowed iterations.

Unlike all of the other iterative linear solvers supplied with SUNDIALS, PCG should only be used on *symmetric* linear systems (e.g. mass matrix linear systems encountered in ARKODE). As a result, the explanation of the role of scaling and preconditioning matrices given in general must be modified in this scenario. The PCG algorithm solves a linear system Ax = b where A is a symmetric ( $A^T = A$ ), real-valued matrix. Preconditioning is allowed, and is applied in a symmetric fashion on both the right and left. Scaling is also allowed and is applied symmetrically. We denote the preconditioner and scaling matrices as follows:

- P is the preconditioner (assumed symmetric),
- S is a diagonal matrix of scale factors.

The matrices A and P are not required explicitly; only routines that provide A and  $P^{-1}$  as operators are required. The diagonal of the matrix S is held in a single N\_Vector, supplied by the user.

In this notation, PCG applies the underlying CG algorithm to the equivalent transformed system

$$\tilde{A}\tilde{x} = \tilde{b} \tag{8.3}$$

where

$$\tilde{A} = SP^{-1}AP^{-1}S,$$

$$\tilde{b} = SP^{-1}b,$$

$$\tilde{x} = S^{-1}Px.$$
(8.4)

The scaling matrix must be chosen so that the vectors  $SP^{-1}b$  and  $S^{-1}Px$  have dimensionless components.

The stopping test for the PCG iterations is on the L2 norm of the scaled preconditioned residual:

$$\|\tilde{b} - \tilde{A}\tilde{x}\|_{2} < \delta$$

$$\Leftrightarrow \qquad \|SP^{-1}b - SP^{-1}Ax\|_{2} < \delta$$

$$\Leftrightarrow \qquad \|P^{-1}b - P^{-1}Ax\|_{S} < \delta$$

where  $||v||_S = \sqrt{v^T S^T S v}$ , with an input tolerance  $\delta$ .

# 8.10.1 SUNLinSol\_PCG Usage

The header file to be included when using this module is sunlinsol\_pcg.h. The SUNLinSol\_PCG module is accessible from all SUNDIALS solvers *without* linking to the libsundials\_sunlinsolpcg module library.

The module SUNLinSol\_PCG provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_PCG(N\_Vector y, int pretype, int maxl, SUNContext sunctx)

This constructor function creates and allocates memory for a PCG SUNLinearSolver.

### **Arguments:**

- y a template vector.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH
- maxl the maximum number of linear iterations to allow.
- *sunctx* the *SUNContext* object (see §4.2)

### **Return value:**

If successful, a SUNLinearSolver object. If either y is incompatible then this routine will return NULL.

#### **Notes:**

This routine will perform consistency checks to ensure that it is called with a consistent N\_Vector implementation (i.e. that it supplies the requisite vector operations).

A max1 argument that is  $\leq 0$  will result in the default value (5).

Since the PCG algorithm is designed to only support symmetric preconditioning, then any of the pretype inputs SUN\_PREC\_LEFT, SUN\_PREC\_RIGHT, or SUN\_PREC\_BOTH will result in use of the symmetric preconditioner; any other integer input will result in the default (no preconditioning). Although some SUN-DIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL), PCG should *only* be used with these packages when the linear systems are known to be *symmetric*. Since the scaling of matrix rows and columns must be identical in a symmetric matrix, symmetric preconditioning should work appropriately even for packages designed with one-sided preconditioning in mind.

### SUNErrCode SUNLinSol\_PCGSetPrecType(SUNLinearSolver S, int pretype)

This function updates the flag indicating use of preconditioning.

### **Arguments:**

- *S* SUNLinSol\_PCG object to update.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH

### **Return value:**

A SUNErrCode

#### **Notes:**

As above, any one of the input values, SUN\_PREC\_LEFT, SUN\_PREC\_RIGHT, or SUN\_PREC\_BOTH will enable preconditioning; SUN\_PREC\_NONE disables preconditioning.

### SUNErrCode SUNLinSol\_PCGSetMaxl(SUNLinearSolver S, int maxl)

This function updates the number of linear solver iterations to allow.

### **Arguments:**

- *S* SUNLinSol\_PCG object to update.
- *maxl* maximum number of linear iterations to allow. Any non-positive input will result in the default value (5).

### Return value:

• A SUNErrCode

# 8.10.2 SUNLinSol\_PCG Description

The SUNLinSol\_PCG module defines the content field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_PCG {
  int maxl;
  int pretype;
  sunbooleantype zeroguess;
  int numiters;
  sunrealtype resnorm;
  int last_flag;
  SUNATimesFn ATimes;
  void* ATData;
  SUNPSetupFn Psetup;
  SUNPSolveFn Psolve;
  void* PData;
  N_Vector s;
  N_Vector r;
  N_Vector p;
 N_Vector z;
  N_Vector Ap;
};
```

These entries of the *content* field contain the following information:

- max1 number of PCG iterations to allow (default is 5),
- pretype flag for use of preconditioning (default is none),
- numiters number of iterations from the most-recent solve,
- resnorm final linear residual norm from the most-recent solve,
- last\_flag last error return flag from an internal function,
- ATimes function pointer to perform Av product,
- ATData pointer to structure for ATimes,
- Psetup function pointer to preconditioner setup routine,
- Psolve function pointer to preconditioner solve routine,
- PData pointer to structure for Psetup and Psolve,
- s vector pointer for supplied scaling matrix (default is NULL),
- r a N\_Vector which holds the preconditioned linear system residual,
- p, z, Ap N\_Vector used for workspace by the PCG algorithm.

This solver is constructed to perform the following operations:

- During construction all N\_Vector solver data is allocated, with vectors cloned from a template N\_Vector that is input, and default solver parameters are set.
- User-facing "set" routines may be called to modify default solver parameters.
- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLinSol\_PCG to supply the ATimes, PSetup, and Psolve function pointers and s scaling vector.
- In the "initialize" call, the solver parameters are checked for validity.

- In the "setup" call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).
- In the "solve" call the PCG iteration is performed. This will include scaling and preconditioning if those options have been supplied.

The SUNLinSol PCG module defines implementations of all "iterative" linear solver operations listed in §8.1:

- SUNLinSolGetType\_PCG
- SUNLinSolInitialize\_PCG
- SUNLinSolSetATimes\_PCG
- SUNLinSolSetPreconditioner PCG
- SUNLinSolSetScalingVectors\_PCG since PCG only supports symmetric scaling, the second N\_Vector argument to this function is ignored.
- SUNLinSolSetZeroGuess\_PCG note the solver assumes a non-zero guess by default and the zero guess flag is reset to SUNFALSE after each call to SUNLinSolSolve\_PCG.
- SUNLinSolSetup\_PCG
- SUNLinSolSolve PCG
- SUNLinSolNumIters\_PCG
- SUNLinSolResNorm PCG
- SUNLinSolResid\_PCG
- SUNLinSolLastFlag\_PCG
- SUNLinSolSpace\_PCG
- SUNLinSolFree\_PCG

# 8.11 The SUNLinSol\_SPBCGS Module

The SUNLinSol\_SPBCGS implementation of the SUNLinearSolver class performs a Scaled, Preconditioned, Bi-Conjugate Gradient, Stabilized [51] method; this is an iterative linear solver that is designed to be compatible with any N\_Vector implementation that supports a minimal subset of operations (N\_VClone(), N\_VDotProd(), N\_VScale(), N\_VLinearSum(), N\_VProd(), N\_VDiv(), and N\_VDestroy()). Unlike the SPGMR and SPFGMR algorithms, SP-BCGS requires a fixed amount of memory that does not increase with the number of allowed iterations.

# 8.11.1 SUNLinSol\_SPBCGS Usage

The header file to be included when using this module is sunlinsol\_spbcgs.h. The SUNLinSol\_SPBCGS module is accessible from all SUNDIALS solvers *without* linking to the libsundials\_sunlinsolspbcgs module library.

The module SUNLinSol SPBCGS provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_SPBCGS (N\_Vector y, int pretype, int maxl, SUNContext sunctx)

This constructor function creates and allocates memory for a SPBCGS SUNLinearSolver.

## **Arguments:**

• y - a template vector.

- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN PREC BOTH
- maxl the maximum number of linear iterations to allow.
- *sunctx* the *SUNContext* object (see §4.2)

#### **Return value:**

If successful, a SUNLinearSolver object. If either y is incompatible then this routine will return NULL.

### **Notes:**

This routine will perform consistency checks to ensure that it is called with a consistent N\_Vector implementation (i.e. that it supplies the requisite vector operations).

A max1 argument that is  $\leq 0$  will result in the default value (5).

Some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLinSol\_SPBCGS object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

**Note:** With SUN\_PREC\_RIGHT or SUN\_PREC\_BOTH the initial guess must be zero (use *SUNLinSolSetZe-roGuess()* to indicate the initial guess is zero).

# SUNErrCode SUNLinSol\_SPBCGSSetPrecType(SUNLinearSolver S, int pretype)

This function updates the flag indicating use of preconditioning.

### **Arguments:**

- S SUNLinSol\_SPBCGS object to update.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH

### **Return value:**

• A SUNErrCode

# SUNErrCode SUNLinSol\_SPBCGSSetMaxl(SUNLinearSolver S, int maxl)

This function updates the number of linear solver iterations to allow.

#### **Arguments:**

- S SUNLinSol\_SPBCGS object to update.
- *maxl* maximum number of linear iterations to allow. Any non-positive input will result in the default value (5).

### **Return value:**

A SUNErrCode

# 8.11.2 SUNLinSol\_SPBCGS Description

The SUNLinSol\_SPBCGS module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SPBCGS {
  int maxl;
  int pretype;
  sunbooleantype zeroguess;
  int numiters;
  sunrealtype resnorm;
  int last_flag;
  SUNATimesFn ATimes;
  void* ATData;
  SUNPSetupFn Psetup;
  SUNPSolveFn Psolve;
  void* PData;
  N_Vector s1;
  N_Vector s2;
  N_Vector r;
  N_Vector r_star;
  N_Vector p;
 N_Vector q;
 N_Vector u;
  N_Vector Ap;
  N_Vector vtemp;
};
```

These entries of the *content* field contain the following information:

- max1 number of SPBCGS iterations to allow (default is 5),
- pretype flag for type of preconditioning to employ (default is none),
- numiters number of iterations from the most-recent solve,
- resnorm final linear residual norm from the most-recent solve,
- last\_flag last error return flag from an internal function,
- ATimes function pointer to perform Av product,
- ATData pointer to structure for ATimes,
- Psetup function pointer to preconditioner setup routine,
- Psolve function pointer to preconditioner solve routine,
- PData pointer to structure for Psetup and Psolve,
- s1, s2 vector pointers for supplied scaling matrices (default is NULL),
- r a N\_Vector which holds the current scaled, preconditioned linear system residual,
- r\_star a N\_Vector which holds the initial scaled, preconditioned linear system residual,
- p, q, u, Ap, vtemp N\_Vector used for workspace by the SPBCGS algorithm.

This solver is constructed to perform the following operations:

- During construction all N\_Vector solver data is allocated, with vectors cloned from a template N\_Vector that is input, and default solver parameters are set.
- User-facing "set" routines may be called to modify default solver parameters.

- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLinSol\_SPBCGS to supply
  the ATimes, PSetup, and Psolve function pointers and s1 and s2 scaling vectors.
- In the "initialize" call, the solver parameters are checked for validity.
- In the "setup" call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).
- In the "solve" call the SPBCGS iteration is performed. This will include scaling and preconditioning if those options have been supplied.

The SUNLinSol\_SPBCGS module defines implementations of all "iterative" linear solver operations listed in §8.1:

- SUNLinSolGetType\_SPBCGS
- SUNLinSolInitialize\_SPBCGS
- SUNLinSolSetATimes\_SPBCGS
- SUNLinSolSetPreconditioner\_SPBCGS
- SUNLinSolSetScalingVectors\_SPBCGS
- SUNLinSolSetZeroGuess\_SPBCGS note the solver assumes a non-zero guess by default and the zero guess flag is reset to SUNFALSE after each call to SUNLinSolSolve\_SPBCGS.
- SUNLinSolSetup\_SPBCGS
- SUNLinSolSolve SPBCGS
- SUNLinSolNumIters\_SPBCGS
- SUNLinSolResNorm\_SPBCGS
- SUNLinSolResid\_SPBCGS
- SUNLinSolLastFlag\_SPBCGS
- SUNLinSolSpace\_SPBCGS
- SUNLinSolFree\_SPBCGS

# 8.12 The SUNLinSol SPFGMR Module

The SUNLinSol\_SPFGMR implementation of the SUNLinearSolver class performs a Scaled, Preconditioned, Flexible, Generalized Minimum Residual [45] method; this is an iterative linear solver that is designed to be compatible with any N\_Vector implementation that supports a minimal subset of operations (N\_VClone(), N\_VDotProd(), N\_-VScale(), N\_VLinearSum(), N\_VProd(), N\_VConst(), N\_VDiv(), and N\_VDestroy()). Unlike the other Krylov iterative linear solvers supplied with SUNDIALS, FGMRES is specifically designed to work with a changing preconditioner (e.g. from an iterative method).

# 8.12.1 SUNLinSol\_SPFGMR Usage

The header file to be included when using this module is sunlinsol\_spfgmr.h. The SUNLinSol\_SPFGMR module is accessible from all SUNDIALS solvers *without* linking to the libsundials\_sunlinsolspfgmr module library.

The module SUNLinSol\_SPFGMR provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_SPFGMR(N\_Vector y, int pretype, int maxl, SUNContext sunctx)

This constructor function creates and allocates memory for a SPFGMR SUNLinearSolver.

### **Arguments:**

- y a template vector.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH
- *maxl* the number of Krylov basis vectors to use.
- *sunctx* the *SUNContext* object (see §4.2)

#### **Return value:**

If successful, a SUNLinearSolver object. If either y is incompatible then this routine will return NULL.

#### Notes:

This routine will perform consistency checks to ensure that it is called with a consistent N\_Vector implementation (i.e. that it supplies the requisite vector operations).

A max1 argument that is  $\leq 0$  will result in the default value (5).

Since the FGMRES algorithm is designed to only support right preconditioning, then any of the pretype inputs SUN\_PREC\_LEFT, SUN\_PREC\_RIGHT, or SUN\_PREC\_BOTH will result in use of SUN\_PREC\_RIGHT; any other integer input will result in the default (no preconditioning). We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS). While it is possible to use a right-preconditioned SUNLinSol\_SPFGMR object for these packages, this use mode is not supported and may result in inferior performance.

# SUNErrCode SUNLinSol\_SPFGMRSetPrecType(SUNLinearSolver S, int pretype)

This function updates the flag indicating use of preconditioning.

#### **Arguments:**

- S SUNLinSol\_SPFGMR object to update.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH

# **Return value:**

A SUNErrCode

#### **Notes:**

Since the FGMRES algorithm is designed to only support right preconditioning, then any of the pretype inputs SUN\_PREC\_LEFT, SUN\_PREC\_RIGHT, or SUN\_PREC\_BOTH will result in use of SUN\_PREC\_RIGHT; any other integer input will result in the default (no preconditioning).

# SUNErrCode SUNLinSol\_SPFGMRSetGSType(SUNLinearSolver S, int gstype)

This function sets the type of Gram-Schmidt orthogonalization to use.

### **Arguments:**

- *S* SUNLinSol\_SPFGMR object to update.
- *gstype* a flag indicating the type of orthogonalization to use:
  - SUN\_MODIFIED\_GS
  - SUN\_CLASSICAL\_GS

#### Return value:

• A SUNErrCode

# SUNErrCode SUNLinSol\_SPFGMRSetMaxRestarts(SUNLinearSolver S, int maxrs)

This function sets the number of FGMRES restarts to allow.

## **Arguments:**

- *S* SUNLinSol\_SPFGMR object to update.
- maxrs maximum number of restarts to allow. A negative input will result in the default of 0.

#### Return value:

• A SUNErrCode

# 8.12.2 SUNLinSol\_SPFGMR Description

The SUNLinSol\_SPFGMR module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SPFGMR {
  int maxl;
  int pretype;
  int gstype;
  int max_restarts;
  sunbooleantype zeroguess;
  int numiters;
  sunrealtype resnorm;
  int last_flag;
  SUNATimesFn ATimes;
  void* ATData;
  SUNPSetupFn Psetup;
  SUNPSolveFn Psolve:
  void* PData;
  N_Vector s1:
  N_Vector s2;
  N_Vector *V;
  N_Vector *Z;
  sunrealtype **Hes;
  sunrealtype *givens;
```

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```
N_Vector xcor;
sunrealtype *yg;
N_Vector vtemp;
};
```

These entries of the *content* field contain the following information:

- max1 number of FGMRES basis vectors to use (default is 5),
- pretype flag for use of preconditioning (default is none),
- gstype flag for type of Gram-Schmidt orthogonalization (default is modified Gram-Schmidt),
- max\_restarts number of FGMRES restarts to allow (default is 0),
- numiters number of iterations from the most-recent solve,
- resnorm final linear residual norm from the most-recent solve,
- last\_flag last error return flag from an internal function,
- ullet ATimes function pointer to perform Av product,
- ATData pointer to structure for ATimes,
- Psetup function pointer to preconditioner setup routine,
- Psolve function pointer to preconditioner solve routine,
- PData pointer to structure for Psetup and Psolve,
- s1, s2 vector pointers for supplied scaling matrices (default is NULL),
- V the array of Krylov basis vectors  $v_1, \ldots, v_{\text{maxl}+1}$ , stored in V[0], ..., V[maxl]. Each  $v_i$  is a vector of type N\_Vector,
- Z the array of preconditioned Krylov basis vectors  $z_1, \ldots, z_{\text{maxl}+1}$ , stored in Z[0], ..., Z[max1]. Each  $z_i$  is a vector of type N\_Vector,
- Hes the  $(\max l + 1) \times \max l$  Hessenberg matrix. It is stored row-wise so that the (i,j)th element is given by Hes[i][j],
- givens a length 2 maxl array which represents the Givens rotation matrices that arise in the FGMRES algorithm. These matrices are  $F_0, F_1, \ldots, F_j$ , where

$$F_i = egin{bmatrix} 1 & & & & & & & & \\ & \ddots & & & & & & & & \\ & & 1 & & & & & & \\ & & & c_i & -s_i & & & & \\ & & & s_i & c_i & & & & \\ & & & & 1 & & & \\ & & & & \ddots & & \\ & & & & 1 \end{bmatrix},$$

are represented in the givens vector as givens[0] =  $c_0$ , givens[1] =  $s_0$ , givens[2] =  $c_1$ , givens[3] =  $s_1$ , ..., givens[2j] =  $c_j$ , givens[2j+1] =  $s_j$ ,

- xcor a vector which holds the scaled, preconditioned correction to the initial guess,
- yg a length (maxl + 1) array of sunrealtype values used to hold "short" vectors (e.g. y and g),
- vtemp temporary vector storage.

This solver is constructed to perform the following operations:

- During construction, the xcor and vtemp arrays are cloned from a template N\_Vector that is input, and default solver parameters are set.
- User-facing "set" routines may be called to modify default solver parameters.
- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLinSol\_SPFGMR to supply the ATimes, PSetup, and Psolve function pointers and s1 and s2 scaling vectors.
- In the "initialize" call, the remaining solver data is allocated (V, Hes, givens, and yg)
- In the "setup" call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).
- In the "solve" call, the FGMRES iteration is performed. This will include scaling, preconditioning, and restarts if those options have been supplied.

The SUNLinSol\_SPFGMR module defines implementations of all "iterative" linear solver operations listed in §8.1:

- SUNLinSolGetType\_SPFGMR
- SUNLinSolInitialize\_SPFGMR
- SUNLinSolSetATimes\_SPFGMR
- SUNLinSolSetPreconditioner\_SPFGMR
- SUNLinSolSetScalingVectors SPFGMR
- SUNLinSolSetZeroGuess\_SPFGMR note the solver assumes a non-zero guess by default and the zero guess flag is reset to SUNFALSE after each call to SUNLinSolSolve\_SPFGMR.
- SUNLinSolSetup\_SPFGMR
- SUNLinSolSolve\_SPFGMR
- SUNLinSolNumIters\_SPFGMR
- SUNLinSolResNorm\_SPFGMR
- SUNLinSolResid\_SPFGMR
- SUNLinSolLastFlag\_SPFGMR
- SUNLinSolSpace\_SPFGMR
- SUNLinSolFree\_SPFGMR

# 8.13 The SUNLinSol SPGMR Module

The SUNLinSol\_SPGMR implementation of the SUNLinearSolver class performs a Scaled, Preconditioned, Generalized Minimum Residual [46] method; this is an iterative linear solver that is designed to be compatible with any N\_Vector implementation that supports a minimal subset of operations (N\_VClone(), N\_VDotProd(), N\_VScale(), N\_VLinearSum(), N\_VProd(), N\_VConst(), N\_VDiv(), and N\_VDestroy()).

# 8.13.1 SUNLinSol\_SPGMR Usage

The header file to be included when using this module is sunlinsol\_spgmr.h. The SUNinSol\_SPGMR module is accessible from all SUNDIALS solvers *without* linking to the libsundials\_sunlinsolspgmr module library.

The module SUNLinSol\_SPGMR provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_SPGMR (N\_Vector y, int pretype, int maxl, SUNContext sunctx)

This constructor function creates and allocates memory for a SPGMR SUNLinearSolver.

### **Arguments:**

- y a template vector.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH
- maxl the number of Krylov basis vectors to use.

#### **Return value:**

If successful, a SUNLinearSolver object. If either y is incompatible then this routine will return NULL.

#### **Notes:**

This routine will perform consistency checks to ensure that it is called with a consistent N\_Vector implementation (i.e. that it supplies the requisite vector operations).

A max1 argument that is  $\leq 0$  will result in the default value (5).

Some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLinSol\_SPGMR object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

#### SUNErrCode SUNLinSol\_SPGMRSetPrecType(SUNLinearSolver S, int pretype)

This function updates the flag indicating use of preconditioning.

### **Arguments:**

- *S* SUNLinSol\_SPGMR object to update.
- *pretype* a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH

#### **Return value:**

• A SUNErrCode

# SUNErrCode SUNLinSol\_SPGMRSetGSType(SUNLinearSolver S, int gstype)

This function sets the type of Gram-Schmidt orthogonalization to use.

### **Arguments:**

- *S* SUNLinSol\_SPGMR object to update.
- gstype a flag indicating the type of orthogonalization to use:
  - SUN\_MODIFIED\_GS
  - SUN\_CLASSICAL\_GS

#### Return value:

A SUNErrCode

## SUNErrCode SUNLinSol\_SPGMRSetMaxRestarts(SUNLinearSolver S, int maxrs)

This function sets the number of GMRES restarts to allow.

# **Arguments:**

- *S* SUNLinSol\_SPGMR object to update.
- maxrs maximum number of restarts to allow. A negative input will result in the default of 0.

#### **Return value:**

• A SUNErrCode

# 8.13.2 SUNLinSol\_SPGMR Description

The SUNLinSol\_SPGMR module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SPGMR {
  int maxl;
  int pretype;
  int gstype;
  int max_restarts;
  sunbooleantype zeroguess;
  int numiters;
  sunrealtype resnorm;
  int last_flag;
  SUNATimesFn ATimes;
  void* ATData;
  SUNPSetupFn Psetup;
  SUNPSolveFn Psolve;
  void* PData;
  N_Vector s1;
  N_Vector s2;
  N_Vector *V;
  sunrealtype **Hes;
  sunrealtype *givens;
  N_Vector xcor;
  sunrealtype *yg;
  N_Vector vtemp;
};
```

These entries of the *content* field contain the following information:

- max1 number of GMRES basis vectors to use (default is 5),
- pretype flag for type of preconditioning to employ (default is none),
- gstype flag for type of Gram-Schmidt orthogonalization (default is modified Gram-Schmidt),

- max\_restarts number of GMRES restarts to allow (default is 0),
- numiters number of iterations from the most-recent solve,
- resnorm final linear residual norm from the most-recent solve,
- last\_flag last error return flag from an internal function,
- ATimes function pointer to perform Av product,
- ATData pointer to structure for ATimes,
- Psetup function pointer to preconditioner setup routine,
- Psolve function pointer to preconditioner solve routine,
- PData pointer to structure for Psetup and Psolve,
- s1, s2 vector pointers for supplied scaling matrices (default is NULL),
- V the array of Krylov basis vectors  $v_1, \ldots, v_{\text{maxl}+1}$ , stored in V[0], ... V[maxl]. Each  $v_i$  is a vector of type N\_Vector.
- Hes the  $(\max l + 1) \times \max l$  Hessenberg matrix. It is stored row-wise so that the (i,j)th element is given by Hes[i][j],
- givens a length 2 maxl array which represents the Givens rotation matrices that arise in the GMRES algorithm. These matrices are  $F_0, F_1, \dots, F_j$ , where

are represented in the givens vector as givens[0] =  $c_0$ , givens[1] =  $s_0$ , givens[2] =  $c_1$ , givens[3] =  $s_1$ , ..., givens[2j] =  $c_j$ , givens[2j+1] =  $s_j$ ,

- xcor a vector which holds the scaled, preconditioned correction to the initial guess,
- yg a length (maxl + 1) array of sunreal type values used to hold "short" vectors (e.g. y and q),
- vtemp temporary vector storage.

This solver is constructed to perform the following operations:

- During construction, the xcor and vtemp arrays are cloned from a template N\_Vector that is input, and default solver parameters are set.
- User-facing "set" routines may be called to modify default solver parameters.
- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLinSol\_SPGMR to supply
  the ATimes, PSetup, and Psolve function pointers and s1 and s2 scaling vectors.
- In the "initialize" call, the remaining solver data is allocated (V, Hes, givens, and yg)
- In the "setup" call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).
- In the "solve" call, the GMRES iteration is performed. This will include scaling, preconditioning, and restarts if those options have been supplied.

The SUNLinSol\_SPGMR module defines implementations of all "iterative" linear solver operations listed in §8.1:

- SUNLinSolGetType\_SPGMR
- SUNLinSolInitialize\_SPGMR
- SUNLinSolSetATimes\_SPGMR
- SUNLinSolSetPreconditioner SPGMR
- SUNLinSolSetScalingVectors\_SPGMR
- SUNLinSolSetZeroGuess\_SPGMR note the solver assumes a non-zero guess by default and the zero guess flag is reset to SUNFALSE after each call to SUNLinSolSolve\_SPGMR.
- SUNLinSolSetup\_SPGMR
- SUNLinSolSolve\_SPGMR
- SUNLinSolNumIters\_SPGMR
- SUNLinSolResNorm\_SPGMR
- SUNLinSolResid SPGMR
- SUNLinSolLastFlag\_SPGMR
- SUNLinSolSpace\_SPGMR
- SUNLinSolFree\_SPGMR

## 8.14 The SUNLinSol\_SPTFQMR Module

The SUNLinSol\_SPTFQMR implementation of the SUNLinearSolver class performs a Scaled, Preconditioned, Transpose-Free Quasi-Minimum Residual [28] method; this is an iterative linear solver that is designed to be compatible with any N\_Vector implementation that supports a minimal subset of operations (N\_VClone(), N\_VDotProd(), N\_VScale(), N\_VLinearSum(), N\_VProd(), N\_VConst(), N\_VDiv(), and N\_VDestroy()). Unlike the SPGMR and SPFGMR algorithms, SPTFQMR requires a fixed amount of memory that does not increase with the number of allowed iterations.

## 8.14.1 SUNLinSol\_SPTFQMR Usage

The header file to be included when using this module is sunlinsol/sunlinsol\_sptfqmr.h. The SUNLinSol\_SPT-FQMR module is accessible from all SUNDIALS solvers without linking to the libsundials\_sunlinsolsptfqmr module library.

The module SUNLinSol\_SPTFQMR provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_SPTFQMR(N\_Vector y, int pretype, int maxl, SUNContext sunctx)

This constructor function creates and allocates memory for a SPTFQMR SUNLinearSolver.

## **Arguments:**

- y a template vector.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT

- SUN\_PREC\_BOTH
- maxl the number of Krylov basis vectors to use.
- *sunctx* the *SUNContext* object (see §4.2)

#### Return value:

If successful, a SUNLinearSolver object. If either y is incompatible then this routine will return NULL.

#### **Notes:**

This routine will perform consistency checks to ensure that it is called with a consistent N\_Vector implementation (i.e. that it supplies the requisite vector operations).

A max1 argument that is  $\leq 0$  will result in the default value (5).

Some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLinSol\_SPTFQMR object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

**Note:** With SUN\_PREC\_RIGHT or SUN\_PREC\_BOTH the initial guess must be zero (use *SUNLinSolSetZe-roGuess()* to indicate the initial guess is zero).

## SUNErrCode SUNLinSol\_SPTFQMRSetPrecType(SUNLinearSolver S, int pretype)

This function updates the flag indicating use of preconditioning.

## **Arguments:**

- *S* SUNLinSol\_SPGMR object to update.
- pretype a flag indicating the type of preconditioning to use:
  - SUN\_PREC\_NONE
  - SUN\_PREC\_LEFT
  - SUN\_PREC\_RIGHT
  - SUN\_PREC\_BOTH

#### Return value:

• A SUNErrCode

#### SUNErrCode SUNLinSol\_SPTFQMRSetMaxl(SUNLinearSolver S, int maxl)

This function updates the number of linear solver iterations to allow.

#### **Arguments:**

- S SUNLinSol\_SPTFQMR object to update.
- *maxl* maximum number of linear iterations to allow. Any non-positive input will result in the default value (5).

#### **Return value:**

A SUNErrCode

## 8.14.2 SUNLinSol\_SPTFQMR Description

The SUNLinSol\_SPTFQMR module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SPTFQMR {
  int maxl;
  int pretype;
  sunbooleantype zeroguess;
  int numiters;
  sunrealtype resnorm;
  int last_flag;
  SUNATimesFn ATimes;
  void* ATData;
  SUNPSetupFn Psetup;
  SUNPSolveFn Psolve;
  void* PData;
  N_Vector s1;
  N_Vector s2;
  N_Vector r_star;
  N_Vector q;
  N_Vector d;
  N_Vector v;
  N_Vector p;
  N_Vector *r;
  N_Vector u;
  N_Vector vtemp1;
  N_Vector vtemp2;
  N_Vector vtemp3;
};
```

These entries of the *content* field contain the following information:

- max1 number of TFQMR iterations to allow (default is 5),
- pretype flag for type of preconditioning to employ (default is none),
- numiters number of iterations from the most-recent solve,
- resnorm final linear residual norm from the most-recent solve,
- last\_flag last error return flag from an internal function,
- ATimes function pointer to perform Av product,
- ATData pointer to structure for ATimes,
- Psetup function pointer to preconditioner setup routine,
- Psolve function pointer to preconditioner solve routine,
- PData pointer to structure for Psetup and Psolve,
- s1, s2 vector pointers for supplied scaling matrices (default is NULL),
- r\_star a N\_Vector which holds the initial scaled, preconditioned linear system residual,
- q, d, v, p, u N\_Vector used for workspace by the SPTFQMR algorithm,
- r array of two N\_Vector used for workspace within the SPTFQMR algorithm,
- vtemp1, vtemp2, vtemp3 temporary vector storage.

This solver is constructed to perform the following operations:

- During construction all N\_Vector solver data is allocated, with vectors cloned from a template N\_Vector that is input, and default solver parameters are set.
- User-facing "set" routines may be called to modify default solver parameters.
- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLinSol\_SPTFQMR to supply the ATimes, PSetup, and Psolve function pointers and s1 and s2 scaling vectors.
- In the "initialize" call, the solver parameters are checked for validity.
- In the "setup" call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).
- In the "solve" call the TFQMR iteration is performed. This will include scaling and preconditioning if those options have been supplied.

The SUNLinSol\_SPTFQMR module defines implementations of all "iterative" linear solver operations listed in §8.1:

- SUNLinSolGetType\_SPTFQMR
- SUNLinSolInitialize\_SPTFQMR
- SUNLinSolSetATimes\_SPTFQMR
- SUNLinSolSetPreconditioner\_SPTFQMR
- SUNLinSolSetScalingVectors\_SPTFQMR
- SUNLinSolSetZeroGuess\_SPTFQMR note the solver assumes a non-zero guess by default and the zero guess
  flag is reset to SUNFALSE after each call to SUNLinSolSolve\_SPTFQMR.
- SUNLinSolSetup\_SPTFQMR
- SUNLinSolSolve\_SPTFQMR
- SUNLinSolNumIters\_SPTFQMR
- SUNLinSolResNorm\_SPTFQMR
- SUNLinSolResid\_SPTFQMR
- SUNLinSolLastFlag\_SPTFQMR
- SUNLinSolSpace\_SPTFQMR
- SUNLinSolFree\_SPTFQMR

# 8.15 The SUNLinSol\_SuperLUDIST Module

The SUNLinsol\_SuperLUDIST implementation of the SUNLinearSolver class interfaces with the SuperLU\_DIST library. This is designed to be used with the SUNMatrix\_SLUNRloc SUNMatrix, and one of the serial, threaded or parallel N\_Vector implementations (NVECTOR\_SERIAL, NVECTOR\_OPENMP, NVECTOR\_PTHREADS, NVECTOR\_PARALLEL, NVECTOR\_PARHYP).

## 8.15.1 SUNLinSol\_SuperLUDIST Usage

The header file to be included when using this module is sunlinsol/sunlinsol\_superludist.h. The installed module library to link to is libsundials\_sunlinsolsuperludist.lib where .lib is typically .so for shared libraries and .a for static libraries.

The module SUNLinSol\_SuperLUDIST provides the following user-callable routines:

**Warning:** Starting with SuperLU\_DIST version 6.3.0, some structures were renamed to have a prefix for the floating point type. The double precision API functions have the prefix 'd'. To maintain backwards compatibility with the unprefixed types, SUNDIALS provides macros to these SuperLU\_DIST types with an 'x' prefix that expand to the correct prefix. E.g., the SUNDIALS macro xLUstruct\_t expands to dLUstruct\_t or LUstruct\_t based on the SuperLU\_DIST version.

```
SUNLinearSolver SUNLinSol_SuperLUDIST(N_Vector y, SuperMatrix *A, gridinfo_t *grid, xLUstruct_t *lu, xScalePermstruct_t *scaleperm, xSOLVEstruct_t *solve, SuperLUStat_t *stat, superlu_dist_options_t *options, SUNContext sunctx)
```

This constructor function creates and allocates memory for a SUNLinSol\_SuperLUDIST object.

#### **Arguments:**

- y a template vector.
- A a template matrix
- grid, lu, scaleperm, solve, stat, options SuperLU\_DIST object pointers.
- *sunctx* the *SUNContext* object (see §4.2)

#### **Return value:**

If successful, a SUNLinearSolver object; otherwise this routine will return NULL.

#### **Notes:**

This routine analyzes the input matrix and vector to determine the linear system size and to assess the compatibility with the SuperLU\_DIST library.

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUN-Matrix implementations. These are currently limited to the SUNMatrix\_SLUNRloc matrix type and the NVECTOR\_SERIAL, NVECTOR\_OPENMP, NVECTOR\_PTHREADS, NVECTOR\_PARALLEL, and NVECTOR\_PARHYP vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

The grid, lu, scaleperm, solve, and options arguments are not checked and are passed directly to SuperLU\_DIST routines.

Some struct members of the options argument are modified internally by the SUNLinSol\_SuperLUDIST solver. Specifically, the member Fact is modified in the setup and solve routines.

#### sunrealtype SUNLinSol\_SuperLUDIST\_GetBerr(SUNLinearSolver LS)

This function returns the componentwise relative backward error of the computed solution. It takes one argument, the SUNLinearSolver object. The return type is sunrealtype.

```
gridinfo_t *SUNLinSol_SuperLUDIST_GetGridinfo(SUNLinearSolver LS)
```

This function returns a pointer to the SuperLU\_DIST structure that contains the 2D process grid. It takes one argument, the SUNLinearSolver object.

#### xLUstruct t \*SUNLinSol\_SuperLUDIST\_GetLUstruct(SUNLinearSolver LS)

This function returns a pointer to the SuperLU\_DIST structure that contains the distributed L and U structures. It takes one argument, the SUNLinearSolver object.

```
superlu_dist_options_t *SUNLinSol_SuperLUDIST_GetSuperLUOptions(SUNLinearSolver LS)
```

This function returns a pointer to the SuperLU\_DIST structure that contains the options which control how the linear system is factorized and solved. It takes one argument, the SUNLinearSolver object.

```
xScalePermstruct_t *SUNLinSol_SuperLUDIST_GetScalePermstruct(SUNLinearSolver LS)
```

This function returns a pointer to the SuperLU\_DIST structure that contains the vectors that describe the transformations done to the matrix A. It takes one argument, the SUNLinearSolver object.

```
xSOLVEstruct_t *SUNLinSol_SuperLUDIST_GetSOLVEstruct(SUNLinearSolver LS)
```

This function returns a pointer to the SuperLU\_DIST structure that contains information for communication during the solution phase. It takes one argument the SUNLinearSolver object.

```
SuperLUStat_t *SUNLinSol_SuperLUDIST_GetSuperLUStat(SUNLinearSolver LS)
```

This function returns a pointer to the SuperLU\_DIST structure that stores information about runtime and flop count. It takes one argument, the SUNLinearSolver object.

## 8.15.2 SUNLinSol\_SuperLUDIST Description

The SUNLinSol\_SuperLUDIST module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SuperLUDIST {
  sunbooleantype
                           first_factorize;
  int
                           last_flag;
  sunrealtype
                           berr;
  gridinfo_t
                           *grid;
  xLUstruct_t
                           *lu;
  superlu_dist_options_t
                           *options;
                           *scaleperm;
  xScalePermstruct_t
  xSOLVEstruct_t
                           *solve:
  SuperLUStat_t
                           *stat;
  sunindextype
                           N;
};
```

These entries of the *content* field contain the following information:

- first\_factorize flag indicating whether the factorization has ever been performed,
- last\_flag last error return flag from internal function evaluations,
- berr the componentwise relative backward error of the computed solution,
- grid pointer to the SuperLU\_DIST structure that strores the 2D process grid
- 1u pointer to the SuperLU\_DIST structure that stores the distributed L and U factors,
- scaleperm pointer to the SuperLU\_DIST structure that stores vectors describing the transformations done to the matrix A,
- options pointer to the SuperLU\_DIST stucture which contains options that control how the linear system is factorized and solved,
- solve pointer to the SuperLU\_DIST solve structure,
- stat pointer to the SuperLU\_DIST structure that stores information about runtime and flop count,

• N – the number of equations in the system.

The SUNLinSol\_SuperLUDIST module is a SUNLinearSolver adapter for the SuperLU\_DIST sparse matrix factorization and solver library written by X. Sherry Li and collaborators [8, 30, 41, 42]. The package uses a SPMD parallel programming model and multithreading to enhance efficiency in distributed-memory parallel environments with multicore nodes and possibly GPU accelerators. It uses MPI for communication, OpenMP for threading, and CUDA for GPU support. In order to use the SUNLinSol\_SuperLUDIST interface to SuperLU\_DIST, it is assumed that SuperLU\_DIST has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with SuperLU\_DIST (see §11.4 for details). Additionally, the wrapper only supports double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to use single or extended precision. Moreover, since the SuperLU\_DIST library may be installed to support either 32-bit or 64-bit integers, it is assumed that the SuperLU\_DIST library is installed using the same integer size as SUNDIALS.

The SuperLU\_DIST library provides many options to control how a linear system will be factorized and solved. These options may be set by a user on an instance of the superlu\_dist\_options\_t struct, and then it may be provided as an argument to the SUNLinSol\_SuperLUDIST constructor. The SUNLinSol\_SuperLUDIST module will respect all options set except for Fact – this option is necessarily modified by the SUNLinSol\_SuperLUDIST module in the setup and solve routines.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNLinSol\_SuperLUDIST module is constructed to perform the following operations:

- The first time that the "setup" routine is called, it sets the SuperLU\_DIST option Fact to DOFACT so that a subsequent call to the "solve" routine will perform a symbolic factorization, followed by an initial numerical factorization before continuing to solve the system.
- On subsequent calls to the "setup" routine, it sets the SuperLU\_DIST option Fact to SamePattern so that a subsequent call to "solve" will perform factorization assuming the same sparsity pattern as prior, i.e. it will reuse the column permutation vector.
- If "setup" is called prior to the "solve" routine, then the "solve" routine will perform a symbolic factorization, followed by an initial numerical factorization before continuing to the sparse triangular solves, and, potentially, iterative refinement. If "setup" is not called prior, "solve" will skip to the triangular solve step. We note that in this solve SuperLU\_DIST operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

The SUNLinSol\_SuperLUDIST module defines implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_SuperLUDIST
- SUNLinSolInitialize\_SuperLUDIST this sets the first\_factorize flag to 1 and resets the internal SuperLU\_DIST statistics variables.
- SUNLinSolSetup\_SuperLUDIST this sets the appropriate SuperLU\_DIST options so that a subsequent solve will perform a symbolic and numerical factorization before proceeding with the triangular solves
- SUNLinSolSolve\_SuperLUDIST this calls the SuperLU\_DIST solve routine to perform factorization (if the setup routine was called prior) and then use the \$LU\$ factors to solve the linear system.
- SUNLinSolLastFlag\_SuperLUDIST
- SUNLinSolSpace\_SuperLUDIST this only returns information for the storage within the solver *interface*, i.e. storage for the integers last\_flag and first\_factorize. For additional space requirements, see the SuperLU\_DIST documentation.
- SUNLinSolFree\_SuperLUDIST

## 8.16 The SUNLinSol\_SuperLUMT Module

The SUNLinSol\_SuperLUMT implementation of the SUNLinearSolver class interfaces with the SuperLU\_MT library. This is designed to be used with the corresponding SUNMATRIX\_SPARSE matrix type, and one of the serial or shared-memory N\_Vector implementations (NVECTOR\_SERIAL, NVECTOR\_OPENMP, or NVECTOR\_PTHREADS). While these are compatible, it is not recommended to use a threaded vector module with SUNLinSol\_SuperLUMT unless it is the NVECTOR\_OPENMP module and the SuperLU\_MT library has also been compiled with OpenMP.

## 8.16.1 SUNLinSol\_SuperLUMT Usage

The header file to be included when using this module is sunlinsol/sunlinsol.SuperLUMT.h. The installed module library to link to is libsundials\_sunlinsolsuperlumt .lib where .lib is typically .so for shared libraries and .a for static libraries.

The module SUNLinSol\_SuperLUMT provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_SuperLUMT(N Vector y, SUNMatrix A, int num threads, SUNContext sunctx)

This constructor function creates and allocates memory for a SUNLinSol\_SuperLUMT object.

#### **Arguments:**

- y a template vector.
- A a template matrix
- num\_threads desired number of threads (OpenMP or Pthreads, depending on how SuperLU\_MT was
  installed) to use during the factorization steps.
- *sunctx* the *SUNContext* object (see §4.2)

#### Return value:

If successful, a SUNLinearSolver object; otherwise this routine will return NULL.

#### **Notes:**

This routine analyzes the input matrix and vector to determine the linear system size and to assess compatibility with the SuperLU\_MT library.

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUN-Matrix implementations. These are currently limited to the SUNMATRIX\_SPARSE matrix type (using either CSR or CSC storage formats) and the NVECTOR\_SERIAL, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

The num\_threads argument is not checked and is passed directly to SuperLU MT routines.

SUNErrCode SUNLinSol\_SuperLUMTSetOrdering(SUNLinearSolver S, int ordering\_choice)

This function sets the ordering used by SuperLU\_MT for reducing fill in the linear solve.

## **Arguments:**

- S the SUNLinSol SuperLUMT object to update.
- ordering\_choice:
  - 0. natural ordering
  - 1. minimal degree ordering on  $A^TA$
  - 2. minimal degree ordering on  $A^T + A$

3. COLAMD ordering for unsymmetric matrices

The default is 3 for COLAMD.

#### Return value:

• A SUNErrCode

## 8.16.2 SUNLinSol\_SuperLUMT Description

The SUNLinSol\_SuperLUMT module defines the content field of a SUNLinearSolver to be the following structure:

```
struct SUNLinearSolverContent SuperLUMT {
  int
               last_flag;
               first_factorize;
  SuperMatrix *A, *AC, *L, *U, *B;
  Gstat_t
               *Gstat:
  sunindextype *perm_r, *perm_c;
  sunindextype N;
  int
               num_threads;
  sunrealtype diag_pivot_thresh;
               ordering;
  int
  superlumt_options_t *options;
};
```

These entries of the *content* field contain the following information:

- last\_flag last error return flag from internal function evaluations,
- first\_factorize flag indicating whether the factorization has ever been performed,
- A, AC, L, U, B SuperMatrix pointers used in solve,
- Gstat GStat\_t object used in solve,
- perm\_r, perm\_c permutation arrays used in solve,
- N size of the linear system,
- num\_threads number of OpenMP/Pthreads threads to use,
- diag\_pivot\_thresh threshold on diagonal pivoting,
- ordering flag for which reordering algorithm to use,
- options pointer to SuperLU MT options structure.

The SUNLinSol\_SuperLUMT module is a SUNLinearSolver wrapper for the SuperLU\_MT sparse matrix factorization and solver library written by X. Sherry Li and collaborators [9, 22, 40]. The package performs matrix factorization using threads to enhance efficiency in shared memory parallel environments. It should be noted that threads are only used in the factorization step. In order to use the SUNLinSol\_SuperLUMT interface to SuperLU\_MT, it is assumed that SuperLU\_MT has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with SuperLU\_MT (see §11.4 for details). Additionally, this wrapper only supports single- and double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to have *sun-realtype* set to extended (see §4.1 for details). Moreover, since the SuperLU\_MT library may be installed to support either 32-bit or 64-bit integers, it is assumed that the SuperLU\_MT library is installed using the same integer precision as the SUNDIALS *sunindextype* option.

The SuperLU\_MT library has a symbolic factorization routine that computes the permutation of the linear system matrix to reduce fill-in on subsequent LU factorizations (using COLAMD, minimal degree ordering on  $A^T*A$ , minimal

degree ordering on  $A^T + A$ , or natural ordering). Of these ordering choices, the default value in the SUNLinSol\_SuperLUMT module is the COLAMD ordering.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNLinSol\_SuperLUMT module is constructed to perform the following operations:

- The first time that the "setup" routine is called, it performs the symbolic factorization, followed by an initial numerical factorization.
- On subsequent calls to the "setup" routine, it skips the symbolic factorization, and only refactors the input matrix.
- The "solve" call performs pivoting and forward and backward substitution using the stored SuperLU\_MT data structures. We note that in this solve SuperLU\_MT operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

The SUNLinSol\_SuperLUMT module defines implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_SuperLUMT
- SUNLinSolInitialize\_SuperLUMT this sets the first\_factorize flag to 1 and resets the internal SuperLU\_MT statistics variables.
- SUNLinSolSetup\_SuperLUMT this performs either a LU factorization or refactorization of the input matrix.
- SUNLinSolSolve\_SuperLUMT this calls the appropriate SuperLU\_MT solve routine to utilize the LU factors to solve the linear system.
- SUNLinSolLastFlag\_SuperLUMT
- SUNLinSolSpace\_SuperLUMT this only returns information for the storage within the solver *interface*, i.e. storage for the integers last\_flag and first\_factorize. For additional space requirements, see the SuperLU MT documentation.
- SUNLinSolFree\_SuperLUMT

# 8.17 The SUNLinSol\_cuSolverSp\_batchQR Module

The SUNLinSol\_cuSolverSp\_batchQR implementation of the SUNLinearSolver class is designed to be used with the SUNMATRIX\_CUSPARSE matrix, and the NVECTOR\_CUDA vector. The header file to include when using this module is sunlinsol/sunlinsol\_cusolversp\_batchqr.h. The installed library to link to is libsundials\_sunlinsolcusolversp.lib where .lib is typically .so for shared libraries and .a for static libraries.

Warning: The SUNLinearSolver\_cuSolverSp\_batchQR module is experimental and subject to change.

## 8.17.1 SUNLinSol\_cuSolverSp\_batchQR description

The SUNLinearSolver\_cuSolverSp\_batchQR implementation provides an interface to the batched sparse QR factorization method provided by the NVIDIA cuSOLVER library [6]. The module is designed for solving block diagonal linear systems of the form

$$\begin{bmatrix} \mathbf{A_1} & 0 & \cdots & 0 \\ 0 & \mathbf{A_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A_n} \end{bmatrix} x_j = b_j$$

where all block matrices  $A_i$  share the same sparsity pattern. The matrix must be the SUNMatrix.cuSparse.

## 8.17.2 SUNLinSol\_cuSolverSp\_batchQR functions

The SUNLinearSolver\_cuSolverSp\_batchQR module defines implementations of all "direct" linear solver operations listed in §8.1:

- SUNLinSolGetType\_cuSolverSp\_batchQR
- SUNLinSolInitialize\_cuSolverSp\_batchQR this sets the first\_factorize flag to 1
- SUNLinSolSetup\_cuSolverSp\_batchQR this always copies the relevant SUNMATRIX\_SPARSE data to the GPU; if this is the first setup it will perform symbolic analysis on the system
- SUNLinSolSolve\_cuSolverSp\_batchQR this calls the cusolverSpXcsrqrsvBatched routine to perform factorization
- SUNLinSolLastFlag\_cuSolverSp\_batchQR
- SUNLinSolFree\_cuSolverSp\_batchQR

In addition, the module provides the following user-callable routines:

SUNLinearSolver SUNLinSol\_cuSolverSp\_batchQR(N\_Vector y, SUNMatrix A, cusolverHandle\_t cusol, SUNContext sunctx)

The function SUNLinSol\_cuSolverSp\_batchQR creates and allocates memory for a SUNLinearSolver object.

#### **Arguments:**

- y a vector for checking compatibility with the solver.
- A a SUNMATRIX cuSparse matrix for checking compatibility with the solver.
- *cusol* cuSolverSp object to use.
- *sunctx* the *SUNContext* object (see §4.2)

#### **Return value:**

If successful, a SUNLinearSolver object. If either A or y are incompatible then this routine will return NULL.

#### Notes:

This routine will perform consistency checks to ensure that it is called with consistent N\_Vector and SUN-Matrix implementations. These are currently limited to the SUNMATRIX\_CUSPARSE matrix type and the NVECTOR\_CUDA vector type. Since the SUNMATRIX\_CUSPARSE matrix type is only compatible with the NVECTOR\_CUDA the restriction is also in place for the linear solver. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

## void SUNLinSol\_cuSolverSp\_batchQR\_GetDescription(SUNLinearSolver LS, char \*\*desc)

The function SUNLinSol\_cuSolverSp\_batchQR\_GetDescription accesses the string description of the object (empty by default).

#### void SUNLinSol\_cuSolverSp\_batchQR\_SetDescription(SUNLinearSolver LS, const char \*desc)

The function SUNLinSol\_cuSolverSp\_batchQR\_SetDescription sets the string description of the object (empty by default).

void **SUNLinSol\_cuSolverSp\_batchQR\_GetDeviceSpace**(*SUNLinearSolver* S, size\_t \*cuSolverInternal, size\_t \*cuSolverWorkspace)

The function SUNLinSol\_cuSolverSp\_batchQR\_GetDeviceSpace returns the cuSOLVER batch QR method internal buffer size, in bytes, in the argument cuSolverInternal and the cuSOLVER batch QR workspace buffer size, in bytes, in the agrument cuSolverWorkspace. The size of the internal buffer is proportional to the number of matrix blocks while the size of the workspace is almost independent of the number of blocks.

## 8.17.3 SUNLinSol\_cuSolverSp\_batchQR content

The SUNLinSol\_cuSolverSp\_batchQR module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_cuSolverSp_batchQR {
                      last_flag;
                                       /* last return flag
   int
                      first_factorize; /* is this the first factorization?
   sunbooleantype
                      internal_size; /* size of cusolver buffer for Q and R
                                                                                     */
   size_t
   size_t
                      workspace_size; /* size of cusolver memory for factorization
   cusolverSpHandle_t cusolver_handle; /* cuSolverSp context
                                       /* opaque cusolver data structure
   csrqrInfo_t
                      info;
   void*
                      workspace;
                                       /* memory block used by cusolver
                                       /* description of this linear solver
   const char*
                      desc;
};
```

## 8.18 The SUNLINEARSOLVER GINKGO Module

New in version 6.4.0.

The SUNLINEARSOLVER\_GINKGO implementation of the SUNLinearSolver API provides an interface to the linear solvers from the Ginkgo linear algebra library [12]. Since Ginkgo is a modern C++ library, SUNLINEAR-SOLVER\_GINKGO is also written in modern C++ (specifically, C++14). Unlike most other SUNDIALS modules, it is a header only library. To use the SUNLINEARSOLVER\_GINKGO SUNLinearSolver, users will need to include sunlinsol\_ginkgo.hpp. The module is meant to be used with the SUNMATRIX\_GINKGO module described in §7.10. Instructions on building SUNDIALS with Ginkgo enabled are given in §11.4. For instructions on building and using Ginkgo itself, refer to the Ginkgo website and documentation.

**Note:** It is assumed that users of this module are aware of how to use Ginkgo. This module does not try to encapsulate Ginkgo linear solvers, rather it provides a lightweight iteroperability layer between Ginkgo and SUNDIALS. Most, if not all, of the Ginkgo linear solver should work with this interface.

## 8.18.1 Using SUNLINEARSOLVER\_GINKGO

After choosing a compatible N\_Vector (see §7.10.1) and creating a Ginkgo-enabled SUNMatrix (see §7.10) to use the SUNLINEARSOLVER\_GINKGO module, we first create a Ginkgo stopping criteria object. Importantly, the sundials::ginkgo::DefaultStop class provided by SUNDIALS implements a stopping criticion that matches the default SUNDIALS stopping criticion. Namely, it checks if the max iterations (5 by default) were reached or if the absolute residual norm was below a speicified tolerance. The criticion can be created just like any other Ginkgo stopping criteria:

```
auto crit{sundials::ginkgo::DefaultStop::build().with_max_iters(max_iters).on(gko_exec)};
```

**Warning:** It is *highly* recommended to employ this criterion when using Ginkgo solvers with SUNDIALS, but it is optional. However, to use the Ginkgo multigrid or cbgmres linear solvers, different Ginkgo criterion must be used.

Once we have created our stopping critierion, we create a Ginkgo solver factory object and wrap it in a *sundi-als::ginkgo::LinearSolver* object. In this example, we create a Ginkgo conjugate gradient solver:

```
using GkoMatrixType = gko::matrix::Csr<sunrealtype, sunindextype>;
using GkoSolverType = gko::solver::Cg<sunrealtype>;
auto gko_solver_factory = gko::share(
   GkoSolverType::build().with_criteria(std::move(crit)).on(gko_exec));
sundials::ginkgo::LinearSolver<GkoSolverType, GkoMatrixType> LS{
   gko_solver_factory, sunctx};
```

Finally, we can pass the instance of *sundials::ginkgo::LinearSolver* to any function expecting a SUNLinear-Solver object through the implicit conversion operator or explicit conversion function.

```
// Attach linear solver and matrix to CVODE.
//
// Implicit conversion from sundials::ginkgo::LinearSolver<GkoSolverType, GkoMatrixType>
// to a SUNLinearSolver object is done.
//
// For details about creating A see the SUNMATRIX_GINKGO module.
CVodeSetLinearSolver(cvode_mem, LS, A);
// Alternatively with explicit conversion of LS to a SUNLinearSolver
// and A to a SUNMatrix:
CVodeSetLinearSolver(cvode_mem, LS->Convert(), A->Convert());
```

**Warning:** SUNLinSolFree() should never be called on a SUNLinearSolver that was created via conversion from a sundials::ginkgo::LinearSolver. Doing so may result in a double free.

#### 8.18.2 SUNLINEARSOLVER GINKGO API

In this section we list the public API of the <code>sundials::ginkgo::LinearSolver</code> class.

template<class GkoSolverType, class GkoMatrixType>

class sundials::ginkgo::LinearSolver: public sundials::ConvertibleTo<SUNLinearSolver>

LinearSolver() = default;

Default constructor - means the solver must be moved to.

**LinearSolver**(std::shared\_ptr<typename *GkoSolverType*::Factory> gko\_solver\_factory, SUNContext sunctx)

Constructs a new LinearSolver from a Ginkgo solver factory.

#### **Parameters**

- **gko\_solver\_factory** The Ginkgo solver factory (typically *gko::matrix::<type>::Factory*`)
- **sunctx** The SUNDIALS simulation context (*SUNContext*)

**LinearSolver** (*LinearSolver* &&that\_solver) noexcept

Move constructor.

*LinearSolver* & operator=(*LinearSolver* & & rhs)

Move assignment.

#### ~LinearSolver() override = default

Default destructor.

## operator SUNLinearSolver() override

Implicit conversion to a SUNLinearSolver.

## operator SUNLinearSolver() const override

Implicit conversion to a SUNLinearSolver.

#### SUNLinearSolver Convert() override

Explicit conversion to a SUNLinearSolver.

#### SUNLinearSolver Convert() const override

Explicit conversion to a SUNLinearSolver.

#### std::shared\_ptr<const gko::Executor> **GkoExec()** const

Get the gko:: Executor associated with the Ginkgo solver.

## std::shared\_ptr<typename *GkoSolverType*::Factory> **GkoFactory**()

Get the underlying Ginkgo solver factory.

## GkoSolverType \*GkoSolver()

Get the underlying Ginkgo solver.

**Note:** This will be *nullptr* until the linear solver setup phase.

#### int NumIters() const

Get the number of linear solver iterations in the most recent solve.

## sunrealtype ResNorm() const

Get the residual norm of the solution at the end of the last solve.

The type of residual norm depends on the Ginkgo stopping criteria used with the solver. With the DefaultStop criteria this would be the absolute residual 2-norm.

#### GkoSolverType \*Setup(Matrix<GkoMatrixType> \*A)

Setup the linear system.

#### **Parameters**

**A** – the linear system matrix

#### Returns

Pointer to the Ginkgo solver generated from the factory

## gko::LinOp \*Solve(N\_Vector b, N\_Vector x, sunrealtype tol)

Solve the linear system Ax = b to the specificed tolerance.

#### **Parameters**

- **b** the right-hand side vector
- $\mathbf{x}$  the solution vector
- tol the tolerance to solve the system to

### Returns

gko::LinOp\* the solution

## 8.19 The SUNLINEARSOLVER\_KOKKOSDENSE Module

New in version 6.4.0.

The SUNLINEARSOLVER\_KOKKOSDENSE SUNLinearSolver implementation provides an interface to KokkosKernels [49] linear solvers for dense and batched dense (block-diagonal) systems. Since Kokkos is a modern C++ library, the module is also written in modern C++ (it requires C++14) as a header only library. To utilize this SUNLinearSolver user will need to include sunlinsol/sunlinsol\_kokkosdense.hpp. More instructions on building SUNDIALS with Kokkos and KokkosKernels enabled are given in §11.4. For instructions on building and using Kokkos and KokkosKernels, refer to the Kokkos and KokkosKernels. documentation.

## 8.19.1 Using SUNLINEARSOLVER\_KOKKOSDENSE

The SUNLINEARSOLVER\_KOKKOSDENSE module is defined by the DenseLinearSolver templated class in the sundials::kokkos namespace:

To use the SUNLINEARSOLVER\_KOKKOSDENSE module, we begin by constructing an instance of a dense linear solver e.g.,

```
// Create a dense linear solver
sundials::kokkos::DenseLinearSolver<> LS{sunctx};
```

Instances of the DenseLinearSolver class are implicitly or explicitly (using the *Convert()* method) convertible to a *SUNLinearSolver* e.g.,

**Warning:** SUNLinSolFree() should never be called on a SUNLinearSolver that was created via conversion from a sundials::kokkos::DenseLinearSolver. Doing so may result in a double free.

The SUNLINEARSOLVER\_KOKKOSDENSE module is compatible with the NVECTOR\_KOKKOS vector module (see §6.14) and SUNMATRIX\_KOKKOSDENSE matrix module (see §7.11).

#### 8.19.2 SUNLINEARSOLVER KOKKOSDENSE API

```
In this section we list the public API of the sundials::kokkos::DenseLinearSolver class.
```

```
template < class ExecSpace = Kokkos::DefaultExecutionSpace, class MemSpace = typename 

ExecSpace::memory_space > class DenseLinearSolver : public sundials::impl::BaseLinearSolver, public sundials::ConvertibleTo < SUNLinearSolver >
```

```
DenseLinearSolver() = default;
```

Default constructor - means the solver must be moved to.

#### DenseLinearSolver(SUNContext sunctx)

Constructs a new DenseLinearSolver.

#### **Parameters**

**sunctx** – The SUNDIALS simulation context (SUNContext)

**DenseLinearSolver**(*DenseLinearSolver* &&that\_solver) noexcept

Move constructor.

DenseLinearSolver & operator=(DenseLinearSolver & & rhs)

Move assignment.

~DenseLinearSolver() override = default

Default destructor.

operator SUNLinearSolver() override

Implicit conversion to a SUNLinearSolver.

operator SUNLinearSolver() const override

Implicit conversion to a SUNLinearSolver.

SUNLinearSolver Convert() override

Explicit conversion to a SUNLinearSolver.

SUNLinearSolver Convert() const override

Explicit conversion to a SUNLinearSolver.

## **8.20 SUNLinearSolver Examples**

There are SUNLinearSolver examples that may be installed for each implementation; these make use of the functions in test\_sunlinsol.c. These example functions show simple usage of the SUNLinearSolver family of modules. The inputs to the examples depend on the linear solver type, and are output to stdout if the example is run without the appropriate number of command-line arguments.

The following is a list of the example functions in test\_sunlinsol.c:

- Test\_SUNLinSolGetType: Verifies the returned solver type against the value that should be returned.
- Test\_SUNLinSolGetID: Verifies the returned solver identifier against the value that should be returned.
- Test\_SUNLinSolInitialize: Verifies that SUNLinSolInitialize can be called and returns successfully.
- Test\_SUNLinSolSetup: Verifies that SUNLinSolSetup can be called and returns successfully.
- Test\_SUNLinSolSolve: Given a SUNMatrix object A, N\_Vector objects x and b (where Ax = b) and a desired solution tolerance tol, this routine clones x into a new vector y, calls SUNLinSolSolve to fill y as the solution to Ay = b (to the input tolerance), verifies that each entry in x and y match to within 10\*tol, and overwrites x with y prior to returning (in case the calling routine would like to investigate further).
- Test\_SUNLinSolSetATimes (iterative solvers only): Verifies that SUNLinSolSetATimes can be called and returns successfully.
- Test\_SUNLinSolSetPreconditioner (iterative solvers only): Verifies that SUNLinSolSetPreconditioner can be called and returns successfully.
- Test\_SUNLinSolSetScalingVectors (iterative solvers only): Verifies that SUNLinSolSetScalingVectors can be called and returns successfully.
- Test\_SUNLinSolSetZeroGuess (iterative solvers only): Verifies that SUNLinSolSetZeroGuess can be called and returns successfully.

- Test\_SUNLinSolLastFlag: Verifies that SUNLinSolLastFlag can be called, and outputs the result to stdout.
- Test\_SUNLinSolNumIters (iterative solvers only): Verifies that SUNLinSolNumIters can be called, and outputs the result to stdout.
- Test\_SUNLinSolResNorm (iterative solvers only): Verifies that SUNLinSolResNorm can be called, and that the result is non-negative.
- Test\_SUNLinSolResid (iterative solvers only): Verifies that SUNLinSolResid can be called.
- Test\_SUNLinSolSpace verifies that SUNLinSolSpace can be called, and outputs the results to stdout.

We'll note that these tests should be performed in a particular order. For either direct or iterative linear solvers, Test\_SUNLinSolInitialize must be called before Test\_SUNLinSolSetup, which must be called before Test\_SUNLinSolSolve. Additionally, for iterative linear solvers Test\_SUNLinSolSetATimes, Test\_SUNLinSolSetPreconditioner and Test\_SUNLinSolSetScalingVectors should be called before Test\_SUNLinSolInitialize; similarly Test\_SUNLinSolNumIters, Test\_SUNLinSolResNorm and Test\_SUNLinSolResid should be called after Test\_SUNLinSolSolve. These are called in the appropriate order in all of the example problems.

# **Chapter 9**

# **Tools for Memory Management**

To support applications which leverage memory pools, or utilize a memory abstraction layer, sundials provides a set of utilities we will collectively refer to as the SUNMemoryHelper API. The goal of this API is to allow users to leverage operations defined by native sundials data structures while allowing the user to have finer-grained control of the memory management.

## 9.1 The SUNMemoryHelper API

• an uninitialized SUNMemory object

```
This API consists of three new SUNDIALS types: SUNMemoryType, SUNMemory, and SUNMemoryHelper:
typedef struct SUNMemory_ *SUNMemory
     The SUNMemory type is a pointer the structure
     struct SUNMemory_
          void *ptr;
              The actual data.
          SUNMemoryType type;
              The data memory type.
          sunbooleantype own;
              A flag indicating ownership.
          size_t bytes;
              The size of the data allocated.
SUNMemory SUNMemoryNewEmpty(SUNContext sunctx)
     This function returns an empty SUNMemory object.
     Arguments:
        • sunctx - the SUNContext object.
     Returns:
```

Changed in version 7.0.0: The function signature was updated to add the SUNContext argument.

```
enum SUNMemoryType
```

The SUNMemoryType type is an enumeration that defines the supported memory types:

```
enumerator SUNMENTYPE_HOST
```

Pageable memory accessible on the host

#### enumerator SUNMEMTYPE\_PINNED

Page-locked memory accesible on the host

#### enumerator SUNMEMTYPE DEVICE

Memory accessible from the device

#### enumerator SUNMEMTYPE\_UVM

Memory accessible from the host or device

#### typedef struct SUNMemoryHelper\_ \*SUNMemoryHelper

The SUNMemoryHelper type is a pointer to the structure

struct SUNMemoryHelper\_

#### void \*content;

Pointer to the implementation-specific member data

SUNMemoryHelper\_Ops ops;

A virtual method table of member functions

SUNContext sunctx;

The SUNDIALS simulation context

## typedef struct SUNMemoryHelper\_Ops\_ \*SUNMemoryHelper\_Ops

The SUNMemoryHelper\_Ops type is defined as a pointer to the structure containing the function pointers to the member function implementations. This structure is define as

#### struct SUNMemoryHelper\_Ops\_

```
SUNErrCode (*alloc)(SUNMemoryHelper, SUNMemory *memptr, size_t mem_size, SUNMemoryType mem_type, void *queue)
```

The function implementing SUNMemoryHelper\_Alloc()

SUNErrCode (\*dealloc)(SUNMemoryHelper, SUNMemory mem, void \*queue)

The function implementing SUNMemoryHelper\_Dealloc()

SUNErrCode (\*copy)(SUNMemoryHelper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

The function implementing SUNMemoryHelper\_Copy()

SUNErrCode (\*copyasync)(SUNMemoryHelper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

The function implementing SUNMemoryHelper\_CopyAsync()

SUNErrCode (\*getallocstats)(SUNMemoryHelper, SUNMemoryType mem\_type, unsigned long \*num\_allocations, unsigned long \*num\_deallocations, size\_t \*bytes\_allocated, size\_t \*bytes\_high\_watermark)

The function implementing SUNMemoryHelper\_GetAllocStats()

SUNMemoryHelper (\*clone)(SUNMemoryHelper)

The function implementing SUNMemoryHelper\_Clone()

#### SUNErrCode (\*destroy)(SUNMemoryHelper)

The function implementing SUNMemoryHelper\_Destroy()

## **9.1.1** Implementation defined operations

The SUNMemory API defines the following operations that an implementation to must define:

```
SUNMemory SUNMemoryHelper_Alloc(SUNMemoryHelper helper, SUNMemory *memptr, size_t mem_size, SUNMemoryType mem_type, void *queue)
```

Allocates a SUNMemory object whose ptr field is allocated for mem\_size bytes and is of type mem\_type. The new object will have ownership of ptr and will be deallocated when SUNMemoryHelper\_Dealloc() is called.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- memptr pointer to the allocated SUNMemory.
- mem\_size the size in bytes of the ptr.
- mem\_type the SUNMemoryType of the ptr.
- queue typically a handle for an object representing an alternate execution stream (e.g., a CUDA/HIP stream or SYCL queue), but it can also be any implementation specific data.

#### **Returns:**

• A new SUNMemory object.

SUNErrCode SUNMemoryHelper\_Dealloc(SUNMemoryHelper helper, SUNMemory mem, void \*queue)

Deallocates the mem->ptr field if it is owned by mem, and then deallocates the mem object.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem the SUNMemory object.
- queue typically a handle for an object representing an alternate execution stream (e.g., a CUDA/HIP stream or SYCL queue), but it can also be any implementation specific data.

#### **Returns:**

• A SUNErrCode indicating success or failure.

SUNErrCode SUNMemoryHelper\_Copy(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

Synchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object should use the memory types of dst and src to determine the appropriate transfer type necessary.

## **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.
- queue typically a handle for an object representing an alternate execution stream (e.g., a CUDA/HIP stream or SYCL queue), but it can also be any implementation specific data.

#### **Returns:**

• A SUNErrCode indicating success or failure.

## **9.1.2 Utility Functions**

The SUNMemoryHelper API defines the following functions which do not require a SUNMemoryHelper instance:

SUNMemory SUNMemoryHelper\_Alias(SUNMemoryHelper helper, SUNMemory mem1)

Returns a SUNMemory object whose ptr field points to the same address as mem1. The new object will not have ownership of ptr, therefore, it will not free ptr when SUNMemoryHelper\_Dealloc() is called.

#### **Arguments:**

- helper a SUNMemoryHelper object.
- mem1 a SUNMemory object.

#### **Returns:**

• A SUNMemory object or NULL if an error occurs.

Changed in version 7.0.0: The SUNMemoryHelper argument was added to the function signature.

## SUNMemory SUNMemoryHelper\_Wrap (SUNMemoryHelper helper, void \*ptr, SUNMemoryType mem\_type)

Returns a SUNMemory object whose ptr field points to the ptr argument passed to the function. The new object will not have ownership of ptr, therefore, it will not free ptr when SUNMemoryHelper\_Dealloc() is called.

#### **Arguments:**

- helper a SUNMemoryHelper object.
- ptr the data pointer to wrap in a SUNMemory object.
- mem\_type the SUNMemoryType of the ptr.

## **Returns:**

• A SUNMemory object or NULL if an error occurs.

Changed in version 7.0.0: The SUNMemoryHelper argument was added to the function signature.

## SUNMemoryHelper SUNMemoryHelper\_NewEmpty(SUNContext sunctx)

Returns an empty SUNMemoryHelper. This is useful for building custom SUNMemoryHelper implementations.

#### **Arguments:**

• helper – a SUNMemoryHelper object.

#### **Returns:**

• A SUNMemoryHelper object or NULL if an error occurs.

Changed in version 7.0.0: The SUNMemoryHelper argument was added to the function signature.

## SUNErrCode SUNMemoryHelper\_CopyOps(SUNMemoryHelper src, SUNMemoryHelper dst)

Copies the ops field of src to the ops field of dst. This is useful for building custom SUNMemoryHelper implementations.

#### **Arguments:**

- src the object to copy from.
- dst the object to copy to.

## **Returns:**

• A SUNErrCode indicating success or failure.

```
SUNErrCode SUNMemoryHelper_GetAllocStats(SUNMemoryHelper helper, SUNMemoryType mem_type, unsigned long *num_allocations, unsigned long *num_deallocations, size_t *bytes_allocated, size_t *bytes_high_watermark)
```

Returns statistics about the allocations performed with the helper.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem\_type the SUNMemoryType to get stats for.
- num\_allocations (output argument) number of allocations done through the helper.
- num\_deallocations (output argument) number of deallocations done through the helper.
- bytes\_allocated (output argument) total number of bytes allocated through the helper at the moment this function is called.
- bytes\_high\_watermark (output argument) max number of bytes allocated through the helper at any moment in the lifetime of the helper.

#### **Returns:**

• A SUNErrCode indicating success or failure.

## 9.1.3 Implementation overridable operations with defaults

In addition, the SUNMemoryHelper API defines the following *optionally overridable* operations which an implementation may define:

```
SUNErrCode SUNMemoryHelper_CopyAsync(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size_t mem_size, void *queue)
```

Asynchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object should use the memory types of dst and src to determine the appropriate transfer type necessary. The ctx argument is used when a different execution stream needs to be provided to perform the copy in, e.g. with CUDA this would be a cudaStream\_t.

## **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.
- queue typically a handle for an object representing an alternate execution stream (e.g., a CUDA/HIP stream or SYCL queue), but it can also be any implementation specific data.

#### **Returns:**

An int flag indicating success (zero) or failure (non-zero).

**Note:** If this operation is not defined by the implementation, then *SUNMemoryHelper\_Copy()* will be used.

#### SUNMemoryHelper SUNMemoryHelper\_Clone(SUNMemoryHelper helper)

Clones the SUNMemoryHelper object itself.

#### **Arguments:**

• helper – the SUNMemoryHelper object to clone.

#### **Returns:**

• A SUNMemoryHelper object.

**Note:** If this operation is not defined by the implementation, then the default clone will only copy the SUNMemoryHelper\_Ops structure stored in helper->ops, and not the helper->content field.

## SUNErrCode SUNMemoryHelper\_Destroy(SUNMemoryHelper helper)

Destroys (frees) the SUNMemoryHelper object itself.

#### **Arguments:**

• helper – the SUNMemoryHelper object to destroy.

#### **Returns:**

• A SUNErrCode indicating success or failure.

**Note:** If this operation is not defined by the implementation, then the default destroy will only free the helper->ops field and the helper itself. The helper->content field will not be freed.

## 9.1.4 Implementing a custom SUNMemoryHelper

A particular implementation of the SUNMemoryHelper API must:

- Define and implement the required operations. Note that the names of these routines should be unique to that implementation in order to permit using more than one SUNMemoryHelper module in the same code.
- Optionally, specify the *content* field of SUNMemoryHelper.
- Optionally, define and implement additional user-callable routines acting on the newly defined SUNMemory-Helper.

An example of a custom SUNMemoryHelper is given in examples/utilities/custom\_memory\_helper.h.

# 9.2 The SUNMemoryHelper\_Cuda Implementation

The SUNMemoryHelper\_Cuda module is an implementation of the SUNMemoryHelper API that interfaces to the NVIDIA [5] library. The implementation defines the constructor

## SUNMemoryHelper SUNMemoryHelper\_Cuda(SUNContext sunctx)

Allocates and returns a SUNMemoryHelper object for handling CUDA memory if successful. Otherwise it returns NULL.

## 9.2.1 SUNMemoryHelper\_Cuda API Functions

The implementation provides the following operations defined by the SUNMemoryHelper API:

SUNMemory SUNMemoryHelper\_Alloc\_Cuda(SUNMemoryHelper helper, SUNMemory memptr, size\_t mem\_size, SUNMemoryType mem\_type, void \*queue)

Allocates a SUNMemory object whose ptr field is allocated for mem\_size bytes and is of type mem\_type. The new object will have ownership of ptr and will be deallocated when SUNMemoryHelper\_Dealloc() is called.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- memptr pointer to the allocated SUNMemory.
- mem\_size the size in bytes of the ptr.
- mem\_type the SUNMemoryType of the ptr. Supported values are:
  - SUNMEMTYPE\_HOST memory is allocated with a call to malloc.
  - SUNMEMTYPE\_PINNED memory is allocated with a call to cudaMallocHost.
  - SUNMEMTYPE\_DEVICE memory is allocated with a call to cudaMalloc.
  - SUNMEMTYPE\_UVM memory is allocated with a call to cudaMallocManaged.
- queue currently unused.

#### **Returns:**

• A new SUNMemory object.

SUNErrCode SUNMemoryHelper\_Dealloc\_Cuda(SUNMemoryHelper helper, SUNMemory mem, void \*queue)

Deallocates the mem->ptr field if it is owned by mem, and then deallocates the mem object.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem the SUNMemory object.
- queue currently unused.

#### **Returns:**

• A SUNErrCode indicating success or failure.

SUNErrCode SUNMemoryHelper\_Copy\_Cuda(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

Synchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object will use the memory types of dst and src to determine the appropriate transfer type necessary.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.
- queue currently unused.

#### **Returns:**

• A SUNErrCode indicating success or failure.

SUNErrCode SUNMemoryHelper\_CopyAsync\_Cuda(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

Asynchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object will use the memory types of dst and src to determine the appropriate transfer type necessary.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.
- queue the cudaStream\_t handle for the stream that the copy will be performed on.

#### **Returns:**

• A SUNErrCode indicating success or failure.

SUNErrCode SUNMemoryHelper\_GetAllocStats\_Cuda(SUNMemoryHelper helper, SUNMemoryType

mem\_type, unsigned long \*num\_allocations, unsigned long \*num\_deallocations, size\_t \*bytes\_allocated, size\_t \*bytes\_high\_watermark)

Returns statistics about memory allocations performed with the helper.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem\_type the SUNMemoryType to get stats for.
- num\_allocations (output argument) number of memory allocations done through the helper.
- num\_deallocations (output argument) number of memory deallocations done through the helper.
- bytes\_allocated (output argument) total number of bytes allocated through the helper at the moment this function is called.
- bytes\_high\_watermark (output argument) max number of bytes allocated through the helper at any moment in the lifetime of the helper.

#### **Returns:**

• A SUNErrCode indicating success or failure.

# 9.3 The SUNMemoryHelper\_Hip Implementation

The SUNMemoryHelper\_Hip module is an implementation of the SUNMemoryHelper API that interfaces to the AMD ROCm HIP library [2]. The implementation defines the constructor

SUNMemoryHelper SUNMemoryHelper\_Hip(SUNContext sunctx)

Allocates and returns a SUNMemoryHelper object for handling HIP memory if successful. Otherwise it returns NULL.

## 9.3.1 SUNMemoryHelper\_Hip API Functions

The implementation provides the following operations defined by the SUNMemoryHelper API:

SUNMemory SUNMemoryHelper\_Alloc\_Hip(SUNMemoryHelper helper, SUNMemory memptr, size\_t mem\_size, SUNMemoryType mem\_type, void \*queue)

Allocates a SUNMemory object whose ptr field is allocated for mem\_size bytes and is of type mem\_type. The new object will have ownership of ptr and will be deallocated when SUNMemoryHelper\_Dealloc() is called.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- memptr pointer to the allocated SUNMemory.
- mem\_size the size in bytes of the ptr.
- mem\_type the SUNMemoryType of the ptr. Supported values are:
  - SUNMEMTYPE\_HOST memory is allocated with a call to malloc.
  - SUNMEMTYPE\_PINNED memory is allocated with a call to hipMallocHost.
  - SUNMEMTYPE\_DEVICE memory is allocated with a call to hipMalloc.
  - SUNMEMTYPE\_UVM memory is allocated with a call to hipMallocManaged.
- queue currently unused.

#### **Returns:**

• An int flag indicating success (zero) or failure (non-zero).

SUNErrCode SUNMemoryHelper\_Dealloc\_Hip(SUNMemoryHelper helper, SUNMemory mem, void \*queue)

Deallocates the mem->ptr field if it is owned by mem, and then deallocates the mem object.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem the SUNMemory object.

#### **Returns:**

• An int flag indicating success (zero) or failure (non-zero).

SUNErrCode SUNMemoryHelper\_Copy\_Hip(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

Synchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object will use the memory types of dst and src to determine the appropriate transfer type necessary.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.

#### **Returns:**

• An int flag indicating success (zero) or failure (non-zero).

SUNErrCode SUNMemoryHelper\_CopyAsync\_Hip(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size t mem size, void \*queue)

Asynchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object will use the memory types of dst and src to determine the appropriate transfer type necessary.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.
- queue the hipStream\_t handle for the stream that the copy will be performed on.

#### **Returns:**

• An int flag indicating success (zero) or failure (non-zero).

```
SUNErrCode SUNMemoryHelper_GetAllocStats_Hip(SUNMemoryHelper helper, SUNMemoryType mem_type, unsigned long *num_allocations, unsigned long *num_deallocations, size_t *bytes_allocated, size_t *bytes_high_watermark)
```

Returns statistics about memory allocations performed with the helper.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem\_type the SUNMemoryType to get stats for.
- num\_allocations (output argument) number of memory allocations done through the helper.
- num\_deallocations (output argument) number of memory deallocations done through the helper.
- bytes\_allocated (output argument) total number of bytes allocated through the helper at the moment this function is called.
- bytes\_high\_watermark (output argument) max number of bytes allocated through the helper at any moment in the lifetime of the helper.

#### **Returns:**

• An int flag indicating success (zero) or failure (non-zero).

# 9.4 The SUNMemoryHelper\_Sycl Implementation

The SUNMemoryHelper\_Sycl module is an implementation of the SUNMemoryHelper API that interfaces to the SYCL abstraction layer. The implementation defines the constructor

```
SUNMemoryHelper SUNMemoryHelper_Sycl(SUNContext sunctx)
```

Allocates and returns a SUNMemoryHelper object for handling SYCL memory using the provided queue. Otherwise it returns NULL.

## 9.4.1 SUNMemoryHelper\_Sycl API Functions

The implementation provides the following operations defined by the SUNMemoryHelper API:

SUNMemory SUNMemoryHelper\_Alloc\_Sycl(SUNMemoryHelper helper, SUNMemory memptr, size\_t mem\_size, SUNMemoryType mem\_type, void \*queue)

Allocates a SUNMemory object whose ptr field is allocated for mem\_size bytes and is of type mem\_type. The new object will have ownership of ptr and will be deallocated when SUNMemoryHelper\_Dealloc() is called.

### **Arguments:**

- helper the SUNMemoryHelper object.
- memptr pointer to the allocated SUNMemory.
- mem\_size the size in bytes of the ptr.
- mem\_type the SUNMemoryType of the ptr. Supported values are:
  - SUNMEMTYPE\_HOST memory is allocated with a call to malloc.
  - SUNMEMTYPE\_PINNED memory is allocated with a call to sycl::malloc\_host.
  - SUNMEMTYPE\_DEVICE memory is allocated with a call to sycl::malloc\_device.
  - SUNMEMTYPE\_UVM memory is allocated with a call to sycl::malloc\_shared.
- queue the sycl::queue handle for the stream that the allocation will be performed on.

#### **Returns:**

• A new SUNMemory object.

SUNErrCode SUNMemoryHelper\_Dealloc\_Sycl(SUNMemoryHelper helper, SUNMemory mem, void \*queue)

Deallocates the mem->ptr field if it is owned by mem, and then deallocates the mem object.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem the SUNMemory object.
- queue the sycl::queue handle for the queue that the deallocation will be performed on.

#### **Returns:**

• A SUNErrCode indicating success or failure.

SUNErrCode SUNMemoryHelper\_Copy\_Sycl(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

Synchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object will use the memory types of dst and src to determine the appropriate transfer type necessary.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.
- queue the sycl::queue handle for the queue that the copy will be performed on.

#### **Returns:**

• A SUNErrCode indicating success or failure.

SUNErrCode SUNMemoryHelper\_CopyAsync\_Sycl(SUNMemoryHelper helper, SUNMemory dst, SUNMemory src, size\_t mem\_size, void \*queue)

Asynchronously copies mem\_size bytes from the the source memory to the destination memory. The copy can be across memory spaces, e.g. host to device, or within a memory space, e.g. host to host. The helper object will use the memory types of dst and src to determine the appropriate transfer type necessary.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- dst the destination memory to copy to.
- src the source memory to copy from.
- mem\_size the number of bytes to copy.
- queue the sycl::queue handle for the queue that the copy will be performed on.

#### **Returns:**

• A SUNErrCode indicating success or failure.

SUNErrCode SUNMemoryHelper\_GetAllocStats\_Sycl(SUNMemoryHelper helper, SUNMemoryType

mem\_type, unsigned long \*num\_allocations, unsigned long \*num\_deallocations, size\_t \*bytes\_allocated, size\_t \*bytes\_high\_watermark)

Returns statistics about memory allocations performed with the helper.

#### **Arguments:**

- helper the SUNMemoryHelper object.
- mem\_type the SUNMemoryType to get stats for.
- num\_allocations (output argument) number of memory allocations done through the helper.
- num\_deallocations (output argument) number of memory deallocations done through the helper.
- bytes\_allocated (output argument) total number of bytes allocated through the helper at the moment this function is called.
- bytes\_high\_watermark (output argument) max number of bytes allocated through the helper at any moment in the lifetime of the helper.

#### **Returns:**

• A SUNErrCode indicating success or failure.

# Chapter 10

# **Acquiring SUNDIALS**

There are two supported ways for building and installing SUNDIALS from source. One option is to use the Spack HPC package manager:

#### spack install sundials

The second supported option for building and installing SUNDIALS is with CMake. Before proceeding with CMake, the source code must be downloaded. This can be done by cloning the SUNDIALS GitHub repository (run git clone https://github.com/LLNL/sundials), or by downloading the SUNDIALS release compressed archives (.tar. gz) from the SUNDIALS website.

The compressed archives allow for downloading of indvidual SUNDIALS packages. The name of the distribution archive is of the form SOLVER-7.1.0.tar.gz, where SOLVER is one of: sundials, cvode, cvodes, arkode, ida, idas, or kinsol, and 7.1.0 represents the version number (of the SUNDIALS suite or of the individual solver). After downloading the relevant archives, uncompress and expand the sources, by running

#### % tar -zxf SOLVER-7.1.0.tar.gz

This will extract source files under a directory SOLVER-7.1.0.

Starting with version 2.6.0 of SUNDIALS, CMake is the only supported method of installation. The explanations of the installation procedure begin with a few common observations:

- 1. The remainder of this chapter will follow these conventions:
  - SOLVERDIR is the directory SOLVER-7.1.0 created above; i.e. the directory containing the SUNDIALS sources.
  - BUILDDIR is the (temporary) directory under which SUNDIALS is built.
  - INSTDIR is the directory under which the SUNDIALS exported header files and libraries will be installed. Typically, header files are exported under a directory INSTDIR/include while libraries are installed under INSTDIR/lib, with INSTDIR specified at configuration time.
- 2. For SUNDIALS' CMake-based installation, in-source builds are prohibited; in other words, the build directory BUILDDIR can **not** be the same as SOLVERDIR and such an attempt will lead to an error. This prevents "polluting" the source tree and allows efficient builds for different configurations and/or options.
- 3. The installation directory INSTDIR can not be the same as the source directory SOLVERDIR.
- 4. By default, only the libraries and header files are exported to the installation directory INSTDIR. If enabled by the user (with the appropriate toggle for CMake), the examples distributed with SUNDIALS will be built together with the solver libraries but the installation step will result in exporting (by default in a subdirectory of the installation directory) the example sources and sample outputs together with automatically generated configuration files that reference the *installed* SUNDIALS headers and libraries. As such, these configuration files for

the SUNDIALS examples can be used as "templates" for your own problems. CMake installs CMakeLists.txt files and also (as an option available only under Unix/Linux) Makefile files. Note this installation approach also allows the option of building the SUNDIALS examples without having to install them. (This can be used as a sanity check for the freshly built libraries.)

Further details on the CMake-based installation procedures, instructions for manual compilation, and a roadmap of the resulting installed libraries and exported header files, are provided in §11 and §11.8.

# Chapter 11

# **Building and Installing with CMake**

CMake-based installation provides a platform-independent build system. CMake can generate Unix and Linux Make-files, as well as KDevelop, Visual Studio, and (Apple) XCode project files from the same configuration file. In addition, CMake also provides a GUI front end and which allows an interactive build and installation process.

The SUNDIALS build process requires CMake version 3.18.0 or higher and a working C compiler. On Unix-like operating systems, it also requires Make (and curses, including its development libraries, for the GUI front end to CMake, ccmake or cmake-gui), while on Windows it requires Visual Studio. While many Linux distributions offer CMake, the version included may be out of date. CMake adds new features regularly, and you should download the latest version from http://www.cmake.org. Build instructions for CMake (only necessary for Unix-like systems) can be found on the CMake website. Once CMake is installed, Linux/Unix users will be able to use ccmake or cmake-gui (depending on the version of CMake), while Windows users will be able to use CMakeSetup.

As previously noted, when using CMake to configure, build and install SUNDIALS, it is always required to use a separate build directory. While in-source builds are possible, they are explicitly prohibited by the SUNDIALS CMake scripts (one of the reasons being that, unlike autotools, CMake does not provide a make distclean procedure and it is therefore difficult to clean-up the source tree after an in-source build). By ensuring a separate build directory, it is an easy task for the user to clean-up all traces of the build by simply removing the build directory. CMake does generate a make clean which will remove files generated by the compiler and linker.

# 11.1 Configuring, building, and installing on Unix-like systems

The default CMake configuration will build all included solvers and associated examples and will build static and shared libraries. The INSTDIR defaults to /usr/local and can be changed by setting the CMAKE\_INSTALL\_PREFIX variable. Support for FORTRAN and all other options are disabled.

CMake can be used from the command line with the cmake command, or from a curses-based GUI by using the ccmake command, or from a wxWidgets or QT based GUI by using the cmake-gui command. Examples for using both text and graphical methods will be presented. For the examples shown it is assumed that there is a top level SUNDIALS directory with appropriate source, build and install directories:

```
$ mkdir (...)/INSTDIR
$ mkdir (...)/BUILDDIR
$ cd (...)/BUILDDIR
```

## 11.1.1 Building with the GUI

Using CMake with the ccmake GUI follows the general process:

- 1. Select and modify values, run configure (c key)
- 2. New values are denoted with an asterisk
- 3. To set a variable, move the cursor to the variable and press enter
  - If it is a boolean (ON/OFF) it will toggle the value
  - If it is string or file, it will allow editing of the string
  - For file and directories, the <tab> key can be used to complete
- 4. Repeat until all values are set as desired and the generate option is available (g key)
- 5. Some variables (advanced variables) are not visible right away; to see advanced variables, toggle to advanced mode (t key)
- 6. To search for a variable press the / key, and to repeat the search, press the n key

Using CMake with the cmake-gui GUI follows a similar process:

- 1. Select and modify values, click Configure
- 2. The first time you click Configure, make sure to pick the appropriate generator (the following will assume generation of Unix Makfiles).
- 3. New values are highlighted in red
- 4. To set a variable, click on or move the cursor to the variable and press enter
  - If it is a boolean (ON/OFF) it will check/uncheck the box
  - If it is string or file, it will allow editing of the string. Additionally, an ellipsis button will appear ... on the far right of the entry. Clicking this button will bring up the file or directory selection dialog.
  - For files and directories, the <tab> key can be used to complete
- 5. Repeat until all values are set as desired and click the Generate button
- 6. Some variables (advanced variables) are not visible right away; to see advanced variables, click the advanced button

To build the default configuration using the curses GUI, from the BUILDDIR enter the ccmake command and point to the SOLVERDIR:

#### \$ ccmake (...)/SOLVERDIR

Similarly, to build the default configuration using the wxWidgets GUI, from the BUILDDIR enter the cmake-gui command and point to the SOLVERDIR:

## \$ cmake-gui (...)/SOLVERDIR

The default curses configuration screen is shown in the following figure.

The default INSTDIR for both SUNDIALS and the corresponding examples can be changed by setting the CMAKE\_-INSTALL\_PREFIX and the EXAMPLES\_INSTALL\_PATH as shown in the following figure.

Pressing the g key or clicking generate will generate Makefiles including all dependencies and all rules to build SUNDIALS on this system. Back at the command prompt, you can now run:

#### \$ make

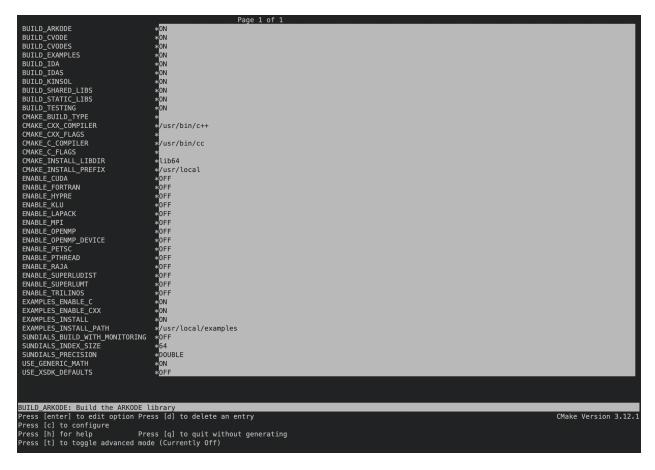


Fig. 11.1: Default configuration screen. Note: Initial screen is empty. To get this default configuration, press 'c' repeatedly (accepting default values denoted with asterisk) until the 'g' option is available.

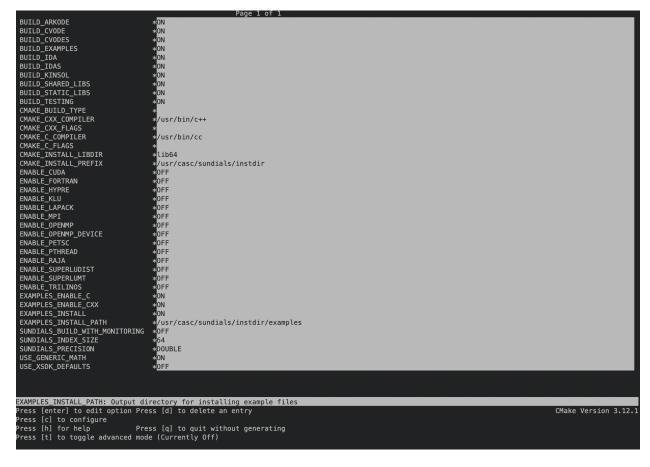


Fig. 11.2: Changing the INSTDIR for SUNDIALS and corresponding EXAMPLES.

or for a faster parallel build (e.g. using 4 threads), you can run

```
$ make -j 4
```

To install SUNDIALS in the installation directory specified in the configuration, simply run:

```
$ make install
```

#### 11.1.2 Building from the command line

Using CMake from the command line is simply a matter of specifying CMake variable settings with the cmake command. The following will build the default configuration:

```
$ cmake -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> ../srcdir
$ make
$ make install
```

## 11.2 Configuration options

A complete list of all available options for a CMake-based SUNDIALS configuration is provide below. Note that the default values shown are for a typical configuration on a Linux system and are provided as illustration only.

#### BUILD\_ARKODE

Build the ARKODE library

Default: ON

#### BUILD\_CVODE

Build the CVODE library

Default: ON

#### BUILD\_CVODES

Build the CVODES library

Default: ON

#### BUILD\_IDA

Build the IDA library

Default: ON

#### BUILD\_IDAS

Build the IDAS library

Default: ON

#### BUILD\_KINSOL

Build the KINSOL library

Default: ON

#### BUILD\_SHARED\_LIBS

Build shared libraries

Default: ON

#### BUILD\_STATIC\_LIBS

Build static libraries

Default: ON

#### CMAKE\_BUILD\_TYPE

Choose the type of build, options are: Debug, Release, RelWithDebInfo, and MinSizeRel

Default: RelWithDebInfo

**Note:** Specifying a build type will trigger the corresponding build type specific compiler flag options below which will be appended to the flags set by CMAKE\_<language>\_FLAGS.

#### CMAKE\_C\_COMPILER

C compiler

Default: /usr/bin/cc

#### CMAKE\_C\_FLAGS

Flags for C compiler

Default:

#### CMAKE\_C\_FLAGS\_DEBUG

Flags used by the C compiler during debug builds

Default: -g

#### CMAKE\_C\_FLAGS\_MINSIZEREL

Flags used by the C compiler during release minsize builds

Default: -Os -DNDEBUG

#### CMAKE\_C\_FLAGS\_RELEASE

Flags used by the C compiler during release builds

Default: -03 -DNDEBUG

#### CMAKE\_C\_STANDARD

The C standard to build C parts of SUNDIALS with.

Default: 99

Options: 99, 11, 17.

#### CMAKE\_C\_EXTENSIONS

Enable compiler specific C extensions.

Default: OFF

#### CMAKE\_CXX\_COMPILER

C++ compiler

Default: /usr/bin/c++

**Note:** A C++ compiler is only required when a feature requiring C++ is enabled (e.g., CUDA, HIP, SYCL, RAJA, etc.) or the C++ examples are enabled.

All SUNDIALS solvers can be used from C++ applications without setting any additional configuration options.

#### CMAKE\_CXX\_FLAGS

Flags for C++ compiler

Default:

#### CMAKE\_CXX\_FLAGS\_DEBUG

Flags used by the C++ compiler during debug builds

Default: -g

#### CMAKE\_CXX\_FLAGS\_MINSIZEREL

Flags used by the C++ compiler during release minsize builds

Default: -Os -DNDEBUG

#### CMAKE\_CXX\_FLAGS\_RELEASE

Flags used by the C++ compiler during release builds

Default: -03 -DNDEBUG

#### CMAKE\_CXX\_STANDARD

The C++ standard to build C++ parts of SUNDIALS with.

Default: 11

Options: 98, 11, 14, 17, 20.

#### CMAKE\_CXX\_EXTENSIONS

Enable compiler specific C++ extensions.

Default: OFF

#### CMAKE\_Fortran\_COMPILER

Fortran compiler

Default: /usr/bin/gfortran

**Note:** Fortran support (and all related options) are triggered only if either Fortran-C support (BUILD\_FORTRAN\_-MODULE\_INTERFACE) or LAPACK (ENABLE\_LAPACK) support is enabled.

#### CMAKE\_Fortran\_FLAGS

Flags for Fortran compiler

Default:

#### CMAKE\_Fortran\_FLAGS\_DEBUG

Flags used by the Fortran compiler during debug builds

Default: -g

#### CMAKE\_Fortran\_FLAGS\_MINSIZEREL

Flags used by the Fortran compiler during release minsize builds

Default: -0s

#### CMAKE\_Fortran\_FLAGS\_RELEASE

Flags used by the Fortran compiler during release builds

Default: -03

#### CMAKE\_INSTALL\_LIBDIR

The directory under which libraries will be installed.

Default: Set based on the system: lib, lib64, or lib/<multiarch-tuple>

#### CMAKE\_INSTALL\_PREFIX

Install path prefix, prepended onto install directories

Default: /usr/local

**Note:** The user must have write access to the location specified through this option. Exported SUNDIALS header files and libraries will be installed under subdirectories include and lib of CMAKE\_INSTALL\_PREFIX, respectively.

#### ENABLE\_CUDA

Build the SUNDIALS CUDA modules.

Default: OFF

#### CMAKE\_CUDA\_ARCHITECTURES

Specifies the CUDA architecture to compile for.

Default: sm\_30

#### EXAMPLES\_ENABLE\_C

Build the SUNDIALS C examples

Default: ON

#### EXAMPLES\_ENABLE\_CXX

Build the SUNDIALS C++ examples

Default: OFF

#### EXAMPLES\_ENABLE\_CUDA

Build the SUNDIALS CUDA examples

Default: OFF

**Note:** You need to enable CUDA support to build these examples.

#### EXAMPLES\_ENABLE\_F2003

Build the SUNDIALS Fortran2003 examples

Default: ON (if BUILD\_FORTRAN\_MODULE\_INTERFACE is ON)

#### EXAMPLES\_INSTALL

Install example files

Default: ON

**Note:** This option is triggered when any of the SUNDIALS example programs are enabled (EXAMPLES\_-ENABLE\_<language> is ON). If the user requires installation of example programs then the sources and sample

output files for all SUNDIALS modules that are currently enabled will be exported to the directory specified by EXAMPLES\_INSTALL\_PATH. A CMake configuration script will also be automatically generated and exported to the same directory. Additionally, if the configuration is done under a Unix-like system, makefiles for the compilation of the example programs (using the installed SUNDIALS libraries) will be automatically generated and exported to the directory specified by EXAMPLES\_INSTALL\_PATH.

#### EXAMPLES\_INSTALL\_PATH

Output directory for installing example files

Default: /usr/local/examples

**Note:** The actual default value for this option will be an examples subdirectory created under CMAKE\_IN-STALL\_PREFIX.

#### BUILD\_FORTRAN\_MODULE\_INTERFACE

Enable Fortran 2003 interface

Default: OFF

**Warning:** There is a known issue with MSYS/gfortran and SUNDIALS shared libraries that causes linking the Fortran interfaces to fail when building SUNDIALS. For now the work around is to only build with static libraries when using MSYS with gfortran on Windows.

#### SUNDIALS\_LOGGING\_LEVEL

Set the maximum logging level for the SUNLogger runtime API. The higher this is set, the more output that may be logged, and the more performance may degrade. The options are:

- 0 no logging
- $1 \log \text{ errors}$
- 2 log errors + warnings
- 3 log errors + warnings + informational output
- 4 log errors + warnings + informational output + debug output
- 5 log all of the above and even more (e.g. vector valued variables may be logged)

Default: 2

#### SUNDIALS\_BUILD\_WITH\_MONITORING

Build SUNDIALS with capabilties for fine-grained monitoring of solver progress and statistics. This is primarily useful for debugging.

Default: OFF

**Warning:** Building with monitoring may result in minor performance degradation even if monitoring is not utilized.

#### SUNDIALS\_BUILD\_WITH\_PROFILING

Build SUNDIALS with capabilties for fine-grained profiling. This requires POSIX timers or the Windows profileapi.h timers.

Default: OFF

Warning: Profiling will impact performance, and should be enabled judiciously.

#### SUNDIALS\_ENABLE\_ERROR\_CHECKS

Build SUNDIALS with more extensive checks for unrecoverable errors.

Default: OFF when CMAKE\_BUILD\_TYPE=Release|RelWithDebInfo `` and ``ON otherwise.

Warning: Error checks will impact performance, but can be helpful for debugging.

#### SUNDIALS\_ENABLE\_EXTERNAL\_ADDONS

Build SUNDIALS with any external addons that you have put in sundials/external.

Default: OFF

**Warning:** Addons are not maintained by the SUNDIALS team. Use at your own risk.

#### ENABLE\_GINKGO

Enable interfaces to the Ginkgo linear algebra library.

Default: OFF

#### Ginkgo\_DIR

Path to the Ginkgo installation.

Default: None

#### SUNDIALS\_GINKGO\_BACKENDS

Semi-colon separated list of Ginkgo target architecutres/executors to build for. Options currenty supported are REF (the Ginkgo reference executor), OMP, CUDA, HIP, and SYCL.

Default: "REF;OMP"

#### ENABLE KOKKOS

Enable the Kokkos based vector.

Default: OFF

#### Kokkos\_DIR

Path to the Kokkos installation.

Default: None

#### ENABLE\_KOKKOS\_KERNELS

Enable the Kokkos based dense matrix and linear solver.

Default: OFF

#### KokkosKernels\_DIR

Path to the Kokkos-Kernels installation.

Default: None

#### ENABLE\_HIP

**Enable HIP Support** 

Default: OFF

#### AMDGPU\_TARGETS

Specify which AMDGPU processor(s) to target.

Default: None

#### **ENABLE HYPRE**

Flag to enable hypre support

Default: OFF

Note: See additional information on building with hypre enabled in §11.4.

#### HYPRE\_INCLUDE\_DIR

Path to hypre header files

Default: none

#### HYPRE\_LIBRARY

Path to hypre installed library files

Default: none

#### ENABLE\_KLU

Enable KLU support

Default: OFF

**Note:** See additional information on building with KLU enabled in §11.4.

#### KLU\_INCLUDE\_DIR

Path to SuiteSparse header files

Default: none

#### KLU\_LIBRARY\_DIR

Path to SuiteSparse installed library files

Default: none

#### ENABLE\_LAPACK

Enable LAPACK support

Default: OFF

**Note:** Setting this option to 0N will trigger additional CMake options. See additional information on building with LAPACK enabled in §11.4.

#### LAPACK\_LIBRARIES

LAPACK (and BLAS) libraries

Default: /usr/lib/liblapack.so;/usr/lib/libblas.so

Note: CMake will search for libraries in your LD\_LIBRARY\_PATH prior to searching default system paths.

#### ENABLE\_MAGMA

Enable MAGMA support.

Default: OFF

Note: Setting this option to ON will trigger additional options related to MAGMA.

#### MAGMA\_DIR

Path to the root of a MAGMA installation.

Default: none

#### SUNDIALS MAGMA BACKENDS

Which MAGMA backend to use under the SUNDIALS MAGMA interface.

Default: CUDA

#### ENABLE\_MPI

Enable MPI support. This will build the parallel nvector and the MPI-aware version of the Many Vector library.

Default: OFF

**Note:** Setting this option to ON will trigger several additional options related to MPI.

#### MPI\_C\_COMPILER

mpicc program

Default:

#### MPI\_CXX\_COMPILER

mpicxx program

Default:

**Note:** This option is triggered only if MPI is enabled (ENABLE\_MPI is ON) and C++ examples are enabled (EXAMPLES\_ENABLE\_CXX is ON). All SUNDIALS solvers can be used from C++ MPI applications by default without setting any additional configuration options other than ENABLE\_MPI.

#### MPI\_Fortran\_COMPILER

mpif90 program

Default:

**Note:** This option is triggered only if MPI is enabled (ENABLE\_MPI is ON) and Fortran-C support is enabled (EXAMPLES\_ENABLE\_F2003 is ON).

#### MPIEXEC\_EXECUTABLE

Specify the executable for running MPI programs

Default: mpirun

**Note:** This option is triggered only if MPI is enabled (ENABLE\_MPI is ON).

#### ENABLE\_ONEMKL

Enable oneMKL support.

Default: OFF

#### ONEMKL DIR

Path to one MKL installation.

Default: none

### SUNDIALS\_ONEMKL\_USE\_GETRF\_LOOP

This advanced debugging option replaces the batched LU factorization with a loop over each system in the batch and a non-batched LU factorization.

Default: OFF

#### SUNDIALS\_ONEMKL\_USE\_GETRS\_LOOP

This advanced debugging option replaces the batched LU solve with a loop over each system in the batch and a non-batched solve.

Default: OFF

#### ENABLE\_OPENMP

Enable OpenMP support (build the OpenMP NVector)

Default: OFF

#### **ENABLE PETSC**

Enable PETSc support

Default: OFF

**Note:** See additional information on building with PETSc enabled in §11.4.

#### PETSC\_DIR

Path to PETSc installation

Default: none

#### PETSC\_LIBRARIES

Semi-colon separated list of PETSc link libraries. Unless provided by the user, this is autopopulated based on the PETSc installation found in PETSC\_DIR.

Default: none

#### PETSC\_INCLUDES

Semi-colon separated list of PETSc include directroies. Unless provided by the user, this is autopopulated based on the PETSc installation found in PETSC\_DIR.

Default: none

#### ENABLE\_PTHREAD

Enable Pthreads support (build the Pthreads NVector)

Default: OFF

#### ENABLE\_RAJA

Enable RAJA support.

Default: OFF

Note: You need to enable CUDA or HIP in order to build the RAJA vector module.

#### SUNDIALS\_RAJA\_BACKENDS

If building SUNDIALS with RAJA support, this sets the RAJA backend to target. Values supported are CUDA,

HIP, or SYCL.

Default: CUDA

#### **ENABLE SUPERLUDIST**

Enable SuperLU\_DIST support

Default: OFF

Note: See additional information on building with SuperLU\_DIST enabled in §11.4.

#### SUPERLUDIST\_DIR

Path to SuperLU\_DIST installation.

Default: none

#### SUPERLUDIST\_OpenMP

Enable SUNDIALS support for SuperLU\_DIST built with OpenMP

Default: none

Note: SuperLU\_DIST must be built with OpenMP support for this option to function. Additionally the environment variable OMP\_NUM\_THREADS must be set to the desired number of threads.

#### SUPERLUDIST\_INCLUDE\_DIRS

List of include paths for SuperLU\_DIST (under a typical SuperLU\_DIST install, this is typically the SuperLU\_DIST SRC directory)

Default: none

**Note:** This is an advanced option. Prefer to use *SUPERLUDIST\_DIR*.

#### SUPERLUDIST\_LIBRARIES

Semi-colon separated list of libraries needed for SuperLU\_DIST

Default: none

Note: This is an advanced option. Prefer to use SUPERLUDIST\_DIR.

#### SUPERLUDIST\_INCLUDE\_DIR

Path to SuperLU\_DIST header files (under a typical SuperLU\_DIST install, this is typically the SuperLU\_DIST SRC directory)

Default: none

Note: This is an advanced option. This option is deprecated. Use SUPERLUDIST\_INCLUDE\_DIRS.

#### SUPERLUDIST\_LIBRARY\_DIR

Path to SuperLU\_DIST installed library files

Default: none

**Note:** This option is deprecated. Use *SUPERLUDIST\_DIR*.

#### ENABLE\_SUPERLUMT

Enable SuperLU\_MT support

Default: OFF

**Note:** See additional information on building with SuperLU\_MT enabled in §11.4.

#### SUPERLUMT\_INCLUDE\_DIR

Path to SuperLU\_MT header files (under a typical SuperLU\_MT install, this is typically the SuperLU\_MT SRC directory)

Default: none

#### SUPERLUMT\_LIBRARY\_DIR

Path to SuperLU\_MT installed library files

Default: none

#### SUPERLUMT\_THREAD\_TYPE

Must be set to Pthread or OpenMP, depending on how SuperLU\_MT was compiled.

Default: Pthread

#### ENABLE\_SYCL

Enable SYCL support.

Default: OFF

**Note:** Building with SYCL enabled requires a compiler that supports a subset of the of SYCL 2020 specification (specifically sycl/sycl.hpp must be available).

CMake does not currently support autodetection of SYCL compilers and CMAKE\_CXX\_COMPILER must be set to a valid SYCL compiler. At present the only supported SYCL compilers are the Intel oneAPI compilers i.e., dpcpp and icpx. When using icpx the -fsycl flag and any ahead of time compilation flags must be added to CMAKE\_CXX\_FLAGS.

#### SUNDIALS\_SYCL\_2020\_UNSUPPORTED

This advanced option disables the use of *some* features from the SYCL 2020 standard in SUNDIALS libraries and examples. This can be used to work around some cases of incomplete compiler support for SYCL 2020.

Default: OFF

#### ENABLE\_CALIPER

Enable CALIPER support

Default: OFF

**Note:** Using Caliper requires setting SUNDIALS\_BUILD\_WITH\_PROFILING to ON.

#### CALIPER\_DIR

Path to the root of a Caliper installation

Default: None

#### ENABLE ADIAK

Enable Adiak support

Default: OFF

#### adiak DIR

Path to the root of an Adiak installation

Default: None

#### SUNDIALS\_LAPACK\_CASE

Specify the case to use in the Fortran name-mangling scheme, options are: lower or upper

Default:

**Note:** The build system will attempt to infer the Fortran name-mangling scheme using the Fortran compiler. This option should only be used if a Fortran compiler is not available or to override the inferred or default (lower) scheme if one can not be determined. If used, SUNDIALS\_LAPACK\_UNDERSCORES must also be set.

#### SUNDIALS\_LAPACK\_UNDERSCORES

Specify the number of underscores to append in the Fortran name-mangling scheme, options are: none, one, or two

Default:

**Note:** The build system will attempt to infer the Fortran name-mangling scheme using the Fortran compiler. This option should only be used if a Fortran compiler is not available or to override the inferred or default (one) scheme if one can not be determined. If used, SUNDIALS\_LAPACK\_CASE must also be set.

#### SUNDIALS\_INDEX\_TYPE

Integer type used for SUNDIALS indices. The size must match the size provided for the SUNDIALS\_INDEX\_SIZE option.

Default: Automatically determined based on SUNDIALS\_INDEX\_SIZE

**Note:** In past SUNDIALS versions, a user could set this option to INT64\_T to use 64-bit integers, or INT32\_T to use 32-bit integers. Starting in SUNDIALS 3.2.0, these special values are deprecated. For SUNDIALS 3.2.0 and up, a user will only need to use the *SUNDIALS\_INDEX\_SIZE* option in most cases.

#### SUNDIALS\_INDEX\_SIZE

Integer size (in bits) used for indices in SUNDIALS, options are: 32 or 64

Default: 64

**Note:** The build system tries to find an integer type of appropriate size. Candidate 64-bit integer types are (in order of preference): int64\_t, \_\_int64, long long, and long. Candidate 32-bit integers are (in order of preference): int32\_t, int, and long. The advanced option, *SUNDIALS\_INDEX\_TYPE* can be used to provide a type not listed here.

#### SUNDIALS\_PRECISION

The floating-point precision used in SUNDIALS packages and class implementations, options are: double, single, or extended

Default: double

#### SUNDIALS\_MATH\_LIBRARY

The standard C math library (e.g., libm) to link with.

Default: -1m on Unix systems, none otherwise

#### SUNDIALS\_INSTALL\_CMAKEDIR

Installation directory for the SUNDIALS cmake files (relative to CMAKE\_INSTALL\_PREFIX).

Default: CMAKE\_INSTALL\_PREFIX/cmake/sundials

#### ENABLE\_XBRAID

Enable or disable the ARKStep + XBraid interface.

Default: OFF

**Note:** See additional information on building with *XBraid* enabled in §11.4.

#### XBRAID\_DIR

The root directory of the XBraid installation.

Default: OFF

#### XBRAID\_INCLUDES

Semi-colon separated list of XBraid include directories. Unless provided by the user, this is autopopulated based on the XBraid installation found in XBRAID\_DIR.

Default: none

#### XBRAID\_LIBRARIES

Semi-colon separated list of XBraid link libraries. Unless provided by the user, this is autopopulated based on the XBraid installation found in XBRAID\_DIR.

Default: none

#### USE\_XSDK\_DEFAULTS

Enable xSDK (see https://xsdk.info for more information) default configuration settings. This sets CMAKE\_BUILD\_TYPE to Debug, SUNDIALS\_INDEX\_SIZE to 32 and SUNDIALS\_PRECISION to double.

Default: OFF

## 11.3 Configuration examples

The following examples will help demonstrate usage of the CMake configure options.

To configure SUNDIALS using the default C and Fortran compilers, and default mpic and mpif90 parallel compilers, enable compilation of examples, and install libraries, headers, and example sources under subdirectories of /home/myname/sundials/, use:

```
% cmake \
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> -DENABLE_MPI=ON \
> /home/myname/sundials/srcdir

% make install
```

To disable installation of the examples, use:

```
% cmake \
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> -DENABLE_MPI=ON \
> -DEXAMPLES_INSTALL=OFF \
> /home/myname/sundials/srcdir

% make install
```

## 11.4 Working with external Libraries

The SUNDIALS suite contains many options to enable implementation flexibility when developing solutions. The following are some notes addressing specific configurations when using the supported third party libraries.

### 11.4.1 Building with Ginkgo

Ginkgo is a high-performance linear algebra library for manycore systems, with a focus on solving sparse linear systems. It is implemented using modern C++ (you will need at least a C++14 compliant compiler to build it), with GPU kernels implemented in CUDA (for NVIDIA devices), HIP (for AMD devices) and SYCL/DPC++ (for Intel devices and other supported hardware). To enable Ginkgo in SUNDIALS, set the <code>ENABLE\_GINKGO</code> to ON and provide the path to the root of the Ginkgo installation in <code>Ginkgo\_DIR</code>. Additionally, <code>SUNDIALS\_GINKGO\_BACKENDS</code> must be set to a list of Ginkgo target architecutres/executors. E.g.,

```
% cmake \
> -DENABLE_GINKGO=ON \
> -DGinkgo_DIR=/path/to/ginkgo/installation \
> -DSUNDIALS_GINKGO_BACKENDS="REF;OMP;CUDA" \
> /home/myname/sundials/srcdir
```

The SUNDIALS interfaces to Ginkgo are not compatible with SUNDIALS\_PRECISION set to extended.

#### 11.4.2 Building with Kokkos

Kokkos is a modern C++ (requires at least C++14) programming model for witting performance portable code for multicore CPU and GPU-based systems including NVIDIA, AMD, and Intel accelerators. To enable Kokkos in SUNDIALS, set the <code>ENABLE\_KOKKOS</code> to ON and provide the path to the root of the Kokkos installation in <code>Kokkos\_DIR</code>. Additionally, the Kokkos-Kernels library provides common computational kernels for linear algebra. To enable Kokkos-Kernels in SUNDIALS, set the <code>ENABLE\_KOKKOS\_KERNELS</code> to ON and provide the path to the root of the Kokkos-Kernels installation in <code>KokkosKernels\_DIR</code> e.g.,

```
% cmake \
> -DENABLE_KOKKOS=ON \
> -DKokkos_DIR=/path/to/kokkos/installation \
> -DENABLE_KOKKOS_KERNELS=ON \
> -DKokkosKernels_DIR=/path/to/kokkoskernels/installation \
> /home/myname/sundials/srcdir
```

**Note:** The minimum supported version of Kokkos-Kernels 3.7.00.

#### 11.4.3 Building with LAPACK

To enable LAPACK, set the ENABLE\_LAPACK option to ON. If the directory containing the LAPACK library is in the LD\_LIBRARY\_PATH environment variable, CMake will set the LAPACK\_LIBRARIES variable accordingly, otherwise CMake will attempt to find the LAPACK library in standard system locations. To explicitly tell CMake what library to use, the LAPACK\_LIBRARIES variable can be set to the desired libraries required for LAPACK.

```
% cmake \
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> -DENABLE_LAPACK=ON \
> -DLAPACK_LIBRARIES=/mylapackpath/lib/libblas.so;/mylapackpath/lib/liblapack.so \
> /home/myname/sundials/srcdir
% make install
```

**Note:** If a working Fortran compiler is not available to infer the Fortran name-mangling scheme, the options SUNDI-ALS\_F77\_FUNC\_CASE and SUNDIALS\_F77\_FUNC\_UNDERSCORES *must* be set in order to bypass the check for a Fortran compiler and define the name-mangling scheme. The defaults for these options in earlier versions of SUNDIALS were lower and one, respectively.

SUNDIALS has been tested with OpenBLAS 0.3.18.

#### 11.4.4 Building with KLU

KLU is a software package for the direct solution of sparse nonsymmetric linear systems of equations that arise in circuit simulation and is part of SuiteSparse, a suite of sparse matrix software. The library is developed by Texas A&M University and is available from the SuiteSparse GitHub repository.

To enable KLU, set ENABLE\_KLU to ON, set KLU\_INCLUDE\_DIR to the include path of the KLU installation and set KLU\_LIBRARY\_DIR to the lib path of the KLU installation. In that case, the CMake configure will result in populating the following variables: AMD\_LIBRARY, AMD\_LIBRARY\_DIR, BTF\_LIBRARY, BTF\_LIBRARY\_DIR, COLAMD\_LIBRARY, COLAMD\_LIBRARY\_DIR, and KLU\_LIBRARY.

For SuiteSparse 7.4.0 and newer, the necessary information can also be gathered from a CMake import target. If SuiteSparse is installed in a non-default prefix, the path to the CMake Config file can be set using CMAKE\_PREFIX\_PATH. In that case, the CMake configure step won't populate the previously mentioned variables. It is still possible to set KLU\_INCLUDE\_DIR and KLU\_LIBRARY\_DIR which take precedence over a potentially installed CMake import target file.

In either case, a CMake target SUNDIALS::KLU will be created if the KLU library could be found. Dependent targets should link to that target.

SUNDIALS has been tested with SuiteSparse version 5.10.1.

#### 11.4.5 Building with SuperLU\_DIST

SuperLU\_DIST is a general purpose library for the direct solution of large, sparse, nonsymmetric systems of linear equations in a distributed memory setting. The library is developed by Lawrence Berkeley National Laboratory and is available from the SuperLU\_DIST GitHub repository.

To enable SuperLU\_DIST, set *ENABLE\_SUPERLUDIST* to ON, set *SUPERLUDIST\_DIR* to the path where SuperLU\_DIST is installed. If SuperLU\_DIST was built with OpenMP then the option *SUPERLUDIST\_OpenMP* and *ENABLE\_OPENMP* should be set to ON.

SUNDIALS supports SuperLU DIST v7.0.0 – v8.x.x and has been tested with v7.2.0 and v8.1.0.

#### 11.4.6 Building with SuperLU\_MT

SuperLU\_MT is a general purpose library for the direct solution of large, sparse, nonsymmetric systems of linear equations on shared memory parallel machines. The library is developed by Lawrence Berkeley National Laboratory and is available from the SuperLU\_MT GitHub repository.

To enable SuperLU\_MT, set ENABLE\_SUPERLUMT to ON, set SUPERLUMT\_INCLUDE\_DIR to the SRC path of the SuperLU\_MT installation, and set the variable SUPERLUMT\_LIBRARY\_DIR to the lib path of the SuperLU\_MT installation. At the same time, the variable SUPERLUMT\_LIBRARIES must be set to a semi-colon separated list of other libraries SuperLU\_MT depends on. For example, if SuperLU\_MT was build with an external blas library, then include the full path to the blas library in this list. Additionally, the variable SUPERLUMT\_THREAD\_TYPE must be set to either Pthread or OpenMP.

Do not mix thread types when building SUNDIALS solvers. If threading is enabled for SUNDIALS by having either ENABLE\_OPENMP or ENABLE\_PTHREAD set to ON then SuperLU MT should be set to use the same threading type.

SUNDIALS has been tested with SuperLU MT version 3.1.

### 11.4.7 Building with PETSc

The Portable, Extensible Toolkit for Scientific Computation (PETSc) is a suite of data structures and routines for simulating applications modeled by partial differential equations. The library is developed by Argonne National Laboratory and is available from the PETSc GitLab repository.

To enable PETSc, set ENABLE\_PETSC to ON, and set PETSC\_DIR to the path of the PETSc installation. Alternatively, a user can provide a list of include paths in PETSC\_INCLUDES and a list of complete paths to the PETSc libraries in PETSC\_LIBRARIES.

SUNDIALS is regularly tested with the latest PETSc versions, specifically up to version 3.18.1 as of SUNDIALS version v7.1.0. SUNDIALS requires PETSc 3.5.0 or newer.

#### 11.4.8 Building with *hypre*

*hypre* is a library of high performance preconditioners and solvers featuring multigrid methods for the solution of large, sparse linear systems of equations on massively parallel computers. The library is developed by Lawrence Livermore National Laboratory and is available from the hypre GitHub repository.

To enable *hypre*, set ENABLE\_HYPRE to ON, set HYPRE\_INCLUDE\_DIR to the include path of the *hypre* installation, and set the variable HYPRE\_LIBRARY\_DIR to the lib path of the *hypre* installation.

**Note:** SUNDIALS must be configured so that SUNDIALS\_INDEX\_SIZE is compatible with HYPRE\_BigInt in the *hypre* installation.

SUNDIALS is regularly tested with the latest versions of *hypre*, specifically up to version 2.26.0 as of SUNDIALS version v7.1.0.

## 11.4.9 Building with MAGMA

The Matrix Algebra on GPU and Multicore Architectures (MAGMA) project provides a dense linear algebra library similar to LAPACK but targeting heterogeneous architectures. The library is developed by the University of Tennessee and is available from the UTK webpage.

To enable the SUNDIALS MAGMA interface set ENABLE\_MAGMA to ON, MAGMA\_DIR to the MAGMA installation path, and SUNDIALS\_MAGMA\_BACKENDS to the desired MAGMA backend to use with SUNDIALS e.g., CUDA or HIP.

SUNDIALS has been tested with MAGMA version v2.6.1 and v2.6.2.

#### 11.4.10 Building with oneMKL for SYCL

The Intel oneAPI Math Kernel Library (oneMKL) includes CPU and SYCL/DPC++ interfaces for LAPACK dense linear algebra routines. The SUNDIALS oneMKL interface targets the SYCL/DPC++ routines, to utilize the CPU routine see §11.4.3.

To enable the SUNDIALS one MKL interface set ENABLE\_ONEMKL to ON and ONEMKL\_DIR to the one MKL installation path.

SUNDIALS has been tested with oneMKL version 2021.4.

#### 11.4.11 Building with CUDA

The NVIDIA CUDA Toolkit provides a development environment for GPU-accelerated computing with NVIDIA GPUs. The CUDA Toolkit and compatible NVIDIA drivers are available from the NVIDIA developer website.

To enable CUDA, set ENABLE\_CUDA to ON. If CUDA is installed in a nonstandard location, you may be prompted to set the variable CUDA\_TOOLKIT\_ROOT\_DIR with your CUDA Toolkit installation path. To enable CUDA examples, set EXAMPLES\_ENABLE\_CUDA to ON.

SUNDIALS has been tested with the CUDA toolkit versions 10 and 11.

#### 11.4.12 Building with HIP

HIP(heterogeneous-compute interface for portability) allows developers to create portable applications for AMD and NVIDIA GPUs. HIP can be obtained from HIP GitHub repository.

To enable HIP, set ENABLE\_HIP to ON and set AMDGPU\_TARGETS to the desired target(ex. gfx705). In addition, set CMAKE\_C\_COMPILER and CMAKE\_CXX\_COMPILER to point to an installation of hipcc.

SUNDIALS has been tested with HIP versions between 5.0.0 to 5.4.3.

#### 11.4.13 Building with RAJA

RAJA is a performance portability layer developed by Lawrence Livermore National Laboratory and can be obtained from the RAJA GitHub repository.

Building SUNDIALS RAJA modules requires a CUDA, HIP, or SYCL enabled RAJA installation. To enable RAJA, set ENABLE\_RAJA to ON, set SUNDIALS\_RAJA\_BACKENDS to the desired backend (CUDA, HIP, or SYCL), and set ENABLE\_CUDA, ENABLE\_HIP, or ENABLE\_SYCL to ON depending on the selected backend. If RAJA is installed in a nonstandard location you will be prompted to set the variable RAJA\_DIR with the path to the RAJA CMake configuration file. To enable building the RAJA examples set EXAMPLES\_ENABLE\_CXX to ON.

SUNDIALS has been tested with RAJA version 0.14.0.

#### 11.4.14 Building with XBraid

XBraid is parallel-in-time library implementing an optimal-scaling multigrid reduction in time (MGRIT) solver. The library is developed by Lawrence Livermore National Laboratory and is available from the XBraid GitHub repository.

To enable XBraid support, set ENABLE\_XBRAID to ON, set XBRAID\_DIR to the root install location of XBraid or the location of the clone of the XBraid repository.

**Note:** At this time the XBraid types braid\_Int and braid\_Real are hard-coded to int and double respectively. As such SUNDIALS must be configured with SUNDIALS\_INDEX\_SIZE set to 32 and SUNDIALS\_PRECISION set to double. Additionally, SUNDIALS must be configured with ENABLE\_MPI set to ON.

SUNDIALS has been tested with XBraid version 3.0.0.

## 11.5 Testing the build and installation

If SUNDIALS was configured with EXAMPLES\_ENABLE\_<language> options to ON, then a set of regression tests can be run after building with the make command by running:

% make test

Additionally, if EXAMPLES\_INSTALL was also set to ON, then a set of smoke tests can be run after installing with the make install command by running:

% make test\_install

## 11.6 Building and Running Examples

Each of the SUNDIALS solvers is distributed with a set of examples demonstrating basic usage. To build and install the examples, set at least of the EXAMPLES\_ENABLE\_<language> options to ON, and set EXAMPLES\_INSTALL to ON. Specify the installation path for the examples with the variable EXAMPLES\_INSTALL\_PATH. CMake will generate CMakeLists.txt configuration files (and Makefile files if on Linux/Unix) that reference the *installed* SUNDIALS headers and libraries.

Either the CMakeLists.txt file or the traditional Makefile may be used to build the examples as well as serve as a template for creating user developed solutions. To use the supplied Makefile simply run make to compile and generate the executables. To use CMake from within the installed example directory, run cmake (or ccmake or cmake-gui to use the GUI) followed by make to compile the example code. Note that if CMake is used, it will overwrite the traditional Makefile with a new CMake-generated Makefile.

The resulting output from running the examples can be compared with example output bundled in the SUNDIALS distribution.

**Note:** There will potentially be differences in the output due to machine architecture, compiler versions, use of third party libraries etc.

## 11.7 Configuring, building, and installing on Windows

CMake can also be used to build SUNDIALS on Windows. To build SUNDIALS for use with Visual Studio the following steps should be performed:

- 1. Unzip the downloaded tar file(s) into a directory. This will be the SOLVERDIR
- 2. Create a separate BUILDDIR
- 3. Open a Visual Studio Command Prompt and cd to BUILDDIR
- 4. Run cmake-gui ../SOLVERDIR
  - a. Hit Configure
  - b. Check/Uncheck solvers to be built
  - c. Change CMAKE\_INSTALL\_PREFIX to INSTDIR
  - d. Set other options as desired
  - e. Hit Generate

- 5. Back in the VS Command Window:
  - a. Run msbuild ALL\_BUILD.vcxproj
  - b. Run msbuild INSTALL.vcxproj

The resulting libraries will be in the INSTDIR.

The SUNDIALS project can also now be opened in Visual Studio. Double click on the ALL\_BUILD.vcxproj file to open the project. Build the whole *solution* to create the SUNDIALS libraries. To use the SUNDIALS libraries in your own projects, you must set the include directories for your project, add the SUNDIALS libraries to your project solution, and set the SUNDIALS libraries as dependencies for your project.

## 11.8 Installed libraries and exported header files

Using the CMake SUNDIALS build system, the command

```
$ make install
```

will install the libraries under LIBDIR and the public header files under INCLUDEDIR. The values for these directories are INSTDIR/lib and INSTDIR/include, respectively. The location can be changed by setting the CMake variable CMAKE\_INSTALL\_PREFIX. Although all installed libraries reside under LIBDIR/lib, the public header files are further organized into subdirectories under INCLUDEDIR/include.

The installed libraries and exported header files are listed for reference in the table below. The file extension .LIB is typically .so for shared libraries and .a for static libraries. Note that, in this table names are relative to LIBDIR for libraries and to INCLUDEDIR for header files.

**Warning:** SUNDIALS installs some header files to INSTDIR/include/sundials/priv. All of the header files in this directory are private and **should not be included in user code**. The private headers are subject to change without any notice and relying on them may break your code.

## 11.9 Using SUNDIALS in your project

After building and installing SUNDIALS, using SUNDIALS in your application involves two steps: including the right header files and linking to the right libraries.

Depending on what features of SUNDIALS that your application uses, the header files needed will vary. For example, if you want to use CVODE for serial computations you need the following includes:

```
#include <cvode/cvode.h>
#include <nvector/nvector_serial.h>
```

If you wanted to use CVODE with the GMRES linear solver and our CUDA enabled vector:

```
#include <cvode/cvode.h>
#include <nvector/nvector_cuda.h>
#include <sunlinsol/sunlinsol_spgmr.h>
```

The story is similar for linking to SUNDIALS. Starting in v7.0.0, all applications will need to link to libsundials\_core. Furthermore, depending on the packages and modules of SUNDIALS of interest an application will need to

link to a few more libraries. Using the same examples as for the includes, we would need to also link to libsundials\_cvode, libsundials\_nvecserial for the first example and libsundials\_cvode, libsundials\_nveccuda, libsundials\_sunlinsolspgmr for the second.

Refer to the documentations sections for the individual packages and modules of SUNDIALS that interest you for the proper includes and libraries to link to.

## 11.10 Using SUNDIALS as a Third Party Library in other CMake Projects

The make install command will also install a CMake package configuration file that other CMake projects can load to get all the information needed to build against SUNDIALS. In the consuming project's CMake code, the find\_package command may be used to search for the configuration file, which will be installed to instdir/SUNDIALS\_INSTALL\_CMAKEDIR/SUNDIALS\_ONFIG.cmake alongside a package version file instdir/SUNDIALS\_INSTALL\_CMAKEDIR/SUNDIALSConfigVersion.cmake. Together these files contain all the information the consuming project needs to use SUNDIALS, including exported CMake targets. The SUNDIALS exported CMake targets follow the same naming convention as the generated library binaries, e.g. the exported target for CVODE is SUNDIALS::cvode. The CMake code snipped below shows how a consuming project might leverage the SUNDIALS package configuration file to build against SUNDIALS in their own CMake project.

```
# Set the variable SUNDIALS_DIR to the SUNDIALS instdir.
# When using the cmake CLI command, this can be done like so:
# cmake -D SUNDIALS_DIR=/path/to/sundials/installation
# Find any SUNDIALS version...
find_package(SUNDIALS REQUIRED)
# ... or find any version newer than some minimum...
find_package(SUNDIALS 7.1.0 REQUIRED)
# ... or find a version in a range
find_package(SUNDIALS 7.0.0...7.1.0 REQUIRED)

# ... or find a version in a range
find_package(SUNDIALS 7.0.0...7.1.0 REQUIRED)

# Link to SUNDIALS libraries through the exported targets.
# This is just an example, users should link to the targets appropriate
# for their use case.
target_link_libraries(myexec PUBLIC SUNDIALS::cvode SUNDIALS::nvecpetsc)
```

**Note:** Changed in version 7.1.0: A single version provided to find\_package denotes the minimum version of SUNDIALS to look for, and any version equal or newer than what is specified will match. In prior versions SUNDIALSConfig. cmake required the version found to have the same major version number as the single version provided to find\_package.

## 11.11 Table of SUNDIALS libraries and header files

Table 11.1: SUNDIALS shared libraries and header files

Core	Libraries	libsundials_core.LIB
	Headers	sundials/sundials_band.h
		sundials/sundials_config.h
		sundials/sundials_context.h
		sundials/sundials_cuda_policies.hpp
		sundials/sundials_dense.h
		sundials/sundials_direct.h
		sundials/sundials_hip_policies.hpp
		sundials/sundials_iterative.h
		sundials/sundials_linearsolver.h
		sundials/sundials_math.h
		sundials/sundials_matrix.h
		sundials/sundials_memory.h
		sundials/sundials_mpi_types.h
		sundials/sundials_nonlinearsolver.h
		sundials/sundials_nvector.h
		sundials/sundials_types.h
		sundials/sundials_version.h
NIVECTOR M. I. I.		sundials/sundials_xbraid.h
NVECTOR Modules SERIAL	Libraries	libsundials_nvecserial.LIB
SERIAL	Headers	nvector/nvector_serial.h
PARALLEL	Libraries	, –
PARALLEL	Headers	libsundials_nvecparallel.LIB
ODENIMO		nvector/nvector_parallel.h
OPENMP	Libraries Headers	libsundials_nvecopenmp.LIB
DELIDEADC		nvector/nvector_openmp.h
PTHREADS	Libraries Headers	libsundials_nvecpthreads.LIB
DADIIVD	Libraries	nvector/nvector_pthreads.h
PARHYP		libsundials_nvecparhyp.LIB
PETSC	Headers	nvector/nvector_parhyp.h
PEISC	Libraries	libsundials_nvecpetsc.LIB
CUDA	Headers Libraries	<pre>nvector/nvector_petsc.h libsundials_nveccuda.LIB</pre>
CODA		
THD	Headers Libraries	nvector/nvector_cuda.h
HIP	Headers	libsundials_nvechip.LIB
DATA		nvector/nvector_hip.h
RAJA	Libraries	libsundials_nveccudaraja.LIB
	17 1	libsundials_nvechipraja.LIB
CVCI	Headers	nvector/nvector_raja.h
SYCL	Libraries	libsundials_nvecsycl.LIB
MANNAGEOR	Headers	nvector/nvector_sycl.h
MANYVECTOR	Libraries	libsundials_nvecmanyvector.LIB
MDDMANNANGEOD	Headers	nvector/nvector_manyvector.h
MPIMANYVECTOR	Libraries	libsundials_nvecmpimanyvector.LIB
A COUNT LIGHT	Headers	nvector/nvector_mpimanyvector.h
MPIPLUSX	Libraries	libsundials_nvecmpiplusx.LIB
	Headers	<pre>nvector/nvector_mpiplusx.h</pre>

Table 11.1 – continued from previous page

	Table 11.1 – continu	ed from previous page
SUNMATRIX Modules		
BAND	Libraries	libsundials_sunmatrixband.LIB
	Headers	sunmatrix/sunmatrix_band.h
CUSPARSE	Libraries	libsundials_sunmatrixcusparse.LIB
	Headers	sunmatrix/sunmatrix_cusparse.h
DENSE	Libraries	libsundials_sunmatrixdense.LIB
	Headers	sunmatrix/sunmatrix_dense.h
Ginkgo	Headers	sunmatrix/sunmatrix_ginkgo.hpp
MAGMADENSE	Libraries	libsundials_sunmatrixmagmadense.LIB
	Headers	sunmatrix/sunmatrix_magmadense.h
ONEMKLDENSE	Libraries	libsundials_sunmatrixonemkldense.LIB
	Headers	sunmatrix/sunmatrix_onemkldense.h
SPARSE	Libraries	libsundials_sunmatrixsparse.LIB
	Headers	sunmatrix/sunmatrix_sparse.h
SLUNRLOC	Libraries	libsundials_sunmatrixslunrloc.LIB
	Headers	sunmatrix/sunmatrix_slunrloc.h
SUNLINSOL Modules		,
BAND	Libraries	libsundials_sunlinsolband.LIB
	Headers	sunlinsol/sunlinsol_band.h
CUSOLVERSP_BATCHQR	Libraries	libsundials_sunlinsolcusolversp.LIB
	Headers	sunlinsol/sunlinsol_cusolversp_batchqr.h
DENSE	Libraries	libsundials_sunlinsoldense.LIB
	Headers	sunlinsol/sunlinsol_dense.h
Ginkgo	Headers	sunlinsol/sunlinsol_ginkgo.hpp
KLU	Libraries	libsundials_sunlinsolklu.LIB
	Headers	sunlinsol/sunlinsol_klu.h
LAPACKBAND	Libraries	libsundials_sunlinsollapackband.LIB
	Headers	sunlinsol/sunlinsol_lapackband.h
LAPACKDENSE	Libraries	libsundials_sunlinsollapackdense.LIB
	Headers	sunlinsol/sunlinsol_lapackdense.h
MAGMADENSE	Libraries	libsundials_sunlinsolmagmadense.LIB
	Headers	sunlinsol/sunlinsol_magmadense.h
ONEMKLDENSE	Libraries	libsundials_sunlinsolonemkldense.LIB
	Headers	sunlinsol/sunlinsol_onemkldense.h
PCG	Libraries	libsundials_sunlinsolpcg.LIB
	Headers	sunlinsol/sunlinsol_pcg.h
SPBCGS	Libraries	libsundials_sunlinsolspbcgs.LIB
	Headers	sunlinsol/sunlinsol_spbcgs.h
SPFGMR	Libraries	libsundials_sunlinsolspfgmr.LIB
	Headers	sunlinsol/sunlinsol_spfgmr.h
SPGMR	Libraries	libsundials_sunlinsolspgmr.LIB
or only	Headers	sunlinsol/sunlinsol_spgmr.h
SPTFQMR	Libraries	libsundials_sunlinsolsptfqmr.LIB
or II Amir	Headers	sunlinsol/sunlinsol_sptfqmr.h
SUPERLUDIST	Libraries	libsundials_sunlinsolsuperludist.LIB
JOI LIKEODISI	Headers	sunlinsol/sunlinsol_superludist.h
SUPERLUMT	Libraries	libsundials_sunlinsolsuperlumt.LIB
SUFERLUMI	Headers	
SUNNONLINSOL Modules	rieauers	sunlinsol/sunlinsol_superlumt.h
NEWTON	Libraries	libsundials_sunnonlinsolnewton.LIB
TABAA TOTA	Headers	sunnonlinsol/sunnonlinsol_newton.h
	11000018	continues on next page

Table 11.1 – continued from previous page

	Table 11.1 – continu	1 10			
FIXEDPOINT	Libraries	libsundials_sunnonlinsolfixedpoint.LIB			
	Headers	<pre>sunnonlinsol/sunnonlinsol_fixedpoint.h</pre>			
PETSCSNES	Libraries	libsundials_sunnonlinsolpetscsnes.LIB			
	Headers	<pre>sunnonlinsol/sunnonlinsol_petscsnes.h</pre>			
SUNMEMORY Modules					
SYSTEM	Libraries	libsundials_sunmemsys.LIB			
	Headers	sunmemory/sunmemory_system.h			
CUDA	Libraries	libsundials_sunmemcuda.LIB			
	Headers	sunmemory/sunmemory_cuda.h			
HIP	Libraries	libsundials_sunmemhip.LIB			
	Headers	sunmemory/sunmemory_hip.h			
SYCL	Libraries	libsundials_sunmemsycl.LIB			
	Headers	sunmemory/sunmemory_sycl.h			
SUNDIALS Packages		, , , , , , , , , , , , , , , , , , , ,			
CVODE	Libraries	libsundials_cvode.LIB			
	Headers	cvode/cvode.h			
		cvode/cvode_bandpre.h			
		cvode/cvode_bbdpre.h			
		cvode/cvode_diag.h			
		cvode/cvode_impl.h			
		cvode/cvode_ls.h			
		cvode/cvode_proj.h			
CVODES	Libraries	libsundials_cvodes.LIB			
CVODES	Headers	cvodes/cvodes.h			
	Treaders	cvodes/cvodes_bandpre.h			
		cvodes/cvodes_bandpre.h			
		cvodes/cvodes_bbdpre.nr			
		cvodes/cvodes_urag.n cvodes/cvodes_impl.h			
ARKODE	Libraries	<pre>cvodes/cvodes_ls.h libsundials_arkode.LIB</pre>			
ARRODE	Libraries	libsundials_xbraid.LIB			
	II 1				
	Headers	arkode/arkode.h			
		arkode/arkode_arkstep.h			
		arkode/arkode_bandpre.h			
		arkode/arkode_bbdpre.h			
		arkode/arkode_butcher.h			
		arkode/arkode_butcher_dirk.h			
		arkode/arkode_butcher_erk.h			
		arkode/arkode_erkstep.h			
		arkode/arkode_impl.h			
		arkode/arkode_ls.h			
		arkode/arkode_mristep.h			
		arkode/arkode_xbraid.h			
IDA	Libraries	libsundials_ida.LIB			
	Headers	ida/ida.h			
		ida/ida_bbdpre.h			
		ida/ida_impl.h			
		ida/ida_ls.h			
IDAS	Libraries	libsundials_idas.LIB			
	Headers	idas/idas.h			
		idas/idas_bbdpre.h			
	ı	continues on next page			

Table 11.1 – continued from previous page

		idas/idas_impl.h
KINSOL	Libraries	libsundials_kinsol.LIB
	Headers	kinsol/kinsol.h
		kinsol/kinsol_bbdpre.h
		kinsol/kinsol_impl.h
		kinsol/kinsol_ls.h

### 11.12 Installing SUNDIALS on HPC Clusters

This section is a guide for installing SUNDIALS on specific HPC clusters. In general, the procedure is the same as described previously for Linux machines. The main differences are in the modules and environment variables that are specific to different HPC clusters. We aim to keep this section as up to date as possible, but it may lag the latest software updates to each cluster.

#### **11.12.1** Frontier

Frontier is an Exascale supercomputer at the Oak Ridge Leadership Computing Facility. If you are new to this system, then we recommend that you review the Frontier user guide.

#### A Standard Installation

Clone SUNDIALS:

```
git clone https://github.com/LLNL/sundials.git && cd sundials
```

Next we load the modules and set the environment variables needed to build SUNDIALS. This configuration enables both MPI and HIP support for distributed and GPU parallelism. It uses the HIP compiler for C and C++ and the Cray Fortran compiler. Other configurations are possible.

```
# required dependencies
module load PrgEnv-cray-amd/8.5.0
module load craype-accel-amd-gfx90a
module load rocm/5.3.0
module load cmake/3.23.2

# GPU-aware MPI
export MPICH_GPU_SUPPORT_ENABLED=1

# compiler environment hints
export CC=$(which hipcc)
export CXX=$(which hipcc)
export FC=$(which ftn)
export CFLAGS="-I${ROCM_PATH}/include"
export CXXFLAGS="-I${ROCM_PATH}/include -Wno-pass-failed"
export LDFLAGS="-L${ROCM_PATH}/lib -lamdhip64 ${PE_MPICH_GTL_DIR_amd_gfx90a} -lmpi_gtl_hsa"
```

Now we can build SUNDIALS. In general, this is the same procedure described in the previous sections. The following command builds and installs SUNDIALS with MPI, HIP, and the Fortran interface enabled, where *<install path>* is your desired installation location, and *<account>* is your allocation account on Frontier:

```
cmake -S . -B builddir -DCMAKE_INSTALL_PREFIX=<install path> -DAMDGPU_TARGETS=gfx90a \
-DENABLE_HIP=ON -DENABLE_MPI=ON -DBUILD_FORTRAN_MODULE_INTERFACE=ON
cd builddir
make -j8 install
# Need an allocation to run the tests:
salloc -A <account> -t 10 -N 1 -p batch
make test
make test_install_all
```

## 11.13 Building with SUNDIALS Addons

SUNDIALS "addons" are community developed code additions for SUNDIALS that can be subsumed by the SUNDIALS build system so that they have full access to all internal SUNDIALS symbols. The intent is for SUNDIALS addons to function as if they are part of the SUNDIALS library, while allowing them to potentially have different licenses (although we encourage BSD-3-Clause still), code style (although we encourage them to follow the SUNDIALS style outlined here).

**Warning:** SUNDIALS addons are not maintained by the SUNDIALS team and may come with different licenses. Use them at your own risk.

To build with SUNDIALS addons,

- 1. Clone/copy the addon(s) into <sundials root>/external/
- 2. Copy the sundials-addon-example block in the <sundials root>/external/CMakeLists.txt, paste it below the example block, and modify the path listed for your own external addon(s).
- 3. When building SUNDIALS, set the CMake option SUNDIALS\_ENABLE\_EXTERNAL\_ADDONS to ON
- 4. Build SUNDIALS as usual.

## **Chapter 12**

# **KINSOL Constants**

Below we list all input and output constants used by the main solver and linear solver modules, together with their numerical values and a short description of their meaning.

## 12.1 KINSOL input constants

Table 12.1: KINSOL Main Solver Input Constants

Constant Name	Value	Description
KIN_ETACHOICE1	1	Use Eisenstat and Walker Choice 1 for $\eta$ .
KIN_ETACHOICE2	2	Use Eisenstat and Walker Choice 2 for $\eta$ .
KIN_ETACONSTANT	3	Use constant value for $\eta$ .
KIN_NONE	0	Use Newton iteration.
KIN_LINESEARCH	1	Use Newton iteration with linesearch globalization.
KIN_PICARD	2	Use Picard iteration.

Table 12.2: Iterative Linear Solver Constants

Constant Name	Value	Description
SUN_PREC_NONE	0	No preconditioning
SUN_PREC_RIGHT	2	Preconditioning on the right.
SUN_MODIFIED_GS	1	Use modified Gram-Schmidt procedure.
SUN_CLASSICAL_GS	2	Use classical Gram-Schmidt procedure.

Table 12.3: Anderson Acceleration Orthogonalization Method Constants

Constant Name	Value	Description
KIN_ORTH_MGS	0	Use Modified Gram-Schmidt for Anderson acceleration.
KIN_ORTH_ICWY	1	Use Inverse Compact WY Modified Gram-Schmidt for Anderson acceleration.
KIN_ORTH_CGS2	2	Use Classical Gram-Schmidt with Reorthogonalization (CGS-2) for Anderson Acceleration.
KIN_ORTH_DCGS2	3	Use CGS-2 with Delayed Reorthogonalization for Anderson acceleration.

## 12.2 KINSOL output constants

Table 12.4: KINSOL Main Solver Output Constants

Constant Name	Value	Description
KIN_SUCCESS	0	Successful function return.
KIN_INITIAL_GUESS_OK	1	The initial user-supplied guess already satisfies the stopping criterion.
KIN_STEP_LT_STPTOL	2	The stopping tolerance on scaled step length was satisfied.
KIN_WARNING	99	A non-fatal warning. The solver will continue.
KIN_MEM_NULL	-1	The kin_mem argument was NULL.
KIN_ILL_INPUT	-2	One of the function inputs is illegal.
KIN_NO_MALLOC	-3	The KINSOL memory was not allocated by a call to KINMalloc.
KIN_MEM_FAIL	-4	A memory allocation failed.
KIN_LINESEARCH_NONCONV	-5	The linesearch algorithm was unable to find an iterate sufficiently distinct from the current iterate.
KIN_MAXITER_REACHED	-6	The maximum number of nonlinear iterations has been reached.
KIN_MXNEWT_5X_EXCEEDED	-7	Five consecutive steps have been taken that satisfy a scaled step length test.
KIN_LINESEARCH_BCFAIL	-8	The linesearch algorithm was unable to satisfy the $\beta$ -condition for nbcfails iterations.
KIN_LINSOLV_NO_RECOVERY	-9	The user-supplied routine preconditioner slve function failed recoverably, but the preconditioner is already current.
KIN_LINIT_FAIL	-10	The linear solver's initialization function failed.
KIN_LSETUP_FAIL	-11	The linear solver's setup function failed in an unrecoverable manner.
KIN_LSOLVE_FAIL	-12	The linear solver's solve function failed in an unrecoverable manner.
KIN_SYSFUNC_FAIL	-13	The system function failed in an unrecoverable manner.
KIN_FIRST_SYSFUNC_ERR	-14	The system function failed with a recoverable error at the first call.
KIN_REPTD_SYSFUNC_ERR	-15	The system function had repeated recoverable errors.

Table 12.5: KINLS Linear Solver Interface Output Constants

Constant Name	Value	Description
KINLS_SUCCESS	0	Successful function return.
KINLS_MEM_NULL	-1	The kin_mem argument was NULL.
KINLS_LMEM_NULL	-2	The KINLS linear solver has not been initialized.
KINLS_ILL_INPUT	-3	The KINLS solver is not compatible with the current N_Vector
		module, or an input value was illegal.
KINLS_MEM_FAIL	-4	A memory allocation request failed.
KINLS_PMEM_NULL	-5	The preconditioner module has not been initialized.
KINLS_JACFUNC_ERR	-6	The Jacobian function failed
KINLS_SUNMAT_FAIL	-7	An error occurred with the current SUNMatrix module.
KINLS_SUNLS_FAIL	-8	An error occurred with the current SUNLinearSolver module.

# Chapter 13

# **Release History**

Jun 2024		ARKODE	CVODE	CVODES	IDA	IDAS	KINSOL
	7.1.0	6.1.0	7.1.0	7.1.0	7.1.0	6.1.0	7.1.0
Feb 2024	7.0.0	6.0.0	7.0.0	7.0.0	7.0.0	6.0.0	7.0.0
Dec 2023	6.7.0	5.7.0	6.7.0	6.7.0	6.7.0	5.7.0	6.7.0
Nov 2023	6.6.2	5.6.2	6.6.2	6.6.2	6.6.2	5.6.2	6.6.2
Sep 2023	6.6.1	5.6.1	6.6.1	6.6.1	6.6.1	5.6.1	6.6.1
Jul 2023	6.6.0	5.6.0	6.6.0	6.6.0	6.6.0	5.6.0	6.6.0
Mar 2023	6.5.1	5.5.1	6.5.1	6.5.1	6.5.1	5.5.1	6.5.1
Dec 2022	6.5.0	5.5.0	6.5.0	6.5.0	6.5.0	5.5.0	6.5.0
Oct 2022	6.4.1	5.4.1	6.4.1	6.4.1	6.4.1	5.4.1	6.4.1
Oct 2022	6.4.0	5.4.0	6.4.0	6.4.0	6.4.0	5.4.0	6.4.0
Aug 2022	6.3.0	5.3.0	6.3.0	6.3.0	6.3.0	5.3.0	6.3.0
Apr 2022	6.2.0	5.2.0	6.2.0	6.2.0	6.2.0	5.2.0	6.2.0
Feb 2022	6.1.1	5.1.1	6.1.1	6.1.1	6.1.1	5.1.1	6.1.1
Jan 2022	6.1.0	5.1.0	6.1.0	6.1.0	6.1.0	5.1.0	6.1.0
Dec 2021	6.0.0	5.0.0	6.0.0	6.0.0	6.0.0	5.0.0	6.0.0
Sep 2021	5.8.0	4.8.0	5.8.0	5.8.0	5.8.0	4.8.0	5.8.0
Jan 2021	5.7.0	4.7.0	5.7.0	5.7.0	5.7.0	4.7.0	5.7.0
Dec 2020	5.6.1	4.6.1	5.6.1	5.6.1	5.6.1	4.6.1	5.6.1
Dec 2020	5.6.0	4.6.0	5.6.0	5.6.0	5.6.0	4.6.0	5.6.0
Oct 2020	5.5.0	4.5.0	5.5.0	5.5.0	5.5.0	4.5.0	5.5.0
Sep 2020	5.4.0	4.4.0	5.4.0	5.4.0	5.4.0	4.4.0	5.4.0
May 2020	5.3.0	4.3.0	5.3.0	5.3.0	5.3.0	4.3.0	5.3.0
Mar 2020	5.2.0	4.2.0	5.2.0	5.2.0	5.2.0	4.2.0	5.2.0
Jan 2020	5.1.0	4.1.0	5.1.0	5.1.0	5.1.0	4.1.0	5.1.0
Oct 2019	5.0.0	4.0.0	5.0.0	5.0.0	5.0.0	4.0.0	5.0.0
Feb 2019	4.1.0	3.1.0	4.1.0	4.1.0	4.1.0	3.1.0	4.1.0
Jan 2019	4.0.2	3.0.2	4.0.2	4.0.2	4.0.2	3.0.2	4.0.2
Dec 2018	4.0.1	3.0.1	4.0.1	4.0.1	4.0.1	3.0.1	4.0.1
Dec 2018	4.0.0	3.0.0	4.0.0	4.0.0	4.0.0	3.0.0	4.0.0
Oct 2018	3.2.1	2.2.1	3.2.1	3.2.1	3.2.1	2.2.1	3.2.1
Sep 2018	3.2.0	2.2.0	3.2.0	3.2.0	3.2.0	2.2.0	3.2.0
Jul 2018	3.1.2	2.1.2	3.1.2	3.1.2	3.1.2	2.1.2	3.1.2
May 2018	3.1.1	2.1.1	3.1.1	3.1.1	3.1.1	2.1.1	3.1.1
Nov 2017	3.1.0	2.1.0	3.1.0	3.1.0	3.1.0	2.1.0	3.1.0

Table 13.1 – continued from previous page

Date	SUNDIALS	ARKODE	CVODE	CVODES	IDA	IDAS	KINSOL
Sep 2017	3.0.0	2.0.0	3.0.0	3.0.0	3.0.0	2.0.0	3.0.0
Sep 2016	2.7.0	1.1.0	2.9.0	2.9.0	2.9.0	1.3.0	2.9.0
Aug 2015	2.6.2	1.0.2	2.8.2	2.8.2	2.8.2	1.2.2	2.8.2
Mar 2015	2.6.1	1.0.1	2.8.1	2.8.1	2.8.1	1.2.1	2.8.1
Mar 2015	2.6.0	1.0.0	2.8.0	2.8.0	2.8.0	1.2.0	2.8.0
Mar 2012	2.5.0	_	2.7.0	2.7.0	2.7.0	1.1.0	2.7.0
May 2009	2.4.0	_	2.6.0	2.6.0	2.6.0	1.0.0	2.6.0
Nov 2006	2.3.0	_	2.5.0	2.5.0	2.5.0	_	2.5.0
Mar 2006	2.2.0	_	2.4.0	2.4.0	2.4.0	_	2.4.0
May 2005	2.1.1	_	2.3.0	2.3.0	2.3.0	_	2.3.0
Apr 2005	2.1.0	_	2.3.0	2.2.0	2.3.0	_	2.3.0
Mar 2005	2.0.2	_	2.2.2	2.1.2	2.2.2	_	2.2.2
Jan 2005	2.0.1	_	2.2.1	2.1.1	2.2.1	_	2.2.1
Dec 2004	2.0.0	_	2.2.0	2.1.0	2.2.0	_	2.2.0
Jul 2002	1.0.0	_	2.0.0	1.0.0	2.0.0	_	2.0.0
Mar 2002	_	_	$1.0.0^{3}$	_	_	_	_
Feb 1999	-	_	-	_	$1.0.0^{4}$	_	_
Aug 1998	_	_	-	_	_	_	$1.0.0^{5}$
Jul 1997	_	_	$1.0.0^{2}$	_	_	_	_
Sep 1994	_	-	1.0.0 1	-	-	_	-

- 1. CVODE written
- 2. PVODE written
- 3. CVODE and PVODE combined
- 4. IDA written
- 5. KINSOL written

## **Chapter 14**

## Changelog

## 14.1 Changes to SUNDIALS in release 7.1.0

#### **Major Features**

Created shared user interface functions for ARKODE to allow more uniform control over time-stepping algorithms, improved extensibility, and simplified code maintenance. The corresponding stepper-specific user-callable functions are now deprecated and will be removed in a future major release.

Added CMake infrastructure that enables externally maintained addons/plugins to be *optionally* built with SUNDIALS. See Contributing for details.

#### **New Features and Enhancements**

Added support for Kokkos Kernels v4.

Added the following Runge-Kutta Butcher tables

- ARKODE\_FORWARD\_EULER\_1\_1
- ARKODE\_RALSTON\_EULER\_2\_1\_2
- ARKODE\_EXPLICIT\_MIDPOINT\_EULER\_2\_1\_2
- ARKODE\_BACKWARD\_EULER\_1\_1
- ARKODE\_IMPLICIT\_MIDPOINT\_1\_2
- ARKODE\_IMPLICIT\_TRAPEZOIDAL\_2\_2

Added the following MRI coupling tables

- ARKODE\_MRI\_GARK\_FORWARD\_EULER
- ARKODE\_MRI\_GARK\_RALSTON2
- ARKODE\_MRI\_GARK\_RALSTON3
- ARKODE\_MRI\_GARK\_BACKWARD\_EULER
- ARKODE\_MRI\_GARK\_IMPLICIT\_MIDPOINT
- ARKODE\_IMEX\_MRI\_GARK\_EULER
- ARKODE\_IMEX\_MRI\_GARK\_TRAPEZOIDAL
- ARKODE\_IMEX\_MRI\_GARK\_MIDPOINT

Added ARKodeButcherTable\_ERKIDToName() and ARKodeButcherTable\_DIRKIDToName() to convert a Butcher table ID to a string representation.

Added the function ARKodeSetAutonomous() in ARKODE to indicate that the implicit right-hand side function does not explicitly depend on time. When using the trivial predictor, an autonomous problem may reuse implicit function evaluations across stage solves to reduce the total number of function evaluations.

Users may now disable interpolated output in ARKODE by passing ARK\_INTERP\_NONE to ARKodeSetInterpolant-Type(). When interpolation is disabled, rootfinding is not supported, implicit methods must use the trivial predictor (the default option), and interpolation at stop times cannot be used (interpolating at stop times is disabled by default). With interpolation disabled, calling ARKodeEvolve() in ARK\_NORMAL mode will return at or past the requested output time (setting a stop time may still be used to halt the integrator at a specific time). Disabling interpolation will reduce the memory footprint of an integrator by two or more state vectors (depending on the interpolant type and degree) which can be beneficial when interpolation is not needed e.g., when integrating to a final time without output in between or using an explicit fast time scale integrator with an MRI method.

Added "Resize" capability to ARKODE's SPRKStep time-stepping module.

Enabled the Fortran interfaces to build with 32-bit sunindextype.

#### **Bug Fixes**

Updated the CMake variable HIP\_PLATFORM default to amd as the previous default, hcc, is no longer recognized in ROCm 5.7.0 or newer. The new default is also valid in older version of ROCm (at least back to version 4.3.1).

Renamed the DPCPP value for the SUNDIALS\_GINKGO\_BACKENDS CMake option to SYCL to match Ginkgo's updated naming convention.

Changed the CMake version compatibility mode for SUNDIALS to AnyNewerVersion instead of SameMajorVersion. This fixes the issue seen here.

Fixed a CMake bug that caused an MPI linking error for our C++ examples in some instances. Fixes GitHub Issue #464.

Fixed the runtime library installation path for windows systems. This fix changes the default library installation path from CMAKE\_INSTALL\_PREFIX/CMAKE\_INSTALL\_BINDIR.

Fixed conflicting .1ib files between shared and static libs when using MSVC on Windows

Fixed invalid SUNDIALS\_EXPORT generated macro when building both shared and static libs.

Fixed a bug in some Fortran examples where c\_null\_ptr was passed as an argument to a function pointer instead of c\_null\_funptr. This caused compilation issues with the Cray Fortran compiler.

Fixed a bug in the HIP execution policies where WARP\_SIZE would not be set with ROCm 6.0.0 or newer.

Fixed a bug that caused error messages to be cut off in some cases. Fixes GitHub Issue #461.

Fixed a memory leak when an error handler was added to a SUNContext. Fixes GitHub Issue #466.

Fixed a bug where MRIStepEvolve() would not handle a recoverable error produced from evolving the inner stepper.

Added missing SetRootDirection and SetNoInactiveRootWarn functions to ARKODE's SPRKStep time-stepping module.

Fixed a bug in ARKodeSPRKTable\_Create() where the coefficient arrays were not allocated.

Fix bug on LLP64 platforms (like Windows 64-bit) where KLU\_INDEXTYPE could be 32 bits wide even if SUNDIALS\_-INT64\_T is defined.

Check if size of SuiteSparse\_long is 8 if the size of sunindextype is 8 when using KLU.

Fixed several build errors with the Fortran interfaces on Windows systems.

#### **Deprecation Notices**

Numerous ARKODE stepper-specific functions are now deprecated in favor of ARKODE-wide functions.

Deprecated the *ARKStepSetOptimalParams* function. Since this function does not have an ARKODE-wide equivalent, instructions have been added to the user guide for how to retain the current functionality using other user-callable functions.

The unsupported implementations of N\_VGetArrayPointer and N\_VSetArrayPointer for the *hypre* and PETSc vectors are now deprecated. Users should access the underlying wrapped external library vector objects instead with N\_VGetVector\_ParHyp and N\_VGetVector\_Petsc, respectively.

## 14.2 Changes to SUNDIALS in release 7.0.0

#### **Major Feature**

SUNDIALS now has more robust and uniform error handling. Non-release builds will be built with additional error checking by default. See §4.3 for details.

#### **Breaking Changes**

Minimum C Standard

SUNDIALS now requires using a compiler that supports a subset of the C99 standard. Note with the Microsoft C/C++ compiler the subset of C99 features utilized by SUNDIALS are available starting with Visual Studio 2015.

Minimum CMake Version

CMake 3.18 or newer is now required when building SUNDIALS.

Deprecated Types and Functions Removed

The previously deprecated types realtype and booleantype were removed from sundials\_types.h and replaced with *sunrealtype* and *sunbooleantype*. The deprecated names for these types can be used by including the header file sundials\_types\_deprecated.h but will be fully removed in the next major release. Functions, types and header files that were previously deprecated have also been removed.

Error Handling Changes

With the addition of the new error handling capability, the \*SetErrHandlerFn and \*SetErrFile functions in CVODE(S), IDA(S), ARKODE, and KINSOL have been removed. Users of these functions can use the functions SUNContext\_PushErrHandler(), and SUNLogger\_SetErrorFilename() instead. For further details see Sections §4.3 and §4.4.

In addition the following names/symbols were replaced by SUN\_ERR\_\* codes:

Removed	Replaced with SUNErrCode
SUNLS_SUCCESS	SUN_SUCCESS
SUNLS_UNRECOV_FAILURE	no replacement (value was unused)
SUNLS_MEM_NULL	SUN_ERR_ARG_CORRUPT
SUNLS_ILL_INPUT	SUN_ERR_ARG_*
SUNLS_MEM_FAIL	SUN_ERR_MEM_FAIL
SUNLS_PACKAGE_FAIL_UNREC	SUN_ERR_EXT_FAIL
SUNLS_VECTOROP_ERR	SUN_ERR_OP_FAIL
SUN_NLS_SUCCESS	SUN_SUCCESS
SUN_NLS_MEM_NULL	SUN_ERR_ARG_CORRUPT
SUN_NLS_MEM_FAIL	SUN_ERR_MEM_FAIL
SUN_NLS_ILL_INPUT	SUN_ERR_ARG_*
SUN_NLS_VECTOROP_ERR	SUN_ERR_OP_FAIL
SUN_NLS_EXT_FAIL	SUN_ERR_EXT_FAIL
SUNMAT_SUCCESS	SUN_SUCCESS
SUNMAT_ILL_INPUT	SUN_ERR_ARG_*
SUNMAT_MEM_FAIL	SUN_ERR_MEM_FAIL
SUNMAT_OPERATION_FAIL	SUN_ERR_OP_FAIL
SUNMAT_MATVEC_SETUP_REQUIRED	SUN_ERR_OP_FAIL

The following functions have had their signature updated to ensure they can leverage the new SUNDIALS error handling capabilities.

- From sundials\_futils.h
  - SUNDIALSFileOpen()
  - SUNDIALSFileClose()
- From sundials\_memory.h
  - SUNMemoryNewEmpty()
  - SUNMemoryHelper\_Alias()
  - SUNMemoryHelper\_Wrap()
- From sundials\_nvector.h
  - N\_VNewVectorArray()

#### SUNComm Type Added

We have replaced the use of a type-erased (i.e., void\*) pointer to a communicator in place of MPI\_Comm throughout the SUNDIALS API with a SUNComm, which is just a typedef to an int in builds without MPI and a typedef to a MPI\_Comm in builds with MPI. As a result:

- When MPI is enabled, all SUNDIALS libraries will include MPI symbols and applications will need to include the path for MPI headers and link against the corresponding MPI library.
- All users will need to update their codes because the call to <code>SUNContext\_Create()</code> now takes a <code>SUNComm</code> instead of type-erased pointer to a communicator. For non-MPI codes, pass <code>SUN\_COMM\_NULL</code> to the <code>comm</code> argument instead of <code>NULL</code>. For MPI codes, pass the <code>MPI\_Comm</code> directly.
- The same change must be made for calls to SUNLogger\_Create() or SUNProfiler\_Create().
- Some users will need to update their calls to *N\_VGetCommunicator()*, and update any custom *N\_Vector* implementations that provide *N\_VGetCommunicator()*, since it now returns a *SUNComm*.

The change away from type-erased pointers for SUNComm fixes problems like the one described in GitHub Issue #275.

The SUNLogger is now always MPI-aware if MPI is enabled in SUNDIALS and the SUNDIALS\_LOGGING\_ENABLE\_-MPI CMake option and macro definition were removed accordingly.

## SUNDIALS Core Library

Users now need to link to sundials\_core in addition to the libraries already linked to. This will be picked up automatically in projects that use the SUNDIALS CMake target. The library sundials\_generic has been superseded by sundials\_core and is no longer available. This fixes some duplicate symbol errors on Windows when linking to multiple SUNDIALS libraries.

Fortran Interface Modules Streamlined

We have streamlined the Fortran modules that need to be included by users by combining the SUNDIALS core into one Fortran module, fsundials\_core\_mod. Modules for implementations of the core APIs still exist (e.g., for the Dense linear solver there is fsunlinsol\_dense\_mod) as do the modules for the SUNDIALS packages (e.g., fcvode\_mod). The following modules are the ones that have been consolidated into fsundials\_core\_mod:

```
fsundials_adaptcontroller_mod
fsundials_context_mod
fsundials_futils_mod
fsundials_linearsolver_mod
fsundials_logger_mod
fsundials_matrix_mod
fsundials_nonlinearsolver_mod
fsundials_nvector_mod
fsundials_profiler_mod
fsundials_types_mod
```

# **Minor Changes**

The CMAKE\_BUILD\_TYPE defaults to RelWithDebInfo mode now i.e., SUNDIALS will be built with optimizations and debugging symbols enabled by default. Previously the build type was unset by default so no optimization or debugging flags were set.

The advanced CMake options to override the inferred LAPACK name-mangling scheme have been updated from SUN-DIALS\_F77\_FUNC\_CASE and SUNDIALS\_F77\_FUNC\_UNDERSCORES to SUNDIALS\_LAPACK\_CASE and SUNDIALS\_LAPACK\_UNDERSCORES, respectively.

As a subset of C99 is now required the CMake option USE\_GENERIC\_MATH as been removed.

The C++ convenience classes (e.g., sundials::Context) have been moved to from SUNDIALS .h headers to corresponding .hpp headers (e.g.,  $sundials/sundials\_context.hpp$ ) so C++ codes do not need to compile with C++14 support when using the C API.

Converted most previous Fortran 77 and 90 examples to use SUNDIALS' Fortran 2003 interface.

## **Bug Fixes**

Fixed GitHub Issue #329 so that C++20 aggregate initialization can be used.

Fixed integer overflow in the internal SUNDIALS hashmap. This resolves GitHub Issues #409 and #249.

## **Deprecation Notice**

The functions in sundials\_math.h will be deprecated in the next release.

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```
sunbooleantype SUNRCompareTol(sunrealtype a, sunrealtype b, sunrealtype tol);
sunrealtype SUNStrToReal(const char* str);
```

Additionally, the following header files (and everything in them) will be deprecated – users who rely on these are recommended to transition to the corresponding *SUNMatrix* and *SUNLinearSolver* modules:

```
sundials_direct.h
sundials_dense.h
sundials_band.h
```

# 14.3 Changes to SUNDIALS in release 6.7.0

## **Major Feature**

Added the SUNAdaptController base class, ported ARKODE's internal implementations of time step controllers to implementations of this class, and updated ARKODE to use these objects instead of its own implementations. Added ARKStepSetAdaptController() and ERKStepSetAdaptController() routines so that users can modify controller parameters, or even provide custom implementations.

### **New Features**

Improved the computational complexity of the sparse matrix ScaleAddI function from  $\mathcal{O}(M*N)$  to  $\mathcal{O}(NNZ)$ .

Added Fortran support for the LAPACK dense linear solver implementation.

Added the routines ARKStepSetAdaptivityAdjustment() and ERKStepSetAdaptivityAdjustment(), that allow users to adjust the value for the method order supplied to the temporal adaptivity controllers. The ARKODE default for this adjustment has been -1 since its initial release, but for some applications a value of 0 is more appropriate. Users who notice that their simulations encounter a large number of temporal error test failures may want to experiment with adjusting this value.

Added the third order ERK method ARKODE\_SHU\_OSHER\_3\_2\_3, the fourth order ERK method ARKODE\_SOFRONIOU\_-SPALETTA\_5\_3\_4, the sixth order ERK method ARKODE\_VERNER\_9\_5\_6, the seventh order ERK method ARKODE\_-VERNER\_10\_6\_7, the eighth order ERK method ARKODE\_VERNER\_13\_7\_8, and the ninth order ERK method ARKODE\_-VERNER\_16\_8\_9.

ARKStep, ERKStep, MRIStep, and SPRKStep were updated to remove a potentially unnecessary right-hand side evaluation at the end of an integration. ARKStep was additionally updated to remove extra right-hand side evaluations when using an explicit method or an implicit method with an explicit first stage.

The MRIStepInnerStepper class in MRIStep was updated to make supplying an MRIStepInnerFullRhsFn optional.

## **Bug Fixes**

Changed the SUNProfiler so that it does not rely on MPI\_WTime in any case. This fixes GitHub Issue #312.

Fixed scaling bug in SUNMatScaleAddI\_Sparse for non-square matrices.

Fixed a regression introduced by the stop time bug fix in v6.6.1 where ARKODE, CVODE, CVODES, IDA, and IDAS would return at the stop time rather than the requested output time if the stop time was reached in the same step in which the output time was passed.

Fixed a bug in ERKStep where methods with  $c_s = 1$  but  $a_{s,j} \neq b_j$  were incorrectly treated as having the first same as last (FSAL) property.

Fixed a bug in ARKODE where ARKStepSetInterpolateStopTime() would return an interpolated solution at the stop time in some cases when interpolation was disabled.

Fixed a bug in ARKStepSetTableNum() wherein it did not recognize ARKODE\_ARK2\_ERK\_3\_1\_2 and ARKODE\_-ARK2\_DIRK\_3\_1\_2 as a valid additive Runge-Kutta Butcher table pair.

Fixed a bug in MRIStepCoupling\_Write() where explicit coupling tables were not written to the output file pointer.

Fixed missing soversions in some SUNLinearSolver and SUNNonlinearSolver CMake targets.

Renamed some internal types in CVODES and IDAS to allow both packages to be built together in the same binary.

# 14.4 Changes to SUNDIALS in release 6.6.2

Fixed the build system support for MAGMA when using a NVIDIA HPC SDK installation of CUDA and fixed the targets used for rocBLAS and rocSPARSE.

# 14.5 Changes to SUNDIALS in release 6.6.1

### **New Features**

Updated the Trilinos Tpetra *N\_Vector* interface to support Trilinos 14.

### **Bug Fixes**

Fixed a memory leak when destroying a CUDA, HIP, SYCL, or system SUNMemoryHelper object.

Fixed a bug in ARKODE, CVODES, IDA, and IDAS where the stop time may not be cleared when using normal mode if the requested output time is the same as the stop time. Additionally, with ARKODE, CVODE, and CVODES this fix removes an unnecessary interpolation of the solution at the stop time that could occur in this case.

# 14.6 Changes to SUNDIALS in release 6.6.0

### **Major Features**

A new time-stepping module, SPRKStep, was added to ARKODE. This time-stepper provides explicit symplectic partitioned Runge-Kutta methods up to order 10 for separable Hamiltonian systems.

Added support for relaxation Runge-Kutta methods in ERKStep and ARKStep, see Relaxation Methods, Relaxation Methods, and Relaxation Methods for more information.

### **New Features**

Updated the default ARKODE, CVODE, and CVODES behavior when returning the solution when the internal time has reached a user-specified stop time. Previously, the output solution was interpolated to the value of tstop; the default is now to copy the internal solution vector. Users who wish to revert to interpolation may call a new routine CVodeSetInterpolateStopTime(), ARKStepSetInterpolateStopTime(), ERKStepSetInterpolateStopTime().

Added the second order IMEX method from [29] as the default second order IMEX method in ARKStep. The explicit table is given by ARKODE\_ARK2\_ERK\_3\_1\_2 (see ARK2-ERK-3-1-2) and the implicit table by ARKODE\_ARK2\_DIRK\_-3\_1\_2 (see ARK2-DIRK-3-1-2).

Updated the F2003 utility routines SUNDIALSFileOpen() and SUNDIALSFileClose() to support user specification of stdout and stderr strings for the output file names.

## **Bug Fixes**

A potential bug was fixed when using inequality constraint handling and calling ARKStepGetEstLocalErrors() or ERKStepGetEstLocalErrors() after a failed step in which an inequality constraint violation occurred. In this case, the values returned by ARKStepGetEstLocalErrors() or ERKStepGetEstLocalErrors() may have been invalid.

# 14.7 Changes to SUNDIALS in release 6.5.1

### **New Features**

Added the following functions to disable a previously set stop time:

- ARKStepClearStopTime()
- ERKStepClearStopTime()
- MRIStepClearStopTime()
- CVodeClearStopTime()
- IDAClearStopTime()

The default interpolant in ARKODE when using a first order method has been updated to a linear interpolant to ensure values obtained by the integrator are returned at the ends of the time interval. To restore the previous behavior of using a constant interpolant call ARKStepSetInterpolantDegree(), ERKStepSetInterpolantDegree(), or MRIStepSetInterpolantDegree() and set the interpolant degree to zero before evolving the problem.

### **Bug Fixes**

Fixed build errors when using SuperLU\_DIST with ROCM enabled to target AMD GPUs.

Fixed compilation errors in some SYCL examples when using the icx compiler.

# 14.8 Changes to SUNDIALS in release 6.5.0

### **New Features**

A new capability to keep track of memory allocations made through the <code>SUNMemoryHelper</code> classes has been added. Memory allocation stats can be accessed through the <code>SUNMemoryHelper\_GetAllocStats()</code> function. See §9.1 for more details.

Added the following functions to assist in debugging simulations utilizing matrix-based linear solvers:

- ARKStepGetJac()
- ARKStepGetJacTime()
- ARKStepGetJacNumSteps()
- MRIStepGetJac()
- MRIStepGetJacTime()
- MRIStepGetJacNumSteps()
- CVodeGetJac()
- CVodeGetJacTime()
- CVodeGetJacNumSteps()
- IDAGetJac()
- IDAGetJacCj()

- IDAGetJacTime()
- IDAGetJacNumSteps()
- KINGetJac()
- KINGetJacNumIters()

Added support for CUDA 12.

Added support for the SYCL backend with RAJA 2022.x.y.

### **Bug Fixes**

Fixed an underflow bug during root finding in ARKODE, CVODE, CVODES, IDA and IDAS. This fixes GitHub Issue #57

Fixed an issue with finding oneMKL when using the icpx compiler with the -fsycl flag as the C++ compiler instead of dpcpp.

Fixed the shape of the arrays returned by the Fortran interfaces to N\_VGetArrayPointer(), SUNDenseMatrix\_Data(), SUNSparseMatrix\_Data(), SUNSparseMatrix\_IndexValues(), and SUNSparseMatrix\_IndexPointers(). Compiling and running code that uses the SUNDIALS Fortran interfaces with bounds checking will now work.

Fixed an implicit conversion error in the Butcher table for ESDIRK5(4)7L[2]SA2.

# 14.9 Changes to SUNDIALS in release 6.4.1

Fixed a bug with the Kokkos interfaces that would arise when using clang.

Fixed a compilation error with the Intel one API 2022.2 Fortran compiler in the Fortran 2003 interface test for the serial  $N_{-}Vector$ .

Fixed a bug in the LAPACK band and dense linear solvers which would cause the tests to fail on some platforms.

# 14.10 Changes to SUNDIALS in release 6.4.0

### **New Requirements**

CMake 3.18.0 or newer is now required for CUDA support.

A C++14 compliant compiler is now required for C++ based features and examples e.g., CUDA, HIP, RAJA, Trilinos, SuperLU\_DIST, MAGMA, Ginkgo, and Kokkos.

## **Major Features**

Added support for the Ginkgo linear algebra library. This support includes new SUNDIALS matrix and linear solver implementations, see the sections §7.10 and §8.18.

Added new SUNDIALS vector, dense matrix, and dense linear solver implementations utilizing the Kokkos Ecosystem for performance portability, see sections §6.14, §7.11, and §8.19 for more information.

### **New Features**

Added support for GPU enabled SuperLU\_DIST and SuperLU\_DIST v8.x.x. Removed support for SuperLU\_DIST v6.x.x or older. Fix mismatched definition and declaration bug in SuperLU\_DIST matrix constructor.

Added the functions following functions to load a Butcher table from a string:

• ARKStepSetTableName()

- ERKStepSetTableName()
- MRIStepCoupling\_LoadTableByName()
- ARKodeButcherTable\_LoadDIRKByName()
- ARKodeButcherTable\_LoadERKByName()

### **Bug Fixes**

Fixed a bug in the CUDA and HIP vectors where *N\_VMaxNorm()* would return the minimum positive floating-point value for the zero vector.

Fixed memory leaks/out of bounds memory accesses in the ARKODE MRIStep module that could occur when attaching a coupling table after reinitialization with a different number of stages than originally selected.

Fixed a memory leak where the projection memory would not be deallocated when calling CVodeFree().

# 14.11 Changes to SUNDIALS in release 6.3.0

### **New Features**

Added the following functions to retrieve the user data pointer provided with SetUserData functions:

- ARKStepGetUserData()
- ERKStepGetUserData()
- MRIStepGetUserData()
- CVodeGetUserData()
- IDAGetUserData()
- KINGetUserData()

Added a variety of embedded DIRK methods from [37] and [38].

Updated MRIStepReset() to call the corresponding MRIStepInnerResetFn with the same tR and yR arguments for the MRIStepInnerStepper object that is used to evolve the MRI "fast" time scale subproblems.

Added a new example (examples/cvode/serial/cvRocket\_dns.c) which demonstrates using CVODE with a discontinuous right-hand-side function and rootfinding.

### **Bug Fixes**

Fixed a bug in ERKStepReset(), ERKStepReInit(), ARKStepReset(), ARKStepReInit(), MRIStepReset(), and MRIStepReInit() where a previously-set value of tstop (from a call to ERKStepSetStopTime(), ARK-StepSetStopTime(), or MRIStepSetStopTime(), respectively) would not be cleared.

Fixed the unituitive behavior of the USE\_GENERIC\_MATH CMake option which caused the double precision math functions to be used regardless of the value of *SUNDIALS\_PRECISION*. Now, SUNDIALS will use precision appropriate math functions when they are available and the user may provide the math library to link to via the advanced CMake option *SUNDIALS\_MATH\_LIBRARY*.

Changed SUNDIALS\_LOGGING\_ENABLE\_MPI CMake option default to be OFF. This fixes GitHub Issue #177.

# 14.12 Changes to SUNDIALS in release 6.2.0

## **Major Features**

Added the *SUNLogger* API which provides a SUNDIALS-wide mechanism for logging of errors, warnings, informational output, and debugging output.

Added support to CVODES for integrating IVPs with constraints using BDF methods and projecting the solution onto the constraint manifold with a user defined projection function. This implementation is accompanied by additions to the CVODES user documentation and examples.

#### **New Features**

Added the function SUNProfiler\_Reset() to reset the region timings and counters to zero.

Added the following functions to output all of the integrator, nonlinear solver, linear solver, and other statistics in one call:

- ARKStepPrintAllStats()
- ERKStepPrintAllStats()
- MRIStepPrintAllStats()
- CVodePrintAllStats()
- IDAPrintAllStats()
- KINPrintAllStats()

The file scripts/sundials\_csv.py contains functions for parsing the comma-separated value (CSV) output files when using the CSV output format.

Added functions to CVODE, CVODES, IDA, and IDAS to change the default step size adaptivity parameters. For more information see the documentation for:

- CVodeSetEtaFixedStepBounds()
- CVodeSetEtaMaxFirstStep()
- CVodeSetEtaMaxEarlyStep()
- CVodeSetNumStepsEtaMaxEarlyStep()
- CVodeSetEtaMax()
- CVodeSetEtaMin()
- CVodeSetEtaMinErrFail()
- CVodeSetEtaMaxErrFail()
- CVodeSetNumFailsEtaMaxErrFail()
- CVodeSetEtaConvFail()
- IDASetEtaFixedStepBounds()
- IDASetEtaMax()
- IDASetEtaMin()
- IDASetEtaLow()
- IDASetEtaMinErrFail()
- IDASetEtaConvFail()

Added the functions ARKStepSetDeduceImplicitRhs() and MRIStepSetDeduceImplicitRhs() to optionally remove an evaluation of the implicit right-hand side function after nonlinear solves. See Nonlinear solver methods, for considerations on using this optimization.

Added the function MRIStepSetOrder() to select the default MRI method of a given order.

Added the functions CVodeSetDeltaGammaMaxLSetup() and CVodeSetDeltaGammaMaxBadJac() in CVODE and CVODES to adjust the  $\gamma$  change thresholds to require a linear solver setup or Jacobian/precondition update, respectively.

Added the function IDASetDeltaCjLSetup() in IDA and IDAS to adjust the parameter that determines when a change in  $c_i$  requires calling the linear solver setup function.

Added the function IDASetMinStep() to set a minimum step size.

## **Bug Fixes**

Fixed the SUNContext convenience class for C++ users to disallow copy construction and allow move construction.

The behavior of N\_VSetKernelExecPolicy\_Sycl() has been updated to be consistent with the CUDA and HIP vectors. The input execution policies are now cloned and may be freed after calling N\_VSetKernelExecPolicy\_-Sycl(). Additionally, NULL inputs are now allowed and, if provided, will reset the vector execution policies to the defaults.

A memory leak in the SYCL vector was fixed where the execution policies were not freed when the vector was destroyed.

The include guard in nvector\_mpimanyvector.h has been corrected to enable using both the Many Vector and MPI-Many Vector vector implementations in the same simulation.

A bug was fixed in the ARKODE, CVODE(S), and IDA(S) functions to retrieve the number of nonlinear solver failures. The failure count returned was the number of failed *steps* due to a nonlinear solver failure i.e., if a nonlinear solve failed with a stale Jacobian or preconditioner but succeeded after updating the Jacobian or preconditioner, the initial failure was not included in the nonlinear solver failure count. The following functions have been updated to return the total number of nonlinear solver failures:

- ARKStepGetNumNonlinSolvConvFails()
- ARKStepGetNonlinSolvStats()
- MRIStepGetNumNonlinSolvConvFails()
- MRIStepGetNonlinSolvStats()
- CVodeGetNumNonlinSolvConvFails()
- CVodeGetNonlinSolvStats()
- CVodeGetSensNumNonlinSolvConvFails()
- CVodeGetSensNonlinSolvStats()
- CVodeGetStgrSensNumNonlinSolvConvFails()
- CVodeGetStgrSensNonlinSolvStats()
- IDAGetNumNonlinSolvConvFails()
- IDAGetNonlinSolvStats()
- IDAGetSensNumNonlinSolvConvFails()
- IDAGetSensNonlinSolvStats()

As a result of this change users may see an increase in the number of failures reported from the above functions. The following functions have been added to retrieve the number of failed steps due to a nonlinear solver failure i.e., the counts previously returned by the above functions:

• ARKStepGetNumStepSolveFails()

- MRIStepGetNumStepSolveFails()
- CVodeGetNumStepSolveFails()
- CVodeGetNumStepSensSolveFails()
- CVodeGetNumStepStgrSensSolveFails()
- IDAGetNumStepSolveFails()
- IDAGetNumStepSensSolveFails()

Changed exported SUNDIALS PETSc CMake targets to be INTERFACE IMPORTED instead of UNKNOWN IMPORTED.

### **Deprecation Notice**

Deprecated the following functions, it is recommended to use the SUNLogger API instead.

- ARKStepSetDiagnostics
- ERKStepSetDiagnostics
- MRIStepSetDiagnostics
- KINSetInfoFile
- SUNNonlinSolSetPrintLevel\_Newton
- SUNNonlinSolSetInfoFile\_Newton
- SUNNonlinSolSetPrintLevel FixedPoint
- SUNNonlinSolSetInfoFile\_FixedPoint
- SUNLinSolSetInfoFile\_PCG
- SUNLinSolSetPrintLevel\_PCG
- SUNLinSolSetInfoFile\_SPGMR
- SUNLinSolSetPrintLevel\_SPGMR
- SUNLinSolSetInfoFile\_SPFGMR
- SUNLinSolSetPrintLevel\_SPFGMR
- SUNLinSolSetInfoFile\_SPTFQM
- SUNLinSolSetPrintLevel\_SPTFQMR
- SUNLinSolSetInfoFile\_SPBCGS
- SUNLinSolSetPrintLevel\_SPBCGS

The SUNLinSolSetInfoFile\_\* and SUNNonlinSolSetInfoFile\_\* family of functions are now enabled by setting the CMake option SUNDIALS\_LOGGING\_LEVEL to a value >= 3.

# 14.13 Changes to SUNDIALS in release 6.1.1

### **New Feature**

Added new Fortran example program, examples/arkode/F2003\_serial/ark\_kpr\_mri\_f2003.f90 demonstrating MRI capabilities.

### **Bug Fixes**

Fixed exported SUNDIALSConfig.cmake.

Fixed Fortran interface to MRIStepInnerStepper and MRIStepCoupling structures and functions.

# 14.14 Changes to SUNDIALS in release 6.1.0

#### **New Features**

Added new reduction implementations for the CUDA and HIP vectors that use shared memory (local data storage) instead of atomics. These new implementations are recommended when the target hardware does not provide atomic support for the floating point precision that SUNDIALS is being built with. The HIP vector uses these by default, but the <code>N\_VSetKernelExecPolicy\_Cuda()</code> and <code>N\_VSetKernelExecPolicy\_Hip()</code> functions can be used to choose between different reduction implementations.

SUNDIALS::lib> targets with no static/shared suffix have been added for use within the build directory (this mirrors the targets exported on installation).

CMAKE\_C\_STANDARD is now set to 99 by default.

#### **Bug Fixes**

Fixed exported SUNDIALSConfig. cmake when profiling is enabled without Caliper.

Fixed sundials\_export.h include in sundials\_config.h.

Fixed memory leaks in the SuperLU\_MT linear solver interface.

# 14.15 Changes to SUNDIALS in release 6.0.0

# **Breaking Changes**

SUNContext Object Added

SUNDIALS v6.0.0 introduces a new *SUNContext* object on which all other SUNDIALS objects depend. As such, the constructors for all SUNDIALS packages, vectors, matrices, linear solvers, nonlinear solvers, and memory helpers have been updated to accept a context as the last input. Users upgrading to SUNDIALS v6.0.0 will need to call *SUNContext\_Create()* to create a context object with before calling any other SUNDIALS library function, and then provide this object to other SUNDIALS constructors. The context object has been introduced to allow SUNDIALS to provide new features, such as the profiling/instrumentation also introduced in this release, while maintaining thread-safety. See the §4.2 for more details.

The scripts/upgrade-to-sundials-6-from-5.sh has been provided with this release (and obtainable from the GitHub release page) to help ease the transition to SUNDIALS v6.0.0. The script will add a SUNCTX\_PLACEHOLDER argument to all of the calls to SUNDIALS constructors that now require a SUNContext object. It can also update deprecated SUNDIALS constants/types to the new names. It can be run like this:

./upgrade-to-sundials-6-from-5.sh <files to update>

## Updated SUNMemoryHelper Function Signatures

The SUNMemoryHelper functions SUNMemoryHelper\_Alloc(), SUNMemoryHelper\_Dealloc(), and SUNMemory-Helper\_Copy() have been updated to accept an opaque handle as the last input. At a minimum, user-defined SUN-MemoryHelper implementations will need to update these functions to accept the additional argument. Typically, this handle is the execution stream (e.g., a CUDA/HIP stream or SYCL queue) for the operation. The CUDA, HIP, and SYCL implementations have been updated accordingly. Additionally, the constructor SUNMemoryHelper\_Sycl() has been updated to remove the SYCL queue as an input.

### Deprecated Functions Removed

The previously deprecated constructor N\_VMakeWithManagedAllocator\_Cuda and the function N\_VSetCudaS-tream\_Cuda have been removed and replaced with N\_VNewWithMemHelp\_Cuda() and N\_VSetKernelExecPolicy\_Cuda() respectively.

The previously deprecated macros PVEC\_REAL\_MPI\_TYPE and PVEC\_INTEGER\_MPI\_TYPE have been removed and replaced with MPI\_SUNREALTYPE and MPI\_SUNINDEXTYPE respectively.

The following previously deprecated *SUNLinearSolver* functions have been removed:

Removed	Replacement
SUNBandLinearSolver	SUNLinSol_Band()
SUNDenseLinearSolver	SUNLinSol_Dense()
SUNKLU	SUNLinSol_KLU()
SUNKLUReInit	<pre>SUNLinSol_KLUReInit()</pre>
SUNKLUSetOrdering	<pre>SUNLinSol_KLUSetOrdering()</pre>
SUNLapackBand	<pre>SUNLinSol_LapackBand()</pre>
SUNLapackDense	<pre>SUNLinSol_LapackDense()</pre>
SUNPCG	SUNLinSol_PCG()
SUNPCGSetPrecType	<pre>SUNLinSol_PCGSetPrecType()</pre>
SUNPCGSetMaxl	<pre>SUNLinSol_PCGSetMaxl()</pre>
SUNSPBCGS	SUNLinSol_SPBCGS()
SUNSPBCGSSetPrecType	<pre>SUNLinSol_SPBCGSSetPrecType()</pre>
SUNSPBCGSSetMaxl	<pre>SUNLinSol_SPBCGSSetMax1()</pre>
SUNSPFGMR	SUNLinSol_SPFGMR()
SUNSPFGMRSetPrecType	<pre>SUNLinSol_SPFGMRSetPrecType()</pre>
SUNSPFGMRSetGSType	<pre>SUNLinSol_SPFGMRSetGSType()</pre>
SUNSPFGMRSetMaxRestarts	<pre>SUNLinSol_SPFGMRSetMaxRestarts()</pre>
SUNSPGMR	SUNLinSol_SPGMR()
SUNSPGMRSetPrecType	<pre>SUNLinSol_SPGMRSetPrecType()</pre>
SUNSPGMRSetGSType	<pre>SUNLinSol_SPGMRSetGSType()</pre>
SUNSPGMRSetMaxRestarts	SUNLinSol_SPGMRSetMaxRestarts()
SUNSPTFQMR	SUNLinSol_SPTFQMR()
SUNSPTFQMRSetPrecType	<pre>SUNLinSol_SPTFQMRSetPrecType()</pre>
SUNSPTFQMRSetMaxl	<pre>SUNLinSol_SPTFQMRSetMaxl()</pre>
SUNSuperLUMT	SUNLinSol_SuperLUMT()
SUNSuperLUMTSetOrdering	<pre>SUNLinSol_SuperLUMTSetOrdering()</pre>

The deprecated functions MRIStepGetCurrentButcherTables and MRIStepWriteButcher and the utility functions MRIStepSetTable and MRIStepSetTableNum have been removed. Users wishing to create an MRI-GARK method from a Butcher table should use MRIStepCoupling\_MIStoMRI() to create the corresponding MRI coupling table and attach it with MRIStepSetCoupling().

The previously deprecated functions ARKStepSetMaxStepsBetweenLSet and ARKStepSetMaxStepsBetweenJac have been removed and replaced with ARKStepSetLSetupFrequency() and ARKStepSetJacEvalFrequency()

respectively.

The previously deprecated function CVodeSetMaxStepsBetweenJac has been removed and replaced with CVode-SetJacEvalFrequency().

The ARKODE, CVODE, IDA, and KINSOL Fortran 77 interfaces has been removed. See §4.7 and the F2003 example programs for more details using the SUNDIALS Fortran 2003 module interfaces.

Namespace Changes

The CUDA, HIP, and SYCL execution policies have been moved from the sundials namespace to the sundials::cuda, sundials::hip, and sundials::sycl namespaces respectively. Accordingly, the prefixes "Cuda", "Hip", and "Sycl" have been removed from the execution policy classes and methods.

The Sundials namespace used by the Trilinos Tpetra *N\_Vector* implementation has been replaced with the sundials::trilinos::nvector\_tpetra namespace.

## **Major Features**

**Profiling Capability** 

A capability to profile/instrument SUNDIALS library code has been added. This can be enabled with the CMake option *SUNDIALS\_BUILD\_WITH\_PROFILING*. A built-in profiler will be used by default, but the Caliper library can also be used instead with the CMake option *ENABLE\_CALIPER*. See the documentation section on profiling for more details.

Warning: Profiling will impact performance, and should be enabled judiciously.

## IMEX MRI Methods and MRIStepInnerStepper Object

The MRIStep module has been extended to support implicit-explicit (ImEx) multirate infinitesimal generalized additive Runge-Kutta (MRI-GARK) methods. As such, MRIStepCreate() has been updated to include arguments for the slow explicit and slow implicit ODE right-hand side functions. MRIStepCreate() has also been updated to require attaching an MRIStepInnerStepper for evolving the fast time scale. MRIStepReInit() has been similarly updated to take explicit and implicit right-hand side functions as input. Codes using explicit or implicit MRI methods will need to update MRIStepCreate() and MRIStepReInit() calls to pass NULL for either the explicit or implicit right-hand side function as appropriate. If ARKStep is used as the fast time scale integrator, codes will need to call ARKStepCreateMRIStepInnerStepper() to wrap the ARKStep memory as an MRIStepInnerStepper object. Additionally, MRIStepGetNumRhsEvals() has been updated to return the number of slow implicit and explicit function evaluations. The coupling table, MRIStepCoupling, and the functions MRIStepCoupling\_Alloc() and MRIStepCoupling\_Create() have also been updated to support IMEX-MRI-GARK methods.

### **New Features**

Two new optional vector operations,  $N\_VDotProdMultiLocal()$  and  $N\_VDotProdMultiAllReduce()$ , have been added to support low-synchronization methods for Anderson acceleration.

The implementation of solve-decoupled implicit MRI-GARK methods has been updated to remove extraneous slow implicit function calls and reduce the memory requirements.

Added a new function CVodeGetLinSolveStats() to get the CVODES linear solver statistics as a group.

Added a new function, CVodeSetMonitorFn(), that takes a user-function to be called by CVODES after every nst successfully completed time-steps. This is intended to provide a way of monitoring the CVODES statistics throughout the simulation.

New orthogonalization methods were added for use within the KINSOL Anderson acceleration routine. See *Anderson Acceleration QR Factorization* and *KINSetOrthAA()* for more details.

## **Deprecation Notice**

The serial, PThreads, PETSc, *hypre*, Parallel, OpenMP\_DEV, and OpenMP vector functions N\_VCloneVectorArray\_\* and N\_VDestroyVectorArray\_\* have been deprecated. The generic N\_VCloneVectorArray() and N\_VDestroyVectorArray() functions should be used instead.

Many constants, types, and functions have been renamed so that they are properly namespaced. The old names have been deprecated and will be removed in SUNDIALS v7.0.0.

The following constants, macros, and typedefs are now deprecated:

Deprecated Name	New Name
realtype	sunrealtype
booleantype	sunbooleantype
RCONST	SUN_RCONST
BIG_REAL	SUN_BIG_REAL
SMALL_REAL	SUN_SMALL_REAL
UNIT_ROUNDOFF	SUN_UNIT_ROUNDOFF
PREC_NONE	SUN_PREC_NONE
PREC_LEFT	SUN_PREC_LEFT
PREC RIGHT	SUN_PREC_RIGHT
PREC_BOTH	SUN PREC BOTH
MODIFIED_GS	SUN_MODIFIED_GS
CLASSICAL_GS	SUN_CLASSICAL_GS
ATimesFn	SUNATimesFn
PSetupFn	SUNPSetupFn
PSolveFn	SUNPSolveFn
DlsMat	SUNDIsMat
DENSE_COL	SUNDLS_DENSE_COL
DENSE_ELEM	SUNDLS_DENSE_ELEM
BAND_COL	SUNDLS_BAND_COL
BAND_COL_ELEM	SUNDLS_BAND_COL_ELEM
BAND_ELEM	SUNDLS_BAND_ELEM
SDIRK_2_1_2	ARKODE_SDIRK_2_1_2
BILLINGTON_3_3_2	ARKODE_BILLINGTON_3_3_2
TRBDF2_3_3_2	ARKODE_TRBDF2_3_3_2
KVAERNO_4_2_3	ARKODE_KVAERNO_4_2_3
ARK324L2SA_DIRK_4_2_3	ARKODE_ARK324L2SA_DIRK_4_2_3
CASH_5_2_4	ARKODE_CASH_5_2_4
CASH_5_3_4	ARKODE_CASH_5_3_4
SDIRK_5_3_4	ARKODE_SDIRK_5_3_4
KVAERNO_5_3_4	ARKODE_KVAERNO_5_3_4
ARK436L2SA_DIRK_6_3_4	ARKODE_ARK436L2SA_DIRK_6_3_4
KVAERNO_7_4_5	ARKODE_KVAERNO_7_4_5
ARK548L2SA_DIRK_8_4_5	ARKODE_ARK548L2SA_DIRK_8_4_5
ARK437L2SA_DIRK_7_3_4	ARKODE_ARK437L2SA_DIRK_7_3_4
ARK548L2SAb_DIRK_8_4_5	ARKODE_ARK548L2SAb_DIRK_8_4_5
MIN_DIRK_NUM	ARKODE_MIN_DIRK_NUM
MAX_DIRK_NUM	ARKODE_MAX_DIRK_NUM
MIS_KW3	ARKODE_MIS_KW3
MRI_GARK_ERK33a	ARKODE_MRI_GARK_ERK33a
MRI_GARK_ERK45a	ARKODE_MRI_GARK_ERK45a
MRI_GARK_IRK21a	ARKODE_MRI_GARK_IRK21a
MRI_GARK_ESDIRK34a	ARKODE_MRI_GARK_ESDIRK34a
MRI_GARK_ESDIRK46a	ARKODE_MRI_GARK_ESDIRK46a
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Table 14.1 – continued from previous page

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Deprecated Name	New Name
IMEX_MRI_GARK3a	ARKODE_IMEX_MRI_GARK3a
IMEX_MRI_GARK3b	ARKODE_IMEX_MRI_GARK3b
IMEX_MRI_GARK4	ARKODE_IMEX_MRI_GARK4
MIN_MRI_NUM	ARKODE_MIN_MRI_NUM
MAX_MRI_NUM	ARKODE_MAX_MRI_NUM
DEFAULT_MRI_TABLE_3	MRISTEP_DEFAULT_TABLE_3
DEFAULT_EXPL_MRI_TABLE_3	MRISTEP_DEFAULT_EXPL_TABLE_3
DEFAULT_EXPL_MRI_TABLE_4	MRISTEP_DEFAULT_EXPL_TABLE_4
DEFAULT_IMPL_SD_TABLE_2	MRISTEP_DEFAULT_IMPL_SD_TABLE_2
DEFAULT_IMPL_SD_TABLE_3	MRISTEP_DEFAULT_IMPL_SD_TABLE_3
DEFAULT_IMPL_SD_TABLE_4	MRISTEP_DEFAULT_IMPL_SD_TABLE_4
DEFAULT_IMEX_SD_TABLE_3	MRISTEP_DEFAULT_IMEX_SD_TABLE_3
DEFAULT_IMEX_SD_TABLE_4	MRISTEP_DEFAULT_IMEX_SD_TABLE_4
HEUN_EULER_2_1_2	ARKODE_HEUN_EULER_2_1_2
BOGACKI_SHAMPINE_4_2_3	ARKODE_BOGACKI_SHAMPINE_4_2_3
ARK324L2SA_ERK_4_2_3	ARKODE_ARK324L2SA_ERK_4_2_3
ZONNEVELD_5_3_4	ARKODE_ZONNEVELD_5_3_4
ARK436L2SA_ERK_6_3_4	ARKODE_ARK436L2SA_ERK_6_3_4
SAYFY_ABURUB_6_3_4	ARKODE_SAYFY_ABURUB_6_3_4
CASH_KARP_6_4_5	ARKODE_CASH_KARP_6_4_5
FEHLBERG_6_4_5	ARKODE_FEHLBERG_6_4_5
DORMAND_PRINCE_7_4_5	ARKODE_DORMAND_PRINCE_7_4_5
ARK548L2SA_ERK_8_4_5	ARKODE_ARK548L2SA_ERK_8_4_5
VERNER_8_5_6	ARKODE_VERNER_8_5_6
FEHLBERG_13_7_8	ARKODE_FEHLBERG_13_7_8
KNOTH_WOLKE_3_3	ARKODE_KNOTH_WOLKE_3_3
ARK437L2SA_ERK_7_3_4	ARKODE_ARK437L2SA_ERK_7_3_4
ARK548L2SAb_ERK_8_4_5	ARKODE_ARK548L2SAb_ERK_8_4_5
MIN_ERK_NUM	ARKODE_MIN_ERK_NUM
MAX_ERK_NUM	ARKODE_MAX_ERK_NUM
DEFAULT_ERK_2	ARKSTEP_DEFAULT_ERK_2
DEFAULT_ERK_3	ARKSTEP_DEFAULT_ERK_3
DEFAULT_ERK_4	ARKSTEP_DEFAULT_ERK_4
DEFAULT_ERK_5	ARKSTEP_DEFAULT_ERK_5
DEFAULT_ERK_6	ARKSTEP_DEFAULT_ERK_6
DEFAULT_ERK_8	ARKSTEP_DEFAULT_ERK_8
DEFAULT_DIRK_2	ARKSTEP_DEFAULT_DIRK_2
DEFAULT_DIRK_3	ARKSTEP_DEFAULT_DIRK_3
DEFAULT_DIRK_4	ARKSTEP_DEFAULT_DIRK_4
DEFAULT_DIRK_5	ARKSTEP_DEFAULT_DIRK_5
DEFAULT_ARK_ETABLE_3	ARKSTEP_DEFAULT_ARK_ETABLE_3
DEFAULT_ARK_ETABLE_4	ARKSTEP_DEFAULT_ARK_ETABLE_4
DEFAULT_ARK_ETABLE_5	ARKSTEP_DEFAULT_ARK_ETABLE_4
DEFAULT_ARK_ITABLE_3	ARKSTEP_DEFAULT_ARK_ITABLE_3
DEFAULT_ARK_ITABLE_4	ARKSTEP_DEFAULT_ARK_ITABLE_4
DEFAULT_ARK_ITABLE_5	ARKSTEP_DEFAULT_ARK_ITABLE_5
DEFAULT_ERK_2	ERKSTEP_DEFAULT_2
DEFAULT_ERK_3	ERKSTEP_DEFAULT_3
DEFAULT_ERK_4	ERKSTEP_DEFAULT_4
DEFAULT_ERK_5	ERKSTEP_DEFAULT_5

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Table 14.1 – continued from previous page

<b>Deprecated Name</b>	New Name
DEFAULT_ERK_6	ERKSTEP_DEFAULT_6
DEFAULT_ERK_8	ERKSTEP_DEFAULT_8

In addition, the following functions are now deprecated (compile-time warnings will be printed if supported by the compiler):

<b>Deprecated Name</b>	New Name
DenseGETRF	SUND1sMat_DenseGETRF
DenseGETRS	SUND1sMat_DenseGETRS
denseGETRF	SUND1sMat_denseGETRF
denseGETRS	SUND1sMat_denseGETRS
DensePOTRF	SUND1sMat_DensePOTRF
DensePOTRS	SUND1sMat_DensePOTRS
denseP0TRF	SUND1sMat_densePOTRF
densePOTRS	SUND1sMat_densePOTRS
DenseGEQRF	SUND1sMat_DenseGEQRF
DenseORMQR	SUND1sMat_DenseORMQR
denseGEQRF	SUND1sMat_denseGEQRF
denseORMQR	SUND1sMat_denseORMQR
DenseCopy	SUND1sMat_DenseCopy
denseCopy	SUND1sMat_denseCopy
DenseScale	SUND1sMat_DenseScale
denseScale	SUND1sMat_denseScale
denseAddIdentity	SUND1sMat_denseAddIdentity
DenseMatvec	SUND1sMat_DenseMatvec
denseMatvec	SUND1sMat_denseMatvec
BandGBTRF	SUND1sMat_BandGBTRF
bandGBTRF	SUND1sMat_bandGBTRF
BandGBTRS	SUND1sMat_BandGBTRS
bandGBTRS	SUND1sMat_bandGBTRS
BandCopy	SUND1sMat_BandCopy
bandCopy	SUND1sMat_bandCopy
BandScale	SUND1sMat_BandScale
bandScale	SUND1sMat_bandScale
bandAddIdentity	SUND1sMat_bandAddIdentity
BandMatvec	SUND1sMat_BandMatvec
bandMatvec	SUND1sMat_bandMatvec
ModifiedGS	SUNModifiedGS
ClassicalGS	SUNClassicalGS
QRfact	SUNQRFact
QRsol	SUNQRsol
DlsMat_NewDenseMat	SUND1sMat_NewDenseMat
DlsMat_NewBandMat	SUND1sMat_NewBandMat
DestroyMat	SUND1sMat_DestroyMat
NewIntArray	SUND1sMat_NewIntArray
NewIndexArray	SUND1sMat_NewIndexArray
NewRealArray	SUND1sMat_NewRealArray
DestroyArray	SUND1sMat_DestroyArray
AddIdentity	SUND1sMat_AddIdentity
	continues on next page

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	1 1 6
<b>Deprecated Name</b>	New Name
SetToZero	SUND1sMat_SetToZero
PrintMat	SUND1sMat_PrintMat
newDenseMat	SUND1sMat_newDenseMat
newBandMat	SUND1sMat_newBandMat
destroyMat	SUND1sMat_destroyMat
newIntArray	SUND1sMat_newIntArray
newIndexArray	SUND1sMat_newIndexArray
newRealArray	SUND1sMat_newRealArray
destroyArray	SUND1sMat_destroyArray

Table 14.2 – continued from previous page

In addition, the entire sundials\_lapack.h header file is now deprecated for removal in SUNDIALS v7.0.0. Note, this header file is not needed to use the SUNDIALS LAPACK linear solvers.

Deprecated "bootstrap" and "minimum correction" predictors in ARKStep (options 4 and 5 to ARKStepSetPredictorMethod()) and the "bootstrap" predictor in MRIStep (option 4 to MRIStepSetPredictorMethod()). These functions will output a deprecation warning message and will be removed in a future release.

# 14.16 Changes to SUNDIALS in release 5.8.0

#### **New Features**

The *RAJA vector* implementation has been updated to support the SYCL backend in addition to the CUDA and HIP backend. Users can choose the backend when configuring SUNDIALS by using the *SUNDIALS\_RAJA\_BACKENDS* CMake variable. This vector remains experimental and is subject to change from version to version.

New *SUNMatrix* and *SUNLinearSolver* implementation were added to interface with the Intel oneAPI Math Kernel Library (oneMKL). Both the matrix and the linear solver support general dense linear systems as well as block diagonal linear systems. See §8.9 for more details. This matrix is experimental and is subject to change from version to version.

Added a new *optional* function to the SUNLinearSolver API, *SUNLinSolSetZeroGuess()*, to indicate that the next call to *SUNLinSolSolve()* will be made with a zero initial guess. SUNLinearSolver implementations that do not use the *SUNLinSolNewEmpty()* constructor will, at a minimum, need set the setzeroguess function pointer in the linear solver ops structure to NULL. The SUNDIALS iterative linear solver implementations have been updated to leverage this new set function to remove one dot product per solve.

The time integrator packages (ARKODE, CVODE(S), and IDA(S)) all now support a new "matrix-embedded" *SUN-LinearSolver* type. This type supports user-supplied SUNLinearSolver implementations that set up and solve the specified linear system at each linear solve call. Any matrix-related data structures are held internally to the linear solver itself, and are not provided by the SUNDIALS package.

Added functions to ARKODE and CVODE(S) for supplying an alternative right-hand side function and to IDA(S) for supplying an alternative residual for use within nonlinear system function evaluations:

- ARKStepSetNlsRhsFn()
- MRIStepSetNlsRhsFn()
- CVodeSetNlsRhsFn()
- IDASetNlsResFn()

Support for user-defined inner (fast) integrators has been to the MRIStep module. See MRIStep Custom Inner Steppers for more information on providing a user-defined integration method.

Added specialized fused HIP kernels to CVODE which may offer better performance on smaller problems when using CVODE with the HIP vector. See the optional input function CVodeSetUseIntegratorFusedKernels() for more information. As with other SUNDIALS HIP features, this capability is considered experimental and may change from version to version.

New KINSOL options have been added to apply a constant damping factor in the fixed point and Picard iterations (see KINSetDamping()), to delay the start of Anderson acceleration with the fixed point and Picard iterations (see KINSetDelayAA()), and to return the newest solution with the fixed point iteration (see KINSetReturnNewest()).

The installed SUNDIALSConfig.cmake file now supports the COMPONENTS option to find\_package. The exported targets no longer have IMPORTED\_GLOBAL set.

# **Bug Fixes**

A bug was fixed in SUNMatCopyOps() where the matrix-vector product setup function pointer was not copied.

A bug was fixed in the *SPBCGS* and *SPTFQMR* solvers for the case where a non-zero initial guess and a solution scaling vector are provided. This fix only impacts codes using SPBCGS or SPTFQMR as standalone solvers as all SUNDIALS packages utilize a zero initial guess.

A bug was fixed in the ARKODE stepper modules where the stop time may be passed after resetting the integrator.

A bug was fixed in IDASetJacTimesResFn() in IDAS where the supplied function was used in the dense finite difference Jacobian computation rather than the finite difference Jacobian-vector product approximation.

A bug was fixed in the KINSOL Picard iteration where the value of KINSetMaxSetupCalls() would be ignored.

# 14.17 Changes to SUNDIALS in release 5.7.0

A new *N\_Vector* implementation based on the SYCL abstraction layer has been added targeting Intel GPUs. At present the only SYCL compiler supported is the DPC++ (Intel oneAPI) compiler. See §6.12 for more details. This vector is considered experimental and is subject to major changes even in minor releases.

A new *SUNMatrix* and *SUNLinearSolver* implementation were added to interface with the MAGMA linear algebra library. Both the matrix and the linear solver support general dense linear systems as well as block diagonal linear systems, and both are targeted at GPUs (AMD or NVIDIA). See §8.8 for more details.

# 14.18 Changes to SUNDIALS in release 5.6.1

Fixed a CMake bug which caused an error if the CMAKE\_CXX\_STANDARD and SUNDIALS\_RAJA\_BACKENDS options were not provided.

Fixed some compiler warnings when using the IBM XL compilers.

# 14.19 Changes to SUNDIALS in release 5.6.0

A new *N\_Vector* implementation based on the AMD ROCm HIP platform has been added. This vector can target NVIDIA or AMD GPUs. See §6.11 for more details. This vector is considered experimental and is subject to change from version to version.

The *RAJA vector* implementation has been updated to support the HIP backend in addition to the CUDA backend. Users can choose the backend when configuring SUNDIALS by using the *SUNDIALS\_RAJA\_BACKENDS* CMake variable. This vector remains experimental and is subject to change from version to version.

A new optional operation, *N\_VGetDeviceArrayPointer()*, was added to the N\_Vector API. This operation is useful for vectors that utilize dual memory spaces, e.g. the native SUNDIALS CUDA N\_Vector.

The SUNDIALS matrix and linear solver interfaces to the *cuSparse matrix* and *cuSolver batched QR solver* no longer require using the CUDA *N\_Vector*. Instead, they require that the vector utilized provides the *N\_VGetDeviceArrayPointer()* operation, and that the pointer returned by *N\_VGetDeviceArrayPointer()* is a valid CUDA device pointer.

# 14.20 Changes to SUNDIALS in release 5.5.0

Refactored the SUNDIALS build system. CMake 3.12.0 or newer is now required. Users will likely see deprecation warnings, but otherwise the changes should be fully backwards compatible for almost all users. SUNDIALS now exports CMake targets and installs a SUNDIALSConfig.cmake file.

Added support for SuperLU DIST 6.3.0 or newer.

# 14.21 Changes to SUNDIALS in release 5.4.0

# **Major Features**

A new class, *SUNMemoryHelper*, was added to support **GPU users** who have complex memory management needs such as using memory pools. This is paired with new constructors for the CUDA and RAJA vectors that accept a *SUNMemoryHelper* object. Refer to §4.8, §9, §6.10 and §6.13 for more information.

Added full support for time-dependent mass matrices in ARKStep, and expanded existing non-identity mass matrix infrastructure to support use of the fixed point nonlinear solver.

An interface between ARKStep and the XBraid multigrid reduction in time (MGRIT) library [1] has been added to enable parallel-in-time integration. See the Multigrid Reduction in Time with XBraid section for more information and the example codes in examples/arkode/CXX\_xbraid. This interface required the addition of three new N\_-Vector operations to exchange vector data between computational nodes, see N\_VBufSize(), N\_VBufPack(), and N\_VBufUnpack(). These N\_Vector operations are only used within the XBraid interface and need not be implemented for any other context.

#### **New Features**

The *RAJA vector* has been updated to mirror the CUDA vector. Notably, the update adds managed memory support to the RAJA vector. Users of the vector will need to update any calls to the *N\_VMake\_Raja()* function because that signature was changed. This vector remains experimental and is subject to change from version to version.

The expected behavior of SUNNonlinSolGetNumIters() and SUNNonlinSolGetNumConvFails() in the SUNNonlinearSolver API have been updated to specify that they should return the number of nonlinear solver iterations and convergence failures in the most recent solve respectively rather than the cumulative number of iterations and failures across all solves respectively. The API documentation and SUNDIALS provided SUNNonlinearSolver implementations have been updated accordingly. As before, the cumulative number of nonlinear iterations and failures may be retrieved with the following functions:

- ARKStepGetNumNonlinSolvIters()
- ARKStepGetNumNonlinSolvConvFails()
- ARKStepGetNonlinSolvStats()
- MRIStepGetNumNonlinSolvIters()
- MRIStepGetNumNonlinSolvConvFails()
- MRIStepGetNonlinSolvStats()

- CVodeGetNumNonlinSolvIters()
- CVodeGetNumNonlinSolvConvFails()
- CVodeGetNonlinSolvStats()
- IDAGetNumNonlinSolvIters()
- IDAGetNumNonlinSolvConvFails()
- IDAGetNonlinSolvStats()

Added the following the following functions that advanced users might find useful when providing a custom SUNNon-linSolSysFn():

- ARKStepComputeState()
- ARKStepGetNonlinearSystemData()
- MRIStepComputeState()
- MRIStepGetNonlinearSystemData()
- CVodeComputeState()
- CVodeGetNonlinearSystemData()
- IDAGetNonlinearSystemData()

Added new functions to CVODE(S), ARKODE, and IDA(S) to to specify the factor for converting between integrator tolerances (WRMS norm) and linear solver tolerances (L2 norm) i.e., tol\_L2 = nrmfac \* tol\_WRMS:

- ARKStepSetLSNormFactor()
- ARKStepSetMassLSNormFactor()
- MRIStepSetLSNormFactor()
- CVodeSetLSNormFactor()
- IDASetLSNormFactor()

Added new reset functions ARKStepReset(), ERKStepReset(), and MRIStepReset() to reset the stepper time and state vector to user-provided values for continuing the integration from that point while retaining the integration history. These function complement the reinitialization functions ARKStepReInit(), ERKStepReInit(), and MRIStepReInit() which reinitialize the stepper so that the problem integration should resume as if started from scratch.

Updated the MRIStep time-stepping module in ARKODE to support higher-order MRI-GARK methods [47], including methods that involve solve-decoupled, diagonally-implicit treatment of the slow time scale.

The function CVodeSetLSetupFrequency() has been added to CVODE(S) to set the frequency of calls to the linear solver setup function.

The Trilinos Tpetra *N\_Vector* interface has been updated to work with Trilinos 12.18+. This update changes the local ordinal type to always be an int.

Added support for CUDA 11.

### **Bug Fixes**

A minor inconsistency in CVODE(S) and a bug ARKODE when checking the Jacobian evaluation frequency has been fixed. As a result codes using using a non-default Jacobian update frequency through a call to CVodeSetMaxStepsBetweenJac or ARKStepSetMaxStepsBetweenJac will need to increase the provided value by 1 to achieve the same behavior as before.

In IDAS and CVODES, the functions for forward integration with checkpointing (IDASolveF(), CVodeF()) are now subject to a restriction on the number of time steps allowed to reach the output time. This is the same restriction

applied to IDASolve() and CVode(). The default maximum number of steps is 500, but this may be changed using the CVodeSetMaxNumSteps() and IDASetMaxNumSteps() function. This change fixes a bug that could cause an infinite loop in IDASolveF() and CVodeF(). This change may cause a runtime error in existing user code.

Fixed bug in using ERK method integration with static mass matrices.

### **Deprecation Notice**

For greater clarity the following functions have been deprecated:

- CVodeSetMaxStepsBetweenJac
- ARKStepSetMaxStepsBetweenJac
- ARKStepSetMaxStepsBetweenLSet

The following functions should be used instead:

- CVodeSetJacEvalFrequency()
- ARKStepSetJacEvalFrequency()
- ARKStepSetLSetupFrequency()

# 14.22 Changes to SUNDIALS in release 5.3.0

### **Major Feature**

Added support to CVODE for integrating IVPs with constraints using BDF methods and projecting the solution onto the constraint manifold with a user defined projection function. This implementation is accompanied by additions to user documentation and CVODE examples. See CVodeSetProjFn() for more information.

#### **New Features**

Added the ability to control the CUDA kernel launch parameters for the CUDA vector and spare matrix implementations. These implementations remain experimental and are subject to change from version to version. In addition, the CUDA vector kernels were rewritten to be more flexible. Most users should see equivalent performance or some improvement, but a select few may observe minor performance degradation with the default settings. Users are encouraged to contact the SUNDIALS team about any performance changes that they notice.

Added new capabilities for monitoring the solve phase in the Newton and fixed-point SUNNonlinearSolver, and the SUNDIALS iterative linear solvers. SUNDIALS must be built with the CMake option SUNDIALS\_BUILD\_WITH\_-MONITORING to use these capabilities.

Added specialized fused CUDA kernels to CVODE which may offer better performance on smaller problems when using CVODE with the CUDA vector. See the optional input function CVodeSetUseIntegratorFusedKernels() for more information. As with other SUNDIALS CUDA features, this is feature is experimental and may change from version to version.

Added a new function, CVodeSetMonitorFn(), that takes a user-function to be called by CVODE after every nst successfully completed time-steps. This is intended to provide a way of monitoring the CVODE statistics throughout the simulation.

Added a new function CVodeGetLinSolveStats() to get the CVODE linear solver statistics as a group.

Added the following optional functions to provide an alternative ODE right-hand side function (ARKODE and CVODE(S)), DAE residual function (IDA(S)), or nonlinear system function (KINSOL) for use when computing Jacobian-vector products with the internal difference quotient approximation:

- ARKStepSetJacTimesRhsFn()
- CVodeSetJacTimesRhsFn()

- CVodeSetJacTimesRhsFnB()
- IDASetJacTimesResFn()
- IDASetJacTimesResFnB()
- KINSetJacTimesVecSysFn()

### **Bug Fixes**

Fixed a bug in the iterative linear solvers where an error is not returned if the Atimes function is NULL or, if preconditioning is enabled, the PSolve function is NULL.

Fixed a bug in ARKODE where the prototypes for ERKStepSetMinReduction() and ARKStepSetMinReduction() were not included in arkode\_erkstep.h and arkode\_arkstep.h respectively.

Fixed a bug in ARKODE where inequality constraint checking would need to be disabled and then re-enabled to update the inequality constraint values after resizing a problem. Resizing a problem will now disable constraints and a call to ARKStepSetConstraints() or ERKStepSetConstraints() is required to re-enable constraint checking for the new problem size.

# 14.23 Changes to SUNDIALS in release 5.2.0

#### **New Features**

The following functions were added to each of the time integration packages to enable or disable the scaling applied to linear system solutions with matrix-based linear solvers to account for lagged matrix information:

- ARKStepSetLinearSolutionScaling()
- CVodeSetLinearSolutionScaling()
- CVodeSetLinearSolutionScalingB()
- IDASetLinearSolutionScaling()
- IDASetLinearSolutionScalingB()

When using a matrix-based linear solver with ARKODE, IDA(S), or BDF methods in CVODE(S) scaling is enabled by default.

Added a new *SUNMatrix* implementation that interfaces to the sparse matrix implementation from the NVIDIA cuS-PARSE library, see §7.7 for more details. In addition, the CUDA Sparse linear solver has been updated to use the new matrix, as such, users of this matrix will need to update their code. This implementations are still considered to be experimental, thus they are subject to breaking changes even in minor releases.

Added a new "stiff" interpolation module to ARKODE, based on Lagrange polynomial interpolation, that is accessible to each of the ARKStep, ERKStep and MRIStep time-stepping modules. This module is designed to provide increased interpolation accuracy when integrating stiff problems, as opposed to the ARKODE-standard Hermite interpolation module that can suffer when the IVP right-hand side has large Lipschitz constant. While the Hermite module remains the default, the new Lagrange module may be enabled using one of the routines ARKStepSetInterpolantType(), ERKStepSetInterpolantType(), or MRIStepSetInterpolantType(). The serial example problem ark\_brusselator.c has been converted to use this Lagrange interpolation module. Created accompanying routines ARKStepSetInterpolantDegree(), ARKStepSetInterpolantDegree() and ARKStepSetInterpolantDegree() to provide user control over these interpolating polynomials.

Added two new functions, ARKStepSetMinReduction() and ERKStepSetMinReduction(), to change the minimum allowed step size reduction factor after an error test failure.

#### **Bug Fixes**

Fixed a build system bug related to the Fortran 2003 interfaces when using the IBM XL compiler. When building the Fortran 2003 interfaces with an XL compiler it is recommended to set CMAKE\_Fortran\_COMPILER to £2003, x1£2003, or x1£2003\_r.

Fixed a bug in how ARKODE interfaces with a user-supplied, iterative, unscaled linear solver. In this case, ARKODE adjusts the linear solver tolerance in an attempt to account for the lack of support for left/right scaling matrices. Previously, ARKODE computed this scaling factor using the error weight vector, ewt; this fix changes that to the residual weight vector, rwt, that can differ from ewt when solving problems with non-identity mass matrix.

Fixed a linkage bug affecting Windows users that stemmed from dllimport/dllexport attribute missing on some SUN-DIALS API functions.

Fixed a memory leak in CVODES and IDAS from not deallocating the atolSmin0 and atolQSmin0 arrays.

Fixed a bug where a non-default value for the maximum allowed growth factor after the first step would be ignored.

### **Deprecation Notice**

The routines ARKStepSetDenseOrder(), ARKStepSetDenseOrder() and ARKStepSetDenseOrder() have been deprecated and will be removed in a future release. The new functions ARKStepSetInterpolantDegree(), ARK-StepSetInterpolantDegree(), and ARKStepSetInterpolantDegree() should be used instead.

# 14.24 Changes to SUNDIALS in release 5.1.0

#### **New Features**

Added support for a user-supplied function to update the prediction for each implicit stage solution in ARKStep. If supplied, this routine will be called *after* any existing ARKStep predictor algorithm completes, so that the predictor may be modified by the user as desired. The new user-supplied routine has type ARKStagePredictFn, and may be set by calling ARKStepSetStagePredictFn().

The MRIStep module has been updated to support attaching different user data pointers to the inner and outer integrators. If applicable, user codes will need to add a call to ARKStepSetUserData() to attach their user data pointer to the inner integrator memory as MRIStepSetUserData() will not set the pointer for both the inner and outer integrators. The MRIStep examples have been updated to reflect this change.

Added support for damping when using Anderson acceleration in KINSOL. See the *Mathematical Considerations* and the description of the *KINSetDampingAA()* function for more details.

Added support for constant damping to the fixed-point SUNNonlinearSolver when using Anderson acceleration. See SUNNonlinSol FixedPoint description and the SUNNonlinSolSetDamping\_FixedPoint() for more details.

Added two utility functions, *SUNDIALSFileOpen()* and *SUNDIALSFileClose()* for creating/destroying file pointers. These are useful when using the Fortran 2003 interfaces.

Added a new build system option, CUDA\_ARCH, to specify the CUDA architecture to target.

# **Bug Fixes**

Fixed a build system bug related to finding LAPACK/BLAS.

Fixed a build system bug related to checking if the KLU library works.

Fixed a build system bug related to finding PETSc when using the CMake variables *PETSC\_INCLUDES* and *PETSC\_LIBRARIES* instead of *PETSC\_DIR*.

Fixed a bug in the Fortran 2003 interfaces to the ARKODE Butcher table routines and structure. This includes changing the ARKodeButcherTable type to be a type(c\_ptr) in Fortran.

# 14.25 Changes to SUNDIALS in release 5.0.0

## **Build System**

Increased the minimum required CMake version to 3.5 for most SUNDIALS configurations, and 3.10 when CUDA or OpenMP with device offloading are enabled.

The CMake option BLAS\_ENABLE and the variable BLAS\_LIBRARIES have been removed to simplify builds as SUN-DIALS packages do not use BLAS directly. For third party libraries that require linking to BLAS, the path to the BLAS library should be included in the \_LIBRARIES variable for the third party library e.g., SUPERLUDIST\_LIBRARIES when enabling SuperLU DIST.

#### **NVector**

Two new functions were added to aid in creating custom  $N\_Vector$  objects. The constructor  $N\_Vewempty()$  allocates an "empty" generic  $N\_Vector$  with the object's content pointer and the function pointers in the operations structure initialized to NULL. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the  $N\_Vector$  API by ensuring only required operations need to be set. Additionally, the function  $N\_VecpyOps()$  has been added to copy the operation function pointers between vector objects. When used in clone routines for custom vector objects these functions also will ease the introduction of any new optional operations to the  $N\_Vector$  API by ensuring all operations are copied when cloning objects.

Added new *N\_Vector* implementations, *ManyVector* and *MPIManyVector*, to support flexible partitioning of solution data among different processing elements (e.g., CPU + GPU) or for multi-physics problems that couple distinct MPI-based simulations together (see the §6.17 and §6.18 for more details). This implementation is accompanied by additions to user documentation and SUNDIALS examples.

Additionally, an *MPIPlusX vector* implementation has been created to support the MPI+X paradigm where X is a type of on-node parallelism (e.g., OpenMP, CUDA, etc.). The implementation is accompanied by additions to user documentation and SUNDIALS examples.

One new required vector operation and ten new optional vector operations have been added to the *N\_Vector* API. The new required operation, *N\_VGetLength()*, returns the global vector length. The optional operations have been added to support the new MPIManyVector implementation. The operation *N\_VGetCommunicator()* must be implemented by subvectors that are combined to create an MPIManyVector, but is not used outside of this context. The remaining nine operations are optional local reduction operations intended to eliminate unnecessary latency when performing vector reduction operations (norms, etc.) on distributed memory systems. The optional local reduction vector operations are *N\_VDotProdLocal*, *N\_VMaxNormLocal*, *N\_VMinLocal*, *N\_VL1NormLocal*, *N\_VWSqrSumLocal*, *N\_-VWSqrSumLocal*, *N\_-V* 

The \*\_MPICuda and \*\_MPIRaja functions have been removed from the CUDA and RAJA vector implementations respectively. Accordingly, the nvector\_mpicuda.h, nvector\_mpiraja.h, libsundials\_nvecmpicuda.lib, and libsundials\_nvecmpicudaraja.lib files have been removed. Users should use the MPI+X vector in conjunction with the CUDA and RAJA vectors to replace the functionality. The necessary changes are minimal and should require few code modifications. See the example programs in examples/ida/mpicuda and examples/ida/mpiraja for examples of how to use the MPI+X vector with the CUDA and RAJA vectors, respectively.

Made performance improvements to the CUDA vector. Users who utilize a non-default stream should no longer see default stream synchronizations after memory transfers.

Added a new constructor to the CUDA vector that allows a user to provide custom allocate and free functions for the vector data array and internal reduction buffer.

Added three new  $N_Vector$  utility functions,  $N_VGetVecAtIndexVectorArray()$ ,  $N_VSetVecAtIndexVectorArray()$ , and  $N_VNewVectorArray()$ , for working with  $N_Vector$  arrays when using the Fortran 2003 interfaces.

## **SUNMatrix**

Two new functions were added to aid in creating custom <code>SUNMatrix</code> objects. The constructor <code>SUNMatnewEmpty()</code> allocates an "empty" generic <code>SUNMatrix</code> with the object's content pointer and the function pointers in the operations structure initialized to <code>NULL</code>. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the <code>SUNMatrix</code> API by ensuring only required operations need to be set. Additionally, the function <code>SUNMatCopyOps()</code> has been added to copy the operation function pointers between matrix objects. When used in clone routines for custom matrix objects these functions also will ease the introduction of any new optional operations to the <code>SUNMatrix</code> API by ensuring all operations are copied when cloning objects.

A new operation, <code>SUNMatMatvecSetup()</code>, was added to the <code>SUNMatrix</code> API to perform any setup necessary for computing a matrix-vector product. This operation is useful for <code>SUNMatrix</code> implementations which need to prepare the matrix itself, or communication structures before performing the matrix-vector product. Users who have implemented a custom <code>SUNMatrix</code> will need to at least update their code to set the corresponding ops structure member, <code>matvecsetup</code>, to <code>NULL</code>.

The generic *SUNMatrix* API now defines error codes to be returned by matrix operations. Operations which return an integer flag indiciating success/failure may return different values than previously.

A new *SUNMatrix* (and *SUNLinearSolver*) implementation was added to facilitate the use of the SuperLU\_DIST library with SUNDIALS.

#### **SUNLinearSolver**

A new function was added to aid in creating custom <code>SUNLinearSolver</code> objects. The constructor <code>SUNLinSol-NewEmpty()</code> allocates an "empty" generic <code>SUNLinearSolver</code> with the object's content pointer and the function pointers in the operations structure initialized to <code>NULL</code>. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the <code>SUNLinearSolver</code> API by ensuring only required operations need to be set.

The return type of the SUNLinSolLastFlag in the SUNLinearSolver has changed from long int to sunindextype to be consistent with the type used to store row indices in dense and banded linear solver modules.

Added a new optional operation to the *SUNLinearSolver* API, *SUNLinSolGetID()*, that returns a *SUNLinear-Solver\_ID* for identifying the linear solver module.

The SUNLinearSolver API has been updated to make the initialize and setup functions optional.

A new *SUNLinearSolver* (and *SUNMatrix*) implementation was added to facilitate the use of the SuperLU\_DIST library with SUNDIALS.

Added a new *SUNLinearSolver* implementation, *cuSolverSp\_batchQR*, which leverages the NVIDIA cuSOLVER sparse batched QR method for efficiently solving block diagonal linear systems on NVIDIA GPUs.

Added three new accessor functions to the KLU linear solver to provide user access to the underlying KLU solver structures: SUNLinSol\_KLUGetSymbolic(), SUNLinSol\_KLUGetNumeric(), and SUNLinSol\_KLUGetCommon().

### SUNNonlinearSolver

A new function was added to aid in creating custom SUNNonlinearSolver objects. The constructor SUNNonlinSolNewEmpty() allocates an "empty" generic SUNNonlinearSolver with the object's content pointer and the function pointers in the operations structure initialized to NULL. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the SUNNonlinearSolver API by ensuring only required operations need to be set.

To facilitate the use of user supplied nonlinear solver convergence test functions the SUNNonlinSolSetConvTestFn() function in the SUNNonlinearSolver API has been updated to take a void\* data pointer as input. The supplied data pointer will be passed to the nonlinear solver convergence test function on each call.

The inputs values passed to the first two inputs of the SUNNonlinSolSolve() function in the SUNNonlinearSolver have been changed to be the predicted state and the initial guess for the correction to that state. Additionally, the definitions of SUNNonlinSolLSetupFn() and SUNNonlinSolLSolveFn() in the SUNNonlinearSolver API have

been updated to remove unused input parameters. For more information on the nonlinear system formulation and the API functions see Nonlinear Algebraic Solvers.

Added a new SUNNonlinearSolver implementation for interfacing with the PETSc SNES nonlinear solver.

#### **New Features**

A new linear solver interface functions, ARKLsLinSysFn and CVLsLinSysFn, as added as an alternative method for evaluating the linear systems  $M - \gamma J$  or  $I - \gamma J$ .

Added the following functions to get the current state and gamma value to ARKStep, CVODE and CVODES that may be useful to users who choose to provide their own nonlinear solver implementation:

- ARKStepGetCurrentState()
- ARKStepGetCurrentGamma()
- CVodeGetCurrentGamma()
- CVodeGetCurrentState()
- CVodeGetCurrentGamma()
- CVodeGetCurrentStateSens()
- CVodeGetCurrentSensSolveIndex()
- IDAGetCurrentCj()
- IDAGetCurrentY()
- IDAGetCurrentYp()
- IDAComputeY()
- IDAComputeYp()

Removed extraneous calls to  $N_{VMin}()$  for simulations where the scalar valued absolute tolerance, or all entries of the vector-valued absolute tolerance array, are strictly positive. In this scenario ARKODE, CVODE(S), and IDA(S) steppers will remove at least one global reduction per time step.

The ARKODE, CVODE(S), IDA(S), and KINSOL linear solver interfaces have been updated to only zero the Jacobian matrix before calling a user-supplied Jacobian evaluation function when the attached linear solver has type SUNLINEARSOLVER\_DIRECT.

Added new Fortran 2003 interfaces to all of the SUNDIALS packages (ARKODE, CVODE(S), IDA(S), and KINSOL as well as most of the *N\_Vector*, *SUNMatrix*, *SUNLinearSolver*, and SUNNonlinearSolver implementations. See §4.7 section for more details. These new interfaces were generated with SWIG-Fortran and provide a user an idiomatic Fortran 2003 interface to most of the SUNDIALS C API.

The MRIStep module has been updated to support explicit, implicit, or IMEX methods as the fast integrator using the ARKStep module. As a result some function signatures have been changed including MRIStepCreate() which now takes an ARKStep memory structure for the fast integration as an input.

The reinitialization functions ERKStepReInit(), ARKStepReInit(), and MRIStepReInit() have been updated to retain the minimum and maximum step size values from before reinitialization rather than resetting them to the default values.

Added two new embedded ARK methods of orders 4 and 5 to ARKODE (from [39]).

Support for optional inequality constraints on individual components of the solution vector has been added the ARKODE ERKStep and ARKStep modules. See the descriptions of ERKStepSetConstraints() and ARKStepSetConstraints() for more details. Note that enabling constraint handling requires the *N\_Vector* operations *N\_VMin-Quotient()*, *N\_VConstrMask()*, and *N\_VCompare()* that were not previously required by ARKODE.

Add two new 'Set' functions to MRIStep, MRIStepSetPreInnerFn() and MRIStepSetPostInnerFn(), for performing communication or memory transfers needed before or after the inner integration.

### **Bug Fixes**

Fixed a bug in the build system that prevented the PThreads NVECTOR module from being built.

Fixed a memory leak in the PETSc *N\_Vector* clone function.

Fixed a memeory leak in the ARKODE, CVODE, and IDA F77 interfaces when not using the default nonlinear solver.

Fixed a bug in the ARKStep time-stepping module in ARKODE that would result in an infinite loop if the nonlinear solver failed to converge more than the maximum allowed times during a single step.

Fixed a bug in ARKODE that would result in a "too much accuracy requested" error when using fixed time step sizes with explicit methods in some cases.

Fixed a bug in ARKStep where the mass matrix linear solver setup function was not called in the Matrix-free case.

Fixed a minor bug in ARKStep where an incorrect flag is reported when an error occurs in the mass matrix setup or Jacobian-vector product setup functions.

Fixed a bug in the CVODE and CVODES constraint handling where the step size could be set below the minimum step size.

Fixed a bug in the CVODE and CVODES nonlinear solver interfaces where the norm of the accumulated correction was not updated when using a non-default convergence test function.

Fixed a bug in the CVODES cvRescale function where the loops to compute the array of scalars for the fused vector scale operation stopped one iteration early.

Fixed a bug in CVODES and IDAS where CVodeF() and IDASolveF() would return the wrong flag under certain circumstances.

Fixed a bug in CVODES and IDAS where CVodeF() and IDASolveF() would not return a root in NORMAL\_STEP mode if the root occurred after the desired output time.

Fixed a bug in the IDA and IDAS linear solver interfaces where an incorrect Jacobian-vector product increment was used with iterative solvers other than SPGMR and SPFGMR.

Fixed a bug the IDAS IDAQuadReInitB() function where an incorrect memory structure was passed to IDAQuadReInit().

Fixed a bug in the KINSOL linear solver interface where the auxiliary scalar sJpnorm was not computed when necessary with the Picard iteration and the auxiliary scalar sFdotJp was unnecessarily computed in some cases.

# 14.26 Changes to SUNDIALS in release 4.1.0

# **Removed Implementation Headers**

The implementation header files (\*\_impl.h) are no longer installed. This means users who are directly accessing or manipulating package memory structures will need to update their code to use the package's public API.

### **New Features**

An additional *N\_Vector* implementation was added for interfacing with the Tpetra vector from Trilinos library to facilitate interoperability between SUNDIALS and Trilinos. This implementation is accompanied by additions to user documentation and SUNDIALS examples.

### **Bug Fixes**

The EXAMPLES\_ENABLE\_RAJA CMake option has been removed. The option EXAMPLES\_ENABLE\_CUDA enables all examples that use CUDA including the RAJA examples with a CUDA back end (if RAJA is enabled).

Python is no longer required to run make test and make test\_install.

A bug was fixed where a nonlinear solver object could be freed twice in some use cases.

Fixed a bug in ARKodeButcherTable\_Write() when printing a Butcher table without an embedding.

# 14.27 Changes to SUNDIALS in release 4.0.2

Added information on how to contribute to SUNDIALS and a contributing agreement.

Moved the definitions of backwards compatibility functions for the prior direct linear solver (DLS) and scaled preconditioned iterarive linear solvers (SPILS) to a source file. The symbols are now included in the appropriate package library, e.g. libsundials\_cvode.lib.

# 14.28 Changes to SUNDIALS in release 4.0.1

A bug in ARKODE where single precision builds would fail to compile has been fixed.

# 14.29 Changes to SUNDIALS in release 4.0.0

The direct and iterative linear solver interfaces in all SUNDIALS packages have been merged into a single unified linear solver interface to support any valid *SUNLinearSolver*. This includes the DIRECT and ITERATIVE types as well as the new MATRIX\_ITERATIVE type. Details regarding how SUNDIALS packages utilize linear solvers of each type as well as a discussion regarding the intended use cases for user-supplied linear solver implementations are included in §8. All example programs have been updated to use the new unified linear solver interfaces.

The unified linear solver interface is very similar to the previous DLS (direct linear solver) and SPILS (scaled preconditioned iterative linear solver) interface in each package. To minimize challenges in user migration to the unified linear solver interfaces, the previous DLS and SPILS functions may still be used however, these are now deprecated and will be removed in a future release. Additionally, that Fortran users will need to enlarge their array of optional integer outputs, and update the indices that they query for certain linear solver related statistics.

The names of all SUNDIALS-provided *SUNLinearSolver* constructors have been updated to follow the naming convention SUNLinSol\_\* where \* is the name of the linear solver. The new constructor names are:

- SUNLinSol\_Band()
- SUNLinSol\_Dense()
- SUNLinSol\_KLU()
- SUNLinSol\_LapackBand()
- SUNLinSol\_LapackDense()
- SUNLinSol\_PCG()
- SUNLinSol SPBCGS()
- SUNLinSol\_SPFGMR()
- SUNLinSol\_SPGMR()
- SUNLinSol\_SPTFQMR()
- SUNLinSol\_SuperLUMT()

Linear solver-specific "set" routine names have been similarly standardized. To minimize challenges in user migration to the new names, the previous function names may still be used however, these are now deprecated and will be removed in a future release. All example programs and the standalone linear solver examples have been updated to use the new naming convention.

The SUNLinSol\_Band() constructor has been simplified to remove the storage upper bandwidth argument.

SUNDIALS integrators (ARKODE, CVODE(S), and IDA(S)) have been updated to utilize generic nonlinear solvers defined by the SUNNonlinearSolver API. This enables the addition of new nonlinear solver options and allows for external or user-supplied nonlinear solvers. The nonlinear solver API and SUNDIALS provided implementations are described in Nonlinear Algebraic Solvers and follow the same object oriented design used by the N\_Vector, SUN-Matrix, and SUNLinearSolver classes. Currently two nonlinear solver implementations are provided, Newton and fixed-point. These replicate the previous integrator-specific implementations of Newton's method and a fixed-point iteration (previously referred to as a functional iteration), respectively. Note the new fixed-point implementation can optionally utilize Anderson's method to accelerate convergence. Example programs using each of these nonlinear solvers in a standalone manner have been added and all example programs have been updated accordingly.

The SUNDIALS integrators (ARKODE, CVODE(S), and IDA(S)) all now use the Newton SUNNonlinearSolver by default. Users that wish to use the fixed-point SUNNonlinearSolver will need to create the corresponding nonlinear solver object and attach it to the integrator with the appropriate set function:

- ARKStepSetNonlinearSolver()
- CVodeSetNonlinearSolver()
- IDASetNonlinearSolver()

Functions for setting the nonlinear solver options or getting nonlinear solver statistics remain unchanged and internally call generic SUNNonlinearSolver functions as needed.

With the introduction of the SUNNonlinearSolver class, the input parameter iter to CVodeCreate() has been removed along with the function CVodeSetIterType and the constants CV\_NEWTON and CV\_FUNCTIONAL. While SUNDIALS includes a fixed-point nonlinear solver, it is not currently supported in IDA.

Three fused vector operations and seven vector array operations have been added to the *N\_Vector* API. These *optional* operations are disabled by default and may be activated by calling vector specific routines after creating a vector (see §6.1 for more details). The new operations are intended to increase data reuse in vector operations, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. The fused operations are:

- N\_VLinearCombination()
- N VScaleAddMulti()
- N\_VDotProdMulti()

and the vector array operations are:

- N\_VLinearCombinationVectorArray()
- N\_VScaleVectorArray()
- N\_VConstVectorArray()
- N\_VWrmsNormVectorArray()
- N\_VWrmsNormMaskVectorArray()
- N\_VScaleAddMultiVectorArray()
- N\_VLinearCombinationVectorArray()

If an *N\_Vector* implementation defines the implementation any of these operations as NULL, then standard vector operations will automatically be called as necessary to complete the computation.

A new N\_Vector implementation, OpenMPDEV, leveraging OpenMP device offloading has been added.

Multiple updates to the CUDA vector were made:

- Changed the N\_VMake\_Cuda() function to take a host data pointer and a device data pointer instead of an N\_-VectorContent\_Cuda object.
- Changed N\_VGetLength\_Cuda to return the global vector length instead of the local vector length.
- Added N\_VGetLocalLength\_Cuda to return the local vector length.
- Added N\_VGetMPIComm\_Cuda to return the MPI communicator used.
- Removed the accessor functions in the suncudavec namespace.
- Added the ability to set the cudaStream\_t used for execution of the CUDA kernels. See the function N\_-VSetCudaStreams\_Cuda.
- Added N\_VNewManaged\_Cuda(), N\_VMakeManaged\_Cuda(), and N\_VIsManagedMemory\_Cuda() functions to accommodate using managed memory with the CUDA vector.

Multiple updates to the *RAJA* vector were made:

- Changed N\_VGetLength\_Raja to return the global vector length instead of the local vector length.
- Added N\_VGetLocalLength\_Raja to return the local vector length.
- Added N\_VGetMPIComm\_Raja to return the MPI communicator used.
- Removed the accessor functions in the sunrajavec namespace.

Two changes were made in the ARKODE and CVODE(S) initial step size algorithm:

- Fixed an efficiency bug where an extra call to the RHS function was made.
- Changed the behavior of the algorithm if the max-iterations case is hit. Before the algorithm would exit with the step size calculated on the penultimate iteration. Now it will exit with the step size calculated on the final iteration.

Fortran 2003 interfaces to CVODE, the fixed-point and Newton nonlinear solvers, the dense, band, KLU, PCG, SP-BCGS, SPFGMR, SPGMR, and SPTFQMR linear solvers, and the serial, PThreads, and OpenMP vectors have been added.

The ARKODE library has been entirely rewritten to support a modular approach to one-step methods, which should allow rapid research and development of novel integration methods without affecting existing solver functionality. To support this, the existing ARK-based methods have been encapsulated inside the new ARKStep time-stepping module. Two new time-stepping modules have been added:

- The ERKStep module provides an optimized implementation for explicit Runge–Kutta methods with reduced storage and number of calls to the ODE right-hand side function.
- The MRIStep module implements two-rate explicit-explicit multirate infinitesimal step methods utilizing different step sizes for slow and fast processes in an additive splitting.

This restructure has resulted in numerous small changes to the user interface, particularly the suite of "Set" routines for user-provided solver parameters and "Get" routines to access solver statistics, that are now prefixed with the name of time-stepping module (e.g., ARKStep or ERKStep) instead of ARKODE. Aside from affecting the names of these routines, user-level changes have been kept to a minimum. However, we recommend that users consult both this documentation and the ARKODE example programs for further details on the updated infrastructure.

As part of the ARKODE restructuring an ARKodeButcherTable structure has been added for storing Butcher tables. Functions for creating new Butcher tables and checking their analytic order are provided along with other utility routines. For more details see the Butcher Table Data Structure section.

ARKODE's dense output infrastructure has been improved to support higher-degree Hermite polynomial interpolants (up to degree 5) over the last successful time step.

# 14.30 Changes to SUNDIALS in release 3.2.1

Fixed a bug in the CUDA vector where the N\_VInvTest() operation could write beyond the allocated vector data.

Fixed the library installation path for multiarch systems. This fix changes the default library installation path from CMAKE\_INSTALL\_PREFIX/CMAKE\_INSTALL\_LIBDIR. The default value library directory name is automatically set to lib, lib64, or lib/<multiarch-tuple> depending on the system, but maybe be overridden by setting CMAKE\_INSTALL\_LIBDIR.

# 14.31 Changes to SUNDIALS in release 3.2.0

# **Library Name Change**

Changed the name of the RAJA nvector library from libsundials\_nvecraja.lib to libsundials\_nvecrudaraja.lib to better reflect that the RAJA vector only support the CUDA backend currently.

#### **New Features**

Added hybrid MPI+CUDA and MPI+RAJA vectors to allow use of more than one MPI rank when using a GPU system. The vectors assume one GPU device per MPI rank.

Support for optional inequality constraints on individual components of the solution vector has been added to CVODE and CVODES. For more details see the Mathematical Considerations and Optional input functions sections. Use of CVodeSetConstraints() requires the  $N_Vector$  operations  $N_VMinQuotient()$ ,  $N_VConstrMask()$ , and  $N_Vector$  operations of CVODES.

## **CMake Updates**

CMake 3.1.3 is now the minimum required CMake version.

Deprecated the behavior of the SUNDIALS\_INDEX\_TYPE CMake option and added the SUNDIALS\_INDEX\_SIZE CMake option to select the sunindextype integer size.

The native CMake FindMPI module is now used to locate an MPI installation.

If MPI is enabled and MPI compiler wrappers are not set, the build system will check if CMAKE\_<language>\_COM-PILER can compile MPI programs before trying to locate and use an MPI installation.

The previous options for setting MPI compiler wrappers and the executable for running MPI programs have been have been deprecated. The new options that align with those used in native CMake FindMPI module are MPI\_C\_COMPILER, MPI\_CXX\_COMPILER, MPI\_Fortran\_COMPILER, and MPIEXEC\_EXECUTABLE.

When a Fortran name-mangling scheme is needed (e.g., <code>ENABLE\_LAPACK</code> is ON) the build system will infer the scheme from the Fortran compiler. If a Fortran compiler is not available or the inferred or default scheme needs to be overridden, the advanced options <code>SUNDIALS\_F77\_FUNC\_CASE</code> and <code>SUNDIALS\_F77\_FUNC\_UNDERSCORES</code> can be used to manually set the name-mangling scheme and bypass trying to infer the scheme.

Parts of the main CMakeLists.txt file were moved to new files in the src and example directories to make the CMake configuration file structure more modular.

### **Bug Fixes**

Fixed a problem with setting *sunindextype* which would occur with some compilers (e.g. armclang) that do not define \_\_STDC\_VERSION\_\_.

Fixed a thread-safety issue in CVODES and IDAS when using adjoint sensitivity analysis.

Fixed a bug in IDAS where the saved residual value used in the nonlinear solve for consistent initial conditions was passed as temporary workspace and could be overwritten.

# 14.32 Changes to SUNDIALS in release 3.1.2

# **CMake Updates**

Updated the minimum required version of CMake to 2.8.12 and enabled using rpath by default to locate shared libraries on OSX.

#### **New Features**

Added the function SUNSparseMatrix\_Reallocate() to allow specification of the matrix nonzero storage.

Added named constants for the two reinitialization types for the KLU SUNLinearSolver.

Updated the SUNMatScaleAdd() and SUNMatScaleAddI() implementations in the sparse SUNMatrix to more optimally handle the case where the target matrix contained sufficient storage for the sum, but had the wrong sparsity pattern. The sum now occurs in-place, by performing the sum backwards in the existing storage. However, it is still more efficient if the user-supplied Jacobian routine allocates storage for the sum  $M + \gamma J$  or  $M + \gamma J$  manually (with zero entries if needed).

The following examples from the usage notes page of the SUNDIALS website, and updated them to work with SUNDIALS 3.x:

- cvDisc\_dns.c demonstrates using CVODE with discontinuous solutions or RHS.
- cvRoberts\_dns\_negsol.c illustrates the use of the RHS function return value to control unphysical negative concentrations.
- cvRoberts\_FSA\_dns\_Switch.c demonstrates switching on/off forward sensitivity computations. This example came from the usage notes page of the SUNDIALS website.

# **Bug Fixes**

Fixed a Windows specific problem where *sunindextype* was not correctly defined when using 64-bit integers. On Windows *sunindextype* is now defined as the MSVC basic type \_\_int64.

Fixed a bug in the full KLU SUNLinearSolver reinitialization approach where the sparse SUNMatrix pointer would go out of scope on some architectures.

The misnamed function CVSpilsSetJacTimesSetupFnBS has been deprecated and replaced by CVSpilsSetJacTimesBS. The deprecated function CVSpilsSetJacTimesSetupFnBS will be removed in the next major release.

Changed LICENSE install path to instdir/include/sundials.

# **14.33** Changes to SUNDIALS in release **3.1.1**

## **Bug Fixes**

Fixed a minor bug in the CVODE and CVODES cvSLdet, where a return was missing in the error check for three inconsistent roots.

Fixed a potential memory leak in the *SPGMR* and *SPFGMR* linear solvers. If "Initialize" was called multiple times then the solver memory was reallocated (without being freed).

Fixed a minor bug in ARKReInit, where a flag was incorrectly set to indicate that the problem had been resized (instead of just re-initialized).

Fixed C++11 compiler errors/warnings about incompatible use of string literals.

Updated the KLU SUNLinearSolver to use a typedef for the precision-specific solve functions to avoid compiler warnings.

Added missing typecasts for some (void\*) pointers to avoid compiler warnings.

Fixed bug in the sparse SUNMatrix where int was used instead of sunindextype in one location.

Fixed a minor bug in KINPrintInfo where a case was missing for KIN\_REPTD\_SYSFUNC\_ERR leading to an undefined info message.

Added missing #include <stdio.h> in N\_Vector and SUNMatrix header files.

Added missing prototypes for ARKSpilsGetNumMTSetups in ARKODE and IDASpilsGetNumJTSetupEvals in IDA and IDAS.

Fixed an indexing bug in the CUDA vector implementation of  $N_{VWrmsNormMask}()$  and revised the RAJA vector implementation of  $N_{VWrmsNormMask}()$  to work with mask arrays using values other than zero or one. Replaced double with realtype in the RAJA vector test functions.

Fixed compilation issue with GCC 7.3.0 and Fortran programs that do not require a *SUNMatrix* or *SUNLinearSolver* e.g., iterative linear solvers, explicit methods in ARKODE, functional iteration in CVODE, etc.

# 14.34 Changes to SUNDIALS in release 3.1.0

Added N\_Vector print functions that write vector data to a specified file (e.g., N\_VPrintFile\_Serial()).

Added make test and make test\_install options to the build system for testing SUNDIALS after building with make and installing with make install respectively.

# 14.35 Changes to SUNDIALS in release 3.0.0

## **Major Feature**

Added new linear solver and matrix interfaces for all SUNDIALS packages and updated the existing linear solver and matrix implementations. The goal of the redesign is to provide greater encapsulation and ease interfacing custom linear solvers with linear solver libraries. Specific changes include:

- Added a *SUNMatrix* interface with three provided implementations: dense, banded, and sparse. These replicate previous SUNDIALS direct (Dls) and sparse (Sls) matrix structures.
- Added example problems demonstrating use of the matrices.
- Added a SUNLinearSolver interface with eleven provided implementations: dense, banded, LAPACK dense, LAPACK band, KLU, SuperLU\_MT, SPGMR, SPBCGS, SPTFQMR, SPFGMR, PCG. These replicate previous SUNDIALS generic linear solvers.
- Added example problems demonstrating use of the linear solvers.
- Expanded package-provided direct linear solver (Dls) interfaces and scaled, preconditioned, iterative linear solver (Spils) interfaces to utilize SUNMatrix and SUNLinearSolver objects.
- Removed package-specific, linear solver-specific, solver modules (e.g., CVDENSE, KINBAND, IDAKLU, ARK-SPGMR) since their functionality is entirely replicated by the generic Dls/Spils interfaces and SUNLinear-Solver / SUNMatrix classes. The exception is CVDIAG, a diagonal approximate Jacobian solver available to CVODE and CVODES.
- Converted all SUNDIALS example problems to utilize new the new matrix and linear solver objects, along with updated Dls and Spils linear solver interfaces.

Added Spils interface routines to ARKODE, CVODE, CVODES, IDA and IDAS to allow specification of a user-provided JTSetup routine. This change supports users who wish to set up data structures for the user-provided Jacobian-times-vector (JTimes) routine, and where the cost of one JTSetup setup per Newton iteration can be amortized between multiple JTimes calls.

Corresponding updates were made to all the example programs.

#### **New Features**

CUDA and RAJA N\_Vector implementations to support GPU systems. These vectors are supplied to provide very basic support for running on GPU architectures. Users are advised that these vectors both move all data to the GPU device upon construction, and speedup will only be realized if the user also conducts the right-hand-side function evaluation on the device. In addition, these vectors assume the problem fits on one GPU. For further information about RAJA, users are referred to the RAJA web site.

Added the type *sunindextype* to support using 32-bit or 64-bit integer types for indexing arrays within all SUNDIALS structures. *sunindextype* is defined to int32\_t or int64\_t when portable types are supported, otherwise it is defined as int or long int. The Fortran interfaces continue to use long int for indices, except for the sparse matrix interface that now uses *sunindextype*. Interfaces to PETSc, hypre, SuperLU\_MT, and KLU have been updated with 32-bit or 64-bit capabilities depending how the user configures SUNDIALS.

To avoid potential namespace conflicts, the macros defining booleantype values TRUE and FALSE have been changed to *SUNTRUE* and *SUNFALSE* respectively.

Temporary vectors were removed from preconditioner setup and solve routines for all packages. It is assumed that all necessary data for user-provided preconditioner operations will be allocated and stored in user-provided data structures.

The file include/sundials\_fconfig.h was added. This file contains SUNDIALS type information for use in Fortran programs.

Added support for many xSDK-compliant build system keys. For more information on on xSDK compliance the xSDK policies. The xSDK is a movement in scientific software to provide a foundation for the rapid and efficient production of high-quality, sustainable extreme-scale scientific applications. For more information visit the xSDK web site.

Added functions SUNDIALSGetVersion() and SUNDIALSGetVersionNumber() to get SUNDIALS release version information at runtime.

Added comments to arkode\_butcher.c regarding which methods should have coefficients accurate enough for use in quad precision.

## **Build System**

Renamed CMake options to enable/disable examples for greater clarity and added option to enable/disable Fortran 77 examples:

- Changed EXAMPLES\_ENABLE to EXAMPLES\_ENABLE\_C
- Changed CXX\_ENABLE to EXAMPLES\_ENABLE\_CXX
- Changed F90\_ENABLE to EXAMPLES\_ENABLE\_F90
- Added EXAMPLES\_ENABLE\_F77 option

Added separate BLAS\_ENABLE and BLAS\_LIBRARIES CMake variables.

Fixed minor CMake bugs and included additional error checking during CMake configuration.

#### **Bug Fixes**

## **ARKODE**

Fixed RCONST usage in arkode\_butcher.c.

Fixed bug in arkInitialSetup to ensure the mass matrix vector product is set up before the "msetup" routine is called.

Fixed ARKODE printf-related compiler warnings when building SUNDIALS with extended precision.

CVODE and CVODES

CVodeFree() now calls 1free unconditionally (if non-NULL).

IDA and IDAS

Added missing prototype for IDASetMaxBacksIC() in ida.h and idas.h.

KINSOL

Corrected KINSOL Fortran name translation for FKIN\_SPFGMR.

Renamed KINLocalFn and KINCommFn to KINBBDLocalFn and KINBBDCommFn respectively in the BBD preconditioner module for consistency with other SUNDIALS solvers.

# 14.36 Changes to SUNDIALS in release 2.7.0

### **New Features and Enhancements**

Two additional  $N_{Vector}$  implementations were added – one for *hypre parallel vectors* and one for *PETSc vectors*. These additions are accompanied by additions to various interface functions and to user documentation.

Added a new N\_Vector function, N\_VGetVectorID(), that returns an identifier for the vector.

The sparse matrix structure was enhanced to support both CSR and CSC matrix storage formats.

Various additions were made to the KLU and SuperLU\_MT sparse linear solver interfaces, including support for the CSR matrix format when using KLU.

In all packages, the linear solver and preconditioner free routines were updated to return an integer.

In all packages, example codes were updated to use N\_VGetArrayPointer\_\* rather than the NV\_DATA macro when using the native vectors shipped with SUNDIALS.

Additional example programs were added throughout including new examples utilizing the OpenMP vector.

### **ARKODE**

The ARKODE implicit predictor algorithms were updated: methods 2 and 3 were improved slightly, a new predictor approach was added, and the default choice was modified.

The handling of integer codes for specifying built-in ARKODE Butcher tables was enhanced. While a global numbering system is still used, methods now have #defined names to simplify the user interface and to streamline incorporation of new Butcher tables into ARKODE.

The maximum number of Butcher table stages was increased from 8 to 15 to accommodate very high order methods, and an 8th-order adaptive ERK method was added.

Support was added for the explicit and implicit methods in an additive Runge-Kutta method with different stage times to support new SSP-ARK methods.

The FARKODE interface was extended to include a routine to set scalar/array-valued residual tolerances, to support Fortran applications with non-identity mass-matrices.

## IDA and IDAS

The optional input function IDASetMaxBacksIC() was added to set the maximum number of linesearch backtracks in the initial condition calculation.

### **Bug Fixes**

Various minor fixes to installation-related files.

Fixed some examples with respect to the change to use new macro/function names e.g., SUNRexp, etc.

In all packages, a memory leak was fixed in the banded preconditioner and banded-block-diagonal preconditioner interfaces.

Corrected name N\_VCloneEmptyVectorArray to N\_VCloneVectorArrayEmpty in all documentation files.

Various corrections were made to the interfaces to the sparse solvers KLU and SuperLU\_MT.

For each linear solver, the various solver performance counters are now initialized to 0 in both the solver specification function and in the solver linit function. This ensures that these solver counters are initialized upon linear solver instantiation as well as at the beginning of the problem solution.

### **ARKODE**

The missing ARKSpilsGetNumMtimesEvals function was added — this had been included in the previous documentation but had not been implemented.

The choice of the method vs embedding the Billington and TRBDF2 explicit Runge–Kutta methods were swapped, since in those the lower-order coefficients result in an A-stable method, while the higher-order coefficients do not. This change results in significantly improved robustness when using those methods.

A bug was fixed for the situation where a user supplies a vector of absolute tolerances, and also uses the vector Resize functionality.

A bug was fixed wherein a user-supplied Butcher table without an embedding is supplied, and the user is running with either fixed time steps (or they do adaptivity manually); previously this had resulted in an error since the embedding order was below 1.

#### **CVODE**

Corrections were made to three Fortran interface functions.

In FCVODE, fixed argument order bugs in the FCVKLU and FCVSUPERLUMT linear solver interfaces.

Added missing Fortran interface routines for supplying a sparse Jacobian routine with sparse direct solvers.

### **CVODES**

A bug was fixed in the interpolation functions used in solving backward problems for adjoint sensitivity analysis.

In the interpolation routines for backward problems, added logic to bypass sensitivity interpolation if input sensitivity argument is NULL.

Changed each the return type of \*FreeB functions to int and added return(0) to each.

#### IDA

Corrections were made to three Fortran interface functions.

Corrected the output from the idaFoodWeb\_bnd.c example, the wrong component was printed in PrintOutput.

## **IDAS**

In the interpolation routines for backward problems, added logic to bypass sensitivity interpolation if input sensitivity argument is NULL.

Changed each the return type of \*FreeB functions to int and added return(0) to each.

Corrections were made to three Fortran interface functions.

Added missing Fortran interface routines for supplying a sparse Jacobian routine with sparse direct solvers.

## KINSOL

The Picard iteration return was changed to always return the newest iterate upon success.

A minor bug in the line search was fixed to prevent an infinite loop when the beta condition fails and lambda is below the minimum size.

Corrections were made to three Fortran interface functions.

The functions FKINCREATE and FKININIT were added to split the FKINMALLOC routine into two pieces. FKINMALLOC remains for backward compatibility, but documentation for it has been removed.

Added missing Fortran interface routines for supplying a sparse Jacobian routine with sparse direct solvers.

#### **Matlab Interfaces Removed**

Removed the Matlab interface from distribution as it has not been updated since 2009.

# 14.37 Changes to SUNDIALS in release 2.6.2

### **New Features and Enhancements**

Various minor fixes to installation-related files

In KINSOL and ARKODE, updated the Anderson acceleration implementation with QR updating.

In CVODES and IDAS, added ReInit and SetOrdering wrappers for backward problems.

In IDAS, fixed for-loop bugs in IDAAckpntAllocVectors that could lead to a memory leak.

#### **Bug Fixes**

Updated the BiCGStab linear solver to remove a redundant dot product call.

Fixed potential memory leak in KLU ReInit functions in all solvers.

In ARKODE, fixed a bug in the Cash-Karp Butcher table where the method and embedding coefficient were swapped.

In ARKODE, fixed error in arkDoErrorTest in recovery after failure.

In CVODES, added CVKLUB prototype and corrected CVSuperLUMTB prototype.

In the CVODES and IDAS header files, corrected documentation of backward integration functions, especially the which argument.

In IDAS, added missing backward problem support functions IDALapackDenseB, IDALapackDenseFreeB, IDALapackBandB, and IDALapackBandFreeB.

In IDAS, made SuperLUMT call for backward problem consistent with CVODES.

In CVODE, IDA, and ARKODE, fixed Fortran interfaces to enable calls to GetErrWeights, GetEstLocalErrors, and GetDky within a time step.

# 14.38 Changes to SUNDIALS in release 2.6.1

Fixed loop limit bug in SlsAddMat function.

In all six solver interfaces to KLU and SuperLUMT, added #include lines, and removed redundant KLU structure allocations.

Minor bug fixes in ARKODE.

### 14.39 Changes to SUNDIALS in release 2.6.0

### **Autotools Build Option Removed**

With this version of SUNDIALS, support and documentation of the Autotools mode of installation is being dropped, in favor of the CMake mode, which is considered more widely portable.

### **New Package: ARKODE**

Addition of ARKODE package of explicit, implicit, and additive Runge-Kutta methods for ODEs. This package API is close to CVODE so switching between the two should be straightforward. Thanks go to Daniel Reynolds for the addition of this package.

### **New Features and Enhancements**

Added *OpenMP* and *Pthreads N\_Vector* implementations for thread-parallel computing environments.

Two major additions were made to the linear system solvers available in all packages. First, in the serial case, an interface to the sparse direct solver KLU was added. Second, an interface to SuperLU\_MT, the multi-threaded version of SuperLU, was added as a thread-parallel sparse direct solver option, to be used with the serial version of the N\_-Vector module. As part of these additions, a sparse matrix (CSC format) structure was added to CVODE.

#### **KINSOL**

Two major additions were made to the globalization strategy options (KINSol argument strategy). One is fixed-point iteration, and the other is Picard iteration. Both can be accelerated by use of the Anderson acceleration method. See the relevant paragraphs in Chapter *Mathematical Considerations*.

An interface to the Flexible GMRES iterative linear solver was added.

#### **Bug Fixes**

In order to avoid possible name conflicts, the mathematical macro and function names MIN, MAX, SQR, RAbs, RSqrt, RExp, RPowerI, and RPowerR were changed to SUNMIN, SUNMAX, SUNSQR, SUNRabs, SUNRsqrt, SUNRexp, SRpowerI, and SUNRpowerR, respectively. These names occur in both the solver and example programs.

In the LAPACK banded linear solver interfaces, the line smu = MIN(N-1, mu+ml) was changed to smu = mu + ml to correct an illegal input error for to DGBTRF and DGBTRS.

In all Fortran examples, integer declarations were revised so that those which must match a C type long int are declared INTEGER\*8, and a comment was added about the type match. All other integer declarations are just INTEGER. Corresponding minor corrections were made to the user guide.

### CVODE and CVODES

In cvRootFind, a minor bug was corrected, where the input array was ignored, and a line was added to break out of root-search loop if the initial interval size is below the tolerance ttol.

Two minor bugs were fixed regarding the testing of input on the first call to CVode – one involving tstop and one involving the initialization of \*tret.

The example program cvAdvDiff\_diag\_p was added to illustrate the use of in parallel.

In the FCVODE optional input routines FCVSETIIN and FCVSETRIN, the optional fourth argument key\_length was removed, with hardcoded key string lengths passed to all tests.

In order to eliminate or minimize the differences between the sources for private functions in CVODE and CVODES, the names of many private functions were changed from CV\* to cv\* and a few other names were also changed.

An option was added in the case of Adjoint Sensitivity Analysis with dense or banded Jacobian. With a call to CVDlsSetDenseJacFnBS or CVDlsSetBandJacFnBS, the user can specify a user-supplied Jacobian function of type CVDls\*\*\*JacFnBS, for the case where the backward problem depends on the forward sensitivities.

In CVodeQuadSensInit, the line cv\_mem->cv\_fQS\_data = ... was corrected (missing Q).

In the CVODES User Guide, a paragraph was added in Section 6.2.1 on CVodeAdjReInit, and a paragraph was added in Section 6.2.9 on CVodeGetAdjY. In the example cvsRoberts\_ASAi\_dns, the output was revised to include the use of CVodeGetAdjY.

For the Adjoint Sensitivity Analysis case in which the backward problem depends on the forward sensitivities, options have been added to allow for user-supplied pset, psolve, and jtimes functions.

In the example cvsHessian\_ASA\_FSA, an error was corrected in the function fB2, y2 in place of y3 in the third term of Ith(yBdot,6).

#### IDA and IDAS

In IDARootfind, a minor bug was corrected, where the input array rootdir was ignored, and a line was added to break out of root-search loop if the initial interval size is below the tolerance ttol.

A minor bug was fixed regarding the testing of the input tstop on the first call to IDASolve().

In the FIDA optional input routines FIDASETIIN, FIDASETRIN, and FIDASETVIN, the optional fourth argument key\_length was removed, with hardcoded key string lengths passed to all strncmp tests.

An option was added in the case of Adjoint Sensitivity Analysis with dense or banded Jacobian. With a call to IDADlsSetDenseJacFnBS or IDADlsSetBandJacFnBS, the user can specify a user-supplied Jacobian function of type IDADls\*\*\*JacFnBS, for the case where the backward problem depends on the forward sensitivities.

#### **KINSOL**

In function KINStop, two return values were corrected to make the values of uu and fval consistent.

A bug involving initialization of mxnewtstep was fixed. The error affects the case of repeated user calls to KINSol with no intervening call to KINSetMaxNewtonStep.

A bug in the increments for difference quotient Jacobian approximations was fixed in function kinDlsBandDQJac.

In the FKINSOL module, an incorrect return value ier in FKINfunc was fixed.

In the FKINSOL optional input routines FKINSETIIN, FKINSETRIN, and FKINSETVIN, the optional fourth argument key\_length was removed, with hardcoded key string lengths passed to all strncmp tests.

### 14.40 Changes to SUNDIALS in release 2.5.0

### **Integer Type Change**

One significant design change was made with this release, the problem size and its relatives, bandwidth parameters, related internal indices, pivot arrays, and the optional output lsflag have all been changed from type int to type long int, except for the problem size and bandwidths in user calls to routines specifying BLAS/LAPACK routines for the dense/band linear solvers. The function NewIntArray is replaced by a pair NewIntArray / NewLintArray, for int and long int arrays, respectively.

### **Bug Fixes**

In the installation files, we modified the treatment of the macro SUNDIALS\_USE\_GENERIC\_MATH, so that the parameter GENERIC\_MATH\_LIB is either defined (with no value) or not defined.

In all packages, after the solver memory is created, it is set to zero before being filled.

In each linear solver interface function, the linear solver memory is freed on an error return, and the function now includes a line setting to NULL the main memory pointer to the linear solver memory.

### Rootfinding

In CVODE(S) and IDA(S), in the functions Rcheck1 and Rcheck2, when an exact zero is found, the array glo of g values at the left endpoint is adjusted, instead of shifting the t location tlo slightly.

### CVODE and CVODES

In CVSetTqBDF, the logic was changed to avoid a divide by zero.

In a minor change to the CVODES user interface, the type of the index which was changed from long int to int.

Errors in the logic for the integration of backward problems in CVODES were identified and fixed.

#### IDA and IDAS

To be consistent with IDAS, IDA uses the function IDAGetDky for optional output retrieval.

A memory leak was fixed in two of the IDASp\*\*\*Free functions.

A missing vector pointer setting was added in IDASensLineSrch.

In IDACompleteStep, conditionals around lines loading a new column of three auxiliary divided difference arrays, for a possible order increase, were fixed.

#### **KINSOL**

Three major logic bugs were fixed - involving updating the solution vector, updating the linesearch parameter, and a missing error return.

Three minor errors were fixed - involving setting etachoice in the Matlab/KINSOL interface, a missing error case in KINPrintInfo, and avoiding an exponential overflow in the evaluation of omega.

### 14.41 Changes to SUNDIALS in release 2.4.0

Added a CMake-based build option in addition to the one based on autotools.

The user interface has been further refined. Some of the API changes involve:

- (a) a reorganization of all linear solver modules into two families (besides the existing family of scaled preconditioned iterative linear solvers, the direct solvers, including new LAPACK-based ones, were also organized into a *direct* family);
- (b) maintaining a single pointer to user data, optionally specified through a Set-type function; and
- (c) a general streamlining of the preconditioner modules distributed with the solvers.

Added interfaces to LAPACK linear solvers for dense and banded matrices to all packages.

An option was added to specify which direction of zero-crossing is to be monitored while performing rootfinding in CVODE(S) and IDA(S).

CVODES includes several new features related to sensitivity analysis, among which are:

- (a) support for integration of quadrature equations depending on both the states and forward sensitivity (and thus support for forward sensitivity analysis of quadrature equations),
- (b) support for simultaneous integration of multiple backward problems based on the same underlying ODE (e.g., for use in an *forward-over-adjoint* method for computing second order derivative information),
- (c) support for backward integration of ODEs and quadratures depending on both forward states and sensitivities (e.g., for use in computing second-order derivative information), and
- (d) support for reinitialization of the adjoint module.

Moreover, the prototypes of all functions related to integration of backward problems were modified to support the simultaneous integration of multiple problems.

All backward problems defined by the user are internally managed through a linked list and identified in the user interface through a unique identifier.

### 14.42 Changes to SUNDIALS in release 2.3.0

#### **New Features and Enhancements**

The main changes in this release involve a rearrangement of the entire SUNDIALS source tree. At the user interface level, the main impact is in the mechanism of including SUNDIALS header files which must now include the relative path e.g., #include <cvode/cvode.h> as all exported header files are now installed in separate subdirectories of the installation *include* directory.

The functions in the generic dense linear solver (sundials\_dense and sundials\_smalldense) were modified to work for rectangular  $m \times n$  matrices ( $m \le n$ ), while the factorization and solution functions were renamed to DenseGETRF / denGETRF and DenseGETRS / denGETRS, respectively. The factorization and solution functions in the generic band linear solver were renamed BandGBTRF and BandGBTRS, respectively.

In IDA, the user interface to the consistent initial conditions calculations was modified. The IDACalcIC() arguments t0, yy0, and yp0 were removed and a new function, IDAGetConsistentIC() is provided.

### **Bug Fixes**

In the CVODES adjoint solver module, the following two bugs were fixed:

- In CVodeF the solver was sometimes incorrectly taking an additional step before returning control to the user (in CV\_NORMAL mode) thus leading to a failure in the interpolated output function.
- In CVodeB, while searching for the current check point, the solver was sometimes reaching outside the integration interval resulting in a segmentation fault.

In IDA, a bug was fixed in the internal difference-quotient dense and banded Jacobian approximations, related to the estimation of the perturbation (which could have led to a failure of the linear solver when zero components with sufficiently small absolute tolerances were present).

### 14.43 Changes to SUNDIALS in release 2.2.0

#### **New Header Files Names**

To reduce the possibility of conflicts, the names of all header files have been changed by adding unique prefixes (e.g., cvode\_ and sundials\_). When using the default installation procedure, the header files are exported under various subdirectories of the target include directory. For more details see Appendix §10.

### **Build System Changes**

Updated configure script and Makefiles for Fortran examples to avoid C++ compiler errors (now use CC and MPICC to link only if necessary).

The shared object files are now linked into each SUNDIALS library rater than into a separate libsundials\_shared library.

### **New Features and Enhancements**

Deallocation functions now take the address of the respective memory block pointer as the input argument.

Interfaces to the Scaled Preconditioned Bi-CGstab (SPBCG) and Scaled Preconditioned Transpose-Free Quasi-Minimal Residual (SPTFQMR) linear solver modules have been added to all packages. At the same time, function type names for Scaled Preconditioned Iterative Linear Solvers were added for the user-supplied Jacobian-times-vector and preconditioner setup and solve functions. Additionally, in KINSOL interfaces have been added to the SUNDIALS DENSE, and BAND linear solvers and include support for nonlinear residual monitoring which can be used to control Jacobian updating.

A new interpolation method was added to the CVODES adjoint module. The function CVadjMalloc has an additional argument which can be used to select the desired interpolation scheme.

FIDA, a Fortran-C interface module, was added.

The rootfinding feature was added to IDA, whereby the roots of a set of given functions may be computed during the integration of the DAE system.

In IDA a user-callable routine was added to access the estimated local error vector.

In the KINSOL Fortran interface module, FKINSOL, optional inputs are now set using FKINSETIIN (integer inputs), FKINSETRIN (real inputs), and FKINSETVIN (vector inputs). Optional outputs are still obtained from the IOUT and ROUT arrays which are owned by the user and passed as arguments to FKINMALLOC.

### 14.44 Changes to SUNDIALS in release 2.1.1

The function N\_VCloneEmpty was added to the global vector operations table.

A minor bug was fixed in the interpolation functions of the adjoint CVODES module.

### 14.45 Changes to SUNDIALS in release 2.1.0

The user interface has been further refined. Several functions used for setting optional inputs were combined into a single one.

In CVODE(S) and IDA, an optional user-supplied routine for setting the error weight vector was added.

Additionally, to resolve potential variable scope issues, all SUNDIALS solvers release user data right after its use.

The build systems has been further improved to make it more robust.

### 14.46 Changes to SUNDIALS in release 2.0.2

Fixed autoconf-related bug to allow configuration with the PGI Fortran compiler.

Modified the build system to use customized detection of the Fortran name mangling scheme (autoconf's AC\_F77\_-WRAPPERS routine is problematic on some platforms).

A bug was fixed in the CVode function that was potentially leading to erroneous behavior of the rootfinding procedure on the integration first step.

A new chapter in the User Guide was added - with constants that appear in the user interface.

### 14.47 Changes to SUNDIALS in release 2.0.1

### **Build System**

Changed the order of compiler directives in header files to avoid compilation errors when using a C++ compiler.

Changed the method of generating sundials\_config.h to avoid potential warnings of redefinition of preprocessor symbols.

### **New Features**

In CVODES the option of activating and deactivating forward sensitivity calculations on successive runs without memory allocation and deallocation.

#### **Bug Fixes**

In CVODES bug fixes related to forward sensitivity computations (possible loss of accuracy on a BDF order increase and incorrect logic in testing user-supplied absolute tolerances) were made.

### 14.48 Changes to SUNDIALS in release 2.0.0

Installation of all of SUNDIALS packages has been completely redesigned and is now based on configure scripts.

The major changes from the previous version involve a redesign of the user interface across the entire SUNDIALS suite. We have eliminated the mechanism of providing optional inputs and extracting optional statistics from the solver through the iopt and ropt arrays. Instead, packages now provide Set functions to change the default values for various quantities controlling the solver and Get functions to extract statistics after return from the main solver routine.

Additionally, the interfaces to several user-supplied routines (such as those providing Jacobians and preconditioner information) were simplified by reducing the number of arguments. The same information that was previously accessible through such arguments can now be obtained through Get-type functions.

In CVODE and CVODES a rootfinding feature was added, whereby the roots of a set of given functions may be computed during the integration of the ODE system.

### Changes to the NVector:

- Removed machEnv, redefined table of vector operations (now contained in the *N\_Vector* structure itself).
- All SUNDIALS functions create new *N\_Vector* variables through cloning, using an *N\_Vector* passed by the user as a template.
- · A particular vector implementation is supposed to provide user-callable constructor and destructor functions.
- Removed the following functions from the structure of vector operations: N\_VNew, N\_VNew\_S, N\_VFree, N\_VFree\_S, N\_VMake, N\_VDispose, N\_VGetData, N\_VSetData, N\_VConstrProdPos, and N\_VOneMask.
- Added the following functions to the structure of vector operations: N\_VClone, N\_VDestroy, N\_VSpace, N\_-VGetArrayPointer, N\_VSetArrayPointer, and N\_VWrmsNormMask.
- Note that nvec\_ser and nvec\_par are now separate modules outside the shared SUNDIALS module.

### Changes to the linear solvers:

- In SPGMR, added a dummy N\_Vector argument to be used as a template for cloning.
- In SPGMR, removed N (problem dimension) from the argument list of SpgmrMalloc.
- Replaced iterativ.{c,h} with iterative.{c,h}.
- Modified constant names in iterative.h (preconditioner types are prefixed with PREC\_).
- Changed numerical values for MODIFIED\_GS (from 0 to 1) and CLASSICAL\_GS (from 1 to 2).

### Changes to sundialsmath submodule:

- Replaced the internal routine for estimating unit roundoff with definition of unit roundoff from float.h.
- Modified functions to call the appropriate math routines given the precision level specified by the user.

#### Changes to sundialstypes submodule:

- Removed integertype.
- Added definitions for BIG\_REAL, SMALL\_REAL, and UNIT\_ROUNDOFF using values from float.h based on the
  precision.
- Changed definition of macro RCONST to depend on the precision level specified by the user.

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- [2] AMD ROCm Documentation. https://rocmdocs.amd.com/en/latest/index.html.
- [3] Intel oneAPI Programming Guide. https://software.intel.com/content/www/us/en/develop/documentation/oneapi-programming-guide/top.html.
- [4] KLU Sparse Matrix Factorization Library. http://faculty.cse.tamu.edu/davis/suitesparse.html.
- [5] NVIDIA CUDA Programming Guide. https://docs.nvidia.com/cuda/index.html.
- [6] NVIDIA cuSOLVER Programming Guide. https://docs.nvidia.com/cuda/cusolver/index.html.
- [7] NVIDIA cuSPARSE Programming Guide. https://docs.nvidia.com/cuda/cusparse/index.html.
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