

Data	$p_{\text{beam}}$ (MeV/c)	$\sigma_{p_{\text{beam}}}/p_{\text{beam}}$ (%)	Depth $d$ (mm)	Spacing $s$ ( $\mu\text{m}$ )	Diameter $\phi$ ( $\mu\text{m}$ )	Max emission efficiency $R_{\text{vac}}$ (%)
Simulation <sup>a</sup>	28	2.5	1	$45 \pm 5$	$152 \pm 16$	$4.124 \pm 0.007$
			2	$50 \pm 5$	$232 \pm 16$	$4.980 \pm 0.009$
			5	$35 \pm 5$	$360 \pm 16$	$7.92 \pm 0.02$
		5	1	$50 \pm 5$	$168 \pm 16$	$2.762 \pm 0.007$
			2	$50 \pm 5$	$248 \pm 16$	$3.172 \pm 0.008$
			5	$40 \pm 5$	$312 \pm 16$	$4.36 \pm 0.01$
		10	1	$50 \pm 5$	$152 \pm 16$	$2.166 \pm 0.007$
			2	$50 \pm 5$	$232 \pm 16$	$2.451 \pm 0.008$
			5	$45 \pm 5$	$312 \pm 16$	$3.22 \pm 0.01$
Ref. [23] (S18) <sup>a</sup>	23	2	1	85	165	$2.76 \pm 0.02$ <sup>b</sup>
Ref. [24] <sup>a</sup>	28	5	$4.75 \pm 0.25$	30	270	$3.05 \pm 0.03$ <sup>b</sup>
Ref. [25] (Aerogel-1)	12.5	3.4	$4.5 \pm 0.5$	$45 \pm 5$	$105 \pm 5$	$6.72 \pm 0.05^{+1.06}_{-0.76}$ <sup>c</sup>

<sup>a</sup> Only statistical errors are shown.

<sup>b</sup> Includes muonium decays within  $10 \text{ mm} < z < 40 \text{ mm}$  only.

<sup>c</sup> Model-dependent assumptions of the temperature at 400 K and the diffusion time at 200 ns.

TABLE II: Simulation of maximum vacuum muonium yield and corresponding optimal spacing and diameter with different beam condition.

Data	$p_{\text{beam}}$ (MeV/c)	$\sigma_{p_{\text{beam}}}/p_{\text{beam}}$ (%)	Depth $d$ (mm)	Spacing $s$ ( $\mu\text{m}$ )	Diameter $\phi$ ( $\mu\text{m}$ )	Max vacuum yield $Y_{\text{vac}}$ (%)
Simulation <sup>a</sup>	28	2.5	1	$50 \pm 5$	$152 \pm 16$	$1.092 \pm 0.002$
			2	$55 \pm 5$	$184 \pm 16$	$1.134 \pm 0.002$
			5	$55 \pm 5$	$184 \pm 16$	$1.122 \pm 0.002$
		5	1	$50 \pm 5$	$152 \pm 16$	$0.583 \pm 0.001$
			2	$60 \pm 5$	$216 \pm 16$	$0.607 \pm 0.001$
			5	$50 \pm 5$	$184 \pm 16$	$0.604 \pm 0.001$
		10	1	$50 \pm 5$	$152 \pm 16$	$0.305 \pm 0.001$
			2	$55 \pm 5$	$200 \pm 16$	$0.320 \pm 0.001$
			5	$60 \pm 5$	$232 \pm 16$	$0.321 \pm 0.001$
Ref. [23] (S18) <sup>a</sup>	23	2	1	85	165	$0.547 \pm 0.004$ <sup>bc</sup>
Ref. [24] <sup>a</sup>	28	5	$4.75 \pm 0.25$	30	270	$0.265 \pm 0.003$ <sup>bc</sup>
Ref. [25] (Aerogel-1) <sup>a</sup>	12.5	3.4	$4.5 \pm 0.5$	$45 \pm 5$	$105 \pm 5$	$1.22 \pm 0.01$ <sup>bd</sup>