Privacy Proofs for OpenDP: Impute Constant Transformation

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Contents

1 Algorithm Implementation

1.1 Code in Rust

The current OpenDP library contains the `make_impute_constant` function implementing the impute constant function. This is defined in lines 62-75 of the file `impute.rs` in the Git repository (https://github.com/opendp/opendp/blob/21-impute/rust/opendp/src/trans/impute.rs#L62-L75).

```rust
/// A [`Transformation`] that imputes elementwise with a constant value.
/// Maps a Vec<Option<T>> -> Vec<T> if input domain is AllDomain<Option<T>>,
///     or Vec<T> -> Vec<T> if input domain is NullableDomain<AllDomain<T>>
/// Type argument DA is "Domain of the Atom"; the domain type inside VectorDomain.
pub fn make_impute_constant<DA, M>(
    constant: DA::NonNull
) -> Fallible<Transformation<VectorDomain<DA>, VectorDomain<AllDomain<DA::NonNull>>, M, M>>
    where DA: ImputableDomain,
          DA::NonNull: 'static + Clone,
          DA::Carrier: 'static,
          M: DatasetMetric {
    if DA::is_null(&constant) { return fallible!(MakeTransformation, "Constant may not be null.") }

    make_row_by_row(
        DA::new(),
        AllDomain::new(),
        move |v| DA::impute_constant(v, &constant).clone())
}
```

1.2 Pseudo Code in Python

Preconditions

To ensure the correctness of the output, we require the following preconditions:

- **User-specified types:**
  - Variable `constant` must be of type `DA::NonNull`
  - Type `DA` must have traits `ImputableDomain`.
  - `DA::NonNull` must have traits `Clone`

Postconditions

- Either a valid `Transformation` is returned or an error is returned.

```
def make_impute_constant(constant : DA::NonNull):
    # instead of VectorDomain(DA), we add ::new() to get a new instance of DA. This is bcause DA has the ImputableDomain trait. Discuss among interns.
    input_domain = VectorDomain(DA::new())
    output_domain = VectorDomain(AllDomain(DA::NonNull))
    input_metric = SymmetricDistance()
    output_metric = SymmetricDistance()

    # check constant for nullity first
    assert(not constant.is_null); # not DA::is_null(constant)
    def Relation(d_in: u32, d_out: u32) -> bool:
        return d_out >= d_in*1

    def function(data: Vec[DA::Carrier]) -> Vec[DA::NonNull]:
        def impute_constant(x: DA) -> DA::NonNull:
            return constant if x.is_null else x
        return list(map(impute_constant, data))

    return Transformation(input_domain, output_domain, function, input_metric, output_metric, stability_relation=Relation)
    # return make_row_by_row(input_domain, output_domain, impute_constant);
```

(grace) Will need to change pseudocode so that it returns the result of a make row by row transformation (which the code does) instead of a Transformation directly.

1.3 Proof

(grace) Mike's comments: If an instance of DA::NonNull is null, it is either imputed or a construction-time error is raised by 2.

Lemma 1.1 (`DA::NonNull` contains `null`)**.** *`var` of type `DA::NonNull` can be of type `null`.*

Proof. Let the domain of atom variable `DA` be `InherentNullDomain<AllDomain<f64>>`. Recall that `InherentNullDomain` exists for types that can represent null inherently in the carrier type. Then the type

`DA::NonNull == InherentNullDomain<AllDomain<f64>>::NonNull == f64`.

The latter holds because in the `InherentNullDomain` implementation in the rust code https://github.com/opendp/opendp/blob/main/rust/opendp/src/trans/impute.rs#L48-L56, the `type NonNull = Self::Carrier`. The `Carrier` of `VectorDomain<AllDomain<T>>` has type `T`, so in this case the `::Carrier` is type `f64`.

Therefore `var` is also of type `f64`. `f64` can contain `null` values, so we are done. □

Theorem 1.2. *For every setting of the input parameters* `constant` *to* `make_impute_constant` *such that the given preconditions hold, the transformation returned by* `make_impute_constant` *has the following properties:*

1. (Appropriate output domain). *If vector v is in the* `input_domain`*, then* `function(v)` *is in the* `output_domain`*.*

2. (Domain-Metric Compatibility). *The domain* `input_domain` *matches one of the possible domains listed in the definition of* `input_metric`*, and likewise* `output_domain` *matches one of the possible domains listed in the definition of* `output_metric`*.*

3. (Stability Guarantee). *For every pair of elements v, w in* `input_domain` *and for every pair* (`d_in`, `d_out`)*, where* `d_in` *is of the associated type for* `input_metric` *and* `d_out` *is the associated type for* `output_metric`*, if v, w are d_{in}-close under* `input_metric` *and* `Relation`(`d_in`, `d_out`) = `True`*, then* `function`(v), `function`(w) *are d_{out}-close under* `output_metric`*.*

Proof. 1. **(Appropriate output domain).**

In the case of `make_impute_constant`, this corresponds to showing that for every vector v of elements of type `DA::Carrier`, `function`(v) is a vector of elements of type `DA::NonNull`. We can also say that `function`(v) is a vector of elements that does not contain any `NonNull` values.

The `function`(v) has type `Vec[DA::NonNull]` follows from the assumption that element v is in `input_domain` and from the type signature of `function` in line 13 of the pseudocode (Section 1.2). The type signature takes in an element of type `Vec[DA::Carrier]` and returns an element of type `Vec[DA::NonNull]`. If the Rust code compiles correctly, then the type correctness follows from the definition of the type signature enforced by Rust. Otherwise, the code will halt at compile time.

The type signature is not a sufficient check, since by Lemma 1.1, the function's output type can represent a value (e.g. `float nan`) that is not a member in the `output_domain`. (grace) How do I clarify that the output domain should actually contain NO NULLS since in the code it's loose? To ensure that function returns the appropriate output domain at run time, we must ensure that at run time the `function`(v) is not null. To do so, we must check whether the `constant` is not null.

Since the `constant` has type `DA::NonNull` and `DA::NonNull` may itself be `f64` by Lemma 1.1, without an extra check `constant` can be null. We must check whether `constant` is null because otherwise if `constant` is null, the user can nonsensically impute `null` entries in a vector with `null` value.

We check whether the constant is null in pseudo code line 9, before it gets called in `function`. If `constant` is `null`, then the check raises a construction-time error, so the function is never run. Thus we are done.

(grace) I didn't use the fact that v is of type DA::Carrier. (?)

2. **(Domain-metric compatibility).** The Symmetric distance is both the `input_metric` and `output_metric`. Symmetric distance is compatible with `VectorDomain(T)` for any generic type `T`, as stated in "List of definitions used in the pseudocode". The theorem holds because for `make_impute_constant`, the input domain is `VectorDomain(DA)` and the output domain is `VectorDomain(AllDomain(DA::NonNull))`.

3. **(Stability guarantee).** We know that $d_{in} \leq d_{out}$ because `Relation`$(d_{in},\ d_{out})$ `= True`. Since the vectors v, w are d_{in}-close, then $d_{Sym}(v, w) \leq d_{in}$.

   The function transformation just replaces the `null` element in vectors v and w with `constant`. Since the null element is also counted toward the symmetric distance of the transformation, the symmetric distance of `function`(v) and `function`(w) stays the same. Therefore the transformation is d_{out} close: $d_{sym}(\texttt{function}(v), \texttt{function}(w)) = d_{sym}(v, w) \leq d_{in} \leq d_{out}$

   □