# Active filters

## I. Theory

Active filters involve use of operational amplifiers along with resistors and capacitors. These filters provide a response similar to LRC filters without using inductors which make the latter bulky at low frequencies.

### *A. Butterworth Low Pass Filter (LPF)*

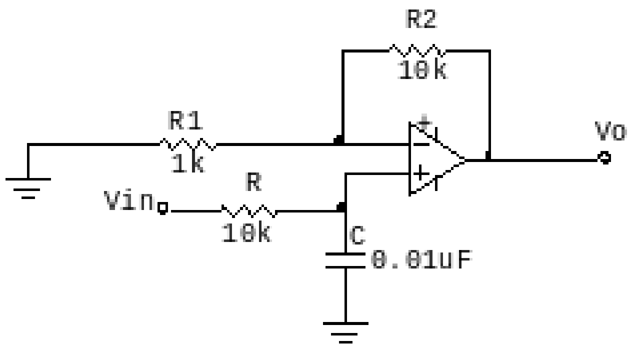


Figure 1

For the Butterworth LPF shown in Fig. 1, the transfer function is as follows:

$$\frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + sRC}\right) \tag{1}$$

where, the first term is the passband gain $k$ given by

$$k = \left(1 + \frac{R_2}{R_1}\right) \tag{2}$$

Solving Eq. 1 for the cutoff frequency results in the following

$$f_c = \frac{1}{2\pi RC} \tag{3}$$

Given the specifications of cutoff frequency $f_c$ and passband gain $k$, the filter can be designed.

*1) Design Steps:*

1) Choose a value of $C$ and get the value of $R$ for a given cutoff frequency using Eq. 3. Choice of $C$ is done first due to the larger available range in $R$ values.
2) The passband gain can be used to find the values of $R_1$ and $R_2$ by choosing one resistor and using Eq. 2 to calculate the other

### *B. Butterworth High Pass Filter (HPF)*

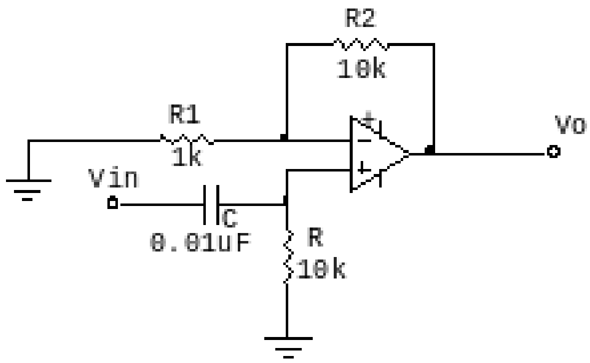


Figure 2

For the Butterworth HPF shown in Fig. 2, the transfer function obtained is

$$\frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{sRC}{1 + sRC}\right) \tag{4}$$

Solving for the cutoff frequency and passband gain results in the same equations as in the case of LPF. Design steps for the filter remain the same as LPF.

### *C. Wide Band Pass Filter (BPF)*

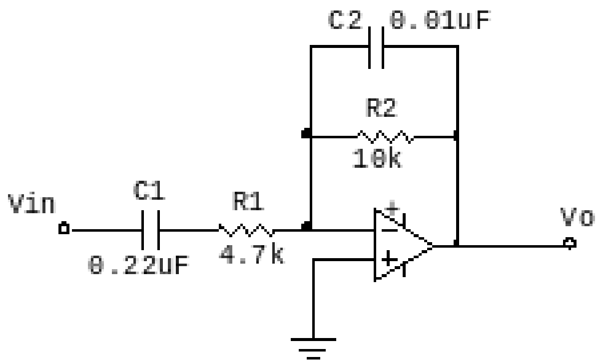


Figure 3

The transfer function for the wide BPF shown in Fig. 3 is as follows

$$\frac{v_o}{v_i} = \frac{-sR_2C_1}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_2C_2) + 1} \tag{5}$$

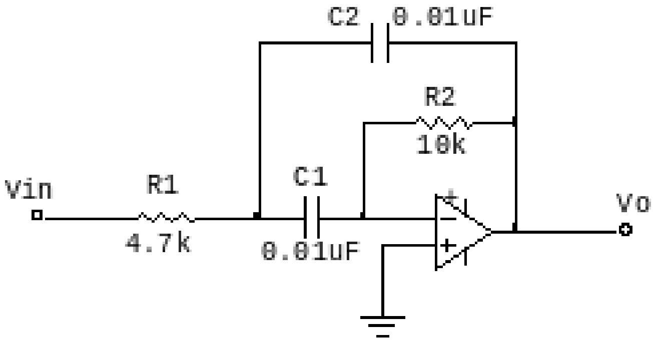


Figure 4

### D. Multiple Feedback BPF

The multiple feedback BPF shown in Fig. 4, has the transfer function as follows,

$$\frac{v_o}{v_i} = \frac{-sR_2C_1}{s^2R_1R_2C_1C_2 + sR_1(C_1 + C_2) + 1} \tag{6}$$

Choosing $C_1$=$C_2$=$C$ and solving Eq. 6 gives us the following

$$f_m = \frac{1}{2\pi C\sqrt{R_1R_2}} \tag{7}$$

$$k = -\frac{R_2}{2R_1} \tag{8}$$

$$B.W. = \frac{1}{\pi R_2 C} \tag{9}$$

where, $k$ is the gain at mid-frequency $f_m$ and $B.W.$ is the bandwidth.

#### 1) Design Steps:

1) Choose a value of $C$ and get the value of $R_1$ and $R_2$ for a given mid-band frequency and mid-band gain using Eq. 7 and 8.

## II. Lab Session

1) Design a Butterworth LPF having cutoff frequency of 1KHz and pass band gain of 10.
2) Apply a sine wave of 1Vp-p at the input and observe the output at 100Hz.
3) Sweep the frequency from 100Hz–500KHz and record observations. Record more readings around the cutoff frequency and less in the passband. Make sure that the input voltage amplitude is maintained at 1Vp-p as the frequency is varied. (This is required since the function generator output voltage at higher frequencies is often slightly smaller than that at lower frequencies).

4) Plot the magnitude plot and verify the frequency response.
5) Apply a square wave of 1Vp-p at the input and observe the output at 1KHz.