ECE 20002 – Spring 2022

Exam #3

April 13, 2022

Section (please mark on scantron)

Prof. Byunghoo Jung (07:30 AM) – Section 0002

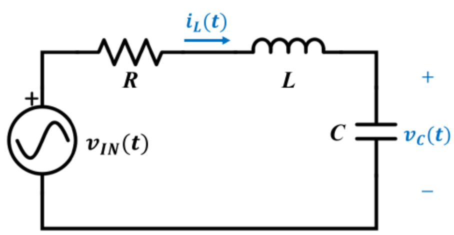
Prof. Michael Capano (03:30 PM) – Section 0003

Instructions

1. DO NOT START UNTIL TOLD TO DO SO.
2. Write your name, section, professor, and student ID# on your **Scantron** sheet. We may check PUIDs.
3. This is a CLOSED BOOKS and CLOSED NOTES exam.
4. The use of a TI-30X IIS calculator is allowed.
5. A formula sheet is provided on the last page. You may tear it out for reference during the exam.
6. Cheating will not be tolerated. Cheating in this exam will result in an F grade for the course. In particular, **continuing to write after the exam time is up is regarded as cheating**.
7. If you cannot solve a question, be sure to look at the other ones and come back if time permits.
8. You may not leave the room during the last 10 minutes of the exam.
9. We collect scantrons only. You may keep the exam booklets.

Question 1. (10 points)

In the circuit below, find $V_C(s)$ in s-domain. Assume the capacitor initial voltage $v_C(0) = 0$ V and the inductor initial current $i_L(0) = 0$ A.
[Note] Do not perform partial fraction.



$$R = 1\ [\Omega]$$
$$L = 1\ [H]$$
$$C = 1\ [F]$$
$$v_{in}(t) = \begin{cases} 2e^{-2t}\ [\mathrm{V}] & for \quad t \ge 0 \\ 0\ [\mathrm{V}] & for \quad t < 0 \end{cases}$$
$$v_C(0) = 0\ [\mathrm{V}]$$
$$i_L(0) = 0\ [\mathrm{A}]$$

$$\mathcal{L}\left[u(t)\right] = \frac{1}{s}$$
$$\mathcal{L}\left[e^{-at}\right] = \frac{1}{s+a}$$

(1) $V_C(s) = \dfrac{s+2}{2(s^2+s+1)}$

(2) $V_C(s) = \dfrac{s+2}{(s^2+s+1)}$

(3) $V_C(s) = \left(\dfrac{s^2+s+1}{s+2}\right)$

(4) $V_C(s) = \dfrac{2(s^2+s+1)}{s+2}$

(5) $V_C(s) = \left(\dfrac{1}{s^2+s+1}\right)\left(\dfrac{1}{s+2}\right)$

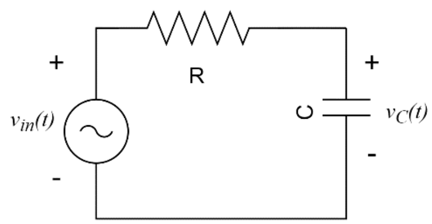
(6) $V_C(s) = \left(\dfrac{1}{s^2+s+1}\right)\left(\dfrac{2}{s+2}\right)$

(7) $V_C(s) = \left(\dfrac{1}{s^2+2s+1}\right)\left(\dfrac{1}{s+2}\right)$

(8) $V_C(s) = \left(\dfrac{1}{s^2+2s+1}\right)\left(\dfrac{2}{s+2}\right)$

Question 2. (10 points)

In the circuit below, find $V_C(s)$ in s-domain. Assume the capacitor initial voltage $v_C(0)$ = 5 V.
[Note] Do not perform partial fraction.



$$R = 1\ [\Omega]$$
$$C = 1\ [F]$$
$$v_{in}(t) = 5e^{-2t}u(t)\ [\mathrm{V}]$$
$$v_C(0) = 5\ [\mathrm{V}]$$

$$\mathcal{L}[u(t)] = \frac{1}{s}$$
$$\mathcal{L}[e^{-at}] = \frac{1}{s+a}$$

(1) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{5}{s+2}\right)$

(2) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{5}{s+2}\right) - \left(\frac{1}{s+1}\right)\left(\frac{5}{s}\right)$

(3) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{5}{s+2}\right) + \frac{5}{s}$

(4) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{5}{s+2}\right) - \left(\frac{1}{s+1}\right)\left(\frac{5}{s}\right) + \frac{5}{s}$

(5) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right)$

(6) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{5}{s+2}\right) - \left(\frac{1}{s+1}\right)5$

(7) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{5}{s+2}\right) + 5$

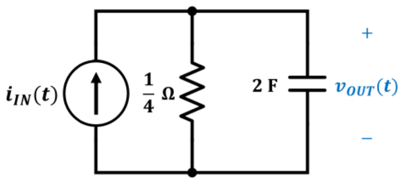
(8) $V_C(s) = \left(\frac{1}{s+1}\right)\left(\frac{5}{s+2}\right) - \left(\frac{1}{s+1}\right)5 + 5$

Question 3. (10 points)

In the circuit below, the output $v_{OUT}(t)$ consists of zero input response (ZIR) and zero state response (ZSR).

$$v_{OUT}(t) = v_{OUT,ZIR}(t) + v_{OUT,ZSR}(t)$$

Find the zero input response (ZIR) $v_{OUT,ZIR}(t)$.



$R = 0.25\ [\Omega]$
$C = 2\ [F]$
$i_{in}(t) = 2u(t)\ [\mathrm{A}]$
$v_C(0^-) = 2\ [\mathrm{V}]$

$\mathcal{L}[u(t)] = \frac{1}{s}$
$\mathcal{L}[e^{-at}] = \frac{1}{s+a}$

(1) $v_{OUT,ZIR}(t) = 0.5\ [\mathrm{V}]$

(2) $v_{OUT,ZIR}(t) = -0.5e^{-2t}\ [\mathrm{V}]$

(3) $v_{OUT,ZIR}(t) = 1.5e^{-2t}\ [\mathrm{V}]$

(4) $v_{OUT,ZIR}(t) = 2e^{-2t}\ [\mathrm{V}]$

(5) $v_{OUT,ZIR}(t) = 2.5e^{-2t}\ [\mathrm{V}]$

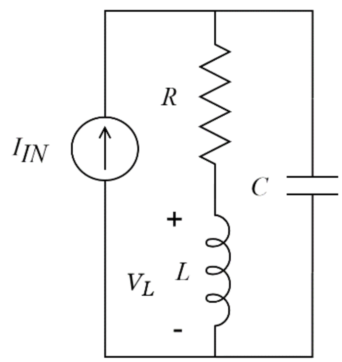
(6) $v_{OUT,ZIR}(t) = 0.5 - 0.5e^{-2t}\ [\mathrm{V}]$

(7) $v_{OUT,ZIR}(t) = 0.5 + 1.5e^{-2t}\ [\mathrm{V}]$

(8) $v_{OUT,ZIR}(t) = 0.5 + 2e^{-2t}\ [\mathrm{V}]$

Question 4. (10 points)

In the circuit below, find H(s) = $V_L(s)/I_{IN}(s)$. Assume all initial conditions are zero.



$R = 0.5\ [\Omega]$
$L = 1\ [H]$
$C = 1\ [F]$

(1) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{0.5}{s^2 + 0.5s + 1}$

(2) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{1}{s^2 + 0.5s + 1}$

(3) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{s}{s^2 + 0.5s + 1}$

(4) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{0.5}{s^2 + 2s + 1}$

(5) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{1}{s^2 + 2s + 1}$

(6) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{s}{s^2 + 2s + 1}$

(7) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{0.5}{s^2 + s + 1}$

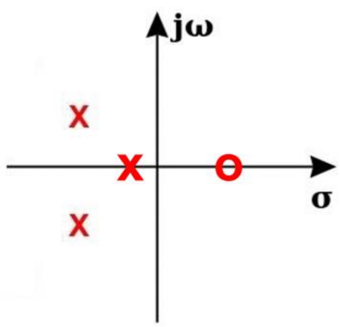
(8) $H(s) = \dfrac{V_L(s)}{I_{IN}(s)} = \dfrac{1}{s^2 + s + 1}$

Question 5. (10 points)

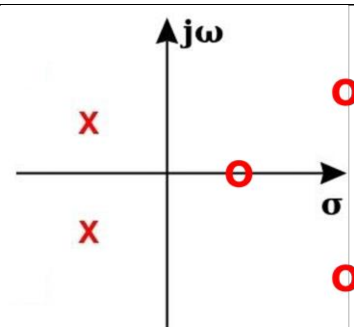
One of the transfer functions or pole-zero diagrams shown below represents an unstable system. Which one?

(1)	$$H(s)=\frac{7(s+3)(s+7)^2}{(s+4)^3(s+2+j3)(s+2-j3)}$$
(2)	$$H(s)=\frac{7(s-3)(s+7)^2}{(s+4)^3(s+2+j3)(s+2-j3)}$$
(3)	$$H(s)=\frac{7(s+3)(s+7)^2}{(s+4)^3(s-2+j3)(s-2-j3)}$$
(4)	$$H(s)=\frac{(s-7)}{(s+4)^3(s+2+j3)(s+2-j3)}$$

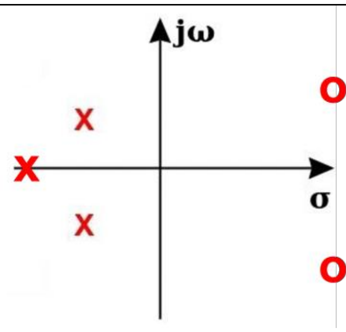
(5)



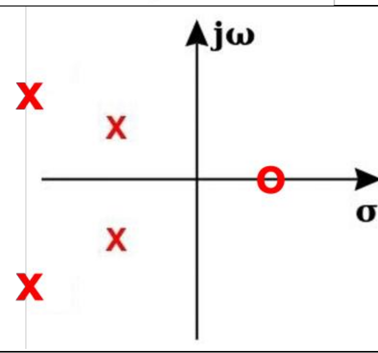
(6)



(7)

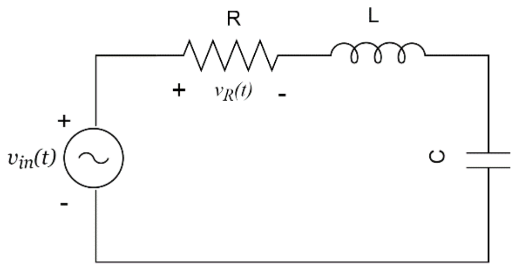


(8)



Question 6. (10 points)

In the circuit below, find $v_R(t)$ in steady-state.
[Hint] Check resonance frequency.

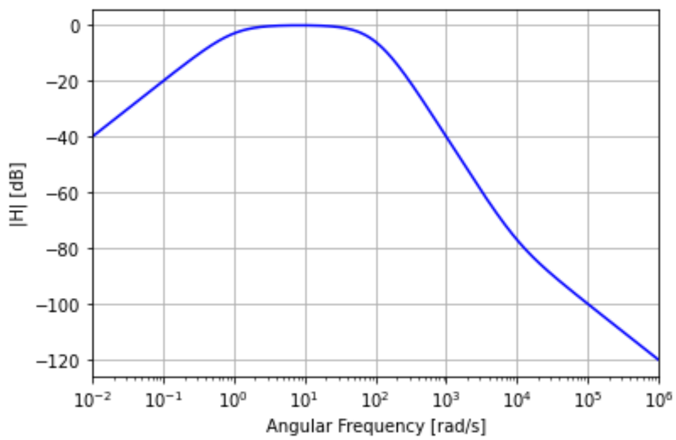


$R = 10\ [\Omega]$
$L = 0.1\ [H]$
$C = 0.1\ [F]$
$v_{in}(t) = 2\cos(10t + 45°)\ [\text{V}]$

1) $v_{in}(t) = 0.2\cos(10t + 45°)\ [\text{V}]$
2) $v_{in}(t) = -0.2\cos(10t + 45°)\ [\text{V}]$
3) $v_{in}(t) = 2\cos(10t + 45°)\ [\text{V}]$
4) $v_{in}(t) = -2\cos(10t + 45°)\ [\text{V}]$
5) $v_{in}(t) = 20\cos(10t + 45°)\ [\text{V}]$
6) $v_{in}(t) = -20\cos(10t + 45°)\ [\text{V}]$
7) $v_{in}(t) = 0.2\cos(10t + 45° + 90°)\ [\text{V}]$
8) $v_{in}(t) = -0.2\cos(10t + 45° + 90°)\ [\text{V}]$

Question 7. (10 points)

Which transfer function H(s) has the Bode magnitude plot shown below?
[Note] The y-axis is in dB scale and the x-axis is in log scale.



1) $H(s)=\dfrac{(s+10000)}{(s+1)(s+100)}$

2) $H(s)=\dfrac{s}{(s+1)(s+100)}$

3) $H(s)=\dfrac{s(s+10000)}{(s+1)(s+100)}$

4) $H(s)=\dfrac{(s+10000)}{(s+1)(s+100)^2}$

5) $H(s)=\dfrac{s}{(s+1)(s+100)^2}$

6) $H(s)=\dfrac{s(s+10000)}{(s+1)(s+100)^2}$

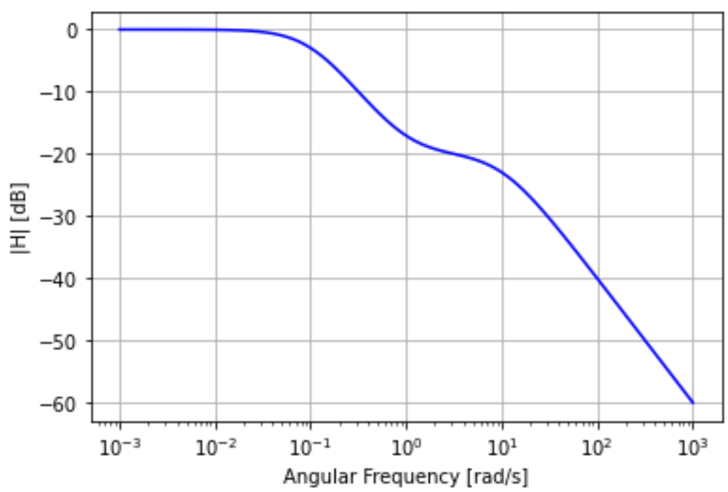
7) $H(s)=\dfrac{(s+10000)}{(s+100)^2}$

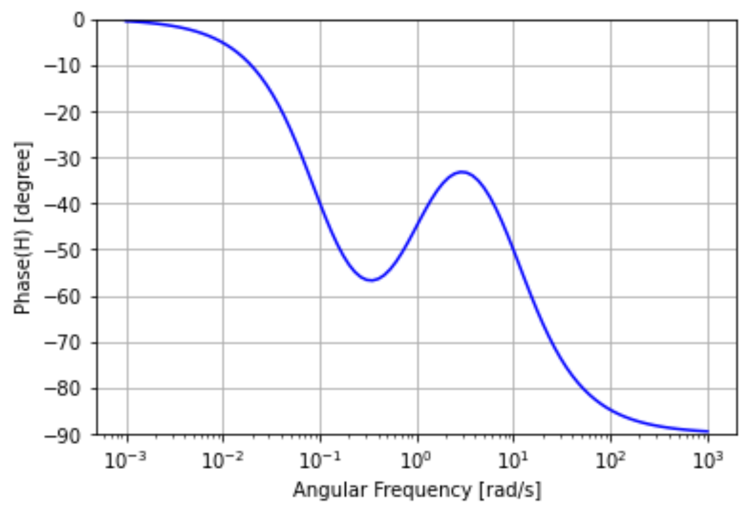
8) $H(s)=\dfrac{s}{(s+100)^2}$

Question 8. (10 points)

Shown below are the Bode plots of a transfer function H(s) = $V_{OUT}(s)/V_{IN}(s)$. When $v_{IN}(t) = 2\cos(0.001t + 15°) + 3\cos(1000t + 45°)$, find $v_{OUT}(t)$ in steady state using the Bode plots.

[Note] In the magnitude plot, the y-axis is in dB scale and the x-axis is in log scale. In the phase plot, the y-axis unit is degree, not radian.

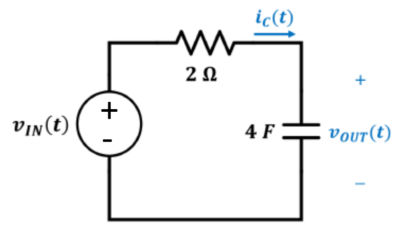




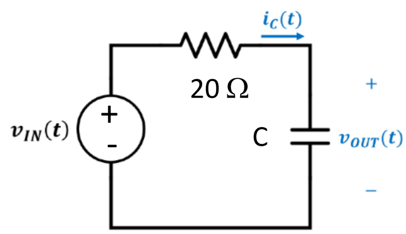
1) $v_{OUT}(t) = 2\cos(0.001t + 15°) + 3\cos(1000t + 45°)$
2) $v_{OUT}(t) = 2\cos(0.001t + 15°) + 0.003\cos(1000t - 45°)$
3) $v_{OUT}(t) = 2\cos(0.001t + 15°) + 0.003\cos(1000t - 90°)$
4) $v_{OUT}(t) = 2\cos(0.001t + 15°) + 0.003\cos(1000t - 135°)$
5) $v_{OUT}(t) = 0.002\cos(0.001t + 15°) + 3\cos(1000t + 45°)$
6) $v_{OUT}(t) = 0.002\cos(0.001t - 75°) + 0.003\cos(1000t - 45°)$
7) $v_{OUT}(t) = 0.002\cos(0.001t - 90°) + 0.003\cos(1000t - 45°)$
8) $v_{OUT}(t) = 0.002\cos(0.001t - 105°) + 0.003\cos(1000t - 45°)$

Question 9. (10 points)

The transfer function of the circuit shown below is $H(s) = \dfrac{V_{OUT}(s)}{V_{IN}(s)} = \dfrac{1}{8\left(s + \dfrac{1}{8}\right)}$.



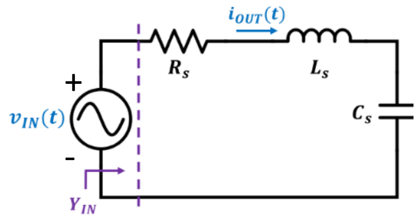
We would like to (a) use a 20 Ω resistor instead of the 2 Ω resistor, (b) keep the passband gain (low frequency gain) unchanged, and (c) increase the 3dB frequency (pole frequency) from 1/8 [rad/s] to 8 [rad/s]. Find the required capacitance value.



1) $C = 6.25\ m\text{F}$
2) $C = 62.5\ m\text{F}$
3) $C = 400\ \text{mF}$
4) $C = 625\ m\text{F}$
5) $C = 40\ \text{F}$
6) $C = 25.6\ \text{F}$
7) $C = 256\ \text{F}$
8) $C = 2560\ \text{F}$

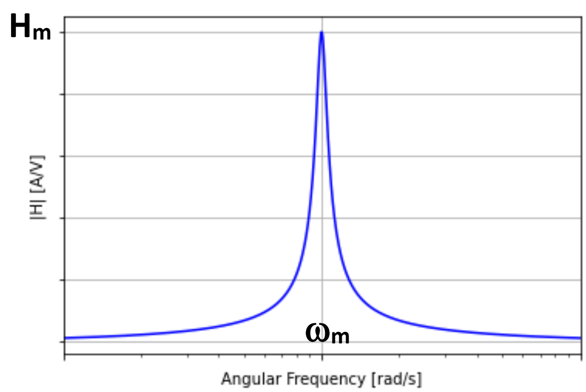
Question 10. (10 points)

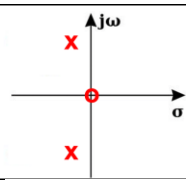
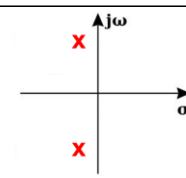
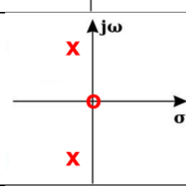
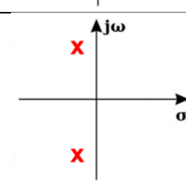
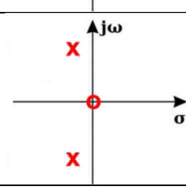
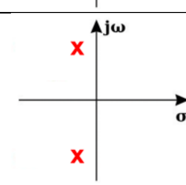
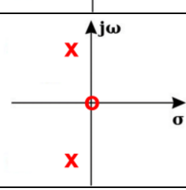
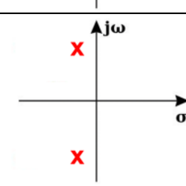
In the circuit below, a transfer function is defined as $H(s) = \dfrac{I_{OUT}(s)}{V_{IN}(s)}$.



$R_S = 0.1\ [\Omega]$
$L_S = 0.1\ [H]$
$C_S = 0.1\ [F]$

Shown below is the Bode magnitude plot of H(s). Find ω_m, H_m, and the pole-zero diagram of H(s).
[Note] The y-axis of the Bode magnitude plot is in linear scale. Do not use [dB].



1)	$\omega_m = 10$ [rad/s] $H_m = 10$ [A/V]		2)	$\omega_m = 10$ [rad/s] $H_m = 10$ [A/V]	
3)	$\omega_m = 100$ [rad/s] $H_m = 10$ [A/V]		4)	$\omega_m = 100$ [rad/s] $H_m = 10$ [A/V]	
5)	$\omega_m = 10$ [rad/s] $H_m = 0.1$ [A/V]		6)	$\omega_m = 10$ [rad/s] $H_m = 0.1$ [A/V]	
7)	$\omega_m = 100$ [rad/s] $H_m = 0.1$ [A/V]		8)	$\omega_m = 100$ [rad/s] $H_m = 0.1$ [A/V]	

Current-Voltage relationships

1.Resistors: $\mathcal{L}\{v_R(t) = Ri_R(t)\} = V_R(s) = RI_R(s)$ (23.1)

$\mathcal{L}\{i_R(t) = Gv_R(t)\} = I_R(s) = GV_R(s)$ (23.2)

2. Inductors:

$$\mathcal{L}\left\{v_L(t) = L\frac{di_L(t)}{dt}\right\} = V_L(s) = L[sI_L(s) - i_L(0^-)] \quad (23.3)$$

$$\mathcal{L}\left\{i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)d\tau\right\} = I_L(s) = \frac{1}{L}\left[\frac{V_L(s)}{s} + \frac{\int_{-\infty}^{0} v_L(\tau)d\tau}{s}\right] \quad (23.4)$$

3. Capacitors:

$$\mathcal{L}\left\{v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(\tau)d\tau\right\} = V_C(s) = \frac{1}{C}\left[\frac{I_C(s)}{s} + \frac{\int_{-\infty}^{0} i_C(\tau)d\tau}{s}\right] \quad (23.5)$$

$$\mathcal{L}\left\{i_C(t) = C\frac{dv_C(t)}{dt}\right\} = I_C(s) = C[sV_C(s) - v_C(0^-)] \quad (23.6)$$