MA 26100
FINAL INSTRUCTIONS
VERSION 01
December 12, 2022

Your name ____________________ Your TA's name ____________________

Student ID # _______________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. While mark all your work on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

9. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

10. If you finish the exam before 9:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 9:55, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

1. Find an equation of the tangent plane to the paraboloid

$$z = 1 - \frac{1}{10}\left(x^2 + 4y^2\right) \ ,$$

at the point $(1, 1, 1/2)$.

A. $\frac{4}{5}x + \frac{1}{5}y + z = \frac{2}{3}$
B. $\frac{1}{5}x + \frac{4}{5}y + z = \frac{3}{2}$
C. $\frac{4}{5}x + \frac{4}{5}y + z = \frac{2}{3}$
D. $\frac{4}{5}x + \frac{1}{5}y + z = \frac{3}{2}$
E. $\frac{1}{5}x + \frac{1}{5}y + z = \frac{2}{3}$

2. Find the points where f has a local extremum, for f given by

$$f(x,y) = -x^3 + 4xy - 2y^2 + 1\,.$$

A. $(0,0)$ local maximum.

B. $(4/3, 4/3)$ local minimum and $(0,0)$ local maximum.

C. $(4/3, 4/3)$ local maximum and $(0,0)$ local minimum.

D. $(4/3, 4/3)$ local minimum.

E. $(4/3, 4/3)$ local maximum.

3. Find the work done by the force

$$\mathbf{F} = -\frac{1}{2}x\,\mathbf{i} - \frac{1}{2}y\,\mathbf{j} + \frac{1}{4}\,\mathbf{k},$$

on a particle as it moves along the helix

$$\mathbf{r}(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j} + t\,\mathbf{k}\,,$$

from the point $(1, 0, 0)$ to the point $(-1, 0, 3\pi)$.

A. $\frac{4\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{4}$

E. $\frac{\pi}{2}$

4. Let $\mathbf{F}$ be the conservative vector field given by $\mathbf{F}(x,y) = \langle y^3+1,\, 3xy^2+1 \rangle$. Consider a semicircular path C_1 from $(0,0)$ to $(2,0)$, that is

$$C_1 : \{\mathbf{r}(t) = \langle 1 - \cos(t),\, \sin(t) \rangle;\, 0 \le t \le \pi\}\ .$$

Evaluate $\int_{C_1} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$.

A. 0

B. 3

C. 1

D. 2

E. -2

5. Consider the circle C centered at 0 with radius 3. A particle travels once around C, counterclockwise. It is subject to the force

$$\mathbf{F}(x, y) = \left\langle y^3,\, x^3 + 3xy^2 + 1 \right\rangle .$$

Use Green's theorem to find the work done by $\mathbf{F}$.

A. $\frac{3\pi}{4}$

B. $\frac{4\pi}{3}$

C. $\frac{243\pi}{4}$

D. $\frac{117\pi}{4}$

E. $\frac{23\pi}{3}$

6. Find the arclength of the curve $\vec{\mathbf{r}}(t) = \langle 12 \sin t, 5 \sin t, 13 \cos t \rangle$ from $t = \frac{\pi}{4}$ to $t = \frac{5\pi}{4}$.

A. π

B. 13π

C. $\frac{13\pi}{4}$

D. 12π

E. $\frac{65\pi}{4}$

7. What value of c makes the function

$$f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y} & \text{if} \quad x \neq y \\ cx & \text{if} \quad x = y \end{cases}$$

continuous?

A. 4

B. 0

C. 2

D. 1

E. Such a c does not exist

8. Choose the triple integral in spherical coordinates that represents the volume of the solid bounded by the cone $z = -\sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2 = 16$.

A. $\displaystyle\int_0^{2\pi}\int_{\frac{3\pi}{4}}^{\pi}\int_0^4 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

B. $\displaystyle\int_0^{2\pi}\int_{\frac{3\pi}{4}}^{\pi}\int_0^4 \rho \sin\phi \, d\rho \, d\phi \, d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^4 \rho \sin^2\phi \, d\rho \, d\phi \, d\theta$

D. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^4 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

E. $\displaystyle\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_0^4 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

9. The integral

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{\sqrt{2x^2+2y^2}}^{\sqrt{27-x^2-y^2}} xyz\ dz\ dy\ dx$$

when converted to cylindrical coordinates becomes

A. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^3 \int_{\sqrt{2}r}^{\sqrt{27-r^2}} r^3 z \cos\theta \sin\theta \, dz \, dr \, d\theta$

B. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^3 \int_{\sqrt{2}r}^{\sqrt{27-r^2}} r^2 z \cos\theta \sin\theta \, dz \, dr \, d\theta$

C. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^3 \int_{\sqrt{2}r}^{\sqrt{27-r^2}} r^3 \cos\theta \sin\theta \, dz \, dr \, d\theta$

D. $\displaystyle\int_0^{\pi} \int_0^3 \int_{\sqrt{2}r}^{\sqrt{27-r^2}} r^3 z \cos\theta \sin\theta \, dz \, dr \, d\theta$

E. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^9 \int_{\sqrt{2}r}^{\sqrt{27-r^2}} r^3 z \cos\theta \sin\theta \, dz \, dr \, d\theta$

10. $\vec{\mathbf{F}} = \langle -y, x, z^3 \rangle$, S is part of the sphere $x^2 + y^2 + z^2 = 4$ above $z = 1$, with upward orientation. Compute $\iint_S (\vec{\nabla} \times \vec{\mathbf{F}}) \cdot d\vec{S}$.

A. -6π

B. 6π

C. 0

D. 3π

E. -3π

11. Find the area of the triangle that has vertices at P(1,2,1),Q(2,3,2), and R(0,2,3).

A. $\sqrt{3}$

B. $\sqrt{2}$

C. $\sqrt{14}$

D. $\frac{\sqrt{14}}{2}$

E. $2\sqrt{17}$

12. Find $\vec{r}(1)$ if $\vec{r''}(t) = 12t\vec{i} + 12t^2\vec{j} + \vec{k}$ and $\vec{r}(0) = \vec{j}$ and $\vec{r'}(0) = -\vec{k}$

A. $2\vec{i} + 2\vec{j} - \frac{1}{2}\vec{k}$

B. $6\vec{i} - \vec{j}$

C. $2\vec{i} - 3\vec{j} + \vec{k}$

D. $2\vec{i} + \vec{k}$

E. $2\vec{i} + \vec{j} + \frac{1}{2}\vec{k}$

13. Use the method of Lagrange multipliers to find the x components only of the points where the absolute maximum and absolute minimum occur for

$$f(x, y) = (x - 2)^2 + (y - 4)^2$$

on the curve

$$x^2 + y^2 = 5$$

A. 2 and -2

B. 0 and -1

C. 1 and -1

D. -2 and 1

E. 1 and 0

14. Reverse the order of integration and evaluate $\int_0^1 \int_{x^{1/3}}^1 \frac{1}{1+y^4} dy\, dx$

A. $\frac{\ln 2}{4}$

B. $\ln 2$

C. $4 \ln 2$

D. $4(\ln 2 - 1)$

E. $\ln 2 - 1$

15. Evaluate the integral $\displaystyle\int_0^{\sqrt{2}}\int_{-3}^{3}\frac{xy^2}{1+x^2}\,dy\,dx$

A. $6\ln 3$

B. $9\ln 3$

C. $12\ln 3$

D. $18\ln 3$

E. $27\ln 3$

16. Suppose that $f(u, v) = e^v \sin(u)$, where $u = s - t$ and $v = t^2$. Find $\frac{\partial f}{\partial t}$ when $(s, t) = (1, 1)$.

A. e

B. $-e$

C. 1

D. $e(\sin(1) - \cos(1))$

E. $e(\sin(1) - 2\cos(1))$

17. Let $f(x, y) = x^2 e^{x+y}$. Find a unit vector in the direction of most rapid decrease for f when $(x, y) = (1, 1)$.

A. $\frac{\langle -3, -1 \rangle}{\sqrt{10}}$

B. $\frac{\langle 3, 1 \rangle}{\sqrt{10}}$

C. $\frac{\langle -3e^2, -e^2 \rangle}{\sqrt{10}}$

D. $\langle 3e^2, e^2 \rangle$

E. $\langle -3e^2, -e^2 \rangle$

18. Find the centroid of the volume bounded by $z = 1$ and $z = x^2 + y^2$.

A. $(1/3, 1/3, 2/3)$

B. $(0, 0, 1/3)$

C. $(0, 0, 2/3)$

D. $(2/3, 0, 2/3)$

E. $(0, 0, 1)$

19. Let $\mathbf{F} = \langle ax, cz - ax, cz + by \rangle$ be a vector field, where $a, b, c \in \mathbb{R}$. Find conditions on a, b and c so that $\mathbf{F}$ is **not** conservative and such that $\text{curl}(\mathbf{F})$ is parallel to $\mathbf{k}$.

A. $b = c$

B. $a = c$

C. $b = c$ and $a = 0$

D. $b = c$ and $a \neq 0$

E. $a = b = c = 0$

20. Find the mass of the part of the plane $z = 8 - 2x - y$ that lies over the square $[0, 1] \times [0, 1]$ when the density function is given by $f(x, y, z) = 12 - z$.

A. 12

B. $11\sqrt{6}$

C. $\sqrt{6}$

D. $\frac{11}{2}$

E. $\frac{11\sqrt{6}}{2}$