CS251 – Data Structures and Algorithms
First Midterm, Fall 2022

Wednesday, September 14^{th}, 2022

Printed Name: ____

PUID: ____

1. You have 50 minutes to finish the exam. Non–authorized electronic devices are not allowed. Sharing material is not allowed and penalized with immediate failure of the exam.
2. If you are not clear about what a question is asking, you can explain what you are answering (e.g., "I think this question is asking for ...") or you can state assumptions that you are making (e.g., "I assume that the entry -1 is equivalent to NULL").
3. Always show as much of your work as practical. Partial credit is largely a function of the clarity and quality of the work shown. Be concise. This exam consists of 14 pages (including this cover sheet); please check to make sure that all of these pages are present before you begin. Credit will not be awarded for pages that are missing. It is your responsibility to make sure that you have a complete copy of the exam.
4. If you finish the test with more than 5 minutes remaining, you may turn in the test and leave. **If you finish with less than 5 minutes remaining, you have to stay until we release the entire class. Stop writing at when the time runs out. If you continue to write, that is considered cheating.**
5. **IMPORTANT**: Write your Purdue username at the TOP of EACH page. Also, be sure to *read* and *sign* the *Academic Honesty Statement* that follows:

Read and sign the Academic Honesty Statement that follows:

"In signing this statement, I hereby certify that the work on this exam is my own, that I have not copied the work of any other student while completing it, that I have not obtained help from any other student, and that I will not provide help to any other student. I understand that if I fail to honor this agreement, I will receive a score of ZERO for this exam and will be subject to disciplinary action as outlined in the course policy."

Signature: ____

If the statement is not signed, the exam will not be graded.

DO NOT BEGIN UNTIL INSTRUCTED TO DO SO ...

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Question	Points	Score
1	2	
2	16	
3	20	
4	12	
5	12	
6	6	
7	12	
8	20	
Total:	100	

Question 1

Before you start:

- Read all questions and budget your time.
- Prioritize the questions you fell secure the most.
- Remember to write your Purdue username at the top right corner of each page.

Also:

(a) (2 points) Do not tear off the final page of the exam. Your answers should be legible, organized, and coherent. Also, they should fit in the given space. In case an answer doesn't fit the given space, indicate that you are continuing the answer in the extra page at the end of the exam (which is why you should not tear it off).

You get one points just for reading and complying with these two instructions.

Question 2

Amy developed an algorithm and ran several experiments on it to analyze its runtime. She collected the following data:

n	$T(n)$
40	15
80	60
160	240
320	960

(a) (8 points) The algorithm took too long for an input size of $n = 640$. Assuming we can use the Power Law $T(n) = an^b$ to propose a runtime expression of the algorithm, make a reasonable prediction for $T(640)$.

(b) (8 points) Assuming we can use the Power Law $T(n) = an^b$ to propose a runtime expression of the algorithm, what is the value of b?

Question 3

Alice, Jane and Elizabeth designed three new algorithms for Mango Enterprises. Let $T_A(n)$ (resp. $T_J(n)$ and $T_E(n)$) denote the worst-case running time of Alice's (resp. Jane's and Elizabeth's) algorithm on any input of size n. Mango Enterprises works with very large datasets so they are only able to use algorithms with worst case running time $T(n) \in O\left(n^2\right)$.

(a) (6 points) Alice runs her algorithm on datasets of size $n = 10^6$ and $n = 10^7$ and finds that the algorithm terminates in 3×10^6 and 3×10^7 steps respectively. Based on the observations above can Alice conclude that $T_A(n) \in O(n)$?

Solution: No, we do not have enough information to conclude that $T_A(n) \in O(n)$. There are several correct explanations: 1) $T_A(n)$ denotes the worst-case running time while the empirical measurements are not necessarily worst case measurements. It is possible, for example, that the average case running time of Alice's algorithm on a random dataset is $O(n)$ but that the worst case running time is larger e.g., $\Omega(n^2)$. 2) Another correct observation is that measuring $T_A(n)$ for $n = 10^6$ and $n = 10^7$ does not tell us how $T_A(n)$ grows as n gets larger e.g., $T_A(n) = 3n + \lfloor 2^{-251} n^2 \rfloor$ which is consistent with our observations, but is not $O(n)$.

(b) (7 points) Bob analyzed Jane's algorithm and claims to have a proof that $T_J(n) \in O(g(n))$ for some function $g(n)$ such that $g(n) \geq n^9$ for all $n \geq 1$. Assuming that Bob's proof is correct is this a sufficient reason for Mango Enterprises to discard Jane's algorithm? Explain your answer.

Solution: We cannot draw any conclusions. First observe that by definition we have $g(n) \in \Omega(n^9)$ or equivalently $n^9 \in O(n^9)$. It is possible that $T_J(n) = n$ which is fast enough for Mango Enterprises to consider and still consistent with the statement $T_j(n) \in O(g(n))$ i.e., in this case we have $T_J(n) = n \in O(n^9)$ and $n^9 \in O(g(n))$ so it follows that $T_J(n) \in O(g(n))$. On the otherhand we would have $T_j(n) = n^3$ which is too slow for Mango and also consistent with Bob's claim that $T_j(n) \in O(g(n))$.

(c) (7 points) Bob analyzed Elizabeth's algorithm and proved that $T_E(n) \in \Omega(g(n))$ for some positive function $g(n)$ such that $g(n) \geq n + 2^{-251}n^3 - 2^{251}n^2$ for all $n \geq 2$. Assuming that Bob's proof is correct show that $T_E(n) = \Omega(n^3)$.

Solution: The algorithm is too slow for Mango. Bob proved that $T_E(n) \in \Omega(g(n))$ and we know that $g(n) \in \Omega(n^3)$ it follows that $T_E(n) \in \Omega(n^3)$. Too see that $g(n) \in \Omega(n^3)$ we can fix the constant $c = 2^{-252}$ and $n_0 = 2^{1+2\times 251}$ and observe that for all $n \geq n_0$ we have $g(n) \geq 2^{-252}n^3$.

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Question 4

Let S be an empty stack. Consider a sequence of intermixed push and pop calls on S. The arguments for the push calls are the numbers 1, 2, 3, 4, 5, and 6 (one at a time in that order). The pop calls happen at arbitrary time (when S is not empty) and each pop prints out the returned value.

(a) (6 points) Is the printed sequence **4 3 5 6 2 1** possible? Explain your answer.

(b) (6 points) Is the printed sequence **5 6 3 4 2 1** possible? Explain your answer.

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Question 5

Let Q be an empty queue implemented using a circular array (as described on class) of capacity $m =$. Q starts with f (front) and b (back) indices at index 0 of the array. We run the following sequence of operations on Q: **enqueue(A), enqueue(B), enqueue(C), dequeue(), dequeue(), enqueue(D), enqueue(E), dequeue(), enqueue(F)**.

(a) (6 points) Show the content of the array and the location of indices f and b after running the sequence.

(b) (6 points) Can we change the order of the three dequeue calls such that $f = 2$ and $b = 0$ after finishing the sequence of operations on Q? Explain your answer.

Question 6

Let Q be an empty queue. Consider a sequence of intermixed enqueue and dequeue calls on Q. The arguments for the enqueue calls are the numbers 1, 2, 3, 4, 5, and 6 (one at a time in that order). The dequeue calls happen at arbitrary time (when Q is not empty) and each dequeue prints out the returned value.

(a) (6 points) Is the printed sequence **1 2 3 4 6 5** possible? Explain your answer.

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Question 7

(a) (6 points) Draw the expression tree for $\left(bx^2 - axy\right) / (by + c)$.

(b) (6 points) Write the preorder, inorder, postorder, and level traversals of the tree from b).

Question 8

Dan and Erin are discussing the behavior of the recursive algorithm below:

```
function Foo(n)
    if n == 1 then
        return 1
    end if
    for i = 1, i ≤ n, i += 1 do
        Bar(i)
    end for
    return n + Foo(b(n))
end function
```

(a) (10 points) Dan says "If $b(n) = n - 1$ then the number of times Foo calls Bar is $O(n)$". Is Dan right? Explain your answer.

(b) (10 points) Erin says "If $b(n) = \frac{n}{2}$ then the number of times Foo calls Bar is $O(n)$. Is Erin right? Explain your answer.

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Extra page #1

You can use this page to write additional work in case the provided space was not enough. You must indicate the question number before elaborating on the answers. YOU MUST NOT REMOVE THIS PAGE FROM THE EXAM.

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Extra page #2

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Extra page #3

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Extra page #4

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