COMP3121: Assignment 2 – Q5

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Let x be a sequence such that $x * \langle 1, 1, -1 \rangle = \langle 1, 0, -1, 2, -1 \rangle$. Since the right hand side contains 5 elements in the sequence, then it follows that x must contain $5 - 3 + 1 = 3$ elements. Hence let $x = \langle \alpha, \beta, \gamma \rangle$. Then the convolution can be written equivalently in polynomial form as

$$(\alpha + \beta x + \gamma x^2)(1 + x - x^2) = 1 + 0x - x^2 + 2x^3 - x^4.$$

Expanding the left hand side, we have

$$\alpha + (\alpha + \beta)x + (-\alpha + \beta + \gamma)x^2 + (-\beta + \gamma)x^3 - \gamma x^4.$$

Thus, comparing coefficients, we attain the following system of equations

$$\begin{aligned} \alpha &= 1 \\ \alpha + \beta &= 0 \\ -\alpha + \beta + \gamma &= -1 \\ -\beta + \gamma &= 2 \\ -\gamma &= -1. \end{aligned}$$

Hence, we see that $\alpha = 1, \beta = -1, \gamma = 1$. So the polynomial version of x is $1 - x + x^2$ and so x can be written as

$$x = \langle 1, -1, 1 \rangle.$$