COMP3121: Assignment 3 – Q3

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Given a timetable schedule of arrivals and departures, we make a note of how many trains have the property such that $a_i > d_i$ where a_i denotes the arrival time of train at index i and d_i denotes the departure time of train at index i. This will be the initial value of a counter. The maximum value of the counter will be the minimum number of platforms required so that the trains can stay on the platform without interfering with arrivals and departures of other trains.

To calculate the counter, we sort the timetable schedule in increasing order of time, making a note of whether the time is an arrival or a departure time of a particular train. If the time is an arrival, then increment the counter to signify that we require another platform. Conversely, if the time is a departure, then decrement the counter to signify that a platform has been freed up. Note that this means that the counter, at any point in time, signifies the number of platforms currently at use.

The maximum counter, as a result, signifies the number of platforms that will accompany the greatest number of trains in the station at any time, and hence, the maximum value of the counter will return the minimum number of platforms required.

Proof of optimality

We shall prove that this strategy is optimal with a proof by contradiction. In particular, we will show that, if the strategy is not optimal, then either it overestimated or underestimated the number of platforms required.

Suppose that the strategy *underestimated* the number of platforms required. Then there is a pair of trains denoted by the tuple (i, j) such that $a_i < a_j$ but all of the platforms are full and so trains i and j must share the same platform. Since each departure decrements the counter which signifies that there is a platform free, then this is only possible if no trains are departing at any point. However, when no train departs, the greedy strategy *increments* the count of platforms to signify that an additional platform is required. As a result, train j must then share a separate platform to train i which contradicts the condition for this to happen. Hence, the maximum is *at least* the strategy's output. We will now show that this strategy *cannot* overestimate the number of platforms needed.

Conversely, suppose that this strategy *overestimated* the number of platforms required. Define the maximum intersection time as k and define the train set $\{1, 2, 3, \ldots, n\}$ to be the number of trains that are concurrently running in that time slot. Then this implies that the greedy solution has picked up *at least* $n+1$ platforms to use. Since the maximum intersection occurring contains n trains, then there must be *at most* n platforms since each departure decremented the counter at any time $< k$. Hence, this implication cannot be made and we, again, arrive at a contradiction.

Thus, in both of these cases, the greedy strategy does not underestimate nor overestimate the minimum number of platforms required.