IGEM, a Model of U.S. Growth and the Environment. Version 20

Dale Jorgenson, Richard Goettle, Mun Ho, Peter Wilcoxen

Appendix A. Equations of the Model and Glossary

March 2017

A.0 Notation:

Time

$t \in I_T$ $\quad I_T = \{1,2,\ldots,T,\ldots\}$

Industry/Producer

$j \in I_{IND}$ $\quad I_{\text{IND}} = \{1,2,\ldots,36\}$

IO Commodities

$i \in I_{COM}$ $\quad I_{\text{COM}} = \{1,2,\ldots,36\}$

Industry Inputs

$i \in I_{INP}$ $\quad I_{\text{INP}} = \{1,2,\ldots,36,K,L\}$

NIPA PCE Commodities

$n \in I_{PCE}$ $\quad I_{\text{PCE}} = \{1,2,\ldots,36\}$ Table 2.3

$n \in I_{\text{PCE-R}}$ $\quad I_{\text{PCE-R}} = \{1,2,\ldots,36,R\}$

Purchasers of domestic output

$j \in I_{BUY}$ $\quad I_{\text{BUY}} = \{1,2,\ldots,36,C,I,G,X\}$

Households

$k \in I_{POP}$

Nodes of production function

$m \in I_{PNODE}$ $\quad I_{PNODE} = \{E,M,\ldots,OS\}$

$i \in I_{PNODEm}$ $\quad I_{PNODEm}$ in Table 2.2

Nodes of consumption function

$m \in I_{CNODE}$ $\quad I_{\text{CNODE}} = \{ND,KS,\ldots,COM\}$

$i \in I_{CNODEm}$ $\quad I_{CNODEm}$ in Table 2.4

Nodes of investment function

$m \in I_{INV}$ $\quad I_{\text{INV}} = \{\text{fixed},\ldots,\text{mining}\}$

$i \in I_{INVm}$ $\quad I_{INVm}$ in Table 5.4

Externalities

$x \in I_{EXT}$ $\quad I_{EXT} = \{1,2,3,4\} = \{CO_2, SO_2, \ldots\}$

Vector of *1*'s

ι

Transpose of matrix *A*

A'

Diagonal matrix of a vector *v*

Diag(*v*)

A.1 Household Sector

***Household first stage decision, Euler equation*:**

$$\text{Max}\sum_{t=1}^{\infty}\frac{N_t^{eq}}{(1+\rho)^t}\left(F_t/N_t^{eq}\right)^{1-\frac{1}{\sigma}} \qquad \text{given } K_0,\{\bar{L}_t\}, \tag{A.1.1}$$

subject to

$$WF \equiv PK_0K_0 + BG_0 + BF_0 + \sum_{t=1}^{\infty}\frac{Y_t^{full}}{\prod_{s=1}^{t}1+r_s} \geq \sum_{t=1}^{\infty}\frac{PF_tF_t}{\prod_{s=1}^{t}1+r_s} \tag{A.1.2}$$

$$Y^{full} = P^h\bar{L} + G^{TRAN} - twW_{t-1} - TLUMP - H^{row} - R^N \tag{A.1.3}$$

$$\left[\frac{F_t/N_t^{\text{eq}}}{F_{t-1}/N_{t-1}^{\text{eq}}}\right]^{1/\sigma} = \frac{1+r_t}{1+\rho}\frac{PF_{t-1}}{PF_t} \tag{A.1.4}$$

Wealth, private income and savings:

$$W_t \equiv PK_tK_t + BG_t + BF_t \tag{A.1.5}$$

$$YF_t = YK_t^{net} + P^h\bar{L} + G^{tran} - TLUMP_t - twW_{t-1} + G_t^{Ktran} + R\_CON^{reb} \tag{A.1.6}$$

$$\begin{aligned} Y_t &= YK_t^{net} + YL_t + G^{tran} - TLUMP_t - twW_{t-1} + G_t^{Ktran} + R\_CON^{reb} \\ &= YF_t - p_t^{leis}L_t^{leis} = YF_t - w_t\psi_C^RC_R^N \end{aligned} \tag{A.1.7}$$

$$YL = P^hLS\frac{1\text{-}(1-\theta^{ephi})tl_{ephi}^a}{1\text{-}tl^m} = P^h(\bar{L} - \psi_C^RN^R) \qquad \text{(from A1.32)} \tag{A.1.8}$$

$$= (1-(1-\theta^{ephi})tl_{ephi}^a)YL^{gross} = (1-(1-\theta^{ephi})tl_{ephi}^a)\sum_j PLD_jLD_j \qquad \text{from A.6.14}$$

$P^hLS = P^h(\bar{L} - \psi_C^RN^R)$ (from A.1.32)

YK^{net} is eq. A.3.15

$$\begin{aligned} S_t &= YF_t - PF_tF_t - H_t^{row} - R_t^N - R\_ITC \\ &= YF_t - w_t\psi_C^RC_R^N - P_t^CC_t - H_t^{row} - R_t^N - R\_ITC \end{aligned} \tag{A.1.9}$$

$$= Y_t - P_t^CC_t - H_t^{row} - R_t^N - R\_ITC \tag{A.1.10}$$

Household and business net transfers to foreigners set exogenous in levels, or as fixed shares of GDP:

$$\bar{H}^{row} \tag{A.1.11a}$$

$$H^{row} = \alpha^{H\_row}GDP \tag{A.1.11b}$$

***Household second stage decision, goods and leisure choice*:**

Rank 2 model estimated for household k from CEX data; indirect utility function is:

$$\ln V_k = \alpha_0 + \alpha^H \ln \frac{p_r}{m_k} + \tfrac{1}{2} \ln \frac{p_r}{m_k}' \mathbf{B}^H \ln \frac{p_r}{m_k} + \ln \frac{p_r}{m_k}' \mathbf{B}_{pA} A_k \qquad \text{(A.1.12)}$$

$\mathbf{c}_k^X = (C_{NDk}^X, C_{Kk}^X, C_{CSk}^X, C_{Rk}^X)'$ consumption vector

$p_r = (p_{ND}^r, p_K^r, p_{SV}^r, p_R^r)'$ price vector indexed by region

$$m_k = p_{ND}^r C_{NDk}^X + p_K^r C_{LKk}^X + p_{CS}^r C_{CSk}^X + p_R^r C_{Rk}^X \qquad \text{(A.1.13)}$$

$$C_{Rk}^X = \sum_{m\ adults} q_{kt}^m (5110 - hoursworked_{kt}^m); \quad q_{kt}^m = p_{Rt}^m / p_{Rt}^r \qquad \text{(A.1.14)}$$

A_k = (0,1) dummies for
{1 child, 2 children, 3+children, 2 adults, 3+adults,
Mid-West, South, West, nonwhite, female, rural}
(left out groups: 0 children, 1 adult,
Northeast, white, male, urban)
e.g. (1,1,0,1,0,0) for "1-child, 2-adults, NEast, nonwhite, male, urban"

The household demand vector in share form:

$$w_k = \frac{1}{D(p_k)} (\alpha_p + B_{pp} \ln \frac{p_k}{M_k} + B_{pA} A_k) \qquad \text{(A.1.15)}$$

$$D(p_k) = -1 + \iota' B_{pp} \ln p_k .$$

Summing over households gives the aggregate demand vector:

$$w = \frac{1}{D(p)} \left[\alpha_p + B_{pp} \ln p - \iota' B_{pp} \frac{\sum_k m_k \ln m_k}{M} + B_{pA} \frac{\sum_k m_k A_k}{M} \right] \qquad \text{(A.1.16)}$$

The price of full consumption:

$$\ln PF = \frac{1}{D(p)} (\alpha_p' \ln p + \tfrac{1}{2} \ln p' B_{pp} \ln p) \qquad \text{(A.1.17)}$$

This is modified for use beyond sample period using exogenous distribution terms expressed as function of the number of households of type k, nf_k :

$$PF.F = M = \sum_k m_k = \sum_K nf_K \bar{m}_K \qquad \text{(A.1.18)}$$

$$\bar{m}_K^0 = \frac{\bar{m}_{K,baseyear}}{M_{baseyear}}; \qquad M_t^0 = \sum_K nf_{Kt} \bar{m}_K^0 M_t \qquad \text{(A.1.19)}$$

$$\begin{aligned} w_t &= \frac{1}{D(p)} \left[\alpha_p + B_{pp} \ln p_t - \iota' B_{pp} \frac{\sum_K nf_{Kt} \bar{m}_K \ln \bar{m}_K}{M} + B_{pA} \frac{\sum_k m_k A_k}{M} \right] \\ &= \frac{1}{D(p)} \left[\alpha_p + B_{pp} \ln p_t - \iota' B_{pp} \xi_t^d + B_{pA} \xi_t^L \right] \end{aligned} \qquad \text{(A.1.20)}$$

We replace $\sum_K nf_{Kt}\bar{m}_K \ln \bar{m}_K / M = \sum_K nf_{Kt} \frac{\bar{m}_K}{M} \ln \frac{\bar{m}_K}{M} + \ln M$ with:

$$\xi_t^d = \sum_K nf_{Kt} \frac{\bar{m}_K^0}{M_t^0} \ln \frac{\bar{m}_K^0}{M_t^0} + \ln M_t = \xi_t^{dd} + \ln PF.F_t$$

$$\xi_t^{dd} = \sum_K nf_{Kt} \frac{\bar{m}_K^0}{M_t^0} \ln \frac{\bar{m}_K^0}{M_t^0}; \qquad \text{(A.1.21)}$$

and replace the vector $\sum_k m_k A_k / M$ with:

$$\xi^L = (\xi_{1child}^L, ..\xi_j^L, ..., \xi_{rural}^L)'$$

j={1 child,….,female, rural}

$$\xi_j^L = \sum_{all\ K \in j} nf_{Kt} \frac{\bar{m}_K^0}{M_t^0} = \text{j}^{\text{th}} \text{ row of } \sum_k \frac{m_k A_k}{M} \qquad \text{(A.1.22)}$$

The demand shares on the CEX basis is:

$$SC^{\text{X}} = \frac{\alpha^H + B^H \ln P^{H1} - B^H \iota \xi^d + B_{pA} \xi^L}{D(p)} \qquad \text{(A.1.23)}$$

$$SC^{\text{X}} \equiv \left(\frac{PC_{ND}^X C_{ND}^X}{MF^X}, \frac{PC_K^X C_K^X}{MF^X}, \frac{PC_{CS}^X C_{CS}^X}{MF^X}, \frac{PC_R^X C_R^X}{MF^X} \right),$$

$$D(p) = -1 + \iota' B^H \ln P^{H1} \qquad \text{(A.1.24)}$$

$$\ln P^{H1} = (\ln PC_{ND}^X, \ln PC_K^X, \ln PC_{CS}^X, \ln PC_R^X)$$

Exogenous bridge equation between CEX units and NIPA units:

$$SC_{it}^N = SC_{it}^X + \Delta \hat{S}C_{it} \qquad \text{i=\{ND,K,CS,R\}} \qquad \text{(A.1.25)}$$

$$\Delta SC_{it} = \alpha + \beta \Delta SC_{it} + \varepsilon_{it} \qquad \varepsilon_{it} = \rho \varepsilon_{it} + u_{it} \qquad \text{(A.1.26)}$$

$$SC^N \equiv \left(\frac{PN^{ND} N^{ND}}{MF^N}, \frac{PN^K N^K}{MF^N}, \frac{PN^{CS} N^{CS}}{MF^N}, \frac{PN^R N^R}{MF^N} \right),$$

$$PC_{ND}^X = PN^{ND} \qquad \text{(A.1.27)}$$

$$PC_{CS}^X = PN^{CS}$$

$$PC_K^X = PN^K = PKD_C$$

$$PC_R^X = PN^R = \psi_C^R P^h$$

$$MF^N = PF * F = PCC.CC + PN^R N^R \qquad \text{(A.1.28)}$$

$$VCC = PCC.CC = PN^{ND} N^{ND} + PN^K N^K + PN^{CS} N^{CS} \qquad \text{(A.1.29)}$$

Time endowment, labor supply, leisure, price of hours, price of leisure:

$$N^R = SC_R^N * MF^N / PN^R \qquad \text{(A.1.30)}$$

$$P^h \bar{L} = P^h LS + PN^R N^R \qquad \text{(A.1.31)}$$

$$LS = \bar{L} - \psi_C^R N^R \qquad \text{(A.1.32)}$$

$$d\ln \bar{L}_t = \sum_k \tfrac{1}{2}(v^L_{kt} + v^L_{kt-1}) d\ln(14*365*POP_{kt})\,; \qquad v^L_{kt} = (1 - tl^m_t)P^L_{kt} \tag{A.1.33}$$

Household third stage decision, allocation of detailed PCE:

NESTED STRUCTURE OF CONSUMPTION (Table 2.4) (A.1.34)

1	$F = F(N^{ND}, N^{KS}, N^{CS}, N_R)$	Aggregate Full consumption
2	$N^{ND} = N^{ND}(N_1, N_5, N^{EN}, N^{CG})$	Nondurables
3	$N^{KS} = N^{KS}(N_{10}, N_{36})$	Capital services
4	$N^{CS} = N^{CS}(N^{MD}, N^{FB}, N^{HO}, N^{RC})$	Consumer services
5	$N^{EN} = N^{EN}(N_3, N_4)$	Energy goods
6	$N^{CG} = N^{CG}(N_2, N_6, N_9, N^{HPG})$	Consumer goods
7	$N^{MD} = N^{MD}(N_{14}, N_{24})$	Medical
8	$N^{FB} = N^{FB}(N_{22}, N_{23}, N_{29})$	Financial & business svcs
9	$N^{HO} = N^{HO}(N_{35}, N^{EDN}, N^{UPS}, N^{TRC})$	Household operation
10	$N^{RC} = N^{RC}(N_{20}, N^{RCS}, N^{TRV})$	Recreation
11	$N^{HPG} = N^{HPG}(N_7, N_8)$	Household & personal goods
12	$N^{EDN} = N^{EDN}(N_{28}, N_{34})$	Education & nonprofit
13	$N^{UPS} = N^{UPS}(N_{11}, N^{ENS}, N^{HPS})$	Utilities & personal svc
14	$N^{TRC} = N^{TRC}(N_5, N_{16}, N_{25}, N^{COM})$	Transportation & commun.
15	$N^{RCS} = N^{RCS}(N_{17}, N_{18}, N_{19})$	Recreation services
16	$N^{TRV} = N^{TRV}(N_{21}, N_{33})$	Travel
17	$N^{ENS} = N^{ENS}(N_{12}, N_{13})$	Energy services
18	$N^{HPS} = N^{HPS}(N_{30}, N_{31}, N_{32})$	Household & pers svc
19	$N^{COM} = N^{COM}(N_{26}, N_{27})$	Communication

subscripts $\in I_{PCE}$

Price dual of **lower** tiers consumption demands $N^m(\ldots)$:

$$\ln PN^m = \alpha^{Hm}{}' \ln P^{Hm} + \frac{1}{2}\ln P^{Hm}{}' B^{Hm} \ln P^{Hm} + \ln P^{Hm}{}' f^{Hm} \qquad m \in I_{\text{CNODE}} \tag{A.1.35}$$

$$\ln P^{Hm} \equiv (\ln PN_{m1}, \ldots, \ln PN_{mi}, \ldots, \ln PN_{m,im})' \qquad i \in I_{\text{CNODE}m} \tag{A.1.36}$$

$$f^{Hm}_t = F^{Hm} f^{Hm}_{t-1} + v^{Hm}_t \tag{A.1.37}$$

$$SN^m = \begin{bmatrix} PN_{m1}N_{m1} / PN^m N^m \\ \cdots \\ PN_{m,im}N_{m,im} / PN^m N^m \end{bmatrix} = \alpha^{Hm} + B^{Hm} \ln PN^{Hm} + f^{Hm} \quad \text{(A.1.38)}$$

$$PN_{mi} \in \left\{ PN_1, \ldots, PN_{34}, PN^{ND}, \ldots, PN^{RC} \right\}$$
$$N_{mi} \in \left\{ N_1, \ldots, N_{34}, N^{ND}, \ldots, N^{RC} \right\}$$

Top tier: $SN_i^{TOP} = SC_i^N$

Adjusting for exogenous consumption demands, $\bar{C}_i$:

$$VCC^{exog} = \sum_{i=1}^{36} PS_i^C \bar{C}_i \qquad i \in I_{COM} \quad \text{(A.1.39)}$$

$$VCC^{net} = VCC - VCC^{exog} \quad \text{(A.1.40)}$$

$$PN_1N_1 = s_1^{fc} PF.F = SN_1^{ND} * SN_1^{\text{TOP}} * PF.F$$
$$PN_2N_2 = s_2^{fc} PF.F = SN_1^{CG} * SN_4^{ND} * SN_1^{\text{TOP}} * PF.F$$
$$\cdots \qquad \text{share of full cons.} \quad \text{(A.1.41)}$$
$$PN_{36}N_{36} = s_{36}^{fc} PF.F = SN_2^{KS} * SN_2^{\text{TOP}} * PF.F$$

$$s_n^{con} = s_n^{fc} PF.F / VCC \quad \text{(A.1.42)}$$

$$PN_nN_n = s_n^{con} VCC^{net} \qquad \text{share of total goods cons.} \quad \text{(A.1.43)}$$

$$VN_{all}^{net} \equiv \left(PN_1N_1, \ldots, PN_{35}N_{35}, PKD_C KD_C, PN^R N^R \right)'$$
$$VN^{net} \equiv \left(PN_1N_1, \ldots, PN_{35}N_{35}, PKD_C KD_C \right)' \qquad \text{goods only} \quad \text{(A.1.44)}$$

Converting from NIPA categories to IO categories:

$$PN = \mathbf{H}' PS^C \qquad \text{where the components of } PS^C: \quad \text{(A.1.45)}$$

$$PS_i^C = (1 + tc_i) PB_i^C \qquad i \in I_{\text{COM}} \quad \text{(A.1.46)}$$

$$PS_K^C = (1 + tc_K) PKD_C \quad \text{(A.1.47)}$$

$$VC^{net} \equiv \left(PS_1^C C_1^{net}, \ldots, PS_{36}^C C_{36}^{net}, PKD_C KD_C \right)' = \mathbf{H} \times VN^{net} \quad \text{(A.1.48)}$$

Allowing for exogenous consumption items:

$$VC_i^{gross} = VC_i^{net} + PS_i^C \bar{C}_i \qquad i \in I_{INP} \quad \text{(A.1.49)}$$

$$VC_i = VC_i^{gross} / (1 + tc_i) \qquad i \in I_{COM}; \quad VC_K = VC_K^{gross} \quad \text{(A.1.50)}$$

$$C_i = VC_i / PB_i^C \qquad i \in I_{INP} \tag{A.1.51}$$

$$C^P \equiv (C_1, C_2, \ldots, C_{36})'$$
$$C \equiv (C_1, \ldots, C_{36}, KD_C)' \quad \text{used in A.6.1}$$

Within period Cobb-Douglas price index of consumption:

$$\ln PCC^{CD} = \sum_{i=1}^{I_{INP}} \frac{VC_i}{VCC} \log PS_i^C \tag{A.1.52}$$

$$CC = VCC / PCC^{CD} \tag{A.1.53}$$

A.2 Producer Model

NESTED STRUCTURE OF PRODUCTION (Table 2.2) (A.2.1)

1	$QI_j = QI^j(KD_j, LD_j, QP^{jE}, QP^{jM})$	Industry output
2	$QP^{jE} = QP^E(QP_6^j, QP^{jFF})$	Energy aggregate
3	$QP^{jM} = QP^M(QP^{jMA}, QP^{jMM}, QP^{jMN}, QP^{jMS})$	Material aggregate
4	$QP^{jFF} = QP^{FF}(QP_4^j, QP^{jOIL}, QP^{jGAS})$	Fossil fuel
5	$QP^{jOIL} = QP^{OIL}(QP_2^j, QP_{23}^j)$	Oil products
6	$QP^{jGAS} = QP^{GAS}(QP_3^j, QP_7^j)$	Gas products
7	$QP^{jMA} = QP^{MA}(QP_1^j, QP_{10}^j, QP_{20}^j, QP_{21}^j)$	Agriculture Intermed.
8	$QP^{jMM} = QP^{MM}(QP_5^j, QP_{12}^j, QP_{13}^j, QP^{jEQ})$	Metallic Intermed.
9	$QP^{jMN} = QP^{MN}(QP_{11}^j, QP_{19}^j, QP_{22}^j, QP_{24}^j)$	Non-metallic Intermed.
10	$QP^{jMS} = QP^{MS}(QP_{32}^j, QP^{jTT}, QP^{jSV})$	Services & Margins
11	$QP^{jEQ} = QP^{EQ}(QP_{14}^j, QP_{15}^j, QP_{16}^j, QP^{jTR})$	Equipment
12	$QP^{jTR} = QP^{TR}(QP_{17}^j, QP_{18}^j)$	Transport equipment
13	$QP^{jTT} = QP^{TT}(QP_{25}^j, QP_{26}^j, QP_{27}^j)$	Trade and Transportation
14	$QP^{jSV} = QP^{SV}(QP^{jBL}, QP^{jOB}, QP^{jOS})$	Services
15	$QP^{jBL} = QP^{BL}(QP_8^j, QP_9^j, QP_{31}^j)$	Building services
16	$QP^{jOB} = QP^{OB}(QP_{28}^j, QP_{29}^j, QP_{30}^j)$	Other business svcs
17	$QP^{jOS} = QP^{OS}(QP_{33}^j, QP_{34}^j, QP_{35}^j, QP_{36}^j)$	Other services

Price dual of **top** tier of production function $QI_j = QI(\ldots)$:

$$PO_j = PO^j(PKD_j, PLD_j, PP^{jE}, PP^{jM}, t; \lambda_j, A^{agg}, T^{agg}) \qquad j \in I_{IND}$$

$$A_t^{agg} = (1 - \Delta A^{agg}) A_{t-1}^{agg} \qquad \text{(A.2.2)}$$

λ_j exogenous productivity shock in industry j

ΔA^{agg} exogenous aggregate productivity shock, common to all

T_t^{agg} exogenous shift term to hit target aggregate GDP growth

$$\ln PO_j = \alpha_0^j + \alpha^{Pj'} \ln P^{Pj0} + \frac{1}{2} \ln P^{Pj0'} B^{Pj} \ln P^{Pj0} + \ln P^{Pj0'} f_t^{Pj} + f_t^j + \ln \lambda_j + \ln A^{agg} + \ln T^{agg} \quad \text{(A.2.3)}$$

$$\xi_t^{Pj} = F^{Pj} \xi_{t-1}^{Pj} + v_t^{Pj} \quad \text{(A.2.4)}$$

$$\xi_t^{Pj} = (1, f_{Kt}^{Pj}, f_{Lt}^{Pj}, f_{Et}^{Pj}, f_{Mt}^{Pj}, \Delta f_t^j)'$$

$$A_j^{TFP} = \ln P^{Pj0'} f_t^{Pj} + f_t^j + \ln \lambda_j + \ln A^{agg} \quad \text{(A.2.5)}$$

$$SP^{j\text{TOP}} = \begin{bmatrix} s_j^K \\ s_j^L \\ s_j^E \\ s_j^M \\ \end{bmatrix} = \begin{bmatrix} PKD_j KD_j / PQ_j QI_j \\ \dots \\ PP^{jM} QP^{jM} / PO_j QI_j \end{bmatrix} = \alpha^{Pj} + B^{Pj} \ln P^{Pj0} + f_t^{Pj} \quad \text{(A.2.6)}$$

Price dual of **lower** tiers of production functions $QP^{jm} = QP(\dots)$:

$$\ln PP^{jm} = \alpha_0^{jm} + \alpha^{Pjm'} \ln P^{Pjm} + \frac{1}{2} \ln P^{Pjm'} B^{Pjm} \ln P^{Pjm} + \ln P^{Pjm'} f_t^{Pjm} \qquad m \in I_{\text{PNODE}} \quad \text{(A.2.7)}$$

$$f_t^{Pjm} = F^{Pjm} f_{t-1}^{Pjm} + v_t^{Pjm} \quad \text{(A.2.8)}$$

$$\ln P^{Pjm} \equiv (\ln PP_{m1}^j, \dots, \ln PP_{mi}^j, \dots, \ln PP_{m,im}^j)' \qquad i \in I_{PNODEm}$$

$$SP^{j\text{m}} = \begin{bmatrix} PP_{m1}^j QP_{m1}^j / PQ^{jm} QP^{jm} \\ \dots \\ PP_{m,im}^j QP_{m,im}^j / PP^{jm} QP^{jm} \end{bmatrix} = \alpha^{Pjm} + B^{Pjm} \ln P^{Pjm} + f_t^{Pjm} \quad \text{(A.2.9)}$$

$$PP_{mi}^j \in \left\{ PB_{1j}, \dots, PB_{36j}, PP^{jFF}, \dots, PP^{jOS} \right\}$$

$$QP_{mi}^j \in \left\{ QP_1^j, \dots, QP_{36}^j, QP^{jFF}, \dots, QP^{jOS} \right\}$$

Vectors for use in formulas below:

$$V^{QI} \equiv \left(PO_1 QI_1, \dots, PO_{36} QI_{36} \right)' \quad \text{(A.2.10)}$$

$$VQI_j = PO_j QI_j \quad \text{(A.2.11)}$$

$$VQI_j^T = PI_j QI_j \quad \text{(A.2.12)}$$

Taxes (net vs. gross output):

$$PI_j = \left(1 + tt_j + tx_j^v\right) PO_j + tu_j + tx_j^u \qquad j \in I_{\text{IND}} \quad \text{(A.2.13)}$$

$$PI_j = \left(1 + tt_j^{full}\right) PO_j \quad \text{(A.2.14)}$$

$$VT^{QI} \equiv \left(PI_1 QI_1, \ldots, PI_{36} QI_{36}\right)' = Diag(\iota + tt^{full}) V^{QI} \quad \text{(A.2.15)}$$

Commodities from industry outputs:

$\mathbf{M} = [M_{ji}]$ = value of commodity i made by industry j (A.2.16)

$$m_{ji}^{col} = \frac{M_{ji}}{\sum_k M_{ki}}; \qquad m_{ji}^{row} = \frac{M_{ji}}{\sum_k M_{jk}} \quad \text{(A.2.17)}$$

$$\ln PC = \mathbf{m}^{\mathbf{col}} ' \ln PI \quad \text{(A.2.18)}$$

$$V^{QC} \equiv \left(PC_1 QC_1, \ldots, PC_{36} QC_{36}\right)' = \mathbf{m}^{\mathbf{row}} ' VT^{QI} \quad \text{(A.2.19)}$$

$$QC_i = V^{QC}{}_i / PC_i \qquad i \in I_{\text{COM}} \quad \text{(A.2.20)}$$

Total supply price from domestic + imports, from eq. A.5.2:

$$PS_i = PS\left(PC_i, PM_i\right) \quad \text{(A.2.21)}$$

Buyer specific externality tax:

$$PB_{ij} = PS_i + tx_x^{Xu} XC_{ijx} \qquad i \in I_{COM} \quad j \in I_{BUY} \quad x \in I_{EXT} \quad \text{(A.2.22)}$$

$$PB = [PB_{ij}]; \quad j \in I_{ind}$$
$$PB^C = [PB_i^C]; \quad PB^I = [PB_i^I]; \quad PB^G = [PB_i^G]; \quad \text{(A.2.23)}$$

The input-output USE matrix, in share terms, used in eq. A.6.4***:***

$$\begin{aligned} A_{1j} &= SP_1^{jMA} * SP_1^{jM} * SP_4^{jTOP} \\ A_{2j} &= SP_1^{jOIL} * SP_2^{jFF} * SP_2^{jE} * SP_3^{jTOP} \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ A_{36j} &= SP_4^{jOS} * SP_3^{jSV} * SP_3^{jMS} * SP_4^{jM} * SP_4^{jTOP} \end{aligned} \quad \text{(A.2.24)}$$

$$A_j \equiv \left(A_{1j}, A_{2j}, \ldots, A_{36j}\right)' \qquad \text{j} \in I_{\text{IND}} \quad \text{(A.2.25)}$$

$$\mathbf{A} \equiv [A_1, A_2, \ldots, A_{36}]' \quad \text{(A.2.26)}$$

$$VU_{ij} = PS_i QP_i^j = A_{ij} VQI_j \qquad i \in I_{\text{COM}} \quad \text{(A.2.27)}$$

$$PKD_j KD_j = SP_1^{jTOP} * VQI_j \quad \text{(A.2.28)}$$

$$PLD_j LD_j = SP_2^{jTOP} * VQI_j \quad \text{(A.2.29)}$$

Intermediate demands when buyer specific prices of input *i* are allowed:

$$VU_{ij} = A_{ij} VQI_j \qquad i \in I_{COM} \qquad j \in I_{IND} \tag{A.2.30}$$

$$QU_{ij} = VU_{ij} / PB_{ij} \tag{A.2.31}$$

For *j*=oil mining, gas mining; the fixed capital option:

$$KD_j = \overline{KD}_j \qquad \text{(j=oil mining, gas mining)} \tag{A.2.32}$$

PKD_j independent endogenous variables

and the mobile mining capital option:

$$PKD_j = \psi_j^K PKD \qquad \text{(j=oil mining, gas mining)} \tag{A.2.33}$$

A.3 Capital and Investment

The owner of aggregate capital:

$$\text{Max}\sum_{t=u}^{\infty}\frac{(1\text{-}tk)(PKD_t\psi^K K_{t\text{-}1}\text{-}tpPK_{t\text{-}1})\text{-}(1\text{-}t^{ITC})PII_t I_t^a}{\prod_{s=u}^{t}1+r_s} \tag{A.3.1}$$

subject to

$$K_t = (1-\delta)K_{t-1} + \psi^I \varepsilon^I I_t^a \tag{A.3.2}$$

ε_t^I investment productivity shock

Hamiltonian:

$$\frac{(1\text{-}tk)(PKD_t\psi^K K_{t\text{-}1}\text{-}tpPK_{t\text{-}1}\text{K}_{t\text{-}1})\text{-}(1-t^{ITC})PII_t I_t^a}{\prod_{s=u}^{t}1+r_s} + \frac{\lambda_t}{\prod_{s=u}^{t}1+r_s}\left((1-\delta)K_{t-1} + \psi^I \varepsilon^I I_t^a - K_t\right) \tag{A.3.3}$$

Euler equation:

$$(1+r_t)\frac{PII_{t-1}}{\psi_{t-1}^I \varepsilon_{t-1}^I} = \frac{1-tk}{1-t^{ITC}}(PKD_t\psi_t^K - tpPK_{t-1}) + (1-\delta)\frac{PII_t}{\psi_t^I \varepsilon_t^I} \tag{A.3.4}$$

Aggregation relationships due to composition differences:

$$PK_t = \psi_t^{PK} PII_t (1-t^{ITC}) \tag{A.3.5}$$

$$KD_t = \psi_t^K K_{t-1} \tag{A.3.6}$$

In the projection period:

$$\psi_t^K = \psi_{t-1}^K + \Delta\psi_t^K \qquad \psi_t^I = \psi_{t-1}^I + \Delta\psi_t^I \tag{A.3.7}$$

$$\Delta\psi_t^K = \alpha_0^{\psi K} + \alpha_1^{\psi K}\Delta\psi_{t-1}^K + \alpha_2^{\psi K}\Delta\psi_{t-2}^K + v_t^{\psi K} \tag{A.3.8a}$$

$$\Delta\psi_t^I = \alpha_0^{\psi I} + \alpha_1^{\psi I}\Delta\psi_{t-1}^I + \alpha_2^{\psi I}\Delta\psi_{t-2}^I + v_t^{\psi I} \tag{A.3.8b}$$

$$\Delta\psi_t^{PK} = \alpha_0^{PK} + \alpha_1^{PK}\Delta\psi_{t-1}^{PK} + \alpha_2^{PK}\Delta\psi_{t-2}^{PK} + v_t^{PK} \tag{A.3.8c}$$

$$VII = PII.I^a \tag{A.3.9}$$

IGEM-N v20 has no corporate-noncorporate distinction.

$$PK^{gain} = (\frac{PK_t - PK_{t-1}}{PK_{t-1}} - \delta) \quad \text{(A.3.10)}$$

$$VK^{gain} = (\frac{PK_t - PK_{t-1}}{PK_{t-1}} - \delta) PK_{t-1} K_{t-1} \quad \text{(A.3.11)}$$

$$Y^I = r(BG + BF) \quad \text{(A.3.12)}$$

$$YK^{gov} = (1 - tk) PKD_{36} KD_{36} \quad \text{(A.3.13)}$$

$$YK = \sum_{j=1}^{C} PKD_j KD_j - RK^{hh} \qquad j \in BUY \quad \text{(A.3.14)}$$

$$YK^{net} = DIV - YK^{gov} + Y^I + (1 - tk)(GINT^{adj} + Y^{ROW,adj}) \quad \text{(A.3.15)}$$

Option exogenous interest, this simplifies to:

$$YK^{net} = DIV - YK^{gov} + (1 - tk)\left(\overline{GINT}^{hh} + \overline{GINT}^{ss} - R\_SS + \bar{Y}^{row}\right) \quad \text{(A.3.15a)}$$

Option endogenous interest, this simplifies to:

$$YK^{net} = DIV - YK^{gov} + Y^I - (1 - tk) R\_SS \quad \text{(A.3.15b)}$$

$$DIV = (1 - tk)[YK - tpPK_{t-1} K_{t-1}] - RCG^h - tkTX^{tot} \quad \text{(A.3.16)}$$

$$\alpha^{div} = DIV / PK_{t-1} K_{t-1} \quad \text{(A.3.17)}$$

$GINT^{adj}$ given in (A.4.49) and (A.4.51)

$Y^{ROW,adj}$ given in (A.5.23); TX^{tot} in (A.4.24)

$$r_t = \alpha^{div} + PK^{gain} \quad \text{(A.3.18)}$$

Above r_t is equivalent to:

$$r_t PK_{t-1} K_{t-1} = (1 - tk)[YK - tpPK_{t-1} K_{t-1}] - RCG^h - tkTX^{tot} + (PK_t - PK_{t-1}) K_{t-1} - \delta PK_{t-1} K_{t-1} \quad \text{(A.3.19)}$$

Programming note: In the IGEM code, kap_income_net = $YK^{net} - twW_{t-1}$

In section 6 we have the rental cost for industry j:

$$PKD_j = \psi_j^K PKD \qquad j \in I_{BUY} \qquad \text{see} \quad \text{(A.6.10)}$$

Household capital equation for special treatment of mortgage deductions, etc.:

$$PKD_{j=C} = (1 + tk_t^{hh}) \psi_C^K PKD \qquad \text{j=C denotes household sector} \quad \text{(A.3.20)}$$

$$PKD_{hh}^{net} = PKD_C / (1 + tk^{hh}) \quad \text{(A.3.21)}$$

$$R\_K^{hh} = PKD_{hh}^{net} KD_C tk^{hh} \quad \text{(A.3.22)}$$

$$VK_{t-1}^{hh} = \frac{KD_C}{\sum_j KD_j} PK_{t-1} K_{t-1} \quad \text{(A.3.23)}$$

NESTED STRUCTURE OF INVESTMENT (A.3.24)

	$I^{a} = I^{a}(I^{\text{fixed}}, I^{\text{inventory}})$	Aggregate investment
	$I^{\text{inventory}} = I^{IY}$	Change in business inventories
1	$I^{\text{fixed}} = I^{FX}(I^{\text{long}}, I^{\text{short}})$	Fixed investment aggregate
2	$I^{\text{long}} = I^{LG}(I_5, I_9, I_{13})$	Long-lived investment aggregate
3	$I^{\text{short}} = I^{SH}(I^{EIT}, I^{TTSVC})$	Short-lived investment aggregate
4	$I^{EIT} = I^{EIT}(I^{MACH}, I^{IT}, I^{TRNSP})$	Equipment & IT aggregate
5	$I^{TTSVC} = I^{TTSVC}(I_{27}, I^{TRD}, I^{SERV})$	Transportation, Trade, Services
6	$I^{MACH} = I^{MACH}(I_{14}, I_{15}, I^{FRAWM})$	Machinery & Furnishings
7	$I^{IT} = I^{IT}(I_{15}, I_{28}, I_{29})$	Information Technology
8	$I^{TRNSP} = I^{TRNSP}(I_{17}, I_{18})$	Transportation equipment
9	$I^{TRD} = I^{TRD}(I_{25}, I_{26})$	Trade aggregate
10	$I^{SERV} = I^{SERV}(I_{22}, I^{MSERV})$	Services aggregate
11	$I^{FRAWM} = I^{FRAWM}(I^{MRAWM}, I^{FURN})$	Furnishings, Metals, Raw Mat.
12	$I^{MSERV} = I^{MSERV}(I_{22}, I_{34}, I_{35})$	Minor services
13	$I^{MRAWM} = I^{MRAWM}(I_{13}, I^{RAWM})$	Metals & raw materials
14	$I^{FURN} = I^{FURN}(I_{19}, I_{21}, I_{24})$	Furnishings
15	$I^{RAWM} = I^{RAWM}(I_{10}, I_{11}, I_{12})$	Raw materials

At **top** tier of investment functions $I = I(\ldots)$:

$$VII = VII^{\text{fixed}} + VII^{\text{invy}} \tag{A.3.25}$$

$$\frac{VII^{\text{invy}}}{VII} = \alpha^{IY} \tag{A.3.26}$$

$$VII_{\text{i}}^{\text{invy}} = \alpha_i^{IY} VII^{\text{invy}} \qquad i \in I_{\text{COM}} \tag{A.3.27}$$

Price dual of fixed investment demand tiers $I^{m} = I^{m}(\ldots)$:

$$\ln PII^{m} = \alpha^{Im} \ln P^{Im} + \tfrac{1}{2} \ln P^{Im} \,' B^{Im} \ln P^{Im} + \ln P^{Im} \,' f_t^{Im} \qquad \text{m} \in \text{I}_{\text{INV}} \tag{A.3.28}$$

$$f_t^{Im} = F^{Im} f_{t-1}^{Im} + v_t^{Im} \tag{A.3.29}$$

$$\ln P^{Im} \equiv \left(\ln PII_{m1}, \ldots, \ln PII_{mi}, \ldots, \ln PII_{m,im}\right) \quad i \in I_{INVm}$$

$$SI^{\mathrm{m}} = \begin{bmatrix} PII_{m1} I^{f}_{m1} / PII^{m} I^{m} \\ \dots \\ PII_{m,im} I^{f}_{m,im} / PII^{m} I^{m} \end{bmatrix} = \alpha^{Im} + B^{Im} \ln PII^{Im} + f_t^{Im} \qquad \begin{matrix} \mathrm{m} \in \mathrm{I_{INV}} \\ \mathrm{mi} \in \mathrm{I_{INVm}} \end{matrix} \qquad \text{(A.3.30)}$$

$$PII_{mi} \in \left\{ PB^{I}_{1}, \dots, PB^{I}_{36}, PII^{\,fixed}, \dots, PII^{\,mining} \right\}$$
$$I_{mi} \in \left\{ I^{f}_{1}, \dots, I^{f}_{35}, I^{\mathrm{fixed}}, \dots, I^{\mathrm{mining}} \right\}$$

Share demands under Cobb-Douglas option:

$$SI = \begin{bmatrix} PB^{I}_{1} I^{f}_{1} / VII \\ \dots \\ PB^{I}_{36} I^{f}_{36} / VII \end{bmatrix} = \alpha^{CD,Im} \qquad \text{(A.3.31)}$$

Values of individual commodities making up aggregate investment demand:

$$VI_i = VI_i^{\,fixed} + VI_i^{\,inventory}:$$

$$VI_1 = 0 + VI_1^{invy}$$
$$VI_5 = SI_1^{LG} * SI_1^{FX} VII^{\,fixed} + VI_2^{invy}$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad \text{(A.3.32)}$$
$$VI_{34} = SI_3^{MSERV} * SI_2^{SERV} * SI_3^{TTSVC} * SI_2^{SH} * SI_2^{FX} VII^{\,fixed} + VI_{34}^{invy}$$
$$VI_{36} = 0$$

$$PII_t = \lambda_t^I PII_t^{m=fixed} \qquad \text{(A.3.33)}$$

$$I_i = VI_i / PB_i^I \qquad \text{(A.3.34)}$$

vectors used in A.6.2:

$$VI \equiv \left(PB^{I}_{1} I_1, \dots, PB^{I}_{36} I_{36} \right)'$$
$$I^P \equiv \left(I_1, \dots, I_{36} \right)'$$
$$I \equiv \left(I_1, \dots, I_{36} \right)'$$

A.4 Government accounts and pollution externalities

Tax rates

$$tc_i = tc + tc^g \qquad i \in I_{\text{COM}} \tag{A.4.1}$$

$$tc_K = tc + tc^K \tag{A.4.2}$$

$$tc_L = tc + tc^L \tag{A.4.3}$$

$$tx_i^v = \sum_{x=1} tx_x^{Xv} XP_{ix} \qquad i \in I_{IND} \quad \text{x} \in I_{EXT} \tag{A.4.4}$$

$$tx_i^u = \sum_{x=1} tx_x^{Xu} XP_{ix} \tag{A.4.5}$$

$$tx_i^{rv} = \sum_{x=1} tx_x^{Xv} XM_{ix} \tag{A.4.6}$$

$$tx_i^{ru} = \sum_{x=1} tx_x^{Xu} XM_{ix} \tag{A.4.7}$$

tu_i unit tax on commodity i (A.4.8)

Buyer-specific externality when using commodity i:

$XC_{ijx} \qquad i \in I_{COM}, j \in I_{BUY}$

$$tx_{ij}^{Bu} = \sum_{x=1} tx_x^{Xu} XC_{ijx} \tag{A.4.9}$$

When unit externality tax is common to all purchasers, full output tax is:

$$tt_i^{full} = tt_i + tx_i^v + \frac{tu_i + tx_i^u}{PO_i} \tag{A.4.10}$$

When externality tax depends on buyer specific factor (see A.4.61)

$$tt_{i,j} = tt_i + tx_i^v + \frac{tu_i}{PO_i} + \frac{tx_x^{Xu} XC_{ijx}}{PS_i} \tag{A.4.11}$$

$$RL^0 = \sum_j PLD_j LD_j \left(1 - \frac{tl^a}{tl^m}\right) \tag{A.4.12}$$

Stock-flow relations

$$BG_t = BG_{t-1} + \Delta G + GFI + \Delta P_t^{BGF} + BG^{disc} \tag{A.4.13}$$

$$BG_t^* = BG_{t-1}^* - GFI - \Delta P_t^{BGF*} \tag{A.4.14}$$

Revenues and expenditures

$$\begin{aligned} R\_TOTAL = {} & R\_SALES + R\_TARIFF + R\_P + R\_K + RK^{hh} + R\_L \\ & + R\_W + R^N + R\_UNIT + R\_EXT + R\_ITC + R\_CON^{net} \\ & + R\_CON^{gov} + TLUMP + YK^{gov} + R\_SS \end{aligned} \tag{A.4.15}$$

Programming note: in IGEM code, rev_total $+YK^{gov} + R\_SS = R\_TOTAL$

$$R\_SALES = \sum_j tt_j PO_j QI_j \quad \text{(A.4.17)}$$

$$R\_TARIFF = \sum_i tr_i PM_i M_i \quad \text{(A.4.18)}$$

$$R\_P = tpPK_{t-1}K_{t-1} \quad \text{(A.4.19)}$$

$$TX^{CGH} = (1-\beta_h)\pi VK_{t-1}^{hh} \quad \text{(A.4.20)}$$

$$TX^{ADE} = (\delta^{std} - \delta^{econ})(r + \delta(1+\pi))PK_{t-1}K_{t-1} \quad \text{(A.4.21)}$$

$$TX^{MID} = r\beta_h VK_{t-1}^{hh} \quad \text{(A.4.22)}$$

$$TX^{PTD} = tpVK_{t-1}^{hh} \quad \text{(A.4.23)}$$

$$TX^{tot} = TX^{ADE} + TX^{MID} + TX^{PTD} \quad \text{(A.4.24)}$$

Under current tax law allowing deductions:

$$R\_KK = tk(YK - R\_P) \quad \text{(A.4.25a)}$$

$$RCG^{hh} = 0 \quad \text{(A.4.25b)}$$

under policy to eliminate tax expenditures for deductions:

$$R\_KK = tk(YK - R\_P + TX^{tot}) \quad \text{(A.4.26a)}$$

$$RCG^{hh} = t_h^g * TX^{CGH} \quad \text{(A.4.26b)}$$

where $YK = \sum_{j=1}^{C} PKD_j KD_j - R\_K^{hh}$ was given in A.3.14

$$R\_K = R\_KK + RCG^{hh} + \frac{tk}{1-tk} r(BG_{t-1} + BF_{t-1}) + tkGINT^{adj} + tkY^{ROW,adj} \quad \text{(A.4.27)}$$

Option exogenous interest, this simplifies to:

$$R\_K = tk(YK - R\_P) + tk(\overline{GINT}^{hh} + \overline{GINT}^{ss} - R\_SS) + tk\bar{Y}^{ROW} \quad \text{(A.4.28)}$$

Option endogenous interest, this simplifies to:

$$R\_K = tk(YK - R\_P) + \frac{tk}{1-tk} r(BG_{t-1} + BF_{t-1}) - tkR\_SS \quad \text{(A.4.28b)}$$

$$RK^{hh} = \frac{tk^{hh}}{1-tk^{hh}} PKD_{37}KD_{37} \qquad 37\text{=C (household)} \quad \text{(A.4.29)}$$

If ignore explicit tax deduction of employer-provided health insurance:

$$R\_L = tl^a P^h LS/(1-tl^m) = tl^a \sum_j PLD_j LD_j \quad \text{(A.4.30)}$$

if explicitly account for EPHI:

$$TX^{ephi} = tl_{ephi}^a \theta^{ephi} \sum_j PLD_j LD_j \quad \text{(A.4.31)}$$

$$R\_L = tl_{ephi}^a (1-\theta^{ephi}) P^h LS/(1-tl^m) = tl_{ephi}^a (1-\theta^{ephi}) \sum_j PLD_j LD_j \quad \text{(A.4.32)}$$

$$R\_W = tw(PK.K + BG + BF) \quad \text{(A.4.33)}$$

$$R\_UNIT = \sum_j tu_j QI_j \quad \text{(A.4.34)}$$

$$R\_EXT = \sum_j tx_j^v PI_j QI_j + \sum_i tx_i^{rv} PM_i M_i + \sum_j tx_j^u QI_j + \sum_i tx_i^{ru} M_i \quad \text{(A.4.35)}$$

$$R\_ITC = -t^{ITC} PII_t I_t^a \quad \text{(A.4.36)}$$

$$R\_CON^{marg} = \sum_{I_{COM}} tc_i PB_i^C C_i + R\_CON^{hk} \quad \text{(A.4.37)}$$

$$R\_CON^{hk} = (tc + tc^K)\frac{\psi_C^K KD_C}{KD} PII_t I_t^a \quad \text{(A.4.38)}$$

$$R\_CON^{reb} = tcVCC^{exempt} \quad \text{(A.4.39)}$$

$$R\_CON^{gov} = \frac{tc^G}{1+tc^G} VGG \quad \text{(A.4.40)}$$

$$R\_CON^{net} = R\_CON^{marg} - R\_CON^{reb} \quad \text{(A.4.41)}$$

YK^{gov} was given in (A.3.13)

These revenue and expenditure variables may be set exogenously:

$R^N = \bar{R}^N$ nontax receipts (A.4.42)
GFI govt foreign net investment
$GINT^{row}$ govt net interest payments to foreigners
G^{tran} govt transfer payments to households (excl social insur)
$G^{tran,row}$ govt transfer payments to foreigners
$\overline{GINT}^{hh}$ govt interest payments to private bond holders
$GINT^{ss}$ investment income of social insur funds
R_SS transfers to govt from social insur funds for admin expenses
ΔG government deficit

or, as shares of GDP:

$$R^N = \alpha^{nontax} GDP \quad \text{(A.4.43a)}$$
$$G^{tran} = \alpha^{GTRAN} GDP \quad \text{(A.4.43b)}$$
$$G^{tran.row} = \alpha^{GTR\_R} GDP \quad \text{(A.4.43c)}$$
$$R\_SS = \alpha^{R\_SS} GDP \quad \text{(A.4.43d)}$$

$$EXP^{gengov} = VGG + G^{tran} + G^{tran,row} + G^{Ktran} + G^{Ktran,row} + GINT^{hh} + GINT^{row} \quad \text{(A.4.44)}$$

$$EXPEND = EXP^{gengov} + GINT^{ss} \quad \text{(A.4.45)}$$

$$G\_SS = GINT^{ss} - R\_SS \quad \text{(A.4.46)}$$

$$\Delta G = EXPEND - R\_TOTAL \quad \text{(A.4.47)}$$

$$GINT = GINT^{hh} + GINT^{ss} = \frac{r}{1-tk} BG \quad \text{(A.4.48)}$$

Option "exogenous interest payments" :

$$GINT^{adj} = \overline{GINT}^{hh} + \overline{GINT}^{ss} - R\_SS - \frac{r}{1-tk} BG_{t-1} \tag{A.4.49}$$

$$\begin{aligned} VGG &= \Delta G + R\_TOTAL \\ &\quad -GINT - GINT^{adj} - GINT^{row} - G^{TRAN} - G^{tran.row} - G^{Ktran} - G^{Ktran,row} \\ &= \Delta G + R\_TOTAL + R\_SS \\ &\quad -\overline{GINT}^{hh} - \overline{GINT}^{ss} - GINT^{row} - G^{TRAN} - G^{tran.row} - G^{Ktran} - G^{Ktran,row} \end{aligned} \tag{A.4.50}$$

Option "endogenous interest payments" :

$$GINT^{adj} = -R\_SS \tag{A.4.51}$$

$$\begin{aligned} VGG &= \Delta G + R\_TOTAL + R\_SS \\ &\quad -GINT - GINT^{row} - G^{TRAN} - G^{tran.row} - G^{Ktran} - G^{Ktran,row} \end{aligned} \tag{A.4.52}$$

$$VG_i = \alpha_i^G VGG / (1+tc^G) \qquad i \in I_{COM} \tag{A.4.53}$$

$$G_i = VG_i / PS_i \qquad i \in I_{COM} \tag{A.4.54}$$

Vectors for use in A.6.2 below:

$$VG \equiv \left(PS_1G_1, \ldots, PS_{36}G_{36}\right)' \tag{A.4.56}$$

$$G^P \equiv \left(G_1, \ldots, G_{36}\right)'$$

$$\ln PGG = \sum_i^{36} \alpha_i^G \ln PS_i (1+tc^{gov}) \tag{A.4.57}$$

$$GG = VGG / PGG \tag{A.4.58}$$

Government closure options:

$$VGG_t = \begin{cases} R\_TOTAL + \Delta G + \ldots & 'resid' \\ \gamma_t^{VGG} GDP_t & 'propr' \end{cases} \tag{A.4.59}$$

$$g^{GDP} = \frac{VGG}{GDP} \tag{A.4.60}$$

Externalities

Quantity of externality *x* (e.g. XP_{jx} =tons of SO2 per billion $ of output of j):

$$EXT_x = \sum_j XP_{jx} QI_j + \sum_j XM_{ix} M_i + \sum_{ij} XC_{ijx} QU_{ij} \qquad x \in I_{EXT} \tag{A.4.61}$$

$$\sigma_t^{GHG} = \frac{E_{GHG,t}}{Y_t^{GDP}} \tag{A.4.62}$$

A.5 The Rest-of-the-World

All commodity imports are regarded as "competitive" in IGEM-N unlike IGEMv18 that includes non-competing imports (NCI). Imports and domestic output make up total supply:

$$QS_i = QS\left(QC_i, M_i\right) \qquad i \in I_{COM} \tag{A.5.1}$$

$$\begin{aligned}\ln PS_{it} &= \alpha_{ct} \ln PC_{it} + \alpha_{mt} \ln PM_{it} + \tfrac{1}{2}(\beta_{cc} \ln^2 PC_{it} + 2\beta_{cm} \ln PC_{it} \ln PM_{it} \\ &\quad + \beta_{mm} \ln^2 PM_{it}) + f_{ct}^{M} \ln PC_{it} + f_{mt}^{M} \ln PM_{it} \\ &\equiv a^{M} \ln P^{M_i} + \ln P^{M_i} \,' B^{M_i} \ln P^{M_i} + \ln P^{M_i} \,' f_t^{M}\end{aligned} \tag{A.5.2}$$

$$\ln P^{M_i} \equiv \left(\ln PC_i, \ln PM_i\right)$$

$$PM_i = e\left(1 + tr_i + tx_i^{rv}\right)PM_i^{*} + tx_i^{ru} \qquad i \in I_{COM} \tag{A.5.3}$$

$$PM_i^{land} = ePM_i^{*} \tag{A.5.4}$$

$$SD^i \equiv \begin{bmatrix} PC_iQC_i / PS_iQS_i \\ PM_iM_i / PS_iQS_i \end{bmatrix} = \alpha^{M_i} + B^{M_i} \ln P^{M_i} + f_t^{Mi} \tag{A.5.5}$$

Cobb-Douglas option:

$$SD^i \equiv \begin{bmatrix} PC_iQC_i / PS_iQS_i \\ PM_iM_i / PS_iQS_i \end{bmatrix} = \alpha^{CD,M_i} \tag{A.5.6}$$

$$PS_iQS_i = PC_iQC_i + PM_iM_i \qquad i \in I_{COM} \tag{A.5.7}$$

Vectors for use in A.6.5:

$$\begin{aligned} VQS &\equiv \left(PS_1QS_1, \ldots, PS_{36}QS_{36}\right)' \\ VM &\equiv \left(PM_1M_1, \ldots, PM_{36}M_{36}\right)' \\ SM &\equiv \left(SD_2^1, SD_2^2, \ldots, SD_2^{36}\right)' \\ M &\equiv \left(M_1, M_2, \ldots, M_{36}\right)' \end{aligned} \tag{A.5.8}$$

Exogenous projection of world prices equal to domestic productivity:

$$\Delta \ln PM_{it}^{*} = \Delta f_{it}^{p} \quad t > 2010; \qquad PM_{it}^{*} = \text{data for } t = .., 2009, 2010 \tag{A.5.11}$$

Exports

$$SX^i \equiv \frac{PC_iX_i}{PC_iQC_i} = \alpha^{X_i} + B^{X_i} \ln P^{X_i} + f_t^{Xi} \tag{A.5.12}$$

$$\ln P^{X_i} \equiv \left(\ln \frac{e_t PM^*_{it}}{1 - tr^*_{iy}}, \ln PC_{it} \right)'$$

$$VX_i = SX^i PC_i QC_i \quad \text{(A.5.13)}$$

Vectors used in A.6.2:

$$X \equiv (X_1, \ldots, X_{36})' \quad \text{(A.5.14)}$$

$$VX \equiv (PC_1 X_1, \ldots, PC_{36} X_{36})'$$

Current account and net foreign assets

$$V^{IMP} = \sum_i ePM^*_i M_i \quad \text{(A.5.15)}$$

$$V^{NCI} = \sum_j ePNCI^*_j NCI_j \quad \text{(A.5.16)}$$

$$V^{EX} = \sum_i PC_i X_i \quad \text{(A.5.17)}$$

$$TB = V^{EX} - V^{IMP} \quad \text{(A.5.18)}$$

$$TB^* = TB / e \quad \text{(A.5.19)}$$

CA current account surplus of the US

Y^{row} net private factor income from rest-of-world

$$CA = TB + \bar{Y}^{row} - GINT^{row} - G^{tran,row} - G^{Ktran,row} - H^{row} \quad \text{(A.5.20)}$$

$$= TB + \frac{r}{1 - tk} BF + Y^{row,adj} - GINT^{row} - G^{tran,row} - G^{Ktran,row} - H^{row} \quad \text{(A.5.21)}$$

$$CA^* = CA / e \quad \text{(A.5.22)}$$

Option "exogenous interest payments" :

$$Y^{ROW,adj} = \bar{Y}^{ROW} - \frac{r}{1 - tk} BF \quad \text{(A.5.23)}$$

Option "endogenous interest payments" :

$$Y^{ROW} = \frac{r}{1 - tk} BF; \quad Y^{ROW,adj} = 0 \quad \text{(A.5.24)}$$

Stock-flow relation:

$$BF_t = BF_{t-1} + CA_t - GFI + BF^{disc} + \Delta P^{BF} \quad \text{(A.5.25)}$$

A.6 Markets, Numeraire and National Accounting

Final demands

$$VFD_i = PS_i\left(C_i^P + I_i^P + G_i^P\right) + PC_i X_i \qquad i \in I_{COM} \tag{A.6.1}$$
$$VFD_i^d = PS_i\left(C_i^P + I_i^P + G_i^P\right)$$

$$\begin{aligned} VFD &\equiv (VFD_1,\ldots,VFD_{36})' \\ &= VC + VI + VG + VX \end{aligned} \tag{A.6.2}$$

Supply equal demand for commodities

$$\begin{aligned} PS_i QS_i &= \sum_{j=1}^{36} PS_i QP_i^j + VFD_i^d + VX_i \\ &= \sum_{j=1}^{36} A_{ij} VQI_j + VFD_i^d + VX_i \end{aligned} \qquad i \in I_{COM} \tag{A.6.3}$$

$$VQS = \mathbf{A}\ VQI + VFD^d + VX \tag{A.6.4}$$

$$VQC = \text{Diag}(SM)VQS \qquad VQS = \text{Diag}(1/SM)VQC \tag{A.6.5}$$

$$\begin{aligned} &\text{Diag}(1/SM)VQC - \text{Diag}(SX)VQC - \mathbf{A}\ VQI = VFD^d \\ &\text{Diag}(1/SM - SX)\mathbf{M}'\text{Diag}(\iota + tt^{full})VQI - \mathbf{A}\ VQI = VFD^d \\ &\left[\text{Diag}(1/SM - SX)\mathbf{M}'\text{Diag}(\iota + tt^{full}) - \mathbf{A}\right] VQI = VFD^d \end{aligned} \tag{A.6.6}$$

With buyer specific externality:

$$diag(1/PS)VQS = [1/PB]\mathbf{A}\ VQI + [1/PB^d]VFD^d + diag(1/PS)VX \tag{A.6.7}$$

$$\begin{aligned} &\left[\text{Diag}(1/(SM.PS) - SX./PS)\mathbf{M}'\text{Diag}(\iota + tt^{full}) - (1/PB)\mathbf{A}\right] VQI \\ &\quad = (VC./PB^C + VI./PB^I + VG./PB^G) \end{aligned} \tag{A.6.8}$$

Saving-investment balance

$$\begin{aligned} VII &= S - \left(BG_t - BG_{t-1} - \Delta P_t^{BGF} - BG^{disc}\right) - \left(BF_t - BF_{t-1} - BF^{disc} - \Delta P^{BF}\right) \\ &= S - (\Delta G + GFI) - (CA - GFI) \\ &= S - \Delta G - CA \end{aligned} \tag{A.6.9}$$

Demand equal supply of private capital

Option "free oil/gas mining capital":

$$PKD_j = \psi_j^K PKD \qquad j \in I_{BUY} \tag{A.6.10}$$

Option “fixed oil/gas mining capital”:

$$PKD_j = \psi_j^K PKD \qquad j \in I_{BUY\setminus\{2,3\}}$$

$$KD_{j=2} = \overline{KD}^{oil}; \quad KD_{j=3} = \overline{KD}^{gas}; \qquad \text{endogenous } PKD_{j=\{2,3\}} \tag{A.6.11}$$

$$\sum_{j=1}^{C} PKD_j KD_j = PKD.KD \qquad j \in I_{BUY} \tag{A.6.12}$$

$$\sum_{j=1}^{C} \psi_j^K KD_j = KD = \psi^K K_{t-1} \tag{A.6.13}$$

Demand equal supply of government capital

$$VG_{GK} = P^{KG} K_{t-1}^{G} \tag{A.6.14}$$

Demand equal supply of labor

$$PLD_j = \psi_j^L \frac{P^h}{\left(1 - tl^m\right)} \qquad j \in I_{\text{BUY}} \tag{A.6.15}$$

$$PN^R = \psi_C^R P^h \tag{A.6.16}$$

$$YL^{gross} = VLD = \sum PLD_j LD_j \tag{A.6.17}$$

$$\left(1 - tl^m\right) \sum PLD_j LD_j = P^h LS \qquad \text{demand=supply}$$
$$= P^h \left(\bar{L}^{ua} - \psi_C^R N^R\right) \tag{A.6.18}$$

Equivalently, $\sum_{j=1}^{36} \psi_j^L LD_j = LS$ (A.6.19)

Business cycle adjustment for large changes in unemployment

$$\bar{L}^{ua} = (1-u)\bar{L} \tag{A.6.20}$$

Disaggregated capital version: Arbitrage between different assets

$$i = \rho^{eq} - \pi^{eq} \tag{A.6.21}$$

$$\rho^e = \bar{r}_0 PKD + \pi \tag{A.6.22}$$

$$\pi^{eq} = i(BAA) - \rho^e \tag{A.6.23}$$

$$\bar{r}_0 \qquad \text{from model simulation trials} \tag{A.6.24}$$

National Accounting

$$GDP = VCC + VII + VGG + V^{EX} - V^{IMP} - V^{NCI} \quad \text{(A.6.25)}$$

$$GNP = GDP + Y^{ROW} - GINT^{row} - G^{tran,row} - H^{row} \quad \text{(A.6.26)}$$

$$CC^{div} = divisia(C_i; PS_i^C) \quad \text{(A.6.27)}$$

$$II^{div} = divisia(I_i; PS_i) \quad \text{(A.6.28)}$$

$$GG^{div} = divisia(G_i; PS_i) \quad \text{(A.6.29)}$$

$$X^{div} = divisia(X_i; PC_i) \quad \text{(A.6.30)}$$

$$M^{div} = divisia(M_i, NCI_j) \quad \text{(A.6.31)}$$

$$Y^{GDP} = divisia(CC^{div}, II^{div}, GG^{div}, X^{div}) - M^{div} \quad \text{(A.6.32)}$$

Formula for $Q = divisia(q_i; p_i)$:

$$\ln \frac{Q_t}{Q_{t-1}} = \sum_{i=1}^{n} \tfrac{1}{2} \left(\frac{p_{it} q_{it}}{V_t} + \frac{p_{i,t-1} q_{i,t-1}}{V_{t-1}} \right) \ln \frac{q_{it}}{q_{i,t-1}}; \qquad V_t = \sum_{i=1}^{n} p_{it} q_{it} \quad \text{(A.6.33)}$$

Within-period simple price indices:

$$\ln PCC^{CD} = \sum_{i=1}^{I_{INP}} \frac{VC_i}{VCC} \log PS_i^C \quad \text{see (A.1.52)} \quad \text{(A.6.34)}$$

$$\ln PII^{CD} = \sum_{i=1}^{I_{INP}} \frac{VI_i}{VII} \log PS_i \quad \text{(A.6.35)}$$

$$\ln PGG^{CD} = \sum_{i}^{NCOM} \alpha_i^G \ln PS_i (1 + tc^{gov}) \quad \text{see (A.4.56)} \quad \text{(A.6.36)}$$

$$\ln PEX^{CD} = \sum_{i=1}^{I_{INP}} \frac{VX_i}{V^{EX}} \log PC_i \quad \text{(A.6.37)}$$

$$\ln PIM^{CD} = \sum_{i=1}^{I_{INP}} \frac{ePM_i^*}{V^{IM}} \log ePM_i^* \quad \text{(A.6.38)}$$

$$\ln P_{CD}^{GDP} = \frac{VCC}{GDP} \log PCC^{CD} + \frac{VII}{GDP} \log PII^{CD} + \frac{VGG}{GDP} \log PGG^{CD} + \frac{V^{EX}}{GDP} \log PEX^{CD} - \frac{V^{IM}}{GDP} \log PIM^{CD} \quad \text{(A.6.39)}$$

Numeraire

$$P_t^h = \bar{P}_t^h \quad \text{(A.6.40)}$$

Walras Law check

$$wal = [P^h \bar{L} - (1 - tl^m) YL^{gross} - P^{leis} L^{leis}] / P^h \bar{L} \quad \text{(A.6.41)}$$

Reporting units. Official NIPA base year = *tbase*

$\bar{P}^{numeraire}_{tbase=2005} = 1; \quad \bar{P}^{numeraire}_{t} = \text{official GDP deflator}$ (A.6.42)

Current dollar values in terms of official NIPA:

$GDP^{cur}_t = \bar{P}^{numeraire}_t GDP_t; \quad VCC^{cur}_t = \bar{P}^{numeraire}_t VCC_t; \ldots$ (A.6.43)

Household Welfare and Social Welfare

$$\ln V_{dt} = \alpha_p' \ln p_t + \tfrac{1}{2} \ln p_t' B_{pp} \ln p_t - D(p) \ln \frac{M_{dt}}{N_{dt}} \tag{A.6.44}$$

$$= \frac{D_t}{D_0} \ln V_{d0} + D_t \ln \left(\frac{D_0 \gamma_t N_{dt} P_t}{\delta^t D_t N_{d0} P_0} \right) \quad \text{(from application of Euler eqn)}$$

$$\ln N_{dt} = \frac{1}{D(p_t)} \ln p_t B_A A_d \tag{A.6.45}$$

$$\ln P_t = \frac{\alpha_p' \ln p_t + \tfrac{1}{2} \ln p_t' B_{pp} \ln p_t}{D_t} \tag{A.6.46}$$

$$V_d = \sum_{t=0}^{\infty} \delta^t \ln V_{dt} \qquad \delta = \frac{1}{1+\rho} \tag{A.6.47}$$

Lifetime budget constraint:

$$\Omega_d = \sum_{t=0}^{\infty} \gamma_t M_{dt}(p_t, V_{dt}, A_d) \qquad \gamma_t = \prod_{s=0}^{t} \frac{1}{1+r_s} \tag{A.6.48}$$

$$\ln \Omega_d(\{p_t\}, \{\gamma_t\}, V_d) = \frac{1}{S} \left[S \ln R + \sum_{t=0}^{\infty} \delta^t D_t \ln \left(\frac{D_0 \gamma_t N_{dt} P_t}{\delta^t D_t P_0} \right) - V_d \right] \tag{A.6.49}$$

$$\Delta W_d = \Omega_d(\{p_t^0\}, \{\gamma_t^0\}, V_d^1) - \Omega_d(\{p_t^0\}, \{\gamma_t^0\}, V_d^0) \tag{A.6.50}$$

A.7 Intertemporal equilibrium and Steady-state

The intertemporal equations for any t-1 and t<T:

$$\left[\frac{F_t / N_t^{\text{eq}}}{F_{t-1} / N_{t-1}^{\text{eq}}}\right]^{1/\sigma} = \frac{1+r_t^{net}}{1+\rho}\frac{PF_{t-1}}{PF_t} \qquad \text{see (A.1.4)} \qquad (A.7.1)$$

$$(1+r_t)\frac{PII_{t-1}}{\psi_{t-1}^I \varepsilon_{t-1}^I} = \frac{1-tk}{1-t^{ITC}}(PKD_t \psi_t^K - tpPK_{t-1}) + (1-\delta)\frac{PII_t}{\psi_t^I \varepsilon_t^I} \qquad \text{see (A.3.4)} \qquad (A.7.2)$$

$$r_t^{net} = r_t - \rho_t^{risk} \qquad (A.7.3)$$

ρ_t^{risk} exogenous risk premium for calibration purposes

Steady-state

T denotes the terminal period that approximates the steady state. The additional equations that hold at T (and does not hold at t<T):

$$\text{Prices}_T\text{-Prices}_{T-1} < tol \qquad (A.7.4)$$

$$\text{Quantities}_T\text{-Quantities}_{T-1} < tol \qquad (A.7.5)$$

$$\Delta G_T = 0 \qquad (A.7.6)$$

$$CA_T = 0 \qquad (A.7.7)$$

$$r_t = \rho \qquad (A.7.8)$$

$$\psi^I I_T^a = \delta K^T \qquad (A.7.9)$$

A.8 Glossary

8.1 Values; 8.2 Quantities; 8.3 Prices, interest; 8.4 Shares and Probabilities
8.5 Behaviorial Parameters; 8.6 Tax rates, Govt spending rates;

A.8.1 Values:

A		IO Use matrix; the use of commodities by each industry
A_j	$j \in I_{IND}$	Columns of **A**
A_{ij}	$j \in I_{COM}$ $j \in I_{IND}$	Share of input *i* in producing output *j*
BB^{flat}		Business tax base (flat tax)
BF		Net US private sector claims on rest-of-world
BF^{disc}		Stock-flow discrepancy in the US external accounts
BG		Government debt to domestic households
BG^{disc}		Stock-flow discrepancy in the US govt accounts
BG^{*}		Government debt to rest-of-world
BN_j	$j \in I_{IND}$	Tax base of noncorp portion of capital income
BQ_j	$j \in I_{IND}$	Tax base of corp portion of capital income
CA		Current account surplus of the US
$debt_h$	`	Debt financed portion of household capital
DEP^{tot}		Value of total depreciation
DIV		"Dividends"; after-tax capital income
EXP^{gengov}		Total expenditures of general govt
$EXPEND$		Total expenditures of government
G^{Ktran}		Capital transfers from government to households
$G^{Ktran,row}$		Capital transfers from government to rest-of-world
G^{tran}		Government transfers to households
$G^{tran,row}$		Government transfers to rest-of-world
$G^{tran,SS}$		Transfers from gen gov to Social Ins Trust Fund
GDP		Value of Gross Domestic Product
GDP^{cur}		Value of GDP in historical current prices
GFI		Government net foreign investment
$GINT$		Government interest payments on public debt to households (including social insurance funds)
$GINT^{adj}$		Arbitrage adjustment for interest income on government bonds
$GINT^{endog}$		Govt. interest payments in the endogenous option
$GINT^{hh}$		Govt interest payments on public debt to households (excluding social insurance funds)
$GINT^{p}$		Govt interest payments on all debt to residents (including social insurance funds)
$GINT^{row}$		Government interest payments to rest-of-world

$GINT^{ss}$	Govt interest payments on public debt to social insurance funds
GNP	Value of Gross National Product
GM	Government net imports
G_SS	Net gen govt payments to Soc Ins Trust Funds
H^{row}	Household transfers to rest-of-world
$\mathbf{M}$	Input-output Make matrix
M_k	Expenditures by household k
MF^X	Full expenditures (incl. leisure), CEX basis
MF^N	Full expenditures (incl. leisure), NIPA basis
R_h^p	Property tax revenue from household
$R^{gengov,TOT}$	Total tax revenues of general govt
R^N	Non-tax receipts of the government
RCG^{hh}	Tax revenue from capital gains on homes
RHK	Revenue from indiv cap taxes; total
RK^{hh}	Revenue from household capital services tax
RKH^{eq}	Revenue from indiv cap taxes on equity
RKH^{hh}	Revenue from indiv cap taxes on HH capital
RKH^{int}	Revenue from indiv cap taxes on HH claims on government and ROW
R_CON^{gov}	Revenue from consumption taxes on government spending
R_CON^{hk}	Revenue from consumption taxes on household capital
R_CON^{marg}	Notional revenue from consumption taxes (ignoring the exemption/rebate)
R_CON^{net}	Revenue from consumption taxes
R_CON^{reb}	Rebate for consumption taxes
R_EXT	Revenue from externality taxes
R_FLAT	Revenue from flat taxes
R_FLAT^{bus}	Revenue from business flat tax
R_FLAT^{hh}	Revenue from household flat tax
R_ITC	Negative revenue from investment tax credit
R_K	Total capital tax revenue
R_KK	Capital tax revenue from physical capital
R_K^{hh}	Revenue from taxes on household capital
R_P	Revenue from property taxes
R_SALES	Revenue from sales taxes
R_SS	Transfers from Soc Ins for admin expenses
R_TARIFF	Revenue from tariffs on imports
R_TOTAL	Total revenues of government
R_UNIT	Revenue from new taxes on unit of output
R_W	Revenue from taxes on wealth (estate tax)
S	Private savings
$TLUMP$	Lump sum tax
TB	Trade balance

TB^*		Trade balance in foreign prices
TX^{tot}		Total tax expenditures adjustment
TX^{ADE}		Tax expenditures: Accelerated depreciation allowances
TX^{MID}		Tax expenditures: Mortgage interest deduction
TX^{PTD}		Tax expenditures: Property tax deduction
V^{EX}		Total value of exports
V^{IMP}		Total value of competitive imports
V^{NCI}		Total value of non-comparable imports
V^{QC}		Vector of values of domestic commodity output
V^{QI}		Vector of values (to producer) of domestic industry output
VC		Vector of values of household purchases of commodities
VCC		Value of aggregate consumption (PCE)
VCC^{exempt}		Consumption tax exemption base
VCC^{net}		Consumption expenditures net of exogenous part
VCC^{exog}		Value of exogenous consumption expenditures
VFD		Vector of values of final demand for commodities
VG_i (VG)	$i \in I_{COM}$	Value of government demand for commodity i (vector)
VG_{GK}		Value of government capital consumption
VGG		Government spending on goods and services
VI		Vector of values of investment inputs
VII		Value of domestic private investment
VII^{bus}		Value of business investment (flat tax)
VII^{invy}		Value of inventory investment
VII^{fixed}		Value of fixed private investment
VK^{gain}		Value of aggregate capital gains
VK_j^{rep}	$j \in I_{IND}$	Replacement cost of capital stock
VN		Vector of values of household purchases of NIPA commodities
VP^j	$j \in I_{IND}$	Vector of values of input into industry j
VT^{QI}		Vector of values of domestic industry output inclusive of sales tax
VU_{ij}	$i \in I_{COM}$	Value of intermediate demand by industry j
VQI_j	$j \in I_{IND}$	Value of industry output (producer price)
VQI_j^T	$j \in I_{IND}$	Value of industry gross output (purchaser price)
VQS		Vector of values of total commodity supply
VX		Vector of values of commodity exports
W		Tangible wealth of private sector (households)
wal		percentage error in Walras Law check
WF		Full wealth of private sector (households)
XR		Travel exports: Expenditures by foreign tourists in U.S.

Y		Private Income
Y^I		Interest from debt portion of claims on all capital
Y^*		Exogenous projected rest-of-world income
Y^{row}		Net income from rest-of-world
$Y^{row,adj}$		Arbitrage adjustment for income from rest-of-world
YF		Full private income (including imputations on leisure)
YK		Gross private capital income
YK^{bus}		Capital income from private business (flat tax)
YK^{gov}		Capital income from govt enterprises
YK^{net}		Private capital income after tax
YL		Labor income after tax
YL^{gross}		Value of labor income
ΔA_j^{TFP}	$j \in I_{IND}$	Total technical change in industry j
ΔG		Government deficit
ΔP^{BF}		Capital gains on net foreign assets
ΔP^{BG}		Capital gains on government bonds
ΔP^{BG*}		Capital gains on government liabilities to Row
$\Delta \hat{S}C_i$	$i \in I_{CNODE=top}$	Projected difference in cons shares, CEX vs NIPA basis

A.8.2 Quantities

A^{agg}		Productivity shift term that applies to all industries
A_j^{TFP}	$j \in I_{IND}$	Productivity in industry j due to both exogenous and induced components; and shocks
C^P		Vector of quantities of consumption of produced commodities
C		Vector of consumption, commodities and non-produced goods
C_i	$i \in I_{INP}$	Consumption of IO commodity *i*
C_i^X	$i \in I_{CNODE=top}$	Consumption CEX basis, top node item i
CC		Aggregate real consumption (from simple Cobb-Douglas index)
CC^{div}		Divisia index of real Consumption
EX_{it}^0	$i \in I_{NCOM}$	Exogenously projected portion of export function
EXT_x	$x \in I_{EXT}$	Quantity of externality of type *x*
F		Full consumption (commodities and leisure)
G^P		Vector of government purchases of commodities
G		Vector of government purchases, commodities and non-produced goods
G_i	$i \in I_{NCOM}$	Government purchases of commodity *i*

GG			Real government final purchases (from CD index)
GG^{div}			Divisia index of real government final purchases
I^a			Aggregate investment in domestic capital stock
I			Vector of commodities used in aggregate investment
I^m	$m \in I_{INV}$		Investment aggregate *m*
I_i^f	$i \in I_{NCOM}$		Investment of commodity *i* in fixed investment
I_i	$i \in I_{NCOM}$		Investment of commodity *i* in domestic capital stock
II^{div}			Divisia index of real aggreg investment
K			Aggregate private domestic capital stock
$K_{4(oil)}$			Capital stock in "oil and gas mining"
KD			Quantity of aggregate capital input normalized such that its rental price is one
KD_j	$j \in I_{NBUY}$		Quantity of capital input into sector *j*
KD_{jcs}	$j \in I_{NBUY}$ $c = \{c, n, h\}$ $s \in I_{ASSET}$		Quantity of capital input into sector *j*, {corp, noncorp}, {short asset, long asset}
$\bar{L}$			Time endowment of economy
$\bar{L}^{ua}$			Time endowment with unemployment adjustment
LD_j	$j \in I_{NBUY}$		Quantity of labor input into sector *j*
LS			Labor supply
M			Vector of competitive imports
M_i	$i \in I_{COM}$		Imports of (competitive) commodities
M^{div}			Divisia index of real Imports (compet and nci)
N^{eq}			Number of household equivalent members in economy
N^m	$m \in I_{CNODE}$		Consumption of NIPA aggregate *m*
N^R			Leisure quantity (NIPA units)
N_i	$i \in I_{PCE}$		Consumption of NIPA commodities
NCI_j	$j \in I_{NBUY}$		Non-competitive imports into sector *j*
QC_i	$i \in I_{COM}$		Total domestic output of commodity *i*
QI_j	$j \in I_{IND}$		Output industry *j*
QP^{jm}	$j \in I_{IND}$	$m \in I_{PNODE}$	Aggregate input *m* into industry *j*
QP_i^j	$i \in I_{COM}$	$j \in I_{IND}$	Input of commodity *i* into industry *j*
QS_i	$i \in I_{COM}$		Total supply of commodity *i*
$rGDP$			Divisia index of real GDP
QU_{ij}	$i \in I_{COM}$		Quantity of intermediate input into industry j
X			Vector of exports
X_i	$i \in I_{COM}$		Exports of commodity *i*
X_i^{IDEN}	$i \in I_{COM}$		Exports that are explicitly identified in IO
X_i^{tr}			Travel exports of commodity *i*

X^{div}		Divisia index of real exports

A.8.3. Prices:

e		"Exchange rate"
i^*		Interest rate on private U.S. owned foreign assets
i		Cost of capital return to debt
P^h		Price of total hours (work and leisure)
P^{Hm}	$m \in I_{CNODE}$	Vector of prices at node m of consumption function
P^{Im}	$m \in I_{INV}$	Vector of prices at node m of investment function
P^{Pjm}	$j \in I_{IND}$; $m \in I_{PNODE}$	Vector of prices at node m of industry j's production function
$\bar{P}^{numeraire}$		Official NIPA GDP deflator
PB_{ij}	$j \in I_{IND}$	Price of commodity i specific to buyer j
$PB^C, PB^I,$ PB^G		Price of commodity specific to Consumption, Investment and Government final demand
PC_i	$i \in I_{COM}$	Price of domestically produced commodities
PC_i^X	$i \in I_{CNODE=top}$	Price of consumption CEX basis
PC_R^X		Price of leisure on CEX basis
PCC^{CD}		Price of aggregate commodity consumption from simple Cobb-Douglas index
PEX^{CD}		Price of aggregate exports from simple Cobb-Douglas index
PF		Price of full consumption
PGG		Price of aggregate government consumption (Cobb-Douglas index)
PI_j	$j \in I_{IND}$	Price of industry output paid by buyers
PII		Price of aggregate investment goods
PII^{CD}		Price of aggregate investment with simple Cobb-Douglas index
PII^m	$m \in I_{INV}$	Price of investment aggregate m
PII_{mi}	$mi \in I_{INVm}$	Union of above aggregate investment prices and supply prices
PIM^{CD}		Price of aggregate imports from simple Cobb-Douglas index
PK		Price of capital stock
PK^{gain}		Capital gain rate for aggregate capital
PKD		Rental price of aggregate capital
PKD_j	$j \in I_{BUY}$	Rental price of capital paid by producer
PLD_j	$j \in I_{BUY}$	Price of labor paid by employers

PM_i	$i \in I_{COM}$	Price of competitive imports paid by importers
PM_i^*	$i \in I_{COM}$	World price of competitive imports
PM_i^{land}	$i \in I_{COM}$	Landed price of imports before tariffs
PN_n	$n \in I_{NIPA}$	Price of NIPA PCE commodity
PN^m	$m \in I_{CNODE}$	Price of consumption aggregate *m*
PN_{mi}	$mi \in I_{CNODEm}$	Union of above 2 sets of consumption prices
PN^R		Price of leisure (NIPA basis)
$PNCI_j^*$	$j \in I_{BUY}$	World price of non-competitive imports
$PNCI_j$	$j \in I_{BUY}$	Price of non-competitive imports paid by importers
$PNCI_j^{land}$	$j \in I_{BUY}$	Landed price of non-competitive imports before tariffs
PO_j	$j \in I_{IND}$	Price of industry output received by producer
PP^{jm}	$j \in I_{IND} \quad m \in I_{PNODE}$	Price of aggregate input *m* into industry *j*
PP_{mi}^j	$mi \in I_{PNODEm}$	Union of above set of aggregate production prices and prices of inputs
PS		Vector of supply prices
PS_i	$i \in I_{COM}$	Price of commodities to buyers
PS_i^C	$i \in I_{COM}$	Prices of commodities for consumption paid by the household sector (after consumption taxes)
r		After tax interest rate used in Euler equation
r_{jc}	$j \in I_{IND} \;\; c \in I_{LEGAL}$	Weighted (equity and debt) rate of return, corp and noncorp
r_h		Weighted (equity and debt) rate of return to household capital
r_{csj}^{net}	$j \in I_{IND}; c = \{c, n, h\}$ $s \in I_{ASSET}$	Net return on capital, {corp, noncorp, household}, {short, long}
r_c^{equ}	c={c,n,h}	Rate of return to equity; {corp, noncorp, household}
π		Inflation rate in cost of capital formula (version 9)
π^{eq}		Equity premium (over debt)
ρ^e		Cost of capital return to equity

A.8.4 Shares and Probabilities

g^{GDP}		government purchases share of GDP
m_{ji}^{col}	$i \in I_{COM}$	share of national commodity i made by industry j
m_{ji}^{row}	$j \in I_{IND}$	share of industry j's output going to commodity i
H		matrix converting NIPA PCE classification to IO commodity classification
s_n^{con}	$n \in I_{PCE}$	NIPA commodity shares of goods consumption

s_n^{fc}	$n \in I_{PCE-R}$	NIPA commodity shares of full consumption
s_j^m	$j \in I_{IND}$; m=K,L,E,M	{capital, labor, energy, material} input cost shares for industry j
SC_i^N	$i \in I_{CNODE=top}$	Shares of household demand, top tier, NIPA basis
SC_i^X	$i \in I_{CNODE=top}$	Shares of household demand, top tier, CEX basis
SD^i	$i \in I_{COM}$	Shares of domestic output, imports in total supply of *i*
SF		Vector of shares of commodities and leisure in full consumption
SI^M	$m \in I_{INV}$	Shares of investment at node *m*
SM		Vector of shares of imports in total supply
SN^m	$m \in I_{CNODE}$	Shares of consumption at node *m*
SP^{jm}	$j \in I_{IND}$ $m \in I_{PNODE}$	Shares of production at node *m* of industry *i*
SX^i	$i \in I_{COM}$	Shares of total supply of *i* exported

A.8.5 Parameters of behavioral equations, Kalman filter terms:

Household functions

ρ		Pure rate of time preference
σ		Household intertemporal elasticity of substitution
α^{Hm}	$m \in I_{CNODE}$	Shares (at unit prices) of consumption at node *m*
B^{Hm}		Share elasticity of consumption (w.r.t. prices) at node *m*
B_{pA}		Coefficients on demographic characteristics of CC function
ξ^{dd}		Distribution coefficient in top tier household demand function
ξ^L		Coefficients of demographic terms in top tier household demand function
f_t^{Hm}	$m \in I_{CNODE}$	Latent variable for bias of consumption change, lower tiers
ψ_C^R		Aggregation constant of leisure
H		Bridge matrix linking NIPA "Personal Constant Expenditures" commodities to IO commodities

Production and commodity functions

α_0^j	$j \in I_{IND}$	Cost function constant
α^{Pjm}	$j \in I_{IND}$ $m \in I_{PNODE}$	Shares (at unit prices) of inputs into industry *j* at node *m*
B^{Pjm}	$j \in I_{IND}$	Share elasticity of input demands (w.r.t.) at node *m*

B_{pt}^{j}	$j \in I_{IND}$		Biases of technical change
f_t^{Pj}	$j \in I_{IND}$		Latent variable for bias of technical change, top tier
f_t^{j}	$j \in I_{IND}$		Latent variable for technical change, top tier
f_t^{Pjm}	$j \in I_{IND}$	$m \in I_{PNODE}$	Latent variable for bias of technical change, lower tiers
A^{agg}			Index of aggregate technology shock
ΔA^{agg}			Aggregate technology improvement
λ_j	$j \in I_{IND}$		Industry technology shock
T^{agg}			Aggregate technology level shifter to hit GDP growth targets
M			IO Make matrix; the contribution of each industry to each commodity
$\boldsymbol{m}^{row}$			Row shares of Make matrix
$\boldsymbol{m}^{col}$			Column shares of Make matrix
δ			Depreciation rate (aggregate capital)

Capital input and cost of capital functions

α_{KD0}^{j}	$j \in I_{IND}$	Constant of industry capital input price function
α_{KD}^{j}	$j \in I_{IND}$	Shares (at unit prices) of inputs of industry capital input
B_{KD}^{j}	$j \in I_{IND}$	Share elasticity of components of industry capital input
α_{KD}^{jc}	$j \in I_{IND}$	Shares (at $p = 1$) of components of indus corporate cap input
B_{KD}^{jc}	$j \in I_{IND}$	Share elasticity of components of indus corporate cap input
α_{KD}^{jn}	$j \in I_{IND}$	Shares (at $p = 1$) of components of indus noncorporate cap input
B_{KD}^{jn}	$j \in I_{IND}$	Share elasticity of components of indus noncorporate cap input
α_{KD0}^{h}		Constant of household capital input price function
α_{KD}^{h}		Shares (at $p = 1$) of components of household capital input
B_{KD}^{h}		Shares of components of household capital input
δ		Depreciation rate (aggregate capital)
δ^{std}		Depreciation rate allowed by law
δ^{econ}		Actual economic depreciation rate
δ_{cs}	$c = c,n,h$	Rate of depreciation of short-lived capital stock
δ_{cl}	$c = c,n,h$	Rate of depreciation of long-lived capital stock
β_{jc}	$j \in I_{IND}$	Corporate debt-equity ratio, industry *j*

β_{jn}	$j \in I_{IND}$	Noncorporate debt-equity ratio, industry *j*
β_h		Debt-equity ratio, household
α^{DIV}		Dividend-payout ratio

Investment functions and capital stock functions

α^{IY}		Share of inventory investment in total investment
α_i^{IY}	$i \in I_{COM}$	Share of inventory investment going to commodity *i*
α^{Im}	$m \in I_{INV}$	Shares (at unit prices) of commodities at investment node *m*
B^{Im}	$m \in I_{INV}$	Shares elasticity of components of total investment at node *m*
λ^I		Shocks to top tier investment cost function
ε^I		Shock to rate of capital formation
f_t^{Im}	$m \in I_{INODE}$	Latent variable for bias of investment change, lower tiers
ψ^K		Aggregation constant of capital services
ψ_j^K	$j \in I_{BUY}$	Aggregation constant of capital
ψ^I		Aggregation constant of investment goods
ψ^{PK}		Aggregation constant of price of capital stock

Trade functions

α^{Mi}	$i \in I_{COM}$	Shares (at unit prices) of domestic commodities and imports in total supply
B^{Mi}	$i \in I_{COM}$	Shares elasticity of components of total supply
f^{Mi}		Latent variable for bias of import change
η^i	$i \in I_{COM}$	Export price elasticities
α^{Xi}	$i \in I_{COM}$	Shares of exports in total supply
B^{Xi}	$i \in I_{COM}$	Shares elasticity of components of exports
f^{Xi}		Latent variable for bias of export change

Government functions

α_i^G	$i \in I_{INP}$	Share of government expenditures on *i*

Labor functions

ψ_j^L	$j \in I_{BUY}$	Aggregation constant of labor
ψ_C^R		Aggregation constant for aggregate leisure

Externalities functions

XP_{jx}	$j \in I_{IND}$	$x \in I_{EXT}$	Production externalities
XM_{ix}	$i \in I_{COM}$	$x \in I_{EXT}$	Import externalities
XC_{ijx}	$i \in I_{COM}$	$x \in I_{EXT}$	Buyer specific externality x

A.8.6 Tax rates, tax parameters, Govt and Funds spending rates:

γ_c^p	$c = c,n,h$	Deduction of property taxes (= 1 in version 9)
γ_c^i	$c = c,n,h$	Proportion of interest payments deducted before tax
γ_c^d		Proportion of dividends deducted before tax on corp.
γ_c^g	$c \in I_{LEGAL}$	Proportion of capital gains on corporate equities excluded from individual income for tax purposes
γ^{VGG}		parameter for setting govt purchases as share of GDP
θ^{ephi}		Employer provided health insurance (ephi) share of labor compensation that is exempted from income tax.c
dhi		Proportion of inflation premium in interest determined by indexing rule of household interest expense
t_c		Tax rate on corporate capital income (federal + S&L)
t_c^e	$c = c,n,h$	Tax on equity income (corporate, noncorporate, household)
t_c^{earn}		Average tax on personal corporate capital income
t_c^g	$c = c,n,h$	Capital gains tax (corporate, noncorporate, household)
t_c^p	$c = c,n,h$	Property tax rate; {corporate, noncorp, household
t^{ITC}		Rate of investment tax credit
t_h		Tax rate on household income used to adjust deductions
tc_i	$i \in I_{COM}$	Total tax rate on consumption commodity
tc		Consumption tax rate
tc^g		Consumption tax on goods only
tc^G		Consumption tax on govt spending
tc^K		Consumption tax on household capital input
tc^L		Consumption tax on private household labor
tc^N		Consumption tax on imports only (NCI)
tl^a		Average tax rate on labor income

tl^{flat}		Flat tax rate on income
tl^{m}		Marginal tax rate on labor income
RL^{0}		Implied taxes on labor income at zero income due to difference between marginal and average rates
tk		Tax rate on aggregate capital income
tk^{hh}		Tax rate on household capital input
tp		Tax rate on aggregate property
tr_i	$i \in I_{COM}$	Tariff rate on competitive imports
tr_i^n	$i \in I_{BUY}$	Tariff rate on noncomp. imports
tr_i^*	$i \in I_{COM}$	World tariff rate on US exports
tt_j	$j \in I_{IND}$	Indirect business tax (sales tax)
tt_j^{full}	$j \in I_{IND}$	The full tax rate on sales
tu_i	$i \in I_{IND}$	Unit tax on quantities sold
tx_i^u	$i \in I_{IND}$	Total unit externalities tax on quantities sold
tx_i^v	$i \in I_{IND}$	Total externalities tax on sales
tx_i^{ru}	$i \in I_{COM}$	Total unit externalities tax on quantities imported
tx_i^{rv}	$i \in I_{COM}$	Total externalities tax on imports
tx_x^{Xu}	$x \in I_{EXT}$	Tax on one unit of externality x
tx_x^{Xv}	$x \in I_{EXT}$	Tax on one dollar of externality x
tw		Wealth tax rate (estate taxes)
z_{cs}	$c \in I_{LEGAL}\ s \in I_{ASSET}$	Depreciation allowances for $1 of investment