

Dark matter and Wigner's third positive-energy representation class

Bert Schroer

permanent address: Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany
present address: CBPF, Rua Dr. Xavier Sigaud 150,
22290-180 Rio de Janeiro, Brazil

June 30, 2013

Abstract

The almost 7 decades lasting futile attempts to understand the possible physical content of the third Wigner representation class (the infinite spin class) came to a partial solution with the 2006 discovery of existence of string-localized spacetime covariantizations . This has led to a still ongoing vast generalization of renormalizability to fields with arbitrary high spin and a better understanding of the origin of partial invisibility as observed in the confinement of gluons and quarks.

The present note explains the total (non-gravitational) invisibility of fields associated to the third Wigner representation class. The last section presents a critical look at the possibility that third class Wigner matter may play a role in dark matter formation.

1 Preparatory remarks: Wigner's positive energy representation classes and perturbative interactions

In a trailblazing 1939 publication on a representation theoretical approach to relativistic quantum theory, Eugene Wigner [1] succeeded to classify all unitary irreducible ray representations of the Poincare group. In particular he found three classes of positive energy representations which, as it turned out later, cover the particle aspects of modern quantum field theory (QFT). One can assume that one of his motivations was to find an intrinsic access to particle theory i.e. one which is not based on a "quantization parallelism" to relativistic classical field theory. This seems likely in view of the fact that his previous collaborator Pascual Jordan already worried about this problem 10 years before in his 1929 status report on QFT [2] (see also [3]) when he suggested to look

for a formulation "without classical crutches". Indeed, Wigner's representation theoretical setting was the first (still limited) attempt to follow the intrinsic logic of covariant and causal relativistic quantum theory.

A second, perhaps more important motivation was the fact that in the 30's new field equations were proposed almost on a monthly basis, many of them with identical physical content but quite different in their appearance. Indeed, one irreducible Wigner representation permits infinitely many covariantizations in terms of generating singular covariant spinorial/tensorial wave functions $\psi^{A,\tilde{B}}(x)$ with different-looking field equations and inner products since the physical spin s is related to the spinorial characters (A, \tilde{B}) by inequalities which permit infinitely many solutions¹. Less than two decades later also their rather singular mathematical status as *wavefunction-valued distributional generators* in the sense of Laurent Schwartz became clear; this phenomenon, which is related to vacuum fluctuations, has no classical counterpart. For the first time the origin of the ultraviolet divergencies, encountered in calculations with quantum fields was understood; although it still took some additional time to adjust the covariant perturbation theory in such a way that all intermediate ultraviolet divergences are avoided [5].

For a long time Wigner's important paper did not receive the attention which it deserved. But in the early 50's, first Arthur Wightman, and a few years later Rudolf Haag began to appreciate its depth and explored it for a general formulation of QFT and for a more foundational route to free fields which avoids quantization. Finally Steven Weinberg [6] provided the most explicit general construction of intertwiners which map Wigner's unitary representations into pointlike covariant fields with the help of group theoretical methods (even though in the later sections of his book he returns to quantization which only uses one of the many covariant field descriptions, namely that which is also "Euler-Lagrange"² (which turned out to be indispensable for Lagrangian or functional integral quantization but plays no role in the more intrinsic setting of "causal perturbation theory" [5]).

For the rest of the present paper it is important to realize that Lagrangian quantization or the closely related functional integral representation are not demanded by any quantum physical principle and certainly do not comply with Wigner's more intrinsic representation-theoretical spirit and its close connection to an intrinsic localization concept (*modular localization*). Without going back to the representation-theoretical roots of fields, one would (and did) miss the fact that the third Wigner ("infinite spin") class does not permit compact causal localization [8] and that its generating fields are covariant operator-valued distributions $\Psi(x, e)$ which are localized on semiinfinite spacelike lines $x + \mathbb{R}_+ e$, $(e, e) = -1$ (see below).

¹These spinorial fields define the linear part of the "local free field equivalence class (Borchers class [4]) whose nonlinear part consists of Wick-ordered polynomials in free fields. The local equivalence classes is much bigger if one also allows spacelike string-localized fields.

²In his previous work on Feynman-rules for any spin, [7] Weinberg he did not impose this restriction.

It turned out that the introduction of "string-local" fields also solves another representation theoretical problem concerning the second ($m = 0$, h) Wigner class. In this case the covariantization only leads to field strengths, but does not permit potentials. The best known illustration is the $h = 1$ absence of covariant potentials in the Wigner setting which is behind the well-known clash between the Hilbert space positivity and point-localized "potentials" for all ($m = 0, h \geq 1$) representations (4). This problem does not exist in the classical Maxwell theory, but the quantization approach requires to do the computations in a (Gupta-Bleuler, BRST) Krein space and to invoke a gauge principle which allows a Hilbert space description for part of the theory which excludes the charged matter fields. The better way is to let the Hilbert space setting of QFT decide what is the best (tightest) field localization consistent with the Hilbert space setting of QT; the result is that potentials (in particular the $h = 1$ vectorpotential $A_\mu(x, e)$) can only be string-local in the previously explained way.

Additional more powerful support for string-localization comes from the power-counting restriction of renormalization theory which for all massive potentials (and a fortiori for all field strengths) with $s \geq 1$ leads to pointlike fields with short-distance dimension $d_{sd} \geq 2$ which increases with s . This leaves only a finite number of pointlike renormalizable couplings between pointlike fields. The lowest $d_{sd} = 2$ occurs for the $s = 1$ Proca potential $A_\mu^P(x)$ and increases with s . On the other hand one always can construct covariant free string-local potentials with $d_{sd} = 1$ independent of s . This leaves an infinite family of stringlike couplings between fields of any s which are renormalizable in the sense of the power-counting criterion. For the special case $s = 1$ the renormalizability can also be achieved in a gauge theoretic Krein space setting at the price of missing the physical description of charged matter. It should be mentioned that the causal localization principle is not only supporting the use of massless stringlocal potentials, but it is also behind the extension of causal renormalizable perturbation for massive fields of arbitrary large spins.

This does not mean that every string-renormalizable interaction is physical. Of primary interest are couplings for which the nonrenormalizable pointlike formulation is in the same relative locality class ("Borchers class" [4]) as its renormalizable stringlike counterpart; such fields are known to describe the same physics (different "field-coordinatizations"). The perturbative implementation is through the *adiabatic equivalence principle* which implements the same locality class membership in every order of perturbation. This principle cannot prevent the increase of the short distance dimensions of the pointlike fields (nonrenormalizability in the sense of losing the temperateness of Schwartz distributions) but it at least keeps the number of coupling parameters the same as in the stringlike formulation; this removes the main reason why nonrenormalizable couplings have been generically dismissed as useless. Nonrenormalizable couplings whose pointlike fields can be shown to be adiabatically equivalent to renormalizable stringlocal fields are physically useful inspite of their worsening short distance behavior which prevents them from being Wightman fields; they are at best strictly localizable fields (SLF) in the sense of Jaffe [17].

In the limit of massless vectormesons pointlike vectorpotentials and pointlike matter fields get lost and only their stringlocal descriptions remain; for massless Yang-Mills interactions all fundamental fields only exist as stringlike operator-valued Schwartz distributions in Hilbert space. It is precisely this kind of QFT with its gluon and quark confinement which will play a useful contrast with the invisibility and inertness of the irreducible strings of the third Wigner class. Confinement in interacting Y-M models and invisibility (darkness) of free infinite spin "stuff" are two sides of the same coin whose unravelling will be mutually supportive; to show that both are a consequence of upholding QFTs foundational principle of modular localization is the main purpose of this note; in the sequel we will follow some of the historical steps to get there.

Despite Wigner's important early joint contributions to field quantization together with Pascual Jordan, he never tried to connect his representation theory with QFT (or better, if there were such attempts we do not know about them). This apparent reservation may be related to his disappointment that his work with R. T. Newton³ [18] did not give any hint for a covariant concept of causal localization which is needed in QFT. Indeed, the foundational solution of that problem in the form of *modular localization* had to await another 6 decades [8]. Among other things this new concept was the key for getting access to the field theoretic content of the third Wigner class (the massless infinite spin representations) [13].

For this it is helpful to remind the reader how the covariantization of the unitary Wigner representations was done in terms of intertwiners i.e. rectangular matrix-valued functions⁴ $u^{A,\dot{B}}(p)$ which intertwine the unitary Wigner representation with the (A, \dot{B}) covariant representation. The covariant positive energy wave function in x-space for the $(m > 0, s)$ Wigner class then have the form

$$\psi^{A,\dot{B}}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3p}{2p_0} e^{ipx} \sum_{s_3} u^{A,\dot{B}}(p)_{s_3} a^*(p, s_3) \quad (1)$$

$$u^{A,\dot{B}}(p) D^s(R(\Lambda, p)) = D^{A,\dot{B}}(\Lambda^{-1}) u^{A,\dot{B}}(\Lambda^{-1}p) \quad (2)$$

where the notation $a^*(p)$ stands for an L^2 -integrable complex wave function in Wigner's representation Hilbert space. The "Wigner rotation" $R(\Lambda, p)$ is an element of the "little group" which for $m > 0$ is the rotation group so that D^s denotes its $2s+1$ dimensional representation whereas $D^{A,\dot{B}}(\Lambda)$ is the double indexed $(2A+1)(2\dot{B}+1)$ dimensional $SL(2, C)$ representation matrix.

For the $m=0$ representations the little group is the noncompact two dimensional Euclidean group $E(2)$; in this case the faithful infinite dimensional "infinite spin" representations (which constitute the third Wigner class) leads to

³The adaptation of Born's quantum mechanical localization (associated with the spectral resolution of the position operator) to the relativistic inner product.

⁴The right lower index is the component index of the spin/helicity, whereas the left lower index labels the different components of the (A, \dot{B}) spinorial representation of the Lorentz group.

totally different local fields than those associated with the finite helicity h representations of the 2-dim. rotations (for which the noncompact $E(2)$ translation subgroup is trivially represented).

The result is that there are three irreducible positive energy Wigner representation classes: massive representations ($m > 0, s=\text{semiinteger}$), the rather "small" massless finite helicity representation class and the big massless third Wigner class⁵ whose $E(2)$ "little mass parameter" κ has a continuous range.

The transition to covariant local quantum fields in case of the first two classes is accomplished by adding a negative frequency contribution with $b(p)$ together with the charge conjugate $v^{A.\dot{B}}(p)$ intertwiner, with $a^\#(p), b^\#(p)$ now being the canonical creation and annihilation operators for Wigner particles. The resulting quantum fields are operator-valued Schwartz distributions whose momentum space polynomial degree is encoded in the intertwiners; it is more convenient to encode it instead into the short distance dimension of their fields which has its lowest possible value $d_{sd} = 1$ for $s = 0, 1/2$ and increases with spin; e.g. the massive $s=1$ Proca potential $A_\mu^P(x)$ has $d_{sd} = 2$. The connection between the spin and the spinorial labels is the following

$$\begin{aligned} \left| A - \dot{B} \right| &\leq s \leq A + \dot{B}, \quad m > 0 \\ h &= A - \dot{B}, \quad m = 0 \end{aligned} \tag{3}$$

The $a^\#, b^\#$ operators have the indices of the $2s+1$ spin component for $m > 0$, respectively 2 helicity components (needed for the representation of TCP in the massless case). Note that although the Wigner rotations associated to the little group are quite different, the covariance properties agree apart from the fact that the relation between the range of spinorial indices and the helicity (4) is more restrictive.

For the following it is convenient to call all covariantizations (independent of the mass) with $s = \left| A - \dot{B} \right|$ *field strengths* and the others *potentials*; e.g. the massive $s=1$ Proca field $A_\mu(x)$ with $d_{sd} = 2$ is a potential, whereas its antisymmetric curl $F_{\mu\nu}(x)$ is a field strength, similarly the massive $g_{\mu\nu}(x)$ is the potential to the field strength $R_{\mu\nu\kappa\lambda}$ (the linearized Riemann tensor). The different covariantizations for given s or h are local fields living in the same Hilbert space and they are relative local with respect to each other (graded commutativity for half-integer s, h); they constitute the linear part of the afore mentioned "locality class" (Borchers class [4]). For massless representation the Wigner classification yields only pointlike field strengths; the missing potentials are a first indication of a *clash between localization and the positivity property of a Hilbert space description*. In order to return to the full (3) spectrum also for massless fields there are two options: either sacrifice the Hilbert space setting (by allowing indefinite metric Krein spaces) for vectorpotentials or let the Hilbert space description in terms of covariant potentials determine its

⁵Called "infinite spin" class by Wigner, a terminology which will not be followed here since it has been the cause of misunderstandings.

tightest possible localization. Quantization methods (Lagrangian or functional) are based on pointlike classical fields and require the Krein space formulation, but here we prefer to maintain the Hilbert space which requires to work with massless string-localized potentials. It turns out that the use of this second more physical option is also much more powerful if it comes to interactions; it does not only allow to calculate the pointlike localized observables as field strength and pointlike composites as (Maxwell) currents (which in the pointlike quantization setting are the gauge invariants), but it also extends the range of renormalized perturbation theory to string-localized potentials which are in the same localization class as their pointlike field strength and currents. The effect of interacting stringlike vectorpotentials is strongest on the matter fields; by transferring their stringlike localization to the matter fields they force the latter⁶ to be stringlike in a stronger sense than themselves [9][10]. All these new objects are string-local physical fields which act in Hilbert space, and the observables are by definition generated by the pointlike generated subalgebras. The gauge principle is replaced by the more foundational causal localization [10] i.e. the role of gauge invariance is replaced by the e -independence (compact localizability) of observables and the gauge concept which in some way affected the unicity of QFT has been incorporated into modular localization as the only principle⁷.

String-localized fields $\Psi^{A,\dot{B}}(x, e)$ with $e = \text{spacelike direction}$ ($e^2 = -1$) are covariant fields which are localized on a semiinfinite line $x + \mathbb{R}_+ e$. They obey spacelike (graded) commutativity for strings [13][14][15]:

$$\Psi^{(A,\dot{B})}(x, e) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3 p}{2p_0} \{ e^{ipx} u^{(A,\dot{B})}(p, e) \cdot a^*(p) + e^{-ipx} v^{(A,\dot{B})}(p, e) \cdot b(p) \} \quad (5)$$

$$[\Psi(x, e), \Phi(x', e')]_{grad} = 0, \quad x + \mathbb{R}_+ e \rangle \langle x' + \mathbb{R}_+ e' \quad (6)$$

$$U(\Lambda, a) \Psi^{(A,\dot{B})}(x, e) U(\Lambda, a)^* = D^{A,\dot{B}}(\Lambda^{-1}) \Psi^{(A,\dot{B})}(\Lambda x + a, \Lambda e) \quad (7)$$

The notations needs some explanation; the dot denotes the inner product in Wigner's "little Hilbert space", i.e. the $2s+1$ component space in the massive case and the two-component helicity space for massless fields (required by representing \mathcal{P}_+). The subscript *grad* in (6) stands for graded (anticommutator for semiinteger spin fields), and the third line expresses the covariance of stringlike fields. The potentials of the second Wigner class ($m=0$, finite helicity) exist only in string-localized form; in this case it is easy to compute their intertwiners [13]. There is an even simpler and more direct method to compute their two-pointfunctions from semiinfinite integrals over field strengths along spacelike halflines [14][15].

⁶For Y-M selfinteractions the gluon fields are stronger stringlike localized than the vector-potential in QED (see later).

⁷Stora's observation [12] that "gauge symmetries", different from genuine global symmetries (which can be spontaneously broken), are determined by the field content + renormalizability, is a special harbinger of this new philosophy.

The third Wigner class contains only noncompact localized wave functions; its $u(p, e), v(p, e)$ intertwiners cannot be computed in terms of Weinberg's [6] group theoretical covariantization. but rather needs the direct help from modular localization [13]; a closed analytic expression of their two-point function is not known. Different from the first two Wigner classes whose relative local field class contains pointlike field strength and stringlike potentials, the Borchers class of generating fields consists of only stringlocal fields. These fields are *irreducible* stringlike in the algebraic sense that (unlike vectorpotentials in QED) they cannot be represented as integrals over pointlike observable fields; in fact not even their non-compact stringlike x -space wave functions can be cut into compact localized pieces. The dot notation in (5) for the third Wigner class refers to the inner product in Wigner's infinite dimensional "little" Hilbert space which carries an irreducible faithful $E(2)$ representation due to the noncompactness of $E(2)$ and the faithfulness of its representation, turns out to be infinite-dimensional.

The main use of stringlike potentials associated to the massive and finite helicity massless class is the *formulation of renormalizable interaction between fields of any spin*. Interacting massive fields (QFTs with mass gaps) always relate to asymptotic particles and comply with the Lehmann-Symanzik-Zimmermann (LSZ) scattering formalism; this is the main content of a structural theorem by Buchholz and Fredenhagen⁸ [16]. It in particular means that one does not need generating fields which are localized on "branes", but the theorem does not distinguish between situations in which the interacting foundational fields (in terms of which the interaction is formulated) only exist in stringlike form or whether its Borchers class contains also pointlike fields.

From perturbative studies of couplings of free fields from the massive and massless finite helicity class one knows that there are 4 Families of renormalizable QFTs. Couplings between pointlike $s < 1$ fields in a in such a way that the power-counting requirement is fulfilled constitute the *standard class of renormalizable models* in a Hilbert space setting; there only exist a finite number of such couplings. Models involving $s \geq 1$ fields are renormalizable in an *extended* sense if the power-counting requirement can be fulfilled for stringlocal $s \geq 1$ fields. Among those (infinitely many) models, only those are of physical interest which also admit compact localized observables generated by pointlike (generally composite) fields in their Borchers class. They can be subdivided into 3 subclasses.

The subclass which is closest to standard renormalizability consists of models which are power-counting renormalizable in the stringlike sense but for which their formally nonrenormalizable pointlike counterparts, whose short distance dimensions d_{sc} increases in an unlimited way with the perturbative order, can be shown to belong to the same local equivalence class as their renormalizable stringlike siblings. It is this formal affiliation to the same Borchers class which gives them a preferred status within the huge general set of nonrenormalizable models; despite the worsening short distance behavior they retain the

⁸Despite intensive research it has not been possible to isolate an analytic S-matrix property which distinguishes genuinely stringlocal massive models from those which also have pointlike fields in their Borchers class. It seems that there are only off-shell distinctions.

dependence on the same (finite number of) coupling parameters as their renormalizable stringlocal siblings. It is this fact which renders them useful as compared to the rather useless (apart from phenomenological use) generically nonrenormalizable couplings. Perturbative illustrations come from "massive gauge theories" i.e. interactions between *massive* vectormesons and $s < 1$ matter; this matter can either be charged (Dirac spinors, complex scalar fields) or neutral (interaction with "Higgs matter"). The pointlike matter fields are certainly not Wightman fields (operator-valued Schwartz distributions) but they may be strictly localizable fields (SLF) in the afore-mentioned sense of Jaffe [17][10]. Under certain restrictive conditions such nontemperate pointlike objects (fields which only allow the smearing with an interaction-dependent dense set of finitely supported test functions) may generate compact localized operator algebras (the framework used in [16]), but more research on this problem⁹ is needed.

The situation changes radically in the zero mass limit of vectormesons, in that case only stringlocal matter survives and there are two kinds (which together with the two previous cases make up the aforementioned 4 possibilities) of models with very different physical properties. For *abelian* couplings as QED, the vectorpotential is a semiinfinite spacelike integral over the pointlocal observable field strength

$$A_\mu(x, e) = \int_0^\infty d\lambda e_\nu F^{\mu\nu}(x + \lambda e) \quad (8)$$

This has the consequence that the tail of the infinitely spacelike extended e -directed infrared photon cloud (which the interaction with $A_\mu(x, e)$ imprints on the in zero order pointlike matter field $\psi_0(x) \rightarrow \psi(x, e)$) can be manipulated (spacetime infrared regularization). Such manipulations are behind the photon-inclusive cross-section prescription which at least saves a certain aspect of the field-particle relation. A conceptually more foundational idea is that in [19] which is based on the direction of the time-arrow which is fundamental for measurements. A more intrinsic and covariant formulation requires a radical change to a more restricted spacetime setting which controls the infinite spacelike tails of infrared photon clouds in the direction e [19]. The breakdown of the standard field-particle relation can be seen in the vanishing of scattering amplitudes computed as large time limits of correlation function. In perturbation theory this manifests itself in infrared divergencies encountered in mass-shell restricted correlations (whose infrared-regularized summation is consistent with the zero scattering amplitudes).

The most radical difference to the particle-field relation arises if the interacting vectorpotential strings cannot be represented as a semiinfinite spacelike integral over a pointlike observable. This happens in Yang-Mills theories for which non of the fundamental fields (gluons, quarks) are linearly related to pointlike observables; the latter can only be found among polynomial com-

⁹This could decide whether the stringlike localization of the B-F theorem is optimal for the physical matter fields of massive gauge theories.

posites of stringlike fundamental fields. In this case the spacelike strings are algebraically *irreducible*; they cannot be chopped into pieces or manipulated at infinity.

The perturbative interaction of massive vectormesons with $s < 1$ matter has traditionally been studied in the setting of Krein spaces (Gupta-Bleuler, BRST). This has the disadvantage that the pointlike localization of fields has no physical meaning; this disadvantage nullifies the formal advantage from the use of the standard pointlike formalism. There exists a cohomological descent to a Hilbert space description based on the use of "ghost charges" but this only works for the massive vectormesons and unfortunately does not include the rather singular pointlike physical matter fields. It says nothing about the breakdown of pointlike localization of matter fields in the massless limit of vectorpotentials. For Y-M couplings it has led to the incorrect claim that "perturbation theory breaks down for long distances", whereas it is only the pointlike Krein space description but not the correctly formulated (stringlike) perturbation theory which ceases to exist [9][10]. For more mathematical details we refer to section 4.

The reader may wonder why so much attention is directed to stringlocal interactions between free fields from the first two Wigner classes in a work whose main theme is the stuff contained in the third Wigner class and its properties of darkness (invisibility, inertness). The reason is simple: the confinement of interacting gluons/quarks shares a foundational property with the darkness of the interaction-free stuff of the third Wigner class in that the appearance of irreducible noncompact localized stuff from an interaction with incoming normal (compact localized) matter is ruled out by the same foundational causal locality principle of QFT which rules out the compact counter registration of free third class Wigner "stuff" which by some cosmological magic has gotten into our universe. The occurrence of noncompact in addition to compact localized quantum matter may be surprising. But what is even more surprising is that the causality principle permits such a wealth of different physical manifestations. Even the full content of a QFT including spacetime properties (the Poincaré group) and inner symmetries can be constructed from the relative modular positioning of a finite number of "monades" (operator algebras without inner structure, similar to points in geometry [20]) in a shared Hilbert space.

Confinement and invisibility are two sides of the same conceptual coin; they are the still unknown noncompact Yin and Yang behind ordinary matter. A description which bans them into "ghostly" corners of a Krein space deprives itself of understanding their physical role. There is presently no reason to believe that their detailed study, in particular their relation with ordinary matter, has to be looked for outside of perturbation theory, but it is clear that noncompact localized matter cannot be studied in terms of perturbing pointlike fields.

The (theoretical) existence of *noninteracting invisible covariant noncompact zero mass matter with string-localized field generators* is a peculiarity of the third Wigner class. Its invisibility and inertness relative to normal matter is the other side of coin which has confinement written on it. Whereas interacting fundamental fields of Y-M interactions are restricted to their interpolating role which they play for their observable pointlike composites, the stringlocal free

fields of the third representation class cannot transmit interactions simply because an irreducible spacelike string cannot change into a compact transmitter of interactions leading to compact localized normal matter.

In fact the state of such noncompact irreducible matter cannot activate a particle counter; this is because such a counter is almost compact [4] and it is impossible to react with a compact "piece of string" if the string state is irreducible in the sense of Wigner. This does of course not answer how such objects come into being starting from interacting normal matter, but once they exist their string-localization acts like a superselection rule with respect to normal matter; in this sense they are inert. In the next section a more detailed mathematical description will be provided.

The Krein description of interacting vectormesons with matter takes pain to convert the unphysical renormalizable matter field into a nonrenormalizable (nontemperate) physical matter field, whereas in the stringlike setting the renormalizable matter field is already physical. On the other hand, the use of stringlike fields poses new problems for the iterative perturbative implementation of causality because the spacelike separation of semiinfinite strings and its consequences in an Epstein-Glaser iterative formulation of causal perturbation is more subtle than that for points. Fortunately most of these additional problems have meanwhile been solved [9].

The reward is the genuine understanding of renormalizability of massive vectormeson interactions with matter (in particular a more foundational understanding of the coupling of neutral scalar matter which plays the main role in the Higgs model [10]). For zero mass abelian vectorpotentials the gain of insight consists in the absence of physical pointlike matter fields whose incorrect use is the origin of on-shell infrared divergencies, whereas in the Y-M case it is the problem of the connection of confinement of interaction gluons and quarks with the causality principle which cannot be understood in the Krein space setting. Finally, as proposed for the first time in this work, there is a promise of a possible explanation of the origin of dark matter in terms of the third Wigner class. In the Hilbert space setting all objects are physical and the difference of observables and confined or invisible objects has its explanation in terms of differences in localization.

A general phenomenon which has no counterpart in the pointlike case is the loss of the connection (34) between the physical spin s and the spinorial indices (A, \dot{B}) . A scalar stringlocal field may carry any integer spin [13].

The existing literature often uses the terminology "(relativistic) quantum mechanics" instead of QFT. Nobody would call classical field theory when he refers to classical mechanics and it is by no means an exaggeration to say that the quantum counterparts of these theories only share the letter \hbar and the concept of operators in Hilbert space. This is not to say that relativistic QM does not exist; it is a quantum mechanical theory which carries a multiparticle representation theory of the Poincaré group which "cluster factorizes" for asymptotically large spatial distances and its only covariant object is a Poincaré-invariant S-matrix [21]. Its vacuum is inert and it does not allow a second quantization representation; unlike Galilean covariant QM its interaction potentials are not

additive but have to be inductively determined from the interaction with one less particle. QM has no intrinsic notion of localization; its "Born localization" which results from the spectral decomposition of a "position operator" can refer to the living space or to an internal space, the interpretation is completely in the hands of the computing physicists. Related to this fact is the possibility to add dimensions; a linear chain of oscillators can be embedded into a space of arbitrary high dimensions; QM and classical field theory (and even quasiclassical approximations of QFT) is the home territory of Kaluza-Klein ideas.

QFT is totally different setting; its causal localization is intrinsic (an inherent part of quantum matter outside the K-K range) and its global vacuum, far from being inert, changes into a KMS state (a kind of intrinsic statistical mechanics state) upon (the immaterial) restriction to ensemble (mathematically an operator algebra) of observables which share the same localization region. The Feynman path integral, which is a rigorous mathematical concept in non-relativistic QM, loses its mathematical/conceptual status in QFT although its intuitive and communicative (and with sufficient artistic physical hindsight) use remains undisputable. The reader who does not appreciate these basic differences [20] has no chance to understand the content of the present work.

2 The "invisibility" of the third Wigner representation class

Whereas string-localized zero mass potentials in case of finite helicities were not introduced for representation theoretical reasons (the first two Wigner classes can be perfectly "covariantized" in terms of pointlike field strengths), the only covariant way to obtain any localized covariant generating wave function/quantum field for the third Wigner class is through covariant string-localized generators [13]. Once one is in the possession of interaction-free stringlike third class quantum potentials, one may ask the question whether such fields permit at least pointlike *composites*.

The non-existence of quadratic pointlike composites has been excluded; more precisely such pointlike fields, if they exist, cannot have nontrivial maltreatments between infinite spin particle states of low particle number [13]. There are good reasons to believe that the lack of pointlike generating Wigner wave functions has a counterpart in the absence of pointlike (or compact-localized) composites [13]. The historical path leading to string-localized fields actually started with the construction of the massless infinite spin strings of the 3rd Wigner class [13]. This idea originated in turn from the investigation of properties of Wigner representations by *combining representation theory with the ideas of modular localization*¹⁰. The latter is an algebraic property which is independent on which field generators are used in the construction of a local observable algebra;

¹⁰This is presumably the *intrinsic form of causal localization* which Wigner was missing in his quest for a covariant localization of wave functions. The "modular" refers to the Tomita-Takesaki modular theory of operator algebras [23]. It is the only theory in which mathematicians and physicists met on par.

it is the analog of a coordinate independent description of geometry and became the hallmark of a foundational understanding and presentation of QFT referred to as *local quantum physics* (LQP) [4].

The application of modular localization to the Wigner representation theory, which has been accomplished in an important paper by Brunetti, Guido and Longo [8] was not the first time that this concept was used for constructive purposes of QFT; in prior work of the author [22] it was realized that the Zamolodchikov-Faddeev algebra creation and annihilation operators $Z^\#(\theta)$ of integrable QFTs [24] are the Fourier transform of generators of wedge-localized algebras; this in turn led to the first existence proofs of (integrable) QFT's with nontrivial (noncanonical) short distance behavior [25]. In that case the aim was to explore the intuitive idea that "the larger the localization region the better the control of localization-generated vacuum-polarization clouds" which render QFT a very different QT from QM¹¹. In QFT only global objects as generators of symmetries or momentum space creation/annihilation operators are "quantum mechanical" (i.e. free of accompanying vacuum polarization clouds). The insufficient appreciation of this difference led to the "ultraviolet catastrophe" of QFT, a historical misunderstanding which still reverberates through part of the literature (the infinities of perturbation theory and their removal through renormalization). The best compromise between particle states (uncontaminated by polarization clouds) and fluctuating interacting fields¹² is achieved by the use of operators which are localized in the noncompact wedge region [24] which is large enough to allow vacuum-polarization free one particle states and yet still permits to use the constructive power of modular localization.

Although it was not seen at the beginning of the constructive use of modular localization, its use has meanwhile spread into perturbation theory where it is presently extending its scope and range. The limitations of perturbations in terms of coupling pointlike fields has been known for more than 5 decades: the Hilbert space setting is limited to fields with spin $s < 1$ and can be extended to $s = 1$ in the form of "operator gauge theory" i.e. a Krein space setting (Gupta-Bleuler, BRST) which uses the same renormalization formalism and yields through its cohomological descent properties to a Hilbert space description of local observables (field strengths, currents) while remaining silent about the nature of charge-carrying matter. The new setting which uses stringlocal fields in Hilbert space did not only enlarge the range of renormalization theory to all $s \geq 1$, but it also incorporates stringlocal matter coupled to $s = 1$ stringlocal vectorpotentials. The matter fields start as pointlike fields in zero order but become stringlike through higher order delocalization-transfer. The stringlocal formalism also fills the gap left in the Krein space treatment by showing that the stringlocal (perturbative) Borchers class also contains pointlike fields whose short distance dimensions increases without bound with growing perturbative order; but as members of the same Borchers class they depend on the same

¹¹It has been known since the work of Furry and Oppenheimer that states obtained by applying interacting fields to the vacuum contain infinite vacuum polarization clouds (particle-antiparticle pairs, photons).

¹²For free fields particles and fields coexist perfectly for any localization region.

few coupling parameters as their stringlocal siblings. The Borchers class loses its pointlike members only in the limit of zero mass vectorpotentials (QED, massless Y-M).

Neither the pointlike setting nor its stringlike extension lead to converging perturbative series; for the control of existence the only known method is that of starting with wedge generators and obtaining compact localized operator algebras through intersections of wedge algebras. It is interesting to note that the stringlocal fields associated to the third Wigner class, for which no pointlike members in their Borchers class were found, and for which Weinberg [6] found no consistent covariantization and as a result declared them to be unphysical ("nature does not use them") played the role of the historical link between modular localization theory and the perturbative use of stringlocal fields. It is precisely their inertness against coupling them with "normal" matter which makes them candidates for the astrophysical dark matter. Thinking about the local (or quasilocal in order to avoid excitations in the vacuum [4]) nature of particle counters it is clear that an irreducible noncompact localized object which upon applying it to the vacuum remains irreducible noncompact as a state (i.e. cannot be chopped into compact pieces) is an excellent candidate for darkness in the sense of escaping counter-detection.

It has been known for a long time that noncompact wedge-localized algebras offer the best compromise between particles and fields. Localization which are tighter than that in wedges blur the particle aspects of interacting theories in the sense that no subwedge localized operator can create a pure one-particle state from the vacuum without the admixture of vacuum polarization clouds; the strongest theorem showing this fact can be found in [26]. This was the reason why existence proofs and explicit analytic constructions of certain interacting models with realistic noncanonical short distance behavior were possible, while the perturbative use of pointlike or stringlike fields has only led to diverging perturbation series (which unfortunately contain no information about the mathematical existence of a model). By starting with vacuum-polarization-wise better behaved generators for wedge algebras and improve of localization by constructing double cone algebras by intersecting wedge algebras (verifying that one still gets operator algebras with "sufficiently many" nontrivial operators), one has solved the existence problem for many integrable models. Covariant fields are objects which generate the algebras for *arbitrary* localizations, they are in this sense "global". They appear in such a *top-to-bottom construction* only at the very end; in most cases they are not really needed to extract the physical properties of a model. Complete absence of vacuum polarization only occurs for fully global objects as generators of symmetries, momentum space creation/annihilation operators and the S-matrix.

That all positive energy Wigner representation are localizable in wedges was already known from the Bisognano-Wichmann work [27], but the proof that they are also localizable in noncompact spacelike cone regions was a nontrivial more recent generalization. Together with the fact that the first and second Wigner classes admit pointlike localized generators, it was suggestive that the third class, which resisted all attempts to force pointlike covariant localization

on it, should possess stringlike generators. Semiinfinite spacelike strings are the cores of (arbitrary narrow) spacelike cones which are the smallest causally closed noncompact regions as points are the cores of arbitrarily small causally closed compact double cone ("diamonds") regions.

The actual construction of covariant intertwiners which connect the unitary third Wigner representations with covariant wave functions and quantum fields requires the direct application of modular localization ideas; whereas for the first two classes Weinberg's group theoretic way [6] of implementing covariance was sufficient. All attempts to force these representations into a pointlike covariant form in analogy to the first two classes failed. The failure of pointlike localization was explained in a theorem [28] which had however little impact on stopping the continuation of other futile attempts. After the construction of string-local free quantum fields as the generators of the QFT associated with the third Wigner class [13] it became clear that there was also not much hope that such a free field theory would contain *pointlike composites* (see remarks made before).

Intertwiners u and their charge conjugates v of the stringlocal fields in the representation for the scalar representation of infinite spin fields where the dot refers to the inner product in the infinite dimensional little Hilbert space and D is the chosen faithful representation of the little group (the analog of spin/helicity) $E(2)$ (Euclidean group in $d=2$). As expected, for a given irreducible representation D the corresponding field is not unique. The above intertwining relation for u, v together with an analytic restriction from the infinite dimensionality of the little group representation determines a family of intertwiners which belong to the same Wigner representation. It is not clear whether the usual setting of implementing interactions by coupling fields makes sense and allows a perturbative formulation. As mentioned before the inertness of registration of states obtained by applying such fields to the vacuum is guaranteed by the irreducibility of their string-localization. such free fields matter is

In this way it becomes that the QFT of the third Wigner class leads to a very peculiar situation; their invisibility is, as in case confinement, related to their irreducible string localization; but opposite from confinement where causality prevents certain "stuff" can not emerge from a reaction, free stringlike matter of the third Wigner class cannot be gotten rid of by interaction with ordinary (compact localizable) matter. Their offering to be identified with inert dark matter leads to the new question: how to hell gets this kind of matter into our universe?

3 Additional remarks on interacting strings

The strength of string-localization depends on the field content of the model. Models of massive vectormesons interacting with matter are based on two formal connections with their point-local counterparts

$$\begin{aligned}
A_\mu(x, e) &= A_\mu^P(x) + \partial_\mu \phi(x, e) \\
\psi(x, e) &= e^{ig\phi(x, s)} \psi(x)
\end{aligned} \tag{9}$$

whose perturbative validity has to be established through the principle of adiabatic equivalence. In zero order g the equation reduces to the free pointlike matter field and the relation of the free Proca potential and its stringlike counterpart

$$\begin{aligned}
\langle A_\mu^P(x) A_\nu^P(y) \rangle &= \frac{1}{(2\pi)} \int \frac{d^3 p}{2p_0} e^{-ip(x-y)} M_{\mu\nu}^{PP}(p) \\
M_{\mu\nu}^{PP}(p) &= -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}, \quad M^{\phi\phi}(p.e.e') = \frac{1}{m^2} - \frac{ee'}{(pe - i\varepsilon)(pe' + i\varepsilon)} \\
M_{\mu\nu}^{AA}(p; e, e') &= -g_{\mu\nu} - \frac{p_\mu p_\nu}{(pe - i\varepsilon)(pe' + i\varepsilon)} + \frac{p_\mu e_\nu}{pe - i\varepsilon} + \frac{p_\mu e'_\nu}{pe' + i\varepsilon}
\end{aligned} \tag{10}$$

where the M-function for the stringlocal Stückelberg field $\phi(x, e)$ appear next to that of the Proca field in the second line whereas the third line is the M of the stringlocal vectormeson (the mixed correlations [9] have been omitted).

The decomposition of the massive vectorpotential $A_\mu^K(x)$ in Krein space would have a similar decomposition into the Hilbert space Proca field and an indefinite metric Stückelberg field $\phi^K(x)$, except that in this case the Stückelberg field adds unphysical extra degrees of freedom (which have to be removed at the end of the perturbative computations) to the physical Proca field whose only purpose is the "softening" of the short distance behavior so that the $A_\mu^K(x)$ has $d_{sd} = 1$ instead of $d_{sd} = 2$ of the Proca field. The linearity of the cohomological descent in terms of the ghost charge formalism is in conflict with a nonlinear relation similar to the second line (9); this explains why the BRST ghost formalism does not lead to a physical matter field which corresponds to $\psi^K(x)$; to avoid misunderstandings, there is a similar connection to a pointlike physical matter field, but this comes from the *formal* requirement¹³ that the looked for physical matter field is in the same "Borchers class" as $\psi^K(x)$. Nevertheless it is believed that the physical matter field obtained by an exponential transformation of ψ^K by the exponential of ϕ^K leads to the same pointlike physical matter field as the one which appears in (9).

The relation (9) looks like an operator gauge transformation between point- and stringlike fields but it is important to not get confused on the logical order between principles and recipes. The principle behind (9) is *not local gauge invariance* but rather the adherence of the pointlike field to the *same Borchers class* as its stringlike sibling; the two descriptions are in the same modular localization class i.e. the two fields are different "coordinatizations" of the same system of local algebras (spacelike cone algebras and their double cone

¹³Since the Krein space locality is not the physical locality, the "Krein Borchers class" is a formal imitation of modular localization (there is no modular operator theory based on Krein spaces).

intersections). There is no gauge principle which separates QFT into standard theories and gauge theories rather the *gauge recipe* permits a computational access to a situation whose conceptional understanding has its origin in modular localization; renormalization theory in Hilbert space only works if one lets the interacting model determine the localization properties it needs in order to be described in a Wightman setting i.e. whether its generating interacting fields are point- or stringlike operator-valued Schwartz distributions. The clear-cut answer in the case of massive gauge theories is that the pointlike non-temperate short distance fluctuations (violation of power-counting in a pointlike setting) require stringlike localization so that at best there exists a pointlike description in terms of SLF Jaffe fields which is in the same locality class as the stringlike Wightman description.

In the zero mass limit of gauge theories the appearance of strong *large distance fluctuations* implies that any pointlike description of the matter fields breaks down i.e. the Borchers class of generating local charged fields contains only string-local fields so that the gauge invariant fields or composites select the pointlike generated observables. charged field strengths and currents. In this case the relations (9) and their massive Y-M counterparts are

$$A_\mu(x, e) = U(g\phi(x, e))A_\mu^P(x) + \partial_\mu\phi(x, e) \quad (11)$$

where the potentials and ϕ are multicomponent and matrix-valued and the U are operator-valued rotation matrices in which the numerical parameters are replaced by the intrinsic multicomponent string-local Stückelberg fields multiplied with the coupling strength¹⁴. A similar relation as in (9) holds for the multi-charged matter field. Again this relations break down in the zero mass limit; there simply exists no pointlike massless Y-M potential and the best one can do is to relate the stringlocal covariant potential to a noncovariant also noncompact localized Coulomb potential plus a Coulomb type Stückelberg field which restores the covariance. It is somewhat ironic that (911), which come close to the gauge prescription, break down in the massless limit which in the Lagrangian quantization is the territory of the gauge recipe. From a philosophical viewpoint this is deeply satisfying since the modular locality principle combines all models under one conceptual roof. What remains however deeply surprising is the infinite number of very different looking realizations of this principle.

Before indicating the origin of gluon and quark confinement from the modular localization principle and relating it to the inertness of 3rd Wigner class quantum stuff, it is informative to compare our Hilbert space setting to the popular Krein space ghost formalism. Actually the terminology "Stückelberg field" for the scalar stringlocal $\phi(x, e)$ (whose derivative removes the bad short distance behavior of the Proca potential and in turn generates the string-localization of the left hand side (911)) comes from the Krein space setting of gauge theory which, especially in the context of massive gauge setting of the standard model,

¹⁴The operator products are "normal-ordered" (generalizations of Wickproducts) whose precise definition follows from the perturbatively implemented *adiabatic equivalence* between the Proca field and its stringlocal counterpart.

stood for an unphysical additional pointlike scalar field which compensates the bad short distance behavior of the Proca field. It does so without changing the pointlike localization, but one should keep in mind that the localization in a Krein setting is a fake; it has no relation to modular localization which combines properties of operator algebras in Hilbert space with causal localization in Minkowski spacetime. It does not lead to physical matter fields¹⁵ but its cohomological descent (operator gauge invariance) to a Hilbert space does lead to compact physically localized observables [29].

The limitation of the indefinite metric setting with respect to physical matter fields can presumably be overcome by constructing "gauge bridges" linking pointlike matter with anti-matter [30]. In fact it may even be possible to obtain such bilocal objects in a completely intrinsic way in terms of a lightlike limiting process applied to products of pointlike currents [31].

As mentioned before, certain aspects of the new philosophy of renormalizability improvement through string-localization were already noticed in the Krein space setting. Stora's observation that the form of Y-M interaction does not need the existence of a "local gauge symmetry" but is a consequence of locality within the renormalization formalism is one of those observations which relativize the view of "gauge" as a symmetry principle. Only *global inner symmetries* are additional impositions which are not already determined by renormalization + field content.

The most radical change of physical content occurs in the zero mass limit of interactions between massive vectormesons with matter or among themselves. In that case the fundamental fields in terms of which the interaction is formulated only exist in the form of stringlike generators leading to noncompact localized operators. Since our attempts of understanding the physical aspects of the 3rd class Wigner stuff rely heavily on analogies with infrared manifestations and confinement, it is helpful to add additional observations to those already made in the first two sections. There is a sharp distinction between reducible strings as they occur in QED and irreducible strings in Y-M couplings. Whereas the interacting stringlocal vectorpotentials are spacelike semiinfinite integrals over field strengths (8) and therefore can be chopped into pieces and infrared regularized by manipulating the infinite tails which at least maintains certain particle aspects of charged matter (photon-inclusive cross-sections), the irreducible Y-M strings do not allow such manipulations. In this case the linearly related interacting "field strengths" are themselves irreducibly stringlocal and the pointlike polynomial composites have no bearing on the nature of the fundamental strings.

Intuitively it is clear that the causality principle prevents the appearance of such noncompact objects from coming out of a compact interaction region associated to the collision region of standard matter. A detailed analysis of confinement of course amounts to investigation going beyond algebraic structures using states and correlation functions. This has not been done, but the good

¹⁵Although the linear ghost charge formalism does not lead to physical matter fields, it is quite plausible that the formal Borchers-class relation between $\psi^K(x)$ and $\psi^{phys}(x)$ may create such a relation.

news is that there is no reason whatsoever to believe that a perturbative access to stringlike Y-M potentials does not exist. The argument one finds in the literature only says that the perturbation theory in terms of interacting *pointlike* gluon/quark fields $A_\mu^K(x), \psi^K(x)$ have incurable infrared problems, but there is no argument why zero mass physical (and hence necessarily stringlike) off shell correlations should suffer from perturbative infrared problems. The (ongoing) clarification of this point is important, because a computation of an asymptotic freedom supporting beta function from "nowhere" (i.e. without perturbative correlation functions and their parametric Callan-Symanzik equations) is not trustworthy. Similarly one cannot convincingly argue that confinement is related to such infrared divergencies.

The idea of irreducible stringlocal Y-M vectorpotentials and quark fields and the contradiction with causal localization one runs into if such noncompact spacelike extended objects could emerge from strongly interacting compact localized nuclear matter are certainly the foundational physical properties behind confinement and the existence of renormalized Hilbert space stringlocal perturbation theory generates ample hope to clarify the confinement issue in the foreseeable future.

The problem of darkness of 3^{rd} class Wigner stuff has two aspects. The easier one is the fact that such irreducibly noncompact localized stuff cannot be registered in a spacetime compact localized counter. Since most calculations in textbooks on QFT are done in momentum space, it is important to realize that this property has no simple understanding in terms of momentum space manipulations. The difficult part (more difficult than confinement) is to understand the behavior of such matter in contact with normal (compact localized) matter. Certainly the causality property preserves the noncompact spacelike localization; the latter acts like a superselection rule.

From the analogy with the perturbative interaction of stringlike massless Y-M potentials with matter (which in zero order starts in its pointlike form, but becomes stringlike from the higher order interaction with the potentials) it is suggestive to expect that the 3^{rd} class Wigner stuff "string-contaminates" the normal matter. But perhaps the desired inertness of dark matter only appears difficult because nobody has attempted to formulate and perturbatively investigate such couplings.

It is the stringlocal formulation which is renormalizable, therefore the second line in (9) should be viewed as a definition of a nonrenormalizable pointlike matter field which is uniquely given in terms of the stringlocal field. Hence it has the same number of coupling parameters as the renormalizable string field even though its short distance dimension keeps increasing with the perturbative order (whereas the pointlike Proca field maintains the finite short distance dimension $d_{s.d.} = 2$). Hence the transfer of string-localization to the matter field creates an extremely singular pointlike field which formally is the renormalizable stringlike field modified with the exponential of the Stückelberg field which converts the stringlocal Wightman field into a pointlike objects with bad short distance properties which is certainly not a Wightman field (a Schwartz distribution). The conjecture is that it is a "strictly localized" (SLF) field in

the sense of Jaffe [17]. This conjecture is based on the fact that the exponential of the $d_{s,d.} = 1$ free Stückelberg field is known to be of Jaffe type. The situation changes radically in the massless limit of vectormesons (see below) when only string-localized matter fields survive which leads to a loss of the interacting pointlike Proca-potential and the SLF pointlike matter field.

This does not only comply with the nonrenormalizability of massive QED, but it also removes the undesired collateral appearance of infinite coupling parameters as a result of infinitely many undetermined counterterms. The adiabatic equivalence principle gives them a perturbative existence in a finite parametric setting which is inherited from the string-localized description. But whether nonperturbative SLF fields, which only allow smearing with a (model-dependent) *dense subset* of finitely supported testfunctions [17], have a joint dense domain of definition (which is necessary to construct operator algebras) remains questionable.

In using stringlocal fields in perturbative calculations one should be aware of the fact (already mentioned at the end of section 1) that the relation between physical spin and spinorial indices (3) gets lost. This explains why the apparently scalar stringlocal Stückelberg field $\phi(x, e)$ applied to the vacuum creates a $s=1$ state and opens completely new ways for the formation of bound states. Hence the use of a scalar 3^{rd} class Wigner field

$$\varphi(x, e) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3 p}{2p_0} \{e^{ipx} u(p, e) \cdot a^*(p) + e^{-ipx} v(p, e) \cdot b(p)\} \quad (12)$$

$$u(\Lambda^{-1}e, \Lambda^{-1}p) = u(e, p) D(R(\Lambda, p)), \quad R(\Lambda, p) \in E(2)$$

where D is an irreducible faithful (and therefore infinite-dimensional) representation of the little group $E(2)$ in studies of interactions with normal matter is quite sufficient

4 Invisibility and dark matter

Up to now a comparison of invisibility/inertness of noncompact 3^{rd} class Wigner matter with observed properties of astrophysical dark matter has been avoided. But since the title of the present work poses this problem, the author is expected to say something. Besides not being able to activate counters, the infinite spin Wigner matter has additional kinematical properties which hide behind this not quite fitting terminology. Although not *directly* measurable, the faithful representations of the noncompact $E(2)$ little group may lead to unknown physical manifestations if further studies show that interactions with normal matter in case they are possible.

Certainly the noncompactness is preserved; an interesting speculation, mentioned before, is that such interactions could "contaminate" ordinary massive matter. Even the problem of how noncompact zero mass matter behaves in a gravitational field is presently unsettled and analogies with standard $m=0$

matter may be misleading. One expects that its kinematical spacelike noncompactness should have consequences for astrophysical lensing.

The astrophysical data (see below) cannot be explained in terms of only standard zero mass matter (see below) and whether ordinary massive matter can fully account for the reactive inertness which is often ascribed to astrophysical dark matter remains an unsolved problem.

The hardest problem (in the opinion of the present author harder than the confinement problem) is how noncompact-localized matter gets into our universe- Whereas the problem with interacting gluons and quarks is why these objects, even though they are represented by operators in a Hilbert space, cannot "get out", the problem with "invisibility" as a result of noncompact spacetime localizability is how such stuff can "get into" our universe.

The main purpose of this paper is to contribute a thought-provoking impulse for looking into QFT's still dark corners.

Added remarks. According to informations which were kindly provided to me by Edward Witten, dark matter is not primarily made of (ordinary!) massless particles, because such particles would not be bound in galaxies or clusters of galaxies, and dark matter is so bound, in large part. In more detail the present situation is as follows.

Cosmological data are usually fitted to (among other things) the total energy density that is contained in massless particles of any kind. The data refer to observations of the universe as it has existed since a temperature of about 1 electron volt (when the cosmic microwave radiation decouples) although some observations involve the universe at a temperature of order 100 KeV (nucleosynthesis). The data are usually expressed in terms of the number of neutrino species; that is, one fits the data on the basis of the massless particles that are contributing being photons; gravitons (whose contribution is very small); and N species of neutrinos. Then one tries to measure N . The data show that N is close to 3 (some observations gave a result consistent with 4, but the recent data from the Planck satellite show that it is much closer to 3 than 4). So to summarize, any unknown massless particles make a contribution to the energy density of the Universe that is less than that of 1 species of neutrino.

Acknowledgement: I am indebted to Edward Witten for an informative and useful email communication about the present state of the dark matter issue. I also thank Jakob Yngvason and Jens Mund for continuous interest in the fate of the third Wigner class whose understanding in terms of modular localization was the result of a past joint collaboration [13].

References

- [1] E.P. Wigner, On unitary representations of the inhomogeneous Lorentz group, *Ann. Math.* **40**, (1939) 149

- [2] P. Jordan, *The Present State of Quantum Electrodynamics*, in *Talks and Discussions of the Theoretical-Physical Conference in Kharkov* (May 19.-25., 1929) Physik.Zeitschr.XXX, (1929) 700
- [3] B. Schroer, *Pascual Jordan's legacy and the ongoing research in quantum field theory*, Eur.Phys.J.H **35**, (2011) 377-434, arXiv:1010.4431
- [4] R. Haag, *Local Quantum Physics*, Springer 1996
- [5] H. Epstein and V. Glaser, Ann. Inst. Henri Poincaré A XIX, (1973) 211
- [6] S. Weinberg, S. Weinberg, *The Quantum Theory of Fields I*, Cambridge University Press 1995
- [7] S. Weinberg, Phys. Rev. **181**, (1969) 1893
- [8] R. Brunetti, D. Guido and R. Longo, *Modular localization and Wigner particles*, Rev. Math. Phys. **14**, (2002) 759
- [9] J. Mund, *String-localized massive vector Bosons without ghosts: the example of massive QED*, to appear
- [10] B. Schroer, *Quadratic dependence on string-localized massive vectormesons: the example of scalar QED*, arXiv:1307.3469
- [11] B. Schroer, *Modular localization and the holistic structure of causal quantum theory, a historical perspective*, to appear
- [12] R. Stora, Int. J. Geom. Meth. Mod. Phys.**5**, (2008) 1345, arXiv:0901.3426
- [13] J. Mund, B. Schroer and J. Yngvason, Commun. Math. Phys. **268**, (2006) 621
- [14] J. Mund, String-localized quantum fields, modular localization, and gauge theories, New Trends in Mathematical Physics (V. Sidoravicius, ed.), Selected contributions of the XVth Int. Congress on Math. Physics, Springer, Dordrecht, 2009, pp. 495–508.
- [15] J. Mund, *String-localized covariant quantum fields*, Prog. Math. 251, (2007) 199
- [16] D. Buchholz and K. Fredenhagen, Nucl. Phys. **B154**, (1979) 226
- [17] A. Jaffe, Phys. Rev. **158**, (1967) 1454
- [18] T. D. Newton and E. P. Wigner, *Localized states for elementary systems*, Review of Modern Physics **21**, (1949) 400-428
- [19] D. Buchholz and J. Roberts, D. Buchholz and J. Roberts, *New Light on Infrared Problems: Sectors, Statistics, Symmetries and Spectrum*, arXiv:1304.2794

- [20] B. Schroer, *Localization and the interface between quantum mechanics, quantum field theory and quantum gravity I*, Stud. Hist. Phil. Mod. Phys. **41**, (2010) 104, arXiv:0912.2874; part II, Stud. Hist. Phil. Mod. Phys. **41**, (2010) 293, arXiv:0912.2886
- [21] F. Coester and W. N. Polyzou, Phys. Rev. D **26**, (1982) 1348-1379 and references therein
- [22] B. Schroer, Modular Wedge Localization and the d=1+1 Formfactor Program, Ann. Phys. 295 (1999), 190, and references therein
- [23] H-J. Borchers, *On revolutionizing quantum field theory with Tomita's modular theory*, J. Math. Phys. **41**, (2000) 8604
- [24] B. Schroer, *The foundational origin of integrability in quantum field theory*, Found. Phys. **43**, (2013) 329; arXiv:1109.1212
- [25] G. Lechner, *On the Construction of Quantum Field Theories with Factorizing S-Matrices*, PhD thesis, arXiv:math-ph/0611050
- [26] J. Mund, Commun. Math. Phys. 315 (2012) 445, arXiv:1012.1454, and references therein
- [27] J. J. Bisognano and E. H. Wichmann, *On the duality condition for quantum fields*, Journal of Mathematical Physics **17**, (1976) 303
- [28] J. Yngvason, Zero-mass infinite spin representations of the Poincaré group and quantum field theory, Commun. Math. Phys. **18** (1970), 195
- [29] G. Scharf, *Quantum Gauge Theory, A True Ghost Story*, John Wiley & Sons, Inc. New York 2001 and references therein.
- [30] M. Duetsch and K. Fredenhagen, Commun.Math.Phys. **203** (1999) 71, arXiv:hep-th/9807078
- [31] J. Langerholc and B. Schroer, *Can Current Operators Determine a Complete Theory?*, Commun. Math. Phys., 4:123–136, 1967. For a recent use of this idea for the intrinsic construction of bilocals with gauge bridges from product of local currents see: <http://unith.desy.de/research/aqft/>, Diploma thesis of S. Jakobs, 2009