

¹ BayesCog: A freely available course in Bayesian statistics and hierarchical Bayesian modeling for psychological science

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Software

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⁸ Summary

⁹ We present [BayesCog](#), an openly-available online course for the computational modeling of human behavior (i.e., cognitive modeling) using Bayesian inference, with reinforcement learning as a core example throughout the course. Assuming little to no prior experience, audience of this course will be formally grounded in key concepts including Bayesian statistics and reinforcement learning, and practically, will build, assess, compare, and validate models using the R interface to the Stan programming language, RStan. Starting with binary choice models, the audience will learn to estimate parameters representing latent components of behavior by fitting reinforcement learning models, both at the individual and group-level, eventually with hierarchical modeling.

¹⁸ The course is generally suitable for those interested in developing models of human cognition at any level of experience. In making the course openly available, we aim for computational modeling under the Bayesian approach to be more strongly represented in the psychological sciences.

²² Statement of Need

²³ Computational modeling is a general framework that uses mathematical equations to infer unobserved latent processes, variables, and parameters from observed data. Whilst implemented in other disciplines (e.g., physics, chemistry, and, astronomy) for centuries, its application specifically towards understanding the human mind (i.e., learning, memory, decision-making, language) ([Farrell & Lewandowsky, 2018](#)) is a relatively recent approach (known as cognitive modeling), one exponentially increasing in popularity ([Palminteri et al., 2017](#)). By formalising cognitive processes as mathematical operations and free parameters, cognitive models generate specific, testable hypotheses about observable behavior, which can be objectively compared, verified, and falsified ([Guest & Martin, 2021; Palminteri et al., 2017; W. Pan et al., 2022; Rocca & Yarkoni, 2021; Zhang et al., 2020](#)). When combined with other modalities of measurement, such as functional magnetic resonance imaging (fMRI), cognitive models present a key framework for understanding how the brain implements cognitive processes such as decision making and (social) learning ([Eckstein et al., 2021; FeldmanHall & Nassar, 2021](#)) and their aberration in mental health disorders ([Hauser et al., 2022; Huys et al., 2016; Sohail & Zhang, 2024](#)).

³⁸ Complementing this approach is the application of Bayesian methods for parameter estimation ([Annis & Palmeri, 2018](#)), which applies the Bayes rule to obtain the posterior distribution of model parameters given the observed data (1):

$$P(\theta|D) = \frac{P(D|\theta) \cdot P(\theta)}{P(D)} \quad (1)$$

⁴¹ where $P(\theta|D)$ is the posterior distribution, $P(D|\theta)$ is the likelihood, $P(\theta)$ is the prior
⁴² distribution, and $P(D)$ is the marginal likelihood.

⁴³ Bayesian methods confer advantages over frequentist approaches (e.g., Maximum Likelihood
⁴⁴ Estimation, MLE), by quantifying the uncertainty, and when implemented hierarchically,
⁴⁵ permit simultaneous estimation of individual and group-level parameters while appropri-
⁴⁶ ately pooling information across participants (M. D. Lee, 2011). Historically restrictive
⁴⁷ due to their computational burden, these methods are now more accessible though the
⁴⁸ development of multiple programming languages and software such as JAGS (Plummer &
⁴⁹ others, 2003) and Stan (Carpenter et al., 2017) which optimize the sampling process used
⁵⁰ for parameter estimation using approaches such as Markov chain Monte Carlo (MCMC).

⁵¹ Using Bayesian models of cognition in one's own research requires a conceptual under-
⁵² standing of both Bayesian statistics and cognitive modeling, as well as the practical skills
⁵³ to translate these models into computer code. Textbooks (Kruschke, 2014; Lambert, 2018;
⁵⁴ M. D. Lee & Wagenmakers, 2014; McElreath, 2018) and tutorial papers (Baribault &
⁵⁵ Collins, 2023; Lockwood & Klein-Flügge, 2021; Wilson & Collins, 2019; Zhang et al.,
⁵⁶ 2020) have made learning these skills more accessible, but are often not freely available,
⁵⁷ and challenging for researchers, especially early career researchers with little to no prior
⁵⁸ experience. Additionally, whilst free, online courses for the computational modeling of
⁵⁹ cognition currently exist, these are few and far between, and exclusively cover non-Bayesian
⁶⁰ implementations in Python (Rhoads & Gan, 2022) and MATLAB (O'Reilly & Ouden,
⁶¹ 2015), as well as Bayesian approaches in Python (Niv Lab, 2021). A full course imple-
⁶² menting Bayesian models of cognition through the open source R programming language
⁶³ is therefore a valuable yet currently non-existent resource.

⁶⁴ Learning Objectives

⁶⁵ In the course, students will:

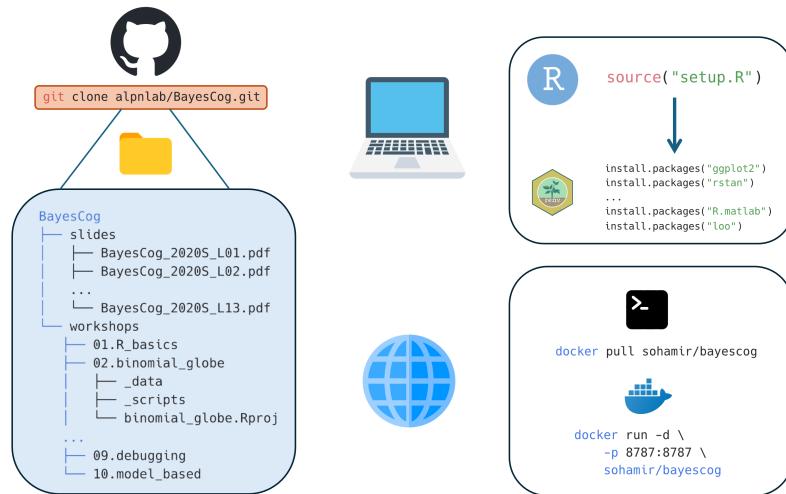
- ⁶⁶ • Build a foundational knowledge of Bayesian statistics and inference, and how it
⁶⁷ differs from frequentist definitions of probability
- ⁶⁸ • Understand Bayesian parameter estimation and the conceptual basis of sampling
⁶⁹ techniques including Markov chain Monte Carlo (MCMC)
- ⁷⁰ • Understand how Bayesian statistics can be applied to uncover latent processes and
⁷¹ parameters of cognition through cognitive modeling
- ⁷² • Learn basic reinforcement learning (RL) concepts and build a simple RL model
⁷³ (Rescorla-Wagner) using RStan
- ⁷⁴ • Be able to evaluate model performance through model comparison, parameter
⁷⁵ recovery and posterior predictive checks

⁷⁶ Prior to the first workshop, the course begins by summarizing the broader philosophy in
⁷⁷ which the techniques and methods are implemented. Specifically, this concerns Marr's
⁷⁸ influential three levels of analysis (Marr, 1982), which describe how algorithmic-level
⁷⁹ (as opposed to computational and implementation levels) models can help understand
⁸⁰ behavior. Doing so shapes the course material within this framework, demonstrating the
⁸¹ necessity for building strong theories in psychology (Press et al., 2022). Assuming no
⁸² prior experience with programming, the course properly (Workshop 01) begins with a
⁸³ basic introduction to the R programming language and the RStudio interface (RStudio
⁸⁴ Team, 2020). This introductory workshop first provides a general introduction to data
⁸⁵ structures, variables, and packages, after which students will work with simulated data
⁸⁶ from a reversal learning task, performing basic summary statistics including correlation
⁸⁷ and regression, and visualizing the results using the ggplot2 package (Wickham, 2016).

88 In Workshop 02, students will be introduced to Bayesian statistics, learning the differences
89 between Bayesian and frequentist definitions of probability. These concepts are put
90 into practical use in Workshop 03, where the Bayesian approach is transformed from a
91 purely mathematical concept to a system where one can determine the values of unknown
92 parameters from data. The goal of Bayesian inference - computing the probability
93 distribution of model parameters given the observed data – is also introduced, together
94 with sampling procedures which approximate the posterior distribution e.g., Markov
95 chain Monte Carlo (MCMC). Workshop 04 subsequently introduces students to the
96 Stan programming language ([Carpenter et al., 2017](#)) and its R interface RStan ([Stan Development Team, 2024](#)). Following an overview of Stan syntax, students will construct
97 a simple binomial model for a coin flipping experiment. Workshop 5 builds upon this
98 introductory workshop by introducing some specific properties and advantageous features
99 of Stan (as opposed to other packages like JAGS), including vectorization and variable
100 declaration, and introduces two more models: the Bernoulli model and linear regression.
101

102 Having built a solid foundation of understanding Bayesian statistics in Workshops 1-5, in
103 Workshops 6-8, students will learn how these methods can be used to infer latent cognitive
104 processes. In Workshop 6, a basic overview to the principles of cognitive modeling is
105 followed by an introduction to reinforcement learning, a popular theory of human behavior
106 that has been widely used in the last decades ([Daw et al., 2011](#); [Dayan & Niv, 2008](#); [D. Lee et al., 2012](#); [Niv, 2009](#)). To this end, we introduce a simple reinforcement learning
107 algorithm consisting of the Rescorla-Wagner model ([Rescorla & Wagner, 1972](#)) that uses
108 an error-driven rule (e.g., through reward prediction error) to update value computation,
109 and a softmax choice rule quantifies the stochasticity and randomness in human action.
110 Students will then practically implement this model in Stan, for simulated choice data for
111 a single subject, before fitting multiple subjects. Workshop 7 directly builds upon this
112 topic by introducing hierarchical Bayesian models ([D. Lee et al., 2012](#)) for simultaneously
113 estimating both group and individual level parameters. Given that Bayesian cognitive
114 models often require troubleshooting for parameter estimation ([Baribault & Collins, 2023](#)),
115 optimization strategies are also introduced in this workshop. Specifically, this covers Stan's
116 sampling parameters and reparameterization; the latter being particularly relevant for
117 hierarchical models. Model comparison is introduced in Workshop 8, with the basis behind
118 model fitting, predictive accuracy and information criterion firstly described. Students will
119 subsequently compare two RL models using the `loo` package ([Vehtari et al., 2015](#)) in R,
120 and plot posterior predictive checks as a measure of model validation. The final workshop
121 (Workshop 9) describes key strategies for code writing styles and code debugging in Stan,
122 using a purposely error-laden delay-discounting model ([M. D. Lee & Wagenmakers, 2014](#))
123 for students to interactively troubleshoot problematic code. To conclude, a published
124 study ([Crawley et al., 2020](#)) where computational models were implemented to uncover
125 learning differences is described, providing real-case examples of model and parameter
126 recovery.

128 All course workshops are accompanied by example data and scripts, the latter provided in
129 both uncompleted and completed versions where appropriate. All software required for
130 the course are open-source and easy to install; instructions are provided in the ‘Course
131 overview’ page. Whilst specific R packages (`rstan`, `loo`, `ggplot2`) are required, the course
132 uses `renv` ([Ushey & Wickham, 2023](#)) to simplify the installation process. Users simply run a
133 single command which installs the required packages for all successive R sessions. However,
134 `renv` only provides a simplified solution to reproducibility and can be mired by system
135 dependencies and versions of RStudio. Therefore, users can alternatively, pull a pre-made
136 Docker image building a container locally to host the RStudio environment. This does not
137 necessitate users to have R or RStudio installed - only Docker Desktop - maintaining a
138 more consistent and reproducible development environment across different systems and
139 platforms. In either case, generating the working environment requires minimal effort
140 (Figure 1.). Detailed guidance on how to recreate the working environment in both cases
141 are provided on the course website.



142
143 **Figure 1.** The course materials are hosted on GitHub and can be downloaded locally
144 using the `git clone` command. Users have two options if wanting to replicate the working
145 environment for the course materials. For working on one's own installation and version
146 of RStudio, `renv` manages all required packages and dependencies. Conversely, if users
147 would like to work on a specific RStudio version, they can pull the provided Docker image,
148 which installs the required R packages on an RStudio server. In either case, the required
149 dependencies are minimal (R/RStudio or Docker), and recreating the environment only
150 involves running one or two simple commands. Icons from icons8.com

151 Future Directions

152 The BayesCog course provides a general introduction to Bayesian statistics and computa-
153 tional modeling within the context of psychological research. Subsequently, the materials
154 and topics could easily be developed further. This includes using computational methods
155 with neuroimaging data (Glascher & O'Doherty, 2010; Hollander et al., 2016), understand-
156 ing behaviors in the social world (Kutlikova et al., 2023; Y. Pan et al., 2023), and the
157 theory-based modeling of psychiatric disorders (Maia et al., 2017; Sohail & Zhang, 2024;
158 Suter et al., 2025). Furthermore, the Stan programming language remains technically
159 challenging, leading to the development of user-friendly packages for computational mod-
160 eling (Ahn et al., 2017). Tutorials on how to implement these packages could further
161 broaden the use of computational methods within the psychological sciences. In the era of
162 large language models (LLMs) and generative AI, the audience are also encouraged to
163 combine this course with LLM tools such as ChatGPT to explore a more tailored learning
164 experience (Sohail & Lin, 2025; Sohail & Zhang, 2025).

165 To this end, we openly receive feedback and suggestions from the wider community.
166 Any tutorial can be adopted, transformed, or built upon for other educational purposes
167 (e.g., courses, single class sessions, workshops) under a Creative Commons Attribution-
168 ShareAlike 4.0 International License. Broader comments can be communicated on the
169 [GitHub repository](#) by either reporting an issue or requesting an enhancement. On-the-
170 other-hand, we recommend major changes to be made communicated beforehand and
171 – if appropriate – made directly by forking the repository and pushing changes to the
172 main branch. A member of the contributing team will then review the changes. Accepted
173 contributions will be credited and following the community guidelines outlined on the
174 [CONTRIBUTORS](#) page.

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179 Social Behavior and Neuroscience' (Rhoads & Gan, 2022), Luke Chang's 'DartBrains:
180 An online open access resource for learning functional neuroimaging analysis methods in
181 Python' (Chang et al., 2020) and Magdalena Chechlacz's 'MRI on BEAR' (Sohail et al.,
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186 Author Contributions

187 L.Z. created, designed and taught the original course materials by developing the syllabus,
188 writing the Stan and R code and creating and interpreting the datasets. A.S. created
189 the website, adding the content by converting, editing and expanding the source material
190 written by L.Z. Both L.Z. and A.S. revised the course materials and wrote the manuscript.

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