

¹ FELiCS: A Versatile Linearized Flow Solver for ² Multi-Physics Applications

³ **S.J. Knechtel¹, S. Demange¹, M. Goldack¹, L.M. Fuchs¹, M. Casel¹, X.
⁴ Song¹, J.S. Müller¹, A. Villié¹, F. Bergner¹, M.T. Degner¹, A.R. Talasikar¹,
⁵ A. Nehls¹, M. Matthaiou¹, P. Kuhn¹, K. Oberleithner¹, and T.L. Kaiser¹**

⁶ **1** Laboratory for Flow Instabilities and Dynamics, Technische Universität Berlin, 10623 Berlin, Germany

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

Software

- [Review](#) ↗
- [Repository](#) ↗
- [Archive](#) ↗

Editor: ↗

Submitted: 02 December 2025

Published: unpublished

License

Authors of papers retain copyright
and release the work under a
Creative Commons Attribution 4.0
International License ([CC BY 4.0](#)).¹⁵¹⁶¹⁷¹⁸¹⁹

⁷ Summary

⁸ The analysis and control of flow instability is crucial in many fields of engineering and science,
⁹ ranging from aerodynamics to combustion. While computational fluid dynamics solvers can
¹⁰ simulate the full nonlinear behavior of flows, the analysis of small perturbations is often more
¹¹ efficiently handled by linearized models. These models provide valuable insights into flow
¹² stability, receptivity, and input-output interactions. Consequently, open-source software (OSS)
¹³ tools for linear stability analysis represent a relevant asset for researchers and engineers studying
¹⁴ complex flow dynamics.

¹⁵ FELiCS is both a monolithic codebase executed via configuration files, as well as a flexible
¹⁶ Python package designed for the linear stability and sensitivity analysis of complex fluid flows.
¹⁷ It solves the linearized equations governing flow motion, turbulence transport, acoustics, and
¹⁸ reacting flows using the Finite Element Method (FEM). Designed as a versatile tool, FELiCS
¹⁹ enables global stability analysis, resolvent analysis, and input-output analysis (or “forcing-
²⁰ response-analysis”) within a single framework. In addition, its modular functionalities can be
²¹ used in the form of a Python package to incorporate linear equations into personal Python
²² scripts, such as for base flow computations, data assimilation or sensitivity calculation among
²³ others. Thus, it addresses the need for a user-friendly tool that combines these diverse analytical
²⁴ capabilities.

²⁵ Statement of need

²⁶ Several tools exist for stability analysis, which are specialized for specific flow regimes, flow
²⁷ equations and/or high performance computing. For instance, NekStab ([Frantz et al., 2023](#))
²⁸ offers efficient parallel computation for incompressible flows while using Nek5000’s spectral
²⁹ elements discretization ([Fischer et al., 2007](#)). Similarly, Nektar++ ([Cantwell et al., 2015](#))
³⁰ supports finite element discretization for various equations, while relying on complex spectral/hp
³¹ elements in C++. Other tools like Linstab2D ([Martini & Schmidt, 2024](#)) and BROADCAST
³² ([Poulain et al., 2023](#)) (based on FOTRAN and Matlab, respectively), discretize with finite
³³ differences, while using structured or quasi-structured meshes. The Python toolkit Resolvent4py
³⁴ ([Padovan et al., 2025](#)) provides matrix-free and high-performant parallelized linear analysis
³⁵ capabilities for large problems which requires the user to supply the linear operator.

³⁶ FELiCS ([Thomas L. Kaiser et al., 2023](#)) addresses a gap in the functionalities of these existing
³⁷ tools by providing a user-friendly, Python-based tool for various types of linear stability
³⁸ calculations on unstructured meshes with various physical models. On the one hand, the
³⁹ monolithic code version can be easily used without any prior knowledge in Python. This
⁴⁰ enables multiple types of analyses, including global modal analysis to assess linear stability,
⁴¹ resolvent analysis to identify amplification mechanisms, and input-output analysis to determine

42 transfer functions between input (boundary) forcing and system output. On the other hand,
43 the FELiCS gives users the freedom to define customized analyses or to integrate FELiCS into
44 external workflows. This significantly simplifies tasks requiring linearized fluid equations, such
45 as data assimilation or sensitivity calculations. Additionally, embedded post-processing scripts
46 allow users to work with analysis outputs directly within the FEM framework.

47 The software serves a wide range of users and is designed to answer the needs of three different
48 types of users:

- 49 ■ For applied users, who run the complete monolithic workflow, pre-written scripts controlled
50 by JSON configuration files handle stability analyses without requiring in-depth Python
51 knowledge.
- 52 ■ Advanced users can import FELiCS as a library within their own Python codes, accessing
53 equations and solvers in an object-oriented manner for complex tasks like data assimilation
54 or gradient-based optimization.
- 55 ■ Finally, the code's modular architecture allows developers to easily adapt and expand
56 the software, such as adding new physical models, following guidelines available in the
57 repository.

58 Features and examples

59 FELiCS implements a wide array of linear and nonlinear equations, including incompressible
60 and compressible Navier-Stokes, density variations in the low-Mach number limit, and chemical
61 reaction models with progress variables, which can be used in any combination. The physical
62 modeling supports heat and mass transport, eddy viscosity for turbulence, sponge layers for
63 boundary damping. Upcoming updates plan to include Reynolds-averaged Navier-Stokes
64 linearized equations and additional reaction models. The software offers an accessible interface,
65 available both as a command-line tool and as a Python-importable module. Users control
66 solver parameters via simple JSON configuration files and can import meshes from GMSH
67 ([Geuzaine & Remacle, 2009](#)) while exporting results to H5 and XDMF formats for visualization
68 in Paraview ([Ahrens et al., 2005](#)). Comprehensive user support, including tutorials, how-to
69 guides, and detailed descriptions of the governing equations and models, are provided on the
70 [documentation website](#). Under the hood, the software ensures high performance and numerical
71 stability by leveraging the PETSc ([Balay et al., 2019](#))/SLEPc ([Hernandez et al., 2005](#)) toolkit
72 for linear algebra and the FEniCS package ([Alnæs et al., 2015](#)) for the FEM framework. The
73 modular, object-oriented architecture further facilitates the extension of new physical models.
74 All these functionalities can also be used by importing FELiCS as a Python package and do
75 computations like solving for a base flow, data assimilation or sensitivity analysis in simple
76 scripts. The code ensures robustness through both minimalist test cases in a separate [repository](#)
77 and automated unit tests.

78 FELiCS supports a diverse range of applications demonstrated in numerous published studies.
79 These include the analysis of combustion instabilities in turbulent jet flames and gas turbines
80 ([Casel et al., 2022](#); [Thomas Ludwig Kaiser, Oberleithner, et al., 2019](#); [Thomas Ludwig Kaiser, Lesshaft, et al., 2019](#); [Thomas Ludwig Kaiser et al., 2023](#); [Saldern et al., 2025](#); [Wang et al., 2022](#)) and aeroacoustic noise prediction for airfoils ([Demange et al., 2024, 2025](#)). It has also
83 been applied to flow instabilities in gas ([Moczarski et al., 2024](#)) and water turbines ([Müller et al., 2022](#)), turbulent mixing ([Thomas Ludwig Kaiser & Oberleithner, 2021a, 2021b; Lückoff et al., 2021](#)), coherent structure modeling in turbulent jets ([Kuhn et al., 2021](#); [Saldern et al., 2024](#)), and linear analysis based on PIV measurements ([Fuchs et al., 2025](#); [Klopsch et al., 2025](#)). Further use cases encompass sensitivity and adjoint-based analyses ([Knechtel et al., 2024](#); [Müller, Saldern, et al., 2024](#)) as well as shape optimization ([Müller, Reumschüssel, et al., 2024](#)).

90 To illustrate the capabilities of the software, [Figure 1](#) presents results from various multi-physics

91 applications and displays incompressible resolvent modes in a cardiovascular flow ([Figure 1\(a\)](#))
 92 ([Villié et al., 2025](#)), compressible resolvent mode of a flow around an airfoil ([Figure 1\(b\)](#))
 93 ([Demange et al., 2025](#)) and an incompressible resolvent mode in a turbulent separation bubble
 94 ([Figure 1\(c\)\)](#) ([Fuchs et al., 2025](#)). [Figure 1\(d\)](#) shows a resolvent response mode of a turbulent
 95 methane-air flame ([Thomas Ludwig Kaiser et al., 2023](#)).

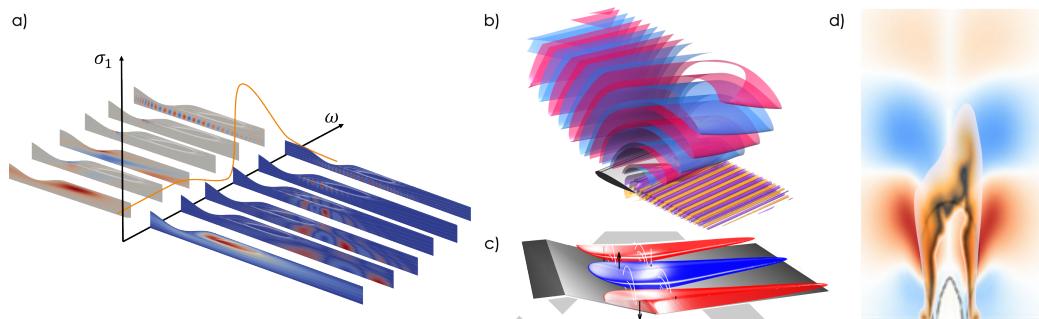


Figure 1: Visualization examples: a) Optimal forcing and response resolvent modes of a stenotic flow; b) Compressible resolvent response mode, two contour levels showing hydrodynamic and acoustic pressure fluctuations around an airfoil; c) Isocontour of the optimal resolvent mode in a turbulent separation bubble; d) Compressible resolvent mode with a flame model around a reacting jet.

96 [Figure 2](#) presents post-processing analyses examples. [Figure 2\(a\)](#) demonstrates shape
 97 optimization applied to an unstable eigenvalue w.r.t. wall-normal shape deformations, while
 98 [Figure 2\(b\)](#) shows the structural sensitivity analysis in a 2D cylinder flow.

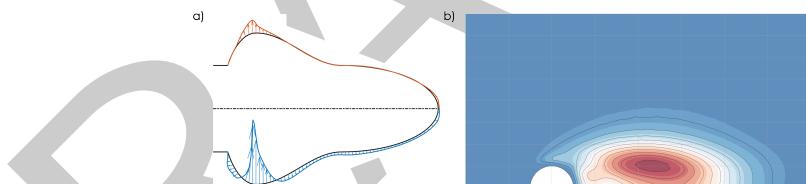


Figure 2: Post-processing examples: a) Growth rate sensitivity of the most unstable eigenvalue w.r.t. wall-normal shape deformations of a runner crown geometry in a turbulent Francis turbine flow; b) Structural sensitivity analysis in a 2D cylinder flow.

99 Acknowledgements

100 The authors thank all current and past contributors to FELiCS. The FELiCS community is
 101 grateful to the developers of FEniCS, PETSc, and SLEPc for their foundational open-source
 102 infrastructure.

103 The authors acknowledge funding from the Deutsche Forschungsgemeinschaft (DFG, German
 104 Research Foundation) through the following project numbers: 458062719 (S.D.), 504349109
 105 (L.M.F.), 441269395 (T.L.K.), 429772199 (J.S.M.). This work also received funding from the
 106 European Union's Horizon Europe research and innovation programme under grant agreement
 107 No. 101137955 (HORIZON-JTI-CLEANH2-2023-04-02), and from the State of Berlin via
 108 the Elsa-Neumann-Fellowship. The work was also supported by the German Federal Ministry
 109 for Economic Affairs and Energy (BMWi, grant 0324314B) and by computing time on
 110 SuperMUC-NG at Leibniz Supercomputing Centre ([www.lrz.de](#)), provided by the Gauss Centre
 111 for Supercomputing e.V. ([www.gauss-centre.eu](#)).

112 References

- 113 Ahrens, J., Geveci, B., & Law, C. (2005). ParaView: An end-user tool for large data
114 visualization. In *Visualization handbook*. Elesvier. <https://doi.org/10.1016/b978-012387582-2/50038-1>
- 116 Alnæs, M. S., Blechta, J., Hake, J., Johansson, A., Kehlet, B., Logg, A., Richardson, C., Ring,
117 J., Rognes, M. E., & Wells, G. N. (2015). The FEniCS project version 1.5. *Archive of
118 Numerical Software*, 3(100). <https://doi.org/10.11588/ans.2015.100.20553>
- 119 Balay, S., Abhyankar, S., Adams, M., Brown, J., Brune, P., Buschelman, K., Dalcin, L.,
120 Dener, A., Eijkhout, V., Gropp, W., & others. (2019). *PETSc users manual*. <https://doi.org/doi.org/10.2172/2998643>
- 122 Cantwell, C. D., Moxey, D., Comerford, A., Bolis, A., Rocco, G., Mengaldo, G., De Grazia,
123 D., Yakovlev, S., Lombard, J.-E., Ekelschot, D., & others. (2015). Nektar++: An open-
124 source spectral/hp element framework. *Computer Physics Communications*, 192, 205–219.
125 <https://doi.org/10.1016/j.cpc.2015.02.008>
- 126 Casel, M., Oberleithner, K., Zhang, F., Zirwes, T., Bockhorn, H., Trimis, D., & Kaiser, T. L.
127 (2022). Resolvent-based modelling of coherent structures in a turbulent jet flame using a
128 passive flame approach. *Combustion and Flame*, 236, 111695. <https://doi.org/10.1016/j.combustflame.2021.111695>
- 130 Demange, S., Oberleithner, K., Yuan, Z., Hanifi, A., & Cavalieri, A. V. G. (2025). Flow
131 Structures Driving Broadband Trailing-Edge Noise: A Resolvent-Based Model. *AIAA
132 Journal*, 1–13. <https://doi.org/10.2514/1.J065730>
- 133 Demange, S., Yuan, Z., Jekosch, S., Hanifi, A., Cavalieri, A. V. G., Sarradj, E., Kaiser, T.
134 L., & Oberleithner, K. (2024). Resolvent model for aeroacoustics of trailing edge noise.
135 *Theoretical and Computational Fluid Dynamics*. <https://doi.org/10.1007/s00162-024-00688-z>
- 137 Fischer, P., Lottes, J., & Tufo, H. (2007). *Nek5000*. Argonne National Laboratory (ANL),
138 Argonne, IL (United States).
- 139 Frantz, R. A., Loiseau, J.-C., & Robinet, J.-C. (2023). Krylov methods for large-scale dynamical
140 systems: Application in fluid dynamics. *Applied Mechanics Reviews*, 75(3), 030802.
- 141 Fuchs, L. M., Steinfurth, B., Saldern, J. G. von, Weiss, J., & Oberleithner, K. (2025). A
142 standing-wave model for the low-frequency dynamics of a turbulent separation bubble.
143 *arXiv Preprint arXiv:2505.24723*.
- 144 Geuzaine, C., & Remacle, J.-F. (2009). Gmsh: A 3-d finite element mesh generator with
145 built-in pre- and post-processing facilities. *International Journal for Numerical Methods in
146 Engineering*, 79(11), 1309–1331. <https://doi.org/10.1002/nme.2579>
- 147 Hernandez, V., Roman, J. E., & Vidal, V. (2005). SLEPc: A scalable and flexible toolkit
148 for the solution of eigenvalue problems. *ACM Transactions on Mathematical Software
(TOMS)*, 31(3), 351–362. <https://doi.org/10.1145/1089014.1089019>
- 150 Kaiser, Thomas L., Demange, S., Müller, J. S., Knechtel, S., & Oberleithner, K. (2023).
151 FELiCS: A versatile linearized solver addressing dynamics in multi-physics flows. *AIAA
152 Aviation 2023 Forum*, 3434.
- 153 Kaiser, Thomas Ludwig, Lesshafft, L., & Oberleithner, K. (2019). Prediction of the flow
154 response of a turbulent flame to acoustic perturbations based on mean flow resolvent
155 analysis. *Journal of Engineering for Gas Turbines and Power*, 141, 111021. <https://doi.org/10.1115/1.4044993>
- 157 Kaiser, Thomas Ludwig, & Oberleithner, K. (2021a). A global linearized framework for

- 158 modelling shear dispersion and turbulent diffusion of passive scalar fluctuations. *Journal of*
 159 *Fluid Mechanics*, 915, A111. <https://doi.org/10.1017/jfm.2021.151>
- 160 Kaiser, Thomas Ludwig, & Oberleithner, K. (2021b). Modeling the transport of fuel mixture
 161 perturbations and entropy waves in the linearized framework. *Journal of Engineering for*
 162 *Gas Turbines and Power*, 143(11), 111001. <https://doi.org/10.1115/1.4051714>
- 163 Kaiser, Thomas Ludwig, Oberleithner, K., Selle, L., & Poinsot, T. (2019). Examining the
 164 effect of geometry changes in industrial fuel injection systems on hydrodynamic structures
 165 with biglobal linear stability analysis. *Journal of Engineering for Gas Turbines and Power*.
 166 <https://doi.org/10.1115/1.4045018>
- 167 Kaiser, Thomas Ludwig, Varillon, G., Polifke, W., Zhang, F., Zirwes, T., Bockhorn, H., &
 168 Oberleithner, K. (2023). Modelling the response of a turbulent jet flame to acoustic forcing
 169 in a linearized framework using an active flame approach. *Combustion and Flame*, 253,
 170 112778. <https://doi.org/10.1016/j.combustflame.2023.112778>
- 171 Klopsch, R., Fuchs, L. M., Rigas, G., Oberleithner, K., & Saldern, J. G. von. (2025). Enabling
 172 resolvent analysis through assimilation of experimental mean flows with physics-informed
 173 neural networks: A case study on the boeing gaussian bump. *AIAA AVIATION FORUM*
 174 AND ASCEND 2025, 3604. <https://doi.org/10.2514/6.2025-3604>
- 175 Knechtel, S. J., Kaiser, T. L., Orchini, A., & Oberleithner, K. (2024). Arbitrary-order
 176 sensitivities of the incompressible base flow and its eigenproblem. *Journal of Fluid Mechanics*,
 177 985, A32. <https://doi.org/10.1017/jfm.2024.195>
- 178 Kuhn, P., Soria, J., & Oberleithner, K. (2021). Linear modelling of self-similar jet turbulence.
 179 *Journal of Fluid Mechanics*, 919, A7.
- 180 Lückoff, F., Kaiser, T. L., Paschereit, C. O., & Oberleithner, K. (2021). Mean field coupling
 181 mechanisms explaining the impact of the precessing vortex core on the flame transfer
 182 function. *Combustion and Flame*, 223, 254–266. <https://doi.org/10.1016/j.combustflame.2020.09.019>
- 184 Martini, E., & Schmidt, O. (2024). Linstab2D: Stability and resolvent analysis of compressible
 185 viscous flows in MATLAB. *Theoretical and Computational Fluid Dynamics*, 38(5), 665–685.
 186 <https://doi.org/10.1007/s00162-024-00706-0>
- 187 Moczarski, L., Treleaven, N. C. W., Oberleithner, K., Schmidt, S., Fischer, A., & Kaiser, T. L.
 188 (2024). Interacting linear modes in the turbulent flow of an industrial swirled combustor.
 189 *AIAA Journal*, 62, 979–988. <https://doi.org/10.2514/1.J063639>
- 190 Müller, J. S., Lückoff, F., Kaiser, T. L., & Oberleithner, K. (2022). On the relevance of the
 191 runner crown for flow instabilities in a francis turbine. *IOP Conference Series: Earth and*
 192 *Environmental Science*, 1079, 12053. <https://doi.org/10.1088/1755-1315/1079/1/012053>
- 193 Müller, J. S., Reumschüssel, J. M., Kaiser, T. L., Knechtel, S. J., & Oberleithner, K. (2024).
 194 Combining bayesian optimization with adjoint-based gradients for efficient control of
 195 flow instabilities. *Center for Turbulence Research, Proceedings of the Summer Program*.
 196 https://web.stanford.edu/group/ctr/ctrsp24/ii05_MULLER.pdf
- 197 Müller, J. S., Saldern, J. G. R. von, Kaiser, T. L., & Oberleithner, K. (2024). Linear
 198 amplification of inertial-wave-driven swirl fluctuations in turbulent swirling pipe flows: A
 199 resolvent analysis approach. *Journal of Fluid Mechanics*, 1000, A91. <https://doi.org/10.1017/jfm.2024.679>
- 201 Padovan, A., Anantharaman, V., Rowley, C. W., Vollmer, B., Colonius, T., & Bodony, D. J.
 202 (2025). Resolvent4py: A parallel python package for analysis, model reduction and control
 203 of large-scale linear systems. *SoftwareX*, 31, 102286. <https://doi.org/10.1016/j.softx.2025.102286>
- 205 Poulain, A., Content, C., Sipp, D., Rigas, G., & Garnier, E. (2023). BROADCAST: A high-

- 206 order compressible CFD toolbox for stability and sensitivity using algorithmic differentiation.
207 *Computer Physics Communications*, 283, 108557. <https://doi.org/10.1016/j.cpc.2022.108557>
- 209 Saldern, J. G. von, Beuth, J. P., Reumschüssel, J. M., Jaeschke, A., Paschereit, C. O.,
210 & Oberleithner, K. (2025). Low-frequency streaky structures in turbulent hydrogen jet
211 flames. *Combustion and Flame*, 278, 114231. <https://doi.org/10.1016/j.combustflame.2025.114231>
- 213 Saldern, J. G. von, Schmidt, O. T., Jordan, P., & Oberleithner, K. (2024). On the role of
214 eddy viscosity in resolvent analysis of turbulent jets. *Journal of Fluid Mechanics*, 1000,
215 A51. <https://doi.org/doi.org/10.1017/jfm.2024.922>
- 216 Villié, A., Schmitter, S., Saldern, J. G. von, Demange, S., & Oberleithner, K. (2025). Physics-
217 informed neural networks for enhancing medical flow magnetic resonance imaging: Artifact
218 correction and mean pressure and reynolds stresses assimilation. *Physics of Fluids*, 37(2).
219 <https://doi.org/10.1063/5.0252852>
- 220 Wang, C., Kaiser, T. L., Meindl, M., Oberleithner, K., Polifke, W., & Lesshafft, L. (2022).
221 Linear instability of a premixed slot flame: Flame transfer function and resolvent analysis.
222 *Combustion and Flame*, 240, 112016. <https://doi.org/10.1016/j.combustflame.2022.112016>

DRAFT