

¹ JAX-in-Cell: A Differentiable Particle-in-Cell Code for ² Plasma Physics Applications

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⁶ Summary

⁷ JAX-in-Cell is a fully electromagnetic, multispecies, and relativistic 1D3V Particle-in-Cell (PIC)
⁸ framework implemented entirely in JAX. It provides a modern, Python-based alternative to
⁹ traditional PIC frameworks. It leverages Just-In-Time compilation and automatic vectorization
¹⁰ to achieve the performance of traditional compiled codes on CPUs, GPUs, and TPUs. The
¹¹ resulting framework bridges the gap between educational scripts and production codes, providing
¹² a testbed for differentiable physics and AI integration that enables end-to-end gradient-based
¹³ optimization. The code solves the Vlasov–Maxwell system on a staggered Yee lattice with either
¹⁴ periodic, reflective, or absorbing boundary conditions, allowing both an explicit Boris solver
¹⁵ and an implicit Crank–Nicolson method via Picard iteration to ensure energy conservation.
¹⁶ Here, we detail the numerical methods employed, validate against standard benchmarks, and
¹⁷ showcase the use of its auto-differentiation capabilities.

¹⁸ Statement of Need

¹⁹ A plasma is a collection of free ions and electrons whose self-generated and external
²⁰ electromagnetic fields drive collective behavior. Such behavior can be modelled using
²¹ PIC simulations, which offer a fully kinetic description and enable exploration of complex
²² interactions in modern plasma physics research, such as fusion devices, laser-plasma
²³ interactions, and astrophysical plasmas ([Birdsall & Langdon, 1991](#)). However, such simulations
²⁴ can be computationally very expensive, often requiring hardware-specific implementations in
²⁵ low-level languages. The current landscape of high-performance production codes such as
²⁶ OSIRIS ([Fonseca et al., 2002](#)), EPOCH ([Smith et al., 2021](#)), VPIC ([Le et al., 2023](#)), and
²⁷ WARPX ([Vay et al., 2018](#)), which are written in C++ or Fortran with MPI/CUDA backends,
²⁸ are highly optimized for massively parallel simulations, often with complex compilation chains,
²⁹ and require external adjoint implementations for optimization. This imposes a steep barrier to
³⁰ entry for new developers, making it cumbersome to test new algorithms, as well as to integrate
³¹ with modern data science workflows. On the other hand, open-source educational Python
³² scripts typically lack the performance and capabilities needed to perform cutting-edge research.

³³ JAX-in-Cell is able to fill this gap by implementing a 1D3V PIC framework entirely within the
³⁴ JAX ecosystem ([Bradbury et al., 2018](#)). It is open-source, user-friendly, and developer-friendly,
³⁵ written entirely in Python. It addresses three specific needs not met by existing codes: 1)
³⁶ hardware-agnostic high performance; 2) unified explicit and implicit solvers; 3) differentiable
³⁷ physics and AI integration. This is achieved using JAX’s Just-In-Time (JIT) compilation via
³⁸ XLA, which allows us to achieve performance parity with compiled languages on CPUs, GPUs,
³⁹ and TPUs. Therefore, researchers can prototype new algorithms in Python and immediately use
⁴⁰ them on complex situations and accelerated hardware. Furthermore, JAX-in-Cell is inherently
⁴¹ differentiable due to its automatic differentiation (AD) capabilities. This allows for new research
⁴² directions such as optimization of laser pulse shapes, parameter discovery from experimental
⁴³ data, or embedding PIC simulations in Physics-Informed Neural Network loops. Finally, both an

⁴⁴ explicit scheme using the standard Boris algorithm ([Boris & others, 1970](#)) and a fully implicit,
⁴⁵ energy-conserving scheme ([Chen & Chacón, 2014](#)) are available to cross-verify results and
⁴⁶ perform long simulations with large time steps, a capability often lacking in lightweight tools.

⁴⁷ Structure

⁴⁸ As in standard PIC approaches ([Birdsall & Langdon, 1991](#)), in JAX-in-Cell, particles are
⁴⁹ advanced along characteristics of the Vlasov equation

$$\partial_t f_s + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{u}} f_s = 0,$$

⁵⁰ with the electromagnetic fields governed by the standard Maxwell equations. Here, \mathbf{v} is velocity,
⁵¹ q_s and m_s are the particle charge and mass, \mathbf{E} and \mathbf{B} are the electric and magnetic fields,
⁵² $\mathbf{u} = \mathbf{v}\gamma$ is the proper velocity, and $\gamma = \sqrt{1 + u^2/c^2}$ is the Lorentz factor, with c the speed of
⁵³ light. The distribution function f_s is discretized as

$$f_s(\mathbf{x}, \mathbf{v}) \approx \sum_p w_p \delta(\mathbf{x} - \mathbf{x}_p) \delta(\mathbf{v} - \mathbf{v}_p),$$

⁵⁴ where x_p denotes the position of each pseudo-particle, v_p denotes the velocity of each pseudo-
⁵⁵ particle, and the weight is given by $w_p = n_0 L / N$, with n_0 number density, L the spatial
⁵⁶ domain length and N the number of pseudo-particles for that species. Then, the spatial
⁵⁷ domain is divided into N_x uniform cells with spacing Δx and advanced in time by Δt . To
⁵⁸ mitigate numerical noise, each pseudo-particle is represented by a triangular shape function
⁵⁹ spanning three grid cells, and the same kernel is used consistently for both the particle-to-grid
⁶⁰ charge deposition and the grid-to-particle field interpolation ([Hockney & Eastwood, 1988](#)).
⁶¹ Accordingly, the current density \mathbf{J} is computed from the continuity equation using a discretely
⁶² charge-conserving scheme ([Villasenor & Buneman, 1992](#)) consistent with the shape function.

⁶³ The core logic of JAX-in-Cell is distributed along six specific modules. The first one is
⁶⁴ `simulation.py`, which serves as the high-level entry point, handling parameter initialization
⁶⁵ (via TOML parsing), memory allocation for particle and field arrays, and the execution of
⁶⁶ the main loop. The time-stepping is performed using a JAX primitive `jax.lax.scan`, which
⁶⁷ allows it to be optimized by the XLA compiler. Then, `algorithms.py` implements the time
⁶⁸ integration schemes, which advance the system state by one timestep Δt by a sequence of
⁶⁹ operations, namely particle push, source deposition, and field update. We implement two
⁷⁰ time-evolution methods (Figure 1), an explicit Boris algorithm ([Boris & others, 1970](#)), and
⁷¹ an implicit Crank–Nicolson scheme solved via Picard iteration ([Chen & Chacón, 2014](#)). The
⁷² JIT-compiled particle dynamics is present in `particles.py`, which includes the relativistic and
⁷³ non-relativistic Boris rotation and the field interpolation routines, which are heavily vectorized
⁷⁴ using `jax.vmap`. The electromagnetic solvers are present in `fields.py`, which include the finite-
⁷⁵ difference curl operators for the Faraday and Ampère laws, as well as the divergence cleaning
⁷⁶ routines that enforce charge conservation. The deposition of charge and current densities from
⁷⁷ particle positions onto the grid is handled by `sources.py`, which implements high-order spline
⁷⁸ interpolation and digital filtering to mitigate aliasing noise. Finally, `boundary_conditions.py`
⁷⁹ is the centralized module to enforce boundary constraints, including routines for particle
⁸⁰ reflection/absorption and ghost-cell updates for the electromagnetic fields.

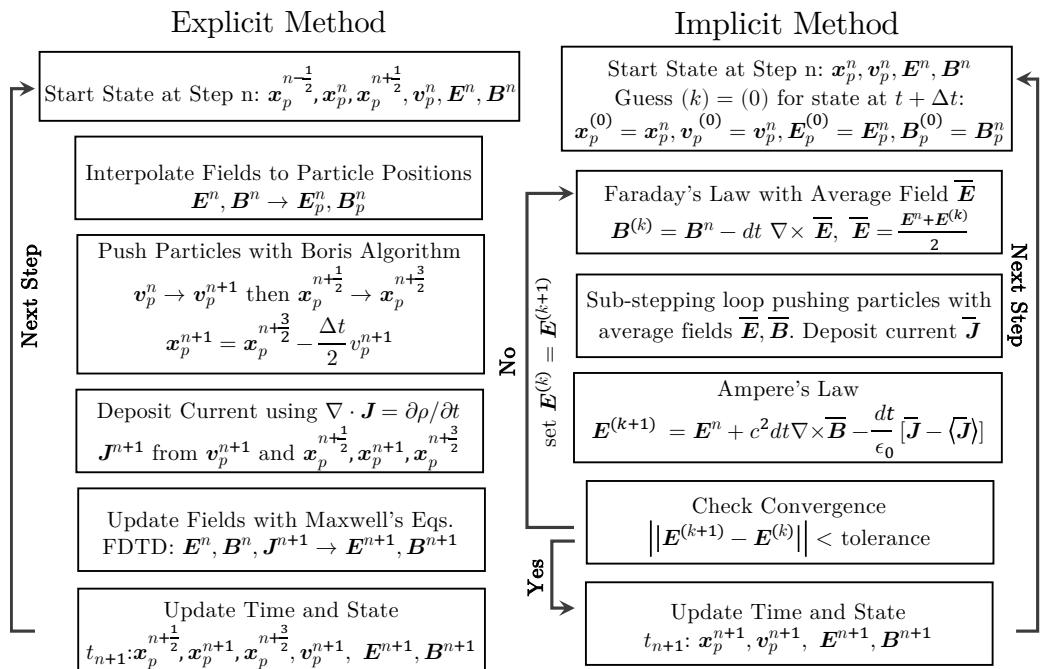


Figure 1: Time-stepping algorithms in JAX-in-Cell. Left: explicit Boris time-stepper and a Finite-Difference Time-Domain (FDTD) method using a staggered Yee grid for the electromagnetic fields. Right: implicit Crank-Nicolson time stepper using a Picard iteration for the electromagnetic system.

81 Due to its simplified design, JAX-in-Cell is able to pass the entire simulation state between
82 functions, which is maintained as an immutable tuple referred to in the code as the carry =
83 $(\mathbf{E}, \mathbf{B}, \mathbf{x}_p, \mathbf{v}_p, q, m)$. This allows the entire simulation to be treated as a single differentiable
84 function, which can facilitate the integration of automatic differentiation workflows. In order to
85 reduce kernel launch overheads on GPUs, as well as the vector lengths for different populations,
86 JAX-in-Cell adopts a monolithic array formulation of the multi-species architecture optimized
87 for the Single-Instruction-Multiple-Data (SIMD) paradigm of JAX. That is, the simulation
88 state, carry, is a single concatenated state of unified, global arrays, regardless of the number
89 of physical species defined in the input configuration. This is a different approach than the one
90 used in traditional PIC codes, which employ an object-oriented design with different species
91 stored in separate containers and iterated over sequentially. During initialization, the code
92 parses the TOML configuration file for the list of species and, for each population, the function
93 `make_particles` generates each phase-space distribution and weights at the initial time, which
94 are then appended to the global state vectors. This allows load balancing across GPU cores
95 and minimizes warp divergence. A comprehensive set of pytest-based unit and integration
96 tests is included in JAX-in-Cell. The test suite is part of a continuous integration workflow on
97 GitHub actions across multiple Python and JAX versions, and uploads coverage reports to
98 Codecov.

99 Capabilities

100 The electromagnetic fields are defined on a staggered Yee lattice. The code then solves the
101 Ampere and Faraday equations

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \frac{\mathbf{J}}{\epsilon_0}, \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E},$$

102 which, in the 1D geometry ($\partial/\partial y = \partial/\partial z = 0$), reduces the curl operator to spatial
103 derivatives along the x lattice. The discrete spatial derivative operator uses central differences
104 for internal points, with ghost cells handling the boundary conditions (periodic, reflective, or

absorbing). JAX-in-Cell allows flexible boundary conditions by handling particle trajectories and electromagnetic fields independently at the domain edges. Charge conservation is ensured using a divergence cleaning step, where the longitudinal component E_x is projected to satisfy Gauss' law, $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$. The charge density ρ and current density \mathbf{J} are deposited onto the grid using a quadratic (3-point) spline shape function $S(x)$, where a multi-pass binomial digital filter is applied to the source terms to mitigate grid-heating instability. This effectively suppresses high wavenumbers near the Nyquist limit while preserving the macroscopic (low wave-number) plasma dynamics.

While the user can initialize their own distribution functions, JAX-in-Cell allows users to use a predefined perturbed Maxwellian

$$f_s(x, \mathbf{v}, t = 0) = f_{s0}(\mathbf{v}) \left[1 + a \cos \left(\frac{2\pi kx}{L} \right) \right],$$

where a is the perturbation amplitude and k the perturbation wavenumber. The Maxwellian background f_{s0} is given by the following anisotropic, drifting distribution

$$f_{s0}(\mathbf{v}) = \frac{n_0}{2\pi^{3/2} v^3} \left[e^{-\frac{(v_x - v_{bx})^2}{v_{th,x}^2} - \frac{(v_y - v_{by})^2}{v_{th,y}^2} - \frac{(v_z - v_{bz})^2}{v_{th,z}^2}} + e^{-\frac{(v_x + v_{bx})^2}{v_{th,x}^2} - \frac{(v_y + v_{by})^2}{v_{th,y}^2} - \frac{(v_z + v_{bz})^2}{v_{th,z}^2}} \right],$$

with $v^3 = v_{th,x} v_{th,y} v_{th,z}$ the product of each thermal velocity along (x, y, z) directions, v_{bi} the drift velocities along each direction i , and n_0 the background density. The addition of an opposite sign drift velocity $\pm v_b$ is controlled using the `velocity_plus_minus` input parameter. Such a perturbed Maxwellian allows us to perform many benchmark simulations, such as Landau damping, the two-stream instability, the bump-on-tail instability, and the Weibel instability.

We first validate JAX-in-Cell by performing such simulations in a periodic boundary of length L , and compare with the corresponding linear theory. Linearizing the Vlasov–Maxwell system around this initial distribution (with equal thermal velocities $v_{th,x} = v_{th,y} = v_{th,z}$), yields the dispersion relation

$$1 + \frac{1}{2k^2 \lambda_D^2} [2 + \xi_1 Z(\xi_1) + \xi_2 Z(\xi_2)] = 0, \quad \xi_i = \frac{\omega}{kv_{th}} - \frac{v_{bi}}{v_{th}},$$

where λ_D is Debye length and Z is the Fried–Conte plasma dispersion function. The complex frequency ω determines both the oscillation frequency and the damping or growth rate γ . We use such a theoretical model to demonstrate two representative test cases using non-relativistic algorithms: 1) Landau damping using a perturbation: $a = 0.025$, $k\lambda_D = 1/2$, zero drift velocities $v_{bx} = v_{by} = v_{bz} = 0$, $v_{th} = 0.35 c$, resolution of $N = 40000$ particles, $N_x = 32$ grid cells, $\Delta x = 0.4\lambda_D$ gird spacing and $\Delta t = 0.1\omega_{pe}^{-1}$ timestep; 2) two-stream instability using a perturbation $a = 5 \times 10^{-7}$, $k\lambda_D = 1/8$, velocities $v_{bx} = 0.2 c$ with `velocity_plus_minus` set to true, $v_{th} = 0.05 c$, $N = 10000$, $N_x = 100$, $\Delta x = 0.5\lambda_D$ and $\Delta t = 0.1\omega_{pe}^{-1}$. We show in Figure 2 (top) the evolution of the electric field energy, as well as the fitted damping/growth rates, showing good agreement with the analytical prediction. We also show in Figure 2 (bottom) the resulting relative energy error between the explicit and implicit methods to demonstrate the precision of the implicit method.

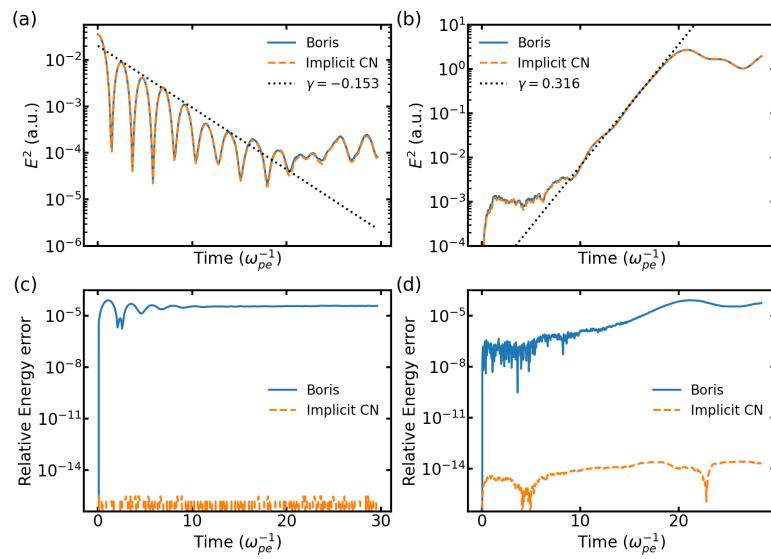


Figure 2: Electric field energy evolution for Landau damping and the two-stream instability. (a) Landau damping with analytical damping rate $\gamma = 0.153\omega_{pe}$. (b) Two-stream instability showing fitted exponential growth rate. (c-d) Relative total energy deviation $|E_{total} - E_{total}(0)|/E_{total}(0)$ demonstrating energy conservation.

139 Next, we perform a simulation of the Weibel instability, which arises in anisotropic plasmas and
 140 may be a mechanism for magnetic field generation and application. The plasma is initialized
 141 with an anisotropic velocity distribution with $v_{thz}/v_{thx} = \sqrt{T_z/T_x} = 10$. We show in Figure
 142 3 the evolution of the magnetic field energy and the energy relative error (left) and the
 143 magnetic field strength in the y direction (right). As expected, the magnetic field organizes
 144 into filamentary structures perpendicular to the velocity anisotropy. Initially, multiple small
 145 filaments form, which subsequently merge into larger-scale structures as the system evolves. In
 146 this case, the implicit method shows a bounded relative energy error of at most 10^{-11} , while
 147 the explicit method appears to have unbounded energy errors.

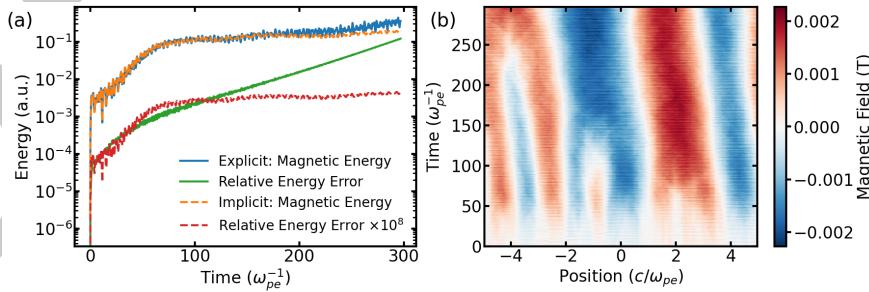


Figure 3: Weibel instability. (a) Evolution of the magnetic field energy and the relative energy error of the simulation during the Weibel instability. (b) Spatial profile of the magnetic field B_y .

148 We demonstrate the multi-species capability of JAX-in-Cell by performing a simulation of the
 149 bump-on-tail instability (Figure 4). This instability arises when a high-velocity electron beam
 150 creates a positive slope in the electron velocity distribution. We initialize the plasma with a bulk
 151 Maxwellian electron population and a small, high-velocity beam that produces a pronounced
 152 bump in the tail of the distribution, and we track the evolution of the phase-space density
 153 and the associated electric field. During the linear growth phase, resonant electrons exchange
 154 energy with Langmuir waves, leading to exponential amplification of the electric field. As the
 155 instability evolves, the initially smooth electron distribution develops coherent phase-space

156 structures, illustrating the code's ability to capture nonlinear wave-particle interactions.

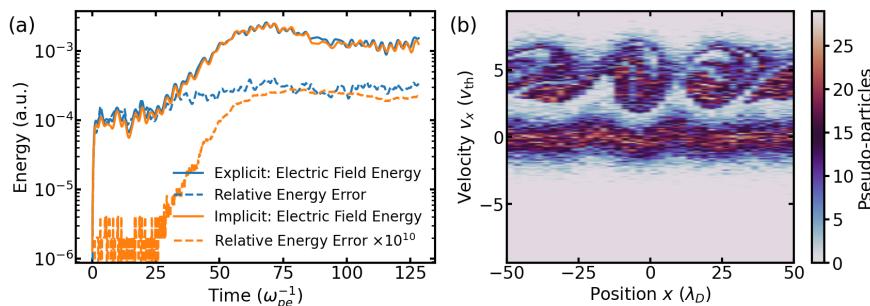


Figure 4: Simulation of the bump-on-tail instability. The number of pseudo-particles in the bulk and beam populations is equal, with a beam-to-bulk weight ratio of 3×10^{-2} . (a) Time evolution of the electric field energy and relative energy error. (b) Snapshot of phase space at $80 \omega_{pe}^{-1}$.

157 Additionally, we evaluate the computational performance of the code by comparing CPU and
 158 GPU run times. For this test, we use the hardware present in the NERSC Perlmutter HPE
 159 Cray EX supercomputer, that is, an AMD EPYC 7763 CPU and an NVIDIA A100 GPU. We
 160 assess how the total runtime scales with the number of pseudo-particles. As a representative
 161 benchmark, we simulate ten drift velocities drawn from the two-stream dispersion relation.
 162 The results are shown in Figure 5. We find that, for the same workload, the GPU executes
 163 the simulation approximately two orders of magnitude faster than the CPU. In particular,
 164 for a system of 64000 pseudo-particles, the GPU completes the full drift-scan in about six
 165 seconds after the initial compilation. The benchmarks also indicate that GPU results depend
 166 on floating-point precision: running in 32-bit mode by manually disabling JAX's x64 option
 167 reduces memory usage and improves speed, but can introduce deviations when compared to
 168 64-bit results.

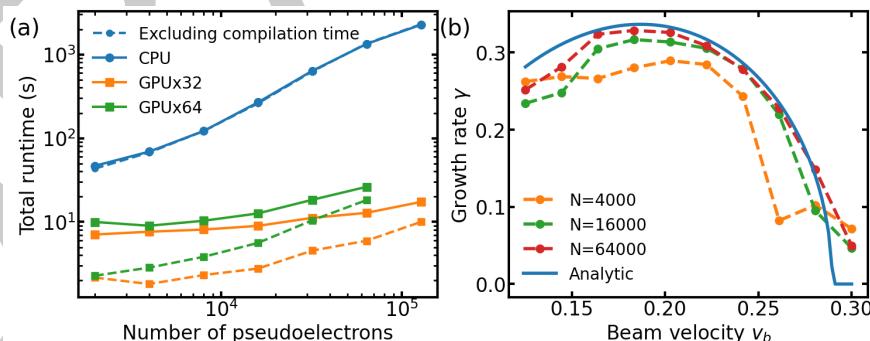


Figure 5: (a) Comparison of total runtime between CPU and GPU. (b) Influence of pseudo-particle number on the two-stream instability sampling results. Growth rates computed from exponential fits.

169 Finally, we showcase in Figure 6 one of the central motivations for JAX-in-Cell, that is, to
 170 make plasma physics simulations natively differentiable. This allows us to take gradients of
 171 high-level parameters used for diagnostics with respect to any number of physical or numerical
 172 parameters without resorting to finite differencing. Such a capability is increasingly important
 173 for tasks such as Bayesian inference of transport coefficients, real-time control, design of
 174 laser pulses, and training of hybrid physics-ML surrogates for fusion devices or astrophysical
 175 plasmas. However, we note that automatic differentiation in PIC codes might be challenging
 176 due to noise, integer indexing, and discontinuous shape functions. We show how some of these
 177 challenges can be overcome. We optimize the dimensionless growth rate $\hat{\gamma} = \gamma/\omega_{pe}$ of the
 178 two-stream instability by applying a damped Newton update to the drift velocity v_d using JAX

179 to evaluate both $\hat{\gamma}(v_d)$ and its derivative $d\hat{\gamma}/dv_d$ via a single forward-mode Jacobian-vector
 180 product, which allows for efficient memory handling. The drift speed (squares) converges from
 181 2×10^7 m/s to 3.95×10^7 m/s in six iterations, while $\hat{\gamma}$ (circles) rapidly approaches the target
 182 growth rate, and the sensitivity $|d\hat{\gamma}/dv_d|$ (diamonds) decreases as the optimizer approaches
 183 the solution. Future work will involve generalizing such an optimization to inverse problems,
 184 such as the possibility of forward-modelling electron beams responsible for Type III solar radio
 185 bursts, and the design of new experiments.

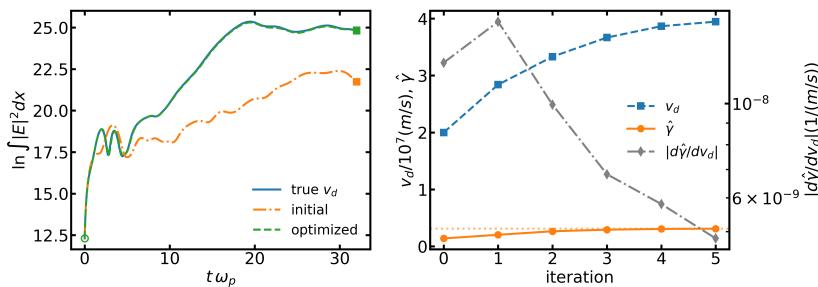


Figure 6: Demonstration of autodifferentiation capabilities in JAX-in-Cell for optimization using the two-stream instability. Left: time evolution of the electric field energy for the true drift speed v_d , the initial guess (dashed), and the optimized value. Right: optimization history of the drift speed v_d , the dimensionless growth rate $\hat{\gamma} = \gamma/\omega_p$, and the auto-differentiated sensitivity $|d\hat{\gamma}/dv_d|$. The algorithm converges in six iterations.

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