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GRTresna: An open-source code to solve the initial data constraints in numerical relativity

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GRTresna is a multigrid solver designed to solve the constraint equations for the initial data required in numerical relativity simulations. In particular it is focussed on scenarios with fundamental fields around black holes and inhomogeneous cosmological spacetimes. The following overview has been prepared as part of the submission of the code to the Journal of Open Source Software. The code is based on the formalism in Aurrekoetxea, Clough & Lim (Aurrekoetxea, Clough, & Lim, 2023) and can be found at https://github.com/GRTLCollaboration/GRTresna

Summary

Numerical relativity (NR) is a tool for the solution of the Einstein Equations, which describe gravity in strong field regimes. The equations can be expressed as a set of coupled partial differential equations (PDEs) for the 10 metric quantities $g_{\mu\nu}$ and their time derivatives $\partial_t g_{\mu\nu}$. NR is primarily focussed on the hyperbolic PDEs that describe their time evolution from an initial data set, but the initial data itself must satisfy a set of four coupled non-linear elliptic PDEs known as the Hamiltonian and momentum constraints. Whilst these constraints can be solved more straightforwardly by making certain assumptions, this significantly restricts the range of physical scenarios that can be studied. A general solver therefore expands the physics that NR evolutions can be used to probe.

In the ADM form of the Einstein Equations (Arnowitt et al., 2008), we slice the spacetime into 3-dimensional hypersurfaces

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij}dx^i dx^j$$

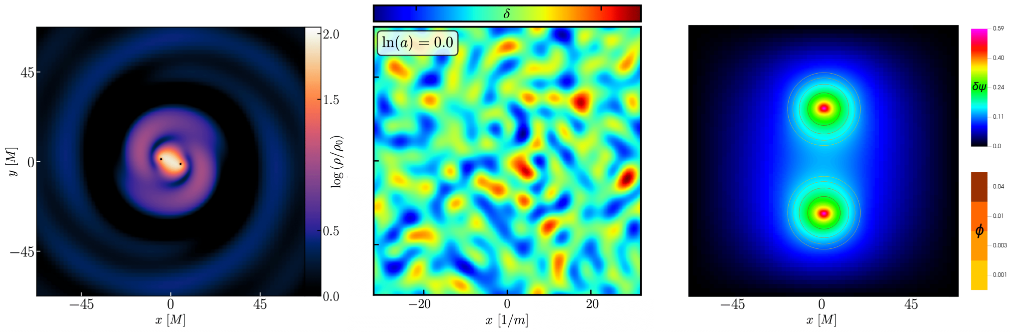
and the elliptic constraints are expressed as

$$H \equiv R + K^2 - K_{ij}K^{ij} - 16\pi\rho = 0,$$

$$M_i \equiv D^j(K_{ij} - \gamma_{ij}K) - 8\pi S_i = 0.$$

Here, γ_{ij} is the 3-metric of the hypersurface, R is the Ricci scalar associated to this metric, and $K_{ij} \sim \partial_t \gamma_{ij}$ is the extrinsic curvature tensor, with $K = \gamma^{ij}K_{ij}$ its trace. The decomposed components of the stress-energy tensor of matter (measured by normal observers) are defined as $\rho = n_\mu n_\nu T^{\mu\nu}$ and $S_i = -\gamma_{i\mu} n_\nu T^{\mu\nu}$, where $n_\mu = (-\alpha, 0, 0, 0)$. These equations constitute the set of four PDEs to be solved. There are 16 unknowns: 6 in γ_{ij}, 6 in K_{ij}, 1 in ρ and 3 in S_i. Usually the matter configuration is set by the physical scenario, which determines ρ and S_i. The constraints only determine 4 quantities, and the remaining 8 (4 of which are physical degrees of freedom, and 4 gauge choices) must be chosen according to physical principles or knowledge about the system. In this short paper, we introduce GRTresna, an open-source code to solve these equations.

The two main methods for finding initial conditions in numerical relativity are the *conformal transverse-traceless* (CTT) and the *conformal thin sandwich* (CTS) approaches. We refer the reader to the standard NR texts (Alcubierre, 2008; Baumgarte & Shapiro, 2010, 2021; Gourgoulhon, 2007; Shibata, 2016) for more details about these. GRTresna implements two variations of the CTT method recently introduced in Aurrekoetxea, Clough & Lim (Aurrekoetxea, Clough, & Lim, 2023): the CTTK and CTTK-Hybrid methods, which are particularly well-suited to cases with fundamental fields in the matter content. Documentation about using and modifying GRTresna can be found in the code wiki https://github.com/GRTLCollaboration/GRTresna/wiki.



Some highlights of work using GRTresna to date: (Left:) Dark matter around binary black holes, from (Aurrekoetxea, Clough, Bamber, et al., 2024; Aurrekoetxea, Marsden, et al., 2024; Bamber et al., 2023) (Middle:) Evolution of inflationary perturbations during preheating, from (Aurrekoetxea, Clough, & Muia, 2023). (Right:) Scalar fields around black holes in $4\partial ST$ gravity, from (Brady et al., 2023)

Key features of GRTresna

The key features of GRTresna are as follows

- Flexibility: GRTresna is designed to be extended to various physical scenarios, including different matter types and gravitational theories beyond GR. It currently supports cosmological-type periodic spacetimes and a superposition of two boosted and/or spinning black holes (Bowen-York initial data), with fully general scalar field matter source configurations and the flexibility to adapt to other setups. While scalar fields are the only matter sources included in the current version of the code, the templated methods allow users to easily replace them with other matter types by copying the scalar field implementation and modifying the methods to compute the corresponding energy and momentum densities.
- Methods: GRTresna incorporates the CTTK and CTTK-Hybrid methods to solve the

Hamiltonian and momentum constraints. These methods offer several advantages when dealing with fundamental fields, as discussed in (Aurrekoetxea, Clough, & Lim, 2023). The method code is also templated, so users can easily implement their preferred methods.

- Initial conditions: The code supports analytical initial data for the matter fields, as well as the option to read grids and data from an existing HDF5 file. This functionality is especially useful when combined with our code that evolves matter on fixed metric backgrounds GRDzhadzha (Aurrekoetxea, Bamber, et al., 2024), meaning that we can upgrade the resulting matter configurations to full NR simulations with backreaction.
- Boundary conditions: The code implements extrapolating, reflective, and periodic boundary conditions, compatible with those in the NR evolution code GRChombo (Andrade & others, 2021; Clough et al., 2015).
- Diagnostics: The code computes the Hamiltonian and momentum constraint errors at each iteration step, and outputs the norm of these values across the grid to a text file.
- Compatibility: As GRTresna is developed on top of Chombo, the solver is primarily designed to be compatible with GRChombo (Andrade & others, 2021; Clough et al., 2015) and the family of codes developed by the GRTL Collaboration. We provide two examples that integrate directly with existing examples in the GRChombo evolution code (via the output of a checkpoint file for restart at $t = 0$), and provide guidance and tools to validate the results. However, the code outputs data in the standard HDF5 data format, which should be straightforward to adapt to other NR codes that support HDF5 input or can be accessed using Python.

Other features that are inherited from Chombo include

- C++ class structure: GRTresna is written in the C++ language, and makes heavy use of object-oriented programming (OOP) and templating.
- Parallelism: GRTresna uses hybrid OpenMP/MPI parallelism.
- Adaptive Mesh Refinement: The code inherits the flexible AMR grid structure of Chombo, with block-structured Berger-Rigoutsos grid generation (Berger & Rigoutsos, 1991). The tagging of refinement regions is fully flexible and while it is based on the sources of the elliptic equations by default, other user-defined measures can be defined (Radia et al., 2022).
- Fast: The code uses a multigrid method to efficiently reduce errors across a hierarchy of discretizations, enabling the solver to achieve rapid convergence while minimizing computational costs. This makes GRTresna highly optimized for handling the demanding computations of initial data in the presence of AMR.

Forthcoming features currently under development include the addition of other methods, in particular the Extended Conformal Thin Sandwich (XCTS) method, non-conformally flat metric data, new matter types including vector fields, the modified scalar-tensor gravity formalism of Brady et. al. (Brady et al., 2023) and dimensional reduction to 2D using the cartoon formalism (Alcubierre et al., 2001; Cook et al., 2016).

Statement of Need

There are a number of existing initial data solvers for numerical relativity, most of which are primarily designed to solve for initial conditions in compact object mergers (i.e. neutron stars and black holes). These include TwoPunctures (Ansorg et al., 2004), SGRID (Tichy, 2009), BAM (Bruegmann et al., 2008), LORENE (Gourgoulhon et al., 2001; *LORENE Website*, n.d.), Spells (Pfeiffer et al., 2003), SpECTRE (Vu, 2024; Vu & others, 2022) ((Nee et al., 2025) for modified gravity) COCAL (Tsokaros et al., 2015; Tsokaros & Uryu, 2012; Uryu & Tsokaros, 2012), PCOCAL (Boukas et al., 2024), Elliptica (Rashti et al., 2022), NRPyElliptic (Assumpcao et al., 2022), KADATH/FUKA (*FUKA Website*, n.d.; Grandclement, 2010; Papenfort et al., 2021}), SPHINCS_ID (Diener et al., 2022; Rosswog et al., 2023; *SPHINCS_ID Website*, n.d.), and the solver of East *et al.* (East et al., 2012). Many of these codes, particularly those using spectral methods like TwoPunctures and SpECTRE, provide a higher accuracy in the solution compared to GRTresna, which is limited to second order accuracy by the multigrid method used. They are therefore better suited to initial data for waveform generation where precision is key. GRTresna is, however, designed to be more flexible and general purpose, tackling both cosmological and black hole spacetimes in a range of scenarios beyond GR and the Standard Model.

In particular, to the best of our knowledge, there is no fully general, publicly available initial condition solver for inhomogeneous cosmological spacetimes. One exception is FLRWSolver, developed by Macpherson *et al.* (Macpherson et al., 2017) as part of the Einstein Toolkit (Loffler & others, 2012), which specializes in initializing data for cosmological perturbations arising from inflation for studies of late-time cosmology. However, this is limited to only weakly non-linear initial data. GRTresna aims to provide an open-source tool that not only incorporates the general features of existing initial data solvers for compact objects in GR but also extends their capabilities to cosmological spacetimes (see (Aurrekoetxea, Clough, & Lim, 2024) for a review of the application of numerical relativity in cosmology). GRTresna is particularly well-suited for fundamental field matter types, such as scalar and vector fields. Its flexible design allows users to implement new solver methods, additional matter types, or extend the code to study theories beyond GR. It is fully compatible with the GRTL Collaboration's ecosystem of codes but can also serve as a complementary tool for generating constraint-satisfying initial data for other numerical relativity codes.

Key research projects using GRChombo

The code has already been used successfully to study a range of problems in fundamental physics, including: - The robustness of inflation to inhomogeneities in the scalar field (Aurrekoetxea et al., 2020; Elley et al., 2025). - The formation of oscillons during inflationary preheating (Aurrekoetxea, Clough, & Muia, 2023). - Formation of spinning primordial black holes (De Jong et al., 2023). - The effect of scalar dark matter environments around binary black holes (Aurrekoetxea, Clough, Bamber, et al., 2024; Aurrekoetxea, Marsden, et al., 2024; Bamber et al., 2023). - The general relativistic evolution of polarized Proca stars (Wang et al., 2024). - Solving the initial conditions problem for modified gravity theories (Brady et al., 2023).

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