

# BiGSTARS.jl: A Julia package for bi-global stability analysis for rotating stratified flows

Subhajit Kar <sup>1</sup>

<sup>1</sup> Tel Aviv University, Israel

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## Software

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## Summary

BiGSTARS.jl is a Julia-based package ([Bezanson et al., 2017](#)) specifically developed for conducting bi-global linear stability analyses of rotating and stratified flows relevant to atmospheric and oceanic dynamics.

## Statement of need

Linear stability analysis investigates the growth or decay of small perturbations about a basic state by linearizing the governing equations and solving the resulting eigenvalue problem ([Drazin & Reid, 2004](#)). In geophysical fluid dynamics, the interplay between rotation (the Coriolis force) and stratification (the buoyancy force) gives rise to multiple instability mechanisms, including baroclinic instability driven by vertical shear and barotropic instability driven by horizontal shear ([Pedlosky, 2013](#)). Such analyses are fundamental for understanding the onset of turbulence, the formation of eddies, and the associated energy transfer across scales in both oceanic and atmospheric systems.

The complexity and dimensionality of stability analyses have evolved substantially, giving rise to distinct methodological approaches with varying computational requirements ([Theofilis, 2011](#)). The classical one-dimensional (1D) stability analysis assumes that the basic state varies along only a single spatial direction. While computationally efficient, this approach presumes spatial homogeneity in the remaining directions and therefore often fails to capture the full dynamics of realistic geophysical flows. At the opposite extreme, the tri-global (3D global) stability analysis allows variations in all three spatial directions, providing the most complete representation of the underlying physics. However, this generality requires substantial computational resources, often beyond the reach of standard research computing infrastructures.

Bridging these two extremes, bi-global (2D global) linear stability analysis occupies an optimal middle ground between 1D and tri-global frameworks ([Theofilis, 2011](#)). In this approach, the basic state varies in two spatial directions while remaining homogeneous in the third, striking a balance between computational tractability and physical realism. Many geophysical flows naturally exhibit this structure, including atmospheric and oceanic jets ([Pedlosky, 2013](#)) as well as submesoscale oceanic fronts and filaments ([McWilliams, 2016](#)), where classical 1D analyses omit key dynamics and fully 3D approaches remain computationally prohibitive.

To address this need, we present BiGSTARS.jl, a Julia-based bi-global stability solver designed to integrate seamlessly with the broader Julia scientific computing ecosystem. The package is distributed with validated benchmark examples and is intended to be readily accessible to the geophysical fluid dynamics community. BiGSTARS.jl implements a spectral-collocation framework in which the governing equations are discretized using Chebyshev polynomials in the vertical direction and Fourier modes in the horizontal direction, applied to a two-dimensional basic state defined over a rectangular domain.

## State of the field

Although several open-source packages exist for one-dimensional stability analyses (e.g., `pyqg` (Abernathy et al., 2022), `eigntools` (Oishi et al., 2021)), to the best of our knowledge, no fully documented open-source software currently offers a comprehensive bi-global eigenvalue solver capable of treating the linearized rotating Boussinesq equations of motion under the  $f$ -plane approximation (Vallis, 2017).

## Key features

`BiGSTARs.jl` leverages Chebyshev-Fourier discretization to handle vertically bounded and horizontally periodic domains — an optimal configuration for linear stability analyses of geophysical flows. The framework also provides flexible boundary condition handling, allowing different mathematical boundary types (e.g., Dirichlet, Neumann) for each variable.

Additionally, the package is based on the shift-and-invert technique, which enables the efficient computation of eigenvalues in targeted regions of the complex plane, crucial for obtaining the most unstable modes. Users can seamlessly switch among multiple Julia eigen-solver backends — `ArnoldiMethod.jl` (Stoppels & Nyman, n.d.), `Arpack.jl` (Shah, 2018), and `KrylovKit.jl` (Haegeman, 2025) — with built-in performance benchmarking tools. To address convergence challenges in large problems, `BiGSTARs.jl` employs adaptive convergence strategies, including automatic shift adjustments and retry logic, thus reducing the need for manual parameter tuning.

The package documentation includes a collection of validated examples that illustrate the key functionalities of the solver, such as setting up a basic state, specifying boundary conditions, and visualizing the results. These examples not only provide an accessible starting point for new users unfamiliar with bi-global stability analysis but also serve as reference cases for developing customized modules.

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## References

- Abernathy, R., Rocha, C., Ross, A., Jansen, M., Li, Z., & Poulin, F. (2022). `Pyqg/pyqg`: v0.7.2. *Zenodo*.
- Bezanson, J., Edelman, A., Karpinski, S., & Shah, V. B. (2017). Julia: A fresh approach to numerical computing. *SIAM Review*, 59(1), 65–98. <https://doi.org/10.1137/141000671>
- Drazin, P. G., & Reid, W. H. (2004). *Hydrodynamic stability*. Cambridge university press.
- Haegeman, J. (2025). `KrylovKit` (Version v0.10.0). *Zenodo*. <https://doi.org/10.5281/zenodo.16710486>
- McWilliams, J. C. (2016). Submesoscale currents in the ocean. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 472(2189), 20160117.
- Oishi, J. S., Burns, K. J., Clark, S. E., Anders, E. H., Brown, B. P., Vasil, G. M., & Lecoanet, D. (2021). `eigntools`: A Python package for studying differential eigenvalue problems with an emphasis on robustness. *Journal of Open Source Software*, 6(62), 3079. <https://doi.org/10.21105/joss.03079>
- Pedlosky, J. (2013). *Geophysical fluid dynamics*. Springer Science & Business Media.
- Shah, V. B. (2018). `Arpack.jl`. In *GitHub repository*. GitHub. <https://github.com/JuliaLinearAlgebra/Arpack.jl>

- 84 Stoppels, H. T., & Nyman, L. (n.d.). *ArnoldiMethod.jl: Arnoldi method with krylov-schur*  
85 *restart natively in julia*. <https://github.com/JuliaLinearAlgebra/ArnoldiMethod.jl>
- 86 Theofilis, V. (2011). Global linear instability. *Annual Review of Fluid Mechanics*, 43(1),  
87 319–352.
- 88 Vallis, G. K. (2017). *Atmospheric and oceanic fluid dynamics*. Cambridge University Press.

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