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emg3d: A multigrid solver for 3D electromagnetic diffusion

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Software

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Summary

Controlled-source electromagnetic (CSEM) surveys are a common geophysical investigation tool in the search for, amongst other, groundwater, hydrocarbons, and minerals. The numerical modelling of CSEM data requires the solution of the Maxwell equations. These can be simplified in the particular case of CSEM, as the frequencies used in surveys are usually sufficiently low to ignore any displacement currents. A diffusive problem remains, which has the resulting system of equations given in the frequency domain by

$$\eta \mathbf{E} - \nabla \times \mu_\mathrm{r}^{-1} \nabla \times \mathbf{E} = -\mathrm{i}\omega\mu_0 \mathbf{J}_\mathrm{s} \, ,$$

where $\eta = \mathrm{i}\omega\mu_0(\sigma - \mathrm{i}\omega\varepsilon)$. The electric field and the current source are denoted as $\mathbf{E}$ and $\mathbf{J}_\mathrm{s}$, respectively, σ is the conductivity, ω is the angular frequency, $\varepsilon = \varepsilon_0\varepsilon_\mathrm{r}$ is the electric permittivity, and $\mu = \mu_0\mu_\mathrm{r}$ is the magnetic permeability.

Various open-source codes exist to model CSEM responses for a layered Earth, e.g., *DIPOLE1D* (Key, 2009) and *empymod* (Werthmüller, 2017), and for a two-dimensional Earth, e.g., *MARE2DEM* (Key & Ovall, 2011). Open-source modellers for a three-dimensional (3D) Earth only recently became available, notably *SimPEG* (Cockett, Kang, Heagy, Pidlisecky, & Oldenburg, 2015), *PETGEM* (Castillo-Reyes, Puente, & Cela, 2018), and *custEM* (Rochlitz, Skibbe, & Günther, 2019). SimPEG is a framework that not only includes CSEM but also other geophysical methods and can model them on various types of regular grids. It currently primarily uses the direct solver *PARDISO* (Schenk & Gärtner, 2004). PETGEM and custEM use finite elements with the *FEniCS* solver (Alnaes et al., 2015). All three codes require substantial memory and are not easily run on a laptop for models with several million cells.

Mulder (2006) has shown that the multigrid method (Briggs, Henson, & McCormick, 2000), with its optimal scaling for both runtime and memory consumption as shown in Figure 1, works fine for diffusive CSEM problems. This was later also confirmed by others (Jaysaval, Shantsev, Kethulle de Ryhove, & Bratteland, 2016). However, the multigrid CSEM codes discussed in these publications are proprietary. The code *emg3d* is a multigrid solver for 3D CSEM diffusion with tri-axial electrical anisotropy using a staggered grid (Mulder, 2006, 2007). It can act as a solver on its own, or be used as a preconditioner for various Krylov subspace methods. Multigrid solvers can struggle to converge with strong grid-stretching or strong anisotropy. We implemented *semicoarsening* (coarsening the grid only in some coordinate directions) and *line relaxation* (solving for the values of a whole gridline simultaneously in some directions) inside the multigrid technique to deal with these issues (Jönsthövel, Oosterlee, & Mulder, 2006).

The code is written completely in Python using the NumPy/SciPy stack (Jones, Oliphant, Peterson, & others, 2001; Walt, Colbert, & Varoquaux, 2011), where the most time- and

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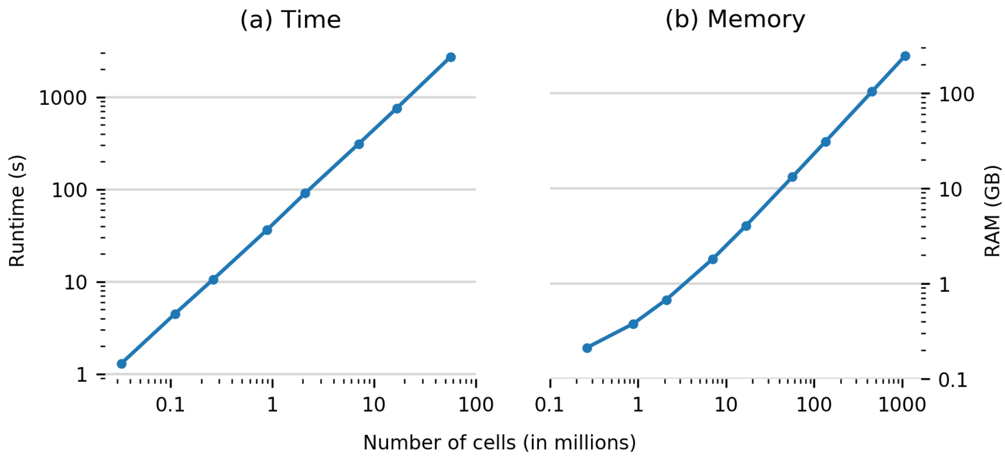

Figure 1: Example showing the optimal scaling of emg3d for both (a) runtime and (b) memory consumption. The model is a homogeneous fullspace of 1 S/m and frequency is 1 Hz (running as a single thread on an Intel(R) Xeon(R) CPU @ 2.50GHz).

memory-consuming parts are sped up through jitted functions using Numba (Lam, Pitrou, & Seibert, 2015). It can currently be used as a stand-alone modeller or as a solver in conjunction with the SimPEG-framework.

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