

# MultiHazard: Copula-based Joint Probability Analysis in R

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## Summary

Compound events occur when combinations of drivers and/or hazards contribute to a societal/environmental impact (Zscheischler et al., 2020). Even if none of the individual drivers or hazards are extreme, their combination can produce extreme impacts. Assessing the potential for compound extreme events is therefore critical for effective risk management and mitigation planning. To determine the probability of compound events, statistical models are applied to time series data of the drivers or hazards, typically as the first step in the risk-analysis modeling chain.

The MultiHazard R package is designed to enable practitioners to estimate the likelihood of compound events. Although the methods in the package are well-established in the scientific literature, they are not widely adopted by the engineering community despite guidelines increasingly mandating the estimation of compound event likelihoods. Functions within the package are designed to allow practitioners to apply their best judgement in making subjective choices. Inputs are time series representing the drivers/hazards; these may be historical observations or numerically generated with models (for the past or future). Outputs are:

- Estimates of the joint return periods for specific combinations of drivers/hazards.
- Isolines with uncertainty bounds containing drivers/hazards with a specified joint return period along with the “most-likely” or an ensemble of events sampled along an isoline.
- Synthetic sets of events where the peak magnitude of at least one driver/hazard is extreme.

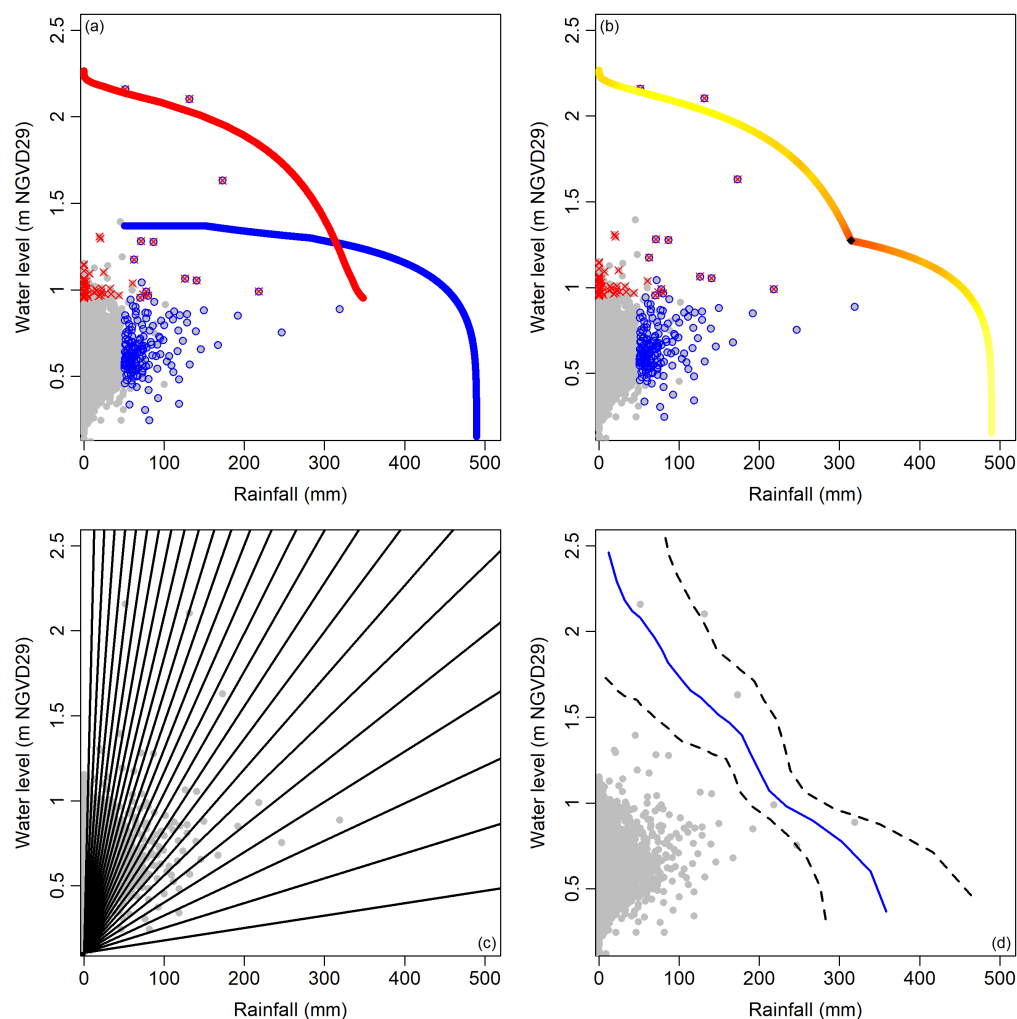
## Statement of need

The MultiHazard package was developed in collaboration with the South Florida Water Management District (SFWMD) to improve the level of service assessments for coastal infrastructure affected by both inland and coastal drivers (Jane et al., 2020). Initially, the package was created to implement the conditional sampling - copula theory approach. In two-sided conditional sampling, a driver is conditioned to be extreme and paired with the maximum value of the other driver within a specified lag-time. The process is repeated with the drivers reversed yielding two conditional samples. The best fitting of 40 copulas are tested to model the dependence between drivers in each conditional sample. The isoline corresponding to a user specified return period is given by the outer envelope when overlaying the (conditional) contours from the copula model fit to each sample Bender et al. (2016), see Figure 1a,b. To obtain a single design event, the probability of events on the isolines is calculated using

42 an empirical density estimate, selecting the event with the highest likelihood. Some experts  
43 recommend sampling an ensemble of events from the isoline to account for uncertainty in  
44 design event selection which is also possible in the package. The conditional sampling – copula  
45 theory approach has continued to gain traction (Kim et al., 2023; Maduwantha et al., 2024)  
46 and in a review of the best-available, actionable science was highlighted as an approach that  
47 Federal agencies in the United States may wish to develop detailed technical guidance on how  
48 to use it meet their needs (FFRMS, 2023).

49 The package possesses the functionality to compute the desired isoline by interpolating joint  
50 return periods calculated over a user-defined grid which can result in smoother curves than  
51 overlaying the partial isolines. Isoline generation for time series comprising events from multiple  
52 (independent) populations e.g. different types of storms is facilitated by multiplying independent  
53 annual-non exceedance probabilities associated with each population as outlined in Maduwantha  
54 et al. (2024). Also coded in the package are the methods proposed in Murphy-Barltrop et  
55 al. (2023) and Murphy-Barltrop et al. (2024) to derive isolines using the HT04 (Heffernan &  
56 Tawn, 2004) and WT13 (Wadsworth & Tawn, 2013) models, along with a novel bootstrap  
57 procedure for evaluating isoline uncertainty (see Figure 1c,d), and a diagnostic tool proposed  
58 for evaluating the goodness of fit for a curve estimate. The HT04 and WT13 models allow for  
59 the analysis of compound events without assuming any copula form. Furthermore, the results  
60 from Murphy-Barltrop et al. (2023) guarantee that the estimated isolines from the HT04 and  
61 WT13 approaches satisfy certain theoretical properties.

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**Figure 1:** The 100-year isoline at case study site derived using two methods. Two-sided conditional sample - copula theory method: (a) Partial isolines from the samples conditioned on rainfall (red line) and water level (blue line) whose outer envelope when overlaid gives (b) the full isoline. Heffernan and Tawn model method: (c) Sample uncertainties are calculated along 100 rays emanating from the point comprising the minimum observed values of both drivers. (d) The median isoline (red line) and associated 90% confidence interval (dashed black lines) obtained via a block bootstrap procedure .

Compound events are of increasing concern for entities responsible for managing flood risk. MultiHazard is designed as a comprehensive user-friendly tool for copula-based joint probability analysis in R. As such, the package provides functions for pre-processing data including imputing missing values, detrending and declustering time series, as well as exploratory data analysis, e.g., analyzing pairwise correlations over a range of time-lags between the two drivers/hazards. The package also contains two automated threshold selection method for the Generalized Pareto distribution [Solari et al. (2017); Murphy:2024] and approaches for robustly capturing the dependence structure when there are more than two relevant drivers/hazards, namely, standard (elliptic/Archimedean) copulas, Pair Copula Constructions (PCCs) and the HT04 model. For the analysis undertaken for the SFWMD, the higher dimensional approaches enabled groundwater level to be included in the analysis. More recently, an approach to generate time varying synthetic events, i.e., hyetographs and hydrographs, was added. Time varying conditions are a prerequisite for non-steady state hydrodynamic modeling.

## Related packages

Several packages employ copula-based approaches to derive isolines. The MATLAB toolbox MvCAT (Multivariate Copula Analysis Toolbox) (Sadegh et al., 2017) utilizes up to 26 copula families to model the dependence structure between a pair of random variables. MvCAT includes fewer copula families than MultiHazard and does not consider the two-sided conditional sampling - copula theory methodology but rather uses a one-sided sampling approach. On the other hand, MvCAT adopts multiple criteria to select among the candidate copula families and a Bayesian framework to account for the uncertainty range for the copula parameters. Parts of the recently released ReturnCurves (André & Murphy-Barltrop, 2024) R package that implement the methods outlined in Murphy-Barltrop et al. (2023) and Murphy-Barltrop et al. (2024) have been subsumed into the MultiHazard package with permission from the authors. The ReturnCurves package only allows practitioners to use the WT13 model to evaluate isolines, while our package incorporates a range of different modelling frameworks.

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