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TomoSphero: Fast Differentiable Tomographic Projector in Spherical Coordinates

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Software

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Summary

Computational tomography is a tool for determining the internal structure of objects from a set of projections, typically taken along some regular path (e.g. circular, helical). In recent years, methods and GPU-accelerated libraries have emerged that allow for fast reconstruction from projections along more complicated paths. Most of these libraries rely on a Cartesian discretization of the object. In planetary and solar tomography, projections are taken along an irregular spacecraft orbit of spherical bodies not well-suited to Cartesian grids.

Statement of need

We present `TomoSphero`, a differentiable tomographic projector over spherical grids which are often used in planetary and solar tomography. `TomoSphero` is designed to be used as a building block in reconstruction algorithms and includes common projection types such as cone-beam and parallel-beam, but is flexible enough to accommodate arbitrary projections. `TomoSphero` is implemented in PyTorch, which allows for fast projection computation on GPUs, easy integration into machine learning algorithms, and automatic differentiation for reconstruction algorithms which require access to gradients.

Tomographic Inversion

Tomography is a method for determining the internal structure of objects from a set of measurements that penetrate into the object being measured. These measurements (sometimes called projections or sinograms) are usually captured from a variety of locations and times which are collectively referred to as the *view geometry*. Measurements are typically modeled as

$$y = Fx + \epsilon$$

where y is a collection of measurements, F is a linear projection operator, x is the object under study, and ϵ is noise.

Tomography has found application in a vast number of domains such as medical imaging, crystallography, and remote sensing, utilizing modalities like X-ray, ultraviolet (UV), ultrasound, seismic waves, and many more. In this paper we discuss TomoSphero, a Python library for planetary and solar tomography.

Fast tomographic reconstruction algorithms that implement explicit inversion formulas typically work only for specific view geometries (such as circular or helical view geometry) and are referred to as *filtered back projection* (FBP) algorithms (Willemink & Noël, 2019). However, some situations (like an orbiting spacecraft) necessitate more complicated measurement paths than are allowed by FBP-type algorithms. For these situations requiring more flexible view geometries where an exact inverse solution is not available, *iterative reconstruction* (IR) algorithms prevail, usually solving an optimization problem of the form

$$\hat{x} = \arg\min_{c} \|y - FM(c)\|_2^2 + \mathcal{R}(...) + ...$$

where M is a parametric model for the object under construction and $\mathcal{R}$ is a regularization term.

Examples include SIRT (Pryse et al., 1993), TV-MIN (Sidky & Pan, 2008), ART (Austen et al., 1986), CGLS (Scales, 1987), Plug-and-play (Venkatakrishnan et al., 2013) and many others. These algorithms obtain synthetic projections of a candidate object using a tomographic operator (sometimes called a *raytracer*) that simulates waves traveling through the object medium. They produce a reconstruction by repeatedly tweaking the candidate object to minimize discrepancy between synthetic and actual projections, and they stand to benefit the most from a fast operator implementation.

TomoSphero is parallelized and GPU-enabled, and its speed has been benchmarked as described in the companion paper. In cases where a simultaneous computation for every pixel of every measurement would consume more memory than is available, some algorithms operate *out-of-core*, where they parallelize as many tasks as will fit into available memory, then serially queue the remaining tasks for processing after current tasks are complete. TomoSphero is not capable of out-of-core operation.

Another consideration in tomographic reconstruction is the choice of grid type for discretization of the reconstructed object. Most publications consider a regular rectilinear grid, which is a reasonable choice when the underlying structure of the object is completely unknown or the scale of features is uniform throughout the object. The primary focus of TomoSphero is in the domain of atmospheric tomography, where regular spherical grids are well-suited for modeling solar and planetary atmospheres that exhibit spherical symmetries (Butala et al., 2010) (Jackson et al., 2011).

Many reconstruction algorithms rely on gradient-based optimization to solve for an object whose structure corresponds to measurement data. Automatic differentiation (*autograd*) is a class of techniques that convert an arbitrary expression into a computational graph of simpler functions, then compute the overall derivative by applying chain rule at each node. Modern machine learning libraries such as PyTorch (Paszke et al., 2019) and Jax (Bradbury et al., 2018) provide such capabilities for building this computational graph. TomoSphero is implemented on top of PyTorch and its autograd capabilities enable rapid prototyping of different parametric models and regularizations.

TomoSphero development was motivated by the Carruthers Geocorona Observatory, a spacecraft containing UV imagers which will survey the Earth's exosphere.

A non-exhaustive comparison of TomoSphero's capabilities against other popular libraries is shown below:

Name	Grid Type	GPU Support	Autograd	Visualization	Out-of-Core
TIGRE (Biguri et al., 2016)	Cartesian	Yes	No	No	Yes
LEAP (Kim & Champley, 2023)	Cartesian	Yes	Yes	No	Yes
ASTRA (Aarle et al., 2016)	Cartesian	Yes	Yes	No	Yes
mbirjax (Bouman & Buzzard, 2024)	Cartesian	Yes	No	No	Yes

Name	Grid Type	GPU Support	Autograd	Visualization	Out-of-Core
ToMoBAR (Kazantsev & Wadeson, 2020)	Cartesian	yes	No	No	Yes
CIL (Jørgensen et al., 2021)	Cartesian	Yes	No	Yes	Yes
Tomosipo (Hendriksen et al., 2021)	Cartesian	Yes	Yes	Yes	Yes

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