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NGSTrefftz: Add-on to NGSolve for Trefftz methods

Paul Stocker[1¶]

1 Georg-August-Universität, Göttingen, Germany ¶ Corresponding author

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Summary

NGSTrefftz is an add-on to Netgen/NGSolve, a finite element software for the numerical treatment of partial differential equations (PDEs). The package implements Trefftz based discontinuous Galerkin (DG) methods in NGSolve. Trefftz methods reduce the number of unknowns in the discretization of PDE problems by injecting knowledge of the PDE into the approximation functions. Like NGSolve, NGSTrefftz is written in C++ and integrates seamlessly with the easy-to-use Python interface of NGSolve.

Trefftz methods originate from Trefftz (1926) and have since been developed for a wide range of problems, for an overview see Zienkiewicz (1997), Hiptmair et al. (2016), Qin (2005), Li et al. (2008), Kita & Kamiya (1995). The central principle of Trefftz methods is the construction of a discrete basis of solutions to the differential operator under consideration, making the space of Trefftz functions problem dependent. In combination with finite elements the Trefftz basis is constructed locally, on each mesh element, continuity and boundary conditions are then enforced in the variational formulation.

Statement of need

NGSTrefftz provides a framework to implement Trefftz finite element spaces for NGSolve, with spaces for Laplace equation, Helmholtz equation, and acoustic wave equation already implemented. NGSolve provides a flexible framework to implement variational formulations for arbitrary physical models. The Trefftz finite element spaces can then be used with the (bi-)linear forms generated in NGSolve. The focus lies on the combination of Trefftz functions with DG methods or least-squares formulations, this approach is also often referred to as frameless Trefftz elements.

On top of that, the package provides unique features that are:

- A quasi-Trefftz space that provides Trefftz-like properties for the acoustic wave equation with smooth coefficients. The method and results are presented in Imbert-Gérard et al. (2021).
- A space–time Trefftz method for the acoustic wave equation on tent-pitched meshes. Tent-pitching is a space–time meshing strategy that provides mesh elements that conform to the causality constraint of a hyperbolic system. The meshes are generated using ngstents and can be used with the Trefftz and quasi-Trefftz space for the acoustic wave equation, results are shown in Perugia et al. (2020) and Imbert-Gérard et al. (2021).
- A general framework to produce Trefftz spaces implicitly is provided by an implementation of the embedded Trefftz method, see Lehrenfeld & Stocker (2022). The approach produces a Galerkin projection of an underlying discontinuous Galerkin method onto a subspace of Trefftz-type. It can be applied to very general cases, including inhomogeneous sources and non-constant coefficient differential operators.

The aim of this package is to facilitate research into Trefftz methods and to make them more accessible to a broad audience. To the best of our knowledge, the only other open source software package that provides Trefftz finite element methods is FreeHyTe (Moldovan & Cismașiu, 2018), implemented in MATLAB. Examples for the usage of all the features in NGSTrefftz are provided in the form of jupyter notebooks.

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