

¹ GaussianMarkovRandomFields.jl: Flexible Latent Gaussian Modeling with Sparse Precision in Julia

³ **Tim Weiland**  ¹

⁴ 1 University of Tübingen, Germany

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

Software

- [Review](#) 
- [Repository](#) 
- [Archive](#) 

Editor: 

Submitted: 29 December 2025

Published: unpublished

License

Authors of papers retain copyright

and release the work under a

Creative Commons Attribution 4.0

International License ([CC BY 4.0](https://creativecommons.org/licenses/by/4.0/))

⁵ Summary

⁶ Bayesian inference provides a strong theoretical foundation to draw conclusions from real-world
⁷ data, with applications across all of the sciences. Yet in practice, algorithms for Bayesian
⁸ inference often require excessive computational resources, limiting their applicability to large-
⁹ scale problems. Particularly in the context of spatial and spatiotemporal settings, Gaussian
¹⁰ Markov Random Fields (GMRFs) ([Havard Rue & Held, 2005](#)) offer a remedy to this issue.
¹¹ GMRFs are Gaussian distributions with sparse precision matrices. Inference under these models
¹² scales particularly favorably due to the use of sparse linear algebra routines. This computational
¹³ efficiency has led to their widespread adoption across various statistical applications ([Lindgren
et al., 2022](#)).

¹⁴ GaussianMarkovRandomFields.jl provides a flexible and efficient Julia implementation of
¹⁵ GMRFs. The package includes various methods to construct GMRFs, including the SPDE
¹⁶ approach ([Lindgren et al., 2011](#)), and provides efficient, customizable routines for GMRF
¹⁷ computations. It is designed to be intuitive to use for rapid prototyping, yet sufficiently flexible
¹⁸ to empower expert users to solve advanced problems. As such, it is suitable for both research
¹⁹ and teaching in spatial statistics and Bayesian modeling.

²¹ Statement of Need

²² GMRFs have become a cornerstone of modern spatial statistics, yet practical barriers limit
²³ their accessibility. Implementing GMRF-based models requires navigating sparse linear algebra
²⁴ libraries, finite element discretizations, and problem-specific solver selection—all of which
²⁵ create a steep barrier to entry for domain scientists.

²⁶ GaussianMarkovRandomFields.jl eliminates these barriers by providing a feature-rich toolkit
²⁷ for creation and manipulation of GMRFs. The package offers utilities to create well-known
²⁸ latent models, ranging from simple autoregressive and Besag ([Besag & Kooperberg, 1995](#))
²⁹ models to more involved spatial and spatiotemporal finite element-based models ([Clarotto et
al., 2024; Lindgren et al., 2011](#)). For users requiring continuous spatial domains, the SPDE
³⁰ approach is implemented through integration with Ferrite.jl ([Carlsson et al., 2025](#)), enabling
³¹ principled approximations of Matérn and related Gaussian processes on arbitrary geometries.
³² As an alternative to SPDE-based constructions, the package also implements KL-minimizing
³³ sparse Cholesky approximations ([Schäfer et al., 2021](#)), which construct sparse precision matrices
³⁴ directly from covariance specifications and are particularly effective for covariance functions
³⁵ with strong screening effects.

³⁶ The package extends beyond Gaussian observations through Gaussian approximation methods.
³⁷ When working with non-Gaussian data such as counts or binary outcomes, the package
³⁸ seamlessly handles exponential family likelihoods and custom likelihood functions, exploiting
³⁹ sparsity structure through automatic differentiation to maintain computational efficiency

even for complex observation models. For large-scale applications, the package provides efficient marginal variance computation methods (Lin et al., 2011; H. Rue, 2005; Sidén et al., 2018). Figure 1 demonstrates this capability on a spatial binary classification task. Automatic differentiation support via ForwardDiff.jl (Revels et al., 2016) and Enzyme.jl (Moses & Churavy, 2020) enables gradient-based optimization and inference methods, making the package compatible with modern probabilistic programming frameworks including Turing.jl (Ge et al., 2018).

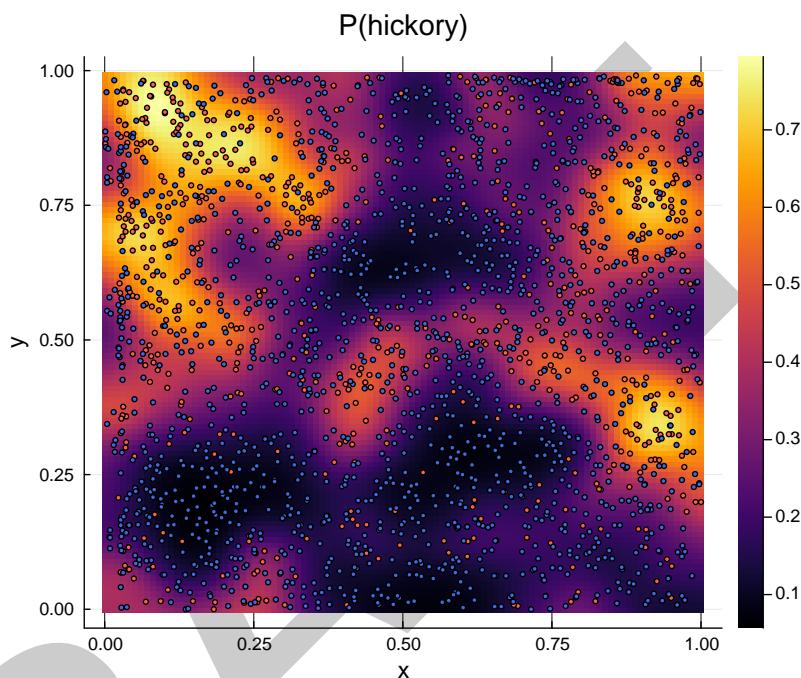


Figure 1: Spatial binary classification of tree species in the Lansing Woods dataset using a Matérn latent field with Bernoulli observations. Points show observed trees: **hickory** and **other species**. The heatmap shows predicted probabilities, demonstrating the package's support for non-Gaussian likelihoods through efficient Gaussian approximation.

Existing GMRF software faces trade-offs between ease of use and flexibility. R-INLA (Håvard Rue et al., 2009) provides comprehensive functionality for latent Gaussian models but is limited to pre-defined model structures and lacks flexibility for custom applications. The inlabru package (Bachl et al., 2019) extends R-INLA with support for non-linear predictors and a more flexible interface, but remains tied to INLA's underlying computational framework. The rSPDE package (Bolin & Simas, 2025) enables fractional SPDE models with non-integer smoothness parameters. General-purpose probabilistic programming frameworks like PyMC (Abril-Pla et al., 2023) provide some GMRF functionality, but lack the specialized treatment of dedicated GMRF packages. GaussianMarkovRandomFields.jl bridges this gap by combining ease of use with full extensibility, allowing users to leverage pre-built components while maintaining the flexibility to implement custom models and solvers.

The package is designed for researchers and practitioners working in spatial statistics, epidemiology, environmental science, and related fields requiring efficient Bayesian inference for spatially or temporally correlated data. It serves both as a research tool for developing novel GMRF-based methods and as a teaching resource for courses in spatial statistics and Bayesian modeling. The package has been used in methodological research on probabilistic PDE solvers (Weiland et al., 2025).

65 Acknowledgements

66 The author gratefully acknowledges co-funding by the European Union (ERC, ANUBIS,
67 101123955). Views and opinions expressed are however those of the author only and do not
68 necessarily reflect those of the European Union or the European Research Council. Neither
69 the European Union nor the granting authority can be held responsible for them. The author
70 also gratefully acknowledges the German Federal Ministry of Education and Research (BMBF)
71 through the Tübingen AI Center (FKZ:01IS18039A); and funds from the Ministry of Science,
72 Research and Arts of the State of Baden-Württemberg. The author further thanks the
73 International Max Planck Research School for Intelligent Systems (IMPRS-IS) for their support.

74 References

- 75 Abril-Pla, O., Andreani, V., Carroll, C., Dong, L., Fonnesbeck, C. J., Kochurov, M., Kumar,
76 R., Lao, J., Luhmann, C. C., Martin, O. A., Osthege, M., Vieira, R., Wiecki, T., & Zinkov,
77 R. (2023). PyMC: A modern and comprehensive probabilistic programming framework in
78 Python. *PeerJ Computer Science*, 9, e1516. <https://doi.org/10.7717/peerj-cs.1516>
- 79 Bachl, F. E., Lindgren, F., Borchers, D. L., & Illian, J. B. (2019). inlabru: An R package for
80 Bayesian spatial modelling from ecological survey data. *Methods in Ecology and Evolution*,
81 10, 760–766. <https://doi.org/10.1111/2041-210X.13168>
- 82 Besag, J., & Kooperberg, C. (1995). On conditional and intrinsic autoregressions. *Biometrika*,
83 82(4), 733–746. <https://doi.org/10.1093/biomet/82.4.733>
- 84 Bolin, D., & Simas, A. B. (2025). *rSPDE: Rational approximations of fractional stochastic
85 partial differential equations*. <https://cran.r-project.org/package=rSPDE>
- 86 Carlsson, K., Ekre, F., & Ferrite.jl contributors. (2025). *Ferrite.jl*. <https://doi.org/10.5281/zenodo.13862652>
- 88 Clarotto, L., Allard, D., Romary, T., & Desassis, N. (2024). The SPDE approach for
89 spatio-temporal datasets with advection and diffusion. *Spatial Statistics*, 62, 100847.
90 <https://doi.org/10.1016/j.spasta.2024.100847>
- 91 Ge, H., Xu, K., & Ghahramani, Z. (2018). Turing: A language for flexible probabilistic
92 inference. In A. Storkey & F. Perez-Cruz (Eds.), *Proceedings of the twenty-first international
93 conference on artificial intelligence and statistics* (Vol. 84, pp. 1682–1690). PMLR.
94 <https://proceedings.mlr.press/v84/ge18b.html>
- 95 Lin, L., Yang, C., Meza, J. C., Lu, J., Ying, L., & E, W. (2011). SellInv—an algorithm for
96 selected inversion of a sparse symmetric matrix. *ACM Transactions on Mathematical
97 Software*, 37(4), 1–19. <https://doi.org/10.1145/1916461.1916464>
- 98 Lindgren, F., Bolin, D., & Rue, H. (2022). The SPDE approach for Gaussian and non-Gaussian
99 fields: 10 years and still running. *Spatial Statistics*, 50, 100599. <https://doi.org/10.1016/j.spasta.2022.100599>
- 101 Lindgren, F., Rue, H., & Lindström, J. (2011). An explicit link between Gaussian fields
102 and Gaussian Markov random fields: The stochastic partial differential equation approach.
103 *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 73(4), 423–498.
104 <https://doi.org/10.1111/j.1467-9868.2011.00777.x>
- 105 Moses, W., & Churavy, V. (2020). Instead of rewriting foreign code for machine learning,
106 automatically synthesize fast gradients. In H. Larochelle, M. Ranzato, R. Hadsell, M. F.
107 Balcan, & H. Lin (Eds.), *Advances in neural information processing systems* (Vol. 33, pp.
108 12472–12485). Curran Associates, Inc.
- 109 Revels, J., Lubin, M., & Papamarkou, T. (2016). Forward-mode automatic differentiation in

- 110 Julia. *arXiv Preprint arXiv:1607.07892*. <https://arxiv.org/abs/1607.07892>
- 111 Rue, H. (2005). *Marginal Variances for Gaussian Markov Random Fields*. Trondheim TU. Inst.
112 Math.
- 113 Rue, Havard, & Held, L. (2005). *Gaussian Markov Random Fields: Theory and Applications*.
114 Chapman and Hall/CRC. <https://doi.org/10.1201/9780203492024>
- 115 Rue, Håvard, Martino, S., & Chopin, N. (2009). Approximate Bayesian inference for latent
116 Gaussian models by using integrated nested Laplace approximations. *Journal of the
117 Royal Statistical Society: Series B (Statistical Methodology)*, 71(2), 319–392. <https://doi.org/10.1111/j.1467-9868.2008.00700.x>
- 119 Schäfer, F., Katzfuss, M., & Owhadi, H. (2021). Sparse Cholesky factorization by Kullback-
120 Leibler minimization. *SIAM Journal on Scientific Computing*, 43(3), A2019–A2046. <https://doi.org/10.1137/20M1336254>
- 122 Sidén, P., Lindgren, F., Bolin, D., & Villani, M. (2018). Efficient Covariance Approximations
123 for Large Sparse Precision Matrices. *Journal of Computational and Graphical Statistics*,
124 27(4), 898–909. <https://doi.org/10.1080/10618600.2018.1473782>
- 125 Weiland, T., Pförtner, M., & Hennig, P. (2025). Flexible and efficient probabilistic PDE solvers
126 through gaussian markov random fields. In Y. Li, S. Mandt, S. Agrawal, & E. Khan (Eds.),
127 *Proceedings of the 28th international conference on artificial intelligence and statistics*
128 (Vol. 258, pp. 2746–2754). PMLR. <https://proceedings.mlr.press/v258/weiland25a.html>

DRAFT