

# r3f: Rotation of 3-dimensional Reference Frames

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## Summary

The r3f library includes attitude-representation conversions, reference-frame conversions, and rotation matrix (direction cosine matrix) utilities. All twenty possible conversions among the following five attitude representations are provided: rotation vector, rotation axis and angle, roll and pitch and yaw (RPY) Euler angles, direction cosine matrix (DCM), and quaternion. This library also includes all twelve possible conversions among the following four frames: ECEF (Earth-centered, Earth-fixed), geodetic (latitude, longitude, and height above ellipsoid), local-level tangent, and local-level curvilinear. Both local-level frames can be in either NED (North, East, Down) or ENU (East, North, Up) orientation. A general dcm function will generate any arbitrary rotation matrix based on an arbitrary sequence of Euler rotation angles. This can be used to convert between ECEF and ECI (Earth-centered Inertial) frames.

A few additional utility functions are included. The orthonormalize\_dcm function will normalize and orthogonalize a rotation matrix. The two Rodrigues's rotation functions, rodrigues\_rotation and inverse\_rodrigues\_rotation, are meant for converting a vector to the matrix exponential of the skew-symmetric matrix of that vector and back again.

This library can be installed with `pip install r3f`.

## Statement of need

Navigation algorithms heavily rely on reference frame conversions and various attitude representations. Key resources, such as the works of Titterton et al. (2004) and Groves (2013), cover inertial navigation, global navigation satellite systems, and other navigation instruments, emphasizing these conversions and representations. The r3f library offers efficient tools for handling these tasks, with functions capable of processing individual vectors or batches of vectors stored as matrices. Built on NumPy (Harris et al., 2020), r3f maintains simplicity with a single dependency.

The following is a list of some other Python libraries with similar functionality. Except for gps-frames it does not appear that any of these has both the standard Earth reference frame conversions as well as attitude representation conversions.

Name	Author	License
PyMap3D	Hirsch (2018)	BSD
transforms	Rivera (2021)	GNU GPL
gps-frames	The Aerospace Corporation (2022)	GNU GPL
rotations	Mom's Friendly Robot Company (2022)	MIT
gtFrame	Jukic (2023)	MIT
PyGeodesy	Ippen (2024)	MIT

Here is an example in which a flight path defined in a local-level curvilinear frame is converted

to geodetic coordinates:

```
import numpy as np
import r3f

K = 1000
theta = np.linspace(np.pi, -np.pi, K)
xc = 1000e3 * (1 + np.cos(theta))
yc = 1000e3 * np.sin(theta)
zc = -(np.cos(theta) + 1) * 1e3/2

[lat, lon, hae] = r3f.curvilinear_to_geodetic(
    [xc, yc, zc],
    [39.783, -84.083, 317.0], degs=True)
```

This library is currently used by students at the Air Force Institute of Technology and in research software.

## References

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