The Journal of Open Source Software

adrt: approximate discrete Radon transform for Python

Karl Otness [1] and **Donsub Rim** [2]

1 New York University, New York, NY, USA **2** Washington University in St. Louis, St. Louis, MO, USA

DOI: 10.21105/joss.05083

Software

- Review
- Repository
- Archive

Editor: Patrick Diehl

Reviewers:

- @zhangjy-ge
- @fruzsinaagocs

Submitted: 06 January 2023
Published: 16 March 2023

License
Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License (CC BY 4.0).

Summary

The Radon transform is a fundamental integral transform that arises in many different fields including medical/seismic tomography, signal/image processing, and the analysis of partial differential equations (Natterer, 2001). The forward transform computes integrals over lines of an input image at various angles and offsets. This package implements a discretization of this transform called the approximate discrete Radon transform (ADRT) which computes integrals over pixel line segments allowing for a faster evaluation over digital images (Brady, 1998; Götz & Druckmüller, 1996). We provide an implementation of the ADRT and related transforms including a back-projection operation, a single-quadrant inverse, and the full multigrid inverse described in Press (2006). Each of these routines is accessible from Python, operates on NumPy arrays (Harris et al., 2020), and is implemented in C++ with optional OpenMP multithreading.

Statement of need

This package, `adrt`, aims to facilitate numerical experimentation with the ADRT by providing production-ready implementations of the ADRT algorithm and related transforms. We expect it to be useful in several broad respects: in scientific computing applications, in studying the properties of the ADRT, and in preparing new specialized software implementations.

The ADRT has demonstrated usefulness in scientific computing (Rim, 2018) and has applications in imaging, image processing, and machine learning which can benefit from the increased performance of the ADRT, which has a time complexity of $\mathcal{O}(N^2 \log N)$ for an $N \times N$ image (see Figure 1), compared to $\mathcal{O}(N^3)$ for the standard Radon transform (Press, 2006). The ADRT approximates the Radon transform with $\mathcal{O}(N^{-1} \log N)$ error, and it possesses important inversion properties which enable it to be used to approximate the inverse Radon transform (Press, 2006; Rim, 2020). Our documentation includes examples of the application of these routines to sample problems in tomography and PDEs, as well as recipes for implementing other transforms with our core routines, including an iterative inverse using the conjugate gradient iteration.

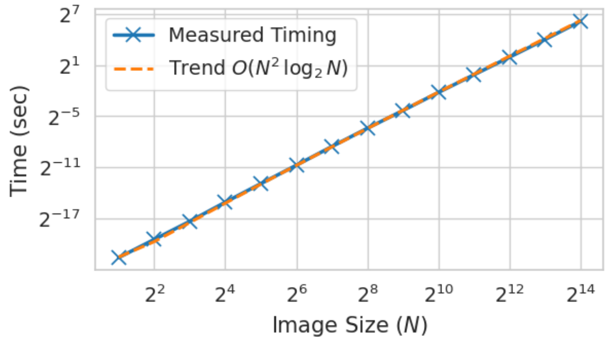

Figure 1: Running time of the ADRT for several image sizes with asymptotic trendline. Tests were run single-threaded on an Intel Xeon Platinum 8268 processor.

These routines also support research into the ADRT itself. While some private implementations exist (Bücker et al., 2015), to the best of our knowledge this is the only publicly available, open source implementation packaged for general use. This implementation provides a testbed for studying the ADRT, including routines exposing the progress of internal iterations. This package can also assist the development of specialized implementations, either by serving as a reference for new development or through reuse of the core C++ source which is independent of Python.

Related research and software

A variety of other discretizations and approximations of the Radon transform exist, such as a linear interpolation and filtered back-projection in Walt et al. (2014); the discrete Radon transform (Beylkin, 1987); a fast transform based on the pseudo-polar Fourier transform (Averbuch et al., 2008); and the non-uniform fast Fourier transform (NUFFT) (Barnett et al., 2019; Greengard & Lee, 2004). However, the ADRT has unique properties that distinguish it from other discretizations, such as its localization property and range characterization (Li et al., 2023).

Acknowledgments

This work was partially supported by the Department of Defense through the National Defense Science & Engineering Graduate (NDSEG) Fellowship program, by the Air Force Center of Excellence on Multi-Fidelity Modeling of Rocket Combustor Dynamics under Award Number FA9550-17-1-0195, and by AFOSR MURI on multi-information sources of multi-physics systems under Award Number FA9550-15-1-0038.

References

Averbuch, A., Coifman, R. R., Donoho, D. L., Israeli, M., Shkolnisky, Y., & Sedelnikov, I. (2008). A framework for discrete integral transformations II—the 2D discrete radon transform. *SIAM Journal on Scientific Computing*, *30*(2), 785–803. https://doi.org/10.1137/060650301

Barnett, A. H., Magland, J., & af Klinteberg, L. (2019). A parallel nonuniform fast fourier transform library based on an “exponential of semicircle” kernel. *SIAM Journal on Scientific Computing*, *41*(5), C479–C504. https://doi.org/10.1137/18M120885X

Beylkin, G. (1987). Discrete radon transform. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, *35*(2), 162–172. https://doi.org/10.1109/TASSP.1987.1165108

Brady, M. L. (1998). A fast discrete approximation algorithm for the Radon transform. *SIAM Journal on Computing*, *27*(1), 107–119. https://doi.org/10.1137/S0097539793256673

Bücker, H. M., Seidler, R., Neuhäuser, D., & Beier, T. (2015). The approximate discrete radon transform: A case study in auto-tuning of OpenCL implementations. *2015 IEEE 9th International Symposium on Embedded Multicore/Many-Core Systems-on-Chip*, 219–226. https://doi.org/10.1109/MCSoC.2015.38

Götz, W. A., & Druckmüller, H. J. (1996). A fast digital radon transform: An efficient means for evaluating the hough transform. *Pattern Recognition*, *29*(4), 711–718. https://doi.org/10.1016/0031-3203(95)00057-7

Greengard, L., & Lee, J.-Y. (2004). Accelerating the nonuniform fast fourier transform. *SIAM Review*, *46*(3), 443–454. https://doi.org/10.1137/S003614450343200X

Harris, C. R., Millman, K. J., Walt, S. J. van der, Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E., Taylor, J., Berg, S., Smith, N. J., Kern, R., Picus, M., Hoyer, S., Kerkwijk, M. H. van, Brett, M., Haldane, A., Río, J. F. del, Wiebe, M., Peterson, P., … Oliphant, T. E. (2020). Array programming with NumPy. *Nature*, *585*(7825), 357–362. https://doi.org/10.1038/s41586-020-2649-2

Li, W., Ren, K., & Rim, D. (2023). A range characterization of the single-quadrant ADRT. *Mathematics of Computation*, *92*, 283–306. https://doi.org/10.1090/mcom/3750

Natterer, F. (2001). *The mathematics of computerized tomography*. Society for Industrial; Applied Mathematics. https://doi.org/10.1137/1.9780898719284

Press, W. H. (2006). Discrete Radon transform has an exact, fast inverse and generalizes to operations other than sums along lines. *Proceedings of the National Academy of Sciences*, *103*(51), 19249–19254. https://doi.org/10.1073/pnas.0609228103

Rim, D. (2018). Dimensional splitting of hyperbolic partial differential equations using the radon transform. *SIAM Journal on Scientific Computing*, *40*(6), A4184–A4207. https://doi.org/10.1137/17M1135633

Rim, D. (2020). Exact and fast inversion of the approximate discrete Radon transform from partial data. *Applied Mathematics Letters*, *102*, 106159. https://doi.org/10.1016/j.aml.2019.106159

Walt, S. van der, Schönberger, J. L., Nunez-Iglesias, J., Boulogne, F., Warner, J. D., Yager, N., Gouillart, E., Yu, T., & contributors, the scikit-image. (2014). Scikit-image: Image processing in Python. *PeerJ*, *2*, e453. https://doi.org/10.7717/peerj.453