

¹ EnAlnem: Non-Negative Tensor Factorization for Multiway and Multi-View Data

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Software

- ⁵ [Review](#) 
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Summary

EnAlnem is a Python package for decomposing non-negative multiway arrays and heterogeneous multi-view datasets into rank-1 nonnegative tensors. It extends scikit-learn's NMF framework to support tensor factorization (NTF), robust modeling via Target Polish, and integration of incomplete or noisy views using the Integrated Sources Model (ISM). EnAlnem is designed for scalable, interpretable machine learning in domains like survival analysis, clustering, and visualization.

EnAlnem internally calls scikit-learn's `_update_cdnmf_fast` C function, which performs coordinate descent updates for NMF. This ensures EnAlnem inherits the same computational efficiency as its ancestor.

EnAlnem is released under the MIT License.

Statement of Need

Modern datasets often contain multiple views or modalities, each offering distinct perspectives on the same observations. Traditional matrix factorization techniques struggle to capture shared latent structures across such views, especially when data is noisy or incomplete. EnAlnem addresses this gap by offering:

- ²¹  Non-negative tensor factorization for multiway data ([Cichocki & Phan, 2009](#))
- ²²  ISM for integrating heterogeneous views ([Fogel et al., 2024](#))
- ²³  Robustness to outliers via Target Polish ([Fogel et al., 2025b](#))
- ²⁴  Random completions for missing data (experimental)

These capabilities are critical for applications in health analytics, bioinformatics, and social sciences, where interpretability and robustness are paramount.

EnAlnem has been successfully applied to survival analysis in the CoxNTF framework ([Fogel et al., 2025a](#)), demonstrating its utility for joint clustering and prediction. This application highlights EnAlnem's ability to extract interpretable latent structures from multiway clinical data, supporting both visualization and outcome modeling.

These methods are described in more detail in the Functionality section below.

Functionality

EnAlnem provides a suite of tools for scalable and interpretable decompositions of multiway and multi-view data:

- ³⁵  **Fast HALS for tensors of any order** ([Cichocki & Phan, 2009](#)): Implements Hierarchical Alternating Least Squares (HALS) for efficient non-negative tensor factorization. This
- ³⁶

37 method generalizes NMF to higher-order tensors and is known for its fast convergence
38 under non-negativity constraints.

39 ■ **Tensor-aware NNDSVD Initialization**

40 To support robust initialization for higher-order tensors, EnAlnem extends the classical
41 NNDSVD method using a structured two-step approach:

42 – **Step 1: Matrix-based initialization**

43 The input tensor is unfolded along the axis with the largest dimension, producing a
44 matrix view. NNDSVD is then applied to this matrix to initialize the factor matrix
45 corresponding to that axis via standard NMF.

46 – **Step 2: Sequential SVD unraveling**

47 For the remaining dimensions, and for each component, EnAlnem performs rank-
48 1 SVDs across the tensor's modes in descending order of their dimensionality.
49 This sequential process "unravels" the structure of the tensor, yielding coherent
50 initializations for all factor matrices.

51 This strategy preserves the interpretability and sparsity benefits of NNDSVD while
52 generalizing it to higher-order data structures, making it suitable for tensor decomposition
53 tasks, as illustrated in the following diagram:

54 Inputs include multiway tensors and multi-view matrices. These are processed through Fast
55 HALS, ISM integration, Target Polish, and optional Random Completions. The outputs include
56 factor matrices, latent representations, and relative error metrics.

- 57 ■ **C-level performance via scikit-learn's optimized gradient descent:** EnAlnem leverages
58 scikit-learn's `_update_cdnmf_fast` C function, which implements coordinate descent
59 updates for NMF. This allows EnAlnem to match the speed and scalability of scikit-learn's
60 native routines.

61 A small benchmarking experiment comparing both implementations on a synthetic matrix
62 (10000×1000 , `n_components=10`) yielded:

63 ==== Performance Comparison ====
64 scikit-learn NMF: 5.691 sec | Rel. Error: 0.5979
65 EnAlnem (NMF mode): 5.753 sec | Rel. Error: 0.5979

66 This confirms that EnAlnem achieves identical reconstruction quality while maintaining
67 competitive performance.

68 For full reproducibility, see the `compare_performances` notebook included in the package,
69 which details the benchmarking setup and results.

- 70 ■ **ISM integration for multi-view learning** (Fogel et al., 2024): The Integrated Sources
71 Model (ISM) enables joint factorization of multiple heterogeneous views by learning a
72 shared latent representation. It is particularly effective when views are noisy, incomplete,
73 or measured on different scales.

- 74 ■ **Target Polish for robust factorization** (Fogel et al., 2025b): Offers resistance to outliers
75 and corrupted data through robust loss functions including CIM, Huber, L1, and L21
76 norms. These options allow users to tailor the decomposition to the noise characteristics
77 of their data.

- 78 ■ **Random Completions for missing data:** An experimental feature that performs multiple
79 random imputations followed by ensemble decomposition using ISM, NMF, or averaging.
80 This approach is useful for exploratory analysis of datasets with missing values, though
81 it is disabled by default.

- 82 ■ **Scikit-learn compatible API:** EnAlnem follows the estimator design pattern used in scikit-
83 learn, exposing methods like `fit_transform`, `get_param`, and `set_param`. It inherits

84 scikit-learn's NMF base classes, ensuring consistent parameter validation, input checking,
85 and seamless integration into ML pipelines.

86 EnAInem Workflow

87 The following diagram illustrates the core components of EnAInem and how they interact to
88 process multiway and multi-view data:

89 Inputs include multiway tensors and multi-view matrices. These are processed through Fast
90 HALS, ISM integration, Target Polish, and optional Random Completions. The outputs include
91 factor matrices, latent representations, and relative error metrics.

92 Installation

93 To install EnAInem from source, follow these steps:

94 1. **Clone the GitHub repository into a new directory:** To clone the repository into a local
95 directory of your choice, run the following commands in your terminal:

```
mkdir <clone_directory>
cd <clone_directory>
git clone https://github.com/Advestis/enAInem.git
cd enAInem
```

96 Replace with the path or name of the directory where you want to clone the repository.

97 2. **Install the package:**

```
pip install .
```

98 3. **(Optional) Install development dependencies:**

99 If you plan to contribute or run tests, install the development extras:

```
pip install .[dev]
```

100 4. **Using enAInem within a project with a virtual environment**

101 If you plan to use enAInem within a project that has its own virtual environment, activate
102 the environment and install the package from the local clone:

```
pip install /absolute/path/to/enAInem
```

103 Replace /absolute/path/to/enAInem with the actual path to the cloned repository.

104 **Note:** EnAInem requires **Python 3.11** or higher. Ensure your environment meets
105 this requirement before installation.

106 Usage

107 EnAInem supports tensor input (np.ndarray):

```
import numpy as np
from enainem import EnAInem

# Simulate a 3-way tensor (e.g., patients x genes x timepoints)
X = np.abs(np.random.randn(100, 50, 10)) # ensure non-negativity

# Instantiate and fit the model
model = EnAInem(n_components=5)
result = model.fit_transform(X)
```

```
# Access factor matrices and reconstruction error
A, B, C = result["A"], result["B"], result["C"]
print("Relative error:", result["relative_error"])

108 EnAlnem also supports multi-view input as a list of matrices (list[np.ndarray]):

views = [np.abs(np.random.randn(100, 20)), np.abs(np.random.randn(100, 30))]
model = EnAInem(n_components=4)
result = model.fit_transform(views)
```

109 Handling Missing Data

110 EnAlnem includes an experimental feature for handling missing values via Random Completions.
111 This functionality is disabled by default: any dataset containing missing values (e.g., NaN) will
112 be rejected unless explicitly allowed.

113 To activate this feature, set the `force_all_finite` parameter to `False` when instantiating the
114 `EnAInem` class:

```
from enainem import EnAInem

model = EnAInem(n_components=5, force_all_finite=False)
result = model.fit_transform(X_with_missing_values)
```

115 When enabled, `EnAInem` performs multiple random imputations and integrates the resulting
116 decompositions using ISM, NMF, or averaging. This approach is under active development
117 and should be used with caution.

118 Examples

- 119 ■ `simple_ntf.py`: Demonstrates tensor decomposition with synthetic data
- 120 ■ `simple_ism.py`: Shows ISM integration across noisy views
- 121 ■ `coxntf_demo.ipynb`: Demonstrates joint clustering and survival prediction using
122 `EnAInem` within the CoxNTF framework ([Fogel et al., 2025a](#))

125 Dependencies

- 126 ■ Python 3.11 or higher
- 127 ■ NumPy, SciPy, pandas, scikit-learn, Dask
- 128 ■ Optional: Matplotlib, IPython, pytest

130 Acknowledgements

131 EnAlnem builds on scikit-learn's NMF infrastructure and is supported by Advestis (ForvisMazars
132 R&D). It incorporates algorithms from:

- 133 ■ Fast HALS for NMF/NTF ([Cichocki & Phan, 2009](#))
- 134 ■ Integrated Sources Model ([Fogel et al., 2024](#))
- 135 ■ Target Polish for robust factorization ([Fogel et al., 2025b](#))

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