

# polyCub: An R package for Integration over Polygons

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#### **Software**

■ Review 🗗

■ Repository 🗗

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## Summary

The integral of a continuously differentiable function f(x,y) over a domain  $W \subset \mathbb{R}^2$  can be approximated using an n-point cubature rule of the form

$$\iint_W f(x, y) \, dx \, dy \approx \sum_{i=1}^n w_i f(x_i, y_i) ,$$

i.e., a weighted sum of function values at an appropriate set of nodes. In the special but common case of integration along the axes, i.e.,  $W = (x_l, x_u) \times (y_l, y_u)$ , the domain is rectangular. Several software packages implement numerical integration over such rectangles (or hypercubes in higher dimensions), for example, the Cuba (Hahn, 2005) or cubature (Johnson, 2017) libraries, which are both interfaced from the cubature package (Narasimhan, Johnson, Hahn, Bouvier, & Kiêu, 2018) in R (R Core Team, 2018).

In spatial statistics, however, the domains of interest typically correspond to geographic regions (administrative districts, lakes, etc.), which are described by *polygons*. Solving integrals over such complex domains requires specialized cubature methods, thus the R package polyCub. A simple graphical summary of the purpose of polyCub is given by its logo (see below).

polyCub implements the following methods for numerical integration over polygons:

- General-purpose product Gauss cubature (Sommariva & Vianello, 2007)
- Simple two-dimensional midpoint rule via spatstat (Baddeley & Turner, 2005)
- Adaptive cubature for radially symmetric functions  $f(x,y) = f_r(\|(x-x_0,y-y_0)\|)$  via integration along the polygon boundary (Meyer & Held, 2014, Supplement B)
- Accurate (but slow) integration of the *bivariate Gaussian density* based on polygon triangulation (Abramowitz & Stegun, 1972, Section 26.9, Example 9)

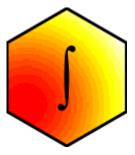


Figure 1: polyCub: cubature over polygonal domains.



## **Usage**

The R package polyCub is released on the Comprehensive R Archive Network (CRAN) and can thus be easily installed using install.packages("polyCub") in R. After that, the basic usage is

```
library("polyCub")
polyCub(polyregion, f)
```

where polyregion is the integration domain and f is the integrand. Details are given in

```
vignette("polyCub")
```

which exemplifies the implemented cubature methods by solving the integral displayed in the package logo.

polyCub is currently used by at least two other R packages: in surveillance, to evaluate the likelihood of self-exciting spatio-temporal point process models for infectious disease spread (Meyer, Held, & Höhle, 2017), and in rase, to integrate bivariate Gaussian densities for phylogeographic analyses (Quintero, Keil, Jetz, & Crawford, 2015).

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