fuzzyclara: Efficient Medoid-based Clustering Algorithms for Large and Fuzzy Data

Maximilian Weigert [1], **Alexander Bauer** [1], **Jana Gauss** [1], and **Asmik Nalmpatian** [1]

1 Statistical Consulting Unit StaBLab, Department of Statistics, LMU Munich, Germany

Summary

Cluster analysis identifies groupings of observations that share similar characteristics. One popular approach are medoid-based methods where each cluster center is represented by one *typical* observation (Kaufman & Rousseeuw, 2005). The R package fuzzyclara makes a wide range of clustering algorithms conveniently available. It covers classical *hard clustering* methods (where one observation belongs to exactly one cluster) and alternative *fuzzy clustering* methods (where each observation is shaped by its partial membership to different clusters), and further implements the option to combine such algorithms with subsampling-based estimation techniques to make the estimation on large data feasible. The package additionally provides convenience functionalities and visualization techniques to cover the whole workflow for real-world clustering applications.

Statement of Need

Partitioning clustering algorithms aim to find reasonable groupings (*clusters*) of a set of observations based on a predefined number of clusters. Medoid-based versions of this strategy build clusters based on *medoids*, i.e. one observation per cluster best representing its typical characteristics. The most prominent representative of medoid-based clustering is the *partitioning around medoids* (PAM) algorithm (Kaufman & Rousseeuw, 2005).

The PAM algorithm, however, has two drawbacks. First, its estimation can become hardly feasible in data situations with thousands of observations, scaling quadratically ($O(n^2)$) in terms of runtime and memory usage. Sampling-based algorithms like CLARA (Kaufman & Rousseeuw, 1986) or CLARANS (Ng & Han, 2002) allow for a more efficient estimation in such situations. Second, PAM is a hard clustering algorithm where each observation is assigned to a single cluster. This assumption often does not resemble reality when observations share characteristics of several *typical* clusters. Such structures are taken into account by *fuzzy clustering* methods, which compute membership scores for each observation to every cluster.

The statistical software R already provides a wide range of packages implementing clustering algorithms for large or fuzzy data. The cluster package (Maechler et al., 2022) contains diverse clustering routines developed by Kaufman & Rousseeuw (2005) including the CLARA algorithm for large data and the FANNY algorithm for fuzzy data. The CLARANS algorithm as an extension of CLARA is implemented in the package qtcat (Klasen, 2022). The package fastkmedoids (Li, 2021) provides computationally more efficient versions of the CLARA and CLARANS algorithms. A variety of medoid-based fuzzy clustering methods are further available in packages vegclust (De Caceres et al., 2010) and fclust (Ferraro et al., 2019).

All above implementations either allow for the application of fuzzy clustering or of subsampling approaches, but not both simultaneously. While the fuzzyclara package includes the option

to perform hard clustering using the classical PAM algorithm and to use all implemented algorithms with any kind of (dis)similarity metric, the package also allows for simultaneously analyzing large and fuzzy data, by combining the CLARA / CLARANS algorithms with the fuzzy-k-medoids algorithm by Krishnapuram et al. (1999). The package further provides routines to cover the whole workflow for real-world clustering applications, including the use of user-defined distance functions.

Combination of fuzzy and CLARA clustering

We combine the CLARA clustering algorithm of Kaufman & Rousseeuw (1986) with the principle of fuzzy clustering. The original CLARA algorithm consists of the following steps, given a predefined number of J clusters:

1. Determination of K random subsamples of the data

2. For each subsample $k = 1, ..., K$:

   (a) Application of PAM clustering on the subsample.

   (b) Assignment of each observation in the complete dataset to the cluster with the closest medoid.

   (c) Computation of the average distance of each observation to its closest clustering medoid as clustering criterion C_p:

   $$C_p = \frac{1}{n} \sum_{i=1}^{n} d_{ij_{min}p},$$ (1)

   where d_{ijp} denotes the distance of observation i to its closest medoid (i.e. observation j_{min}) based on the clustering solution on subsample k.
3. Selection of the optimal set of clusters according to the minimal clustering criterion.

We adapt this scheme to allow for fuzziness by applying the fuzzy-k-medoids algorithm (Krishnapuram et al., 1999) in step 2a. Each observation of the complete dataset is then assigned a membership score to all clusters j:

$$u_{ijp} = \frac{\left(\frac{1}{d_{ijp}}\right)^{\frac{1}{m-1}}}{\sum_{j=1}^{J} \left(\frac{1}{d_{ijp}}\right)^{\frac{1}{m-1}}},$$ (2)

where m controls the degree of fuzziness. The clustering criterion C_p is accordingly defined as the weighted sum of all distances to the medoids of all clusters, with weights according to the membership scores:

$$C_p = \frac{1}{n} \sum_{i=1}^{n} \sum_{k=1}^{K} u_{ijp}^m d_{ijp}.$$ (3)

The optimal cluster solution is the subsample solution which minimizes this average weighted distance. Note that this simplifies to the hard clustering CLARA algorithm when only allowing membership scores of 0 and 1. The implementation of a fuzzy version of the more efficient CLARANS algorithm (which iteratively evaluates random pairs of medoids and non-medoids) (Ng & Han, 2002) follows the same scheme of adaptation.

fuzzyclara further covers the whole clustering workflow, allowing for the interpretation of clustering solutions (e.g. with principal components plots), the analysis of silhouette scores or the determination of the number of clusters.

Application

We demonstrate the functionality by clustering German tourists from the included `travel` dataset. The data originates from an annual cross-sectional study on pleasure travel and comprises 11,871 travelers between 2009 to 2018. Included variables cover the travel year, the annual number of trips, the overall travel expenses and the maximum travel distance.

Due to the large data size of over 10,000 observations, we apply the CLARA algorithm with 20 randomly drawn samples of size 1,000. As tourists typically share characteristics of several tourist types, we use the fuzzy version with a membership exponent of $m = 1.5$. The elbow criterion (see Figure 1) suggests the use of a five cluster solution.
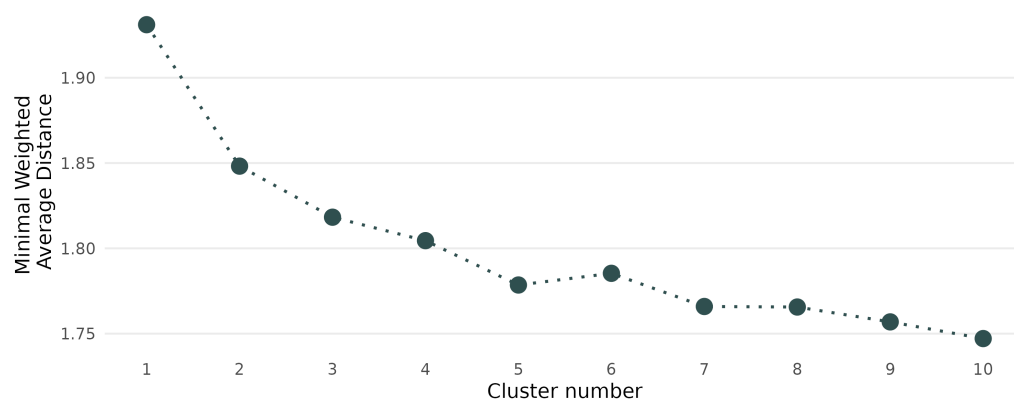

Figure 1: Elbow plot of clustering solutions with 1 to 10 clusters, depicting the minimal average weighted distance.

Figure 2 highlights the characteristics of the different clusters. While clusters 1 and 3 show a tendency to trips of shorter length, lower distance and lower costs, the tourists of cluster 4 tend to travel most frequently and spend the most money for travelling. Less salient differences exist between clusters 2 and 5.
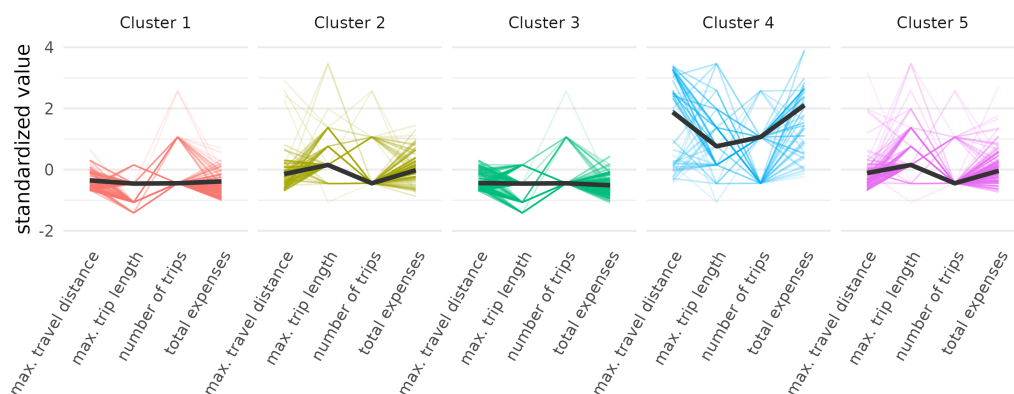

Figure 2: Parallel coordinate plot showing 500 randomly sampled observations, with cluster medoids in bold black lines. The lines' transparency encode the respective observation's membership score, with less transparency encoding a lower degree of fuzziness and thus a clearer membership.

Acknowledgments

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