

PyRoLL - An Extensible OpenSource Framework for Rolling Simulation

Max Weiner^{1*}, Christoph Renzing^{1*}, Max Stirl¹, Matthias Schmidtchen¹, and Ulrich Prahl¹

¹ Institute of Metal Forming, TU Bergakademie Freiberg, Germany * These authors contributed equally.

DOI: [10.21105/joss.06200](https://doi.org/10.21105/joss.06200)

Software

- [Review](#) ↗
- [Repository](#) ↗
- [Archive](#) ↗

Editor: [Philip Cardiff](#) ↗ 

Reviewers:

- [@philipcardiff](#)
- [@rboman](#)

Submitted: 10 October 2023

Published: 17 January 2024

License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](#)).

Summary

Groove rolling is one of the main process routes for the production of metal long products, such as bars, beams, wires and rods. In this process, a workpiece is deformed by two or more rotating rolls to reduce the cross-section achieve a desired cross-section shape. The shape is determined by the contour of the rolls' surfaces, called the groove or calibre. The process is usually accomplished in multiple steps, each called a pass. The industry is currently under heavy pressure to optimize its processes regarding energy consumption while maintaining or increasing product quality. The introduction of new materials and alloys challenges production and technology engineers. In the past decades, numerical simulation tools have become an integral part of process development and maintenance. There are used to estimate forces and engine torques, as well as geometrical and microstructural evolution of the workpiece for a given pass sequence and input workpiece geometry and material. This knowledge is important to design or optimize a process, identify sources of errors, increase product quality or extend the product portfolio.

PyRoLL is an open-source, modular and extensible framework aiming at the numerical simulation of groove rolling processes and accompanying processes, such as heating, cooling and transportation. PyRoLL Core serves as the basis for model and application development by defining needed data structures and solution algorithms and providing a versatile plugin system. Rolling processes are quite a complex issue since a simulation has to regard the mechanical behavior of the workpiece and plant, as well as thermodynamic and chemical processes occurring within the workpiece. The plugin system enables modular simulation setup, where the user can choose from a growing library of state-of-the-art model approaches published in scientific literature that target specific aspects of the process such as material flow, stress state, thermal evolution, microstructure evolution or elastic tool response. Additional and new model approaches can be implemented as plugin packages and used just the same as officially provided ones. By this concept, the ecosystem can grow and thus avoid the need to implement the basic stuff every time, so the user or developer can concentrate on the actual focus of his work.

Statement of need

Established in the late 19th century, mathematical modelling of groove rolling and optimization of used grooves, known as groove or roll pass design, was investigated by a variety of authors. Geuze ([1900](#)), Brovot ([1903](#)) and Mercader ([1924](#)) gave the first set of rules for the design of grooves as well as simple equations for the calculation of material spread in groove rolling. Further investigations regarding this topic were carried out by Siebel ([1932](#)), who introduced the equivalent rectangle approach to derive the material spread in groove rolling from an equivalent flat roll pass. This method has been constantly supplemented by various authors ([Dong et al.](#),

2006, 2008; Lee et al., 2000; Lendl, 1948a, 1948b, 1949; Spittel et al., 1984). Furthermore empirical and analytical models have been developed, focusing on different aspects of groove rolling as spreading (Angott et al., 1986; Dixon & Yuen, 1996; Ekelund, 1927; El-Nikhaily, 1979; Geleji, 1967; Klasen, 1984; Marini, 1941; O. Pawelski et al., 1988; Roux, 1939; Siebel, 1932; Sparling, 1961; Tafel & Sedlaczek, 1925; Vater & Schütza, 1972; Wusatowski, 1969), longitudinal tension (Jäckel, 1991; Lommatzsch, 1991; Nikkilä, 1977; Schulze, 1986; Shokhin & Permyakova, 2015; Treis, 1968; Voigtländer, 1984), strain rate (Hensel & Gehre, 1985), contact area between work roll and rolled stock (Zouhar, 1960, 1966) and power and work needed (Goldhahn, 1981; Hensel & Gehre, 1985; Hensel & Kunzmann, 1977; Zouhar, 1960, 1966). Since 1990, research also focuses on the modelling of microstructure evolution in groove rolling (Blinov, 2004; Cuong, 1991; Krause et al., 2007; Lehnert et al., 1991; Lehnert & Cuong, 1995). Besides the approach of tracing the groove rolling back to an equivalent flat rolling pass, there were also investigations focusing on the modelling of material flow, including the actual shape of the groove (Goldhahn, 1981; Hensel et al., 1987; Hensel & Goldhahn, 1981; Hensel & Spittel, 1978; Kopp et al., 1985; Mauk & Kopp, 1982). To make optimum use of these models, attempts to automatic groove pass design were always an issue of development (Betshammer, 2006; Eriksson, 2004, 2005; Eriksson & Lundberg, 2004; Hensel, 1987; Hensel & Großmann, 1987; Hensel & Wehage, 1988; Körmer, 1987; Krause et al., 2007; Kunzman, 1977; Lambiase & Langella, 2009; Malmgren, 2000; Ogoltcov et al., 2015; Overhagen, Braun, Gamal, et al., 2020; Overhagen, Braun, & Deike, 2020; Overhagen, 2021; Schmidt, 1997; Schmidtchen et al., 2020; Schmidtchen & Prahl, 2021). An exhaustive overview of general tasks in roll pass design and published models is given by Oduguwa & Roy (2006).

The mentioned models and simulation programs use empirical, analytical or semi-analytical approaches to describe the groove rolling process. Aside from these models, there is a huge amount of research focusing on the usage of Finite-Element-Method (FEM) based models for groove rolling and groove pass design. The finite element theory is actively developed since the 1980s for use in groove rolling (Bernhardt & Philipp, 2013; Bontcheva & Petzov, 2005; Glowacki, 2005; Kim et al., 2005; Liu & Yanagimoto, 2002; Macura & Petruska, 1996; Shivpuri & Shin, 1992; Takashima & Nakata, 2014; Vallellano et al., 2008; Yanagimoto et al., 2000). It provides a general approach for complex problems in two or three dimensions and offers simulation results of high accuracy and depth. The main disadvantage is the high computational effort in solving the equation systems, increasing rapidly with the non-linearity of the model equations and the resolution of the solution space, typically in the range from hours to days. Analytical and empirical models, however, do not provide that high accuracy and depth, but are usually computable within fractions of seconds. So they are suitable for interactive design with quick feedback loops and numerical applications, which require large count of evaluations, such as optimization routines.

According to the authors' experience, it is not common in the field of rolling simulation to provide source code and input files alongside journal publications, so the work cannot be reproduced directly and easily. Notable exceptions from this are Alexander (1972) and a textbook by H. Pawelski & Pawelski (2000). Also, the field is characterized by the usage of handcrafted, specialized and not reusable self-made tools, or, by the usage of large commercial FE packages, either specialized or general-purpose. In effect, every research project has to start from scratch to build up a simulation, even if only small partial models have to be investigated. The authors propose a new open and extensible rolling simulation framework to support future research and development. The framework is designed to allow a modular exchange of model approaches describing partial problems of the highly complex groove rolling process. The whole project aims to provide a growing library of model approaches to reflect the state of the art found in scientific literature and make it available to the public. So new research can start on a growing base to explore the actual topic without implementing the same stuff again and again. The primary focus lies hereby on analytical, empirical and semi-empirical approaches, although interfacing with third-party software such as FE or CALPHAD suites, as well as material databases, is possible and planned. PyRoll is deeply integrated with Python's scientific ecosystem, which allows efficient and user-friendly post-processing of simulation

results, as well as application of numerical routines wrapped around the simulation core, such as optimization and fitting of metamodels.

Acknowledgements

The authors thank the following people for their valuable feedback and/or testing efforts:

- Jennifer Mantel (Student, TU Bergakademie Freiberg)
- Richard Pfeifer (Student, TU Bergakademie Freiberg)
- Frank Gerlach (VFUP Riesa e.V.)
- Gerald Rothenbucher (Plansee SE)
- Koos van Putten (SMS Group GmbH)
- Louisa Preis (FNsteel B.V.)
- Christian Overhagen (Universität Duisburg-Essen)
- Tomas Kubina (Liberty a.s.)

The authors thank the following industrial partners for supporting the software development by submitting feedback on usage, simulation result quality and rewarding questions to answer:

- ESF Elbe-Stahlwerke Feralpi GmbH
- FNsteel B.V.
- BGH Edelstahl Freital GmbH
- SMS Group GmbH
- Plansee SE
- Liberty Ostrava a.s.

Conflict of Interest

The authors declare, that they have no conflict of interest.

Research Projects

The software development was or is supported by the following research projects:

- Development and Modelling of Wear of Grooved Rolls for Finishing Blocks (Industrial Funding by ESF Elbe-Stahlwerke Feralpi Riesa GmbH)
- Investigations on Material Flow and Forming Conditions (Industrial Funding by ESF Elbe-Stahlwerke Feralpi Riesa GmbH)
- Design and Numerical Investigations of Different Leader Passes for Rebars (Industrial Funding by ESF Elbe-Stahlwerke Feralpi Riesa GmbH)

The software development will be supported by the following coming research projects:

- “Validierungsförderung” by Sächsische Aufbaubank (SAB) and European Union (EU)
- Implementation and Validation of a Calculation Model for Angular Sections produced by Rolling (RISE DAAD)



Co-funded by
the European Union



This project is co-financed from tax revenues on the basis of the budget adopted by the Saxon State Parliament.

References

- Alexander, J. M. (1972). On the theory of rolling. *Proceedings of the Royal Society A*, 326(1567), 535–563. <https://doi.org/10.1098/rspa.1972.0025>
- Angott, P. B., Berger, B., Neuschütz, E., & Pütz, P. D. (1986). *Untersuchungen des Werkstoffflusses beim Walzen in Kalibern*. Verein Deutscher Eisenhüttenleute (VDEh).
- Bernhardt, R., & Philipp, F.-D. (2013). Modellierung des Kaliberwalzens und der Gefügeausbildung mit modernsten Simulationstools. *Der Kalibreur*, 74.
- Betshammer, J. (2006). *Improved Billet Shape Modeling in Optimization of the Hot Rod and Wire Rolling Process* [Candidate thesis]. Linköpings university.
- Blinov, V. (2004). *Modellierung des Stoffflusses, der Gefüge- und Eigenschaftsentwicklung beim Warmwalzen von Flachprofilen aus Draht ausgewählter Stahlgüten* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Bontcheva, N., & Petzov, G. (2005). Total simulation model of the thermo-mechanical process in shape rolling of steel rods. *Computational Materials Science*, 34(4), 377–388. <https://doi.org/10.1016/j.commatsci.2005.01.009>
- Brovot, A. (1903). *Das Kalibrieren der Walzen*. Verlag von Arthur Felix.
- Cuong, N. D. (1991). *Mathematische Modellierung und Simulierung der Gefügebildungsvorgänge beim Warmwalzen in Kalibern, vorzugsweise beim Walzen von Stabstahl und Draht* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Dixon, A. E., & Yuen, W. Y. D. (1996). A Computationally fast method to model thin strip rolling. *Computational Techniques And Applications: Ctac 95-Proceedings Of The Seventh Biennial Conference*. The Seventh Biennial Conference.
- Dong, Y., Zhang, W., & Song, J. (2006). A New Analytical Model for the Calculation of Mean Roll Radius in Round-oval-round Alloy Bar Rolling. *ISIJ International*, 46(10), 1458–1466. <https://doi.org/10.2355/isijinternational.46.1458>
- Dong, Y., Zhang, W., & Song, J. (2008). An analytical model for the prediction of cross-section profile and mean roll radius in alloy bar rolling. *Journal of University of Science and Technology Beijing, Mineral, Metallurgy, Material*, 15(3), 344–351. [https://doi.org/10.1016/S1005-8850\(08\)60065-1](https://doi.org/10.1016/S1005-8850(08)60065-1)
- Ekelund, S. (1927). Några dynamiska förhållanden vid valsning. *Jernkontorets Annaler*.
- El-Nikhaily, A. E. G. (1979). *Stoffflußmodelle für das Streckkaliberwalzen* [PhD thesis]. RWTH Aachen.
- Eriksson, C. (2004). *Roll Pass Design for improved flexibility and quality in wire rod rolling* [PhD thesis]. Royal Institue of Technology.
- Eriksson, C. (2005). Flexibility and Utilization of Roll Pass Sequences in some Swedish Wire Rod Mills. *Steel Research*, 76(9), 635–643. <https://doi.org/10.1002/srin.200506068>
- Eriksson, C., & Lundberg, S. E. (2004). Working range of roll pass sequences in wire rod rolling with interstand tensions. *Ironmaking & Steelmaking*, 31(1), 81–92. <https://doi.org/10.1179/030192304225011016>
- Geleji, A. (1967). Räumliche Verteilung des Werkstoffflusses und des Walzdruckes im Walzspalt. *Archiv für das Eisenhüttenwesen*, 38(2), 99–103. <https://doi.org/10.1002/srin.196705760>
- Geuze, L. (1900). *Traité théorique et pratique du laminage du fer et de l'acier* (Vol. 1). C. Béranger.

- Glowacki, M. (2005). The mathematical modelling of thermo-mechanical processing of steel during multi-pass shape rolling. *Journal of Materials Processing Technology*, 168(2), 336–343. <https://doi.org/10.1016/j.jmatprotec.2004.12.007>
- Goldhahn, G. (1981). *Formänderungsverteilung sowie Kraft- und Arbeitsbedarf beim Walzen in der Streckkaliberreihe Rund-Oval* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Hensel, A. (1987). Ein neues Rechenverfahren für das Walzen von Feinstahl und Draht. *Neue Hütte*, 32(5).
- Hensel, A., & Gehre, F. (1985). Formänderungsverteilung sowie Kraft und Arbeitsbedarf beim Walzen im Streckkalibern bei höheren Umformgeschwindigkeiten. *Neue Hütte*, 30(1), 12–17.
- Hensel, A., & Goldhahn, G. (1981). Untersuchungen des Werkstoffflusses beim Walzen in Streckkaliberreihe Rund-Oval. *Neue Hütte*, 26(2), 51–55.
- Hensel, A., & Großmann, L. (1987). Untersuchungen des Stoffflusses in der Streckkaliberreihe Quadrat - Oval. *Neue Hütte*, 32(11), 410–412.
- Hensel, A., & Kunzmann, E. (1977). Werkstofffluss und Kräfte beim Walzen in ausgewählten Streckkalibern. *Der Kalibreur*, 26, 3–46.
- Hensel, A., & Spittel, T. (1978). *Kraft- und Arbeitsbedarf bildsamer Formgebungsverfahren*. VEB Deutscher Verlag für Grundstoffindustrie.
- Hensel, A., & Wehage, H. (1988). Rechnergestützte Ermittlung des Temperaturverlaufs in Primärkühlstrecken und beim Walzen mit Zwischengerüstkühlung. *Neue Hütte*, 33(2), 55–57.
- Hensel, A., Wehage, H., & Wehage, J. (1987). Weiterentwicklung eines Rechenmodells auf dem Gebiet des kontinuierlichen Walzens in Streckkaliberreihen. *Neue Hütte*, 32(11), 406–410.
- Jäckel, I. (1991). *Beitrag zur Analyse des Walzens von Vollquerschnitten unter besonderer Berücksichtigung umformwirksamer Längszüge* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Kim, S. Y., Lee, H. W., Min, J. H., & Im, Y. T. (2005). Steady state finite element simulation of bar rolling processes based on rigid-viscoplastic approach. *International Journal for Numerical Methods in Engineering*, 63(11), 1583–1603. <https://doi.org/10.1002/nme.1328>
- Klasen, F. (1984). *Werkstofffluss beim Warmwalzen von Stahl auf geneigter Walzbahn* [PhD thesis]. RWTH Aachen.
- Kopp, R., Helsper, T., Kallabis, H. P., & Osterburg, H. (1985). Modelle zur Berechnung des Stoffflusses für Stabstahl und Formstahlkaliberreihen zur Weiterentwicklung der Kalibrierungstechnik. *Stahl und Eisen*, 105(18), 931–937.
- Körner, A. (1987). *Modellierung von Streckkaliberreihen* [PhD thesis]. TU Bergakademie Freiberg.
- Krause, G., Kawalla, R., Chabbi, L., Urlau, U., Stüber, A., Diederichs, R., & van Hüllen, P. (2007). Simulation of Material Flow, Microstructure and Properties Evolution in Bar and Wire Rolling. *Steel Research International*, 78(10), 745–750. <https://doi.org/10.1002/srin.200706280>
- Kunzman, E. (1977). *Verfahrenoptimierung gezeigt am Beispiel des Streckkaliberwalzens* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Lambiase, F., & Langella, A. (2009). Automated procedure for roll pass design. *Journal of Materials Engineering and Performance*, 18(3), 263–272. <https://doi.org/10.1007/>

s11665-008-9289-2

- Lee, Y., Choi, S., & Kim, Y. H. (2000). Mathematical model and experimental validation of surface profile of a workpiece in round-oval-round pass sequence. *Journal of Materials Processing Technology*, 108(1), 87–96. [https://doi.org/10.1016/S0924-0136\(00\)00734-2](https://doi.org/10.1016/S0924-0136(00)00734-2)
- Lehnert, W., & Cuong, N. D. (1995). Integrated Model for Calculating Parameters of Steel during Rolling Microstructural and Forming in Continuous Mills. *ISIJ International*, 35(9), 1100–1108. <https://doi.org/10.2355/isijinternational.35.1100>
- Lehnert, W., Cuong, N. D., & Zengler, P. (1991). Mathematische und experimentelle Ermittlung der örtlichen Prozess- und Gefügeparameter in der Umformzone beim Warmwalzen. *Neue Hütte*, 36(2), 46–52.
- Lendl, A. E. (1948a). Rolled Bars - Part I - Calculation of Spread between non parallel roll surfaces. *Iron and Steel*, 21(14), 397–402.
- Lendl, A. E. (1948b). Rolled Bars - Part II - Application of Spread Calculation to Pass Design. *Iron and Steel*, 21(14), 601–604.
- Lendl, A. E. (1949). Rolled Bars - Part III - Application of Spread Calculation to Diamond Passes. *Iron and Steel*, 22(12), 499–501.
- Liu, J., & Yanagimoto, J. (2002). Three-dimensional Numerical Analysis of Microstructural Evolution in and after Bar and Shape Rolling Processes. *ISIJ International*, 42(8), 868–875. <https://doi.org/10.2355/isijinternational.42.868>
- Lommatzsch, J. (1991). *Ein Berechnungsmodell zur Beschreibung des Einflusses von Längszügen auf die Walzgutabmessung* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Macura, P., & Petruska, J. (1996). Numerical and experimental simulation of pass rolling. *Journal of Materials Processing Technology*, 60(1-4), 55–60. [https://doi.org/10.1016/0924-0136\(96\)02307-2](https://doi.org/10.1016/0924-0136(96)02307-2)
- Malmgren, N.-G. (2000). Rolling - The computer program library for billet, bar and rod mills. *Rolling Proceedings*, 18.
- Marini, N. (1941). Nuova teoria sulla laminazione. *La Metallurgia Italiana*, 292–309.
- Mauk, P. J., & Kopp, R. (1982). Breitung beim Warmwalzen. *Der Kalibreur*, 37.
- Mercader, E. (1924). Berechnung der Streckovalkaliber. *Stahl und Eisen*, 44(14), 46–48.
- Nikkilä, K. (1977). *On the Effects of Front and Back Tensions on Wire Rod Rolling* [PhD thesis]. Helsinki University of Technology.
- Oduguwa, V., & Roy, R. (2006). A Review of Rolling System Design Optimisation. *International Journal of Machine Tools and Manufacture*, 46(7-8), 912–928. <https://doi.org/10.1016/j.ijmachtools.2005.07.023>
- Ogoltcov, A., Sokolov, D., Sokolov, S., & Vasilyev, A. (2015). STAN 2000: Computer Model For Simulation of Hot Rolling On Mill 2000 Of Severstal. *Conference Proceedings. Metal*. ISBN: 978-80-87294-58-1
- Overhagen, C. (2021). A computational method for pass design of the four-roll rolling process for sizing of round sections. *24th International Conference on Material Forming. ESAFORM 21*. <https://doi.org/10.25518/esaform21.3987>
- Overhagen, C., Braun, R., & Deike, R. (2020). Analysis of elastic rolling stand deformation and interstand tension effects on section faults of hot rolled wire rod and bars. *Tm - Technisches Messen*, 87(5), 343–348. <https://doi.org/10.25518/esaform21.3987>
- Overhagen, C., Braun, R., Gamal, O., Jobst, A., & Radschun, M. (2020). Effizientere Walzprozesse. *Stahl und Eisen*, 9, 57–60.

- Pawelski, H., & Pawelski, O. (2000). *Technische Plastomechanik* (Vol. 1). Verlag Stahleisen. ISBN: 978-3-514-00659-1
- Pawelski, O., Rasp, W., & Klasen, F. (1988). Breitung beim Warmwalzen auf geneigter Bahn. *steel research international*, 59(2), 60–67. <https://doi.org/10.1002/srin.198801607>
- Roux, M. J. (1939). Étude sur le phénomène de l'élargissement dans les laminoirs. *Rev. Metall*, 36(6), 257–270. <https://doi.org/10.1051/metal/193936060257>
- Schmidt, B. (1997). *Entwicklung und Erprobung einer Softwarelösung für die mathematische Simulation des Walzens von Langprodukten* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Schmidtchen, M., Hoffmann, F., & Prah, U. (2020). Development of Fast Simulation Tools with Higher Modelling Depth for Improved Description of Local Material Flow, Stress State and Microstructure in Breakdown and Shaping Grooves. *Der Kalibreur*, 80, 27–40.
- Schmidtchen, M., & Prah, U. (2021). Status of developments in (fast simulation assisted) groove pass design for asymmetric profiles. *Der Kalibreur*, 81, 18.
- Schulze, B. (1986). *Beitrag zur Analyse des Warmwalzens von Massivquerschnitten mit formänderungswirksamen Längszügen* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Shivpuri, R., & Shin, W. (1992). A Methodology for roll pass optimization for multi-pass rolling. *Int. J. Mach. Tools Manufact.*, 32(5), 671–683. [https://doi.org/10.1016/0890-6955\(92\)90022-9](https://doi.org/10.1016/0890-6955(92)90022-9)
- Shokhin, V. V., & Permyakova, O. V. (2015). The Study of Continuous Rolling Mill Interstand Tension Inferential Control Systems. *Procedia Engineering*, 129, 231–238. <https://doi.org/10.1016/j.proeng.2015.12.038>
- Siebel, E. (1932). *Die Formgebung im bildsamen Zustande*. Verlag Stahleisen.
- Sparling, L. G. M. (1961). Formula for “Spread” in Hot Flat Rolling. *Proceedings of the Institution of Mechanical Engineers*, 175(1), 604–640. https://doi.org/10.1243/PIME_PROC_1961_175_043_02
- Spittel, M., Eberlein, L., & Spittel, K. (1984). Berechnung des Umformgrads bei irregulären Kalibrierungen. *Neue Hütte*, 29(7), 259–262.
- Tafel, W., & Sedlacek, H. (1925). Das Breiten beim Walzen. *Stahl und Eisen*, 45(6), 3.
- Takashima, Y., & Nakata, N. (2014). T-bar Rolling Process with Universal and Edger Mills. *Procedia Engineering*, 81, 191–196. <https://doi.org/10.1016/j.proeng.2014.09.149>
- Treis, H. (1968). *Ermittlung der Formänderungsverhältnisse beim Warmwalzen auf der Flachbahn ohne und mit äusserem Langszug* [PhD thesis]. RWTH Aachen.
- Vallellano, C., Cabanillas, P. A., & García-Lomas, F. J. (2008). Analysis of deformations and stresses in flat rolling of wire. *Journal of Materials Processing Technology*, 195(1-3), 63–71. <https://doi.org/10.1016/j.jmatprotec.2007.04.124>
- Vater, M., & Schütza, A. (1972). *Untersuchungen über den Formänderungsverlauf beim Warmwalzen von Stahl in einfachen Streckkalibern in Abhängigkeit von der Kaliberform und den Abmessungsverhältnissen*. VS Verlag für Sozialwissenschaften. <https://doi.org/10.1007/978-3-322-88346-9>
- Voigtländer, J. (1984). *Untersuchungen zum Einfluss von Längszugkräften auf den Umformprozess beim Stabwalzen* [PhD thesis]. Technische Universität Bergakademie Freiberg.
- Wusatowski, Z. (1969). *Fundamentals of Rolling* (Vol. 1). Pergamon Press. ISBN: 978-0-08-012276-2

- Yanagimoto, J., Ito, T., & Liu, J. (2000). FE-based Analysis for the Microstructure Evolution in Hot Bar Rolling. *ISIJ International*, 40(1), 65–70. <https://doi.org/10.2355/isijinternational.40.65>
- Zouhar, G. (1960). *Umformungskräfte beim Walzen in Streckkaliberreihen* (Vol. 1). Akademie Verlag Berlin.
- Zouhar, G. (1966). *Grundlagen der Bildsamen Formgebung* (technical report No. 1). Technische Universität Bergakademie Freiberg.