

¹ conjugate-models: Conjugate Models in Python

² William Dean  ¹

³ 1 Independent Researcher

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Software

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⁴ Summary

⁵ conjugate-models is a modern Python package for Bayesian conjugate inference that prioritizes a clean, idiomatic API and seamless integration with widely used Python data analysis libraries. It implements the conjugate likelihood-prior pairs cataloged in Fink's compendium and Wikipedia's conjugate prior table, making rigorous Bayesian updating, exploration, and visualization accessible for practitioners, educators, and researchers. Comprehensive documentation, interactive examples, and an online Distribution Explorer further support flexible application and learning.

¹² Statement of need

¹³ Bayesian inference with conjugate priors offers a tractable and interpretable approach to updating distributions in light of new evidence. While general-purpose probabilistic programming frameworks exist, they can introduce significant cognitive and computational overhead for common conjugate models. The conjugate-models package is designed to provide efficient, intuitive, and didactic support for Bayesian conjugate workflows across statistics, data science, and education, allowing direct use with array-like objects from libraries such as numpy ([Harris et al., 2020](#)), pandas ([The pandas development team, n.d.](#)), and polars ([Polars Contributors, 2024](#)). The project also provides live interactive documentation and an [online Distribution Explorer](#) for real-time model investigation.

²² A prior distribution is conjugate to a likelihood when the posterior remains in the same distribution family after observing data ([Raiffa & Schlaifer, 1961](#)). Conjugate priors provide closed-form posterior updates and posterior predictive distributions, eliminating the need for numerical integration ([Fink, 1997](#)). Because these updates are analytic rather than iterative, posterior computation is instantaneous regardless of data size—enabling real-time interactive exploration and rapid model iteration. conjugate-models implements the conjugate pairs cataloged in Fink's compendium ([Fink, 1997](#)) and Wikipedia's conjugate prior table ([Wikipedia contributors, 2026](#)). The complete list of supported models is maintained at [the online documentation](#).

³¹ Python lacked a dedicated, user-friendly package making Bayesian conjugate inference accessible, idiomatic, and didactically powerful—especially one offering robust integration with common data tooling and interactive resources for teaching and exploratory work. Existing educational tools for Bayesian statistics often lack interactivity or require complex setup, making the intuitive nature of conjugate priors difficult to convey to students and practitioners new to Bayesian methods.

³⁷ Features & API Overview

³⁸ conjugate-models provides an intuitive, pipeable API compatible with numpy arrays ([Harris et al., 2020](#)), pandas DataFrames/Series ([The pandas development team, n.d.](#)), polars DataFrames

⁴⁰ ([Polars Contributors, 2024](#)) (for element-wise operations), and general numerical types. The
⁴¹ package includes vectorized and indexable operations for batch and multi-arm inference, built-in
⁴² plotting for posterior, prior, and predictive distributions, and connection to scipy distributions
⁴³ for interoperability ([Virtanen et al., 2020](#)).

⁴⁴ A typical workflow follows a consistent pattern:

```
from conjugate.distributions import SomeDistribution
from conjugate.helpers import some_model_inputs
from conjugate.models import some_model

observed_data = ...

prior: SomeDistribution = ...
posterior: SomeDistribution = some_model(
    **some_model_inputs(observed_data),
    prior=prior,
)
```

⁴⁵ Example Usage

⁴⁶ Sequential Bayesian Updates

⁴⁷ The package naturally supports sequential Bayesian learning, where each posterior becomes
⁴⁸ the next prior. For example, with a binomial model and beta prior, users can iteratively update
⁴⁹ beliefs as new trial data arrives, with each posterior distribution serving as the prior for the
⁵⁰ next update. This enables real-time learning applications where beliefs continuously evolve
⁵¹ with incoming evidence. A complete worked example of sequential updating with beta-binomial
⁵² conjugacy is available in the [online documentation](#).

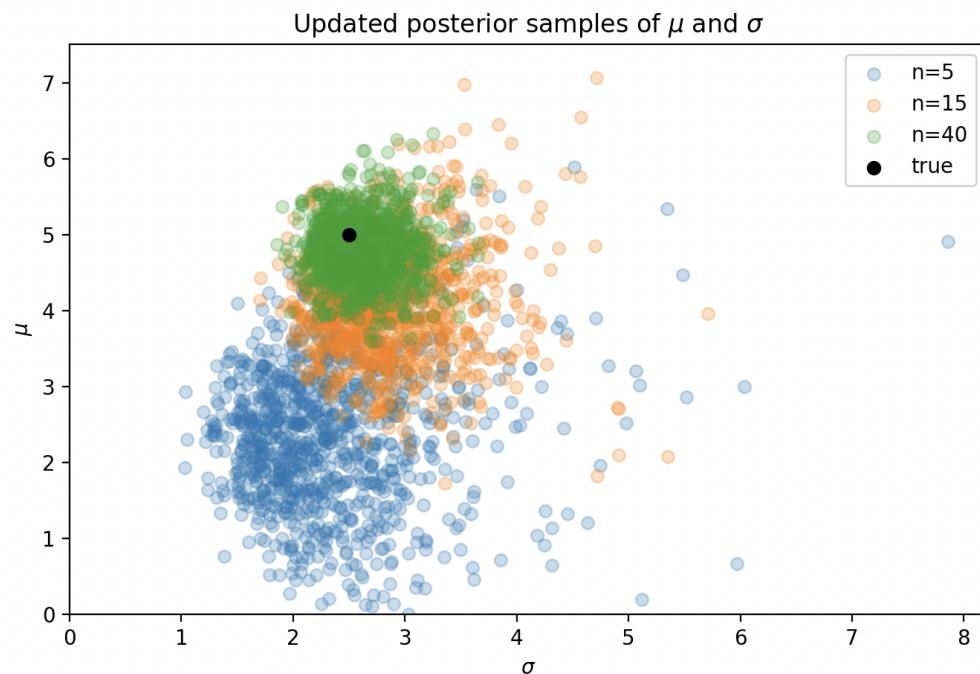
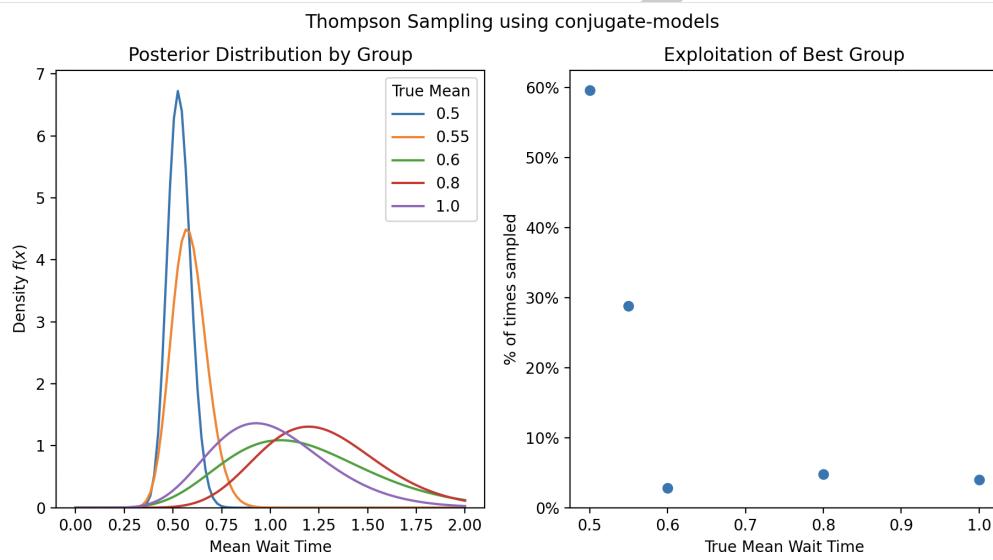


Figure 1: Sequential Bayesian updates showing prior, intermediate posterior, and final posterior distributions. Each update incorporates new evidence while maintaining uncertainty.

53 Thompson Sampling for Minimizing Wait Times

54 Thompson sampling is effective for exploration-exploitation problems where the goal is
 55 optimization. The package's vectorized operations and scipy integration enable sophisticated
 56 applications such as multi-armed bandit problems, where posterior samples from conjugate
 57 updates guide exploration-exploitation decisions across multiple arms simultaneously. For
 58 instance, exponential-gamma conjugate pairs can model wait times across different service
 59 options, with Thompson sampling using posterior rate samples to balance exploration
 60 of uncertain options with exploitation of promising ones. A complete implementation
 61 demonstrating Thompson sampling with exponential-gamma conjugate updates for wait time
 62 minimization is available in the [online documentation](#).



53 **Figure 2:** Thompson sampling results showing posterior distributions and exploitation rates. The algorithm
 54 successfully identifies and favors the group with the lowest wait time (highest rate), demonstrating
 55 effective exploration-exploitation balance.

63 Related Work

64 Several Python packages address aspects of Bayesian inference. ArviZ ([Kumar et al., 2019](#))
 65 excels at posterior analysis and visualization but focuses on MCMC/variational inference
 66 outputs rather than conjugate models. Bambi ([Capretto et al., 2022](#)) provides a high-level
 67 interface for Bayesian linear models via PyMC but targets more complex model specifications.
 68 scikit-learn ([Pedregosa et al., 2011](#)) includes some Bayesian methods but emphasizes frequentist
 69 machine learning. General probabilistic programming languages like PyMC ([Abril-Pla et al.,
 70 2023](#)) and Stan ([Stan Development Team, 2025](#)) offer comprehensive modeling capabilities
 71 but introduce substantial overhead for simple conjugate updates.
 72 conjugate-models distinguishes itself by wrapping `scipy.stats` distributions with conjugate
 73 update semantics, plotting interfaces, and seamless integration into standard Bayesian workflows
 74 ([Gelman et al., 2020](#)). Its focused scope enables immediate posterior computation without
 75 MCMC or optimization, making it ideal for interactive exploration, education, and rapid
 76 prototyping of conjugate models.

77 Limitations

78 This package specifically targets conjugate prior-likelihood pairs, so non-conjugate models
79 require other tools like PyMC ([Abril-Pla et al., 2023](#)) or Stan ([Stan Development Team, 2025](#)). Model updates operate on sufficient statistics rather than raw data, requiring users to
80 compute summary statistics beforehand. Distribution and parameter names follow established
81 conventions from statistical literature ([Fink, 1997](#)), which may differ from other packages.
82 Community support is available through GitHub Issues and Discussions.

84 Software Design

85 conjugate-models adopts a distribution-centric design where each probability distribution is a
86 first-class object wrapping `scipy.stats` distributions while adding conjugate-specific functionality.
87 Key design decisions include:

88 *Compositional API*: Rather than monolithic model classes, the package separates distributions
89 (`conjugate.distributions`) from update logic (`conjugate.models`), allowing users to
90 compose workflows naturally. This mirrors the mathematical structure where conjugate
91 updates transform one distribution into another of the same family.

92 *Scipy Integration*: The `dist` property on each distribution class provides direct access to
93 `scipy.stats` objects, enabling interoperability with the broader scientific Python ecosystem
94 without reimplementing standard functionality like PDF computation, sampling, and statistical
95 methods.

96 *Vectorized Operations*: Parameters accept array-like inputs (numpy arrays, pandas/polars
97 columns), enabling batch inference for multi-arm problems like Thompson sampling without
98 explicit loops. This design choice prioritizes performance for real-world applications involving
99 multiple simultaneous models.

100 *Mixin Architecture*: Plotting capabilities are added via mixins (e.g., `ContinuousPlotDistMixin`,
101 `DiscretePlotMixin`), keeping core distribution classes focused while enabling rich visualization.
102 The `SliceMixin` provides indexing support for vectorized parameters.

103 *Helper Function Design*: The helpers module provides functions to extract sufficient statistics
104 from raw observational data, bridging the gap between real-world datasets and the mathematical
105 abstractions required for conjugate updates.

106 These design choices prioritize clarity and composability over abstraction, making the
107 mathematical structure of conjugate inference explicit in the code while maintaining
108 compatibility with the broader scientific Python ecosystem.

109 Research Impact Statement

110 conjugate-models addresses a specific gap in the Python ecosystem for lightweight, immediate
111 Bayesian inference without MCMC overhead. Evidence of research impact includes:

112 *Community Adoption*: The package is available on PyPI with sustained development over
113 2+ years (270+ commits since June 2023), comprehensive test coverage (94%), and active
114 maintenance demonstrated through regular releases and bug fixes.

115 *Educational Value*: The package's didactic design supports teaching Bayesian concepts through
116 immediate, interactive feedback. The live Distribution Explorer provides hands-on learning
117 without installation requirements, making Bayesian inference more accessible to students and
118 practitioners.

119 *Documentation and Examples*: Comprehensive documentation with 15+ worked examples
120 demonstrates real-world applications from A/B testing to Thompson sampling, supporting

121 both educational use and practical implementation.

122 *Technical Contributions:* The package implements the complete catalog of conjugate pairs
123 from Fink's compendium ([Fink, 1997](#)) with modern Python practices, providing a reference
124 implementation for conjugate Bayesian inference that was previously unavailable in a single,
125 cohesive package.

126 *Integration Capabilities:* Seamless compatibility with numpy, pandas, polars, and scipy enables
127 integration into existing data science workflows without requiring users to learn new data
128 structures or abandon familiar tools.

129 The package serves researchers, educators, and practitioners who need rapid, interpretable
130 Bayesian inference for conjugate models, complementing rather than competing with general-
131 purpose probabilistic programming frameworks.

132 Acknowledgments

133 We thank the scientific Python community, particularly the maintainers and contributors of
134 NumPy ([Harris et al., 2020](#)), SciPy ([Virtanen et al., 2020](#)), and Matplotlib, whose foundational
135 libraries make this package possible. The examples showcase integration with pandas ([The](#)
136 [pandas development team, n.d.](#)), Polars ([Polars Contributors, 2024](#)), and PyMC ([Abril-Pla et](#)
137 [al., 2023](#)), demonstrating the collaborative spirit of the open-source ecosystem.

138 AI Usage Disclosure

139 Generative AI tools were used during the development of this software and the preparation of
140 this manuscript. All AI-assisted outputs were reviewed, edited, and validated by the human
141 author, who made all core design decisions.

142 *Software Development:* GitHub Copilot Pro was used via the GitHub web interface for code
143 suggestions and code review assistance. The majority of the codebase was written directly by
144 the author without AI assistance.

145 *Documentation:* AI tools assisted with portions of the package documentation. The majority
146 of documentation was written directly by the author.

147 *Paper Authoring:* opencode (version 1.0.220) with Claude Opus 4.5 was used to:
148 - Gather and organize information from existing documentation and the codebase during paper drafting
149 - Simulate peer review feedback via agent prompts to identify areas for improvement
150 - Iterate on paper structure, content, and clarity

151 *Human Oversight:* The author made all architectural and design decisions for the software,
152 determined the scientific content and framing of the paper, and reviewed and validated all
153 AI-assisted outputs before inclusion. No AI-generated code or text was included without human
154 verification and editing.

155 Availability

156 conjugate-models is available on PyPI: pip install conjugate-models. The package is
157 open-source under the MIT license at <https://github.com/williambdan/conjugate>, with
158 contribution guidelines at [CONTRIBUTING.md](#). Live documentation and an interactive
159 Distribution Explorer are available at <https://williambdan.github.io/conjugate/>.

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