

¹ macroframe-forecast: Smooth and ² Constraint-Consistent Forecasting

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Software

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⁵ Summary

⁶ The Python package macroframe-forecast generates forecasts that are both temporally
⁷ smooth and consistent with user-specified constraints. The framework consists of model
⁸ estimation and reconciliation: first, unconstrained forecasts are produced using user-specified
⁹ statistical or machine learning models; second, these forecasts are adjusted to satisfy linear
¹⁰ equality and inequality constraints while minimizing volatility over time in the forecast horizon.
¹¹ The package offers a user-friendly interface for specifying constraints as strings and supports
¹² advanced customization of weights and smoothing parameters. An example - forecasting U.S.
¹³ GDP with a terminal GDP growth rate target - illustrates the usage.

Statement of need

¹⁵ In forecasting economic time series, statistical models often need to be supplemented with
¹⁶ procedures that impose constraints while preserving smoothness over time. For example, GDP
¹⁷ forecasts generated using models such as autoregressions or decision trees may not align with
¹⁸ the long-term growth rates anticipated by forecasters. In such cases, forecasters aim to adjust
¹⁹ the time series so that it converges smoothly to the desired long-term growth path. However,
²⁰ ad hoc constraint imposition, such as manually altering only the terminal value in a long time
²¹ series, can introduce undesirable discontinuities between the penultimate and terminal values.
²² Other such examples in economic forecasting can be found in Ando (2024) and Ando & Kim
²³ (2023).

²⁴ Systematically imposing constraints while retaining smoothness is important but challenging.
²⁵ Constraints often stem from accounting identities and expert judgment, making their
²⁶ incorporation essential for internal consistency. Smoothness is equally critical, as optimal
²⁷ forecasts typically exhibit less volatility than historical data, as in random walk. Achieving both
²⁸ objectives manually is resource-intensive, especially when dealing with numerous variables and
²⁹ constraints, raising the question of how to systematically impose constraints and smoothness.

³⁰ Existing packages in R and Python assist forecast reconciliation and smoothing separately
³¹ but not jointly. For instance, the hts (Hyndman et al., 2021) and FoReco (Girolimetto & Di
³² Fonzo, 2023) packages in R support reconciliation, but the reconciled forecast may not be
³³ smooth over time. This is also the case for hierarchicalforecast (Olivares et al., 2024)
³⁴ package in Python. On the other hand, packages, such as smooth (Svetunkov, 2024) and
³⁵ forecast (Hyndman et al., 2024) for R and statsmodels (Seabold & Perktold, 2010) for
³⁶ Python, provide methods to generate smooth forecasts but do not have the functionality to
³⁷ impose constraints.

³⁸ To our knowledge, no package supports the simultaneous application of both reconciliation and
³⁹ smoothing, and this is the gap that macroframe-forecast attempts to fill. A more detailed
⁴⁰ explanation of macroframe-forecast can be found in Ando et al. (2025).

41 State of the field

42 Several tools support macroeconomic forecasting, but none let users produce multiple smooth
43 forecasts that also satisfy various constraints. General-purpose econometric packages like
44 Python's statsmodels ([Seabold & Perktold, 2010](#)) and R's smooth ([Svetunkov, 2024](#)) offer
45 flexible time-series and multivariate models (ARIMA, VAR, VECM), yet enforcing accounting
46 identities or judgmental constraints remains largely manual and ad hoc.

47 Existing reconciliation packages in R and Python — hts ([Hyndman et al., 2021](#)), FoReco
48 ([Girolimetto & Di Fonzo, 2023](#)), hierarchicalforecast ([Olivares et al., 2024](#)) — can impose
49 constraints, but the resulting reconciled series often display unnatural kinks that are difficult to
50 justify. Enforcing constraints typically disrupts smoothness, while smoothing breaks constraints,
51 meaning both properties must be ensured simultaneously, requiring a dedicated theory and a
52 package that implements it.

53 `macroframe-forecast` was developed to fill this gap. It begins with any unconstrained forecast
54 — whether from statistical models in `sktime` ([Löning et al., 2019](#)), expert judgment, or
55 hybrid methods—and systematically enforces high-dimensional accounting identities, inequality
56 constraints, and long-run conditions while preserving temporal smoothness. Users can specify
57 constraints in human-readable strings, which the package automatically converts into the
58 appropriate matrix form, reducing misspecification risk. In this way, `macroframe-forecast`
59 offers a unique scholarly and practical contribution, complementing rather than competing
60 with existing forecasting or reconciliation tools.

61 Software Design

62 The design of `macroframe-forecast` reflects a trade-off between flexibility, transparency, and
63 computational efficiency. The package adopts a two-step architecture - first generating
64 unconstrained forecasts, then adjusting them through reconciliation and smoothing - which
65 clearly separates model estimation from constraint enforcement. To give users flexibility in
66 choosing forecasting method most suitable for their specific context, we deliberately designed
67 the unconstrained forecasting step to explicitly support any forecasting model implemented in
68 `sktime` ([Löning et al., 2019](#)) without modifying the reconciliation logic.

69 The second step is formulated as a single quadratic programming problem with an explicit
70 objective function and clearly defined equality and inequality constraints. This approach avoids
71 ad hoc adjustments such as manually applying smoothing after constraints are imposed. Instead,
72 forecast accuracy, smoothness over time, and consistency with constraints are handled jointly
73 within one optimization problem. Constraints are specified using readable string expressions,
74 reducing the need for users to work directly with matrices as in existing packages.

75 Although existing packages support forecast reconciliation or smoothing individually, there has
76 not been a package that achieves both. Incorporating smoothness penalties into reconciliation-
77 focused packages, or adding general constraint handling to smoothing libraries, would have
78 required substantial structural changes. Building a new package was therefore necessary to
79 implement a transparent, theory-consistent framework that jointly enforces smoothness and
80 constraints while remaining easy to use for applied macroeconomic forecasting.

81 Research Impact Statement

82 `macroframe-forecast` enables economic policy makers to produce multivariate macroeconomic
83 forecasts that are both smooth over time and internally consistent with accounting identities
84 through an intuitive interface. While existing reconciliation tools such as hts ([Hyndman et al.,
85 2021](#)) and FoReco ([Girolimetto & Di Fonzo, 2023](#)) enforce accounting consistency, they often
86 generate unrealistic kinks in forecast paths, limiting their usefulness in policy settings. Such

87 artifacts are difficult for policy makers to justify publicly and weaken the credibility of forecasts
 88 when communicated to the media.

89 In the absence of a systematic solution, macroframework forecasting has typically relied
 90 on manual adjustments in spreadsheet environments, requiring substantial staff time and
 91 limiting scalability and transparency. `macroframe-forecast` is the first open-source package to
 92 jointly ensure smoothness and accounting coherence, allowing experts to incorporate judgment
 93 without sacrificing statistical rigor. This significantly lowers the cost of producing explainable,
 94 high-dimensional forecasts and improves institutional forecasting capacity.

95 Community readiness is demonstrated through comprehensive documentation, reproducible
 96 example workflows, and a permissive open-source license that encourages reuse and extension.
 97 As macroeconomic forecasting increasingly combines expert judgment with data-driven
 98 methods, `macroframe-forecast` provides an infrastructure with strong potential for adoption
 99 and downstream impact across economic policy institutions.

100 AI Usage Disclosure

101 Generative AI tools (Microsoft Copilot, GitHub Copilot) were used to assist the development
 102 of this package, the writing of this manuscript, and the preparation of supporting materials.
 103 The majority of the work was produced by the authors, and the usage of AI tools was to
 104 facilitate code debugging, generate potential solutions, fix typos and assist with other minor
 105 tasks (e.g. formatting). All AI-generated inputs were reviewed and edited by the authors before
 106 incorporation in the code/paper.

107 Method

108 The framework consists of two steps, where the first step provides users with a flexible choice
 109 of forecasting models, and the second step allows users to adjust the first-step forecasts so
 110 that the forecasts are smooth over time and satisfy various constraints, such as accounting
 111 identities and pre-specified targets. For the theoretical properties, see Ando (2024).

112 Suppose there are M time series to forecast, and if we stack the forecast horizon of all the
 113 time series, the size is N . The first-step forecast $\bar{y} \in \mathbb{R}^N$ can be generated using any model in
 114 the `sktime` (Löning et al., 2019) python package. The second-step forecast \tilde{y} is generated by
 115 solving the following quadratic programming problem

$$\tilde{y} = \arg \min_{y \in \mathbb{R}^N} (y - \bar{y})^\top W^{-1} (y - \bar{y}) + y^\top \Phi y \quad \text{s.t.} \quad C_{\text{eq}} y - d_{\text{eq}} = 0, \quad C_{\text{ineq}} y - d_{\text{ineq}} \leq 0.$$

116 The first term penalizes deviations from the first-step forecast \bar{y} , weighted by W^{-1} , where
 117 W is an estimator of the first step forecast error covariance matrix. It ensures that accurate
 118 forecasts are adjusted less than less accurate ones. The second term enforces smoothness
 119 via Φ , a block-diagonal matrix built from a degenerate penta-diagonal matrix used in the
 120 calculation of the Hodrick-Prescott filter (Hodrick & Prescott, 1997). $C_{\text{eq}} y - d_{\text{eq}} = 0$ refers
 121 to equality constraints, and $C_{\text{ineq}} y - d_{\text{ineq}} \leq 0$ refers to inequality constraints.

122 Weight matrix selection

123 By default, W is estimated using time series cross validation and Oracle Approximating
 124 Shrinkage (OAS) (Chen et al., 2010), which shrinks the sample covariance toward a scalar. An
 125 alternative is OAS with diagonal target (Ando & Xiao, 2023), which is robust when variables
 126 differ in scale. Users can also specify identity weighting or custom matrices.

¹²⁷ **Smoothness parameters**
¹²⁸ The smoothness matrix Φ is defined as

$$\Phi = \begin{bmatrix} \lambda_1 F_1 & & \\ & \ddots & \\ & & \lambda_M F_M \end{bmatrix}, \quad F_i = \begin{bmatrix} 1 & -2 & 1 & 0 \\ -2 & 5 & -4 & 1 \\ 1 & -4 & 6 & -4 \\ 0 & 1 & -4 & 6 \\ & \ddots & \ddots & \ddots & \ddots & 0 \\ & & 6 & -4 & 1 & 0 \\ & & -4 & 6 & -4 & 1 \\ & & 1 & -4 & 5 & -2 \\ & & 0 & 1 & -2 & 1 \end{bmatrix}, \quad \lambda_i = \frac{\lambda_i^*}{\sigma_i^2},$$

¹²⁹ where $i = 1, \dots, M$. Default parameter values λ_i^* follow HP filter conventions (e.g., 1600
¹³⁰ for quarterly data). It is then scaled by the first step forecast error variance σ_i^2 , obtained
¹³¹ from the diagonal elements of the weight matrix W for each time series i , to ensure unit
¹³² invariance. Setting $\lambda_i^* = 0$ disables smoothing for time series i , reducing the problem to pure
¹³³ reconciliation.

¹³⁴ **An example: single-variable GDP forecast**
¹³⁵ Using U.S. nominal GDP data (1950-2024), we forecast the values for 2025-2030 under the
¹³⁶ constraint that 2030 growth rate equals 4 percent. When the column name of the variable to
¹³⁷ be forecasted is GDP, the constraint is expressed as $GDP_{2030} - 1.04 * GDP_{2029} = 0$. The
¹³⁸ MFF class generates the first-step forecast via the default pipeline and reconciles them to satisfy
¹³⁹ the constraint while smoothing the trajectory. Assuming that the user has read the GDP data
¹⁴⁰ as a pandas dataframe named `df0` with index year and GDP, the forecasts (both first and
¹⁴¹ second-step) can be generated as follows:

```
from macroframe_forecast import MFF
from macroframe_forecast.examples import generate_example_GDP_df

df0 = generate_example_GDP_df()
m = MFF(df0, equality_constraints=["GDP_2030 - 1.04 * GDP_2029"])
m.fit()
```

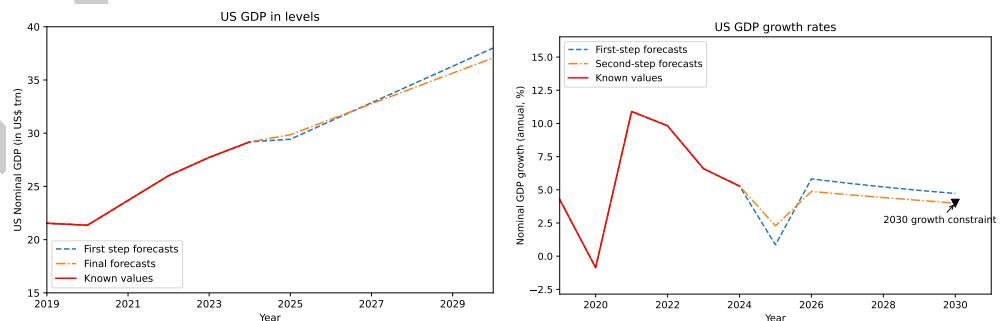


Figure 1: US GDP forecasts: (a) levels and (b) growth.

¹⁴² Figure 1: Forecasts of U.S. nominal GDP, 2025-030, under the 4 percent growth constraint in
¹⁴³ 2030.

¹⁴⁴ Source: IMF April 2025 World Economic Outlook database and authors' calculations.

¹⁴⁵ Note: Panel (a) shows the forecast of annual US GDP in trillion USD. Panel (b) shows the
¹⁴⁶ growth rates computed from the level forecasts.

¹⁴⁷ The second-step forecast can also be customized by providing exogenously defined first-step
¹⁴⁸ forecast, weight matrix, or the smoothness parameters. These can be achieved by using the
¹⁴⁹ function Reconciliation available in the package.

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