

¹ gwsnr: A python package for efficient signal-to-noise calculation of gravitational-waves

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DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

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Editor: 

Submitted: 30 October 2025

Published: unpublished

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Summary

Gravitational waves (GWs), ripples in spacetime predicted by Einstein's theory of General Relativity, have revolutionized astrophysics since their first detection in 2015. Emitted by cataclysmic events such as mergers of binary black holes (BBHs), binary neutron stars (BNSs), and black hole-neutron star pairs (BH-NSs), these waves provide a unique window into the cosmos.

A central quantity in GW analysis is the Signal-to-Noise Ratio (SNR), which measures the strength of a GW signal relative to the background noise in detectors such as LIGO (The LIGO Scientific Collaboration et al. (2015), B. P. Abbott et al. (2020), Buikema et al. (2020)), Virgo (F. Acernese et al. (2014), F. Acernese et al. (2019)), and KAGRA (Akutsu et al. (2020), Aso et al. (2013)). While real detections are established using a False-Alarm Rate (FAR) threshold, under stationary Gaussian noise assumptions the condition that the SNR exceeds a chosen threshold can serve as a practical proxy (Essick (2023), Essick & Fishbach (2024)), especially in simulations of detectable events and in studies aimed at extracting astrophysical information (Abbott, B. P. et al. (2016)).

Applications such as population simulations for rate estimation (B. P. Abbott et al. (2016)) and hierarchical Bayesian inference with selection effects (Thrane & Talbot (2019), Essick & Fishbach (2024)) require repeated and efficient computation of the Probability of Detection (P_{det}), which is generally derived from SNR. However, traditional approaches that rely on noise-weighted inner products for SNR evaluation are computationally demanding and often impractical for such large-scale analyses (Taylor & Gerosa (2018), Gerosa & others (2020)).

Statement of Need

The *gwsnr* Python package addresses this challenge by providing efficient and flexible tools for computing the optimal SNR (ρ_{opt}). This quantity depends on the intrinsic and extrinsic source parameters, the detector antenna response ($F_{+,x}$), and the noise power spectral density (PSD) (Allen et al. (2012)). The primary use case of ρ_{opt} in *gwsnr* is the estimation of P_{det} , which is evaluated against a detection statistics threshold.

The package provides a flexible and user-friendly interface for combining detector noise models, waveform families, detector configurations, and signal parameters. It accelerates ρ_{opt} evaluation using a **partial-scaling interpolation** method for non-precessing binaries and a multiprocessing **inner-product** routine for frequency-domain waveforms implemented in *lalsuite* (LIGO Scientific Collaboration et al. (2018)), including those with spin precession and subdominant modes. For rapid P_{det} estimation, *gwsnr* also supports ANN-based models and a Hybrid SNR recalculation scheme. Finally, using an optimal-SNR threshold $\rho_{\text{opt},\text{thr}}$, the package computes the horizon distance (D_{hor}), a standard measure of detector sensitivity, via both analytical (Allen et al. (2012)) and numerical methods.

41 High performance is achieved through *NumPy* vectorization (*NumPy Community (2022)*) and
 42 Just-in-Time (JIT) compilation with *Numba* (*Lam et al. (2022)*), with optional GPU acceleration
 43 available via *JAX* (*James Bradbury & others (2018)*) and *MLX* (*Hannun et al. (2023)*). These
 44 JIT compilers translate Python code into optimized machine code at runtime, while built-in
 45 parallelization strategies such as `numba.prange`, `jax.vmap`, and `mlx.vmap` maximize efficiency
 46 on both CPUs and GPUs (supported hardware includes NVIDIA and Apple Silicon GPUs).

47 This combination of efficiency and usability makes *gwsnr* a valuable tool for GW data analysis.
 48 It enables large-scale simulations of compact binary mergers, facilitates the estimation of
 49 detectable lensed and unlensed event rates (as demonstrated in the *ler* package; *Phurailatpam*
 50 et al. (2024), *Ng et al. (2024)*, *More & Phurailatpam (2025)*, *Janquart et al. (2023)*,
 51 *R. Abbott et al. (2021)*, *Collaboration et al. (2023)*, *Wierda et al. (2021)*, *Wempe et al.*
 52 (*2022*)), and supports the treatment of selection effects through P_{det} in hierarchical Bayesian
 53 frameworks (*Thrane & Talbot (2019)*, *Essick (2023)*).

54 Mathematical Formulation and Methods Overview

55 Following are the key mathematical formulations and methods implemented in *gwsnr* for SNR
 56 calculation, P_{det} estimation, and D_{hor} computation.

57 Noise-Weighted Inner Product

58 The standard frequency-domain inner product (*Allen et al. (2012)*) between two signals $\tilde{a}(f)$
 59 and $\tilde{b}(f)$ is

$$60 \langle a | b \rangle = 4\Re \int_{f_{\min}}^{f_{\max}} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df,$$

61 where $S_n(f)$ is the detector PSD. The optimal SNR is $\rho = \sqrt{\langle h | h \rangle}$, and for polarizations
 h_+, h_\times :

$$\rho = \sqrt{F_+^2 \langle \tilde{h}_+ | \tilde{h}_+ \rangle + F_\times^2 \langle \tilde{h}_\times | \tilde{h}_\times \rangle}.$$

62 While the inner product method is computationally expensive, *gwsnr* accelerates it through
 63 multiprocessing, `numba.njit`, and optional `jax` backends (with `ripplegw` for waveform
 64 generation; *Edwards et al. (2024)*).

65 Partial Scaling Interpolation

66 For aligned-spin or non-spinning binaries, *gwsnr* adapts FINDCHIRP (*Allen et al. (2012)*) to
 67 precompute a partial-scaled SNR,

$$\rho_{1/2} = \frac{D_{\text{eff}}}{\mathcal{M}^{5/6}} \rho_{\text{opt}},$$

68 where \mathcal{M} is the chirp mass and D_{eff} the effective distance. $\rho_{1/2}$ is stored on a parameter grid
 69 (2D for non-spinning, 4D for aligned spins). New SNRs are recovered by spline interpolation
 70 and rescaling:

$$70 \rho = \rho_{1/2} \frac{\mathcal{M}^{5/6}}{D_{\text{eff}}}.$$

71 This replaces costly inner-product integrations with fast interpolation, yielding significant
 72 speed-ups.

⁷³ **ANN-based P_{det} Estimation**

⁷⁴ *gwsnr* includes an ANN built with tensorflow (Abadi et al. (2015)) and scikit-learn
⁷⁵ (Pedregosa et al. (2011)), trained to approximate ρ_{opt} for BBH systems with the IMRPhenomXPHM waveform, which includes spin precession and subdominant modes. While the
⁷⁶ ANN is poor at estimating ρ_{opt} directly, its outputs are effective for P_{det} , since detectability
⁷⁷ depends on threshold crossing rather than precise values.

⁷⁸ Trained on large *lter* datasets, the model uses partial-scaled SNRs to reduce input dimensionality
⁸⁰ (15 to 5) and accelerate detectability estimates under stationary Gaussian noise. Users can
⁸¹ also retrain the ANN for different detectors or astrophysical settings. Related work includes
⁸² (Chapman-Bird & others (2023), Gerosa & others (2020), Callister & others (2024)).

⁸³ **Hybrid SNR Recalculation for P_{det} Estimation**

⁸⁴ The Partial Scaling method is efficient for aligned-spin systems but unreliable for precessing
⁸⁵ binaries, and the ANN-based approach is less accurate. To address this, *gwsnr* uses a hybrid
⁸⁶ strategy: it first estimates SNRs with Partial Scaling or ANN, identifies signals near the
⁸⁷ threshold ρ_{th} , and then recalculates them with the Noise-Weighted Inner Product.

⁸⁸ This approach retains the speed of approximations while ensuring accuracy for systems close
⁸⁹ to the detection limit, producing more reliable P_{det} estimates.

⁹⁰ **Statistical Models for P_{det}**

⁹¹ In *gwsnr*, estimation of P_{det} is based on a detection threshold for the observed (matched-filter)
⁹² SNR, $\rho_{\text{obs,thr}}$. The observed SNR, ρ_{obs} , is modeled either as a Gaussian random variate
⁹³ centered at ρ_{opt} (or $\rho_{\text{opt,net}}$ for a detector network) with unit variance (Fishbach et al. (2020),
⁹⁴ B. P. Abbott et al. (2019)), or as a non-central χ distribution (`scipy.stats.ncx2`; Virtanen
⁹⁵ et al. (2020)) with non-centrality parameter $\lambda = \rho_{\text{opt}}$ (or $\rho_{\text{opt,net}}$) and two degrees of freedom
⁹⁶ for a single detector, extended to $2N$ for a network of N detectors (Essick (2023)).

⁹⁷ *gwsnr* uses precomputed $\rho_{\text{obs,thr}}$ values derived from semianalytic sensitivity estimates of GW
⁹⁸ transient injection catalogues (following Essick (2023)). The package also supports custom
⁹⁹ threshold computation from user-provided catalogue data, including parameter-dependent
¹⁰⁰ thresholds that vary with intrinsic properties such as the primary mass ($m_{1,\text{src}}$).

¹⁰¹ **Horizon Distance Calculation**

¹⁰² D_{hor} is a standard measure of detector sensitivity, defined as the maximum distance at which
¹⁰³ an optimally oriented source can be detected with a given threshold $\rho_{\text{opt,thr}}$ (Allen et al.
¹⁰⁴ (2012)). *gwsnr* computes D_{hor} using two methods.

¹⁰⁵ The **analytical method** rescales a known D_{eff} by

$$D_{\text{hor}} = \frac{\rho_{\text{opt}}}{\rho_{\text{th}}} D_{\text{eff}}.$$

¹⁰⁶ The **numerical method** maximizes SNR over sky location, then solves for the luminosity distance
¹⁰⁷ (d_L) where

$$\rho(d_L) - \rho_{\text{opt,thr}} = 0.$$

¹⁰⁸ **Acknowledgements**

¹⁰⁹ The author gratefully acknowledges the substantial contributions from all who supported
¹¹⁰ this research. Special thanks go to my academic advisors for their invaluable guidance and

111 unwavering support. The interactions with my research colleagues have greatly enriched this
 112 work. The Department of Physics at The Chinese University of Hong Kong is acknowledged
 113 for the Postgraduate Studentship that made this research possible. Thanks are also due to the
 114 LIGO Laboratory for the computational resources, supported by National Science Foundation
 115 Grants No. PHY-0757058 and No. PHY-0823459.

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