

¹ PinCFlow.jl: An idealized-atmospheric-flow solver ² coupled to the 3D transient gravity-wave model ³ MS-GWaM

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¹⁹ PinCFlow.jl (*Pseudo-inCompressible Flow solver*) is an atmospheric-flow solver that was designed for conducting idealized simulations. It integrates the Boussinesq, pseudo-incompressible, and compressible equations in a conservative flux form (Klein, 2009; Rieper et al., 2013), using a semi-implicit method that combines explicit and implicit time-stepping schemes (Benacchio & Klein, 2019; Chew et al., 2022; Schmid et al., 2021). Spatially, the equations are discretized with a finite-volume method, such that all quantities are represented by spatial averages over grid cells and fluxes are computed on the respective cell interfaces. The grid is staggered (Arakawa & Lamb, 1977) so that the velocity components are defined at the same points as the corresponding fluxes of scalar quantities. PinCFlow.jl operates in a vertically stretched terrain-following coordinate system based on Gal-Chen & Somerville (1975a), Gal-Chen & Somerville (1975b), and Clark (1977).

²⁰ The Lagrangian gravity-wave parameterization MS-GWaM (Multi-Scale Gravity-Wave Model)

²¹ is interactively coupled to the dynamical core of PinCFlow.jl, so that unresolved gravity waves may be parameterized in a manner that accounts for transient wave-mean-flow interaction and horizontal wave propagation. The resolved fields are updated with tendencies computed by MS-GWaM at the beginning of every time step. A description of the theory behind MS-GWaM can be found in Achatz et al. (2017) and Achatz et al. (2023). For a numerical perspective and more information on the development, see Muraschko et al. (2015), Böloni et al. (2016), Wilhelm et al. (2018), Wei et al. (2019), and Jochum et al. (2025).

²⁹ Statement of need

³⁰ Atmospheric gravity waves are generated by various sources, including convection, flow over mountains, and jet-frontal imbalances. As they propagate upward, they transport and deposit momentum and energy, thus impacting the large-scale flow. A significant part of the gravity-wave spectrum cannot be resolved by weather and climate models at their respective standard resolutions and must therefore be parameterized (Achatz et al., 2024; Alexander et al., 2010; Fritts & Alexander, 2003).

³⁶ Most gravity-wave parameterizations rely on single-column and steady-state approximations, since these yield a comparatively simple and efficient method of calculating the mean-flow impact. However, it has become increasingly apparent that transient wave-mean-flow interaction and horizontal gravity-wave propagation need to be accounted for in an accurate description of gravity-wave-mean-flow interaction (Böloni et al., 2016; Ehard et al., 2017; Muraschko et al.,

41 2015; Sato et al., 2009; Senf & Achatz, 2011; Wei et al., 2019). The Multi-Scale Gravity-Wave
 42 Model (MS-GWaM) has been created for this reason. Essential to its development, as to
 43 the improvement of the fundamental understanding of atmospheric dynamics, is the study of
 44 gravity waves in idealized simulations. PinCFlow.jl provides a modeling infrastructure tailored
 45 to this exact need. Moreover, it is interactively coupled to MS-GWaM, thus allowing for
 46 direct comparison between high-resolution wave-resolving simulations and coarse-resolution
 47 wave-parameterizing simulations.

48 Features

49 **Figure 1** shows an overview of PinCFlow.jl's features. As has been mentioned above, the model
 50 can be used to integrate the Boussinesq, pseudo-incompressible or compressible equations. For
 51 each of these, it provides a number of background atmospheres defined by model parameters.
 52 The initial deviations of the prognostic variables from the background state are set with
 53 user-defined functions. In addition to the momentum, continuity, and potential-temperature
 54 equations, the flow solver integrates the prognostic equation for an arbitrary passive tracer with
 55 direct gravity-wave impact parameterized using MS-GWaM according to Knop et al. (2026).

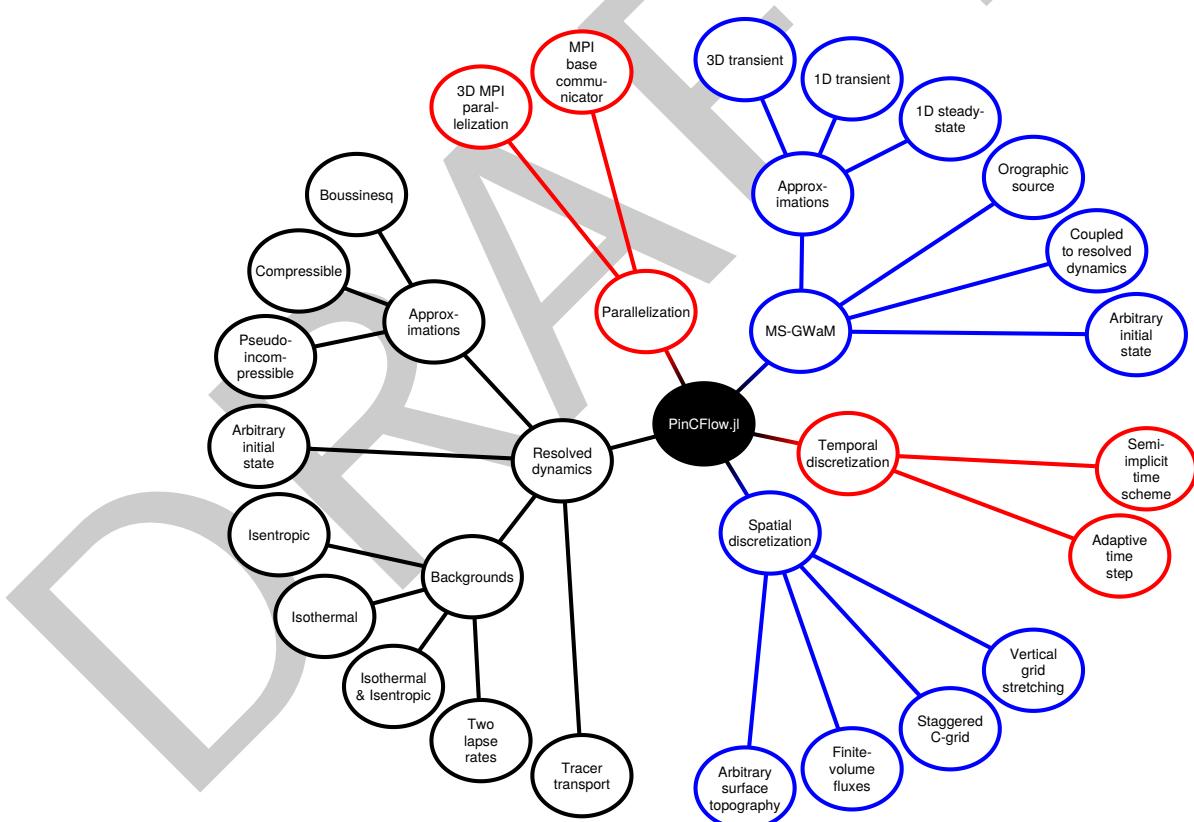


Figure 1: Overview of PinCFlow.jl's features.

56 PinCFlow.jl's grid is determined from a user-defined surface topography, using a height-based
 57 terrain-following coordinate system. Fluxes are computed with a finite-volume (MUSCL)
 58 scheme (e.g., Leer, 2003), using the monotonized-centered variant limiter (e.g., Kemm, 2011).
 59 In addition to the advection terms, the model includes molecular friction, heat conduction,
 60 and a turbulent viscosity.

61 MS-GWaM can be run in a 1D steady-state mode (which represents classic gravity-wave

parameterizations), a 1D transient mode (so that the impact of transient wave-mean-flow interaction may be investigated separately, i.e., without horizontal propagation), and a 3D transient mode (the full implementation of the theory). It is interactively coupled to the resolved dynamics, so that the evolution of the gravity-wave field depends on the current state of the large-scale flow and in turn changes it via tendencies that are added to the prognostic equations. This coupling includes the impact of gravity-wave fluxes on the large-scale tracer. The parameterization has an inbuilt orographic source which launches mountain waves generated by the resolved flow passing over unresolved terrain. The latter is represented by an arbitrary spectrum computed by a user-defined function. Moreover, MS-GWaM may be initialized with any wave field, also via a user-defined function.

By default, the flow solver uses an adaptive time step determined from several stability criteria, including CFL conditions with respect to the resolved flow and the maximum group velocities of the unresolved gravity-wave field computed by MS-GWaM. The CFL numbers are configurable, however, one may also run the model with a fixed time step.

MPI parallelization is supported in all grid dimensions. The number of MPI tasks is set per dimension, using corresponding model parameters. Furthermore, one may pass a base communicator to the model, facilitating the setup of ensemble simulations in a single Julia script.

Software design

PinCFlow.jl's design philosophy aims to balance a user-friendly and minimal Public API with a well-structured and easily extendible model. The code is organized in nested modules that group related functions and types. Most functions take an instance of PinCFlow.jl's type State as input, which contains all information on the model's configuration and current simulation state, organized in nested composite types. Julia's multiple dispatch is heavily used to implement algorithms that are specialized for the configuration provided by the user. This facilitates the continued development of the model, since adding a new feature can be as simple as defining a new type for an existing model parameter and dispatching to an appropriate method where the feature comes into play. All of PinCFlow.jl's functions have docstrings with detailed descriptions of each of their methods, which can be found in the reference section of the documentation.

The Public API consists of all objects that are exported by PinCFlow.jl's main module, including the State type, a composite type called Namelists and the types of its fields (which are used to create a State instance in a particular configuration), model parameter types, and the integrate function (which also takes an instance of Namelists as input). An extension for the visualization package Makie.jl (Danisch & Krumbiegel, 2021) is provided, which exports tools for plotting the model output. A user guide and a number of examples that illustrate the usage of PinCFlow.jl are given in the documentation.

Research impact statement

PinCFlow.jl originated from a pseudo-incompressible flow solver with implicit turbulence parameterization (PincFloit), written in Fortran by Rieper et al. (2013). PincFloit was used by Böloni et al. (2016) to conduct wave-resolving simulations of gravity-wave packets, which provided reference data for the validation of an early version of MS-GWaM. An important result of this study was that, due to its accounting for transience, MS-GWaM yielded a more accurate forcing of the mean flow than the classic steady-state approach. Wilhelm et al. (2018) modified PincFloit to also integrate the Boussinesq equations, coupled MS-GWaM to it, and validated the latter against submesoscale-resolving simulations of mesoscale inertia gravity waves radiated by a submesoscale gravity-wave packet. Wei et al. (2019) used this new framework to show that the common approach of using the gravity-wave pseudomomentum-flux

convergence to force the mean flow and thus neglect elastic and thermal effects is less reliable than using a direct approach which includes these contributions. Due to the implicit turbulence parameterization no longer being used, the model was renamed PincFlow when Schmid et al. (2021) implemented a semi-implicit time scheme (based on Benacchio & Klein, 2019) and verified its accuracy and efficiency in baroclinic life-cycle simulations. Using this scheme in conjunction with a newly added terrain-following coordinate system, Jochum et al. (2025) extended PincFlow's implementation of MS-GWaM with an orographic source and showed that the parameterization is able to capture highly transient interaction between mountain waves and mean flow in an idealized setting, with wave-resolving simulations providing a reference. Finally, Knop et al. (2026) showed that MS-GWaM can improve the representation of gravity-wave-induced tracer transport through the inclusion of a tracer Stokes drift due to unresolved gravity waves.

PinCFlow.jl is based on a translation of PincFlow from Fortran to Julia. Apart from time-dependent orography, heating above flat terrain in the pseudo-incompressible regime, alternative turbulence schemes, and a fully explicit time scheme, it offers all the features that were essential to the studies listed above. In addition, it introduces several new features, most notably access to the compressible regime (allowing for heating above arbitrary terrain), vertical grid stretching, and arbitrary initialization of the deviations from the atmospheric background. The implementation of the compressible mode necessitated slight modifications to the time scheme, based on the temporal discretization in the models introduced by Benacchio & Klein (2019) and Chew et al. (2022). Note that in contrast to these models, PinCFlow.jl does not provide seamless access to soundproof and hydrostatic dynamics (instead, the prognostic equations are set in the initialization and not changed during the simulation) or any data assimilation capabilities.

State of the field

What distinguishes PinCFlow.jl from other software is its implementation of MS-GWaM. The latter was first developed as an isolated code (Bölöni et al., 2016; Muraschko et al., 2015) and later as a part of the predecessor of PinCFlow.jl (by Jochum et al., 2025; Knop et al., 2026; Wei et al., 2019; Wilhelm et al., 2018) and within the NWP-and-climate model ICON (by Bölöni et al., 2021; Kim et al., 2021, 2024; Voelker et al., 2024). PinCFlow.jl's version of MS-GWaM differs from the ICON version in several components, most notably its coordinate system, discretization, and sources, however, the underlying theory (Achatz et al., 2017, 2023) is the same. In contrast to other gravity-wave parameterizations, MS-GWaM relaxes the single-column and steady-state approximations in the description of the gravity-wave field and uses a direct approach (as opposed to the above-mentioned pseudomomentum approach) to calculate the mean-flow impact, resulting in additional force terms in the horizontal-momentum and potential-temperature equations (e.g., Wei et al., 2019). The conservation of gravity-wave energy is encoded in a prognostic equation for a wave-action density in the topological product of physical space and spectral space. This equation is integrated in a Lagrangian manner, so that wave action is essentially traced along rays in a six-dimensional phase space. Via this method, MS-GWaM avoids so-called caustics, i.e., ray intersections leading to an ill-defined wave-action-density equation, in a unique and physically sound manner (e.g., Muraschko et al., 2015). The numerical implementation involves the definition of rectangular ray volumes, which may overlap, move, stretch, split, and merge (e.g., Jochum et al., 2025).

The code presented here provides the first open access to a full implementation of MS-GWaM coupled to a flow solver within an idealized and highly configurable framework.

156 AI usage disclosure

157 No generative AI tools were used in the software creation or writing of the paper. Parts of the
158 reference section of the documentation were initially generated with AI but manually rewritten
159 (in their entirety) later on.

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