

NEoST: A Python package for nested sampling of the neutron star equation of state

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Summary

The Nested Equation of State Sampling (NEoST) package is an open-source code that allows users to infer the parameters of the dense matter Equation of State (EoS) in neutron stars via nested sampling. It provides a Bayesian inference framework that compares pre-existing EoS models (parameterized or tabulated, for both crust and core) to a variety of user-defined astrophysical input data (real or synthetic), namely mass-radius samples, mass-tidal deformability samples, and mass samples. NEoST can also be used to provide a fast solver for the Tolman-Oppenheimer-Volkoff (TOV) equations for neutron star structure (Oppenheimer & Volkoff, 1939; Tolman, 1939). Moreover, NEoST is able to fully account for a possible dark matter component inside neutron stars, where the dark matter is described by the (Nelson et al., 2019) model.

Statement of need

Matter in the cores of neutron stars can reach several times the nuclear saturation density. The EoS of matter under such circumstances is not well understood: in addition to extreme levels of neutron-richness there could also exist stable states of strange matter, in the form of either hyperons or deconfined quarks (Hebeler et al., 2015; Lattimer & Prakash, 2016; Tolos & Fabbietti, 2020). Neutron star properties like mass, radius and tidal deformability depend on the EoS, so measurement of these quantities provides insight into the properties of ultradense nuclear matter.

Astrophysical data sets that can be used to constrain the EoS take the form of posterior distributions that are derived from separate inference analyses. Examples include: mass posteriors from pulsar timing analysis of radio pulsars in binary systems (Fonseca et al., 2021), joint mass-radius posteriors from Pulse Profile Modeling using NICER data (Miller et al., 2021; Riley et al., 2021); and joint mass-tidal deformability posters from gravitational wave observations of neutron star binary mergers (Abbott et al., 2019). NEoST provides a framework for EoS inference that couples these various different types of astrophysical data to either parameterized or tabulated EoS models, e.g. (Keller et al., 2023).

The NEST package and science use

NEoST is an open source Python package for Bayesian inference of EoS parameters (for parameterized models) and/or evidence computation (for parameterized and tabulated models), given astrophysical data sets in the form of posterior distributions. NEoST samples from the prior distribution of the EoS model parameters and central densities, computes the corresponding mass and radius/tidal deformability and then evaluates the likelihood by applying a kernel density estimation to the posterior distributions of the astrophysical data sets using the nested sampling software MultiNest (Buchner et al., 2014; Feroz et al., 2009), Figure 1. The full Bayesian inference framework, including notes relating to prior distributions, is described in detail in (Raaijmakers et al., 2020). It includes a library of existing EoS models for crust and core, and users can easily define their own models.

NEoST also offers various options for post-processing, including generating plots showing the inferred EoS credible regions in pressure-energy density space, and the associated inferred mass-radius relation credible intervals.

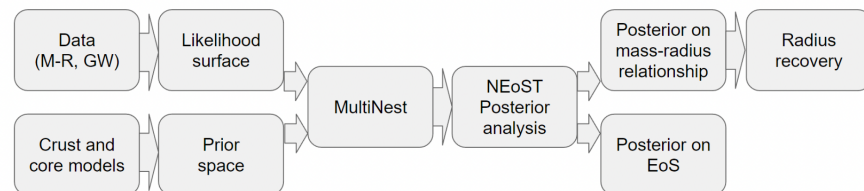


Figure 1: A schematic representation of the inference process using NEoST. It shows how the track for physical measurements and the track for theoretical models are fed through the framework, and what the main steps of analysis are after inference is complete.

NEoST is being used for EoS inference using mass-radius posteriors generated from pulse profile modeling of NICER data (Raaijmakers et al., 2019, 2020, 2021; Rutherford, Mendes, et al., 2024), specifically those generated using the X-PSI package (Choudhury et al., 2024; Riley et al., 2019, 2021, 2023; Salmi et al., 2022, 2023; Salmi, Choudhury, et al., 2024; Salmi, Deneva, et al., 2024; Vinciguerra et al., 2024). It has also been used to study EoS prior sensitivities using synthetic mass-radius posteriors (Greif et al., 2019) and to study the consequences of a potential dark matter component in neutron stars (Rutherford et al., 2023; Rutherford, Prescod-Weinstein, et al., 2024).

The core routines of NEoST are written in Cython (Behnel et al., 2011), and are dependent on the GNU Scientific Library (GSL, Gough, 2009). In case the user does not wish to use cythonised code, there is also an alternative set of routines written purely in Python. High-level object-oriented model construction is performed by a user in the Python language.

Release versions of NEoST are freely available on GitHub under the GNU General Public License. Extensive documentation, step-by-step tutorials, and reproduction code for existing data analyses, are available via the GitHub repository, along with a suite of unit tests. Future plans include tutorials documenting different types of astrophysical data sets, EoS models that include a dark matter component, and options for coupling to different samplers.

Software: Python/C language (Oliphant, 2007), GNU Scientific Library (GSL, Gough, 2009), NumPy (van der Walt et al., 2011), Cython (Behnel et al., 2011), OpenMP (Dagum & Menon, 1998), MPI for Python (Dalcín et al., 2008), Matplotlib (Droettboom et al., 2018; Hunter, 2007), IPython (Perez & Granger, 2007), Jupyter (Kluyver et al., 2016), MultiNest (Feroz et al., 2009), PyMultiNest (Buchner et al., 2014), GetDist (Lewis, 2019), SciPy (Jones et al.,

2001--), Seaborn([Waskom, 2021](#)), corner.py([Foreman-Mackey, 2016](#)), alive-progress([Sampaio de Almeida, 2019 --](#)).

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References

- Abbott, B. P., Abbott, R., Abbott, T. D., Acernese, F., Ackley, K., Adams, C., Adams, T., Addesso, P., Adhikari, R. X., Adya, V. B., Affeldt, C., Agarwal, B., Agathos, M., Agatsuma, K., Aggarwal, N., Aguiar, O. D., Aiello, L., Ain, A., Ajith, P., ... Virgo Collaboration. (2019). Properties of the Binary Neutron Star Merger GW170817. *Physical Review X*, 9(1), 011001. <https://doi.org/10.1103/PhysRevX.9.011001>
- Behnel, S., Bradshaw, R., Citro, C., Dalcin, L., Seljebotn, D. S., & Smith, K. (2011). Cython: The best of both worlds. *Computing in Science Engineering*, 13(2), 31–39. <https://doi.org/10.1109/MCSE.2010.118>
- Buchner, J., Georgakakis, A., Nandra, K., Hsu, L., Rangel, C., Brightman, M., Merloni, A., Salvato, M., Donley, J., & Kocevski, D. (2014). X-ray spectral modelling of the AGN obscuring region in the CDFS: Bayesian model selection and catalogue. *Astronomy and Astrophysics*, 564, A125. <https://doi.org/10.1051/0004-6361/201322971>
- Choudhury, D., Salmi, T., Vinciguerra, S., Riley, T. E., Kini, Y., Watts, A. L., Dorsman, B., Bogdanov, S., Guillot, S., Ray, P. S., Reardon, D. J., Remillard, R. A., Bilous, A. V., Huppenkothen, D., Lattimer, J. M., Rutherford, N., Arzoumanian, Z., Gendreau, K. C., Morsink, S. M., & Ho, W. C. G. (2024). A NICER View of the Nearest and Brightest Millisecond Pulsar: PSR J0437-4715. *Astrophysical Journal, Letters*, 971(1), L20. <https://doi.org/10.3847/2041-8213/ad5a6f>
- Dagum, L., & Menon, R. (1998). OpenMP: An industry standard API for shared-memory programming. *Computational Science & Engineering, IEEE*, 5(1), 46–55. <https://doi.org/10.1109/99.660313>
- Dalcín, L., Paz, R., Storti, M., & D'Elía, J. (2008). MPI for python: Performance improvements and MPI-2 extensions. *Journal of Parallel and Distributed Computing*, 68(5), 655–662. <https://doi.org/10.1016/j.jpdc.2007.09.005>
- Droettboom, M., Caswell, T. A., Hunter, J., Firing, E., Nielsen, J. H., Lee, A., Andrade, E. S. de, Varoquaux, N., Stansby, D., Root, B., Elson, P., Dale, D., Lee, J.-J., May, R., Seppänen, J. K., Klymak, J., McDougall, D., Straw, A., Hobson, P., ... Würtz, P. (2018). *Matplotlib/matplotlib v2.2.2*. Zenodo. <https://doi.org/10.5281/zenodo.1202077>
- Feroz, F., Hobson, M. P., & Bridges, M. (2009). MULTINEST: an efficient and robust Bayesian inference tool for cosmology and particle physics. *Monthly Notices of the RAS*, 398, 1601–1614. <https://doi.org/10.1111/j.1365-2966.2009.14548.x>
- Fonseca, E., Cromartie, H. T., Pennucci, T. T., Ray, P. S., Kirichenko, A. Yu., Ransom, S. M., Demorest, P. B., Stairs, I. H., Arzoumanian, Z., Guillemot, L., Parthasarathy, A., Kerr, M., Cognard, I., Baker, P. T., Blumer, H., Brook, P. R., DeCesar, M., Dolch,

- T., Dong, F. A., ... Zhu, W. W. (2021). Refined Mass and Geometric Measurements of the High-mass PSR J0740+6620. *Astrophysical Journal, Letters*, 915(1), L12. <https://doi.org/10.3847/2041-8213/ac03b8>
- Foreman-Mackey, D. (2016). Corner.py: Scatterplot matrices in python. *Journal of Open Source Software*, 1(2), 24. <https://doi.org/10.21105/joss.00024>
- Gough, B. (2009). *GNU scientific library reference manual - third edition* (3rd ed.). Network Theory Ltd. ISBN: 9780954612078
- Greif, S. K., Raaijmakers, G., Hebeler, K., Schwenk, A., & Watts, A. L. (2019). Equation of state sensitivities when inferring neutron star and dense matter properties. *Monthly Notices of the Royal Astronomical Society*, 485(4), 5363–5376. <https://doi.org/10.1093/mnras/stz654>
- Hebeler, K., Holt, J. D., Menéndez, J., & Schwenk, A. (2015). Nuclear Forces and Their Impact on Neutron-Rich Nuclei and Neutron-Rich Matter. *Annual Review of Nuclear and Particle Science*, 65, 457–484. <https://doi.org/10.1146/annurev-nucl-102313-025446>
- Hunter, J. D. (2007). Matplotlib: A 2D graphics environment. *Computing in Science & Engineering*, 9(3), 90–95. <https://doi.org/10.1109/MCSE.2007.55>
- Jones, E., Oliphant, T., Peterson, P., & others. (2001--). *SciPy: Open source scientific tools for Python*. <http://www.scipy.org/>
- Keller, J., Hebeler, K., & Schwenk, A. (2023). Nuclear Equation of State for Arbitrary Proton Fraction and Temperature Based on Chiral Effective Field Theory and a Gaussian Process Emulator. *Physical Review Letters*, 130(7), 072701. <https://doi.org/10.1103/PhysRevLett.130.072701>
- Kluyver, T., Ragan-Kelley, B., Pérez, F., Granger, B., Bussonnier, M., Frederic, J., Kelley, K., Hamrick, J., Grout, J., Corlay, S., Ivanov, P., Avila, D., Abdalla, S., & Willing, C. (2016). *Jupyter notebooks – a publishing format for reproducible computational workflows* (F. Loizides & B. Schmidt, Eds.; pp. 87–90). IOS Press.
- Lattimer, J. M., & Prakash, M. (2016). The equation of state of hot, dense matter and neutron stars. *Physics Reports*, 621, 127–164. <https://doi.org/10.1016/j.physrep.2015.12.005>
- Lewis, A. (2019). GetDist: a Python package for analysing Monte Carlo samples. *arXiv e-Prints*, arXiv:1910.13970. <https://arxiv.org/abs/1910.13970>
- Miller, M. C., Lamb, F. K., Dittmann, A. J., Bogdanov, S., Arzoumanian, Z., Gendreau, K. C., Guillot, S., Ho, W. C. G., Lattimer, J. M., Loewenstein, M., Morsink, S. M., Ray, P. S., Wolff, M. T., Baker, C. L., Cazeau, T., Manthripragada, S., Markwardt, C. B., Okajima, T., Pollard, S., ... Stairs, I. (2021). The Radius of PSR J0740+6620 from NICER and XMM-Newton Data. 918(2), L28. <https://doi.org/10.3847/2041-8213/ac089b>
- Nelson, A. E., Reddy, S., & Zhou, D. (2019). Dark halos around neutron stars and gravitational waves. *Journal of Cosmology and Astroparticle Physics*, 2019(7), 012. <https://doi.org/10.1088/1475-7516/2019/07/012>
- Oliphant, T. E. (2007). Python for scientific computing. *Computing in Science Engineering*, 9(3), 10–20. <https://doi.org/10.1109/MCSE.2007.58>
- Oppenheimer, J. R., & Volkoff, G. M. (1939). On Massive Neutron Cores. *Physical Review*, 55(4), 374–381. <https://doi.org/10.1103/PhysRev.55.374>
- Perez, F., & Granger, B. E. (2007). IPython: A system for interactive scientific computing. *Computing in Science Engineering*, 9(3), 21–29. <https://doi.org/10.1109/MCSE.2007.53>
- Raaijmakers, G., Greif, S. K., Hebeler, K., Hinderer, T., Nissanke, S., Schwenk, A., Riley, T. E., Watts, A. L., Lattimer, J. M., & Ho, W. C. G. (2021). Constraints on the Dense Matter Equation of State and Neutron Star Properties from NICER’s Mass-Radius Estimate

- of PSR J0740+6620 and Multimessenger Observations. *Astrophysical Journal, Letters*, 918(2), L29. <https://doi.org/10.3847/2041-8213/ac089a>
- Raaijmakers, G., Greif, S. K., Riley, T. E., Hinderer, T., Hebeler, K., Schwenk, A., Watts, A. L., Nissanke, S., Guillot, S., Lattimer, J. M., & Ludlam, R. M. (2020). Constraining the Dense Matter Equation of State with Joint Analysis of NICER and LIGO/Virgo Measurements. *Astrophysical Journal, Letters*, 893(1), L21. <https://doi.org/10.3847/2041-8213/ab822f>
- Raaijmakers, G., Riley, T. E., Watts, A. L., Greif, S. K., Morsink, S. M., Hebeler, K., Schwenk, A., Hinderer, T., Nissanke, S., Guillot, S., Arzoumanian, Z., Bogdanov, S., Chakrabarty, D., Gendreau, K. C., Ho, W. C. G., Lattimer, J. M., Ludlam, R. M., & Wolff, M. T. (2019). A NICER View of PSR J0030+0451: Implications for the Dense Matter Equation of State. *Astrophysical Journal, Letters*, 887(1), L22. <https://doi.org/10.3847/2041-8213/ab451a>
- Riley, T. E., Choudhury, D., Salmi, T., Vinciguerra, S., Kini, Y., Dorsman, B., Watts, A. L., Huppenkothen, D., & Guillot, S. (2023). X-PSI: A Python package for neutron star X-ray pulse simulation and inference. *The Journal of Open Source Software*, 8(82), 4977. <https://doi.org/10.21105/joss.04977>
- Riley, T. E., Watts, A. L., Bogdanov, S., Ray, P. S., Ludlam, R. M., Guillot, S., Arzoumanian, Z., Baker, C. L., Bilous, A. V., Chakrabarty, D., Gendreau, K. C., Harding, A. K., Ho, W. C. G., Lattimer, J. M., Morsink, S. M., & Strohmayer, T. E. (2019). A NICER View of PSR J0030+0451: Millisecond Pulsar Parameter Estimation. *Astrophysical Journal, Letters*, 887(1), L21. <https://doi.org/10.3847/2041-8213/ab481c>
- Riley, T. E., Watts, A. L., Ray, P. S., Bogdanov, S., Guillot, S., Morsink, S. M., Bilous, A. V., Arzoumanian, Z., Choudhury, D., Deneva, J. S., Gendreau, K. C., Harding, A. K., Ho, W. C. G., Lattimer, J. M., Loewenstein, M., Ludlam, R. M., Markwardt, C. B., Okajima, T., Prescod-Weinstein, C., ... Cognard, I. (2021). A NICER View of the Massive Pulsar PSR J0740+6620 Informed by Radio Timing and XMM-Newton Spectroscopy. *Astrophysical Journal, Letters*, 918(2), L27. <https://doi.org/10.3847/2041-8213/ac0a81>
- Rutherford, N., Mendes, M., Svensson, I., Schwenk, A., Watts, A. L., Hebeler, K., Keller, J., Prescod-Weinstein, C., Choudhury, D., Raaijmakers, G., Salmi, T., Timmerman, P., Vinciguerra, S., Guillot, S., & Lattimer, J. M. (2024). Constraining the Dense Matter Equation of State with New NICER Mass-Radius Measurements and New Chiral Effective Field Theory Inputs. *Astrophysical Journal, Letters*, 971(1), L19. <https://doi.org/10.3847/2041-8213/ad5f02>
- Rutherford, N., Prescod-Weinstein, C., & Watts, A. (2024). Probing fermionic asymmetric dark matter cores using global neutron star properties. *arXiv e-Prints*, arXiv:2410.00140. <https://doi.org/10.48550/arXiv.2410.00140>
- Rutherford, N., Raaijmakers, G., Prescod-Weinstein, C., & Watts, A. (2023). Constraining bosonic asymmetric dark matter with neutron star mass-radius measurements. *Physical Review D*, 107(10), 103051. <https://doi.org/10.1103/PhysRevD.107.103051>
- Salmi, T., Choudhury, D., Kini, Y., Riley, T. E., Vinciguerra, S., Watts, A. L., Wolff, M. T., Arzoumanian, Z., Bogdanov, S., Chakrabarty, D., Gendreau, K., Guillot, S., Ho, W. C. G., Huppenkothen, D., Ludlam, R. M., Morsink, S. M., & Ray, P. S. (2024). The Radius of the High-mass Pulsar PSR J0740+6620 with 3.6 yr of NICER Data. *Astrophysical Journal*, 974(2), 294. <https://doi.org/10.3847/1538-4357/ad5f1f>
- Salmi, T., Deneva, J. S., Ray, P. S., Watts, A. L., Choudhury, D., Kini, Y., Vinciguerra, S., Cromartie, H. T., Wolff, M. T., Arzoumanian, Z., Bogdanov, S., Gendreau, K., Guillot, S., Ho, W. C. G., Morsink, S. M., Cognard, I., Guillemot, L., Theureau, G., & Kerr, M. (2024). A NICER View of PSR J1231-1411: A Complex Case. *Astrophysical Journal*, 976(1), 58. <https://doi.org/10.3847/1538-4357/ad81d2>
- Salmi, T., Vinciguerra, S., Choudhury, D., Riley, T. E., Watts, A. L., Remillard, R. A., Ray, P.

- S., Bogdanov, S., Guillot, S., Arzoumanian, Z., Chirenti, C., Dittmann, A. J., Gendreau, K. C., Ho, W. C. G., Miller, M. C., Morsink, S. M., Wadiasingh, Z., & Wolff, M. T. (2022). The Radius of PSR J0740+6620 from NICER with NICER Background Estimates. *Astrophysical Journal*, 941(2), 150. <https://doi.org/10.3847/1538-4357/ac983d>
- Salmi, T., Vinciguerra, S., Choudhury, D., Watts, A. L., Ho, W. C. G., Guillot, S., Kini, Y., Dorsman, B., Morsink, S. M., & Bogdanov, S. (2023). Atmospheric Effects on Neutron Star Parameter Constraints with NICER. *Astrophysical Journal*, 956(2), 138. <https://doi.org/10.3847/1538-4357/acf49d>
- Sampaio de Almeida, R. (2019 --). *Alive-progress*. <https://github.com/rsalmei/alive-progress>
- Tolman, R. C. (1939). Static Solutions of Einstein's Field Equations for Spheres of Fluid. *Physical Review*, 55(4), 364–373. <https://doi.org/10.1103/PhysRev.55.364>
- Tolos, L., & Fabbietti, L. (2020). Strangeness in nuclei and neutron stars. *Progress in Particle and Nuclear Physics*, 112, 103770. <https://doi.org/10.1016/j.ppnp.2020.103770>
- van der Walt, S., Colbert, S. C., & Varoquaux, G. (2011). The NumPy array: A structure for efficient numerical computation. *Computing in Science Engineering*, 13(2), 22–30. <https://doi.org/10.1109/MCSE.2011.37>
- Vinciguerra, S., Salmi, T., Watts, A. L., Choudhury, D., Riley, T. E., Ray, P. S., Bogdanov, S., Kini, Y., Guillot, S., Chakrabarty, D., Ho, W. C. G., Huppenkothen, D., Morsink, S. M., Wadiasingh, Z., & Wolff, M. T. (2024). An Updated Mass-Radius Analysis of the 2017-2018 NICER Data Set of PSR J0030+0451. *Astrophysical Journal*, 961(1), 62. <https://doi.org/10.3847/1538-4357/acf83>
- Waskom, M. L. (2021). Seaborn: Statistical data visualization. *Journal of Open Source Software*, 6(60), 3021. <https://doi.org/10.21105/joss.03021>