

¹ open_nipals: An sklearn-compatible Python package for NIPALS dimensional reduction

³ **Niels Schlusser**  ^{1*}, **Ryan M. Wall**  ^{2*}, and **David R. Ochsenbein**  ¹

⁴ 1 Cilag GmbH International, Schaffhausen, Switzerland ² Johnson & Johnson Innovative Medicine,
⁵ Titusville, New Jersey, USA * These authors contributed equally.

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⁶ Summary

⁷ open_nipals is a Python package that implements the Nonlinear Iterative Partial Least Squares ⁸ (NIPALS) algorithm ([Geladi & Kowalski, 1986](#)) for Partial Least Squares (PLS) regression as ⁹ well as Principal Component Analysis (PCA). It employs the data transformation methods `fit()` ¹⁰ and `transform()` from scikit-learn ([Pedregosa et al., 2011](#)) and leverages Nelson's Single ¹¹ Component Projection (SCP) method for the imputation of missing data ([Nelson et al., 1996](#)). ¹² The NIPALS algorithm represents an alternative to the common Singular Value Decomposition ¹³ (SVD) procedure for both PCA and PLS implemented in scikit-learn ([Pedregosa et al., 2011](#)). ¹⁴ It is an iterative procedure that processes the data and internal matrices vector-wise ¹⁵ and iteratively. When combined with SCP, NIPALS allows natural handling of missing data ¹⁶ and setting tailored accuracy goals.

¹⁷ Statement of Need

¹⁸ Python has emerged as a popular and comparatively simple programming environment for the development of machine learning and data science applications. Packages like numpy for vector operations ([Harris et al., 2020](#)), pandas for the handling of tabular data ([team, 2020](#)), and scikit-learn (abbreviated sklearn in the following) for orthodox machine learning techniques like Random Forests, Support Vector Machines (SVM), and Principal Component Analyses (PCA) ([Pedregosa et al., 2011](#)) promote Python's success in extracting patterns from big ²¹ and complex data sets. However, sklearn relies on Singular Value Decomposition (SVD) for its PCA and PLS classes, with negative effects on performance for applications like batch ²² manufacturing and chemometrics, where missing data is common ([Nelson et al., 1996](#)). PCA ²³ and PLS models require unit scaled and mean centered input data, a feature that is nicely ²⁴ implemented in sklearn's StandardScaler class. ²⁵ To this end, we felt the need to complement sklearn with an implementation of the NIPALS ²⁶ algorithm for PCA and PLS. As will be discussed [below](#), the NIPALS algorithm has especially ²⁷ beneficial properties when it comes to peak memory consumption scaling and accuracy. Since ²⁸ it is an iterative algorithm, the runtime requirement can be balanced with the desired accuracy ²⁹ target. In general, it is a favorable option at small number of latent variables (`n_components` ³⁰ < 10) and a high numerical accuracy requirement.

³⁵ Why a separate package?

³⁶ We decided to wrap the implementation of NIPALS PCA and PLS into a separate package ³⁷ instead of extending sklearn by another model because:

- ³⁸ 1. open_nipals' usecase is very specific to batch manufacturing and chemometrics ³⁹ (cf. [benchmarking](#)). sklearn's audience is a broader Python machine learning ⁴⁰ community.

- 41 2. open_nipals follows a slightly different philosophy in that it integrates the missing value
 42 imputation into the proper package, and therefore does not align with sklearn in that
 43 particular aspect. Integrating the imputation of missing data into the analysis package
 44 comes with a performance advantage.
 45 3. Keeping open_nipals and sklearn apart facilitates maintainability of both packages.

46 Related Software

47 To our knowledge, the only other maintained open-source Python package that implements
 48 the NIPALS algorithm for PCA and PLS is Salvador García Muñoz' pyphi ([Garcia Munoz et al., 2019](#)). Our implementation is different in the following aspects:

- 49 1. open_nipals follows the template of sklearn, which allows:
 50 1. Integration with other sklearn modules, e.g. the StandardScaler
 51 2. Accumulation of multiple transformation steps into a sklearn.pipeline.
 52 2. open_nipals uses Nelson's single component projection method ([Nelson et al., 1996](#))
 53 for score calculation in the face of missing values.
 54 3. The utility class of open_nipals contains ArrangeData, another sklearn style data
 55 transformer object that ensures correct ordering and quantity of input columns.
 56

57 Functionality

58 Wherever possible, open_nipals follows and inherits structures from parent classes in sklearn.
 59 In principle, its functionality can be split into three parts:

- 60 1. Utility functions for data preprocessing
 61 2. Principal Component Analysis
 62 3. Partial Least Squares regression.

63 We decided to combine PCA and PLS functionality into one package, such that they can
 64 share common utility functions, e.g. – but not limited to – the ArrangeData class and matrix
 65 multiplication with missing values.

66 Data Preprocessing, and Utility Functions

67 It is *strongly* encouraged to mean-center the input data for both PCA and PLS, and scale their
 68 variance to unity, e.g. with sklearn's StandardScaler. However, the informed user should still
 69 have the chance to also apply open_nipals to non-standardized data. Therefore, we did not
 70 make standardization a part of open_nipals, unlike sklearn.PCA, which automatically mean-
 71 centers input data when not already done. Moreover, the ArrangeData class of open_nipals
 72 ensures correct ordering of the input columns, as well as proper formatting. A code example
 73 for preprocessing could therefore look like:

```
from open_nipals.utils import ArrangeData
import pandas as pd
from sklearn.preprocessing import StandardScaler

# Load data
df = pd.read_csv('my_data.csv')

# Create objects
arrdat = ArrangeData()
scaler = StandardScaler()

# Fit and transform data using both objects
data = scaler.fit_transform(arrdat.fit_transform(df))
```

PCA

74 Principal Component Analyses with `open_nipals` utilize a `NipalsPCA` transformer object, that
75 can be fitted to and transform input data (and both at once), e.g. with:

```
from open_nipals.nipalsPCA import NipalsPCA
```

```
model = NipalsPCA()  
transformed_data = model.fit_transform(data)
```

77 The number of fitted components can be specified with the `n_components` argument in
78 the constructor, which defaults to `n_components=2`. After having constructed the object,
79 components can be added or subtracted using the `set_components()` function. Once fitted,
80 components are stored so they do not have to be fitted again. This saves compute time should
81 the developer decide to use lower number of components than are fitted and later move back
82 to a higher number of principal components.

83 The following functions and attributes of the `sklearn` API are implemented by `NipalsPCA`:

- 84 ■ `fit(X)` to fit a new `NipalsPCA` model to a data set `X`
- 85 ■ `transform(X)` to transform a data set `X` according to a model previously fitted
- 86 ■ `fit_transform(X)` to do both above steps in one
- 87 ■ `inverse_transform(X)` to predict original data from transformed input data `X`
- 88 ■ `explained_variance_ratio_` to return the variance ratios explained by each component

89 Please note that the NIPALS algorithm does not compute eigenvalues of the covariance matrix,
90 therefore computing them specifically for `explained_variance_` seemed unnatural. Thus, this
91 attribute was not implemented.

92 The distance of a given data point from the average of the training data within the PCA model
93 (in-model distance, IMD) can be calculated with `calc_imd()`, where Hotelling's T^2 ([Hotelling, 1931](#)) is implemented and could be extended to other IMD metrics (e.g. Mahalanobis Distance).
94 Conversely, the out-of-model distance (OOMD, calculated by `calc_oomd()`) gives a measure
95 of the distance to the model hyperplane. This is available as two metrics, DModX and QRes
96 ([Eriksson et al., 1999](#)).

98 Finally, the `calc_limit()` function calculates theoretical limits on both IMD and OOMD such
99 that a specified fraction alpha of the data lies within these limits, assuming the data follows
100 an f-distribution ([Brereton, 2016](#)).

101 Following the data preparation scheme detailed in the [benchmarking section](#), we simulated
102 1000 samples in a 40-dimensional data space, and trained a 4-component PCA model on it.
103 [Figure 1](#) shows the IMD-OOMD plot given this model and the theoretical limits calculated
104 with it.

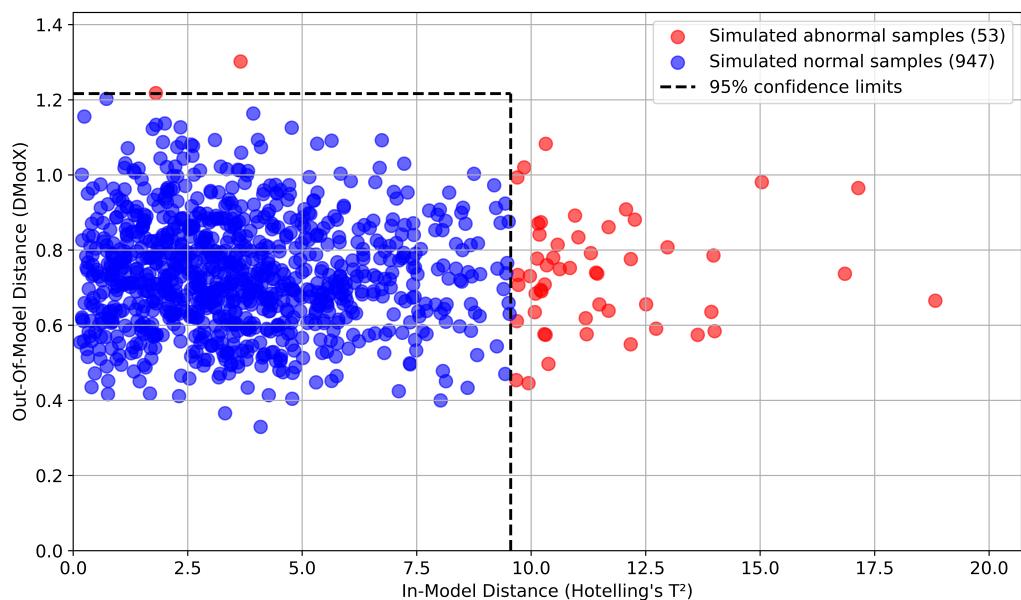


Figure 1: PCA-modelled data points on the IMD-OOMD plane with 0.95 confidence interval.

PLS

105 Beyond projecting input data onto a low-dimensional latent space, as PCA models do, PLS
 106 models can also predict dependent variables.
 107

108 Initializing a basic PLS model with a `NipalsPLS` object and transforming the input data
 109 accordingly looks like:

```
from open_nipals.nipalsPLS import NipalsPLS

model = NipalsPLS()
transformed_x_data, transformed_y_data = model.fit_transform(data_x, data_y)
```

110 The following functions and attributes of the `sklearn` API are implemented by `NipalsPLS`:

- 111 ■ `fit(X)` to fit a new `NipalsPLS` model to data sets `X` and `y`
- 112 ■ `transform(X, y=None)` to transform a data set `X` according to a model previously fitted,
 with the option of adding a `y` data set
- 113 ■ `fit_transform(X,y)` to do both above steps in one
- 114 ■ `inverse_transform(X)` to predict original data from transformed input data `X`
- 115 ■ `predict(X)` to predict dependent variables `y` given the model
- 116 ■ `explained_variance_ratio_` to return the `X` and `y` variance ratios explained by each
 component

117 As for `NipalsPCA`, `NipalsPLS` does not implement `explained_variance_` since the eigenvalues
 118 are not accessible as a byproduct of the NIPALS PLS algorithm

119 Beyond standard `sklearn` functionality, `NipalsPLS` implements `calc_oomd()` for the out-of-
 120 model distance with either QRes or DModX as implemented metrics, `calc_imd()` for the in-model
 121 distance, using the Hotelling's T^2 metric. Summary statistics of PLS models can be displayed
 122 in similar plots to [Figure 1](#).

123 `NipalsPLS` primarily differs from `NipalsPCA` by the inclusion of a `predict()` method to predict
 124 a `y`-matrix from an `x`-matrix with a previously fitted model, and the calculation of the regression
 125 vector with `get_reg_vector()`. The latter serves as a measure for what input features the
 126 model considers predictive of the output, see [Figure 2](#).

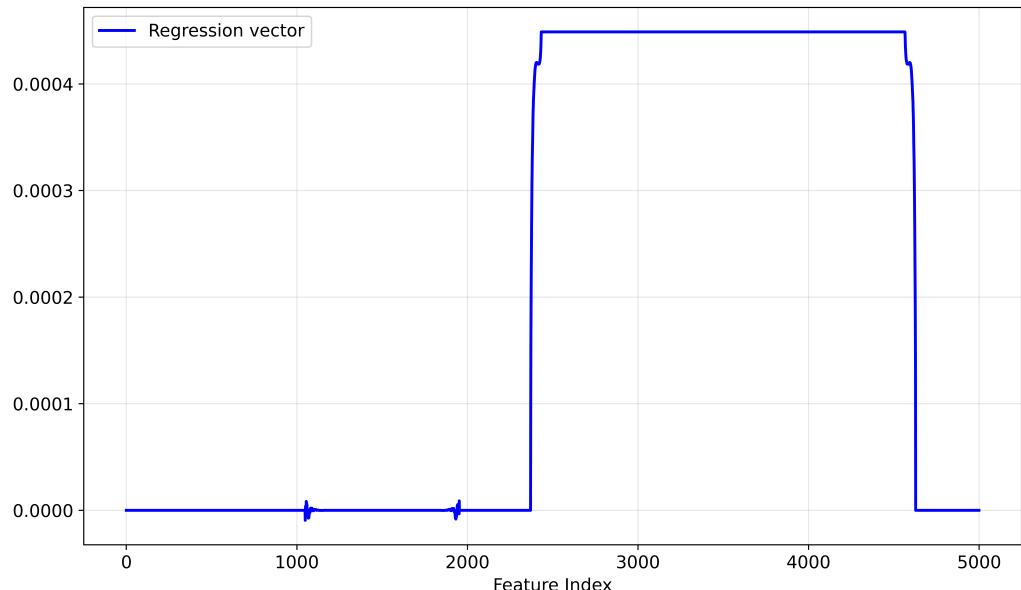


Figure 2: Visualization of a PLS regression vector, generated from simulated test data inspired by spectroscopy.

129 Debugging

130 If the maximum iteration counter is hit during the fitting procedure of new components of
 131 an open_nipals PCA or PLS model, a `max_iter` Reached on LV {ind_LV} warning is raised.
 132 This indicates that the desired numerical tolerance for this component could not be achieved
 133 during the fit procedure. Try to increase `max_iter`, or decrease `tol_criteria` if less numerical
 134 precision is still acceptable in your usecase.

135 Default values for both NipalsPCA and NipalsPLS are `max_iter = 10000` and `tol_criteria`
 136 = `1e-6`. These defaults manifest a slightly higher emphasis on numerical accuracy than
 137 performance.

138 If the user observes any unexpected behavior by an open_nipals PCA or PLS models, it is
 139 recommended to enable the `verbose` flag in the `fit()` and `set_components()` methods. This
 140 will print milestone markers during the fitting procedure.

141 Benchmarking

142 In order to assess open_nipals' performance, we compared it to other common dimensionality
 143 reduction techniques with missing value imputation.

144 Since the PCA and PLS implementations of open_nipals are very similar, and there
 145 are more alternative implementations of PCA than of PLS, we decided to stick to PCA
 146 for the benchmarking, expecting that observed trends translate to the performance of
 147 open_nipals.NipalsPLS module.

148 Our benchmark measures the respective runtime, peak memory consumption, and
 149 data reconstruction accuracy of the execution of the PCA model's `fit()` routine.
 150 Runtime and peak memory allocation are measured by the Python-native `time` and
 151 `tracemalloc` packages. Data reconstruction accuracy is measured as the mean difference
 152 between the (synthetic) input data and the data reconstructed by the model using a
 153 `model.inverse_transform(model.transform(data))` logic.

154 The tested dimensionality reduction and imputation techniques are:

155 1. `open_nipals.NipalsPCA` with its native Nelson's Single Component Projection.
156 2. Adding components to an existing `open_nipals.NipalsPCA` model, leveraging the
157 `set_components()` method. This is only evaluated for adding components given a
158 fixed dataset.
159 3. `sklearn.PCA` with `sklearn.SimpleImputer` for univariate imputation of sparse input
160 matrices.
161 4. `sklearn.PCA` with `sklearn.MatrixImputer` for multivariate imputation of sparse input
162 matrices.
163 5. `sklearn.FactorAnalysis` Expectation Maximization (FA/EM) procedure, combined
164 with `sklearn.SimpleImputer`.
165 6. `sklearn.FastICA` for Independent Component Analysis (ICA), combined with a
166 `sklearn.SimpleImputer`.
167 Fit tolerances were set to $1e-4$ for all methods, a maximum number of iterations was set to
168 1000 where applicable. As mentioned before, one of the upsides of `open_nipals.NipalsPCA`
169 compared to the SVD-based `sklearn.PCA` implementation is that numerical cost and
170 accuracy can be traded off by setting tolerance criteria. We found these settings place
171 `open_nipals.NipalsPCA` in the middle of the pack with regards to numerical cost, justifying
172 the setting *a posteriori*. Wherever necessary, we passed a globally initialized numpy random
173 number generator into the model constructors or fit methods. Bear in mind that the
174 NIPALS algorithm is a deterministic algorithm and therefore did not require any random state
175 initialization.

176 Dataset creation

177 A realistic synthetic dataset was created:

- 178 1. Construct an $M \times M$ random orthogonal matrix using the QR-decomposition, where M is
179 the number of features of the dataset.
180 2. Construct a Gaussian $N \times M$ random matrix, with the number of samples N . Without loss
181 of generality, decrease the variance of each of the M columns by a factor of 0.9 to mimic
182 decreasing variance of the principal components.
183 3. Multiply the matrices built in the previous two steps.
184 4. Add Gaussian random noise with amplitude 0.1.
185 5. Standardize the resulting data matrix with an `sklearn.StandardScaler`.
186 6. Randomly mask 10% of the matrix entries with `np.nan`.

187 This procedure was repeated for any combination of number of samples and features plotted
188 in the sections below.

189 **Parameter scaling of numerical cost and accuracy**

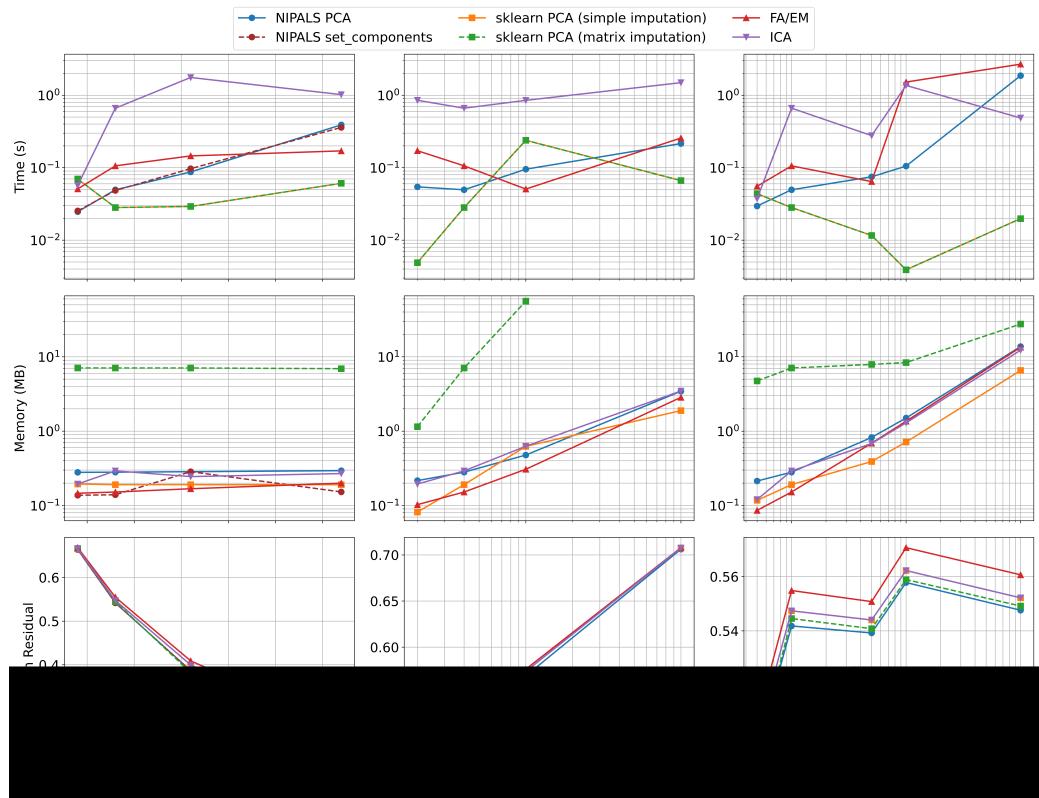


Figure 3: Computational complexity as measured by runtime (upper row), peak memory consumption (middle row), and accuracy in reconstructing the input data measured by mean residual (lower row). Scaling of quantites evaluated with respect to `n_components` (left columns), dimensionality of the dataset (number of features, middle column), and number of samples (right column).

190 Figure [Figure 3](#) compares the performance of `open_nipals.NipalsPCA`, measured by runtime,
 191 memory requirements, and accuracy, for varying `n_components`, dimensionalities, and number
 192 of samples, to alternative dimensionality reduction and imputation techniques. The data was
 193 varied around a center configuration of `n_components=4`, 100 samples, and 40 features. FastICA
 194 raised convergence warnings for `n_samples=100`, `n_features=40`, `n_components=4,8,16`,
 195 `n_samples=1000`, `n_features=40`, `n_components=4`, `n_samples=100`, `n_features=20`,
 196 `n_components=4`, `n_samples=100`, `n_features=100`, `n_components=4`. We found the
 197 accuracy of the fitted ICA models to be comparable to the others, therefore decided to still use
 198 these datapoints. Excessive memory consumption of the experimental `MatrixImputer` made
 199 the computation of the point at `n_features=4`, 100 samples, and 1000 features numerically
 200 impossible. Therefore, this datapoint was omitted in the middle plot of the middle row in
 201 [Figure 3](#).

202 **Computational complexity and accuracy vs. number of latent variables**

203 The leftmost column of [Figure 3](#) shows complexity and accuracy evaluated over a range of 2 -
 204 16 latent variables.

205 The upper left plot of [Figure 3](#) shows the runtime cost of `open_nipals.NipalsPCA` increases
 206 with `n_components` at less-than exponential scaling, with ICA being by far the slowest option
 207 for `n_components>4`, FA/EM being the slowest option at `n_components<=4`, and `sklearn.PCA`
 208 the fastest option globally. `sklearn.PCA` relies on SVD, therefore the numerical cost (runtime

209 and memory) of inferring any number of latent variables is equal, which can be seen from the
 210 upper left and the middle left panel of [Figure 3](#). The two different imputer + sklearn.PCA
 211 combinations require precisely the same runtime at varying n_components, indicating that the
 212 runtime is dominated by the SVD, not the imputation. Interestingly, fitting only additional
 213 components with set_components() has a minor effect on runtime at small n_components,
 214 and is even slower than fitting a new open_nipals.NipalsPCA model at n_components=16.

215 The peak memory consumption vs. n_components is plotted in the left plot of the middle
 216 row of [Figure 3](#). The comparison of sklearn.PCA paired with either MatrixImputer or
 217 SimpleImputer immediately suggests that the memory consumption is dominated by the
 218 imputation method rather than the actual PCA model fitting. The memory consumption of
 219 open_nipals.NipalsPCA can be substantially reduced by adding components to an existing
 220 model with the set_components method, yielding the lowest memory consumption of all
 221 investigated methods at n_components<16.

222 This comes with very competitive accuracy of open_nipals.NipalsPCA, only mildly surpassed
 223 by the MatrixImputation + sklearn.PCA scenario. As expected, the choice of imputation
 224 method starts to matter at higher n_components, i.e. more predictive models.

225 Computational complexity and accuracy vs. number of features

226 The middle column of [Figure 3](#) shows the scaling of computational complexity and accuracy
 227 with the dimensionality of the data space (n_features 20 - 1000). Since a new model has
 228 to be fit every time the dimensionality of the data space changes, there is no usecase for
 229 set_components(), which was consequently not displayed in the plots.

230 The upper middle panel of [Figure 3](#) shows that open_nipals.NipalsPCA' model fitting has a
 231 high but competitive runtime, surpassed by FA/EM at small and ICA at 100 features, while
 232 being the slowest option at 1000 features. Its runtime in seconds vs. dimensionality curve is
 233 quite shallow (5e-2 - 2e-1), even compared to sklearn.PCA (5e-3 - 7e-2), which was faster
 234 than its competitors by 1-2 orders of magnitude throughout the investigated range of features.
 235 However, sklearn.PCA shows a clear exponential scaling in runtime caused by the scaling of
 236 the numerical complexity of the SVD algorithm with the number of features (bear in mind
 237 that sklearn.PCA performs an SVD of the covariance matrix, which is a square matrix of the
 238 size of the number of features).

239 The memory consumption in MB vs. features is plotted in the middle row's middle plot of
 240 [Figure 3](#). It shows a competitive memory usage by open_nipals.NipalsPCA, particularly at
 241 >= 100 features. Memory usage at very small number of features (less than 100) seems to be
 242 dominated by overhead, of which open_nipals.NipalsPCA requires more than more orthodox
 243 algorithms like sklearn.PCA with univariate imputation. However, as the dimensionality
 244 increases, the overhead becomes less relevant, leading to open_nipals.NipalsPCA's memory
 245 consumption curve being the shallowest (2e-1 - 3.5), compared to 8e-2MB - 2MB for
 246 sklearn.PCA with univariate imputation. The matrix imputation technique proves once
 247 more to require the most memory throughout the entire investigated range of dataspace
 248 dimensions by 1 - 2 orders of magnitude, leading to the data point at 1000 features being
 249 impossible to collect.

250 The accuracy is evaluated against different number of features in the lower middle panel of
 251 [Figure 3](#). It proves open_nipals.NipalsPCA to be the most accurate technique, although the
 252 accuracies being very similar among the different methods. All methods lose accuracy as the
 253 number of dimensions increases.

254 Computational complexity and accuracy vs. number of samples

255 The rightmost column of [Figure 3](#) displays computational complexity and accuracy over a
 256 broad range of dataset size (n_samples 50 - 10000).

257 The runtime requirement scaling with n_samples can be found in the upper right plot of [Figure 3](#).
258 It shows competitive, but rather slow fit time by open_nipals.NipalsPCA, comparable to its
259 competitors FA/EM and ICA. Both imputation methods combined with sklearn.PCA are more
260 than one order of magnitude faster and show very shallow profiles. The number of samples
261 does not directly play a role in SVD-based PCA since the bottleneck is the computation of the
262 eigenvalues of the n_features x n_features covariance matrix, thus the shallow sklearn.PCA
263 runtime curves are expected.

264 As can be seen from the middle right panel of [Figure 3](#), the peak memory consumption
265 of open_nipals.NipalsPCA follows exponential scaling with n_samples ([Figure 3](#)), which
266 is competitive with its alternatives. The two sklearn.PCA scenarios form the edge cases,
267 here. SVD-based PCA with univariate imputation consumes a bit less memory than ICA,
268 FA/EM, and open_nipals.NipalsPCA at medium to large sample sizes, while SVD-based PCA
269 with multivariate imputation consumes about one order of magnitude more memory than its
270 competitors, although this gap closes with increasing sample size.

271 The numerical accuracy as a function of n_samples in [Figure 3](#) remains quite constant, with
272 open_nipals.NipalsPCA being slightly more accurate than all of its competitors throughout
273 the entire range.

274 Benchmarking conclusion

275 Compared to other standard dimensionality reduction methods, open_nipals.NipalsPCA
276 convinces with highest accuracy, competitive memory consumption and runtime cost at
277 low to medium n_components. The algorithmic benchmark is amended by the possibility of
278 freely trading numerical accuracy for numerical cost, and minimizing memory consumption
279 through using set_components() to add new principal components to a pre-existing model.
280 This makes it particularly valuable and superior in settings with small- to medium-sized and
281 -dimensional datasets, with low number of latent variables.

282 Availability

283 open_nipals is available open-source under the BSD 3-clause license from this github repository
284 (?). We appreciate your feedback and contributions. The latest version is deployed to the
285 Python Package Index ([Schlusser, 2026b](#)), the documentation was deployed to readthedocs
286 ([Schlusser, 2026a](#)). A jupyter notebook to reproduce [Figure 1](#), another notebook to generate
287 [Figure 2](#), and a notebook to run the benchmark and generate [Figure 3](#) can be found in (?)
288 under the paper branch. A permanent DOI to the version of the code referred to by this paper
289 can be found in ([Schlusser, 2026c](#)).

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