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BellDiagonalQudits: A package for entanglement analyses of mixed maximally entangled qudits

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Software

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Summary

In the field of quantum information and technology, entanglement in quantum systems called qudits is regarded as resource for quantum-based or quantum-assisted information processing tasks. It allows new ways of information processing like quantum computation or quantum teleportation and provides possibilities for speedups in a variety of algorithms with applications like search or optimization. Despite its theoretical and practical relevance, there is no general method to determine whether a given quantum state is entangled or not due to a special and hard to detect form of entanglement called bound entanglement. Bipartite Bell states are maximally entangled states of two qudits and are of high relevance for application in quantum technologies. Mixtures of those states can be entangled, including its bound form, or separable. Leveraging their special properties, Bell states can be numerically generated and analyzed for their entanglement properties by various methods implemented in this Julia package.

Statement of need

`BellDiagonalQudits` is an Quantum Information-affiliated Julia package for generation and entanglement analyses of mixed maximally entangled bipartite qudits in general dimension. The API for `BellDiagonalQudits` provides an user-friendly interface to generate representations of Bell diagonal quantum states and to analyze their entanglement properties with various general or specialized criteria to detect entanglement or separability. Leveraging geometric properties of a certain class of mixed Bell states that are related by Weyl transformations, `BellDiagonalQudits` combines known analytical results (Baumgartner et al., 2007) and numerical methods for quantum state representation and analysis. It leverages and depends on the Julia package `QuantumInformation`, the convex sets of `LazySets` (Forets & Schilling, 2021) and the optimization methods of `Optim` (Mogensen & Riseth, 2018).

`BellDiagonalQudits` was designed to be used by researchers in quantum science and quantum information theory. It has already been used in multiple scientific publications (Popp & Hiesmayr, 2022a; Popp & Hiesmayr, 2022b) in the context of entanglement classification and detection of bipartite qudits in dimension three and four. The combination of efficient state generation via random sampling or deterministic procedures and implementation of both frequently used and specialized entanglement and separability detectors supports the research of entanglement in several ways. From a general point of view, entangled Bell states are well accessible for powerful methods of entanglement and separability detection, leveraging their symmetries and geometric properties. It was shown in (Hiesmayr, 2021; Popp & Hiesmayr, 2022a; Popp & Hiesmayr, 2022b) that a significant share of the group of mixed Bell states related by Weyl transformations are bound entangled, offering a systematic way to generate and investigate those states with respect to the separability problem in different dimensions. In addition to general methods applicable to any Bell diagonal state, `BellDiagonalQudits`

provides features to generate the special symmetries of Bell states that are related via Weyl transformations. These symmetries are leveraged for improved entanglement classification and the numerical generation of specialized entanglement witnesses in any dimension. Furthermore, the implemented methods of `BellDiagonalQudits` can be used in various quantum information processing tasks involving Bell diagonal states in any dimension like Quantum Key Distribution or entanglement verification. `BellDiagonalQudits` uses and integrates well with the general interface of `QuantumInformation` allowing the investigation of Bell states in the context of quantum channels, entanglement measures or entropy.

Relation to research projects

The methods of `BellDiagonalQudits` to generate and analyze Bell diagonal states in general dimension are based on analytical properties summarized in (Baumgartner et al., 2007). Extensions of those methods and efficient implementation in `BellDiagonalQudits` enabled the detailed analysis (Popp & Hiesmayr, 2022a) of Bell diagonal qudits in three dimensions (qutrits), providing an operational solution to the separability problem for those states. Additionally the relative shares of separable and (bound) entangled states were precisely determined among the Bell diagonal states. In (Popp & Hiesmayr, 2022b), higher dimensions were considered, focusing on a detailed comparison and geometric properties of separable states in dimension three and four.

Package information

`BellDiagonalQudits` is available on Github at https://github.com/kungfugo/BellDiagonalQudits.jl. The package documentation is available at https://kungfugo.github.io/BellDiagonalQudits.jl/dev/ and provides examples of usage.

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