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ler : LVK (LIGO-Virgo-KAGRA collaboration) event (compact-binary mergers) rate calculator and simulator

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Summary

Gravitational waves (GWs) are ripples in the fabric of space and time caused by the acceleration of unevenly distributed mass or masses. Observable GWs are created especially during the violent cosmic events of merging compact binaries, such as 'binary black holes' (BBHs), 'binary neutron stars' (BNSs), and 'neutron star and black hole pair' (NSBHs). GWs emitted by these events can be distorted or magnified by the gravitational fields of massive objects such as galaxies or galaxy clusters, a phenomenon known as gravitational lensing. Profound comprehension of gravitational lensing's impact on GW signals is imperative to their accurate interpretation and the extraction of astrophysical insights therein. For this purpose, statistical modelling of GWs lensing can provide valuable insights into the properties of the lensing objects and GW sources. Such statistics require accurate and efficient means to calculate the detectable lensing rates, which depend on up-to-date modeling and implementation of lens and source properties and their distribution. The outcomes of these computational analyses not only contribute to generating dependable forecasts but also play an important role in validating forthcoming lensing events (Janquart et al., 2023) (Collaboration et al., 2023) (Abbott et al., 2021).

Obtaining precise outcomes in statistical analyses of this nature necessitates the utilization of large-scale sampling, often numbering in the millions. However, this process is computationally demanding. The ler framework addresses this by employing innovative techniques to optimize the workflow and computation efficiency required for handling large-scale statistical analyses, essential for modeling detectable events and calculating rates. Its integration of modular statistical components enhances the framework's adaptability and extendability, thus proving to be an invaluable asset in the evolving field of gravitational wave research. Detailed description, source code, and examples are available in ler documentation.

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Statement of need

ler is a statistics-based Python package specifically designed for computing detectable rates of both lensed and unlensed GW events, catering to the requirements of the LIGO-Virgo-KAGRA Scientific Collaboration (The LIGO Scientific Collaboration et al., 2015) (Acernese et al., 2014) (Akutsu et al., 2020) and astrophysics research scholars. The core functionality of ler intricately hinges upon the interplay of various components which include sampling the properties of compact-binary sources, lens galaxies characteristics, solving lens equations to derive properties of resultant images, and computing detectable GW rates. This comprehensive functionality builds on the leveraging of array operations and linear algebra from the *numpy* (NumPy Community, 2022) library, enhanced by interpolation methods from *scipy* (Virtanen et al., 2020) and Python's *multiprocessing* capabilities. Efficiency is further boosted by the *numba* (Lam et al., 2022) library's Just-In-Time (*njit*) compilation, optimizing extensive numerical computations and employing the inverse transform sampling method to replace more cumbersome rejection sampling. The modular design of ler not only optimizes speed and functionality but also ensures adaptability and upgradability, supporting the integration of additional statistics as research evolves. Currently, ler is an important tool in generating simulated GW events—both lensed and unlensed—and provides astrophysically accurate distributions of event-related parameters for both detectable and non-detectable events. This functionality aids in event validation and enhances the forecasting of detection capabilities across various GW detectors to study such events. The architecture of the ler API facilitates seamless compatibility with other software packages, allowing researchers to integrate and utilize its functionalities based on specific scientific requirements.

Design and Structure

The architecture of the ler API is deliberately organized such that each distinct functionality holds its own significance in scientific research. Simultaneously, these functionalities seamlessly integrate and can be employed collectively to accommodate diverse scientific objectives. Key features of ler and its dependencies can be summarized as follows:

- Sampling GW source properties:
  - For the unlensed events, the sampling distribution $(R_m(z_s))$ for the source's redshift (z_s) is derived from the merger rate density of compact binaries, which, in turn, is based on the star formation rate. The code is meticulously designed to enable the straightforward integration of future updates or user-specified distributions of these sources.
  - Intrinsic and extrinsic parameters (θ) of GW sources are sampled using prior distributions $(P(\theta))$ from the *gwcosmo* (gwcosmo Contributors, 2022) and *bilby* (Ashton et al., 2019) packages, with options for users to input custom distributions.
- Sampling of lens galaxy attributes and source red-shifts:
  - For the lensed case, the source redshift (z_s) is sampled under the strong lensing condition (SL), based on the precomputed probability of strong lensing with a source at z_s (optical depth: $P(\text{SL}|z_s)$ or $\tau(z_s)$). This probability can be recalculated for specified configurations of lens galaxies, leveraging *multiprocessing* and *njit* functionalities for enhanced efficiency.
  - Following Wierda et al. (2021), the package utilizes the Elliptical Power Law with external shear (EPL+Shear) model (Wempe et al., 2022) for sampling the galaxy parameters (θ_L). Rejection sampling is applied on the above samples depending on whether the event is strongly lensed or not, $P(\text{SL}|z_s, \theta_L)$.
- Generation of image properties:
  - The source position (β) is sampled from the caustic in the source plane.
  - Sampled lens properties and source positions are fed to *lenstronomy* (Birrer et al., 2021) to generate properties of the images. This is the slowest part of the entire

    simulation, which ler tackles through parallelization with multiprocessing.
  - Image properties like magnification (μ_i) and time delay (Δt_i) modify the original source signal strength, changing the signal-to-noise ratio (SNR) and our ability to detect.
- Calculation of detectable merger rates per year:
  - The calculation of rates necessitates integration over simulated events that meet specific detection criteria. This process includes computing SNRs (ρ) for each event or its lensed images, followed by an assessment against a predetermined threshold(s) (ρ_{th}).
  - SNR calculations are optimized using *gwsnr* python package, leveraging interpolation and multiprocessing for accuracy and speed.
  - Simulated events and rate results, along with input configurations, are systematically archived for easy access and future analysis. Additionally, all interpolators used in the process are preserved for future applications.
  - Most cosmology-related calculations within the *ler* package are performed using the *astropy* library (Astropy Collaboration, 2013). The default cosmological model is LambdaCDM ($H_0 = 70$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$); however, users have the flexibility to employ any cosmology available in *astropy*. All internal calculations in *ler* will then be based on the user-selected cosmological model.

Equations

Detectable Unlensed rates:

$$R_U = \int dz_s \frac{dV_c}{dz_s} \frac{R_m(z_s)}{1+z_s} \left\{ \Theta[\rho(z_s, \theta) - \rho_{th}] P(\theta) d\theta \right\}$$

z_s: GW source redshift, $\frac{dV_c}{dz_s}$: Differential co-moving volume, $\frac{1}{1+z_s}$: Time dilation correction factor, $R_m(z_s)$: source frame merger rate density, θ: GW source parameters, P: probability distribution, ρ: SNR, ρ_{th}: SNR threshold, Θ: Heaviside function to select detectable GW events.

Detectable Lensed rates:

$$\begin{aligned} R_L = \int dz_s \frac{dV_c}{dz_s} \tau(z_s) \frac{R_m(z_s)}{1+z_s} \mathcal{O}_{images}(z_s, \theta, \mu_i, \Delta t_i, \rho_{th}) \\ P(\theta) P(\theta_L | \text{SL}, z_s) P(\beta | \text{SL}) d\theta d\beta d\theta_L dz_s \end{aligned}$$

$\tau(z_s)$: Optical-depth of strong lensing, θ_L: lens parameters, β: source position, μ: image magnification, Δt: image time delay, $\mathcal{O}$: operator to select dectectable lensed GW events, i: index of images of a lensed event, SL: strong lensing condition.

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References

Abbott, R., Abbott, T. D., Abraham, S., Acernese, F., Ackley, K., Adams, A., Adams, C., Adhikari, R. X., Adya, V. B., Affeldt, C., Agarwal, D., Agathos, M., Agatsuma, K., Aggarwal, N., Aguiar, O. D., Aiello, L., Ain, A., Ajith, P., Aleman, K. M., … Zweizig, J. (2021). Search for lensing signatures in the gravitational-wave observations from the first half of LIGO–virgo's third observing run. *The Astrophysical Journal*, *923*(1), 14. https://doi.org/10.3847/1538-4357/ac23db

Acernese, F., Agathos, M., Agatsuma, K., Aisa, D., Allemandou, N., Allocca, A., Amarni, J., Astone, P., Balestri, G., Ballardin, G., Barone, F., Baronick, J.-P., Barsuglia, M., Basti, A., Basti, F., Bauer, T. S., Bavigadda, V., Bejger, M., Beker, M. G., … Zendri, J.-P. (2014). Advanced virgo: A second-generation interferometric gravitational wave detector. *Classical and Quantum Gravity*, *32*(2), 024001. https://doi.org/10.1088/0264-9381/32/2/024001

Akutsu, T., Ando, M., Arai, K., Arai, Y., Araki, S., Araya, A., Aritomi, N., Aso, Y., Bae, S., Bae, Y., Baiotti, L., Bajpai, R., Barton, M. A., Cannon, K., Capocasa, E., Chan, M., Chen, C., Chen, K., Chen, Y., … Zhu, Z.-H. (2020). Overview of KAGRA: Detector design and construction history. *Progress of Theoretical and Experimental Physics*, *2021*(5), 05A101. https://doi.org/10.1093/ptep/ptaa125

Ashton, G., Hübner, M., Lasky, P. D., Talbot, C., Ackley, K., Biscoveanu, S., Chu, Q., Divakarla, A., Easter, P. J., Goncharov, B., Vivanco, F. H., Harms, J., Lower, M. E., Meadors, G. D., Melchor, D., Payne, E., Pitkin, M. D., Powell, J., Sarin, N., … Thrane, E. (2019). Bilby: A user-friendly bayesian inference library for gravitational-wave astronomy. *The Astrophysical Journal Supplement Series*, *241*(2), 27. https://doi.org/10.3847/1538-4365/ab06fc

Astropy Collaboration. (2013). Astropy: A community Python package for astronomy. *Astronomy and Astrophysics*, *558*. https://doi.org/10.1051/0004-6361/201322068

Birrer, S., Shajib, A. J., Gilman, D., Galan, A., Aalbers, J., Millon, M., Morgan, R., Pagano, G., Park, J. W., Teodori, L., Tessore, N., Ueland, M., Vyvere, L. V. de, Wagner-Carena, S., Wempe, E., Yang, L., Ding, X., Schmidt, T., Sluse, D., … Amara, A. (2021). Lenstronomy II: A gravitational lensing software ecosystem. *Journal of Open Source Software*, *6*(62), 3283. https://doi.org/10.21105/joss.03283

Collaboration, T. L. S., Virgo Collaboration, the, KAGRA Collaboration, the, Abbott, R., Abe, H., Acernese, F., Ackley, K., Adhicary, S., Adhikari, N., Adhikari, R. X., Adkins, V. K., Adya, V. B., Affeldt, C., Agarwal, D., Agathos, M., Aguiar, O. D., Aiello, L., Ain, A., Ajith, P., … Zweizig, J. (2023). *Search for gravitational-lensing signatures in the full third observing run of the LIGO-virgo network*. https://arxiv.org/abs/2304.08393

gwcosmo Contributors. (2022). gwcosmo: A Python package for gravitational-wave cosmology.

In *GitHub repository*. GitHub. https://github.com/gwcosmo/gwcosmo

Janquart, J., Wright, M., Goyal, S., Chan, J. C. L., Ganguly, A., Garrón, Á., Keitel, D., Li, A. K. Y., Liu, A., Lo, R. K. L., Mishra, A., More, A., Phurailatpam, H., Prasia, P., Ajith, P., Biscoveanu, S., Cremonese, P., Cudell, J. R., Ezquiaga, J. M., … Veitch, J. (2023). Follow-up analyses to the O3 LIGO–Virgo–KAGRA lensing searches. *Monthly Notices of the Royal Astronomical Society*, *526*(3), 3832–3860. https://doi.org/10.1093/mnras/stad2909

Lam, S., Pitrou, S., & Seibert, M. (2022). Numba: A high performance python compiler. In *Numba Documentation*. Anaconda, Inc. https://numba.pydata.org/

NumPy Community. (2022). NumPy: A fundamental package for scientific computing with python. In *NumPy Website*. NumPy. https://numpy.org/

The LIGO Scientific Collaboration, Aasi, J., Abbott, B. P., Abbott, R., Abbott, T., Abernathy, M. R., Ackley, K., Adams, C., Adams, T., Addesso, P., Adhikari, R. X., Adya, V., Affeldt, C., Aggarwal, N., Aguiar, O. D., Ain, A., Ajith, P., Alemic, A., Allen, B., … Zweizig, J. (2015). Advanced LIGO. *Classical and Quantum Gravity*, *32*(7), 074001. https://doi.org/10.1088/0264-9381/32/7/074001

Virtanen, P., Gommers, R., Oliphant, T. E., Haberland, M., Reddy, T., Cournapeau, D., Burovski, E., Peterson, P., Weckesser, W., Bright, J., Walt, S. J. van der, Brett, M., Wilson, J., Millman, K. J., Mayorov, N., Nelson, A. R. J., Jones, E., Kern, R., Larson, E., … Contributors, S. 1.0. (2020). SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. In *Nature Methods*. SciPy. https://www.scipy.org/

Wempe, E., Koopmans, L. V. E., Wierda, A. R. A. C., Hannuksela, O. A., Agnello, A., Bonvin, C., Bucciarelli, B., Camera, C., Czoske, O., Finke, C., & others. (2022). *A lensing multi-messenger channel: Combining LIGO-virgo-kagra lensed gravitational-wave measurements with euclid observations*. https://arxiv.org/abs/2204.08732

Wierda, A. R. A. C., Wempe, E., Hannuksela, O. A., Koopmans, L. V. E., Agnello, A., Bonvin, C., Bucciarelli, B., Camera, C., Czoske, O., Finke, C., & others. (2021). Beyond the detector horizon: Forecasting gravitational-wave strong lensing. *The Astrophysical Journal*, *921*(1), 154. https://doi.org/10.3847/1538-4357/ac1bb4