

¹ AlgebraicAgents.jl: Hierarchical Composition of Multi-Formalism Dynamical Systems

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⁶ Summary

⁷ AlgebraicAgents.jl is a Julia framework for hierarchical dynamical systems modeling that
⁸ treats formalism as a per-node choice rather than a global constraint. Differential equations,
⁹ discrete-event systems, and agent-based models coexist within a single hierarchy, coupled
¹⁰ through a minimal stepping interface. The framework supports annotation of information
¹¹ flows between components, visualizing model architecture, and querying relationship structure.
¹² Native wrappers for Julia's scientific computing ecosystem enable practitioners to compose
¹³ domain-specific models into representations of complex dynamics.

¹⁴ Statement of Need

¹⁵ Modeling dynamical systems at scale—enterprises with interacting business units, multi-physics engineering systems, coupled socioeconomic networks—requires composing components naturally expressed in different formalisms and operating at different temporal granularities. Continuous dynamics govern reactors, discrete events drive logistics, and agent-based rules capture decisions. These subsystems must integrate while remaining independently developable, often beginning as mockups and refined iteratively as data arrives.

²¹ This challenge decomposes into three sub-problems:

- ²² ▪ **Multi-formalism coupling.** Subsystems should be expressible in different formalisms, such as ODEs, discrete-time systems, or agent-based models, at varying granularity, with the framework facilitating their coupling.
- ²³ ▪ **Hierarchical modularity.** Subsystems should support independent development, validation, and reuse as building blocks within larger models.
- ²⁴ ▪ **Semantic transparency.** Visualizing and querying information flows across models should support both validation and explainability.

²⁹ State of the Field

³⁰ Prior work addresses aspects of this challenge. The Functional Mock-up Interface ([Blochwitz et al., 2011](#)) standardizes co-simulation of black-box models but imposes protocol overhead suited to industrial interoperability rather than rapid prototyping. Meta-modeling frameworks like the Generic Modeling Environment ([Lebedzki et al., 2001](#)) operate at a higher abstraction, enabling construction of domain-specific formalisms. The Ptolemy project ([Eker et al., 2003](#)) and Lingua Franca ([Menard et al., 2023](#)) provide principled foundations for heterogeneous component interaction across concurrent, real-time, and distributed settings.

³⁷ Within the Julia ecosystem ([Bezanson et al., 2017](#)), ModelingToolkit.jl ([Ma et al., 2021](#); [Rackauckas & Nie, 2017](#)), and its commercial extension, JuliaHub's Dyad, excel at symbolic-

39 numeric modeling and equation-based composition but do not naturally accommodate discrete
 40 or agent-based dynamics. AlgebraicDynamics.jl (Baez et al., 2023; Brown et al., 2022) brings
 41 categorical semantics to dynamical systems yet enforces strict interface typing that constrains
 42 exploratory work.
 43 AlgebraicAgents.jl relaxes formalism and shifts focus towards compositional flexibility. Any
 44 agent can access any other agent's state, and synchronization is temporal rather than type-
 45 enforced. This design prioritizes iteration speed and introspection over type-enforced interface
 46 contracts, a trade-off suited to exploratory modeling, where specifications evolve alongside
 47 understanding.

48 Software Design

49 The central abstraction of the framework is the *agent*, which serves a dual role. First, an agent
 50 implements a dynamical system with a custom evolution rule, exposing its internal clock and
 51 state variables as observable quantities to other agents. Second, an agent acts as a node in a
 52 rooted tree hierarchy, serving as a container for nested child agents, each of which is itself an
 53 agent with this dual character.

54 Agents are implemented as Julia structures that subtype AbstractAlgebraicAgent. The
 55 @agent macro provides a lightweight inheritance mechanism, automatically including common
 56 interface fields while permitting user-defined fields:

```

@agent struct InventoryAgent
  stock_level::Int
  reorder_time::Int
end
  
```

57 Synchronized Evolution

58 Each agent implements `_step!`, which advances its state and returns the time on its internal
 59 clock, that is, the furthest point for which its trajectory has been computed. The simulation
 60 loop coordinates agents by identifying the minimum projected time across the hierarchy and
 61 stepping only those agents at that frontier.

62 This mechanism is exemplified below for the case of three agents A, B, and C.

Global Step	Agent A ($\Delta t=1$)	Agent B ($\Delta t=1.5$)	Agent C ($\Delta t=3$)	Frontier (min)	Stepped
0 (init)	0	0	0	0	—
1	1	1.5	3	1	A, B, C
2	2	1.5	3	1.5	A
3	2	3	3	2	B
4	3	3	3	3	A
5	4	4.5	6	4	A, B, C

63 Formally, the single step of the simulator is defined as follows.

```

function step!(a::Agent, t=projected_to(a))
  # Recurse depth-first.
  t_min = minimum(step!(c, t) for c in children(a); init=nothing)

  # Step only agents at the time frontier.
  projected_to(a) == t && _step!(a)
  
```

```

    return something(t_min, projected_to(a))
end

```

- 64 For models where an agent with a shorter step queries another agent already projected further
 65 ahead, *observables* mitigate temporal inconsistency. Agents expose state variables through
 66 `gettimeobservable`, which can implement interpolation or extrapolation logic.
- 67 We note that during the evolutionary step, any agent in the system can be accessed and
 68 modified.
- 69 Beyond evolutionary stepping, the framework supports three callback types:

Callback Type	Execution Timing	Typical Use Case
Futures	At predetermined time	Scheduled events, delayed triggers
Controls	Every solver step	Invariant enforcement, monitoring
Instantaneous	Within current step	Intra-step coordination, priority-ordered effects

70 Topology, Wires, and Relations

- 71 Each agent occupies a node in a tree topology. Path-based references enable navigation:
 72 `getagent(a, ".../sibling/child")`. Container agents with trivial evolution organize
 73 subsystems into logical compartments, enabling modular development and hierarchical
 74 visualization.
- 75 Beyond the execution hierarchy, *wires* explicitly declare directed information flows between
 76 agents. While agents can programmatically access any other agent's state, declared wires serve
 77 as documentation and enable dependency analysis:

```

add_wire!(full_system,
  from=factory_a_main, to=inventory_central_main,
  from_var_name="output_volume", to_var_name="incoming_stock")

```

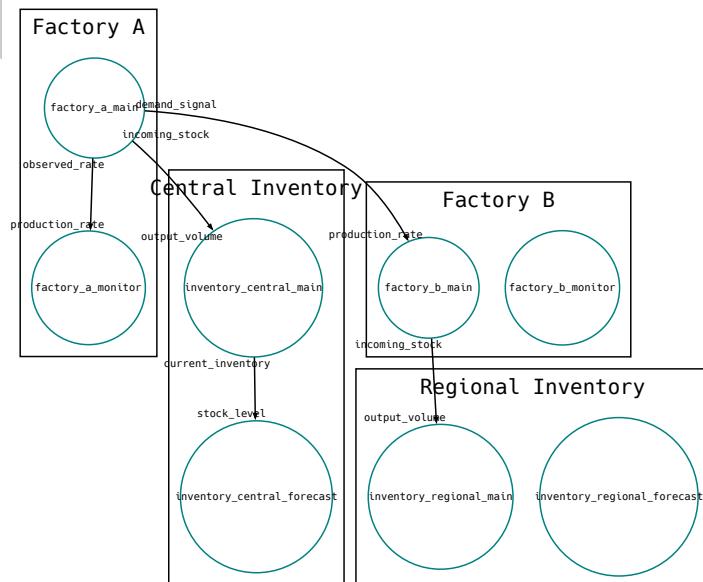


Figure 1: Visualization of agent hierarchy and information flows. Grey dashed arrows represent parent-child edges and solid black arrows represent information flows. Black rectangles group related agents.

78 *Concepts* represent atemporal notions—resources, constraints, abstractions—that participate
 79 in relations alongside agents. This enables modeling of “what” (materials, approvals, markets)
 80 separate from “how” (processes), supporting dependency queries and ontological visualization.
 81 Both agents and concepts belong to the union type `RelatableType`, enabling *relations* to
 82 connect any pair with typed labels:

```
c_finished_good = Concept("FinishedGood", Dict(:type => "product"))
add_relation!(factory_a, c_finished_good, :produces)
```

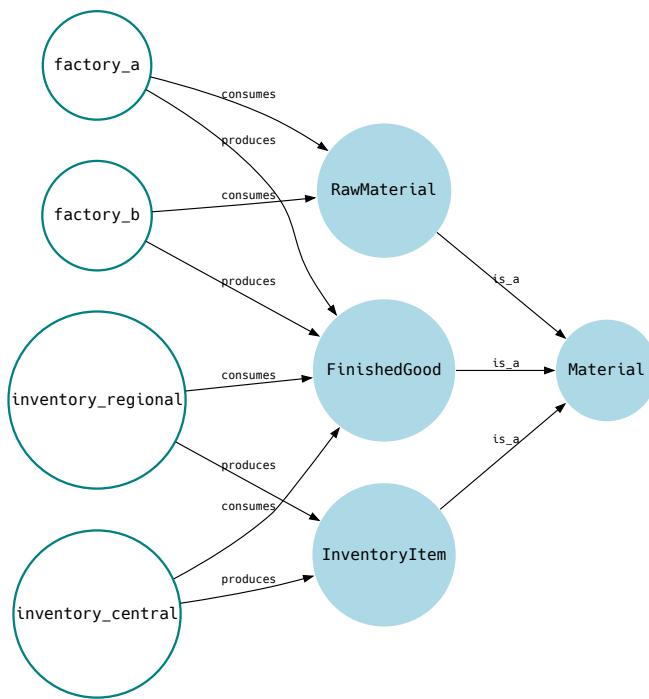


Figure 2: Diagram of concepts and relations. Blue filled circles denote Concepts and green outlined circles denote Agents. Directed arrows indicate relations between two relatable types.

83 These semantic annotations support Graphviz visualization and structured queries over model
 84 architecture.

85 Research Impact Statement

86 Integrations

87 AlgebraicAgents.jl provides native wrappers for Julia’s scientific modeling ecosystem.
 88 DiffEqAgent wraps `DEProblem` instances from DifferentialEquations.jl, enabling ODEs,
 89 SDEs, DDEs, and DAEs to participate in hierarchical simulations. Integration with
 90 Agents.jl (Datseris et al., 2022) allows agent-based models to compose with continuous
 91 or discrete dynamical systems. GraphicalAgent wraps `AbstractResourceSharer` or
 92 `AbstractMachine` from AlgebraicDynamics.jl, providing compatibility with categorical
 93 composition patterns. Visualization functions generate Graphviz DOT and Mermaid diagram
 94 output for documentation.

95 Availability and Documentation

96 AlgebraicAgents.jl is registered in Julia's General registry. [API documentation](#) and a
97 [comprehensive example](#)—a synthetic pharmaceutical company model—are available online.
98 Contributions welcome via [GitHub](#). The framework is licensed as MIT license.

99 Applications and Value Modeling Ecosystem

100 The framework has been applied to develop proprietary pharmaceutical value chain models.
101 Two companion packages by the authors extend this foundation: [ReactiveDynamics.jl](#), providing
102 chemical reaction network–inspired syntax for business process modeling natively compatible
103 with AlgebraicAgents.jl, and [CEEDesigns.jl](#), implementing Bayesian cost-efficient experimental
104 design for drug discovery.

105 AI Usage Disclosure

106 The authors used GitHub Copilot for inline code suggestions and Claude Opus 4.5 (Anthropic)
107 for editorial refinement of the manuscript. The authors reviewed and validated all AI-suggested
108 edits and bear full responsibility for the final work.

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