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polyCub: An R package for Integration over Polygons

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Software

- Review
- Repository
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Summary

The integral of a continuously differentiable function $f(x, y)$ over a domain $W \subset \mathbb{R}^2$ can be approximated using an n-point cubature rule of the form

$$\iint_W f(x, y)\,\mathrm{d}x\,\mathrm{d}y \approx \sum_{i=1}^{n} w_i f(x_i, y_i)\,,$$

i.e., a weighted sum of function values at an appropriate set of nodes. In the special but common case of integration along the axes, i.e., $W = (x_l, x_u) \times (y_l, y_u)$, the domain is *rectangular*. Several software packages implement numerical integration over such rectangles (or *hypercubes* in higher dimensions), for example, the Cuba (Hahn, 2005) or cubature (Johnson, 2017) libraries, which are both interfaced from the `cubature` package (Narasimhan, Johnson, Hahn, Bouvier, & Kiêu, 2018) in R (R Core Team, 2018).

In spatial statistics, however, the domains of interest typically correspond to geographic regions (administrative districts, lakes, etc.), which are described by *polygons*. Solving integrals over such complex domains requires specialized cubature methods, thus the R package `polyCub`. A simple graphical summary of the purpose of `polyCub` is given by its logo (see below).

`polyCub` implements the following methods for numerical integration over polygons:

- General-purpose *product Gauss cubature* (Sommariva & Vianello, 2007)
- Simple *two-dimensional midpoint rule* via `spatstat` (Baddeley & Turner, 2005)
- Adaptive cubature for *radially symmetric functions* $f(x, y) = f_r(\|(x - x_0, y - y_0)\|)$ via integration along the polygon boundary (Meyer & Held, 2014, Supplement B)
- Accurate (but slow) integration of the *bivariate Gaussian density* based on polygon triangulation (Abramowitz & Stegun, 1972, Section 26.9, Example 9)

Figure 1: `polyCub`: cubature over polygonal domains.

Usage

The R package `polyCub` is released on the Comprehensive R Archive Network (CRAN) and can thus be easily installed using `install.packages("polyCub")` in R. After that, the basic usage is

```
library("polyCub")
polyCub(polyregion, f)
```

where `polyregion` is the integration domain and `f` is the integrand. Details are given in

```
vignette("polyCub")
```

which exemplifies the implemented cubature methods by solving the integral displayed in the package logo.

`polyCub` is currently used by at least two other R packages: in `surveillance`, to evaluate the likelihood of self-exciting spatio-temporal point process models for infectious disease spread (Meyer, Held, & Höhle, 2017), and in `rase`, to integrate bivariate Gaussian densities for phylogeographic analyses (Quintero, Keil, Jetz, & Crawford, 2015).

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