BGGM: Bayesian Gaussian Graphical Models in R

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The `R` package **BGGM** provides tools for making Bayesian inference in Gaussian graphical models (GGM). The methods are organized around two general approaches for Bayesian inference: (1) estimation and (2) hypothesis testing. The key distinction is that the former focuses on either the posterior or posterior predictive distribution (Gelman, Meng, & Stern, 1996; see section 5 in Rubin, 1984) , whereas the latter focuses on model comparison with the Bayes factor (Jeffreys, 1961; Kass & Raftery, 1995).

What is a Gaussian Graphical Model ?

A Gaussian graphical model captures conditional (in)dependencies among a set of variables. These are pairwise relations (partial correlations) controlling for the effects of all other variables in the model.

Applications

The Gaussian graphical model is used across the sciences, including (but not limited to) economics (Millington & Niranjan, 2020), climate science (Zerenner, Friederichs, Lehnertz, & Hense, 2014), genetics (Chu, Weiss, Carey, & Raby, 2009), and psychology (Rodriguez, Williams, Rast, & Mulder, 2020).

Overview

The methods in **BGGM** build upon existing algorithms that are well-known in the literature. The central contribution of **BGGM** is to extend those approaches:

1. Bayesian estimation with the novel matrix-F prior distribution (Mulder & Pericchi, 2018)

   - Estimation (Williams, 2018)

2. Bayesian hypothesis testing with the matrix-F prior distribution (Williams & Mulder, 2019)

   - Exploratory hypothesis testing
   - Confirmatory hypothesis testing

3. Comparing Gaussian graphical models (Williams, 2018; Williams, Rast, Pericchi, & Mulder, 2020)

- [Partial correlation differences](#)
- [Posterior predictive check](#)
- [Exploratory hypothesis testing](#)
- [Confirmatory hypothesis testing](#)

4. Extending inference beyond the conditional (in)dependence structure (Williams, 2018)

- [Predictability](#)(e.g., Haslbeck & Waldorp, 2018)
- [Posterior uncertainty](#)[intervals](#) for the partial correlations
- [Custom Network Statistics](#)

Supported Data Types

- **Continuous**: The continuous method was described in Williams (2018). Note that this is based on the customary [Wishart](#)[distribution](#).

- **Binary**: The binary method builds directly upon Talhouk, Doucet, & Murphy (2012) that, in turn, built upon the approaches of Lawrence, Bingham, Liu, & Nair (2008) and Webb & Forster (2008) (to name a few).

- **Ordinal**: The ordinal methods require sampling thresholds. There are two approach included in **BGGM**. The customary approach described in Albert & Chib (1993) (the default) and the 'Cowles' algorithm described in Cowles (1996).

- **Mixed**: The mixed data (a combination of discrete and continuous) method was introduced in Hoff (2007). This is a semi-parametric copula model (i.e., a copula GGM) based on the ranked likelihood. Note that this can be used for *only* ordinal data (not restricted to "mixed" data).

The computationally intensive tasks are written in c++ via the R package **Rcpp** (Eddelbuettel et al., 2011) and the c++ library **Armadillo** (Sanderson & Curtin, 2016). The Bayes factors are computed with the R package **BFpack** (Mulder et al., 2019). Furthermore, there are [plotting](#) functions for each method, control variables can be included in the model (e.g., ~ gender), and there is support for missing values (see bggm_missing).

Comparison to Other Software

BGGM is the only R package to implement all of these algorithms and methods. The mixed data approach is also implemented in the package **sbgcop** (base R, Hoff, 2007). The R package **BDgraph** implements a Gaussian copula graphical model in c++ (Mohammadi & Wit, 2015), but not the binary or ordinal approaches. Furthermore, **BGGM** is the only package for confirmatory testing and comparing graphical models with the methods described in Williams et al. (2020).

Acknowledgements

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