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logKDE: log-transformed kernel density estimation

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Summary

Exploratory data analysis, as proposed in Tukey (1977), is an important paradigm for conducting meaningful and useful applied statistics. According to Chapters 5, 19, and 20 of Tukey (1977), the visualization of data, and their density and distributional characteristics, may provide practitioners with great insight into the necessary processes that are required in order to effectively analyse the various features of the data.

Since its introduction in the pioneering works of Rosenblatt (1956) and Parzen (1962), the method of kernel density estimation (KDE) has become among one of the most popular methods for estimation, interpolation, and visualization of probability density functions (PDFs), due to its effectiveness and simplicity. The manuscripts of Silverman (2018) and Wand and Jones (1994) provide thorough treatments on the topic of KDE.

In the base `stats` package of the *R* programming environment (R Core Team 2016), univariate KDE can be conducted via the function, `density`. The function, `density`, is often taught in introductory *R* classes and presented in popular textbooks. See for example Section 5.6 of Venables and Ripley (2002), Section 7.4 of Trosset (2009), Section 6.3 of Keen (2010), Section 2.1 of Maindonald and Braun (2010), and Section 2.3 of Verzani (2014).

The KDE method implemented in `density` is the standard KDE technique for estimation of univariate PDFs, assuming that the data are real-valued. Unfortunately, `density` is often used to analyze data that are not real-valued, such as income data, which are positive-valued (cf, Charpentier and Flachaire 2015). The use of `density` in the case of positive-valued data causes the obtained estimator of the PDF that characterizes the data to not integrate to one, over the positive domain. This implies that it will provide an incorrect specification of the data generating process.

In order to mitigate against the aforementioned shortcoming of `density`, Charpentier and Flachaire (2015), among others, have suggested the use of the kernel density estimators (KDEs), constructed with log-normal kernel functions. These log-transformed KDEs, or log-KDEs for brevity, are *bona fide* PDFs over the positive domain, and are thus suited for applications to positive-valued data.

The `logKDE` package for *R* provides functions for conducting log-transformed KDE using log-KDEs, constructed with log-normal, as well as log-transformations of the Epanechnikov, Laplace, logistic, triangular, and uniform families of kernel functions, through the main function, `logdensity`. The function allows for a variety of bandwidth methods, including Silverman's rule of thumb (cf. Silverman 2018), cross-validation in both the natural and log-transformation space, and a new rule of thumb that is based on the comparison of the log-KDE to a log-normal fit. Where necessary, we have programmed the various functionalities in *C++*, and integrated the *C++* codes using `Rcpp` (Eddelbuettel

2013). For greater speed, at the expense of some accuracy, we have also implemented a fast Fourier transform version of our procedure, via the function, `logdensity_fft`.

There are two packages that are currently available, which share similar features with `logKDE`. The first is `Ake` (Wansouwe, Some, and Kokonendji 2016), via the `ker = 'LN'` setting of `dke.fun`, and the second is `evmix` (Hu and Scarrott 2018), via the `dbckden`, with the setting `bcmethod = 'logtrans'`. Unlike `logKDE`, `Ake` only offers log-transformed KDE constructed from log-normal kernels. Although `evmix` allows for the construction of log-KDEs using a variety of kernels, including some that are not currently available in `logKDE`, it does not permit the use of the log-Laplace and log-logistic kernels. Thus, with respect to variety of kernels, both packages have something to offer that the other does not.

We believe that the key difference between `evmix` and `logKDE` is that of user experience. In `logKDE`, `logdensity` is designed to closely replicate the syntax and behavior of `density`. Thus, users who have learnt to use `density` will quickly make use of `logdensity` and its features. The syntax of `dbckden` is dramatically different to that of `density`. The function allows for many controls that are meant for higher level users and which may overwhelm someone who is only interested in conducting KDE as an exploratory tool. Therefore, we believe that `logdensity` provides an experience that is more user friendly and familiar than that of `dbckden`.

Users can obtain the latest build of `logKDE` on GitHub (https://github.com/andrewthomasjones/logKDE). The latest stable build can be obtained from CRAN (https://CRAN.R-project.org/package=logKDE), and an archival build can be obtained from Zenodo (https://zenodo.org/record/1320105). A detailed literature review, mathematical study, simulation study, and demonstration of the log-transformed KDE method appear in the vignette of the package, which can be accessed via the command `vignette('logKDE')`. Thorough descriptions of the package functions appear in the manual, which can be accessed at https://cran.r-project.org/web/packages/logKDE/logKDE.pdf. Bug reports and other feedback can be directed to the GitHub issues page (https://github.com/andrewthomasjones/logKDE/issues).

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References

Charpentier, A, and E Flachaire. 2015. "Log-Transform Kernel Density Estimation of Income Distribution." *L'Actualite Economique* 91:141–59. https://www.doi.org/10.7202/1036917ar.

Eddelbuettel, D. 2013. *Seamless R and C++ Integration with Rcpp*. New York: Springer. https://www.doi.org/10.1007/978-1-4614-6868-4.

Hu, Y, and C Scarrott. 2018. "Evmix: An R Package for Extreme Value Mixture Modelling, Threshold Estimation and Boundary Corrected Kernel Density Estimation." *Journal of Statistical Software* 56:1–30. https://www.doi.org/10.18637/jss.v084.i05.

Keen, K J. 2010. *Graphics for Statistics and Data Analysis with R*. Boca Raton: CRC Press.

Maindonald, J, and J Braun. 2010. *Data Analysis and Graphics Using R: An Example-Based Approach*. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9781139194648.

Parzen, E. 1962. "On Estimation of a Probability Density Function and Mode." *Annals of Mathematical Statistics* 33:1065–76. https://www.doi.org/10.1214/aoms/1177704472.

R Core Team. 2016. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. https://www.R-project.org/.

Rosenblatt, M. 1956. "Remarks on Some Nonparametric Estimates of a Density Function." *Annals of Mathematical Statistics* 27:832–37. https://www.doi.org/10.1214/aoms/1177728190.

Silverman, B W. 2018. *Density Estimation for Statistics and Data Analysis*. London: Chapman; Hall.

Trosset, M W. 2009. *An Introduction to Statistical Inference and Its Applications with R*. Boca Raton: CRC Press.

Tukey, J W. 1977. *Exploratory Data Analysis*. Reading: Addison-Wesley.

Venables, W N, and B D Ripley. 2002. *Modern Applied Statistics with S*. New York: Springer. https://doi.org/10.1007/978-0-387-21706-2.

Verzani, J. 2014. *Using R for Introductory Statistics*. Boca Raton: CRC Press.

Wand, M P, and M C Jones. 1994. *Kernel Smoothing*. Boca Raton: CRC Press.

Wansouwe, W E, S M Some, and C C Kokonendji. 2016. "Ake: An R Package for Discrete and Continuous Associated Kernel Estimations." *The R Journal* 8:258–76. https://journal.r-project.org/archive/2016/RJ-2016-045/index.html.