

Spatial sensitivity analysis

Course and practical application

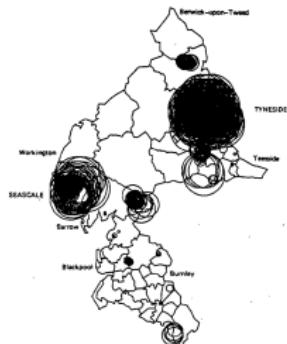
OpenMOLE

September 2nd, 2020

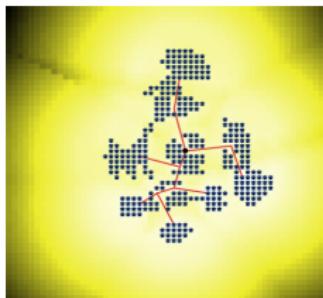
- 1 Scientific context
- 2 Spatial synthetic data
- 3 Perturbation of data
- 4 Sensitivity to spatial configuration
- 5 Spatial indicators for model outputs
- 6 Synthesis
- 7 Application on the Zombie model

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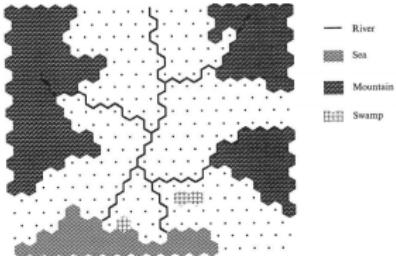
Modeling and simulation in geography



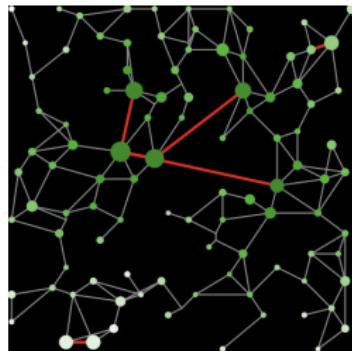
Geographical analysis machine
[Openshaw et al., 1987]



Hybrid urban morphogenesis
[Raimbault et al., 2014]



Simpop 1 model [Sanders et al., 1997]



SimpopNet model [Schmitt, 2014]

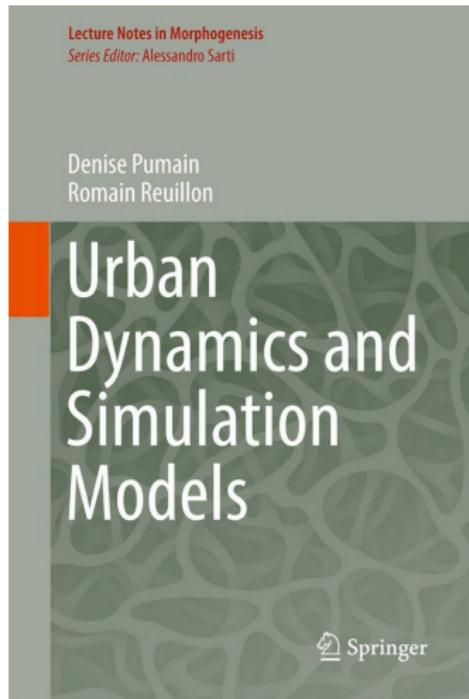
Necessity of simulation models in geography induced by complexities of these systems ?

- ▶ Ontological complexity [Pumain, 2003]
- ▶ Dynamical complexity: non-ergodicity and path-dependency [Pumain, 2012]
- ▶ Complexity and co-evolution [Raimbault, 2019a]
- ▶ Complexity and emergence [Bedau, 2002]

Historical succession of epistemologies in the case of systems of cities [Varenne, 2017]:

1. Deduction from theory (top-down): Christaller
2. Induction from the empirical (bottom-up): Berry
3. Towards an abductive epistemology (iterative interaction theoretical-empirical): Pumain

→ simulation allows synthesis



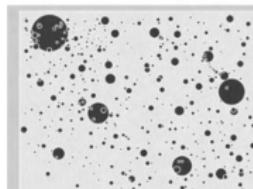
Development of evolutive urban theory
[Pumain, 2018]

- Recurrent stylized facts on main systems of cities
- Construction of simulation models (with an explicative purpose)
- Tools and methods to explore simulation models



Evolutionary Urban Theory

Spatio-temporal scales



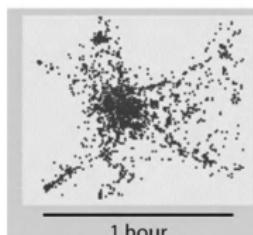
Emerging properties

Hierarchy
Functional diversity
Spatial pattern

Organization levels

Macro: System of cities
(urban networks)

1 day



Centrality
Function
Morphology
"Ambiance urbaine"

Meso: City
(urban areas)

1 hour

Descriptors



Life cycle
Profession
Power

Micro: Actors
(households, firms, institutions)

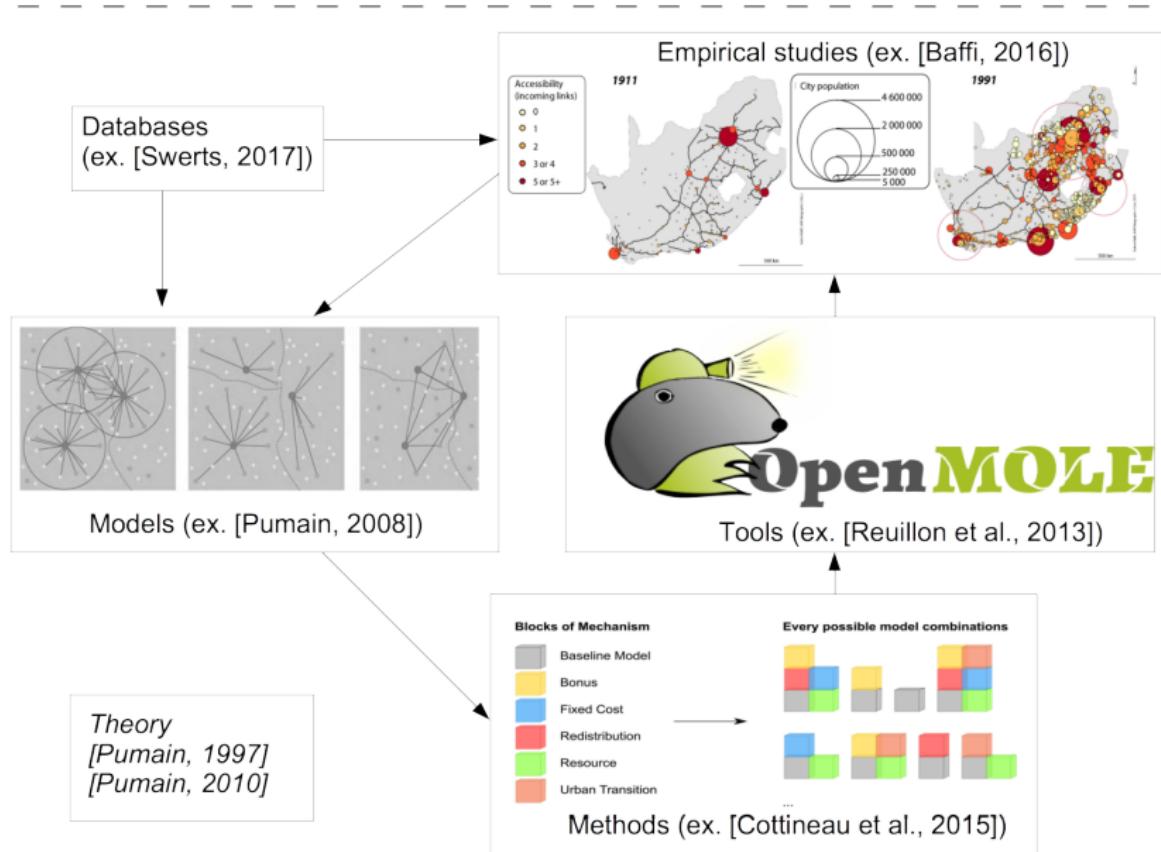
Systems of cities as co-evolutive systems in which interactions are crucial

[Pumain, 1997]

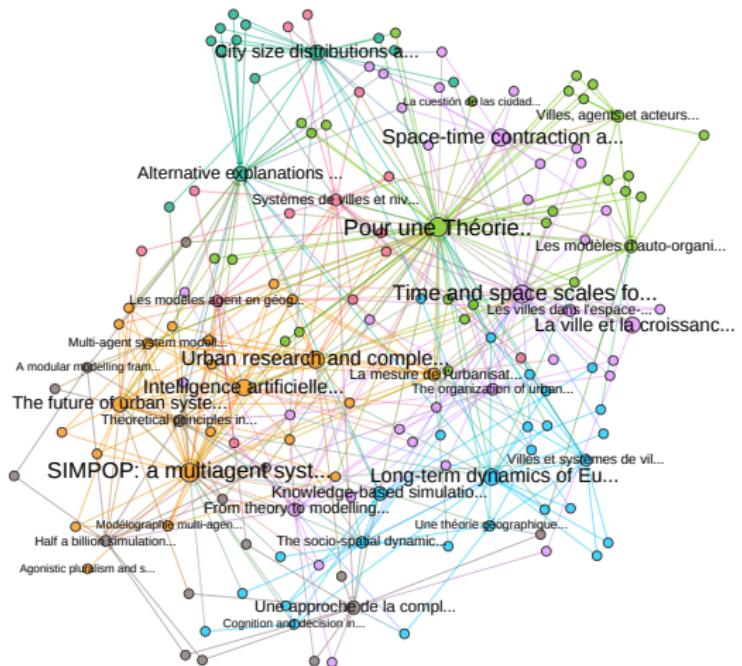
[Pumain, 2008]

[Pumain, 2018]

Construction of Knowledge across Domains



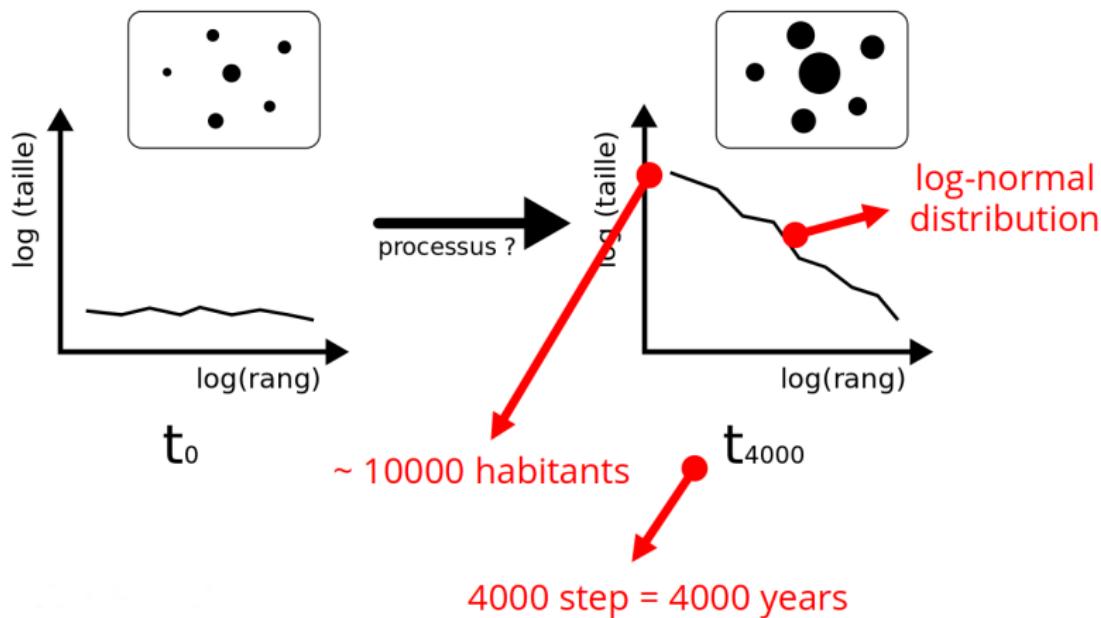
Evolutive urban theory



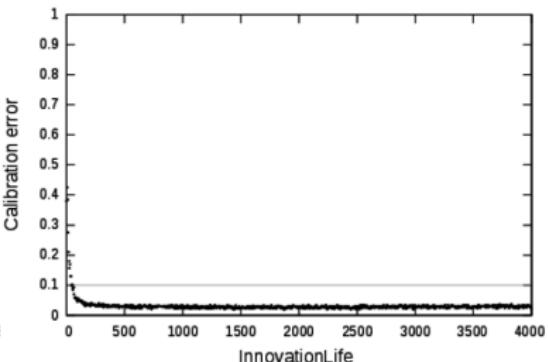
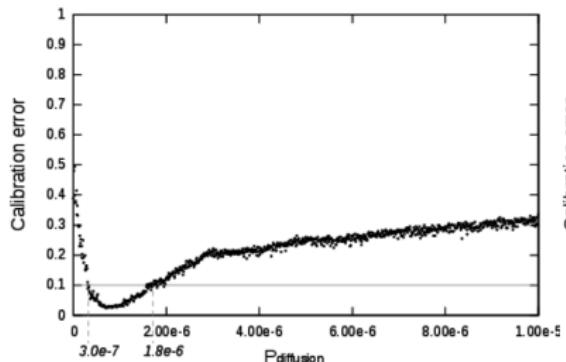
[Raimbault, 2017] Citation network analysis of key publications in the evolutive urban theory

SimpopLocal model calibrated with distributed NSGA2 on grid
[Schmitt et al., 2015]

Formalising the expectations as indicators:

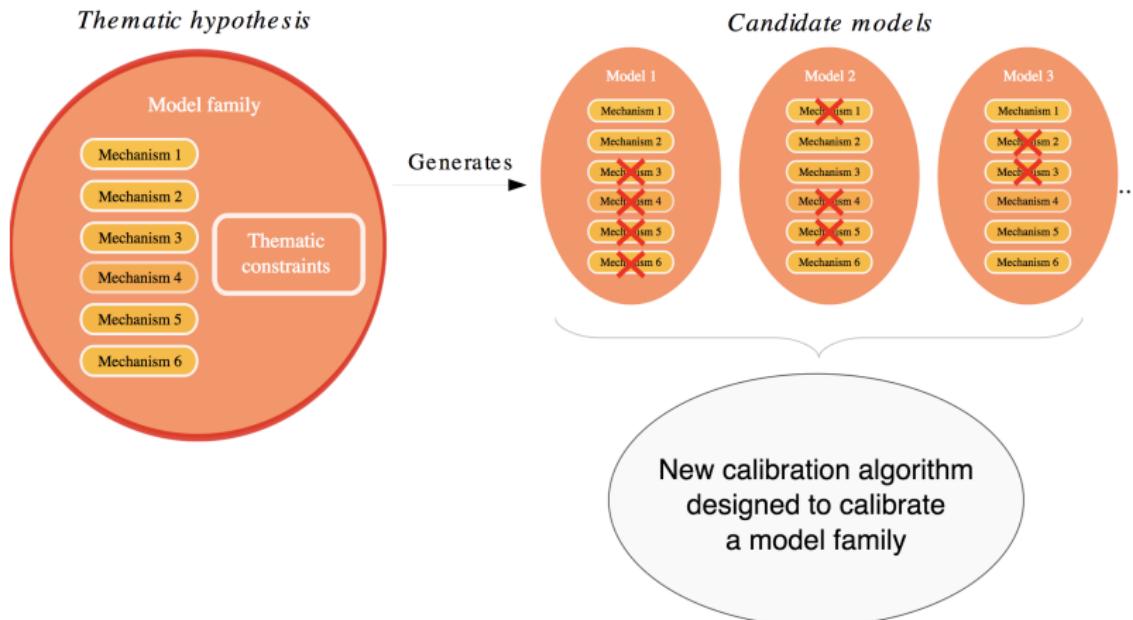


Computes the best calibration at fixed steps along one dimension.
[Reuillon et al., 2015]



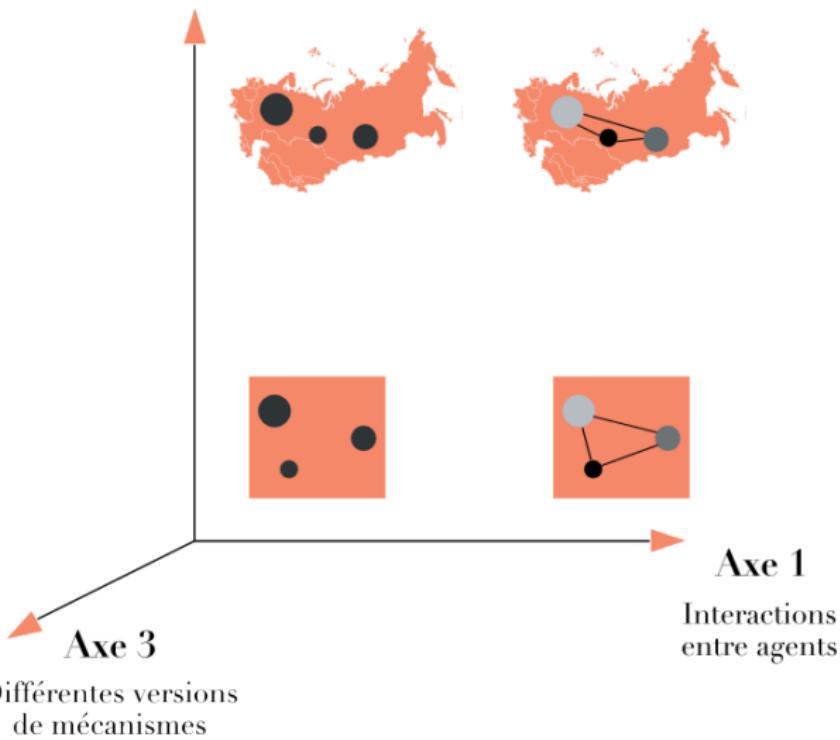
Unicity of mechanisms

Automate the confrontation of alternative hypothesis / mechanisms [Cottineau, 2014]



Axe 2

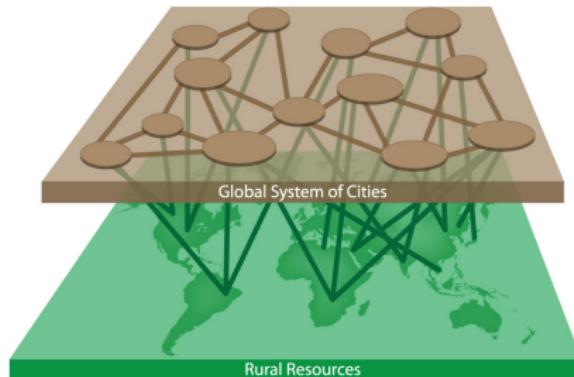
Interactions entre
les agents et
l'environnement



(c) Clémentine Cottineau, UMR Géographie-Cités, P.A.R.I.S., 2014

Exchange mechanism: market vs centralized

City growth: interurban interactions vs environmental situation



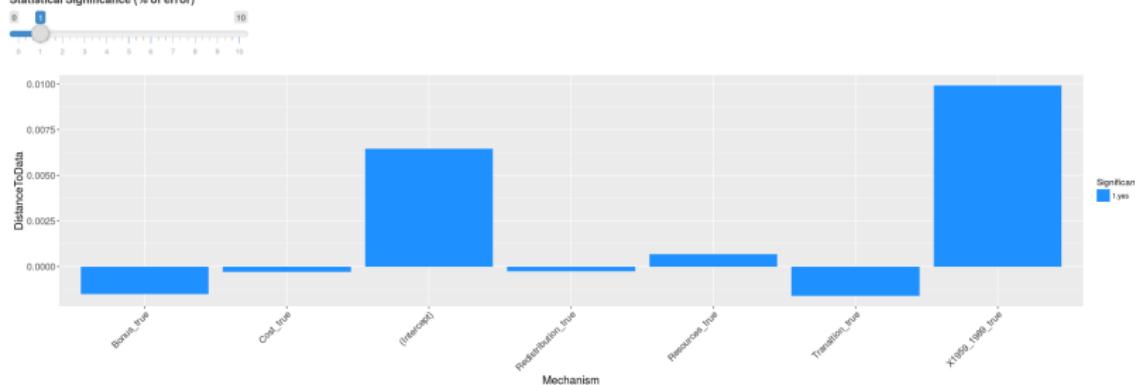
Calibration of model family

Compute the best set of parameters for all 64 models.

Contribution of mechanisms to the quality of simulation (closeness to data)

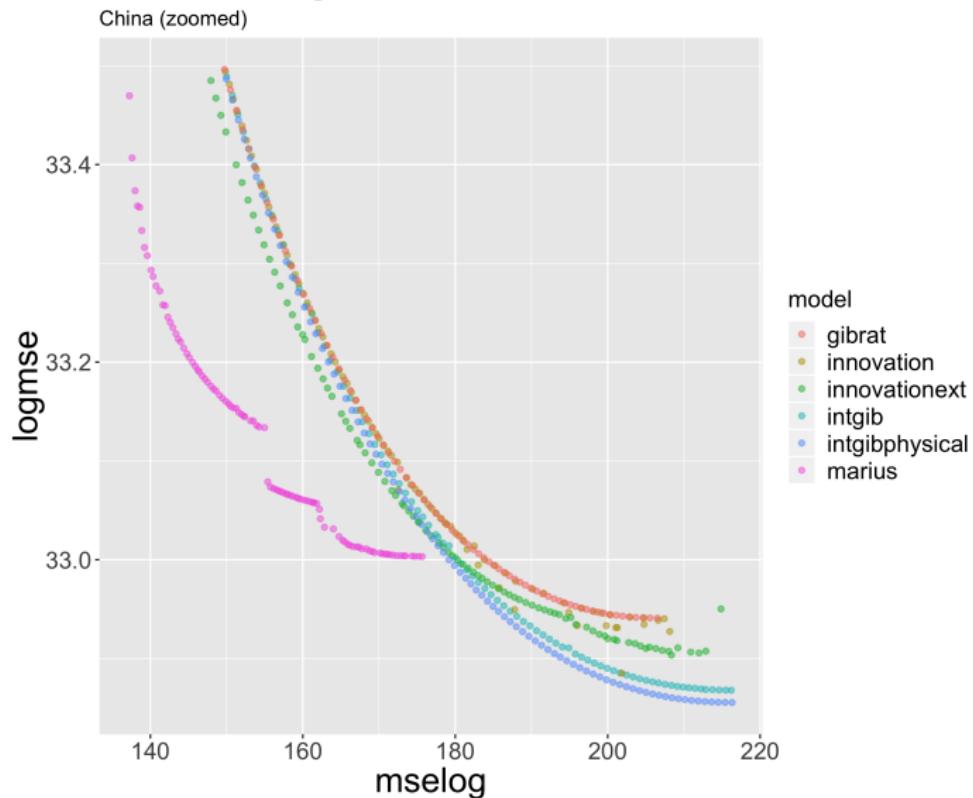
Models with different combination of mechanisms have been calibrated intensively against empirical data, using generic algorithms for more than 100000 generations. This plot shows the results of a regression explaining one measure of the quality of models (a small difference between simulated and empirical urban trajectories) by their mechanisms composition (the fact that any of the supplementary mechanisms is activated or not). Each bar represents the value of the estimated coefficient for each activated mechanism, in comparison with the same model structure without this mechanism, everything else being equal.

Statistical Significance (% of error)

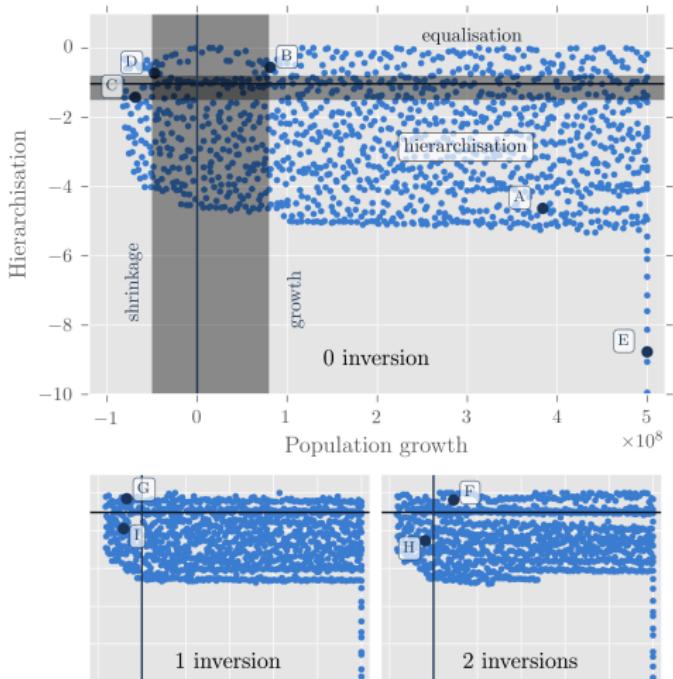


Other example of multi-modeling

Benchmark of growth models for systems of cities
[Raimbault et al., 2020]



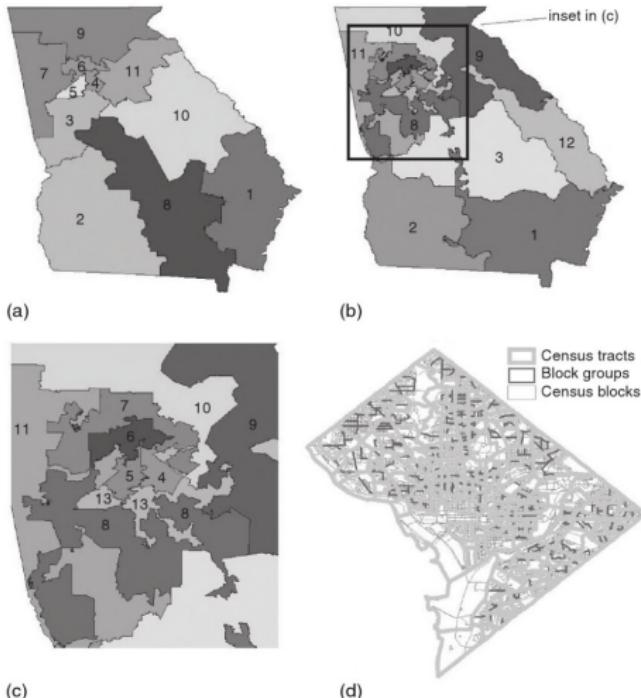
Diversity of urban systems dynamics produced by the MARIUS model [Chérel et al., 2015]



Classical problems in geography and spatial sciences :

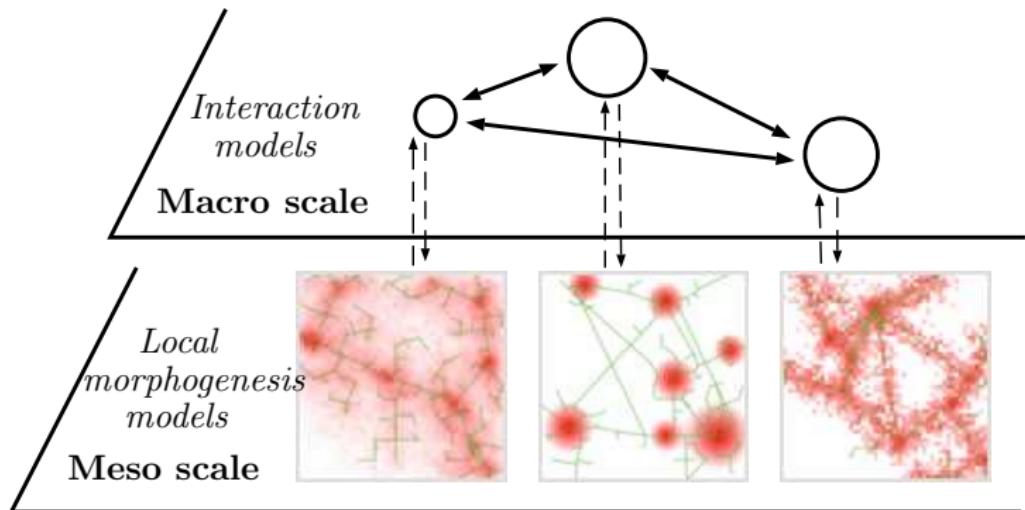
- ▶ Modifiable Areal Unit Problem
- ▶ Dependancy of processes to scale
- ▶ Spatial non-stationarity
- ▶ Fuzzy and noisy data
- ▶ Genericity/particularity

The Modifiable Areal Unit Problem

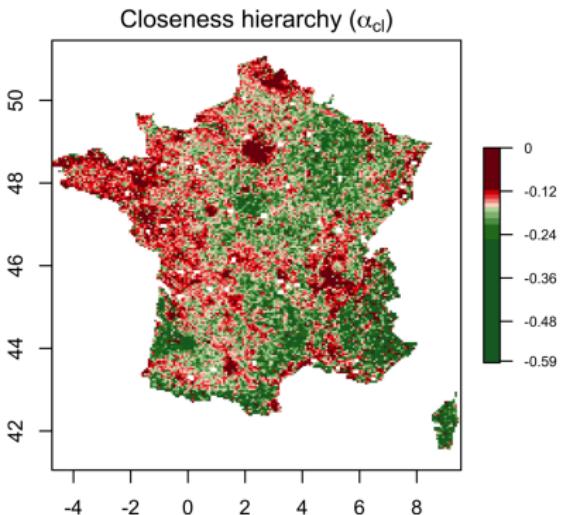
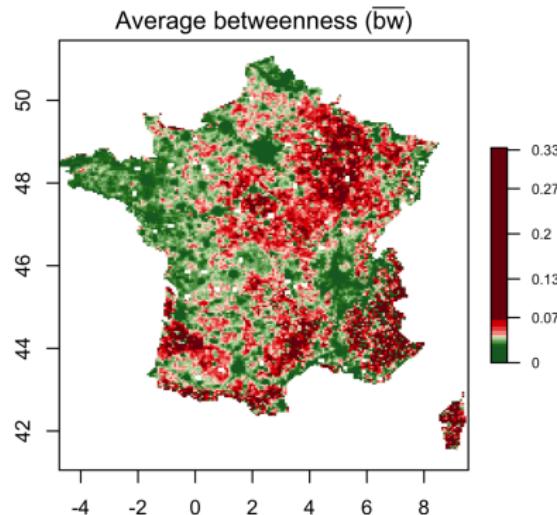


Wong, D. (2009). The modifiable areal unit problem (MAUP). The SAGE handbook of spatial analysis, 105, 23.

Processes specific to scales, coupling implies dedicated ontologies
[Raimbault, 2019b]



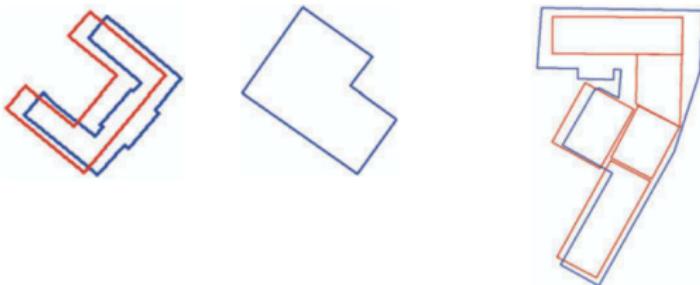
Spatial non-stationarity of road network properties
[Raimbault, 2019c]



Assessment of data quality in OpenStreetMap [Fan et al., 2014]

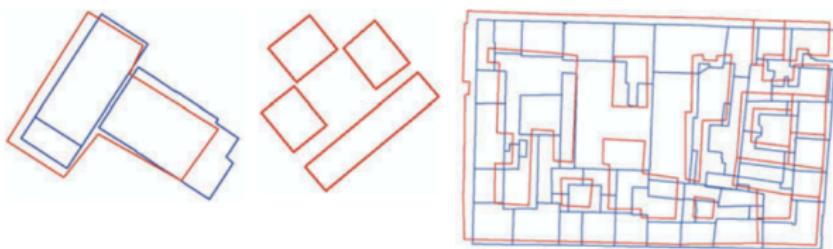
Relation	1:1	1:0	1: n
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Illustration

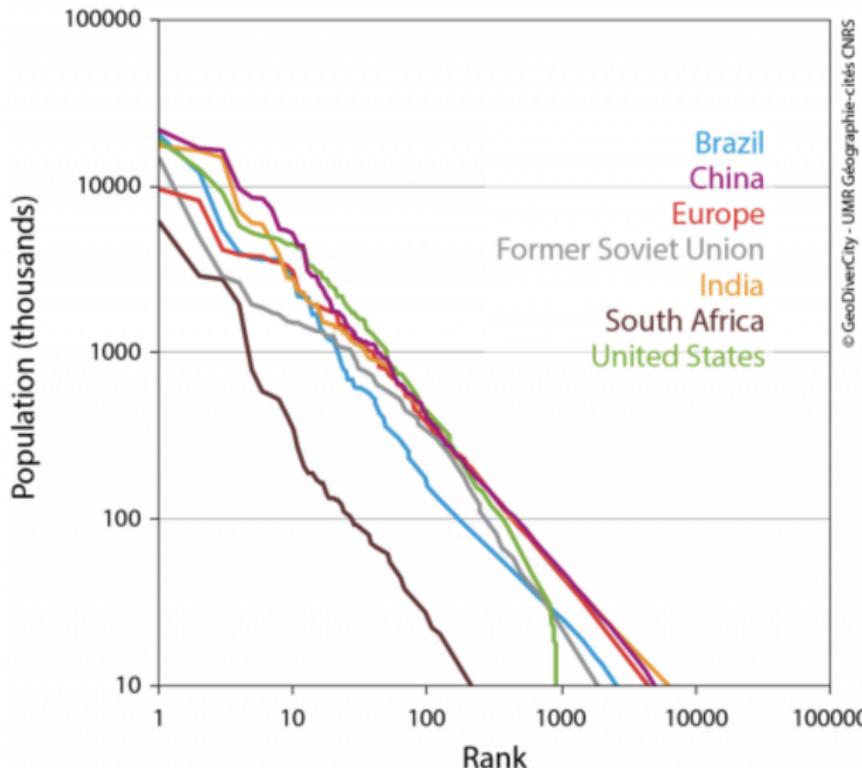


Relation	$n:1$	$0:1$	$n:m$
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Illustration



Urban systems are simultaneously universal and particular
[Pumain et al., 2015]



⇒ **spatial configuration are parameters too**

- ▶ “*Space matters*”: impact of spatial configuration on model behavior
- ▶ Model behaviors which are robust to spatial configuration
- ▶ Model behaviors which are robust to noise in parametrization real datasets

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- coupling models with spatial configuration generators (spatial synthetic data) gives model sensitivity to space through sensitivity analysis of the coupled model
- synthetic urban forms resembling real configurations
- at different scales: microscopic (buildings), mesoscopic (population distribution), macroscopic (system of cities)

At the microscopic scale (district): generating building layouts

Raimbault, J., & Perret, J. (2019). Generating urban morphologies at large scales. In Artificial Life Conference Proceedings (pp. 179-186). Cambridge: MIT Press.

- ▶ systematic comparison of simple processual generators
- ▶ introduction of morphological indicators
- ▶ calibration on sampled layouts from OpenStreetMap

Urban form indicators for building layouts:

- ▶ density, number of buildings, average area
- ▶ Moran index and average distance on rasterized representation
- ▶ average detour in the free space
- ▶ mathematical morphology indicators (steps for erosion and dilation) [Serra, 1994]

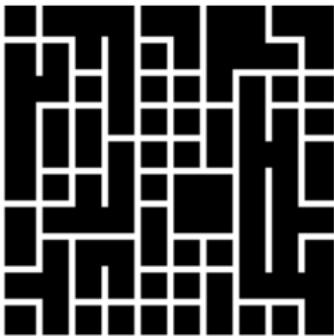
Complementary generators



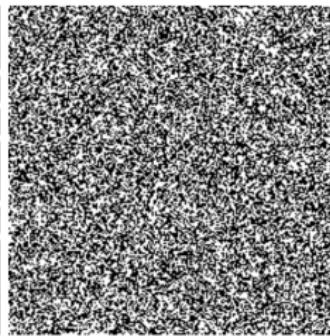
(a) Blocks



(b) Kernel mixture



(c) Network percolation



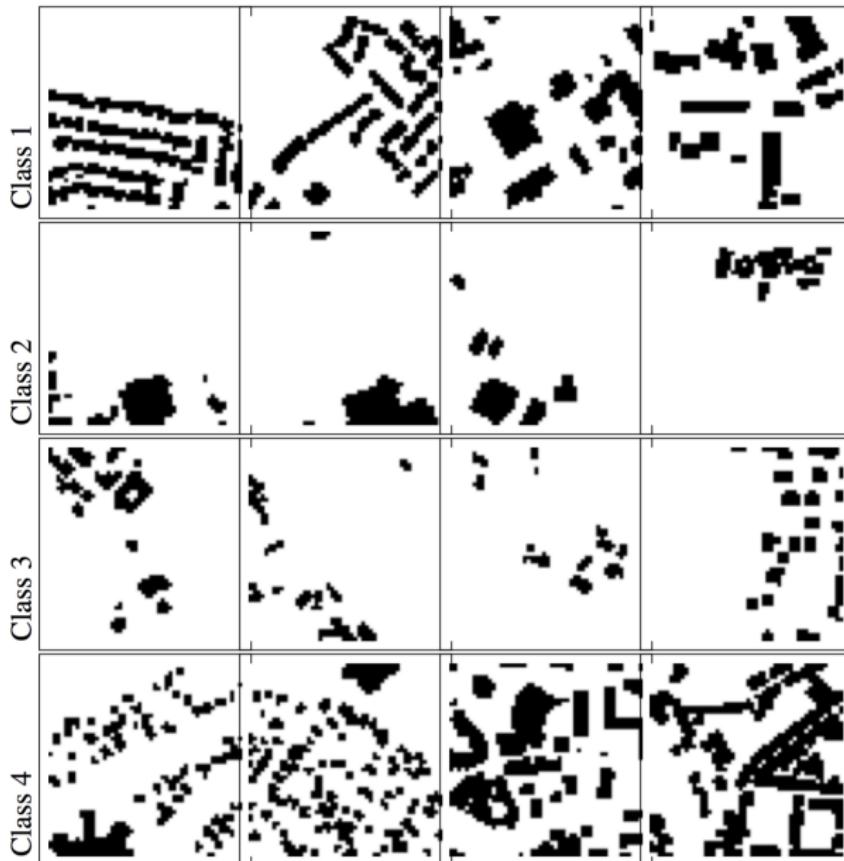
(d) Random

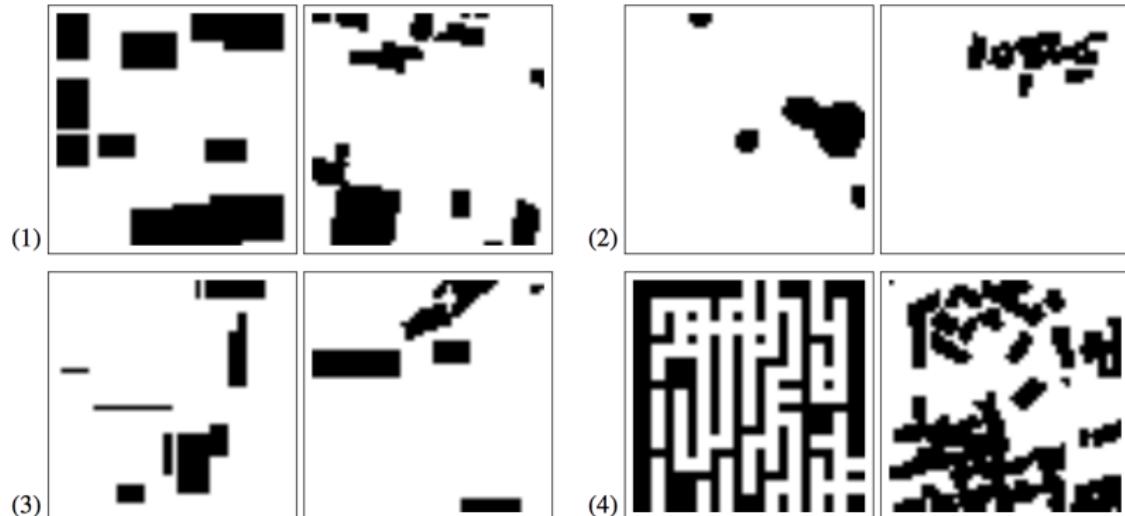
Real configurations

Sampled districts from OpenStreetMap

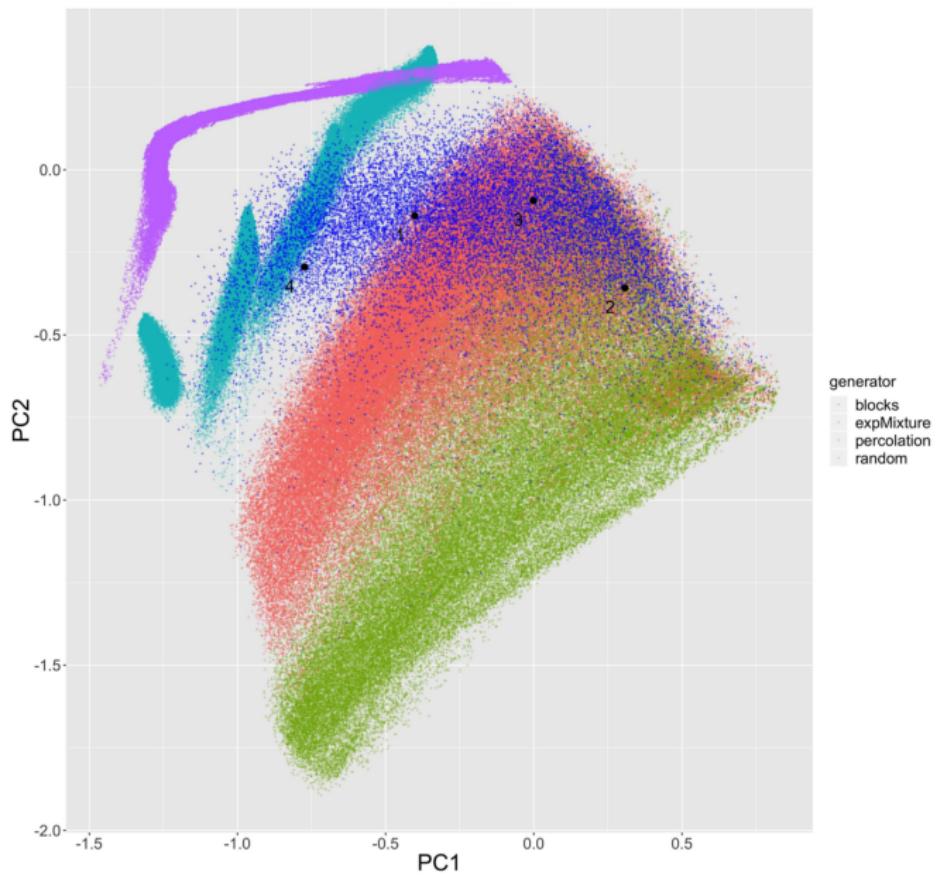


Classification of urban forms





Point cloud



	Random	Blocks	Exp. Mixture	Percolation
Centroid 1	0.424 ± 0.011	0.106 ± 0.063	0.303 ± 0.101	0.325 ± 0.019
Centroid 2	0.809 ± 0.022	0.164 ± 0.099	0.184 ± 0.141	0.947 ± 0.019
Centroid 3	0.428 ± 0.019	0.095 ± 0.054	0.109 ± 0.064	0.541 ± 0.019
Centroid 4	0.515 ± 0.005	0.311 ± 0.077	0.589 ± 0.149	0.083 ± 0.025

Why not use calibration heuristics? Open question of fitting a point cloud; issue of projecting in a reduced dimension space

```
DirectSampling(  
    evaluation = model hook (workDirectory / "grids.csv"),  
    sampling =  
        (myCenters is 3) x  
        (myRadius is 10.0) x  
        (myThreshold is 10.0) x  
        (myGrid is ExpMixtureThresholdSpatialSampling(gridSize = 100,  
            center = myCenters, radius = myRadius, threshold = myThreshold))  
    zipWithIndex id  
)
```

- other generators have their own primitives
(BlocksGridSpatialSampling,
PercolationGridSpatialSampling) and arguments (**see the documentation**)
- explore the script SpatialSampling.oms

Script SpatialSampling.oms

1. Use the MatrixHook and an EmptyTask to visualize different types of grid
2. Explore the generators applied to the ZOMBIE model at <https://om.exmodelo.org/spatialsens/>
3. Plug a hook to retrieve all flat configurations (using the commented ScalaTask)
4. Next step: Modify the script to plug random worlds into the model (later)

At the mesoscopic scale: population grids

- ▶ a reaction-diffusion model for population distributions
- ▶ urban form measures at the mesoscopic scale
- ▶ towards synthetic generators coupling population and road networks

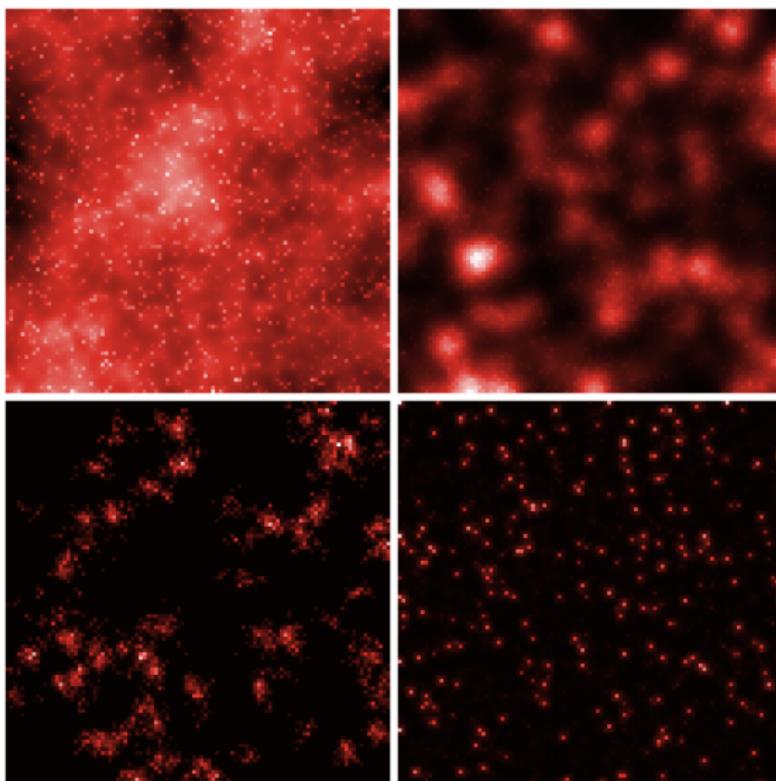
Raimbault, J. (2018). Calibration of a density-based model of urban morphogenesis. *PLoS one*, 13(9), e0203516.

Raimbault, J. (2019). An urban morphogenesis model capturing interactions between networks and territories. In *The Mathematics of Urban Morphology* (pp. 383-409). Birkhäuser, Cham.

- Crucial role of the interplay between concentration forces and dispersion forces [Fujita and Thisse, 1996] in keeping Urban Systems at the border of chaos
- Potentiality of aggregation mechanisms (such as Simon model) to produce power laws
- Link with Reaction-diffusion approaches in Morphogenesis [Turing, 1952]
- Extension of a DLA-type model introduced by [Batty, 1991], with simple abstract processes of population aggregation and diffusion

- Grid world with cell populations $(P_i(t))_{1 \leq i \leq N^2}$.
- At each time step:
 1. Population growth with exogenous rate N_G , attributed independently to a cell following a preferential attachment of strength α
 2. Population is diffused n_d times with strength β
- Stopping criterion: fixed maximal population P_m .
- Output measured by morphological indicators: Moran index, average distance, rank-size hierarchy, entropy.

Generating Population Distributions



Examples of generated territorial shapes

Morphological indicators

1. Rank-size slope γ , given by $\ln(P_{\tilde{i}}/P_0) \sim k + \gamma \cdot \ln(\tilde{i}/i_0)$ where \tilde{i} are the indexes of the distribution sorted in decreasing order.
2. Entropy of the distribution:

$$\mathcal{E} = \sum_{i=1}^M \frac{P_i}{P} \cdot \ln \frac{P_i}{P} \quad (1)$$

$\mathcal{E} = 0$ means that all the population is in one cell whereas
 $\mathcal{E} = M$ means that the population is uniformly distributed.

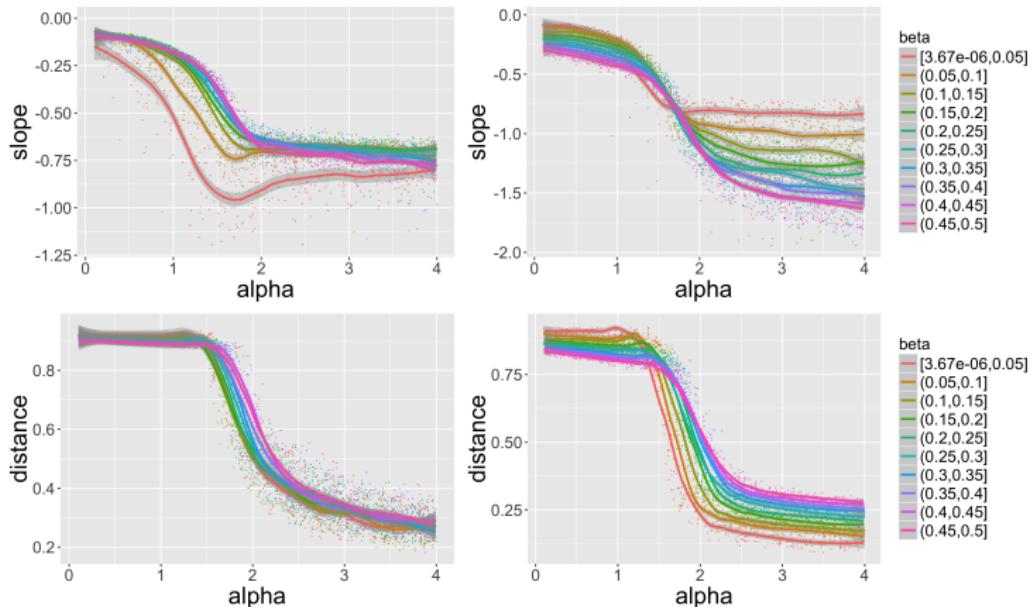
3. Spatial-autocorrelation given by Moran index, with simple spatial weights given by $w_{ij} = 1/d_{ij}$

$$I = M \cdot \frac{\sum_{i \neq j} w_{ij} (P_i - \bar{P}) \cdot (P_j - \bar{P})}{\sum_{i \neq j} w_{ij} \sum_i (P_i - \bar{P})^2}$$

4. Mean distance between individuals

$$\bar{d} = \frac{1}{d_M} \cdot \sum_{i < j} \frac{P_i P_j}{P^2} \cdot d_{ij}$$

where d_M is a normalisation constant



*Phase transitions of indicators unveiled by exploration of the parameter space
 (80000 parameter points, 10 repetitions each)*

Path-dependency and frozen accidents

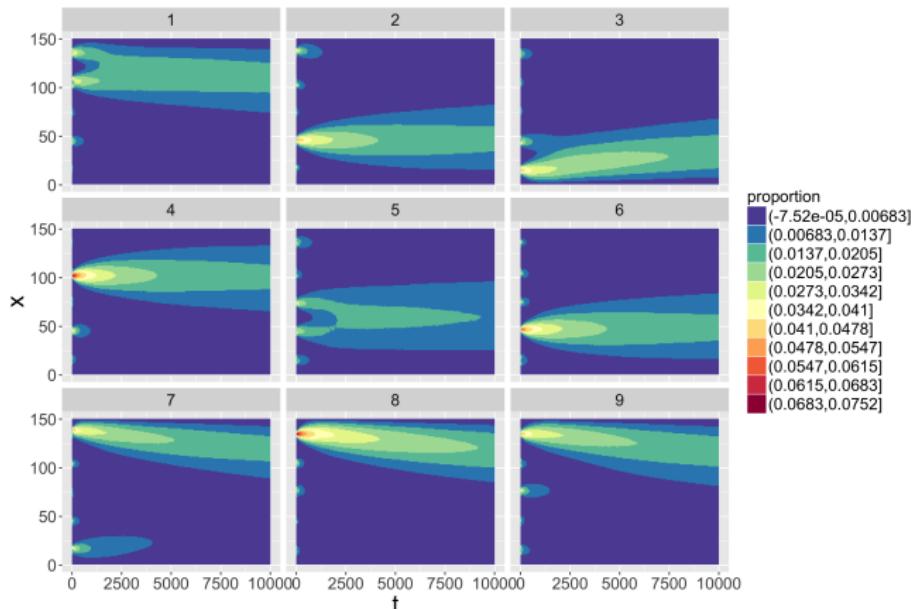
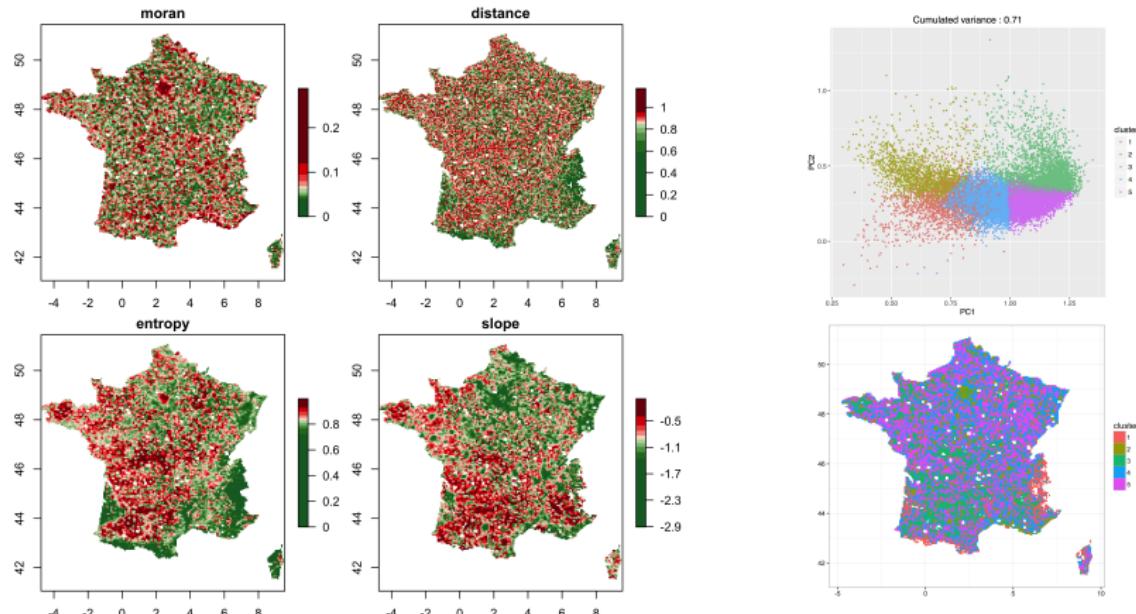


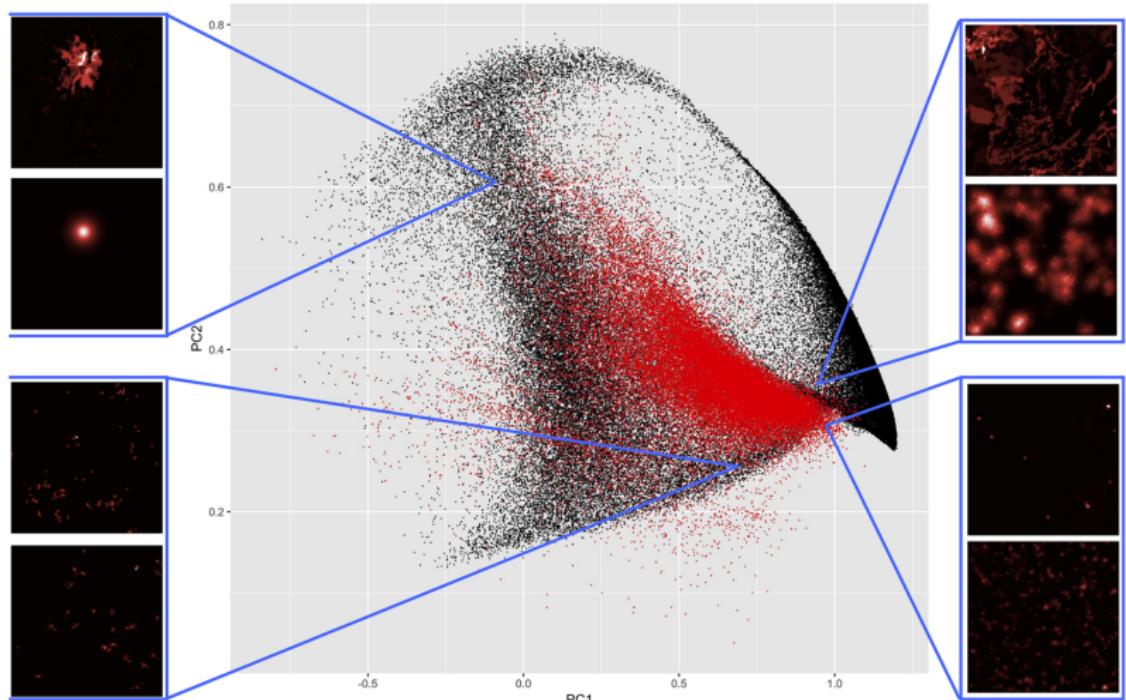
Illustration of path-dependence in a simplified one-dimensional version of the model: cell trajectories in time for 9 independent repetitions from the same initial configuration.

Empirical Data for Calibration



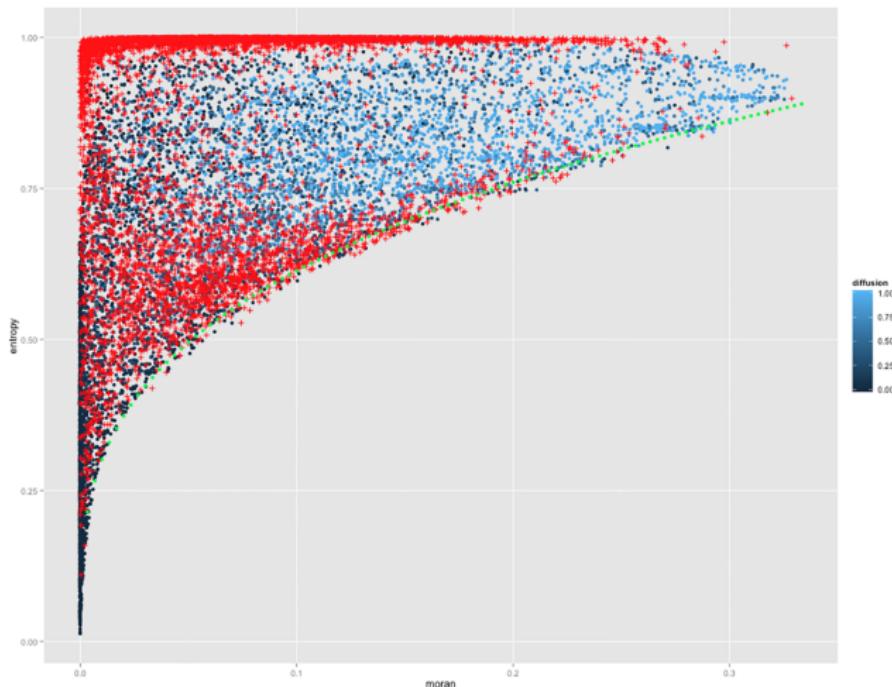
Computation of morphological indicators on population density data for Europe (shown here on France), morphological classification.

Model Calibration



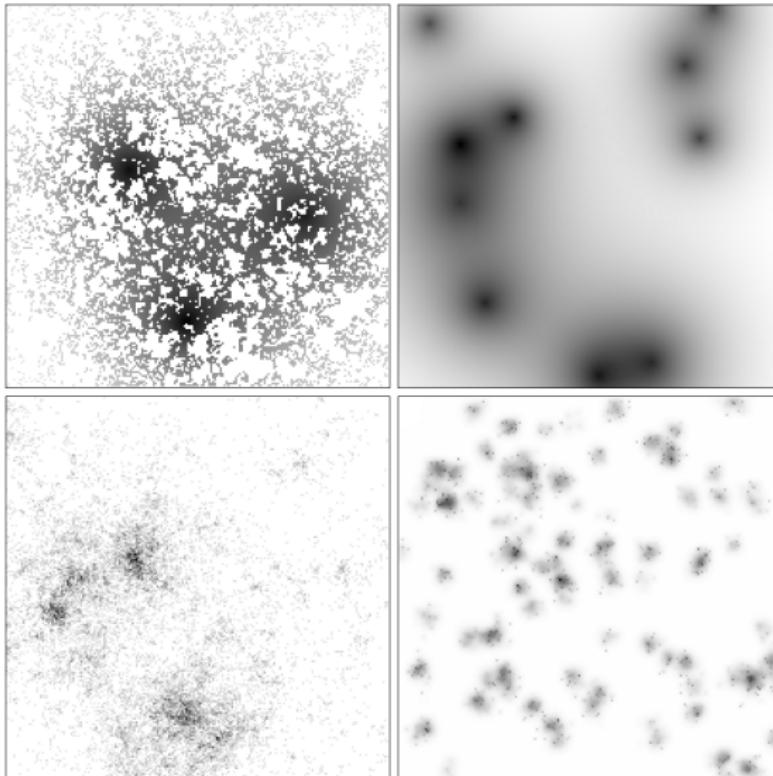
Brute force calibration by exploring the parameter space. Reproduction of most existing configuration in the morphological sense (here in principal plan).

Model Targeted Exploration

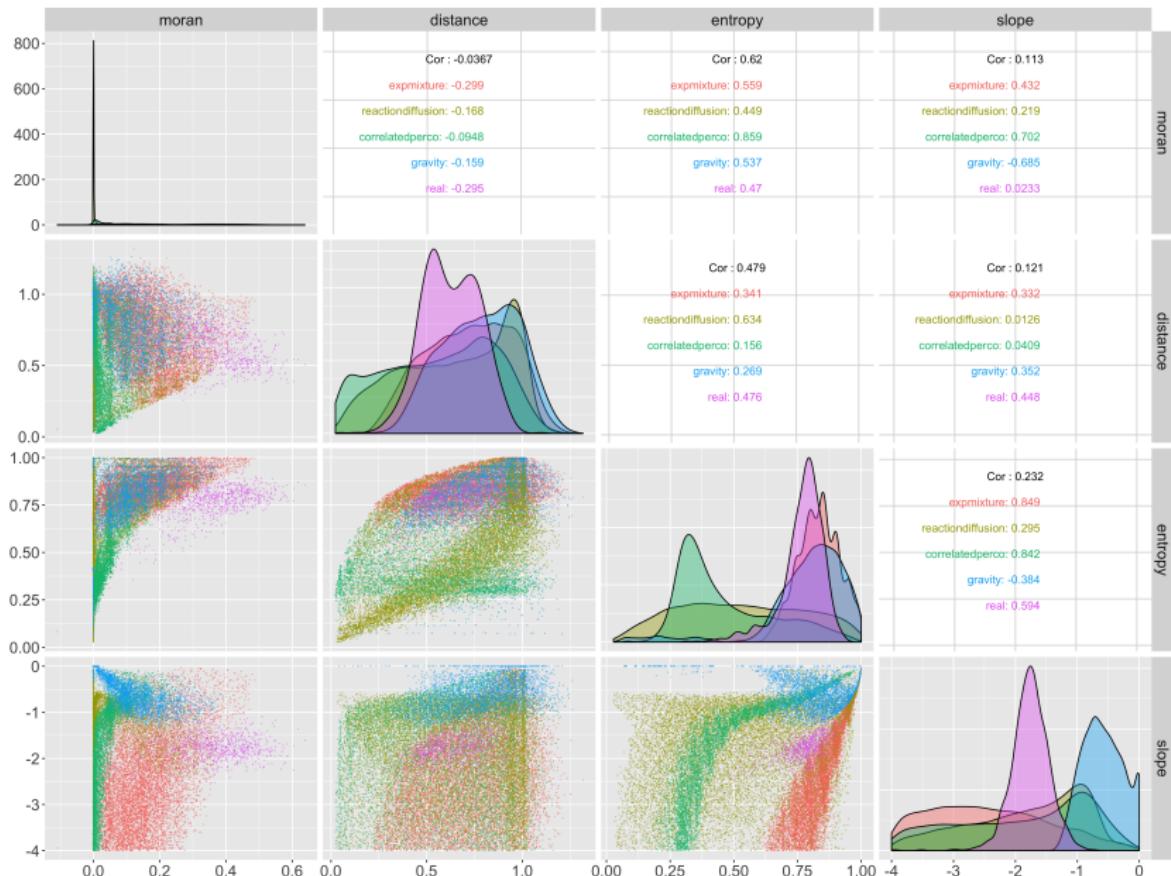


Potentialities of targeted model explorations: here feasible space using Pattern Space Exploration algorithm [Chérel et al., 2015].

Benchmarking exponential mixtures, correlated percolation and gravity-based models [Raimbault, 2020]

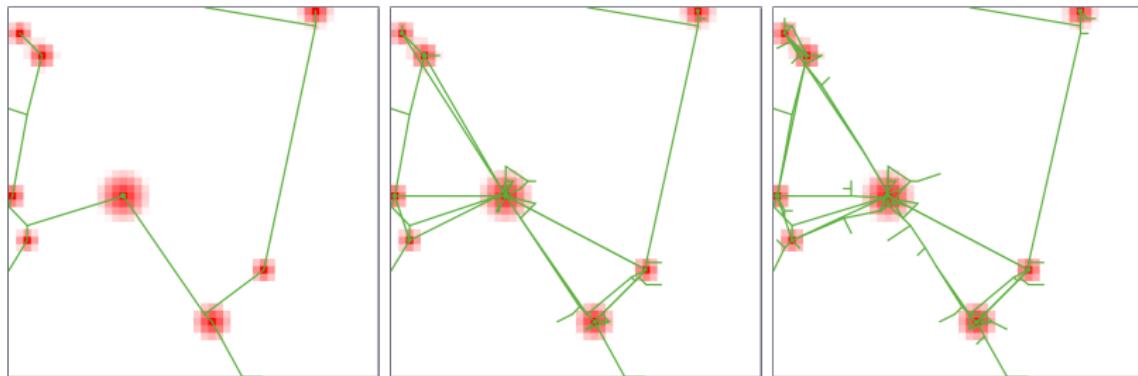


Feasible morphological spaces

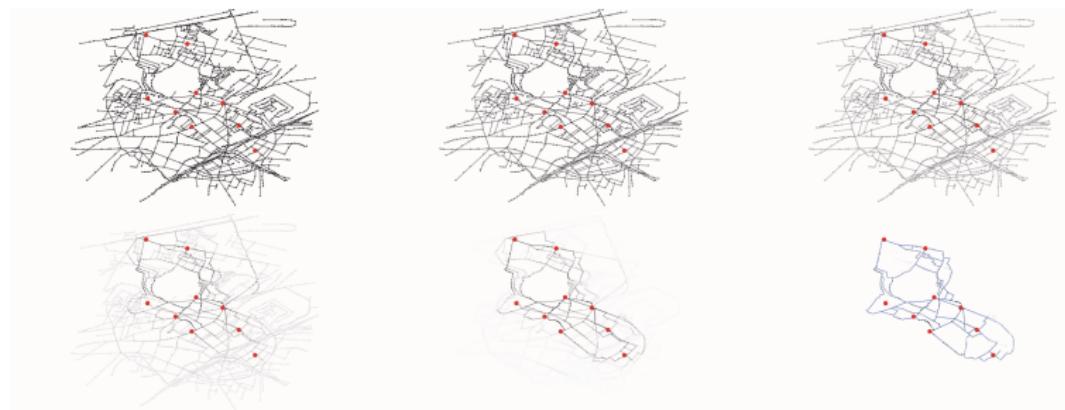


Network generated conditionally to population; at fixed time steps :

1. Add new nodes preferentially to new population and connect them
2. Variable heuristic for new links, among: nothing, random, gravity-based deterministic breakdown, gravity-based random breakdown (from [Schmitt, 2014]), cost-benefits (from [Louf et al., 2013]), biological network generation (based on [Tero et al., 2010])



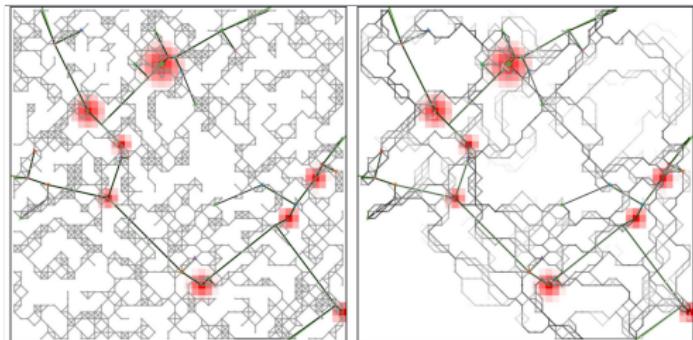
Model studied by [Tero et al., 2010] : exploration and reinforcement by a slime mould searching for resources



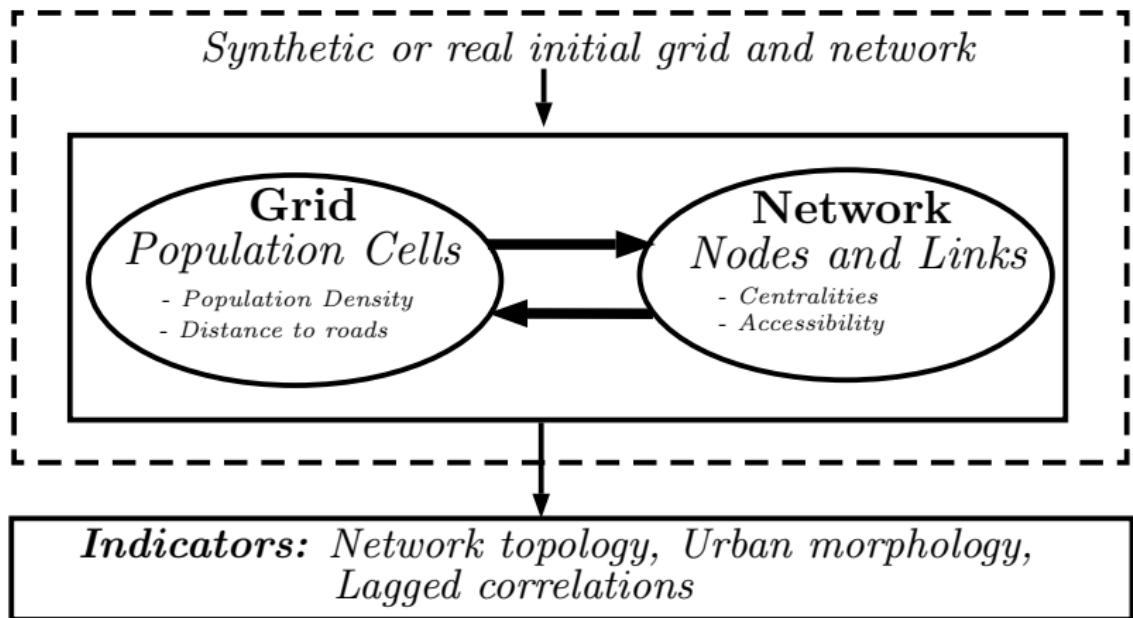
Application to the design of optimal bus routes

Adding new links with biological heuristic:

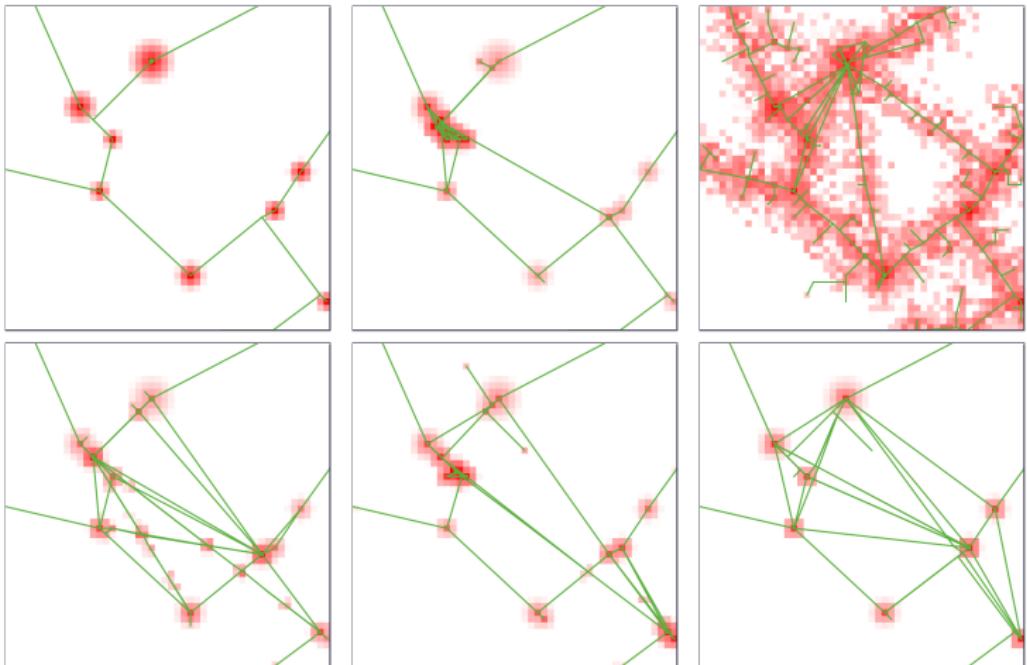
1. Create network of potential new links, with existing network and randomly sampled diagonal lattice
2. Iterate for k increasing ($k \in \{1, 2, 4\}$ in practice) :
 - ▶ Using population distribution, iterate $k \cdot n_b$ times the slime mould model to compute new link capacities
 - ▶ Delete links with capacity under θ_d
 - ▶ Keep the largest connected component
3. Planarize and simplify final network



Intermediate stage for biological network generation

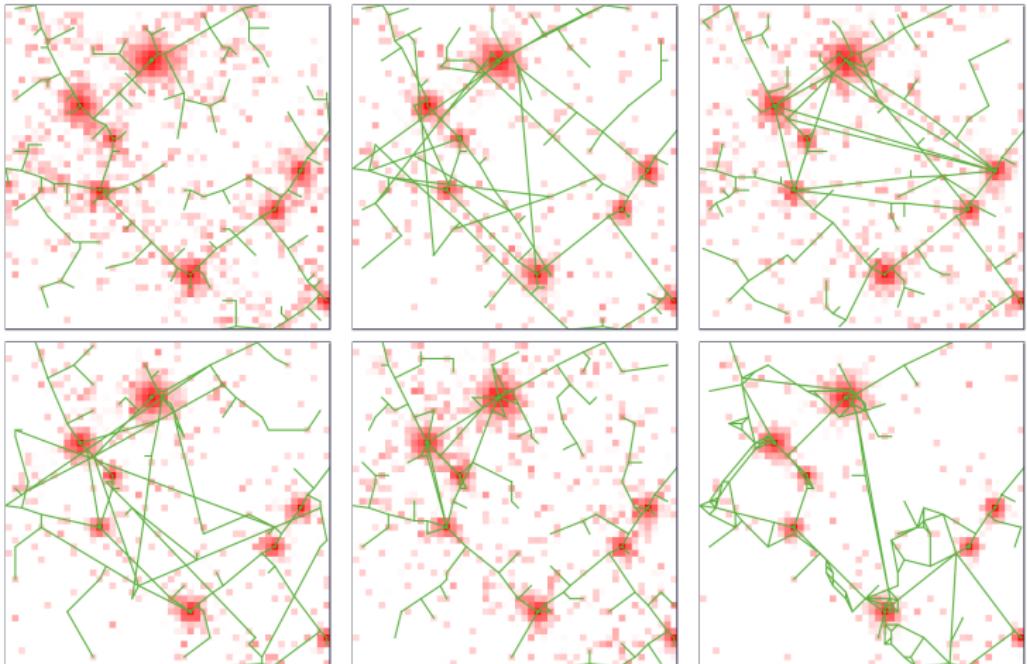


Generated Urban Shapes: Urban Form



*In order: setup; accessibility driven; road distance driven; betweenness driven;
closeness driven; population driven.*

Generated Urban Shapes: Network

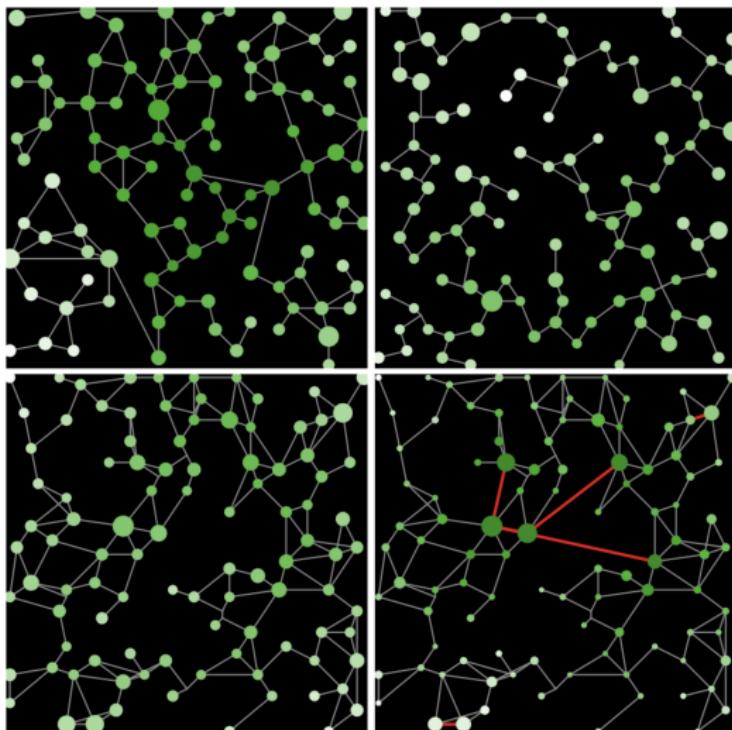


In order: connection; random; deterministic breakdown; random breakdown; cost-driven; biological.

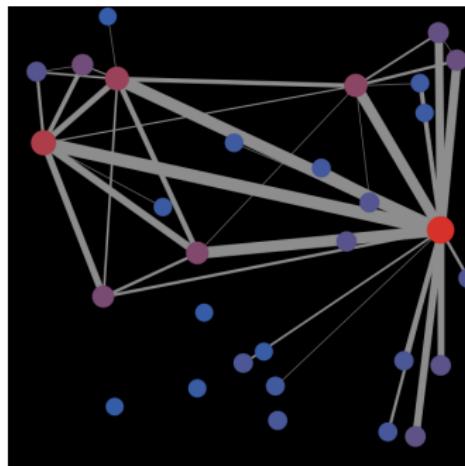
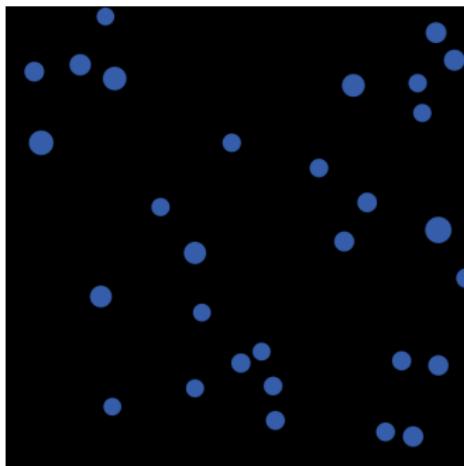
At the macroscopic scale: synthetic systems of cities

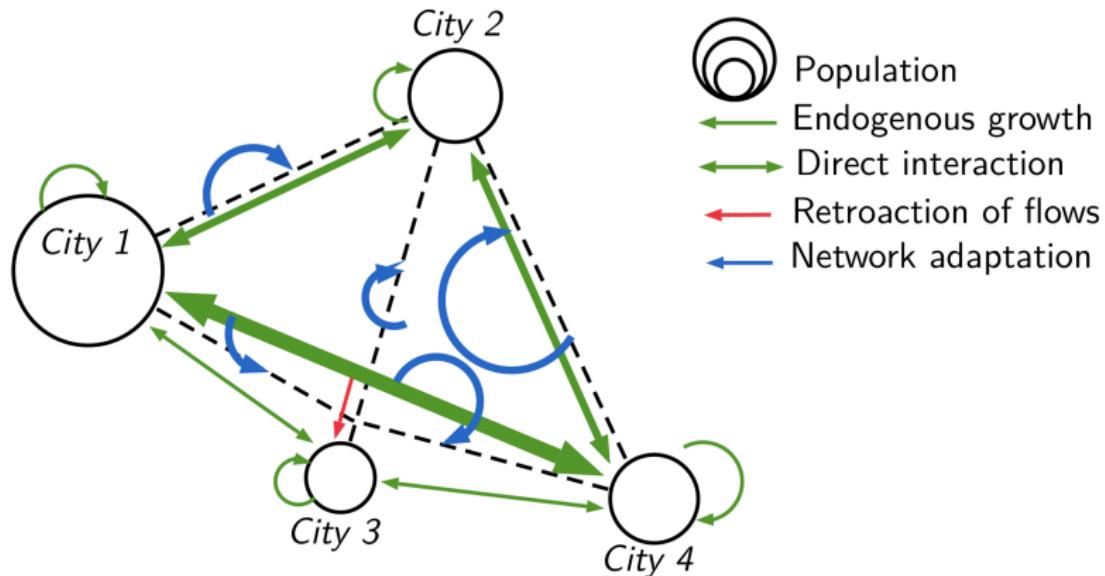
- ▶ Evolutive urban theory: systems of cities follow general stylized facts [Pumain, 2018]
- ▶ Rank-size law [Pumain et al., 2006]
- ▶ Central place theory

Synthetic system of cities and network for the SimpopNet model
[Raimbault, 2018b]

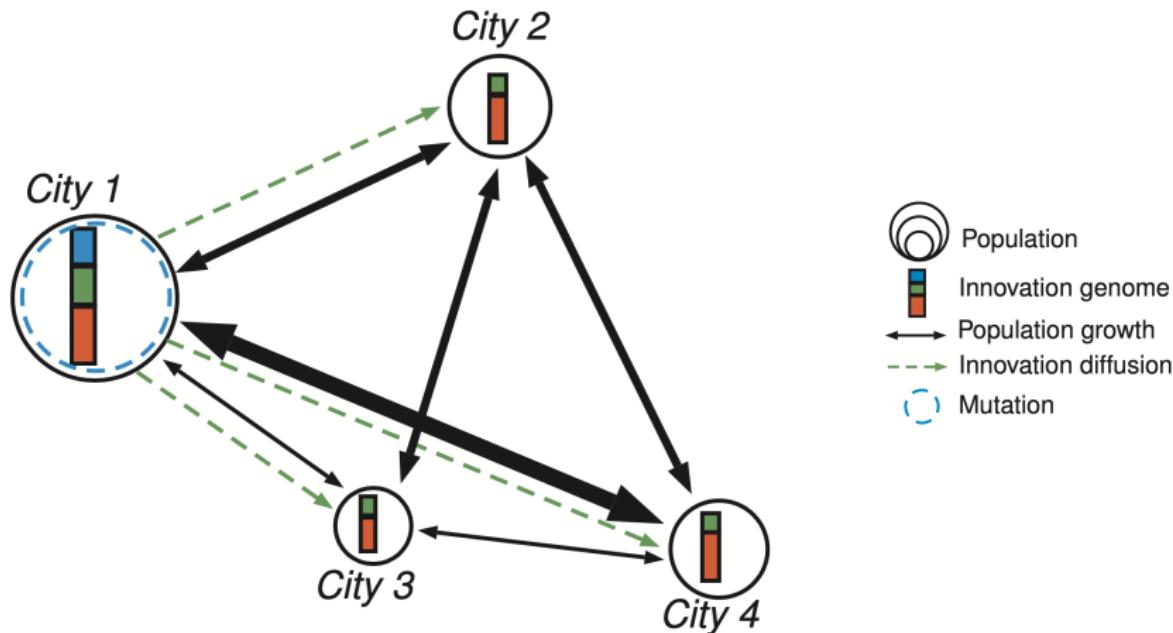


Cities-network co-evolution model explored on synthetic systems of cities [Raimbault, 2019a]





Raimbault, J. (2018). Modeling the co-evolution of cities and networks. Handbook of cities and networks, Rozenblat C., Niel Z., eds. (in press) arXiv:1804.09430.

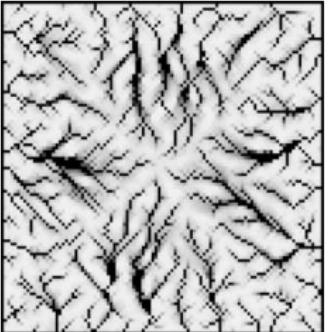
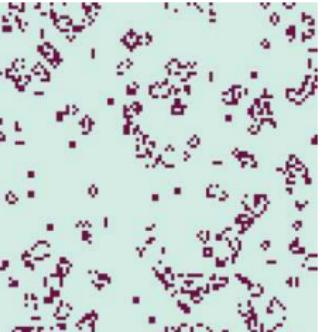
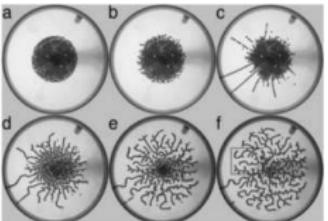
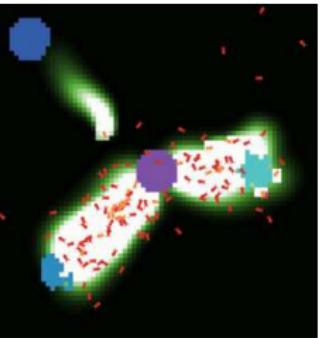


Raimbault, J. (2020). A model of urban evolution based on innovation diffusion. Artificial Life Conference Proceedings 2020 NO. 32, 500-508.

Spatial synthetic data in other disciplines ?

- ▶ spatial network generative models
- ▶ other disciplines ? ecology ? geosciences
[Mogheir et al., 2004] ?
- ▶ interaction with data driven disciplines ? (planning, architecture, spatio-temporal datamining)
- ▶ genericity of some models ? (reaction-diffusion)
- ▶ synthetic data generation methods (synthetic populations)
- ▶ synthetic data at the core of applied statistics methodology
(not much in spatial statistics ?)

Spatial form and function: Morphogenesis

	Physical	Biological	Engineered
Non Functional			
Functional			

Sources (in order by column). Ants, Erosion, Game of Life: NetLogo Library ; Arbotron [Jun and Hübler, 2005]; Industrial design [Aage et al., 2017]; Swarm chemistry [Sayama, 2007]

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→ *How does noise in real data impacts the result ?*

- ▶ Impact of missing elements
- ▶ Impact of imprecise coordinates or topology
- ▶ Optimal matching between spatial datasets

→ *How does perturbation of real data allows to explore scenario*

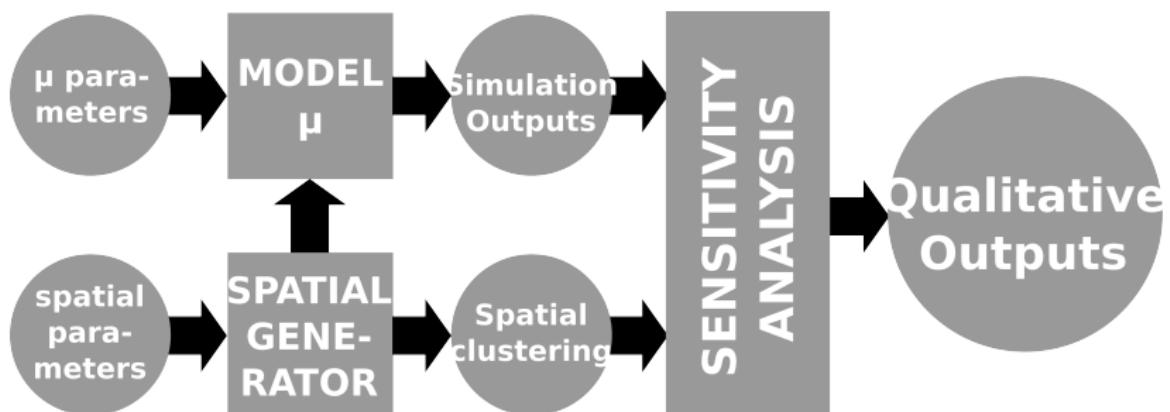
Examples:

- ▶ simulating urban projects by modifying population of areas with a given spatial correlation structure
- ▶ simulating network disruptions or new transportation lines

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General workflow to test the spatial sensitivity of simulation models

Raimbault, J., Cottineau, C., Le Texier, M., Le Néchet, F., & Reuillon, R. (2019). Space Matters: Extending Sensitivity Analysis to Initial Spatial Conditions in Geosimulation Models. *Journal of Artificial Societies and Social Simulation*, 22(4).

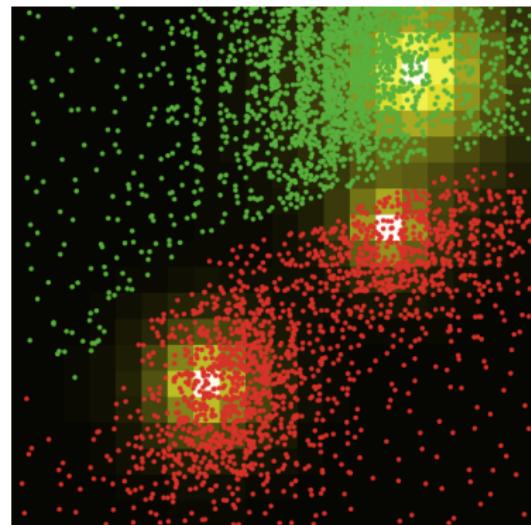
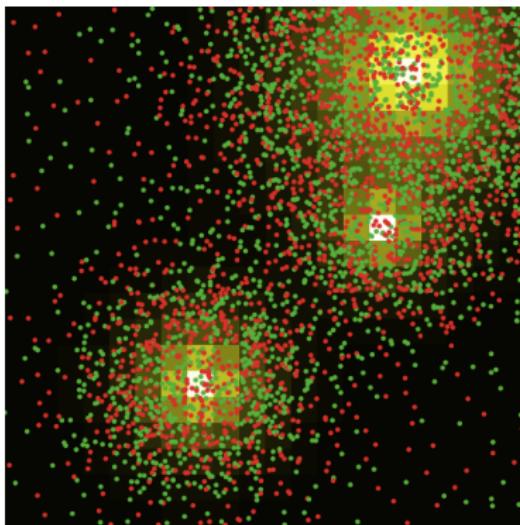


Relative distance of phase diagrams to compare global model behavior when meta-parameters change

$$d_r(\mu_{\vec{\alpha}_1}, \mu_{\vec{\alpha}_2}) = 2 \cdot \frac{d(\mu_{\vec{\alpha}_1}, \mu_{\vec{\alpha}_2})^2}{Var[\mu_{\vec{\alpha}_1}] + Var[\mu_{\vec{\alpha}_2}]}$$

Why could the Schelling model be sensitive to space ?

[Banos, 2012] network effects in Schelling model



Influence of spatial generator parameters on model outputs

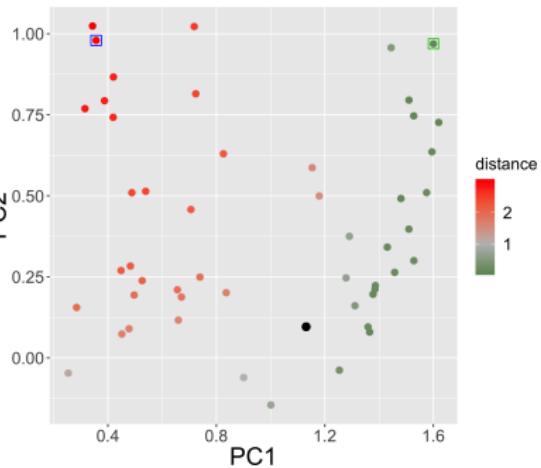
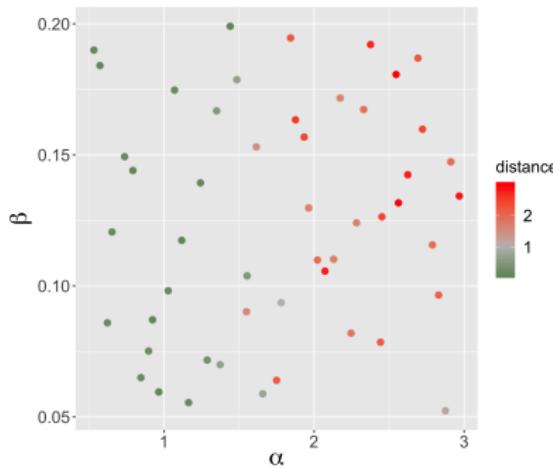
Simulation outcome by segregation index:	Dissimilarity		Entropy		Moran's I	
Intercept	-0.212 ***	-0.141 ***	-0.254 ***	-0.208 ***	-0.036 ***	-0.061 ***
Similarity Wanted (S)	1.212 ***	1.212 ***	1.250 ***	1.250 ***	0.550 ***	0.550 ***
quadratic term (S^2)	-0.942 ***	-0.942 ***	-0.963 ***	-0.963 ***	-0.428 ***	-0.438 ***
Vacancy Rate (V)	0.602 ***	0.602 ***	0.453 ***	0.453 ***	-0.027 ***	-0.027 ***
Minority Index (%Maj - %Min)	0.307 ***	0.307 ***	0.130 ***	0.130 ***	-0.067 ***	-0.067 ***
Density Grid = Polycentric		0.087 ***		0.052 ***		0.001 ***
Density Grid = Discontinuous		0.111 ***		0.068 ***		0.00
Attraction generator parameter α		-0.083 ***		-0.053 ***		0.014 ***
Diffusion generator parameter β		0.323 ***		0.218 ***		0.017 ***
R2 (%)	30.6	34.7	24.1	25.6	23.9	24.0
# of observations (sim. runs)	2,106,000	2,106,000	2,106,000	2,106,000	2,106,000	2,106,000
AIC	-70717.68	-198748.2	208213.8	166048.8	-4385990	-4387816

Application: Sugarscape model

A model of resource collection

- ▶ agents collect a spatial resource
- ▶ the resource regrows at a certain rate only

Relative distances between phase diagrams



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In the spatial approach, spatial model indicators are also important: what kind of spatial structure does the model produce ?

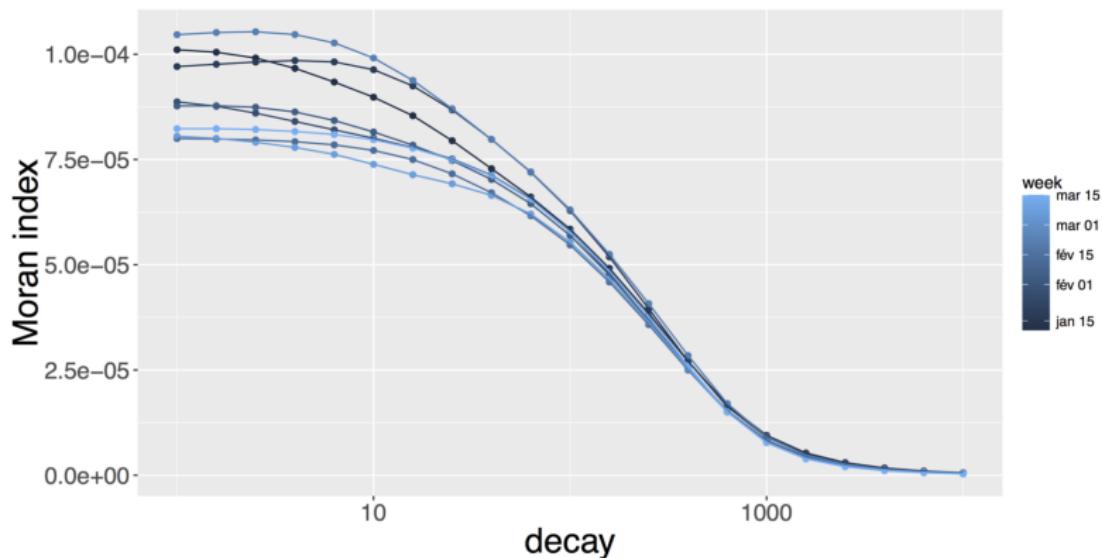
- ▶ previous form indicators at different scales, applied on any spatialized variable or event: quantify level of aggregation, hierarchy, clustering
- ▶ spatial statistics indicators and methods
- ▶ more complicated approaches: fractals and multifractals, spatial datamining

Spatial autocorrelation at a given range

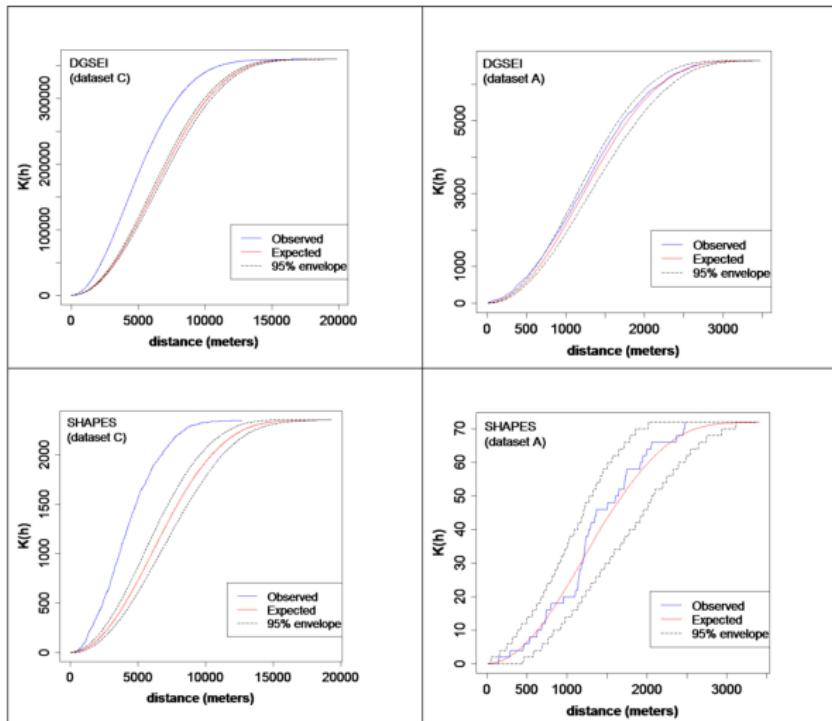
Given spatial weights w_{ij}

$$I = \frac{N}{\sum_{i,j} w_{ij}} \cdot \frac{\sum_{i,j} w_{ij} \cdot (X_i - \bar{X})(X_j - \bar{X})}{\sum_i (X_i - \bar{X})^2}$$

Extracting typical spatial scales of a system using Moran Index
[Bergeaud and Raimbault, 2020]

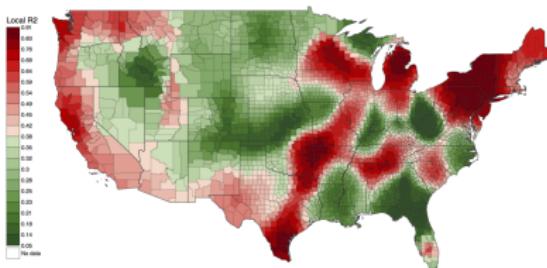
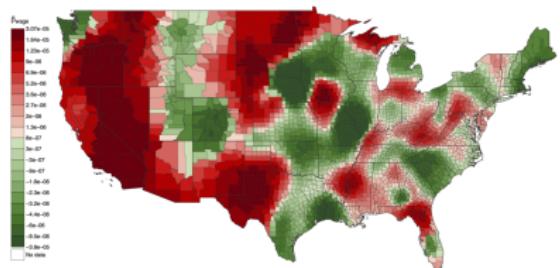
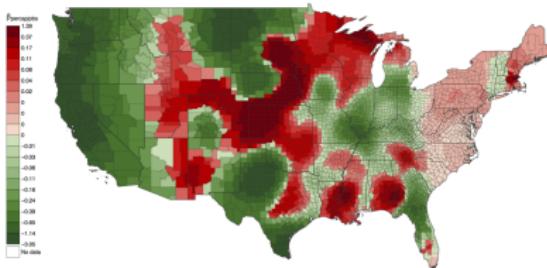
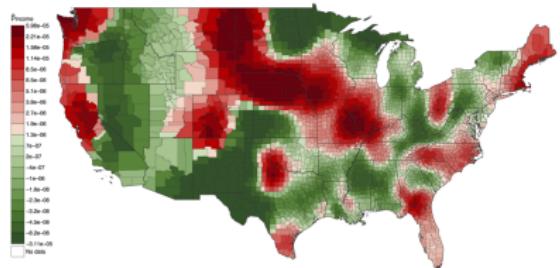


Quantifying the level of clustering of spatial points or points on a network regarding a Poisson process null model [Vandenbulcke et al., 2017]



Geographically Weighted Regression

GWR handles spatial non-stationarity with adaptative scale fitting
[Fotheringham et al., 2003]



Bergeaud, A., & Raimbault, J. (2020). An empirical analysis of the spatial variability of fuel prices in the United States. *Transportation Research Part A: Policy and Practice*, 132, 131-143.

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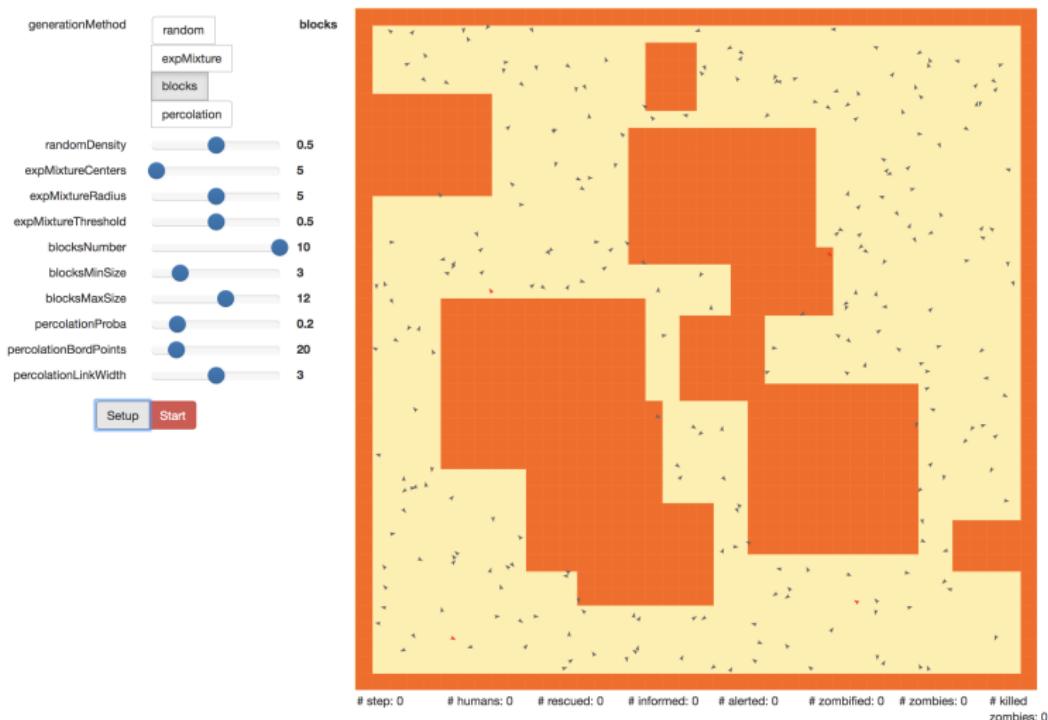
	OpenMOLE	spatialdata	planned
Micro grid spatial samplings	✓	✓	✓
Meso grid spatial samplings	✓	✓	✓
Macro spatial samplings	✗	✓	✓
Spatial network generation	✗	✓	✓
Real data import	✗	✗	✓
Real data perturbations	✗	✗	✓
Spatial statistics	✓	✓	✓
Hybrid methods	✗	✗	✓
Domain models (transportation, land-use)	✗	✓	✓

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Synthetic spaces for ZOMBIE

Try the GUI with synthetic spaces at

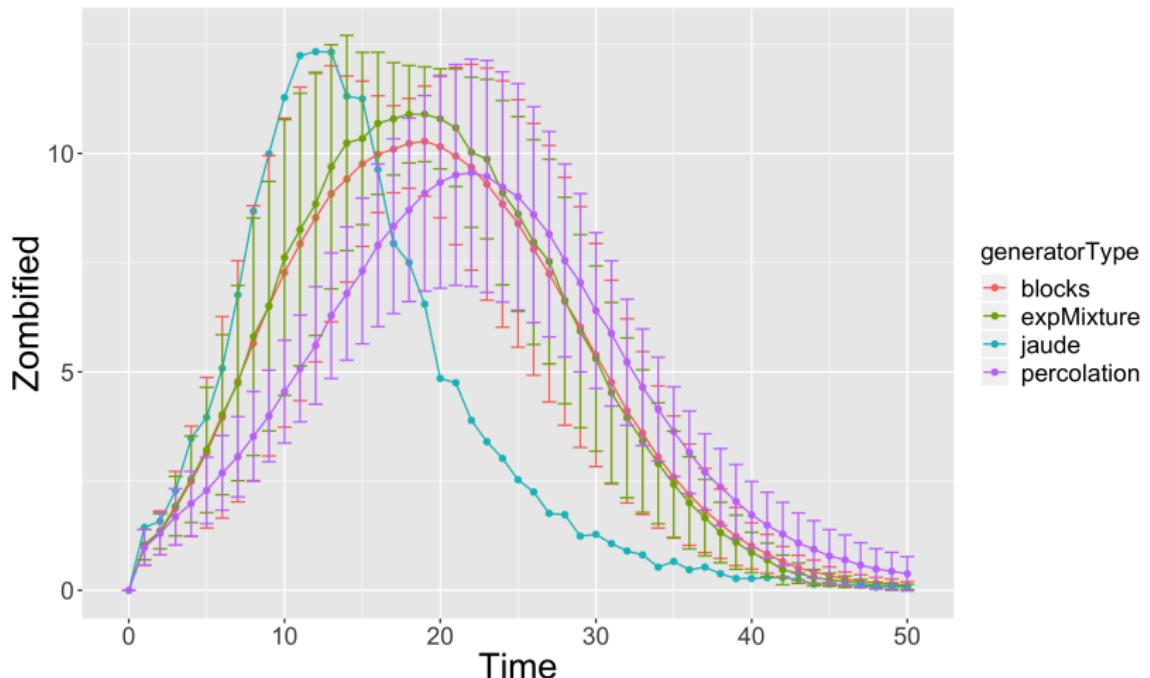
<https://om.exmodelo.org/spatialsens/>



First application: fixed model parameters, sensitivity of time series to spatial configuration

→ explore the script `spatialsens.oms`

Sensitivity of zombified dynamics



Second application: variance-based comparison of phase diagrams for the cooperation submodel, between varying spatial configuration

→ explore the script `phasediag.oms`

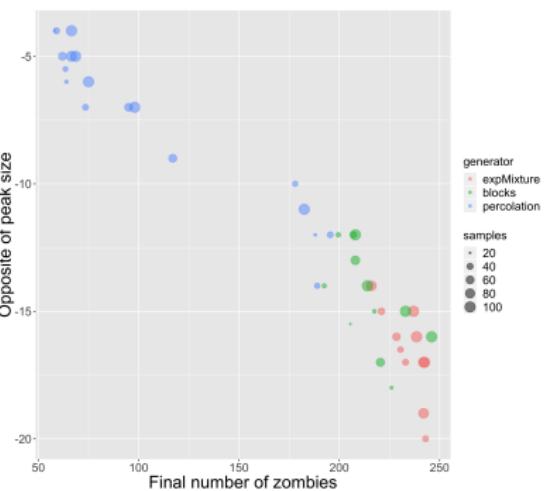
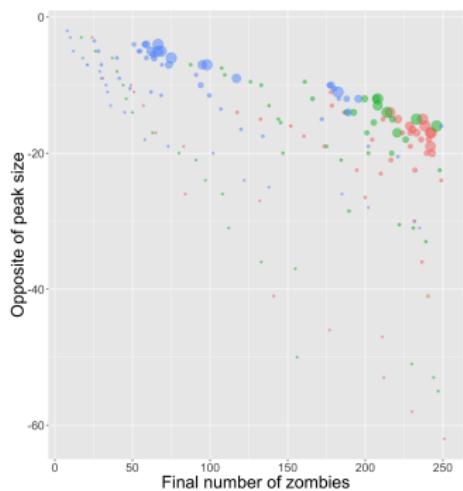
No results for this experiment (problem of memory more than computation time) - method still need to be reformulated/implemented in a “Saltelli-like” manner

Third application: calibration of the default parameters model, for each type of generator, to maximize peak size (time-localized dynamics) and minimize number of zombified

→ explore the script calibrate.oms

Complementary restricted Pareto fronts between the different spatial generators

→ urban planning policies yield different zombie-resilient compromises, and different “planning paradigms” are complementary



- ▶ test the robustness of your qualitative findings to the spatial configuration
- ▶ use spatial metrics to answer specific questions
- ▶ change the initial spatial distribution of agents (***need library/model tuning***)
- ▶ study the influence of spatial distribution of rescues or traps (***need library/model tuning if structured spatial configuration***)
- ▶ compute complicated spatial statistics on agents trajectories e.g. (***need library/model tuning***)

- **Space matters:** relevance of spatially-explicit models and spatial sensitivity analysis.
- **Synthetic data:** first experimental samplings included in OpenMOLE, soon more to come.
- **Disciplinary context:** strong contingency on included models and forms, please provide feedbacks, suggestions, needs, ideas from your viewpoint.

Open issues at <https://github.com/openmole/spatialdata/issues>

References I

-  Aage, N., Andreassen, E., Lazarov, B. S., and Sigmund, O. (2017). Giga-voxel computational morphogenesis for structural design. *Nature*, 550(7674):84–86.
-  Banos, A. (2012). Network effects in schelling's model of segregation: new evidence from agent-based simulation. *Environment and Planning B: Planning and Design*, 39(2):393–405.
-  Batty, M. (1991). Generating urban forms from diffusive growth. *Environment and Planning A*, 23(4):511–544.

References II

-  Bedau, M. (2002).
Downward causation and the autonomy of weak emergence.
Principia: an international journal of epistemology, 6(1):5–50.
-  Bergeaud, A. and Rimbault, J. (2020).
An empirical analysis of the spatial variability of fuel prices in
the united states.
Transportation Research Part A: Policy and Practice,
132:131–143.
-  Chérel, G., Cottineau, C., and Reuillon, R. (2015).
Beyond corroboration: Strengthening model validation by
looking for unexpected patterns.
PLoS ONE, 10(9):e0138212.

References III

-  Cottineau, C. (2014).
L'évolution des villes dans l'espace post-soviétique. Observation et modélisations.
PhD thesis, Université Paris 1 Panthéon-Sorbonne.
-  Fan, H., Zipf, A., Fu, Q., and Neis, P. (2014).
Quality assessment for building footprints data on openstreetmap.
International Journal of Geographical Information Science, 28(4):700–719.
-  Fotheringham, A. S., Brunsdon, C., and Charlton, M. (2003).
Geographically weighted regression: the analysis of spatially varying relationships.
John Wiley & Sons.

References IV

-  Fujita, M. and Thisse, J.-F. (1996).
Economics of agglomeration.
Journal of the Japanese and international economies,
10(4):339–378.
-  Jun, J. K. and Hübler, A. H. (2005).
Formation and structure of ramified charge transportation
networks in an electromechanical system.
*Proceedings of the National Academy of Sciences of the
United States of America*, 102(3):536–540.
-  Louf, R., Jensen, P., and Barthelemy, M. (2013).
Emergence of hierarchy in cost-driven growth of spatial
networks.
Proceedings of the National Academy of Sciences,
110(22):8824–8829.

References V

-  Mogheir, Y., De Lima, J., and Singh, V. (2004).
Characterizing the spatial variability of groundwater quality
using the entropy theory: I. synthetic data.
Hydrological processes, 18(11):2165–2179.
-  Openshaw, S., Charlton, M., Wymer, C., and Craft, A. (1987).
A mark 1 geographical analysis machine for the automated
analysis of point data sets.
International Journal of Geographical Information System,
1(4):335–358.
-  Pumain, D. (1997).
Pour une théorie évolutive des villes.
Espace géographique, 26(2):119–134.

References VI

-  Pumain, D. (2003).
Une approche de la complexité en géographie.
Géocarrefour, 78(1):25–31.
-  Pumain, D. (2008).
The socio-spatial dynamics of systems of cities and innovation processes: a multi-level model.
The Dynamics of Complex Urban Systems, pages 373–389.
-  Pumain, D. (2012).
Urban systems dynamics, urban growth and scaling laws: The question of ergodicity.
In *Complexity Theories of Cities Have Come of Age*, pages 91–103. Springer.

References VII

-  Pumain, D. (2018).
An evolutionary theory of urban systems.
In *International and Transnational Perspectives on Urban Systems*, pages 3–18. Springer.
-  Pumain, D., Paulus, F., Vacchiani-Marcuzzo, C., and Lobo, J. (2006).
An evolutionary theory for interpreting urban scaling laws.
Cybergeo: European Journal of Geography.
-  Pumain, D., Swerts, E., Cottineau, C., Vacchiani-Marcuzzo, C., Ignazzi, C. A., Bretagnolle, A., Delisle, F., Cura, R., Lizzi, L., and Baffi, S. (2015).
Multilevel comparison of large urban systems.
Cybergeo: European Journal of Geography.

References VIII

-  Raimbault, J. (2017).
An applied knowledge framework to study complex systems.
In *Complex Systems Design & Management*, pages 31–45.
-  Raimbault, J. (2018a).
Calibration of a density-based model of urban morphogenesis.
PloS one, 13(9):e0203516.
-  Raimbault, J. (2018b).
Unveiling co-evolutionary patterns in systems of cities: a
systematic exploration of the simpopnet model.
arXiv preprint arXiv:1809.00861.
-  Raimbault, J. (2019a).
Modeling interactions between transportation networks and
territories: a co-evolution approach.
arXiv preprint arXiv:1902.04802.

-  Rimbault, J. (2019b).
Towards multi-scalar models for the co-evolution of
transportation networks and territories.
In *Theo Quant 2019*.
-  Rimbault, J. (2019c).
An urban morphogenesis model capturing interactions between
networks and territories.
In *The Mathematics of Urban Morphology*, pages 383–409.
Springer.
-  Rimbault, J. (2020).
A comparison of simple models for urban morphogenesis.

-  Raimbault, J., Banos, A., and Doursat, R. (2014).
A hybrid network/grid model of urban morphogenesis and optimization.
In *4th International Conference on Complex Systems and Applications*, pages 51–60.
-  Raimbault, J., Denis, E., and Pumain, D. (2020).
Empowering urban governance through urban science:
Multi-scale dynamics of urban systems worldwide.
Sustainability, 12(15):5954.
-  Raimbault, J. and Perret, J. (2019).
Generating urban morphologies at large scales.
Forthcoming in proceedings of Artificial Life 2019.
arXiv:1903.06807.

References XI

-  Reuillon, R., Schmitt, C., De Aldama, R., and Mouret, J.-B. (2015).
A new method to evaluate simulation models: The calibration profile (cp) algorithm.
Journal of Artificial Societies and Social Simulation, 18(1):12.
-  Sanders, L., Pumain, D., Mathian, H., Guérin-Pace, F., and Bura, S. (1997).
Simpop: a multiagent system for the study of urbanism.
Environment and Planning B, 24:287–306.
-  Sayama, H. (2007).
Decentralized control and interactive design methods for large-scale heterogeneous self-organizing swarms.
In *European Conference on Artificial Life*, pages 675–684.
Springer.

References XII

-  Schmitt, C. (2014).
Modélisation de la dynamique des systèmes de peuplement: de SimpopLocal à SimpopNet.
PhD thesis, Paris 1.
-  Schmitt, C., Rey-Coyrehourcq, S., Reuillon, R., and Pumain, D. (2015).
Half a billion simulations: Evolutionary algorithms and distributed computing for calibrating the simpoplocal geographical model.
Environment and Planning B: Planning and Design, 42(2):300–315.
-  Serra, J. (1994).
Morphological filtering: an overview.
Signal processing, 38(1):3–11.

References XIII

-  Tero, A., Takagi, S., Saigusa, T., Ito, K., Bebber, D. P., Fricker, M. D., Yumiki, K., Kobayashi, R., and Nakagaki, T. (2010).
Rules for biologically inspired adaptive network design.
Science, 327(5964):439–442.
-  Turing, A. M. (1952).
The chemical basis of morphogenesis.
Philosophical Transactions of the Royal Society of London B: Biological Sciences, 237(641):37–72.
-  Vandenbulcke, G., Int Panis, L., and Thomas, I. (2017).
On the location of reported and unreported cycling accidents:
A spatial network analysis for brussels.
Cybergeo: European Journal of Geography.



- Varenne, F. (2017).
Théories et modèles en sciences humaines. Le cas de la géographie.
Editions Matériologiques.