

# Spatial sensitivity analysis

## Course and practical application

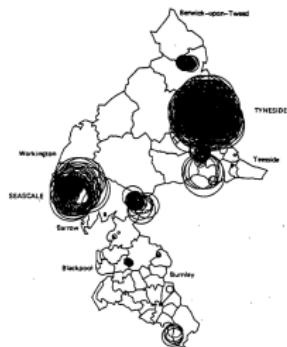
**OpenMOLE**

June 26, 2019

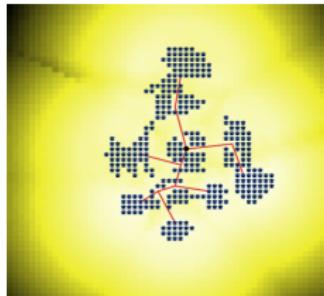
- 1 Scientific context
- 2 Spatial synthetic data
- 3 Perturbation of data
- 4 Sensitivity to spatial configuration
- 5 Spatial indicators for model outputs
- 6 Synthesis
- 7 Application on the Zombie model

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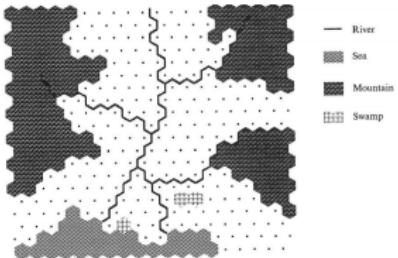
# Modeling and simulation in geography



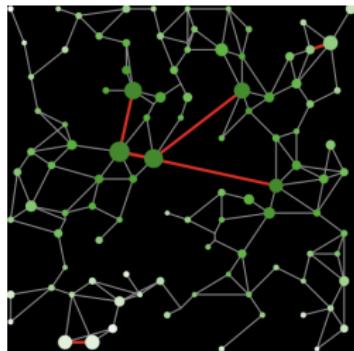
Geographical analysis machine  
[Openshaw et al., 1987]



Hybrid urban morphogenesis  
[Raimbault et al., 2014]



Simpop 1 model [Sanders et al., 1997]



SimpopNet model [Schmitt, 2014]

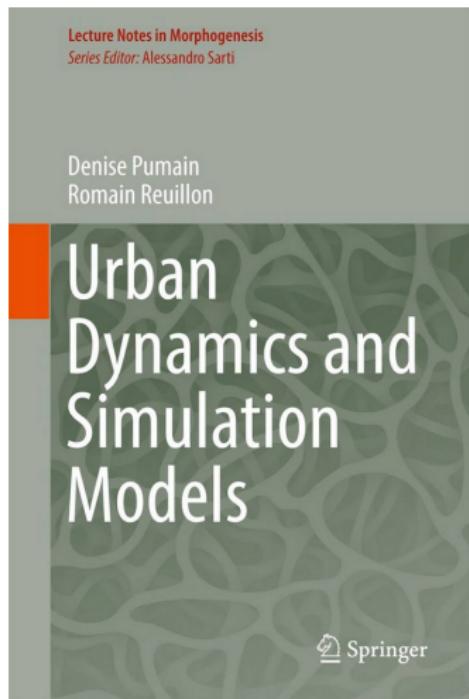
*Necessity of simulation models in geography induced by complexities of these systems ?*

- ▶ Ontological complexity [Pumain, 2003]
- ▶ Dynamical complexity: non-ergodicity and path-dependancy [Pumain, 2012]
- ▶ Complexity and co-evolution
- ▶ Complexity and emergence

Historical succession of epistemologies in the case of systems of cities [Varenne, 2017]:

1. Deduction from theory (top-down): Christaller
2. Induction from the empirical (bottom-up): Berry
3. Towards an abductive epistemology (iterative interaction theoretical-empirical): Pumain

→ simulation allows synthesis



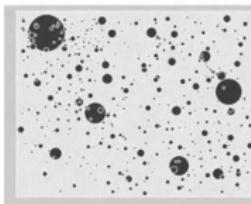
Development of evolutive urban theory  
[Pumain, 2018]

- Recurrent stylized facts on main systems of cities
- Construction of simulation models (with an explicative purpose)
- Tools and methods to explore simulation models



# Evolutive Urban Theory

## Spatio-temporal scales



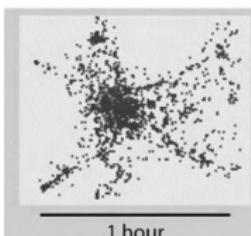
## Emerging properties

Hierarchy  
Functional diversity  
Spatial pattern

## Organization levels

**Macro: System of cities**  
(urban networks)

1 day

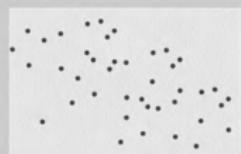


1 hour

Centrality  
Function  
Morphology  
"Ambiance urbaine"

**Meso: City**  
(urban areas)

## Descriptors

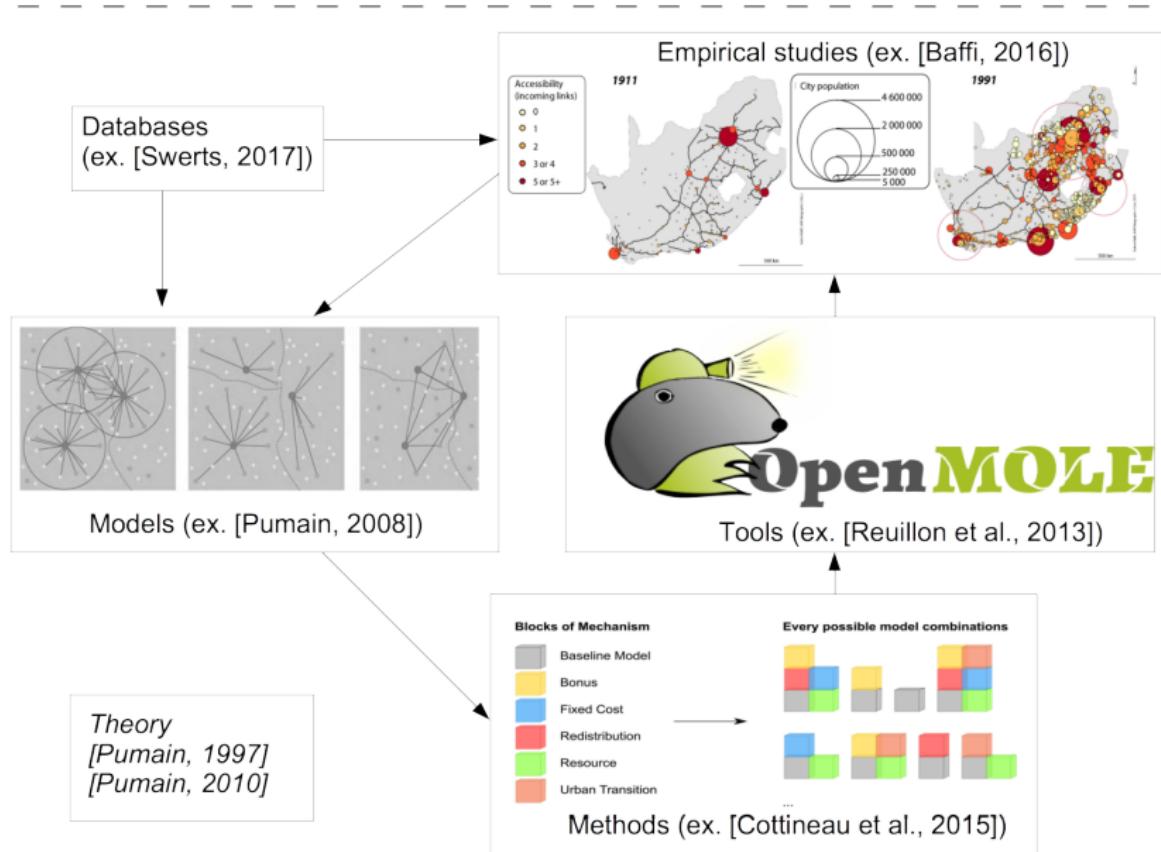


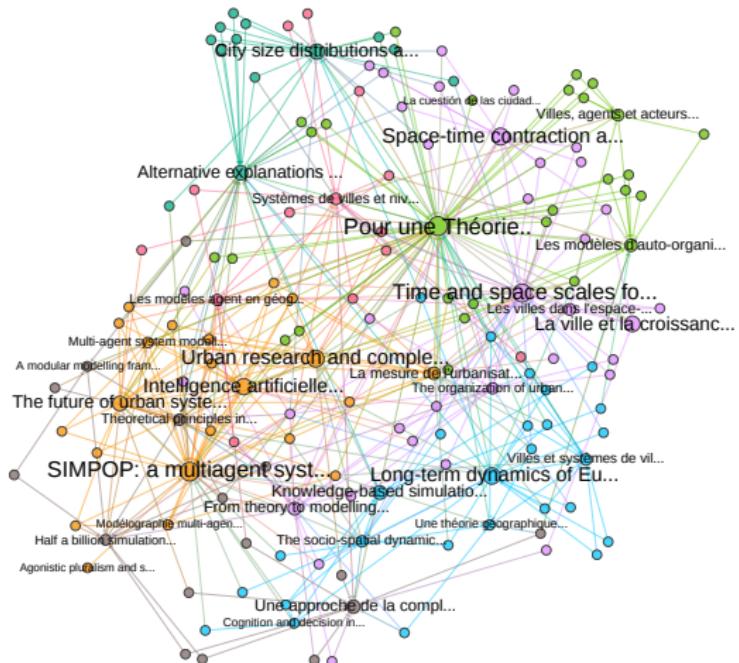
Life cycle  
Profession  
Power

**Micro: Actors**  
(households, firms, institutions)

*Systems of cities as co-evolutive systems in which interactions are crucial*  
[Pumain, 1997]  
[Pumain, 2008]  
[Pumain, 2018]

# Construction of Knowledge across Domains

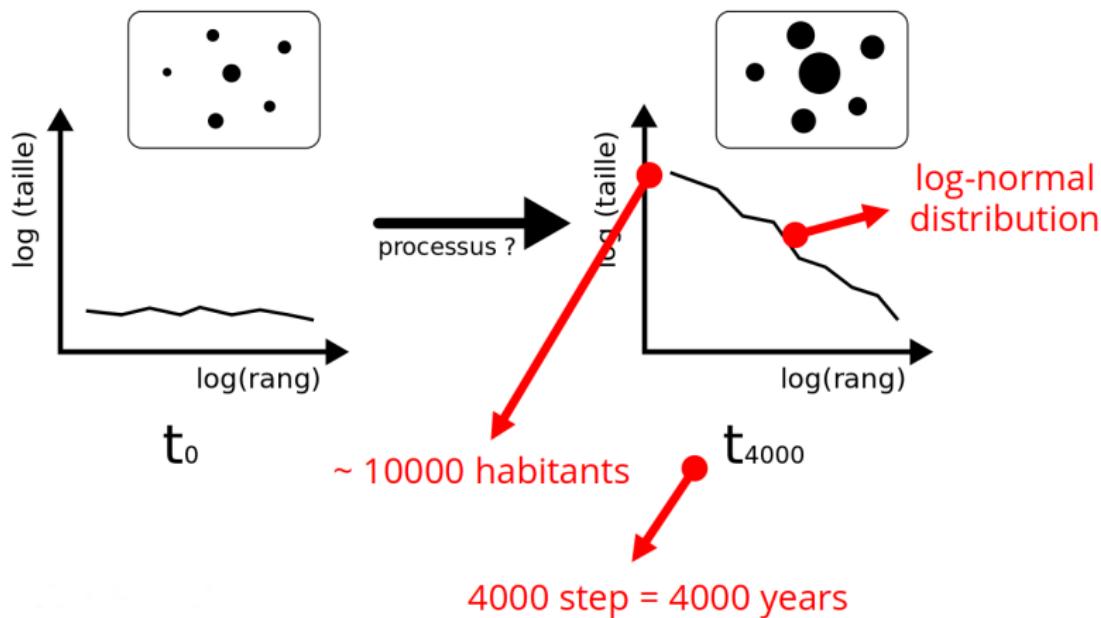




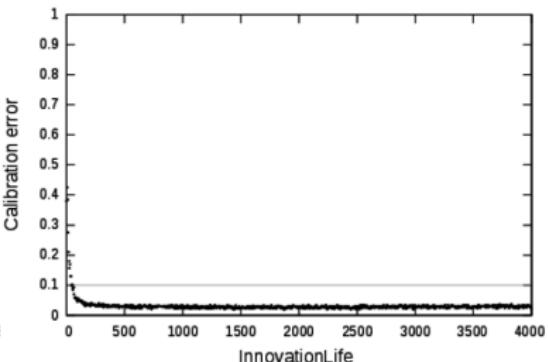
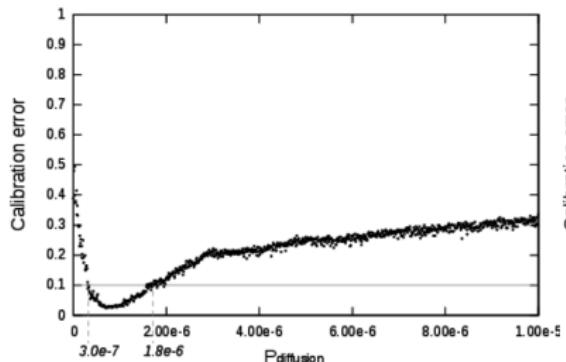
[Raimbault, 2017] Citation network analysis of key publications in the evolutive urban theory

SimpopLocal model calibrated with distributed NSGA2 on grid  
[Schmitt et al., 2015]

*Formalising the expectations as indicators:*

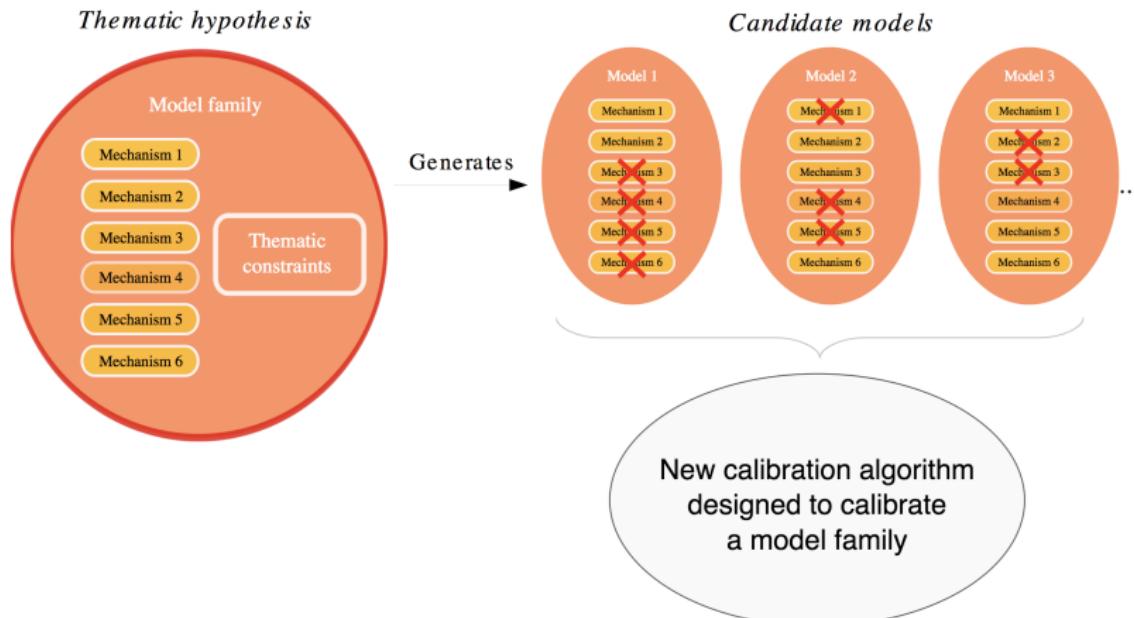


*Computes the best calibration at fixed steps along one dimension.*  
[Reuillon et al., 2015]



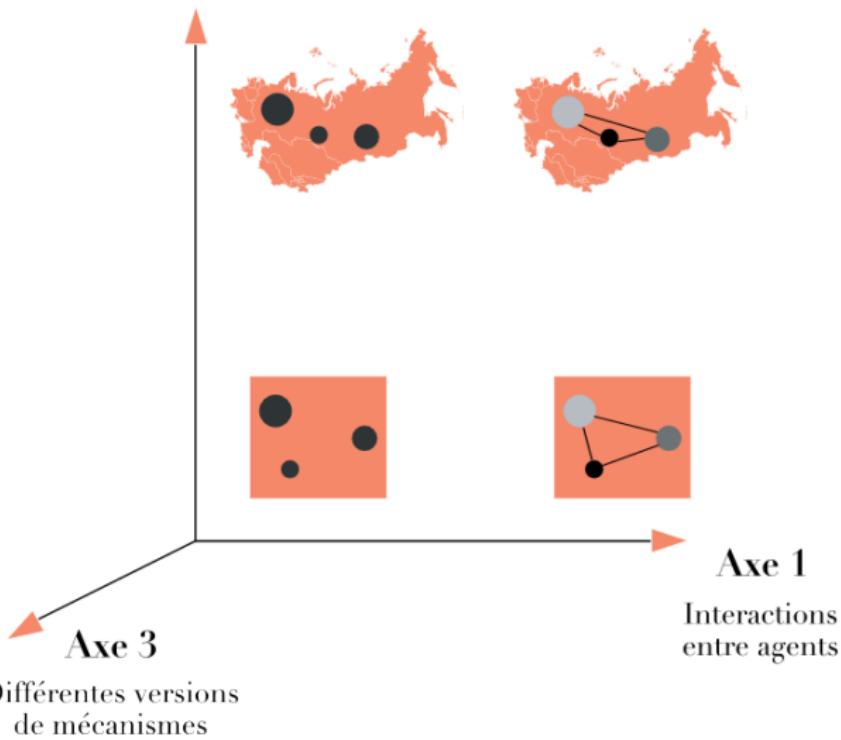
# Unicity of mechanisms

*Automate the confrontation of alternative hypothesis / mechanisms [Cottineau, 2014]*



## Axe 2

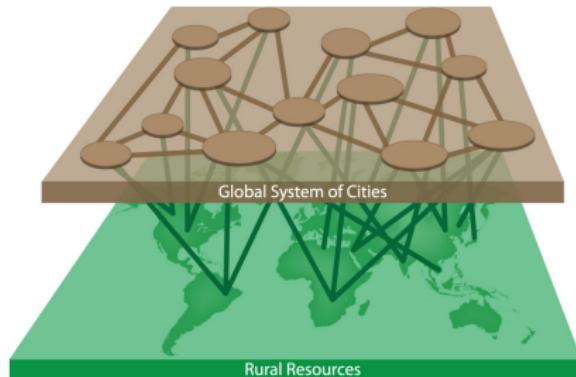
Interactions entre  
les agents et  
l'environnement



(c) Clémentine Cottineau, UMR Géographie-Cités, P.A.R.I.S., 2014

**Exchange mechanism:** market vs centralized

**City growth:** interurban interactions vs environmental situation



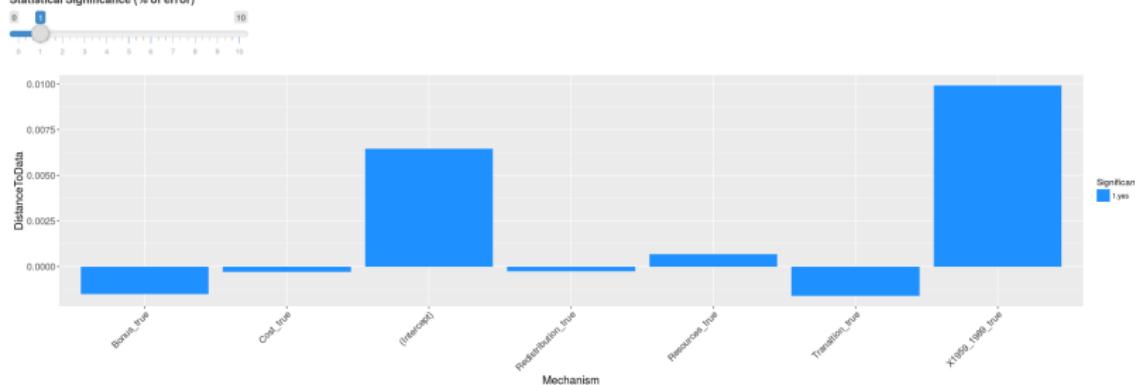
# Calibration of model family

*Compute the best set of parameters for all 64 models.*

## Contribution of mechanisms to the quality of simulation (closeness to data)

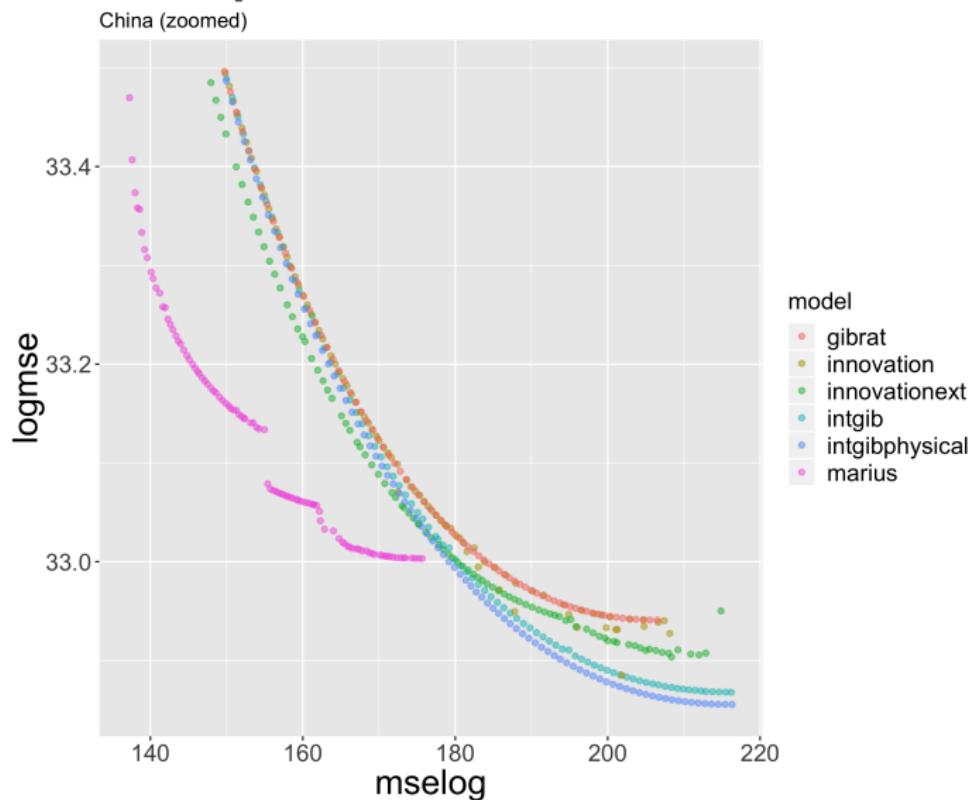
Models with different combination of mechanisms have been calibrated intensively against empirical data, using generic algorithms for more than 100000 generations. This plot shows the results of a regression explaining one measure of the quality of models (a small difference between simulated and empirical urban trajectories) by their mechanisms composition (the fact that any of the supplementary mechanisms is activated or not). Each bar represents the value of the estimated coefficient for each activated mechanism, in comparison with the same model structure without this mechanism, everything else being equal.

Statistical Significance (% of error)

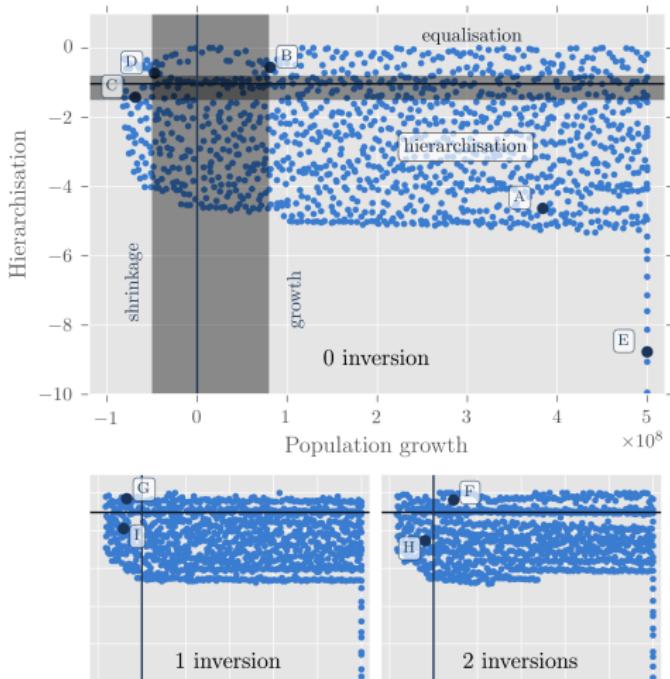


# Other example of multi-modeling

*Benchmark of growth models for systems of cities*  
[Raimbault, 2018b]



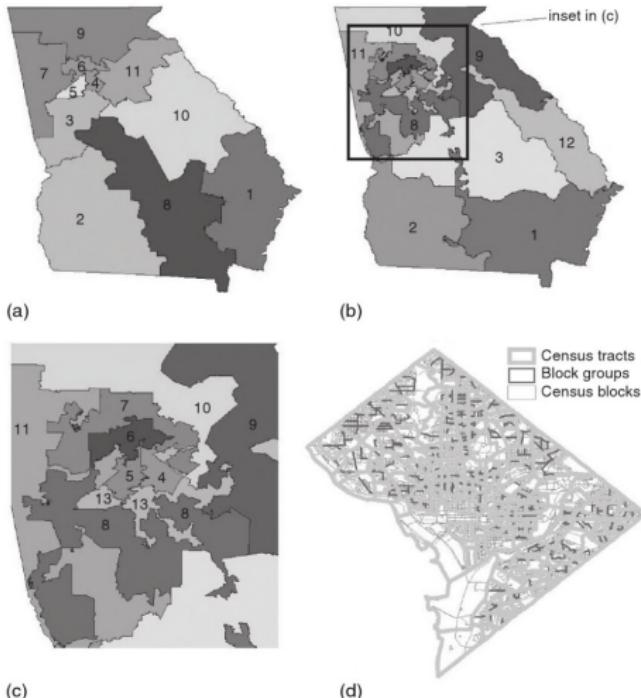
*Diversity of urban systems dynamics produced by the MARIUS model [Chérel et al., 2015]*



*Classical problems in geography and spatial sciences :*

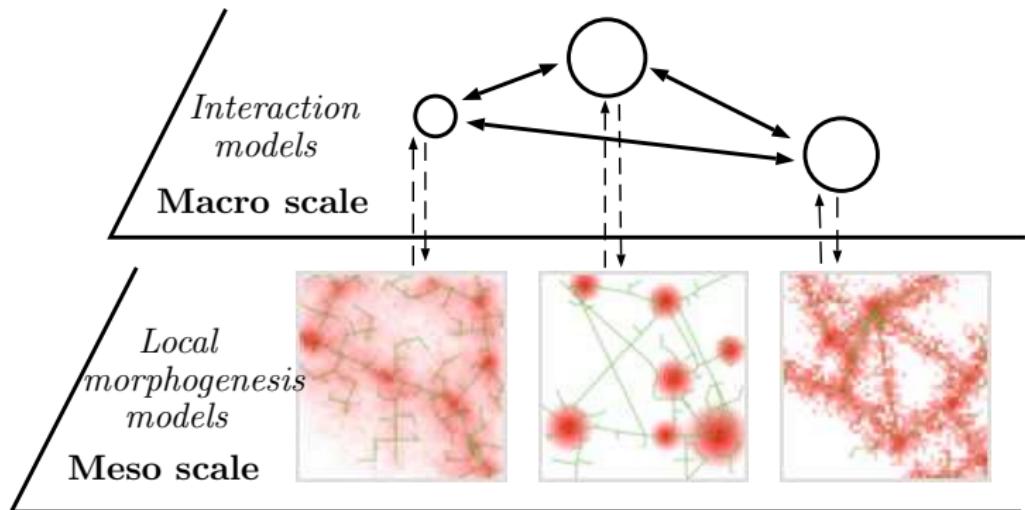
- ▶ Modifiable Areal Unit Problem
- ▶ Dependancy of processes to scale
- ▶ Spatial non-stationarity
- ▶ Fuzzy and noisy data
- ▶ Genericity/particularity

# The Modifiable Areal Unit Problem

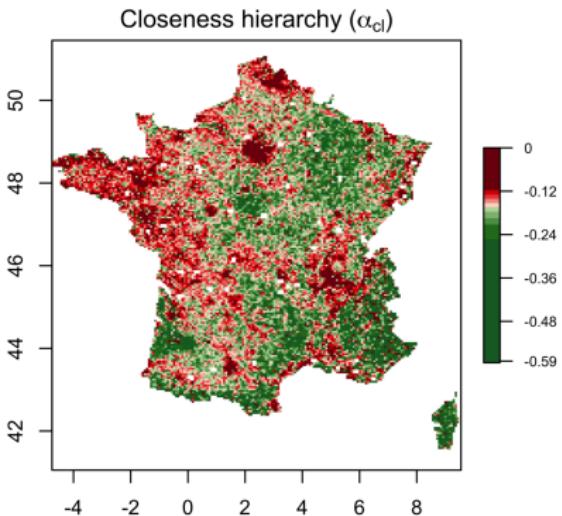
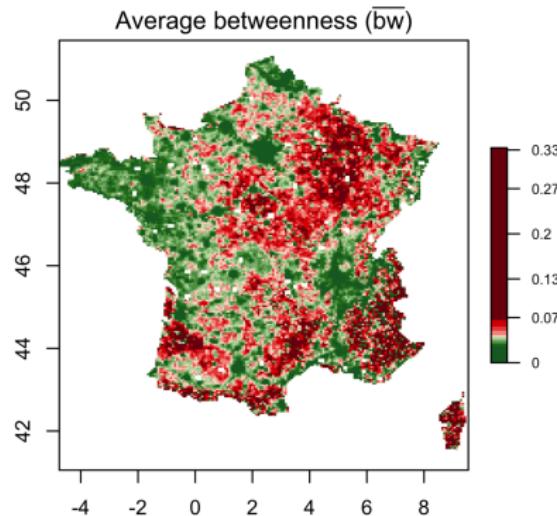


Wong, D. (2009). The modifiable areal unit problem (MAUP). The SAGE handbook of spatial analysis, 105, 23.

*Processes specific to scales, coupling implies dedicated ontologies*  
[Raimbault, 2019b]



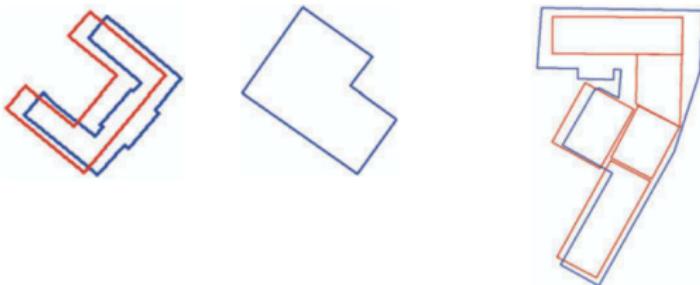
*Spatial non-stationarity of road network properties*  
[Raimbault, 2019c]



## *Assessment of data quality in OpenStreetMap [Fan et al., 2014]*

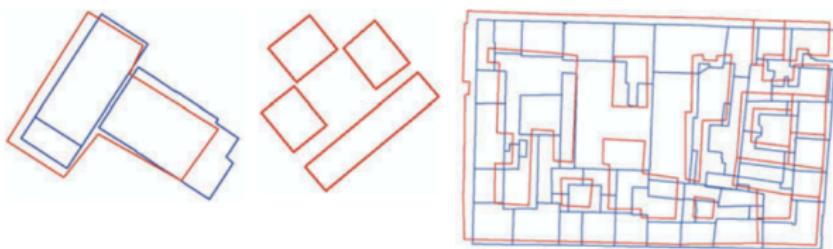
Relation	1:1	1:0	1: $n$
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Illustration



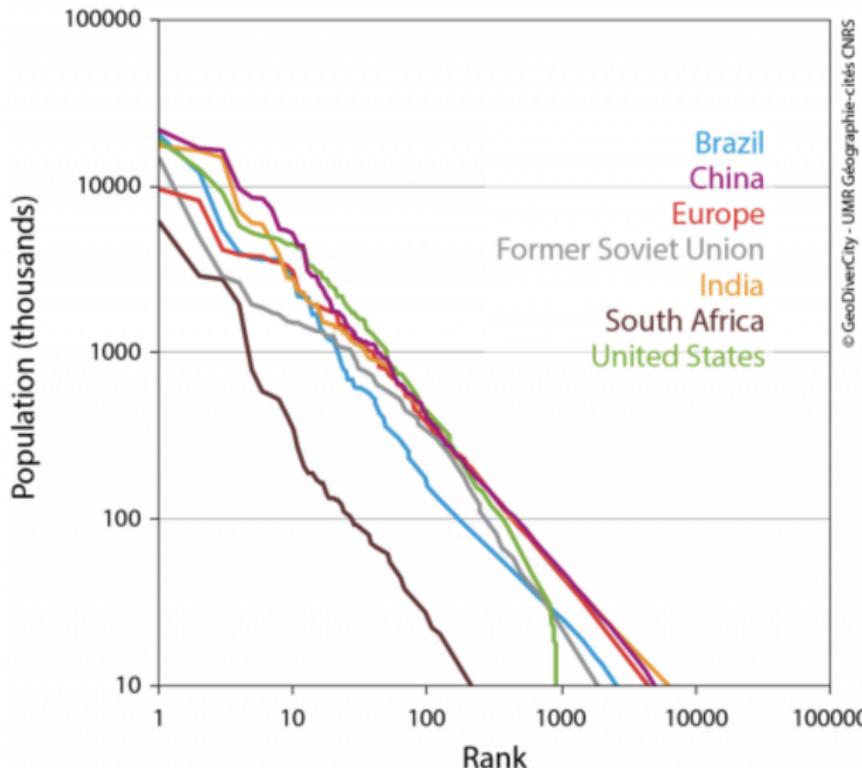
Relation	$n:1$	0:1	$n:m$
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Illustration



# Genericity and specificity

*Urban systems are simultaneously universal and particular*  
[Pumain et al., 2015]



⇒ **spatial configuration are parameters too**

- ▶ “*Space matters*”: impact of spatial configuration on model behavior
- ▶ Model behaviors which are robust to spatial configuration
- ▶ Model behaviors which are robust to noise in parametrization real datasets

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- coupling models with spatial configuration generators (spatial synthetic data) gives model sensitivity to space through sensitivity analysis of the coupled model
- synthetic urban forms resembling real configurations
- at different scales: microscopic (buildings), mesoscopic (population distribution), macroscopic (system of cities)

## *At the microscopic scale (district): generating building layouts*

Rimbault, J. and Perret, J., 2019. Generating urban morphologies at large scales. *Forthcoming in proceedings of Artificial Life 2019.* arXiv:1903.06807

- ▶ systematic comparison of simple processual generators
- ▶ introduction of morphological indicators
- ▶ calibration on sampled layouts from OpenStreetMap

Urban form indicators for building layouts:

- ▶ density, number of buildings, average area
- ▶ Moran index and average distance on rasterized representation
- ▶ average detour in the free space
- ▶ mathematical morphology indicators (steps for erosion and dilation) [Serra, 1994]

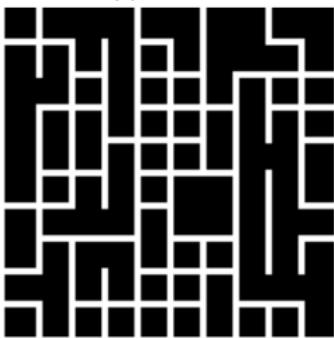
## Complementary generators



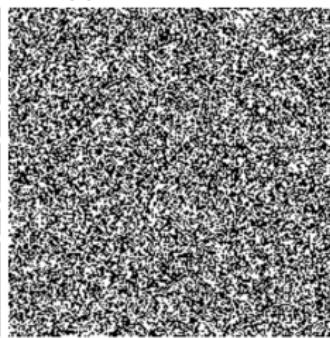
(a) Blocks



(b) Kernel mixture



(c) Network percolation



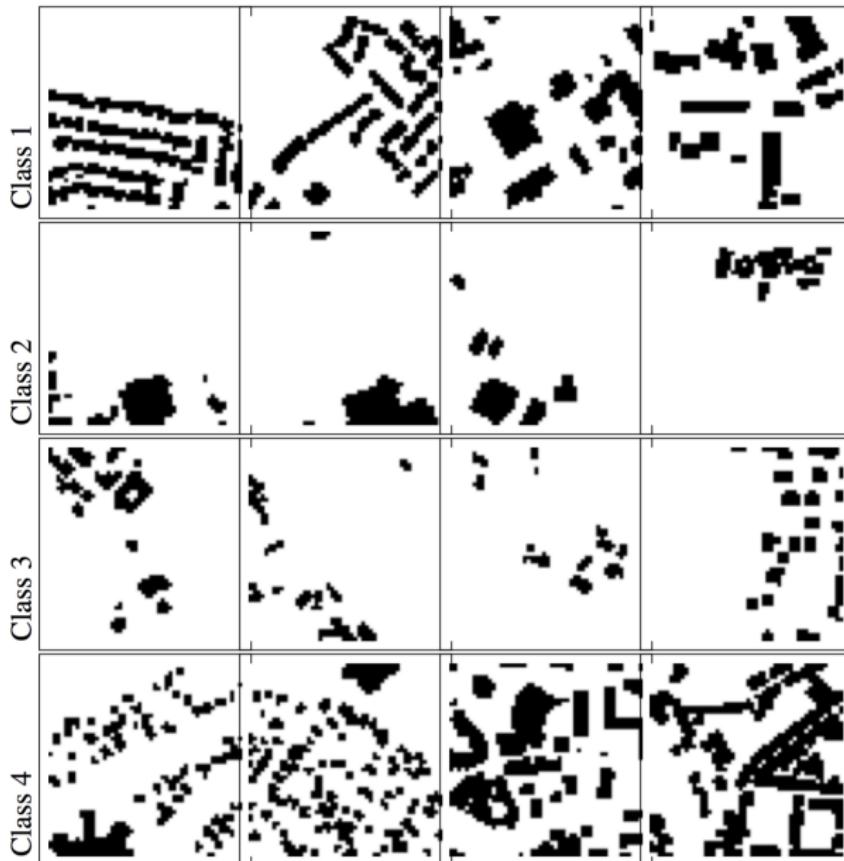
(d) Random

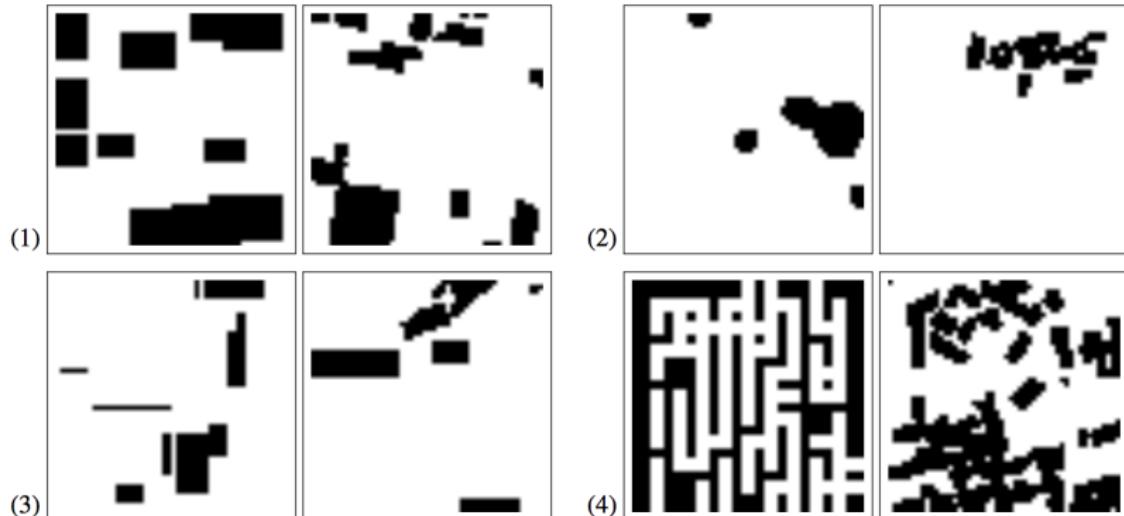
# Real configurations

*Sampled districts from OpenStreetMap*

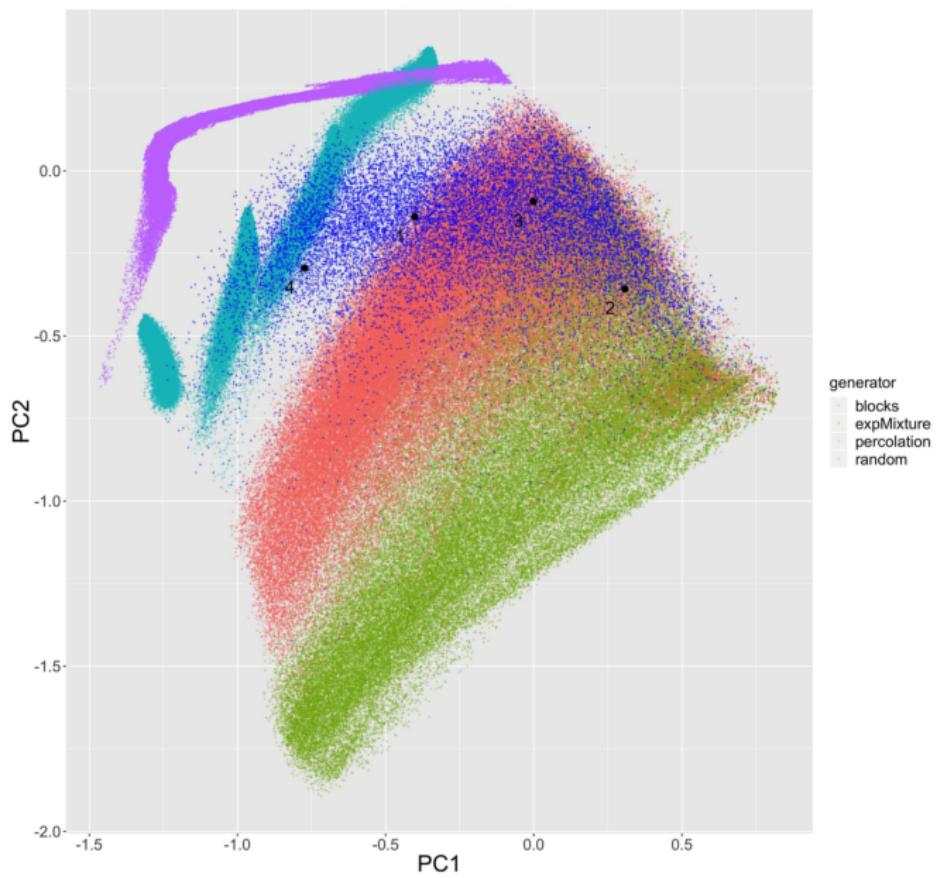


# Classification of urban forms





# Point cloud



	Random	Blocks	Exp. Mixture	Percolation
Centroid 1	$0.424 \pm 0.011$	$0.106 \pm 0.063$	$0.303 \pm 0.101$	$0.325 \pm 0.019$
Centroid 2	$0.809 \pm 0.022$	$0.164 \pm 0.099$	$0.184 \pm 0.141$	$0.947 \pm 0.019$
Centroid 3	$0.428 \pm 0.019$	$0.095 \pm 0.054$	$0.109 \pm 0.064$	$0.541 \pm 0.019$
Centroid 4	$0.515 \pm 0.005$	$0.311 \pm 0.077$	$0.589 \pm 0.149$	$0.083 \pm 0.025$

*Why not use calibration heuristics? Open question of fitting a point cloud; issue of projecting in a reduced dimension space*

```
sampling =  
  BlocksGridSpatialSampling(  
    samples = 2,  
    gridSize = 10,  
    blocks = (p1gen in Range(1.0,10.0)),  
    blockMinSize = (p2gen in Range(2.0,4.0)),  
    blockMaxSize = (p3gen in Range(2.0,4.0)),  
    prototype = w  
)
```

→ other generators have their own primitives  
(ExpMixtureThresholdSpatialSampling,  
PercolationGridSpatialSampling) and arguments (**see the documentation**)

→ explore the script SpatialSampling.oms

<http://nextcloud.openmole.org/s/HsyqS3bSiS7Tb6B/download>

## *At the mesoscopic scale: population grids*

- ▶ a reaction-diffusion model for population distributions
- ▶ urban form measures at the mesoscopic scale
- ▶ towards synthetic generators coupling population and road networks

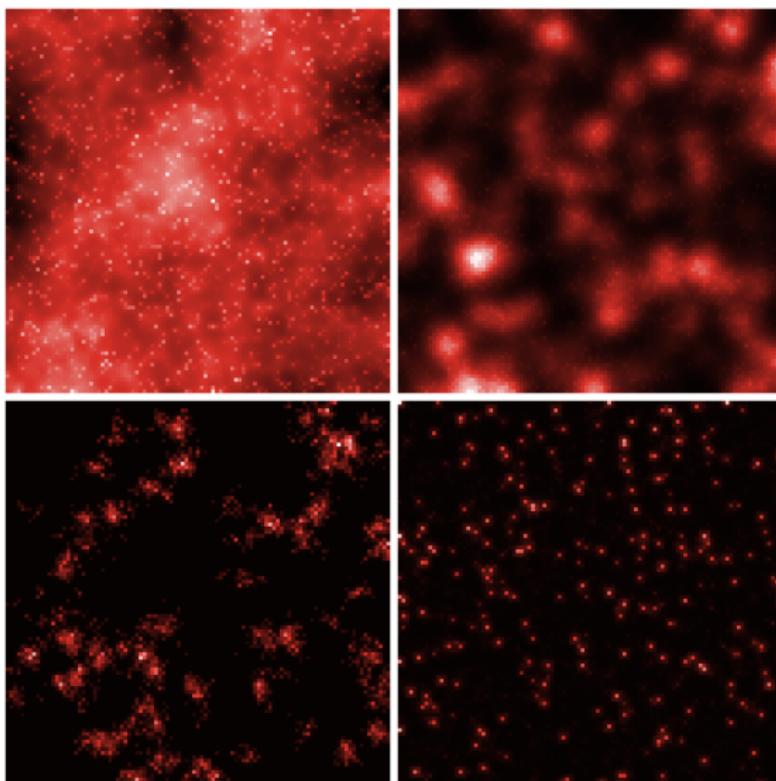
Raimbault, J. (2018). Calibration of a density-based model of urban morphogenesis. *PLoS one*, 13(9), e0203516.

Raimbault, J. (2019). An urban morphogenesis model capturing interactions between networks and territories. In *The Mathematics of Urban Morphology* (pp. 383-409). Birkhäuser, Cham.

- Crucial role of the interplay between concentration forces and dispersion forces [Fujita and Thisse, 1996] in keeping Urban Systems at the border of chaos
- Potentiality of aggregation mechanisms (such as Simon model) to produce power laws
- Link with Reaction-diffusion approaches in Morphogenesis [Turing, 1952]
- Extension of a DLA-type model introduced by [Batty, 1991], with simple abstract processes of population aggregation and diffusion

- Grid world with cell populations  $(P_i(t))_{1 \leq i \leq N^2}$ .
- At each time step:
  1. Population growth with exogenous rate  $N_G$ , attributed independently to a cell following a preferential attachment of strength  $\alpha$
  2. Population is diffused  $n_d$  times with strength  $\beta$
- Stopping criterion: fixed maximal population  $P_m$ .
- Output measured by morphological indicators: Moran index, average distance, rank-size hierarchy, entropy.

# Generating Population Distributions



*Examples of generated territorial shapes*

# Morphological indicators

1. Rank-size slope  $\gamma$ , given by  $\ln(P_{\tilde{i}}/P_0) \sim k + \gamma \cdot \ln(\tilde{i}/i_0)$  where  $\tilde{i}$  are the indexes of the distribution sorted in decreasing order.
2. Entropy of the distribution:

$$\mathcal{E} = \sum_{i=1}^M \frac{P_i}{P} \cdot \ln \frac{P_i}{P} \quad (1)$$

$\mathcal{E} = 0$  means that all the population is in one cell whereas  
 $\mathcal{E} = M$  means that the population is uniformly distributed.

3. Spatial-autocorrelation given by Moran index, with simple spatial weights given by  $w_{ij} = 1/d_{ij}$

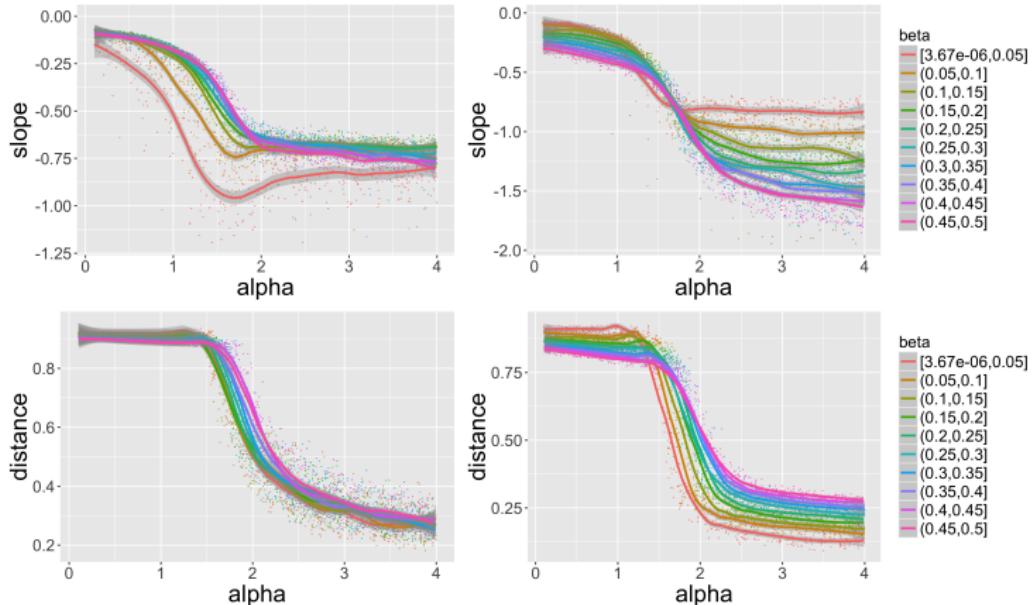
$$I = M \cdot \frac{\sum_{i \neq j} w_{ij} (P_i - \bar{P}) \cdot (P_j - \bar{P})}{\sum_{i \neq j} w_{ij} \sum_i (P_i - \bar{P})^2}$$

4. Mean distance between individuals

$$\bar{d} = \frac{1}{d_M} \cdot \sum_{i < j} \frac{P_i P_j}{P^2} \cdot d_{ij}$$

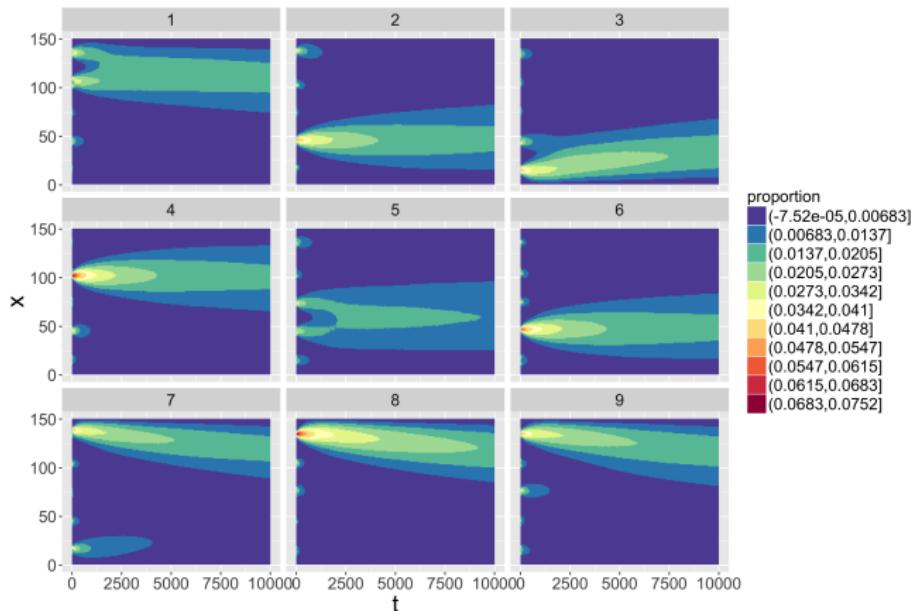
where  $d_M$  is a normalisation constant

# Model behavior



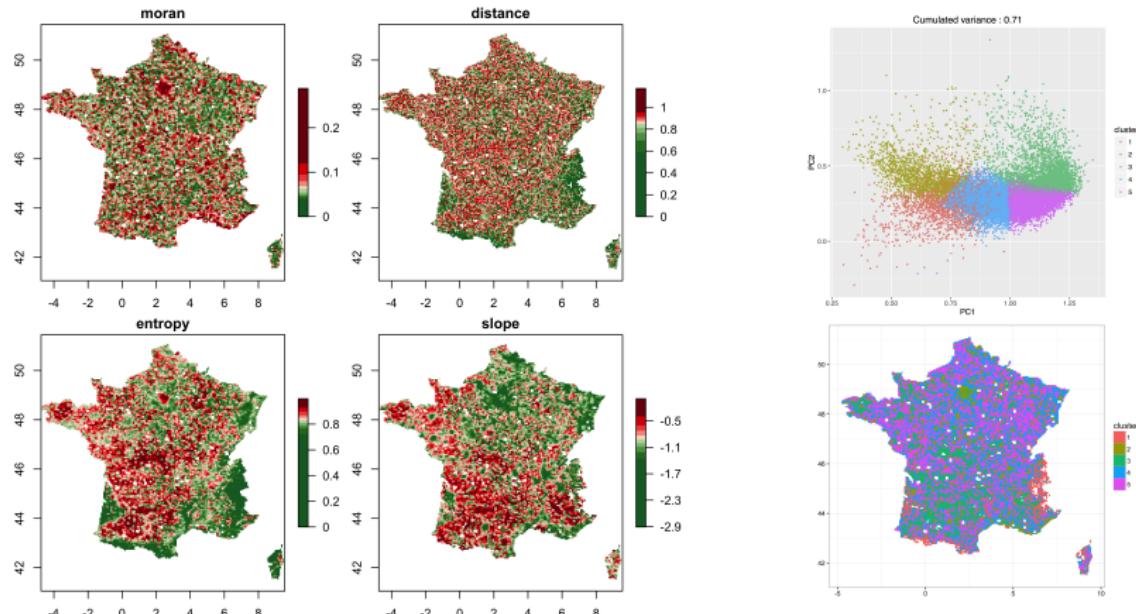
*Phase transitions of indicators unveiled by exploration of the parameter space  
(80000 parameter points, 10 repetitions each)*

# Path-dependence and frozen accidents



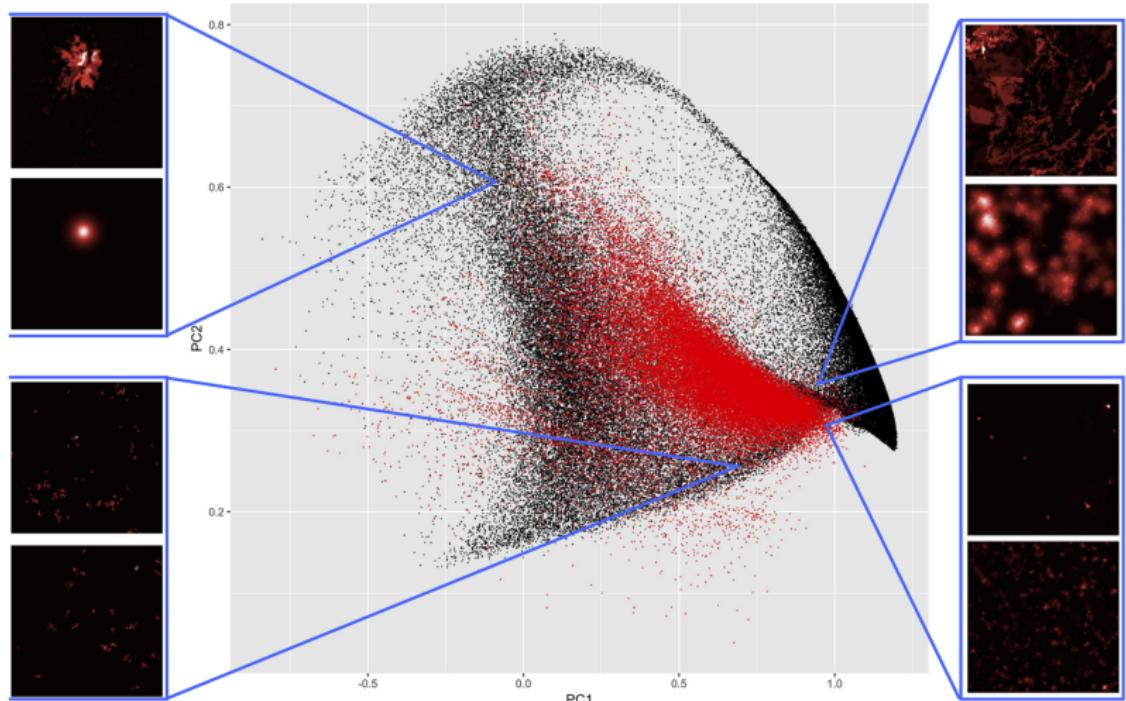
*Illustration of path-dependence in a simplified one-dimensional version of the model: cell trajectories in time for 9 independent repetitions from the same initial configuration.*

# Empirical Data for Calibration



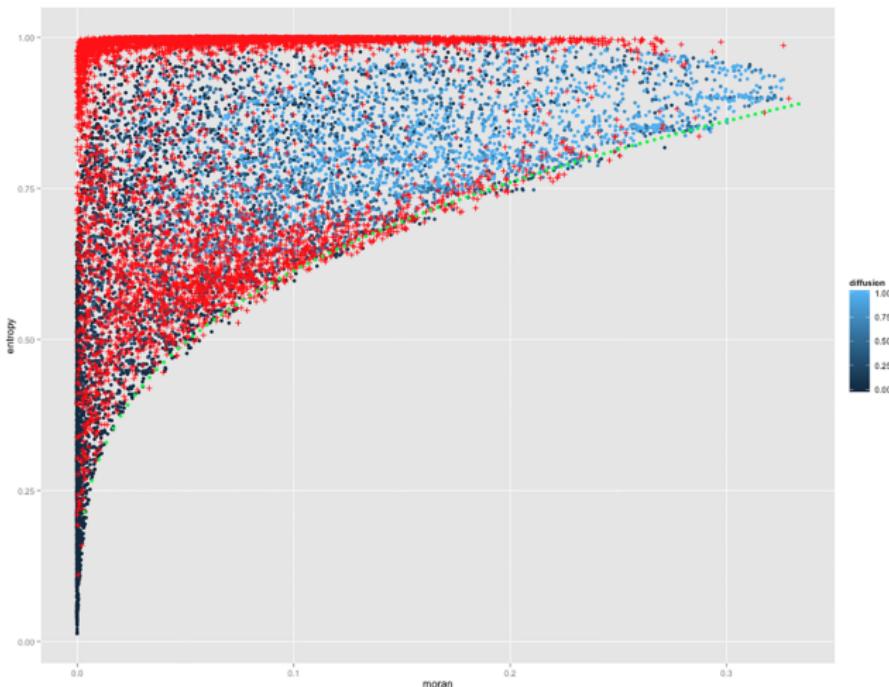
*Computation of morphological indicators on population density data for Europe (shown here on France), morphological classification.*

# Model Calibration



*Brute force calibration by exploring the parameter space. Reproduction of most existing configuration in the morphological sense (here in principal plan).*

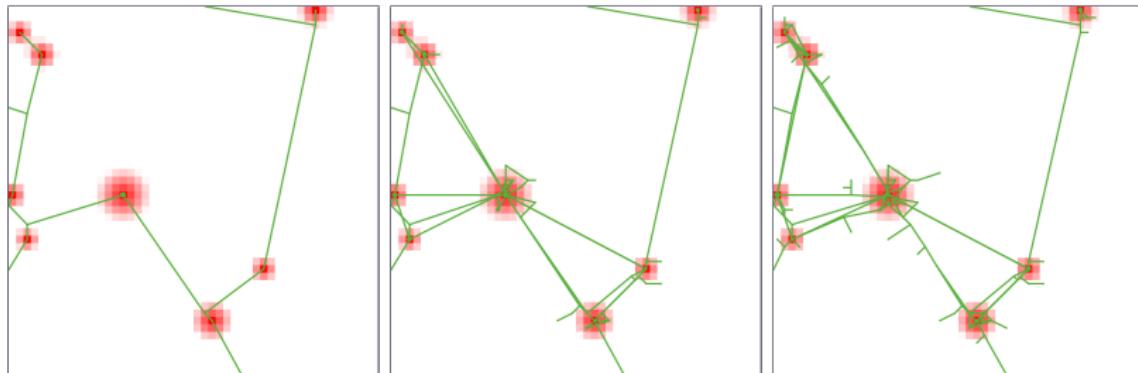
# Model Targeted Exploration



*Potentialities of targeted model explorations: here feasible space using Pattern Space Exploration algorithm [Chérel et al., 2015].*

Network generated conditionally to population; at fixed time steps :

1. Add new nodes preferentially to new population and connect them
2. Variable heuristic for new links, among: nothing, random, gravity-based deterministic breakdown, gravity-based random breakdown (from [Schmitt, 2014]), cost-benefits (from [Louf et al., 2013]), biological network generation (based on [Tero et al., 2010])



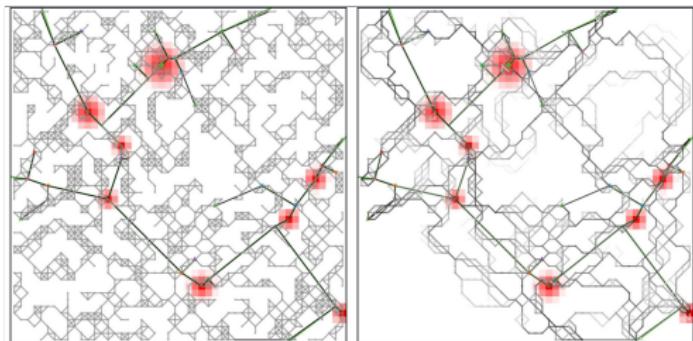
Model studied by [Tero et al., 2010] : exploration and reinforcement by a slime mould searching for resources



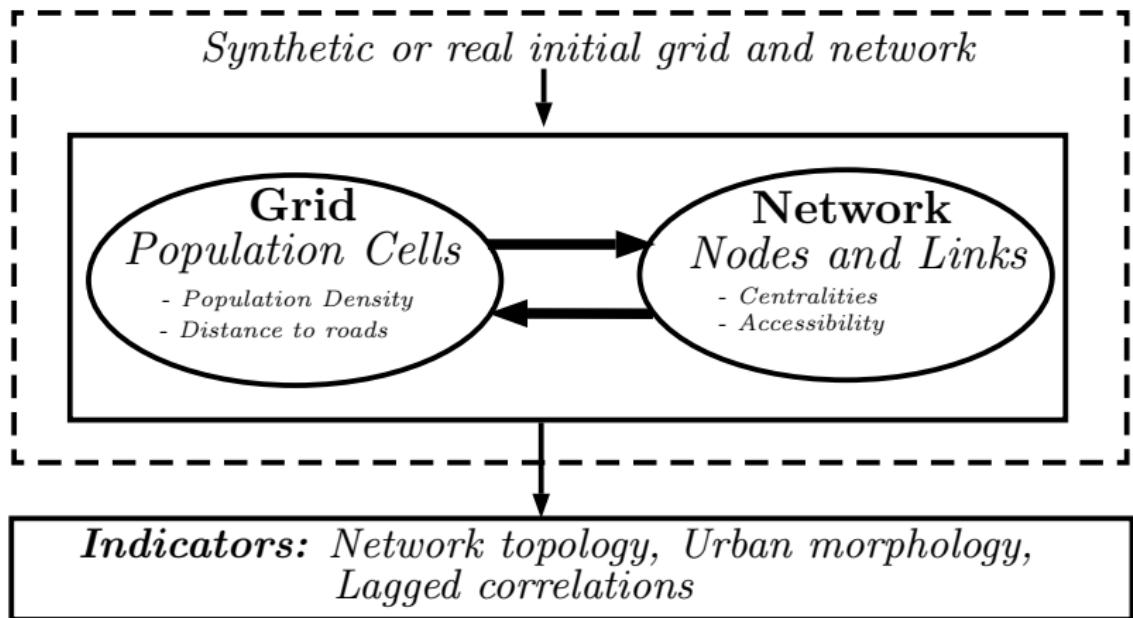
*Application to the design of optimal bus routes*

Adding new links with biological heuristic:

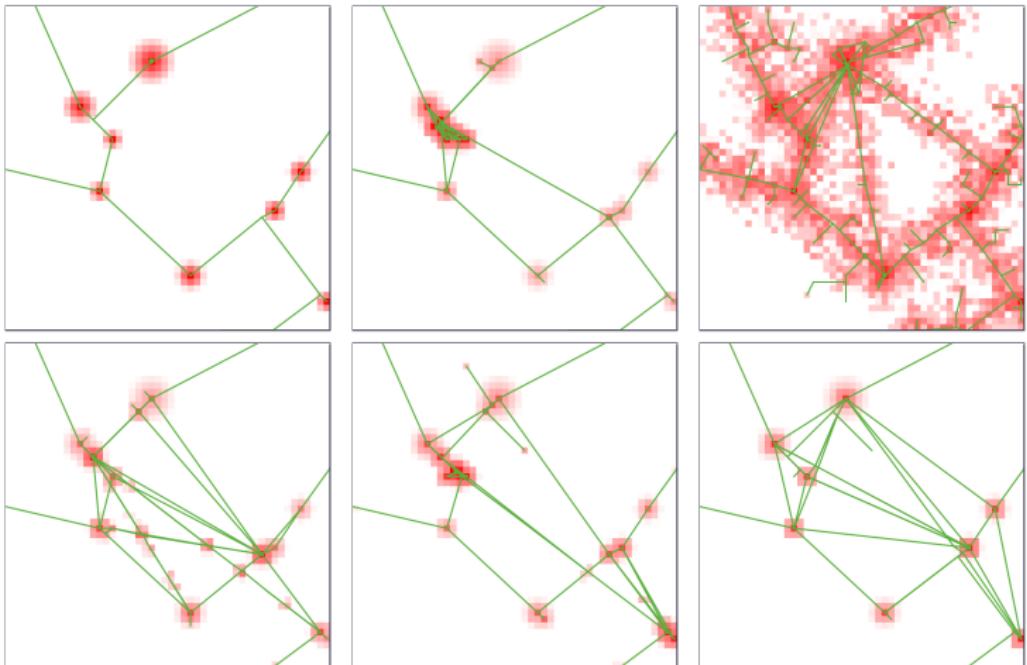
1. Create network of potential new links, with existing network and randomly sampled diagonal lattice
2. Iterate for  $k$  increasing ( $k \in \{1, 2, 4\}$  in practice) :
  - ▶ Using population distribution, iterate  $k \cdot n_b$  times the slime mould model to compute new link capacities
  - ▶ Delete links with capacity under  $\theta_d$
  - ▶ Keep the largest connected component
3. Planarize and simplify final network



Intermediate stage for biological network generation

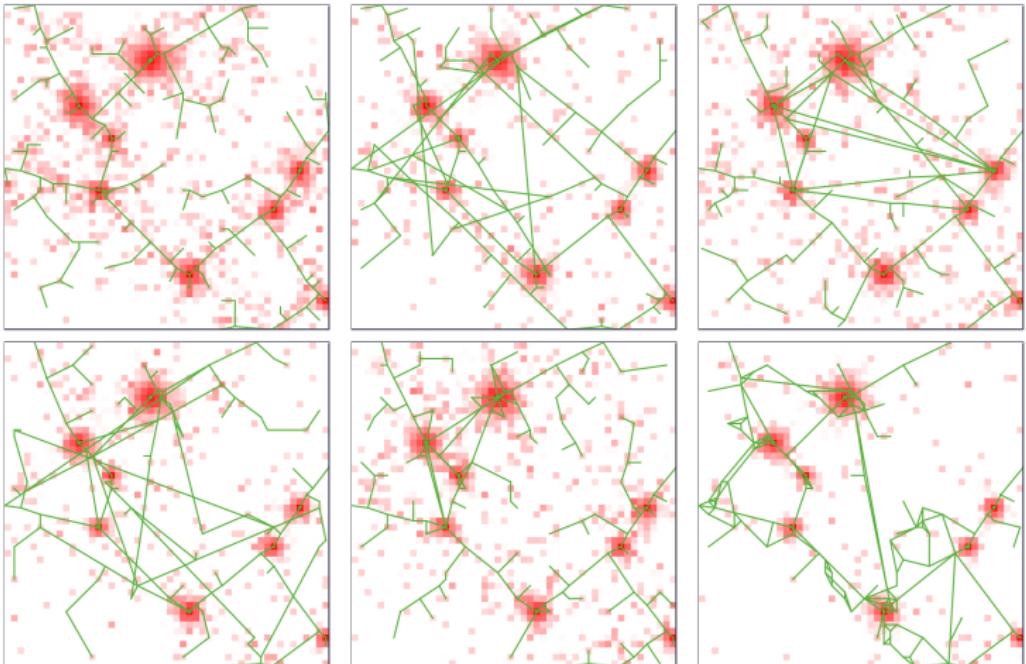


# Generated Urban Shapes: Urban Form



*In order: setup; accessibility driven; road distance driven; betweenness driven; closeness driven; population driven.*

# Generated Urban Shapes: Network

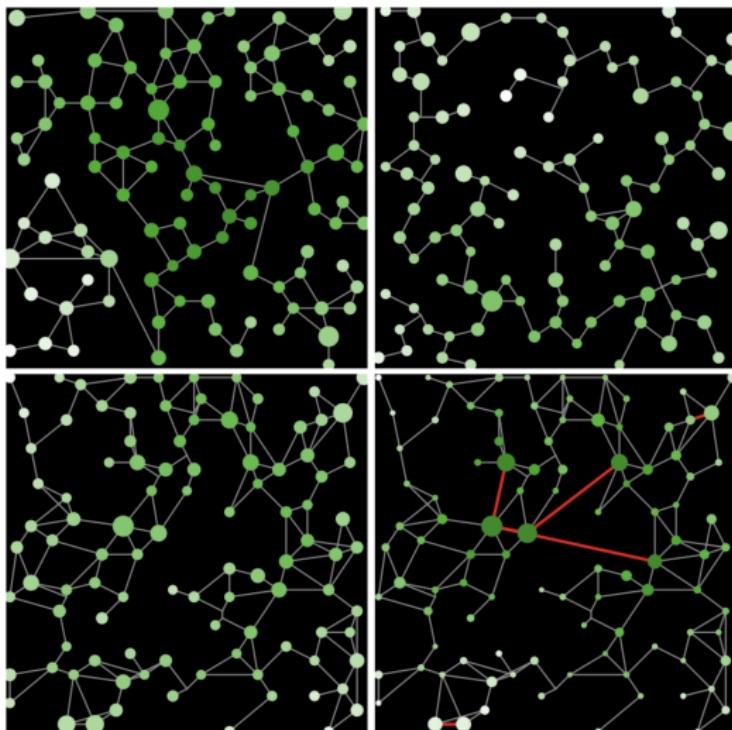


*In order: connection; random; deterministic breakdown; random breakdown; cost-driven; biological.*

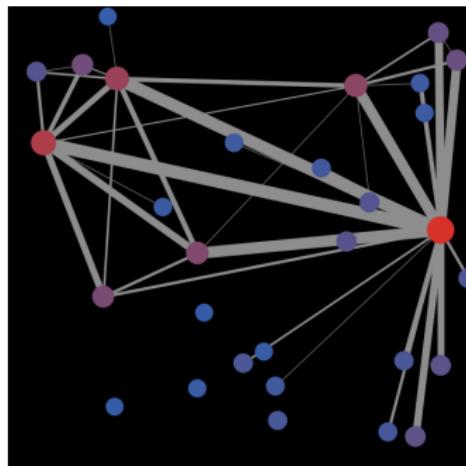
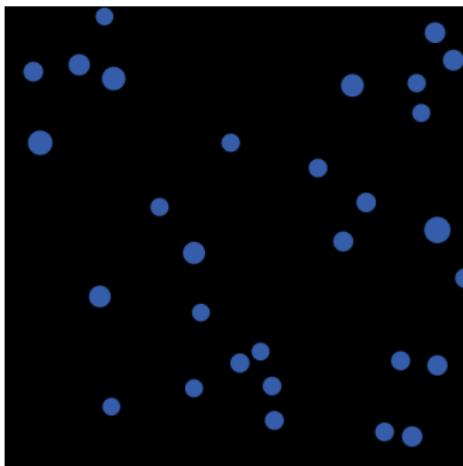
*At the macroscopic scale: synthetic systems of cities*

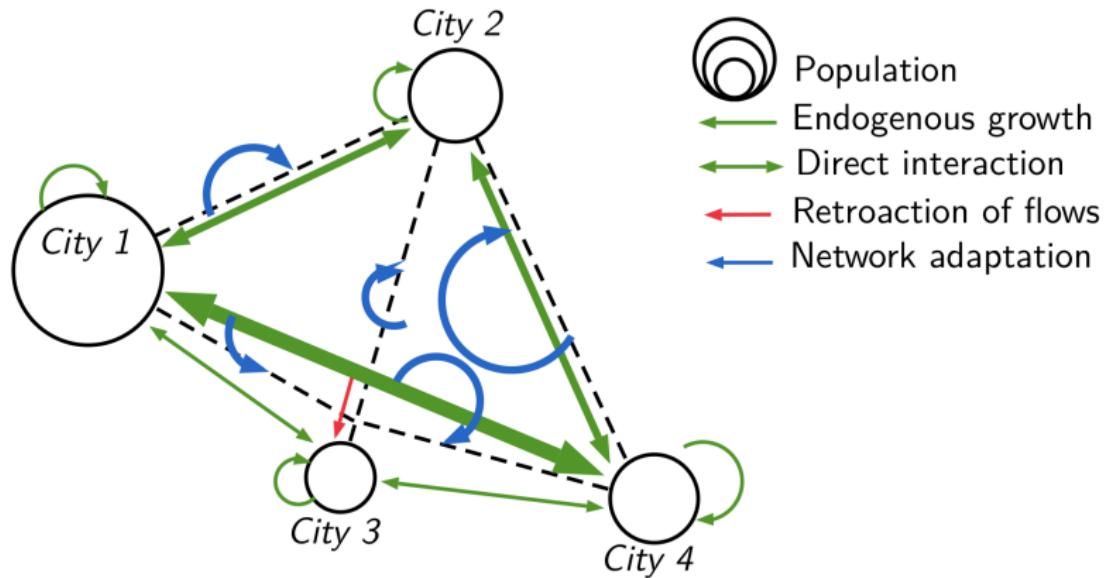
- ▶ Evolutive urban theory: systems of cities follow general stylized facts [Pumain, 2018]
- ▶ Rank-size law [Pumain et al., 2006]
- ▶ Central place theory

*Synthetic system of cities and network for the SimpopNet model*  
[Raimbault, 2018c]



*Cities-network co-evolution model explored on synthetic systems of cities [Raimbault, 2019a]*





## *Spatial synthetic data in other disciplines ?*

- ▶ spatial network generative models
- ▶ other disciplines ? ecology ? geosciences  
[Mogheir et al., 2004] ?
- ▶ interaction with data driven disciplines ? (planning,  
architecture, spatio-temporal datamining)
- ▶ genericity of some models ? (reaction-diffusion)
- ▶ synthetic data generation methods (synthetic populations)
- ▶ synthetic data at the core of applied statistics methodology  
(not much in spatial statistics ?)

# Spatial form and function: Morphogenesis

	Physical	Biological	Engineered
Non Functional			
Functional			

Sources (in order by column). Ants, Erosion, Game of Life: NetLogo Library ; Arbotron [Jun and Hübner, 2005]; Industrial design [Aage et al., 2017]; Swarm chemistry [Sayama, 2007]

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→ *How does noise in real data impacts the result ?*

- ▶ Impact of missing elements
- ▶ Impact of imprecise coordinates or topology
- ▶ Optimal matching between spatial datasets

→ *How does perturbation of real data allows to explore scenario*

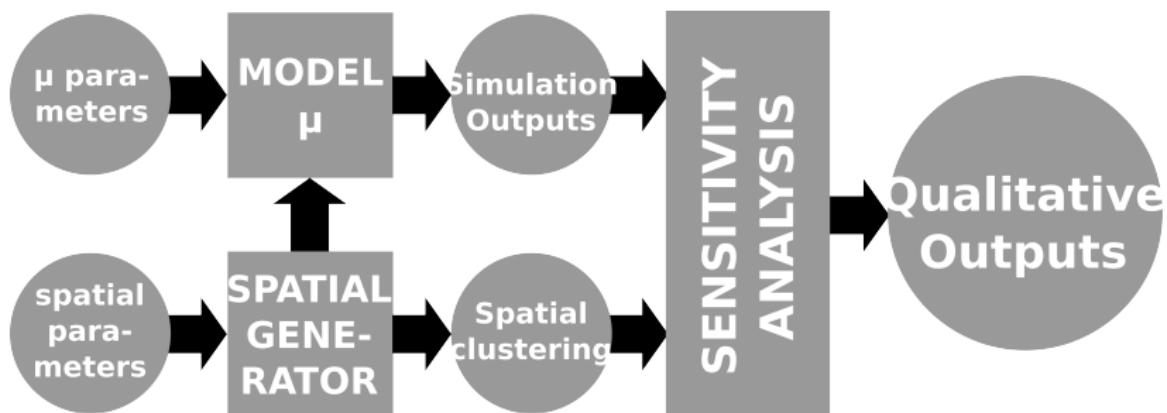
## Examples:

- ▶ simulating urban projects by modifying population of areas with a given spatial correlation structure
- ▶ simulating network disruptions or new transportation lines

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*General workflow to test the spatial sensitivity of simulation models*

Raimbault, J., Cottineau, C., Texier, M. L., Néchet, F. L., & Reuillon, R. (2018). Space Matters: extending sensitivity analysis to initial spatial conditions in geosimulation models. arXiv preprint arXiv:1812.06008.

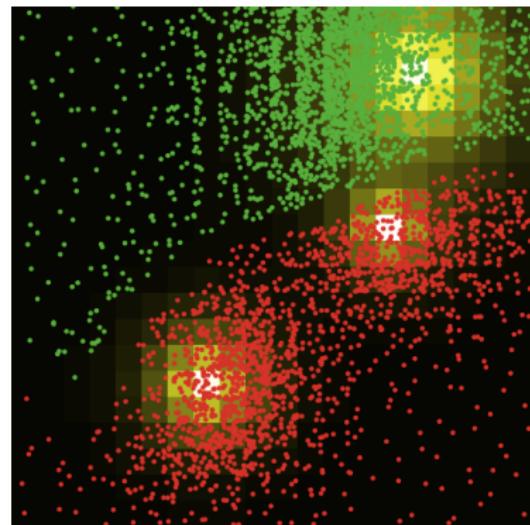
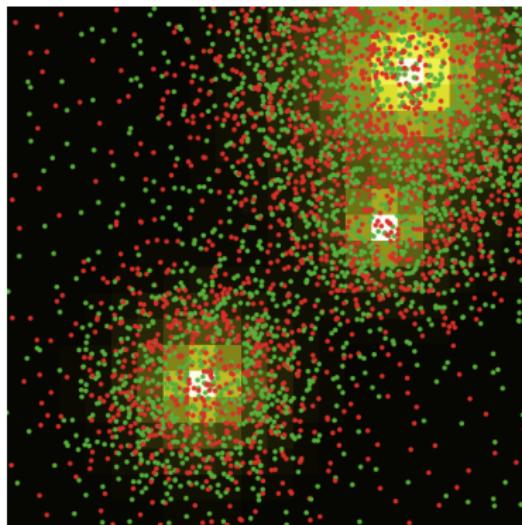


*Relative distance of phase diagrams to compare global model behavior when meta-parameters change*

$$d_r(\mu_{\vec{\alpha}_1}, \mu_{\vec{\alpha}_2}) = 2 \cdot \frac{d(\mu_{\vec{\alpha}_1}, \mu_{\vec{\alpha}_2})^2}{Var[\mu_{\vec{\alpha}_1}] + Var[\mu_{\vec{\alpha}_2}]}$$

*Why could the Schelling model be sensitive to space ?*

[Banos, 2012] network effects in Schelling model



*Influence of spatial generator parameters on model outputs*

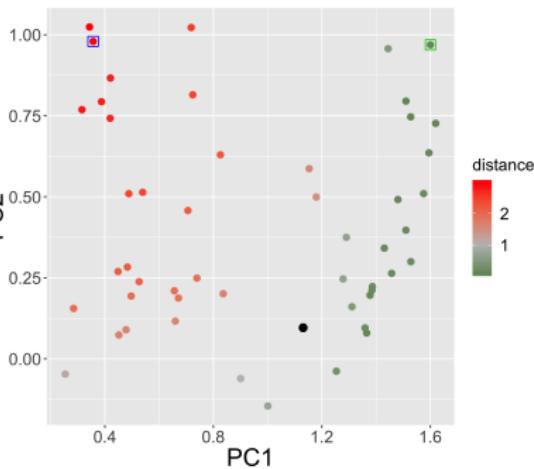
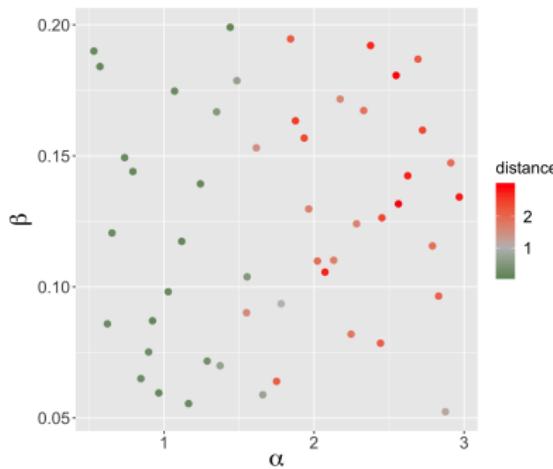
Simulation outcome by segregation index:	Dissimilarity		Entropy		Moran's I	
<b>Intercept</b>	-0.212 ***	-0.141 ***	-0.254 ***	-0.208 ***	-0.036 ***	-0.061 ***
<b>Similarity Wanted (S)</b>	1.212 ***	1.212 ***	1.250 ***	1.250 ***	0.550 ***	0.550 ***
<b>quadratic term (<math>S^2</math>)</b>	-0.942 ***	-0.942 ***	-0.963 ***	-0.963 ***	-0.428 ***	-0.438 ***
<b>Vacancy Rate (V)</b>	0.602 ***	0.602 ***	0.453 ***	0.453 ***	-0.027 ***	-0.027 ***
<b>Minority Index (%Maj - %Min)</b>	0.307 ***	0.307 ***	0.130 ***	0.130 ***	-0.067 ***	-0.067 ***
<b>Density Grid = Polycentric</b>		0.087 ***		0.052 ***		0.001 ***
<b>Density Grid = Discontinuous</b>		0.111 ***		0.068 ***		0.00
<b>Attraction generator parameter <math>\alpha</math></b>		-0.083 ***		-0.053 ***		0.014 ***
<b>Diffusion generator parameter <math>\beta</math></b>		0.323 ***		0.218 ***		0.017 ***
<b>R2 (%)</b>	30.6	34.7	24.1	25.6	23.9	24.0
<b># of observations (sim. runs)</b>	2,106,000	2,106,000	2,106,000	2,106,000	2,106,000	2,106,000
<b>AIC</b>	-70717.68	-198748.2	208213.8	166048.8	-4385990	-4387816

# Application: Sugarscape model

## A model of resource collection

- ▶ agents collect a spatial resource
- ▶ the resource regrows at a certain rate only

## Relative distances between phase diagrams



- 1 Scientific context
- 2 Spatial synthetic data
- 3 Perturbation of data
- 4 Sensitivity to spatial configuration
- 5 Spatial indicators for model outputs
- 6 Synthesis
- 7 Application on the Zombie model

*In the spatial approach, spatial model indicators are also important: what kind of spatial structure does the model produce ?*

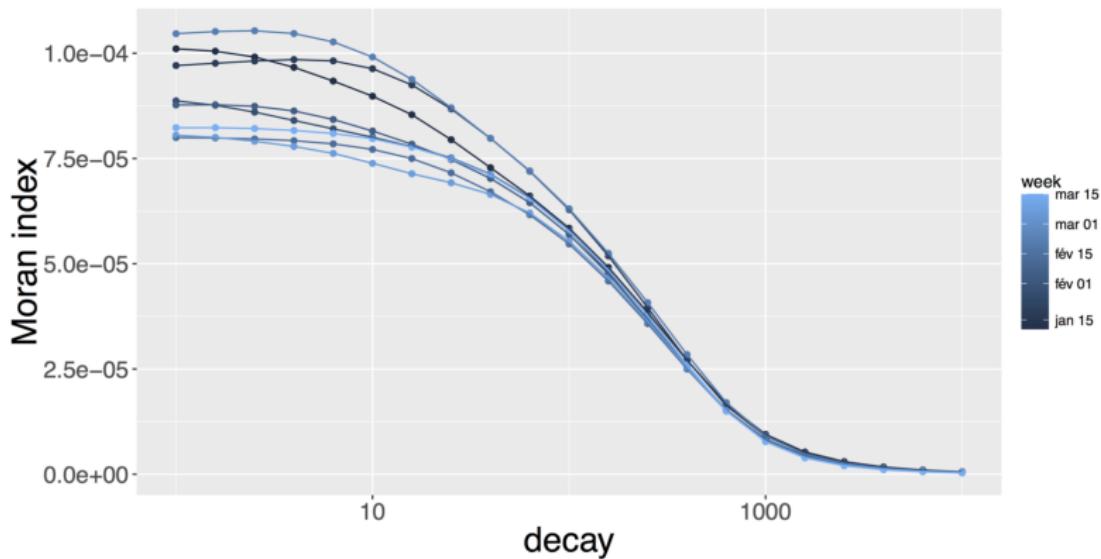
- ▶ previous form indicators at different scales, applied on any spatialized variable or event: quantify level of aggregation, hierarchy, clustering
- ▶ spatial statistics indicators and methods
- ▶ more complicated approaches: fractals and multifractals, spatial datamining

*Spatial autocorrelation at a given range*

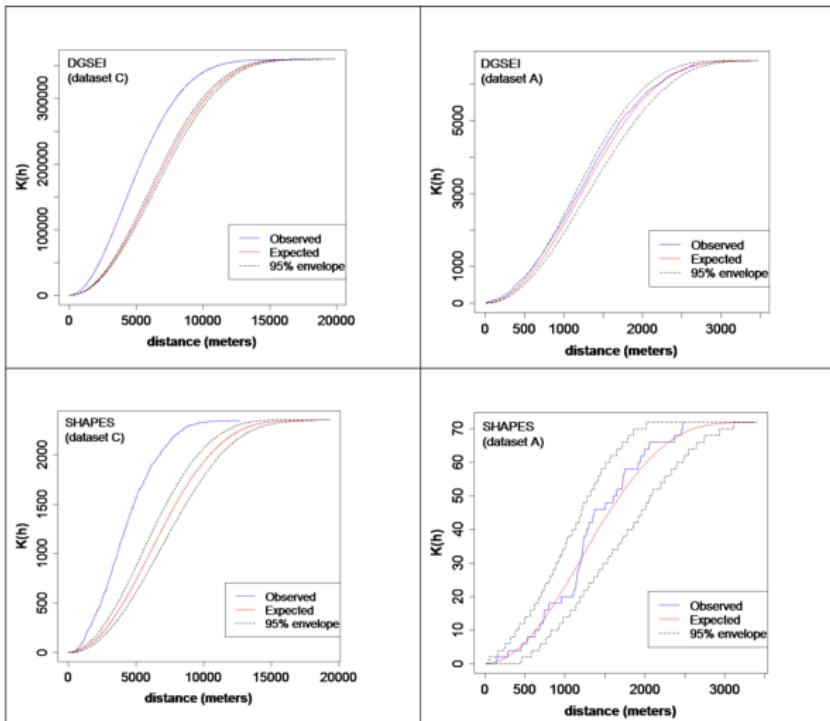
Given spatial weights  $w_{ij}$

$$I = \frac{N}{\sum_{i,j} w_{ij}} \cdot \frac{\sum_{i,j} w_{ij} \cdot (X_i - \bar{X})(X_j - \bar{X})}{\sum_i (X_i - \bar{X})^2}$$

*Extracting typical spatial scales of a system using Moran Index*  
[Raimbault and Bergeaud, 2017]

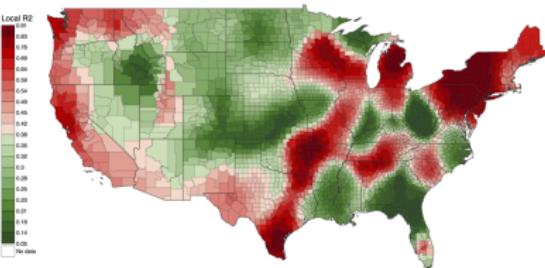
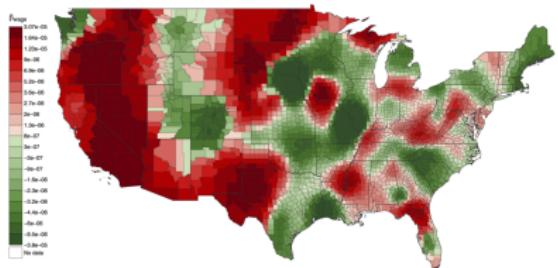
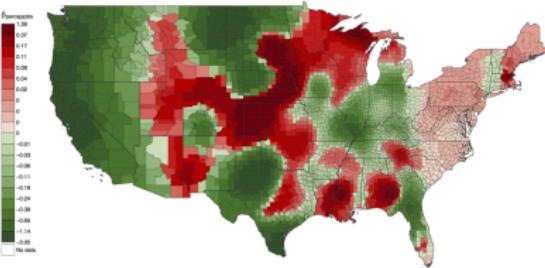
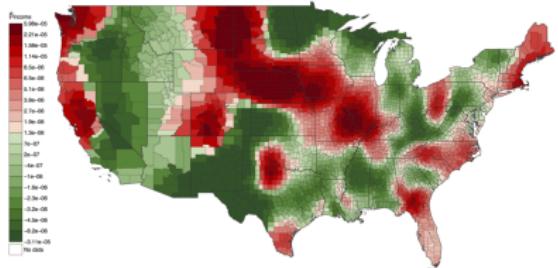


*Quantifying the level of clustering of spatial points or points on a network regarding a Poisson process null model [Vandenbulcke et al., 2017]*



# Geographically Weighted Regression

*GWR handles spatial non-stationarity with adaptative scale fitting*  
[Fotheringham et al., 2003]



Raimbault, J., & Bergeaud, A. (2017). The cost of transportation: Spatial analysis of us fuel prices. arXiv preprint arXiv:1706.07467.

- 1 Scientific context
- 2 Spatial synthetic data
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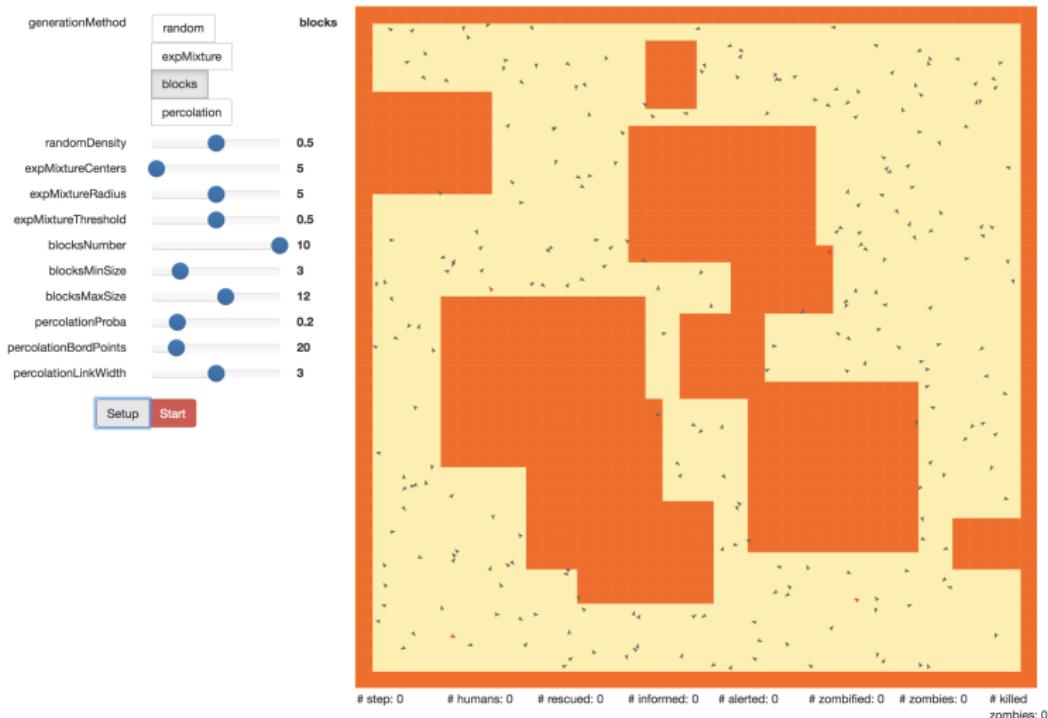
	OpenMOLE	spatialdata	planned
Micro grid spatial samplings	✓	✓	✓
Meso grid spatial samplings	✓	✓	✓
Macro spatial samplings	✗	✓	✓
Spatial network generation	✗	✓	✓
Real data import	✗	✗	✓
Real data perturbations	✗	✗	✓
Spatial statistics	✓	✓	✓
Hybrid methods	✗	✗	✓
Domain models (transportation, land-use)	✗	✓	✓

- 1 Scientific context
- 2 Spatial synthetic data
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# Synthetic spaces for ZOMBIE

*Try the GUI with synthetic spaces at*

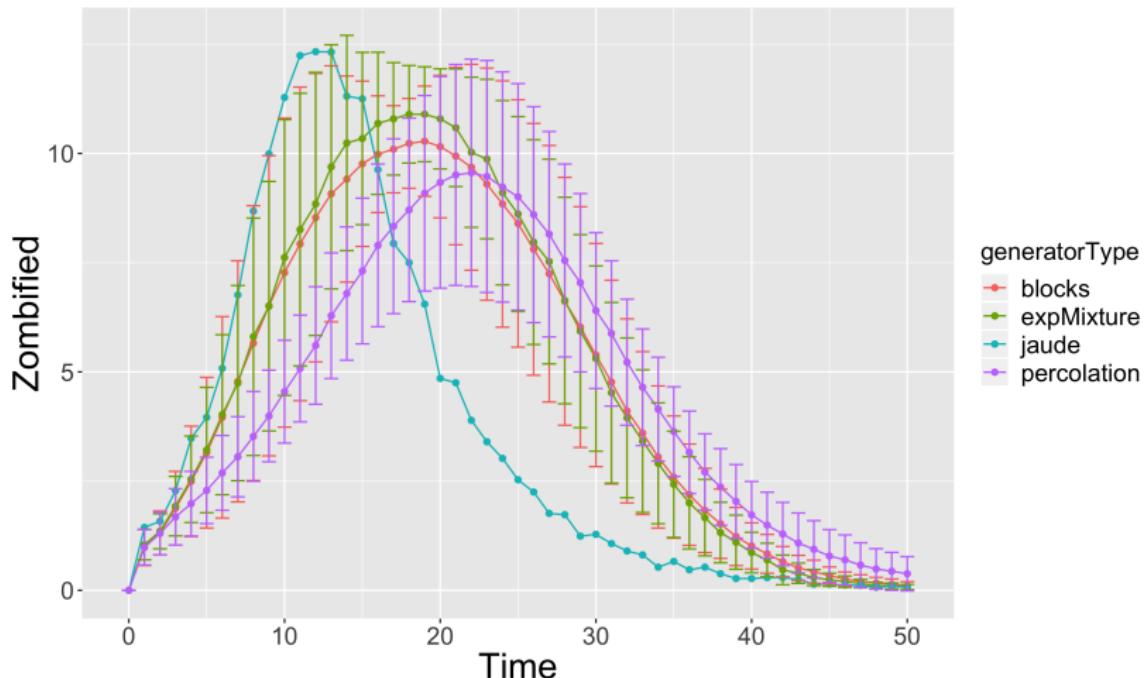
<https://om.exmodelo.org/spatialsens/>



*First application: fixed model parameters, sensitivity of time series to spatial configuration*

→ explore the script `spatialsens.oms`

# Sensitivity of zombified dynamics



*Second application: variance-based comparison of phase diagrams for the cooperation submodel, between varying spatial configuration*

→ explore the script phasediag.oms

**No results** for this experiment (problem of memory more than computation time) - method still need to be reformulated/implemented in a “Saltelli-like” manner

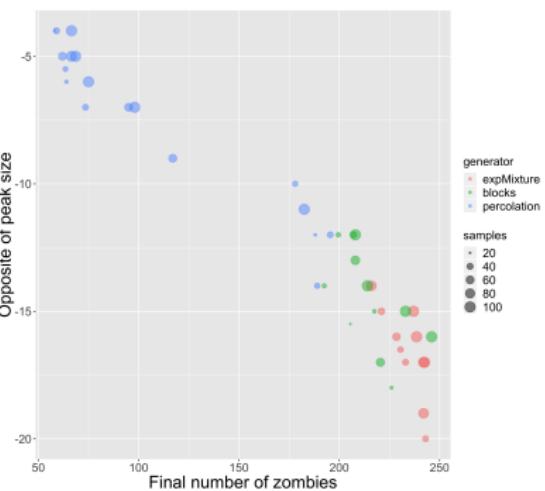
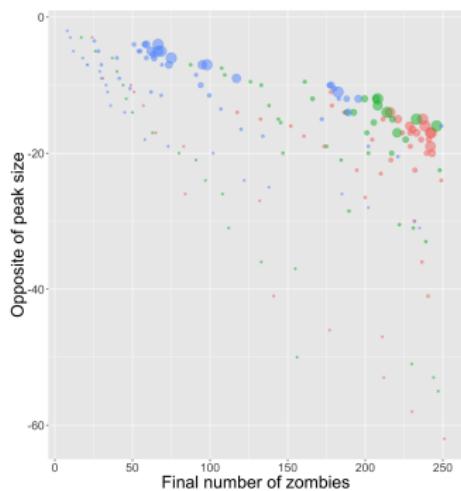
*Third application: calibration of the default parameters model, for each type of generator, to maximize peak size (time-localized dynamics) and minimize number of zombified*

→ explore the script calibrate.oms

# Calibration results

*Complementary restricted Pareto fronts between the different spatial generators*

→ urban planning policies yield different zombie-resilient compromises, and different “planning paradigms” are complementary



- ▶ test the robustness of your qualitative findings to the spatial configuration
- ▶ use spatial metrics to answer specific questions
- ▶ change the initial spatial distribution of agents (***need library/model tuning***)
- ▶ study the influence of spatial distribution of rescues or traps (***need library/model tuning if structured spatial configuration***)
- ▶ compute complicated spatial statistics on agents trajectories e.g. (***need library/model tuning***)

- **Space matters:** relevance of spatially-explicit models and spatial sensitivity analysis.
- **Synthetic data:** first experimental samplings included in OpenMOLE, soon more to come.
- **Disciplinary context:** strong contingency on included models and forms, please provide feedbacks, suggestions, needs, ideas from your viewpoint.

*Open issues at <https://github.com/openmole/spatialdata/issues>*



*Satellite session on methods and epistemology in modeling and simulation, at Conference on Complex Systems, 2nd October 2019*

**Submit your abstract before June 30th !**

https:

//iscpif.fr/ccs-satellite-session-2019-new-methods/

**Submission link:**

<https://easychair.org/conferences/?conf=simexplo2019>

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