

ML Foundation: HW3

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1

TODO: insert screenshot here.

2

First, we prove $H^2 = H$:

$$\begin{aligned} H^2 &= (X(X^T X)^{-1} X^T)^2 \\ &= (X(X^T X)^{-1} X^T)(X(X^T X)^{-1} X^T) \\ &= X(X^T X)^{-1} [(X^T X)(X^T X)^{-1}] X^T \\ &= X(X^T X)^{-1} X^T \\ &= H \end{aligned}$$

Now we can prove that $(I - H)^2 = (I - H)$:

$$\begin{aligned} (I - H)^2 &= I^2 - 2IH + H^2 \\ &= I - 2H + H \\ &= I - H \end{aligned}$$

3

TODO: 看不懂題目在寫什麼？QQ

4

$$\hat{E}_2(\Delta u, \Delta v) = E(u, v) + \nabla E(u, v) \cdot (\Delta u, \Delta v) + \frac{1}{2}(\Delta u, \Delta v)^T \nabla^2 E(u, v)(\Delta u, \Delta v)$$

Let

$$\left\{ \begin{aligned} 0 = \frac{\partial \hat{E}_2(\Delta u, \Delta v)}{\partial \Delta u} &= \frac{\partial E}{\partial u} + \frac{1}{2} \left(2 \frac{\partial^2 E}{\partial u^2} \Delta u + 2 \frac{\partial^2 E}{\partial u \partial v} \Delta v \right) \\ &= \frac{\partial E}{\partial u} + \frac{\partial^2 E}{\partial u^2} \Delta u + \frac{\partial^2 E}{\partial u \partial v} \Delta v \\ 0 = \frac{\partial \hat{E}_2(\Delta u, \Delta v)}{\partial \Delta v} &= \frac{\partial E}{\partial v} + \frac{\partial^2 E}{\partial v^2} \Delta v + \frac{\partial^2 E}{\partial v \partial u} \Delta u \end{aligned} \right.$$

$$\begin{cases} 0 = \frac{\partial E}{\partial u} + \frac{\partial^2 E}{\partial u^2} \Delta u + \frac{\partial^2 E}{\partial u \partial v} \Delta v \\ 0 = \frac{\partial E}{\partial v} + \frac{\partial^2 E}{\partial v^2} \Delta v + \frac{\partial^2 E}{\partial v \partial u} \Delta u \end{cases}$$

We can combine the two equations to only one equation by one vector.

$$\begin{aligned} 0 &= \nabla E(u, v) + \nabla^2 E(u, v) \cdot (\Delta u, \Delta v) \\ -\nabla^2 E(u, v) \cdot (\Delta u, \Delta v) &= \nabla E(u, v) \\ (\Delta u, \Delta v) &= -(\nabla^2 E(u, v))^{-1} \nabla E(u, v) \end{aligned}$$