ML Foundation: HW3

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TODO: insert screenshot here.

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First, we prove $H^2 = H$:

$$\begin{aligned}
H^2 &= (X(X^TX)^{-1}X^T)^2 \\
&= (X(X^TX)^{-1}X^T)(X(X^TX)^{-1}X^T) \\
&= X(X^TX)^{-1}[(X^TX)(X^TX)^{-1}]X^T \\
&= X(X^TX)^{-1}X^T \\
&= H
\end{aligned}$$

Now we can prove that $(I-H)^2 = (I-H)$:

$$\begin{aligned}
(I-H)^2 &= I^2 - 2IH + H^2 \\
&= I - 2H + H \\
&= I - H
\end{aligned}$$

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TODO: 看不懂題目在寫什麼？QQ

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$$\hat{E}_2(\Delta u, \Delta v) = E(u,v) + \nabla E(u,v) \cdot (\Delta u, \Delta v) + \frac{1}{2}(\Delta u, \Delta v)^T \nabla^2 E(u,v)(\Delta u, \Delta v)$$

Let

$$\begin{cases}
0 = \dfrac{\partial \hat{E}_2(\Delta u, \Delta v)}{\partial \Delta u} = \dfrac{\partial E}{\partial u} + \dfrac{1}{2}\left(2\dfrac{\partial^2 E}{\partial u^2}\Delta u + 2\dfrac{\partial^2 E}{\partial u \partial v}\Delta v\right) \\
\qquad\qquad\qquad\quad = \dfrac{\partial E}{\partial u} + \dfrac{\partial^2 E}{\partial u^2}\Delta u + \dfrac{\partial^2 E}{\partial u \partial v}\Delta v \\
0 = \dfrac{\partial \hat{E}_2(\Delta u, \Delta v)}{\partial \Delta v} = \dfrac{\partial E}{\partial v} + \dfrac{\partial^2 E}{\partial v^2}\Delta v + \dfrac{\partial^2 E}{\partial v \partial u}\Delta u
\end{cases}$$

$$\begin{cases} 0 = \dfrac{\partial E}{\partial u} + \dfrac{\partial^2 E}{\partial u^2}\Delta u + \dfrac{\partial^2 E}{\partial u \partial v}\Delta v \\ 0 = \dfrac{\partial E}{\partial v} + \dfrac{\partial^2 E}{\partial v^2}\Delta v + \dfrac{\partial^2 E}{\partial v \partial u}\Delta u \end{cases}$$

We can combine the two equations to only one equation by one vector.

$$\begin{aligned} 0 &= \nabla E(u,v) + \nabla^2 E(u,v) \cdot (\Delta u, \Delta v) \\ -\nabla^2 E(u,v) \cdot (\Delta u, \Delta v) &= \nabla E(u,v) \\ (\Delta u, \Delta v) &= -\left(\nabla^2 E(u,v)\right)^{-1} \nabla E(u,v) \end{aligned}$$

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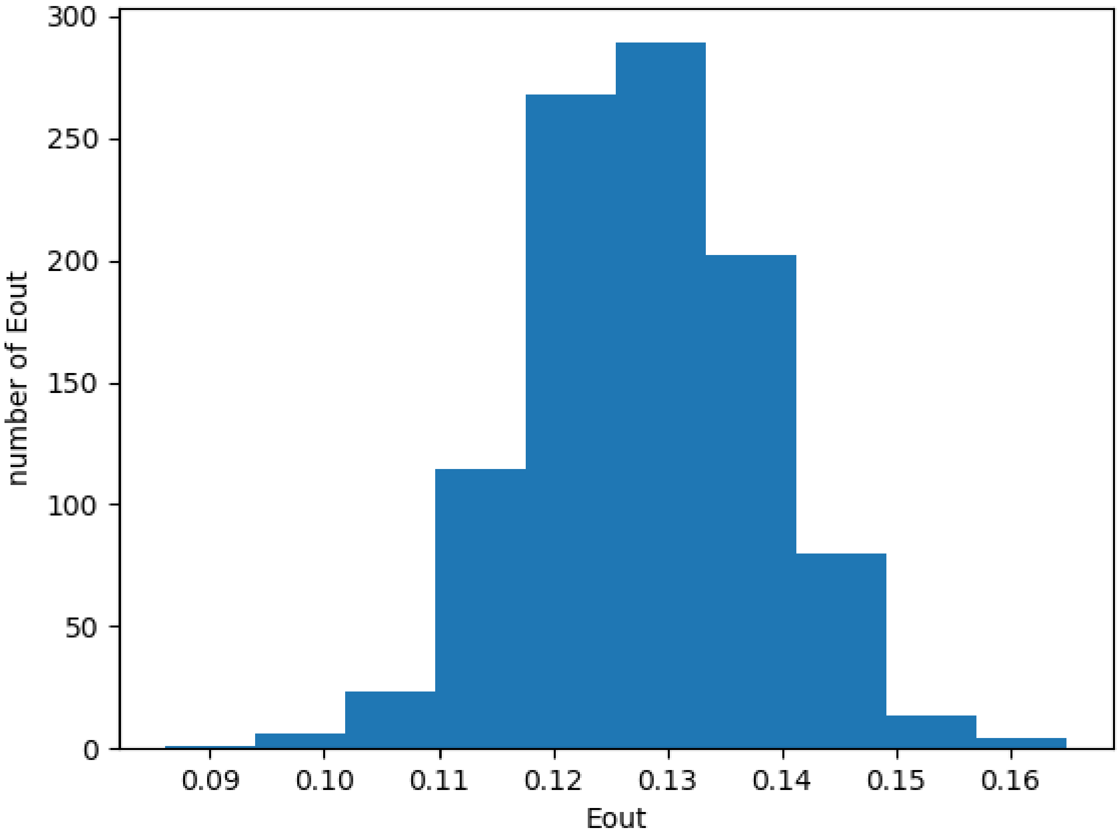

Figure 1: Histogram of E_{out}

Figure 1 shows the histogram of E_{out}.