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# ***Introduction to Probability***



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## *Preface*

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Placeholder



# 1

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## *Fundamentals of Probability and its Axioms*

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Placeholder

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### 1.1 Combinations and Permutations

#### 1.1.1 Permutations

#### 1.1.2 Combinations

#### 1.1.3 Worked Examples

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### 1.2 Review of Set Theory

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### 1.3 Laws of Set Theory

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### 1.4 Axioms of Probability

#### 1.4.1 Sample spaces with equally likely outcomes

#### 1.4.2 Worked Examples





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## *Conditional Probability and Independence*

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### 2.1 Conditional Probability

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### 2.2 The Multiplication Rule

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### 2.3 Independence of Events

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### 2.4 The Law of Total Probability

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### 2.5 Bayes' Theorem

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### 2.6 Applications

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## *Discrete Random Variables and Distributions*

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### 3.1 Random Variables

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### 3.2 Probability Mass Functions (PMFs)

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### 3.3 Cumulative Distribution Functions (CDFs)

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### 3.4 Expected Value and Properties

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### 3.5 Variance and Standard Deviation

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### 3.6 Common Discrete Distributions: Binomial, Geometric, Poisson, Negative Binomial, multinomial



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## *Continuous Random Variables and Distributions*

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### 4.1 Continuous Random Variables

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### 4.2 Probability Density Functions (PDFs)

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### 4.3 Cumulative Distribution Functions (CDFs)

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### 4.4 Expected Value and Variance (Continuous Case)

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### 4.5 Uniform, Exponential, and Normal Distributions

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### 4.6 Change of Variables and Applications

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## *Joint Distributions and Independence*

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### 5.1 Joint Discrete Distributions

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### 5.2 Joint Continuous Distributions

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### 5.3 Marginal and Conditional Distributions

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### 5.4 Independence of Random Variables

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### 5.5 Covariance and Correlation

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### 5.6 Sums of Random Variables





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## *Limit Theorems*

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### 6.1 Sequences of Random Variables

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### 6.2 Convergence in Probability and Distribution

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### 6.3 The Law of Large Numbers (LLN)

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### 6.4 Central Limit Theorem (CLT)

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### 6.5 Applications of the CLT

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### 6.6 Approximations Using the Normal Distribution

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## *Appendix*

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Algebra Review

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Calculus Review



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## *Bibliography*

- Sheldon, Ross. *A First Course in Probability*. Pearson, 2016.
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