**Unified Mathematical Model [Steady-state + Dynamic Voltage–Current Characteristics of PEMFC]**

For PEM fuel cells, steady-state V–I characteristics of a fuel cell are determined by:





























As we Know, a change in the current does not result in an immediate change in the voltage for a practical fuel cell due to the existence of the charge double layer in the fuel cell. The charge layer on (or close to) the interface of electrode/electrolyte acts a storage of electrical charges and energy and hence it behaves as an electrical capacitor. Consequently, if the current changes, there will be some time for the voltage to reduce when the current increases or to increase when the current decreases. Thus, an accurate model of V–I characteristics for PEM fuel cells must include such a dynamic.

If the cell operating temperature and the partial pressures of the hydrogen and oxygen keep constant, the dynamic change of V–I characteristics depends on the activation and concentration overvoltages. Thus, their sum is defined as the transient component of the PEMFC voltage:



Both the reversible voltage and the ohmic overvoltage are independent of the dynamic response of the fuel cell. Hence, the sum of both is defined as the steady component of the PEMFC voltage:



Hence, the cell voltage includes the two components which are the steady and transient components.

















**Nomenclature [Constant Values and Parametric Values (User Inputs and Middle Values)]**

A: cell active area (cm2) [User]

: concentration of O2 in the catalytic interface (mol/cm3) [System]

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Dmax: maximum current density (A/cm2) [System]

D: actual current density of the cell (A/cm2) [System]

H: cell parametric coefficient (V) [System]

L: thickness of PEM (cm) [User]

N: number of cells [User]

OT: cell operating temperature (K) [User]

: partial pressure of H2 (atm) [User]

: partial pressure of O2 (atm) [User]

: resistance to the transfer of protons through PEM () [System]

Re: electronic resistance () [User]

Greek letters α1, α2, α3, α4: empirical coefficients for activation overvoltage

* α1: -0.948
* α2: [System]
* α3: 
* α4: 

: membrane parameter [An adjustable parameter with a possible minimum value of 14 and a maximum value of 23]

Uc: universal gas constant () [8314.47]

Fc: Faraday’s constant () [96484600]

n: Number of moles of electrons transferred in the balanced equation occurring in the fuel cell [2]

: change in free Gibbs energy for balanced equation occurring in the fuel cell () [User]

: change in entropy for balanced equation occurring in the fuel cell () [User]

: reference temperature (K) [User]

EN: reversible voltage of the fuel cell (V) [System]

Va: activation overvoltage (V) [System]

Vc: concentration overvoltage (V) [System]

Vohm: ohmic overvoltage (V) [System]

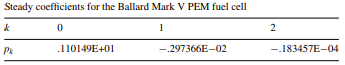
Nst: the number of the given discrete current data for the steady component (Nst ≥ 3). [System]

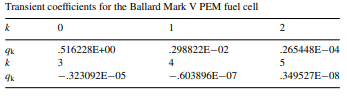
Pk: coefficients determined by using the least squares technique [System]

Icellm:  known discrete current values [A] [System]

qk: coefficients determined by using the least squares technique [System]

Ntr: the number of the given discrete current data for the transient component (Ntr ≥ 6). [System]





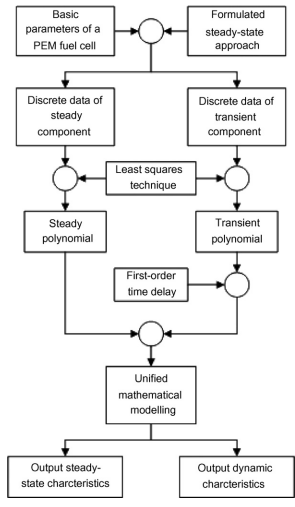
++The dynamics of the fuel cell can be considered that there is a first-order time delay in the activation and concentration overvoltages.

TC : first-order time delay constant [s] [System]

Rac: equivalent resistance (ohm) [System]

Cac:: the value of the equivalent capacitance (F) [User]

**Block Diagram**

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