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Anaglyph Stereo Without Ghosting

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Abstract

Anaglyph stereo provides a low-budget solution to viewing stereoscopic images. However, it may suffer from ghosting and bad color reproduction. Here we address the first issue. We present a novel technique to perceptually calibrate an anaglyph stereoscopic system and to use the calibration to eliminate ghosting from the anaglyph image. We build a model based on luminance perception by the left and right eyes through the anaglyph glasses. We do not rely on power spectra of a monitor or on transmission spectra of anaglyph glasses, but show how the five parameters of our model can be captured with just a few measurements within a minute. We present how full color, half color, and gray anaglyphs can be rendered with our technique and compare them to the traditional method.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Color, shading, shadowing, and texture—

1. Introduction

There are several ways to use stereoscopy to provide 3D depth perception for images, videos, or computer graphics: side-by-side viewing, anaglyphs, shutter glasses, polarized light, Infitec glasses, and autostereoscopic displays. Anaglyphs use colored filters to separate the left and the right image. Nowadays red-cyan glasses are most common, which can be purchased for just a few cents.

However, anaglyph stereo suffers from ghosting, limited color reproduction, and retinal rivalry. Ghosting results from crosstalk, which refers to the fact that a color channel of the image that should be filtered out for an eye passes the filter partially. Crosstalk comes from the imperfection of the anaglyph filters and the display device and cannot be eliminated by an algorithm. However, we present a technique to reduce ghosting substantially, so that it becomes negligible.

We want the luminance of images perceived through the glasses to correspond to the value perceived without the other image. We do not want to rely on emission spectra of displays or on transmission spectra of filters, since these can seldom be obtained without measurements, neither do we rely on a properly calibrated monitor. Instead we target low-cost environments and, therefore, will acquire the relevant data through simple and easy-to-use perceptual measurements that come at no extra cost. First we will present how to measure perceived luminance through anaglyph glasses. Next we will show how to use the measurements to compensate for crosstalk and eliminate the aspect of ghosting caused by luminance. Our technique can render anaglyphs, similar to the anaglyphs known as full color (Photoshop algorithm), half color (modified Photoshop method), and grayscale, but with removed ghosting artifacts.

After the discussion of related work, we will present our model and show how to derive the parameters by just a few measurements. In Section 4, we will show how ghosting can be corrected given the model parameters. Section 5 generalizes our model to arbitrary filter colors. Section 6 illustrates the results of the correction process using two idealized models for the sources of crosstalk. In Section 7, we analyze a combination of monitor and glasses based on transmission and emission spectra and compare the result to the idealized models. Section 8 describes how luminance ghosting can be eliminated even for high contrast input images.

2. Related Work

Dubois [Dub01] considers the spectral distribution of the display colors and the transmission of the filters to calculate anaglyphs based on minimizing projection error between the original stereo pair and the anaglyph seen. This is closely related to our method; however, we do not need the spectral distributions and the transmission curves but derive our anaglyphs based on few measurements capturing the relevant interrelations. Dubois uses a weighted square error in CIE XYZ space, which is not perceptually uniform. A perceptually uniform space would be more appropriate, but these spaces are non-linear. Hence, a non-linear transformation would be required since a simple linear transformation cannot capture the non-linearity. In contrast to this approach, we focus on luminance, which is most important to perception of high special frequencies [Mul85]. Additionally, our method does not try to minimize the error between the original stereo image and the final anaglyph but the error between the stereo image seen through the filter and the final anaglyph.

Sanders and McAllister [SM03] compare three stereo generation approaches: the Photoshop algorithm, the algorithm proposed by Dubois [Dub01], and the midpoint algorithm operating in CIE L*a*b* space. They put a threshold on the value of the red channel of the anaglyph and claim that an algorithm that produces a red channel value above the threshold will cause ghosting. This is a heuristic approach that has little validity without considering the colors in the two images.

Ideses and Yaroslavsky [IY05] point out that anaglyphs are one of the most economical methods for stereoscopic presentation and propose several methods to reduce ghosting in anaglyph stereo images. These rely on stereo image registration, defocusing, and a non-linear operation on depth maps. Ghosting can be eliminated by registering the images and bringing them to alignment. Since accommodation is related to convergence [OUW*06, Uka06], aligning the images is beneficial. However, only one depth plane can be aligned at once, which limits the utility of this technique. Ideses and Yaroslavsky [IY05] also show how blurring the image color components can reduce ghosting effects in anaglyphs. Lobel [Lob09] carries this to extremes by presenting magenta-cyan anaglyphs, for which both eyes receive the blue channel that is blurred to reduce ghosting. However, Lobel reports ghosting as a limitation of his approach; our technique could be used to solve this issue.

Some methods rely on explicit knowledge of the spectral distributions of the display device and the transmission functions of the filters, which is not required for our method. Such a technique was introduced by McAllister et al. [MZS10] and compared to different other techniques. Similarly, Sorensen et al. [SHS04] designed a special amber-blue filter pair, known as ColorCode 3-D, with the goal to separate the color information and the depth information to the two eyes.

Woods et al. [WYK07] discuss crosstalk in anaglyph stereoscopic images and also explain crosstalk, the source of ghosting [WT02], for different stereoscopic displays [Woo10]. According to Woods et al., three factors play an important role when looking at crosstalk in anaglyph stereo: the spectral quality of the display, the spectral quality of the glasses, and the quality of the anaglyph image generation matrix. Woods and Harris [WH10] compare the crosstalk of different anaglyph glasses on different displays and recommend good matches for a given monitor to minimize crosstalk. Using the “right” anaglyphs for a given monitor is much less important with our technique. If it happens that you have the “right” glasses for your monitor, you will simply not notice any difference using our technique, since there is no need to compensate for non-existent ghosting. However, if you wear the “wrong” glasses, our technique can significantly reduce ghosting. Bloos [Blo08] designed a test pattern for ghosting determination. In contrast to our calibration method, he uses a static image to determine two measurement values for both eyes; these, in contrast to our method, do not serve a calibration purpose but are a quality measure to enable the comparison of two stereo renderings.

There are several software applications for the generation of anaglyph stereo images. One typical example is StereoPhoto Maker [SS10], which allows the user to generate anaglyphs for different filters using a variety of algorithms and also by providing a custom matrix.

3. Determining Filter Parameters

We use tuples instead of column vectors throughout the paper:

$$(a,b,c) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

3.1. Model

With our model we want to determine how we can account the different color channels for the received luminance through a filter of some anaglyph glasses. With such a model we want to answer questions of the following kind: how much of the luminance coming from the green channel is leaking through the red filter?

First, let us consider the following model:

$$a_l x_{lr}^{\gamma} + b_l x_{lg}^{\gamma} + (1 - a_l - b_l) x_{lb}^{\gamma} = Y \tag{1}$$

Here, we denoted $x_{lr} x_{lg} x_{lb}$ as the non-linear RGB values, ranging from 0 to 1, Y is the (normalized) luminance and a_l, b_l, and γ are our model parameters. If we could obtain measurements of Y for varying RGB values, we could fit our model by least squares fitting on Y and obtain our model parameters. Unfortunately, we cannot measure luminance without additional hardware. However, we can modify our model instead:

$$a_l x_{lr}^{\gamma} + b_l x_{lg}^{\gamma} + (1 - a_l - b_l) x_{lb}^{\gamma} = y^{\gamma}$$

Here y denotes a gray value (red, green, and blue have identical values); therefore, we can obtain our model parameters without requiring any additional hardware. So far, the model is designed for the luminance arriving at one eye. We could build an identical model for the other eye's filter. However, since γ is identical for the two eyes, we can also combine them into one model:

$$(a_l x_{lr}^{\gamma} + b_l x_{lg}^{\gamma} + (1 - a_l - b_l) x_{lb}^{\gamma} +$$
$$a_r x_{rr}^{\gamma} + b_r x_{rg}^{\gamma} + (1 - a_r - b_r) x_{rb}^{\gamma})^{1/\gamma} = y$$

The (leading) subscripts l and r refer to quantities related to the left and right eyes, respectively. When measuring for the left eye, we set $x_{rr} = x_{rg} = x_{rb} = 0$, resulting in Eq. (1) for the single-eye model. When measuring for the right eye, we set the left eye's values to zero. y is always used for the measured gray value.

3.2. Measurement Process

To calculate the luminance parameters we conducted measurements on the left and right eyes while wearing the anaglyph glasses. The user had to adjust the luminance of red, green, blue, cyan, magenta, and yellow to match a gray value by face-based luminance matching [KRC02]. We performed this process not just for one but for 20 gray values, resulting in 120 measurement points for each eye.

3.3. Model Evaluation

The sRGB standard uses a slightly different model for gamma than we. The sRGB color system uses a piecewise function with a linear and a power function part. However, since modern LCD monitors often have a gamma substantially different from the one in sRGB, our model allows us to fit measurements better. Figure 1 shows the differences between the measured data values and our fitted model for the red-cyan glasses (the model parameters will be presented in Section 6.2). The estimated population variance is 0.00660=1.68/255. This low variance indicates that our model is valid. We tested the model with different persons, even with one red-green blind person; each of them had very similar calibration results. We also tested it with different LCD monitors and a CRT monitor and obtained low variances.

3.4. Reduced Calibration

Since it is time-consuming to take 240 samples, we have reduced the number of samples to 3 for each eye for practical calibration purposes. We have developed the following heuristics for good sample positions. Since we adjust the intensity of colors to match that of a gray sample, we should choose colors that are far away from the main diagonal of the color cube, where the gray colors are. At the same time, we should choose colors that are far away from each other. Therefore, pure red, green, and blue are good sample positions for both eyes.

Having few sample positions is more likely to cause problems with the fitting algorithm, which requires the first-order

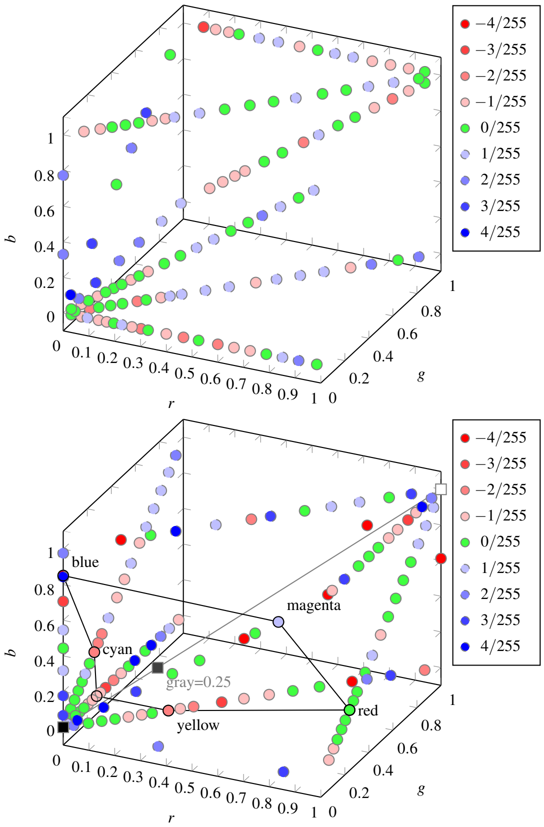

Figure 1: *Difference between model and measured values for the left (red) and right (cyan) eyes, in the top and bottom image, respectively. The axes are scaled between 0 and 1. The bottom image additionally illustrates the color values whose luminance is perceived equal to gray=0.25.*

derivatives of the model function with respect to the model parameters. These derivatives have terms x^{β} with fractional β (γ and $1/\gamma$) and—due to inaccuracies—negative x. Such functions cannot be evaluated since x^{β} is undefined, causing the fitting algorithm to stuck. Our solution is to extend the function C^1 continuously at 0 to negative values by a linear function.

4. Correction of Ghosting

The main idea behind our ghosting correction technique is that we want the anaglyph image to be identically perceived through the glasses as if the display showed the original images.

If we had ideal red-cyan glasses together with a corresponding display device, the red filter would entirely filter out the green and blue components for the left eye and the cyan filter would entirely filter out the red component for the right eye and we could fulfill our desiderata by just tak-

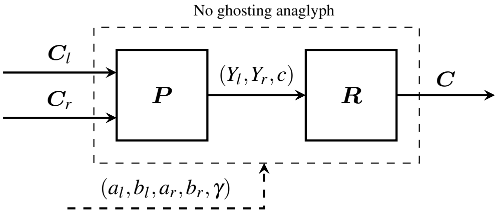

Figure 2: *First the luminance values for the input images (Y_l and Y_r) and a chromaticity value (c) are calculated by the transformation $\boldsymbol{P}$. Next an output color is calculated that has the desired luminance values and the desired chromaticity value by the transformation $\boldsymbol{R}$.*

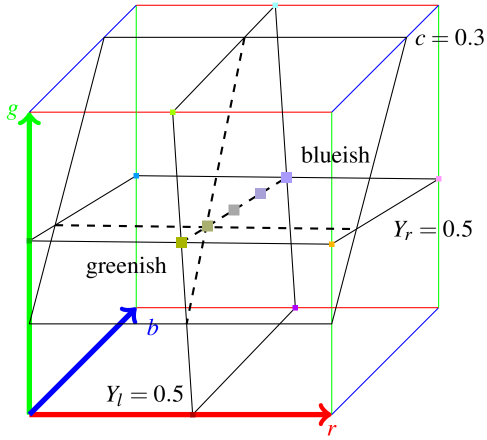

Figure 3: *Points in linear RGB space, where luminance for the left eye (Y_l)=0.5 and luminance for the right eye (Y_r)=0.5 and chroma (c)=0.3 lie on planes. The plane normals are the row entries of $\boldsymbol{R^{-1}}$. The figure illustrates the red-cyan glasses that are measured in Section 6.2.*

ing the red component from the left eye image and the green and blue components from the right eye image.

Since neither anaglyph filters nor displays are ideal, there always is some leakage from the one image to the other one, which may cause ghosting. The color difference between the desired and the perceived image can be described by the luminance and the chromaticity difference. Luminance differences result in much more distracting ghosting than chromaticity differences. Therefore, our main goal is to eliminate luminance differences. We have three degrees of freedom when rendering the anaglyph image, two of them are fixed due to luminance. Thus, we have one remaining degree of freedom that we use to minimize the most significant chromaticity difference.

4.1. Anaglyphs Without Ghosting

In this subsection, we present our color anaglyph generation technique that does not produce luminance ghosting. Each pixel is processed independently. We take as input RGB colors for the left (R'_l, G'_l, B'_l) and right (R'_r, G'_r, B'_r) eyes and produce an RGB anaglyph color (R', G', B') that does not exhibit luminance ghosting.

First we transform the non-linear R′G′B′ colors to linear RGB space by using the γ value measured:

$$C = C'^{\gamma}$$

where $C \in \{R_l, G_l, B_l, R_r, G_r, B_r\}$ and $C' \in \{R'_l, G'_l, B'_l, R'_r, G'_r, B'_r\}$.

For both eyes, we reproduce luminance exactly. Additionally, one chromaticity channel is reproduced for one eye. Figure 2 shows how the general approach can be described by two transformations $\boldsymbol{P}$ and $\boldsymbol{R}$; in this section, we will derive $\boldsymbol{R}$.

Luminance can be calculated, using the calibrated values that exactly capture the luminance through the three color channels, by:

$$Y_{\text{EYE}}((R,G,B)) = a_{\text{EYE}}R + b_{\text{EYE}}G + c_{\text{EYE}}B$$

where $c_{\text{EYE}} = 1 - a_{\text{EYE}} - b_{\text{EYE}}$.

Some of the following considerations are specific to red-cyan filter pairs, the obtained solution is generalized for arbitrary filters in Section 5. Using red-cyan glasses, the most prominent chromaticity channel we can perceive is the green-blue channel seen through the cyan glass on the right eye. We therefore set $c = G - B$. Hereby, we obtain the linear transform that maps a color vector to a (luminance left, luminance right, chromaticity) vector (YYc space) by:

$$\boldsymbol{R^{-1}} = \begin{bmatrix} z_r & z_g & z_b \\ y_r & y_g & y_b \\ 0 & 1 & -1 \end{bmatrix}$$

where $y_r = a_r$, $y_g = b_r$, $y_b = 1 - a_r - b_r$, $z_r = a_l$, $z_g = b_l$, and $z_b = 1 - a_l - b_l$. Thus we can take the inverse of $\boldsymbol{R^{-1}}$, which is $\boldsymbol{R}$, to calculate no ghosting anaglyphs.

$$\boldsymbol{R} = \frac{1}{z_r - y_r} \begin{bmatrix} y_g + y_b & -z_g - z_b & y_g z_b - y_b z_g \\ -y_r & z_r & y_b z_r - y_r z_b \\ -y_r & z_r & y_r z_g - y_g z_r \end{bmatrix}$$

The transformation from YYc space to RGB space is illustrated in Figure 3. In the following, we will present full color, half color, and gray variants that only differ in $\boldsymbol{P}$ but have the same $\boldsymbol{R}$.

4.2. Full Color Anaglyphs

In the same manner as $\boldsymbol{R^{-1}}$ maps a color to YYc space, $\boldsymbol{P}$ needs to map two colors to YYc space. Y_l has to be calculated using the color of the left, Y_r and c using the color of the right images. The transformation that maps the input colors for the images can be described by a linear transformation.

As input, we take the vector $(R_l, G_l, B_l, R_r, G_r, B_r)$. Hence $\boldsymbol{P}$ can be represented by the following 3×6 matrix:

$$\boldsymbol{P} = \begin{bmatrix} z_r & z_g & z_b & 0 & 0 & 0 \\ 0 & 0 & 0 & y_r & y_g & y_b \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

4.2.1. Nice Properties

Our full color anaglyphs have some nice properties. First, in case the input for the left and right eyes is identical ($\boldsymbol{C}_l = \boldsymbol{C}_r$), the anaglyph will just reproduce the input color. Second, the luminance for both eyes is preserved. The last property we describe for red-cyan glasses: in case the green and blue components are identical in the input image for the right eye (cyan filter), they will be identical in the anaglyph image. From these properties, we could derive some equations and obtain the same overall transformation ($\boldsymbol{RP}$) like we presented above. However, the introduction of the YYc space helps us to obtain an intuitive understanding of our transformation, which also holds for the half color and gray anaglyphs presented below.

4.3. Half Color Anaglyphs

Full color anaglyphs cause binocular rivalry (aka retinal rivalry) for some people. Half color anaglyphs are designed to reduce binocular rivalry by taking the gray value instead of the red component of the left image as red component for the anaglyph image. Considering our technique, we can replace the individual luminance calculation for the left and right images by a calculation that treats the left and right image channels identically. A reasonable calculation can be based on the luminance from the RGB calculation of the sRGB model, given by $Y((R,G,B)) = \boldsymbol{sRGB_Y}(R,G,B)$ with the row vector $[x_r, x_g, x_b] = \boldsymbol{sRGB_Y} \approx [0.2126, 0.7152, 0.0722]$. Hence $\boldsymbol{P}$ can be represented by:

$$\boldsymbol{P} = \begin{bmatrix} x_r & x_g & x_b & 0 & 0 & 0 \\ 0 & 0 & 0 & x_r & x_g & x_b \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

4.4. Gray Anaglyphs

For gray anaglyphs, the green and blue components need to be identical, which is the case when c is zero. Hence $\boldsymbol{P}$ is given by:

$$\boldsymbol{P} = \begin{bmatrix} x_r & x_g & x_b & 0 & 0 & 0 \\ 0 & 0 & 0 & x_r & x_g & x_b \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4.5. Removing Ghosting from Anaglyphs Generated by Naïve Methods

Now we will provide a method to correct ghosting in anaglyph images generated by naïve methods, see Figure 4. The naïve methods use the red component (full color) or the luma (half color and gray) of the left image and the green and blue components (full and half color) of the right image, which are assigned to the red, green, and blue components of the anaglyph image. The green and blue components are replaced by the luma of the right image in the gray anaglyph case. We interpret the red component of the anaglyph image as luminance of the left image, since this is the only information we have from the left image; therefore, it is the best estimation of luminance available. To estimate the luminance of the right image we use the green and blue components weighted by the filter values, which need to be normalized to account for the unknown red component. Hence $\boldsymbol{P}$ is given by:

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{y_g}{y_g + y_b} & \frac{y_b}{y_g + y_b} \\ 0 & 1 & -1 \end{bmatrix}$$

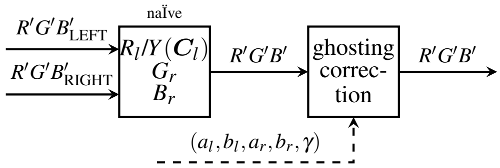

Figure 4: *First the images for the left and right eyes are composed by the naïve approach to an anaglyph stereo image, by selecting the red channel or the luma of the left and the green and blue channels of the right image. We take the naïve anaglyph image and eliminate ghosting by using the filter values obtained in Section 3.*

5. Filter Color Estimation

Our technique is not limited to red-cyan glasses. The only thing we need to fix is which chromaticity channel we want to preserve. This is the *green-blue channel* of the *right eye* in the red-cyan case. In the general case, we will make the decision based on the filters.

The vector $(a_{\text{EYE}}, b_{\text{EYE}}, 1 - (a_{\text{EYE}} + b_{\text{EYE}}))$ actually is the luminance from the red, green, and blue color channels, respectively, as perceived through the filters. From the sRGB model, we can take the linear RGB to luminance transformation $Y((R,G,B)) = \boldsymbol{sRGB_Y}(R,G,B)$. Since we have a filter between the display and our eyes, not all of the luminance arrives: $(a_{\text{EYE}}, b_{\text{EYE}}, 1 - (a_{\text{EYE}} + b_{\text{EYE}})) = f_{\text{EYE}} \boldsymbol{f}_{\text{EYE}} \circ [0.2126, 0.7152, 0.0722]^T$, where $\circ$ denotes the entry-wise vector product and f_{EYE} is used to normalize $\boldsymbol{f}_{\text{EYE}}$ in the way that the largest component is 1. Given this relationship, we can calculate the filter values $\boldsymbol{f}_{\text{EYE}}$.

Anaglyph filters are designed in a way that one from the filter pair filters out all but one color channel (this is the red filter for the red-cyan filter pair, which filters out all but the red channel); we choose to preserve the other two color channels of the other eye. Algorithmically, we make the decision which eye we choose based on the second largest filter component. We do not need to derive new equations but simply interchange the input channels when we calculate the correction matrix.

6. Results

We developed an application using C++, OpenGL, and Qt, which is included in the supplementary material and for which we provide updates on our website `www.vis.uni-stuttgart.de` (in "Research"). We use GLSL for rendering the anaglyph image out of the left and right images. The 3×6 transformation matrix ($\boldsymbol{RP}$) can be decomposed into a dot product of a 3-component vector from one image and a 3×4 matrix multiplication taking the result of the dot product and the other source image, which can be efficiently evaluated in the shader.

We measured ColorCode 3-D and red-cyan paper-frame glasses on a Dell U2410 monitor, with the Standard Preset Mode.

6.1. ColorCode 3-D (Amber-Blue)

For the ColorCode 3-D glasses, we obtain the following results: $a_l = 0.4581$, $b_l = 0.5183$, $a_r = 0.0076$, $b_r = 0.0552$, $\gamma = 1.856$. Using this, we obtain $\boldsymbol{f}_l = (1, 0.336, 0.152)$ and $\boldsymbol{f}_r = (0.003, 0.006, 1)$. This means, the left eye glasses let pass the entire red component, 34% of the green, and 15% of the blue component; and the right eye filter lets pass the blue component, 0.3% of the red, and 0.6% of the green component. This is remarkable, since on the left eye we can potentially distinguish all colors and only have a limited luminance leaking consisting of 2.4% ($= 1-(a_l+b_l)$), which causes ghosting. On the right eye, we have 6.3% luminance leaking. With our technique, we can eliminate the luminance leakages, but it is not possible to eliminate the color leakage of the blue component to the left eye.

6.2. Red-Cyan Filter

For the red-cyan glasses, we obtain the following results: $a_l = 0.9260$, $b_l = 0.0612$, $a_r = 0.0094$, $b_r = 0.8705$, $\gamma = 1.668$; $\boldsymbol{f}_l = (1, 0.020, 0.041)$; $\boldsymbol{f}_r = (0.027, 0.732, 1)$. 2% of the green and 4% of the blue channel are not enough to see green or blue on the left eye. The 3% red on the right eye is also not apparent. Considering the ghosting, we have 7% for the left eye's luminance but just 1% for the right eye's luminance, which means there is strong ghosting on the left eye.

6.3. Illustration

In the previous section, we measured how much luminance is perceived from the primaries through the filters, but we did not consider the chromaticity perceived (i.e., as which color this luminance is perceived). Given the filter values, we can estimate the color of an image seen through the glasses. However, without having further measurement data capturing color perception, not just luminance, we need to make some assumptions.

When we assume that the filter only affects the luminance but not the color hue and saturation, we obtain the images shown in the left columns of Figure 5, which are calculated by simply multiplying the RGB values with the filter transmission values. This behavior would be obtained by monochromatic primaries.

Monitor primaries usually are not monochromatic but extended and overlap partially [BDM04]. Let us assume that we have anaglyph filters that clearly separate. In such a case, light from the red, green, and blue channels passing the red filter appears red, since the blue and green frequencies are totally removed by the ideal filter. Light from the red, green, and blue channels passing the cyan filter does not appear red. This means, we can collect the luminance from the green and blue color channels passing through the red filter and replace it with a luminance that we add to the red channel; and the same way we distribute the luminance from the red channel passing through the cyan filter to the green and blue channels. Contrary to the previous assumptions, which represent the worst case for our ghosting removal technique, this model leads to the best case. The resulting images are shown in the right columns of Figure 5.

The first thing to note is that there are no visible differences between the cyan images; this could be expected, since just 1% of the luminance from the red channel leaks through the cyan filter. However, there is visible ghosting in the anaglyph image generated by the naïve method as seen through the red filter in both cases. This ghosting is mainly caused by the luminance differences to the original image as seen through the glasses. With our technique, we eliminate the luminance differences. In the first case, there may remain differences in color hue and saturation—depending on your display the compensation shown in the figure may be too large or to small, resulting in remaining luminance differences—but in the second case, we can eliminate ghosting entirely (if the dynamic range is sufficient, see Section 8).

7. Analysis

In the previous section, we made some assumptions on the filter transmission and the monitor primaries spectra and obtained two different estimates. The second one (idealized filter) obtained very good results, while the first one (idealized device) suffered from quite strong color leakage. Here, we will analyze a real system to identify which of the two models comes closer to the real situation.

To simulate the colors the observer sees through the fil-

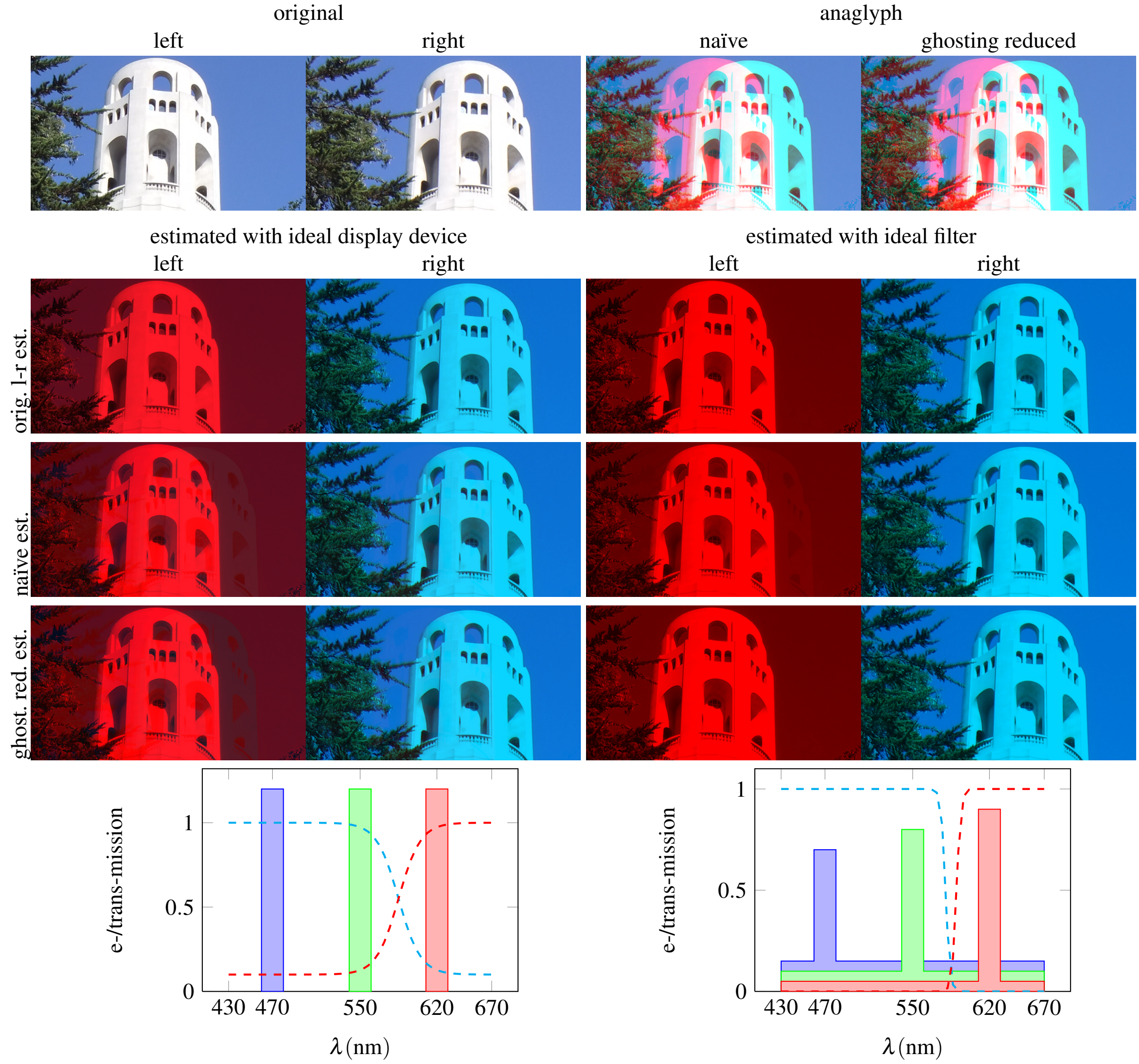

Figure 5: *The diagrams illustrate the transmission spectra of the red-cyan filters as dashed lines, and the emission spectra of the monitor primaries as shaded areas. The left diagram uses an idealization with respect to the display device in the sense that the spectra are narrow; however the red-cyan filters do not separate well. The right diagram uses an idealization with respect to the filter, in the sense that the filter has a clear separation; however the display spectra overlap. From top to bottom, we see the original left and right images and the naïve and ghosting reduced anaglyphs in the first row; the (non-anaglyph) images as they look through red-cyan glasses (second row), the naïve anaglyph images as they look through the glasses (third row), our ghosting reduced anaglyph images as they look through the glasses (forth row). (Colors may be strongly distorted in printouts.)*

ters, we analyze the power spectrum of a monitor together with a pair of red-cyan anaglyph glasses. CRT monitors have quite similar spectra [WT02]—which differ from LCD spectra [Sha02]. We used the monitor spectra of the right Sony Trinitron Multiscan GDM-F500 monitor presented by Boyaci et al. [BDM04] and the transmission spectra of the paper-frame glasses (perspektrum.de) presented by Wieser [Wie06] for the calculations. Using the CIE 1931 2° Standard Observer color matching functions, we calculated the gamut and white point of the GDM-F500, see Figure 6. The cyan and red gamuts are calculated the same way, but with multiplying the monitor spectra with the filters transmission spectra beforehand. Both the red and the cyan gamuts lie completely outside the GDM-F500 and the sRGB

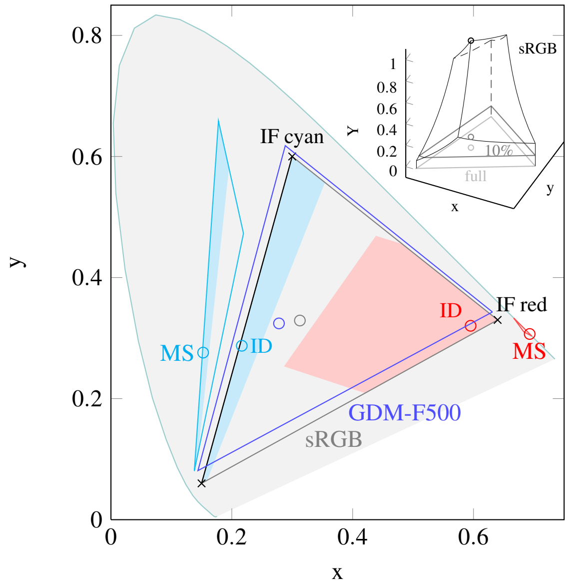

Figure 6: *Gamuts (as triangles) and white points (as circles) of the idealized display model (ID), the idealized filter model (IF), and the spectral measurement (MS) in the CIExy coordinate system. Gamut parts reaching 10% of the maximal possible luminance are drawn as shaded areas. In blue, the gamut and white point of the Sony Trinitron Multiscan GDM-F500 are shown; the ID gamuts are identical to the sRGB gamut (in gray) at* $Y \approx 0$.

gamuts, and can therefore not be reproduced on these monitors. Color gamuts are 3D structures, which can be presented in xyY space. At $Y \approx 0$ they are defined by the primaries. As luminance increases, the gamut shrinks and collapses to the white point at $Y = 1$. Gamuts are usually presented as projection to the xy plane, which corresponds to the $Y \approx 0$ gamut. At $Y \approx 0$, the gamuts used by the idealized display device model correspond to the full monitor gamut. As additional information we show gamut parts reaching 10% of the maximal luminance as shaded areas in Figure 6.

Notice that for the red filter the spectral measurement's gamut significantly differs from the idealized display device model's gamut but comes close to the idealized filter model's gamut. This means, the idealized filter model is more adequate and we should expect low color leakage in the real world.

8. Dynamic Range

With imperfect filters, luminance intended for one eye reaches the other eye and vice versa. This implies that we cannot achieve each combination of luminance for the two eyes, as illustrated in Figure 7. The gamut directly corresponds to the measured filter values for the red-cyan glasses (see Section 6.2).

After applying the ghosting correction matrix, resulting colors do not necessary lie within $[0,1]$. Concrete example: the left input image is dark $\boldsymbol{C}_l = (0.02, 0.02, 0.02)$ and the right one is bright $\boldsymbol{C}_r = (0.9, 0.9, 0.9)$, resulting in point A. The anaglyph color after ghosting correction would be $\boldsymbol{C} = (-0.051, 0.909, 0.909)$. Unfortunately, the red component needs to be reduced to negative values to compensate for the luminance coming from the green and blue channels of the output image shining through the red filter. Next we present two solutions that bring the colors to the $[0,1]$ interval.

8.1. Clamp to Range

The easiest way to deal with the problem is to clamp the output color values to $[0,1]$, resulting in (0, 0.909, 0.909) for A. Actually, this is a projection to the closest point inside the gamut. This way, luminance for inside gamut points is corrected for leakage and reproduced exactly and luminance for outside gamut points is maintained as close as possible. Ghosting can appear for outside gamut points.

8.2. Complete Ghosting Elimination

To be able to completely eliminate ghosting we need to remove points that get mapped outside the gamut. This can be done by applying a transformation to the input color values. We restrict the luminance values to lie within given intervals in a way that the resulting combinations of luminance values (forming an axis-aligned rectangle) lie completely inside the gamut. By mapping $\boldsymbol{C}_l^{new} = 1 - z_r + (2z_r - 1)\boldsymbol{C}_l^{old}$ and $\boldsymbol{C}_r^{new} = 1 - y_r + (2y_r - 1)\boldsymbol{C}_r^{old}$, we obtain a range having the luminance of pure red and pure cyan in its corners for the red-cyan glasses, shown as dashed line in Figure 7. For A^{new}, we obtain $Y_l = 0.091$, $Y_r = 0.892$, and (0.026, 0.901, 0.901) as resulting anaglyph color. The dynamic range with respect to the luminance on the right eye still is large. Unfortunately, it significantly reduces the dynamic range with respect to the left eye (Y_l). We cannot produce as bright colors as before, neither can we produce dark color on the red eye. Nevertheless, we can guarantee that no ghosting will appear in the output image. As noted above the intervals are not predefined; usually, dynamic range for one eye can be traded for dynamic range for the other one. The dotted line in Figure 7 shows intervals that slightly improve the range for the left but significantly reduce the range for the right eye.

9. Conclusions and Future Work

We have presented a model for luminance perceived through anaglyph glasses that can be used to substantially remove ghosting artifacts from anaglyph stereo images. We have

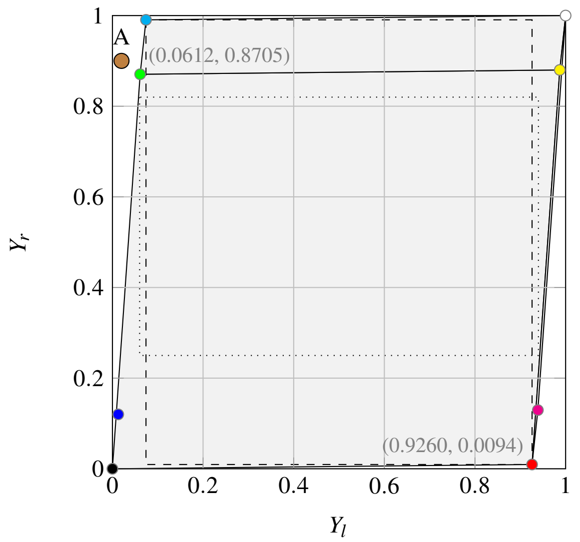

Figure 7: *This figure illustrates the perceived luminance through the left and right glasses of the red-cyan filter (presented in Section 6.2) for different colors displayed on the screen. Luminance combinations that cannot be obtained through the glasses are shown outside the gray area. In particular, low luminance on the left eye cannot be obtained when we have high luminance on the right eye (point A). Restricted working spaces are illustrated by dashed and dotted lines.*

showed how the model parameters can be estimated with just six measurements. Our method can be used to render full color, half color, and gray anaglyphs without luminance ghosting, as well as for removing luminance ghosting artifacts from anaglyphs created with a naïve method. Our method is not limited to red-cyan glasses, it determines which glasses are in use during calibration, no further information is required. We have illustrated the results of our method for different display devices and filters and related the results to a real monitor and filter combination. Finally, we have showed how ghosting can be eliminated independently of the luminance combinations in the source images by sacrificing dynamic range.

We believe that our method be of interest since it greatly enhances anaglyph stereo rendering. It is versatile and can also be combined with current approaches. A combination with the magenta-cyan technique targeting reduction of retinal rivalry is a good example of an interesting combination.

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