**Reasenberg & Jones with time-dependent magnitude of completeness**

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*Revised July 9 (change A&B to G&H, rewrite eq 7-9 in terms of function F in eq 10)*

Reasenberg and Jones (1989) give the equation:

(1)

to specify the modeled rate of aftershocks with magnitude ≥Mmin at time t following a mainshock of magnitude Mm. The parameter *a* represents the productivity of the sequence, *b* describes the magnitude distribution, *p* describes the rate of decay of the aftershock sequence, and *c* prevents the function from blowing up at the time of the mainshock. The c-value is usually fixed, and that will be assumed here. The b-value is fixed or determined from the magnitude-frequency distribution. The a-value and p-value are to be found by maximum likelihood.

Due to changing magnitude of completeness following a large earthquake, we would like to revise this equation to be in terms of a time-dependent magnitude of completeness, Mc(t,Mm), instead of a constant minimum magnitude Mmin. The rate of aftershock larger than Mc(t,Mm) is then:

. (2)

Helmstetter et al. (2006) give a formula for the time-dependent magnitude of completeness, Mc(t,Mm), following a mainshock of magnitude Mm in southern California:

, and . (3)

To generalize this equation for use in other regions with different catalog completeness characteristics, and to make analytic solution of the log-likelihood possible, I rewrite this equation as:

, and , (4)

where Mcat is the known completeness of the catalog when a large earthquake has not recently occurred, and G and H are constants describing the catalog completeness following a mainshock. The parameter *c* is the same as in the modified Omori decay (equations 1 and 2). This may underestimate Mc at times t<c, which should be at least partially corrected if the c-value in the Omori decay is to be understood as accounting for catalog incompleteness at short times. The cross-over time between time-dependent Mc and constant Mc=Mcat can be given as:

. (5)

The log-likelihood of any set of parameter values, a and p, is:

(6)

Where tbeg and tend are the beginning and end of the time period used to fit the data, respectively, and n is the number of earthquake of magnitude greater than Mc(t, Mm) at the time of their occurrence. The integral can be solved analytically, giving:

(7)

If tbeg>tc, there is no time-dependence to the magnitude of completeness, the familiar form of the log-likelihood should be used:

(8)

where . If tend<tc, instead use:

. (9)

In equations 7-9, the function F(c,x,t1,t2), is defined as:

, for x=1 (10)

, for x1.

In equation 10, x may stand for p or for p-bH. The limits of integration, t1 and t2, stand for some combination of tbeg, tc, and tend.

References:

Helmstetter, A., Y. Y. Kagan, and D. D. Jackson, Comparison of Short-Term and Time-Independent Earthquake Forecast Models for Southern California, ￼Bulletin of the Seismological Society of America, 96, 90–106, 2006.

Reasenberg, P.A., and L.M. Jones, Earthquake hazard after a mainshock in California, Science, 243, 1173-1176, 1989.