<CHT>Chapter 5 - Algebra

The jump from arithmetic to algebra can be a difficult one for many students. Many students struggle with the idea that mathematics can include situations that aren’t static and do change. In elementary arithmetic, a situation such as:



This equation is a static situation and will yield the answer of 8 every time. However, a situation such as:



can yield many different answers because the answer depends on what amount (number) that *x* represents. Since the value of can vary (represent different values), it is known as a variable.

Algebra is useful to better model real life situations. In the first equation shown,  can only model situations where you add those two numbers together. For example, if your uncle gives you five dollars and your aunt give you three dollars, then you will always receive eight dollars. The second equation can model more complex situations. For example, you wish to buy a game that costs $38 but you only have three dollars. Your uncle will pay you five dollars an hour to work for him. If you’ve worked five hours, have you earned enough money? If not, how many hours will you have to work?

Algebra and algebraic thinking open up a world of possibilities that arithmetic alone cannot do.

\*\*\*\*INTRO END\*\*\*\*

<D>Module 1—Algebraic Expressions

**After completing this module, you should be able to:**

<NL>

 Convert between written and symbolic algebraic expressions and equations.

 Simplify and evaluate algebraic expressions.

 Add and subtract algebraic expressions.

 Multiply and divide algebraic expressions.

</NL>

Algebraic expressions are the building blocks of algebra. While a numerical expression (also known as an arithmetic expression) like can represent only a single number, an algebraic expression such as can represent many different numbers. This module will introduce you to algebraic expressions, how to create them, simplify them, and perform arithmetic operations on them.

[H2] Algebraic Expressions and Equations

<FIG/>

CS\_Photo\_05\_01\_001: A diverse image of two college-aged people, one woman and one man, close to the same age.

Xavier and Yasenia have the same birthday, but they were born in different years. This year Xavier is 20 years old and Yasenia is 23, so Yasenia is three years older than Xavier. When Xavier was 15, Yasenia was 18. When Xavier will be 33, Yasenia will be 36. No matter what Xavier’s age is, Yasenia’s age will always be three years more.

In the language of algebra, we say that Xavier’s age and Yasenia’s age are variable and the three is a constant. The ages change, or vary, so age is a <term>variable</term>. The three years between them always stays the same, so the age difference is the <term>constant</term>. In algebra, letters of the alphabet are used to represent variables. Suppose we call Xavier’s age *x*. Then we could use to represent Yasenia’s age. See Table 5.1. Letters are used to represent variables. The letters most often used for variables are *x*, *y*, *z*, *a*, *b*, and *c*.

<DEF>[DEFINITION]

A **variable** is a letter that represents a number or quantity with a value that may change.

A **constant** is a number that always has the same value.

</BOX>

<TB>Table 5.1: Xavier’s and Yasenia’s Ages

|  |  |
| --- | --- |
| **Xavier’s age** | **Yasenia’s age** |
| 15 | 18 |
| 20 | 23 |
| 33 | 36 |
| *x* |  |

To write algebraically, we need some symbols as well as numbers and variables. The symbols for the four basic arithmetic operations: addition, subtraction, multiplication, and division are summarized in Table 5.2, along with words we use for the operations and the result.

<TB>Table 5.2: Symbols for Operations

|  |  |  |  |
| --- | --- | --- | --- |
| **Operation** | **Notation** | **Say:** | **The result is…** |
| Addition |  | *a* plus *b* | The sum of *a* and *b* |
| Subtraction | *a* − *b* | *a* minus *b* | The difference of *a* and *b* |
| Multiplication | *a* • *b*, (*a*)(*b*), (*a*)*b*, *a*(*b*), *ab, ba* | *a* times *b* | The product of *a* and *b* |
| Division | *a* ÷ *b*, *a*/*b* | *a* divided by *b* | The quotient of *a* and *b* |

<LIGHT> *In algebra, the cross symbol (×) is normally not used to show multiplication because that symbol could cause confusion. For example, does*  *mean* *y (three times y) or* *(three times x times y)? To make it clear, use • or parentheses for multiplication.*

<END>

We perform these operations on two numbers. When translating from symbolic form to words, or from words to symbolic form, pay attention to the words *of* or *and* to help you find the numbers.

<BL>

● The *sum* ***of*** 5 ***and*** 3 means add 5 plus 3, which we write as .

● The *difference* ***of*** 9 ***and*** 2 means subtract 9 minus 2, which we write as 

● The *product* ***of*** 4 ***and*** 8 means multiply 4 times 8, which we can write as .

● The *quotient* ***of*** 20 ***and*** 5 means divide 20 by 5, which we can write as .

</BL>

<example>EXAMPLE 1—<title>Translating from Algebra to Words</title>

<EXERCISE>

<PROBLEM>

<NL>

* 
* 
* 
* 

</NL>

**<SOLUTION>**

<NL>

* According to Table 5.2, this could be translated as 12 plus 14 OR the sum of 12 and 14.
* According to Table 5.2, this could be translated as 30 times 5 OR the product of 30 and 5.
* According to Table 5.2, this could be translated as 64 divided by 8 OR the quotient of 64 and 8.
* According to Table 5.2, this could be translated as *x* minus *y* OR the difference of *x* and *y.*

</NL>

<END>

<BOX>[YOUR TURN] 1

<EXERCISE>

<PROBLEM>

1. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

**<SOLUTION>**

<ANS>1. 18 plus 11 OR the sum of 18 and 11

<ANS>2. 27 times 9 OR the product of 27 and 9

<ANS>3. 84 divided by 7 OR the quotient of 84 and 7

<ANS>4. *p* minus *q* OR the difference of *p* and *q*

<END>

<example>EXAMPLE 2—<title>Translating from Words to Algebra</title>

<EXERCISE>

<PROBLEM>

<NL>

* The difference of 47 and 19
* 72 divided by 9
* The sum of *m* and *n*
* 13 times 7

</NL>

**<SOLUTION>**

<NL>

* According to Table 5.2, these words could be translated as 
* According to Table 5.2, these words could be translated as 
* According to Table 5.2, these words could be translated as 
* According to Table 5.2, these words could be translated as 

</NL>

<END>

<BOX>[YOUR TURN] 2

<EXERCISE>

<PROBLEM>

1. 43 plus 67

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. The product of 45 and 3

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. The quotient of 45 and 3

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. 89 minus 42

</SOLUTION>

</EXERCISE>

**<SOLUTION>**

<ANS>1. 

<ANS>2. 

<ANS>3. 

<ANS>4. 

<END>

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have <term>expressions</term> and equations. In Examples 1 and 2 we used expressions. An expression is like an English phrase. Notice that the English phrases do not form a complete sentence because the phrase does not have a verb. Table 5.3 has examples of expressions.

<BOX>[DEFINITION]

An **expression** is a number, a variable, or a combination of numbers and variables using operation symbols.

</BOX>

<TB>Table 5.3: Examples of Expressions

|  |  |  |
| --- | --- | --- |
| **Expression** | **Words** | **English Phrase** |
|  | 3 plus 5 | The sum of three and five |
|  | *n* minus one | The difference of *n* and one |
|  | 6 times 7 | The product of six and seven |
|  | *x* divided by *y* | The quotient of *x* and *y* |

<example>EXAMPLE 3—<title>Translating From an English Phrase to an Expression</title>

<EXERCISE>

<PROBLEM>

<NL>

1. Seven more than a number *n*.

2. A number *n* times itself.

3. Six times a number *n*, plus two more.

4. The cost of postage is a flat rate of 10 cents for every parcel, plus 34 cents per ounce *x*.

</NL>

**<SOLUTION>**

<NL>

1. **

2. **or 

3. 

4. 

</NL>

<END>

<BOX>[YOUR TURN] 3

<EXERCISE>

<PROBLEM>

1. Twenty less than a number *n*. (Hint: you have a number *n* and you want 20 less than it)

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. Add two to a number *n*, then multiply it by six.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. A number *n* to the third power minus five.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. A plumber charges $60 per hour *h*, plus a $40 flat fee for every job.

</SOLUTION>

</EXERCISE>

**<SOLUTION>**

<ANS>1. **

<ANS>2.  OR 

<ANS>3. **

<ANS>4. 

<END>

An <term>equation</term> is two expressions linked with an <term>equal sign</term>. When two quantities have the same value, we say they are equal and connect them with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equals sign gives the verb. Table 5.4 has some examples of equations.

<DEF>[DEFINITION]

An **equation** is two expressions connected by an equal sign.

The symbol “=” is called the **equal sign**.  is read “a isequal to b”.

</BOX>

<TB>Table 5.4: Examples of Equations

|  |  |
| --- | --- |
| **Equation** | **English Sentence** |
|  | The sum of three and five is equal to eight. |
|  | *n* minus one equals fourteen. |
|  | The product of six and seven is equal to forty-two. |
|  | *x* is equal to fifty-three. |
|  | *y* plus nine is equal to two *y* minus three. |

<example>EXAMPLE 4—<title>Translating From an English Sentence to an Equation</title>

<EXERCISE>

<PROBLEM>

<NL>

* Two times *x* is 6.
* *n* plus 2 is equal to *n* times 3.
* The quotient of 35 and 7 is 5.
* Sixty-seven minus *x* is 56.

</NL>

**<SOLUTION>**

<NL>

* 
* **
* 
* 

</NL>

<END>

<BOX>[YOUR TURN] 4

<EXERCISE>

<PROBLEM>

1. Five times *y* is 50.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. Half of a number *n* is 30.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. The difference of three times a number *n* and 7 is 2.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. Two times *x* plus 7 is 21.

</SOLUTION>

</EXERCISE>

**<SOLUTION>**

<ANS>1. 

<ANS>2. 

<ANS>3. 

<ANS>4. 

<END>

<BOX>[WHO KNEW?]

<T>The Use of Variables

French philosopher and mathematician René Descartes (1596–1650) is usually given credit for the use of the letters *x*, *y*, and *z* to represent unknown quantities in algebra. He introduced these ideas in his publication of *La Geometrie*, which was printed in 1637. In this publication, he also used the letters *a*, *b*, and *c* to represent known quantities. There is a (possibly fictitious) story that, when the book was being printed for the first time, the printer began to run short of the last three letters of the alphabet. So the printer asked Descartes if it mattered which of *x*, *y*, or *z* were used for the mathematical equations in the book. Descartes decided it made no difference to him; so the printer decided to use *x* predominantly for the mathematics in the book, because the letters *y* and *z* would occur more often in the body of the text (written in French) then the letter *x* would! This might explain why the letter *x* is still used today as the most common variable to represent unknown quantities in algebra.

</BOX>

[H2] Simplifying and Evaluating Algebraic Expressions

To simplify an expression means to do all the math possible. For example, to simplify  we would first multiply to get 8 and then add 1 to get 9. We have introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values. Consider . Do you add first or multiply first? Do you get different answers?

|  |  |  |
| --- | --- | --- |
| Add first: | Multiply first: | Which one is correct? |

Early on, mathematicians realized the need to establish some guidelines when performing arithmetic operations to ensure that everyone would get the same answer. Those guidelines are called the Order of Operations and are listed in Table 5.5.

<TB>Table 5.5: The Order of Operations

|  |  |
| --- | --- |
| Step 1. Parentheses and Other Grouping Symbols | Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first. |
| Step 2. Exponents | Simplify all expressions with exponents. |
| Step 3. Multiplication and Division | Perform all multiplication and division in order from left to right. These operations have equal priority. |
| Step 4. Addition and Subtraction | Perform all addition and subtraction in order from left to right. These operations have equal priority. |

<LIGHT> *You may have heard about Please Excuse My Dear Aunt Sally or PEMDAS. Be careful to notice in Steps 3 and 4 in Table 5.5 that multiplication and division, as well as addition and subtraction, happen in order from LEFT to RIGHT. It is possible, for example, to have PEDMAS or PEMDSA. The PEMDAS trick can be misleading if not fully understood!*

<example>EXAMPLE 5—<title>Making a Numerical Equation True Using the Order of Operations</title>

<EXERCISE>

<PROBLEM>

Use parentheses to make the following statements true.

<NL>

* 
* 
* 
* 

</NL>

**<SOLUTION>**

<NL>

* Add the parentheses around the  . Then you have .
* Add the parentheses around the . Then you have .
* Add the parentheses around the . Then you have .
* Add the parentheses around the . Then you have 

</NL>

<END>

<BOX>[YOUR TURN] 5

<EXERCISE>

<PROBLEM>

1. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. 

</SOLUTION>

</EXERCISE>

**<SOLUTION>**

<ANS>1. 

<ANS>2. 

<ANS>3. 

<ANS>4. 

<END>

<BOX>[TECH CHECK]

The Desmos activity called “Twin Puzzles” is a way for students to practice working with the order of operations. For teachers, the activity can be found at: <https://teacher.desmos.com/activitybuilder/custom/57ae458a697f767c75597801?r=w.hd>. Teachers will need a Desmos account to assign the activity for student use; once they have assigned the activity to their students, teachers need to share the code for the activity with their students. Students will input the code at <https://student.desmos.com/> to work on the activity.

</BOX>

<example>EXAMPLE 6—<title>Simplifying an Expression Using the Order of Operations</title>

<EXERCISE>

<PROBLEM>

<NL>

* 
* 
* 
* 
* 

</NL>

**<SOLUTION>**

<NL>

* 
* 
* 
* 
* 

</NL>

<END>

<BOX>[YOUR TURN] 6

<EXERCISE>

<PROBLEM>

1. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

5. 

</SOLUTION>

</EXERCISE>

**<SOLUTION>**

<ANS>1. 

<ANS>2. 

<ANS>3. 

<ANS>4. 

<ANS>5. 

<END>

<example>EXAMPLE 7—<title>Evaluating and Simplifying an Expression</title>

<EXERCISE>

<PROBLEM>

<LAL>

* Evaluate  when .
* Evaluate  when  </LAL>

**<SOLUTION>**

<LAL>

* To evaluate, let  in the expression, then solve: 
* To evaluate, let  in the expression, then solve: 

</LAL>

<END>

<BOX>[YOUR TURN] 7

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

a. Evaluate 5*x* – 6 when .

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

b. Evaluate  when.

</SOLUTION>

</EXERCISE>

**<SOLUTION>**

<ANS>a. 

<ANS>b. 

<END>

[H2] Operations of Algebraic Expressions

Algebraic expressions are made up of <term>terms</term>. Examples of terms are 7, y, 5*x*2, 9*a*, and *b*5. The constant that multiplies the variable is called the <term>coefficient</term>. Think of the coefficient as the number in front of the variable. Consider the algebraic expressions 5*x*2 that has a coefficient of 5, and 9*a* has a coefficient of 9. If there is no number listed in front of the variable, then the coefficient is 1 since .

Some terms share common traits. When two terms are constants or have the same variable and exponent, we say they are **like terms**. If there are like terms in an expression, you can simplify the expression by combining the like terms. We add the coefficients and keep the same variable.

<DEF>[DEFINITION]

A **term** is a constant or the product of a constant and one or more variables.

The **coefficient** of a term is the constant that multiplies the variable in a term. Like terms are terms that are either constants or have the same variables raised to the same powers.

</BOX>

<example>EXAMPLE 8—<title>Adding Algebraic Expressions</title>

<EXERCISE>

<PROBLEM>

Add 

**<SOLUTION>**

Step 1: We can add the terms in any order and get the same result (think:  ) so we can drop the parentheses:



Step 2: Next we group like terms together:



Step 3: Then we combine the like terms:



<END>

<BOX>[YOUR TURN] 8

<EXERCISE>

<PROBLEM>

Add 

</SOLUTION>

</EXERCISE>

**<SOLUTION>**

<ANS>5*x*2 – 3*x* – 7

<END>

<example>EXAMPLE 9—<title>Subtracting Algebraic Expressions</title>

<EXERCISE>

<PROBLEM>

Subtract .

**<SOLUTION>**

Step 1: Distribute the negative inside the parentheses (think:  , which is the correct answer). You cannot just drop the parentheses (for example,  , which is not correct as we have already verified the answer is 3):



Step 2: Next we group like terms together:



Step 3: Then we combine the like terms:



<END>

<BOX>[YOUR TURN] 9

<EXERCISE>

<PROBLEM>

Subtract .

**<SOLUTION>**



<END>

**[DEFINITION]**

Before looking at multiplying algebraic expressions we look at the <term>distributive property</term>. For example,  which can also be solved as  . If we use a variable, then  . We can extended this example to  which can also be solved as  . If we use variables, then  .

<example>EXAMPLE 10—<title>Multiplying Algebraic Expressions</title>

<EXERCISE>

<PROBLEM>

Multiply .

**<SOLUTION>**

Step 1: Use the distributive property:



Step 2: Next we multiply:



Step 3: Then we combine the like terms:



<END>

<BOX>[YOUR TURN] 10

<EXERCISE>

<PROBLEM>

Multiply .

**<SOLUTION>**



<END>

<LIGHT> *You may have heard the term “FOIL” which stands for: First, Outer, Inner, Last. FOIL essentially describes a way to use the distributive property if you multiply a two-term expression by another two-term expression, but FOIL only works in that specific situation. For example, suppose you have a two-term expression multiplied by a three-term expression, such as* . *What terms qualify as inner terms and what terms qualify as outer terms? In this particular situation, FOIL cannot possibly work; the multiplication of *  *should yield six terms, where FOIL is designed to only give you four! The distributive property works regardless of how many terms there are. FOIL can be misleading and applied inappropriately if not fully understood!*

<END>

<example>EXAMPLE 11—<title>Dividing Algebraic Expressions</title>

<EXERCISE>

<PROBLEM>

Divide .

**<SOLUTION>**

We have to divide EACH term by 4*x*:



<END>

<BOX>[YOUR TURN] 11

<EXERCISE>

<PROBLEM>

Divide 

**<SOLUTION>**

<ANS>

<END>

<LIGHT> *Be careful how you divide! Sometimes students incorrectly divide only one term on top by the bottom term. For example:*  *might turn into if done incorrectly. When we divide expressions, EACH term is divided by the divisor. So,*  *If you forget, it is always a good idea to check these rules by creating an example using numerical expressions. For example,*  *Dividing each term on top by 3 would yield* *which is the correct answer. However, if you just divided the 9 on top by the 3 on the bottom, getting* *this does not result in the correct answer.*

<BOX>[PEOPLE IN MATHEMATICS]

<T>Al-Khwarizmi</title>

<FIG/>

CS\_Photo\_05\_01\_002: Image of Al-Khwarizmi.

Abu Ja’far Muhammad ibn Musa Al-Khwarizmi was born around 780 AD, probably in or around the region of Khwarizm, which is now part of modern-day Uzbekistan. For most of his adult life, he worked as a scholar at the House of Wisdom in Baghdad, Iraq. He wrote many mathematical works during his life, but is probably most famous for his book *Al-kitab al-muhtasar fi hisab al-jabr w’al’muqabalah*, which translates to *The Condensed Book on the Calculation of al-Jabr* (*completion*) *and al’muqabalah* (*balancing*). The word *al-jabr* would eventually become the word we use to describe the topic that he was writing about in this book: *algebra*. From another book of his, with the Latin title *Algoritmi de numero Indorum* (*Al-Khwarizmi on the Hindu Art of Reckoning* ), our word *algorithm* is derived. In addition to writing on mathematics, Al-Khwarizmi wrote works on astronomy, geography, the sundial, and the calendar. <END>

<BOX>VIDEO: Why We Teach Algebra

In 2012, Andrew Hacker wrote an opinion piece in the New York Times Magazine suggesting that teaching algebra in high school was a waste of time. [http://tinyurl.com/9yedr2j](https://www.youtube.com/redirect?event=video_description&redir_token=QUFFLUhqbFFPS0IzVUhZU2NnaklhM1BUSXYxeUdSZUJ5Z3xBQ3Jtc0tuSHcwSTQ4WDRJWURWaXU0X0QzQnEybEEwcS01dzlaQmlRZTU1TzBrZ2dEc3lWcng3dzk1ZGhMcWxISl9MOGFTMDRFaElGNWNYajBSNWlRbk9ZWUxucUx3SXVtUVJNYTEzU1p3N1pERVMzTFFCN0VLNA&q=http%3A%2F%2Ftinyurl.com%2F9yedr2j). Keith Devlin, a British Mathematician, was asked to comment on Hacker's article by his students in his Stanford University’s Continuing Studies course "Mathematics: Making the Invisible Visible" on iTunes University <http://tinyurl.com/b7mnu46>. Dr. Devlin concludes that Hacker was displaying his ignorance of what algebra is.

<<https://www.youtube.com/watch?v=ehT9jgF59rU>>

<H1>[CHECK YOUR UNDERSTANDING]

<EXERCISE>

<PROBLEM>

1. Juliette is two inches taller than her friend Vivian. Which of the following algebraic equations(s) represent their height? Use *J* for Juliette’s height and *V* for Vivian’s height.

A. 

B. 

C. 

D. 

Answer: A and B

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. Which of the following represents an algebraic expression?

A. 

B. 

C. 

D. 

Answer: B and C

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. Which of the following expressions equal 10*x*?

A. 

B. 

C. 

D. 

Answer: D

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. Using the expression , when a certain number is put in for *x*, the result is 50. What is the value of *x*?

A. 

B. 

C. 2

D. 3

Answer: B

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

5. Which of the following expressions equals (*x* – *y*)2? Hint: use the distributive property.

A. 

B. 

C. 

D. 

Answer: D

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

6. Given the expression , the distributive property allows it to be rewritten as:

A. 

B. 

C. 

D. 

Answer: A

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

7. Given the two algebraic expressions  and , the solution is . What mathematical operation was performed on the two algebraic expressions?

A. Addition

B. Subtraction

C. Multiplication

D. Division

Answer: C

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

8. Given the two algebraic expressions  and (6*x*); the solution is ; what mathematical operation was performed on the two algebraic expressions?

A. Addition

B. Subtraction

C. Multiplication

D. Division

Answer: D

<H1>EXERCISES

For the following exercises, translate from algebra to words.

<EXERCISE>

<PROBLEM>

1. 50 – 15

Answer: 50 minus 15 OR the difference of 50 and 15

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. (10)(*x*)

Answer: 10 times *x* OR the product of 10 and *x*

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. 2*a* – *b*

Answer: The difference of 2 times *a* and *b* OR 2 times *a* minus *b*

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. 100 ÷ 33

Answer: The quotient of 100 and 33 OR 100 divided by 33

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

5. 

Answer: The sum of 3 times *x* and 5 OR 3 times *x* plus 5

</SOLUTION>

</EXERCISE>

For the following exercises, translate from words to algebra.

<EXERCISE>

<PROBLEM>

6. 15 divided by 3.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

7. The sum of 13 and 13.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

8. 120 minus 12.

Answer: 120 – 12

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

9. The product of 5 and 4.

Answer: (5)(4)

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

10. The sum of double *x* and 5.

Answer: 

</SOLUTION>

</EXERCISE>

For the following exercises, translate from an English phrase to an expression.

<EXERCISE>

<PROBLEM>

11. Three times *y* minus 7.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

12. *a* divided by 2; then add 4.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

13. *x* squared minus 3.

Answer: *x*2 – 3

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

14. A rental car company charges $0.15 per mile *m*, plus a $40 flat fee for the rental.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

15. A parking garage in New York City charges $20 for the first hour, then $5 per hour *h*.

Answer: 

</SOLUTION>

</EXERCISE>

For the following exercises, use parentheses to make the following statements true.

<EXERCISE>

<PROBLEM>

16. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

17. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

18. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

19. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

20. 

Answer: 

</SOLUTION>

</EXERCISE>

For the following exercises, evaluate and simplify the expression.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

21. 

Answer: 81

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

22. 

Answer: 11

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

23. 

Answer: 14

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

24. 

Answer: 26

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

25. 

Answer: 4

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

26. 

Answer: 9

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

27. 

Answer: 203

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

28. Yasenia is three years older than Xavier. How old is Yasenia when Xavier is 18 years old?

Answer: 21

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

29. A rental car company charges $0.15 per mile *m*, plus a $40 flat fee for the rental. What is the cost of the car rental if one drives 100 miles?

Answer: $55

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

30. A parking garage in New York City charges $20 for the first hour, then $5 per hour *h*. What is the cost of parking for 10 hours?

Answer: $65

</SOLUTION>

</EXERCISE>

For the following exercises, perform the indicated operation for the expressions.

<EXERCISE>

<PROBLEM>

31. Add 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

32. Add 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

33. Subtract 

Answer: 5*x* – 11

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

34. Subtract 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

35. Multiply 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

36. Multiply 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

37. Multiply 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

38. Multiply 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

39. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

40. 

Answer: 

</SOLUTION>

</EXERCISE>

\*\*\*\*MODULE 1 END\*\*\*\*

<D>Module 2 — Linear Equations in One Variable with Applications

<FIG/>

CS\_Photo\_05\_02\_001: A photo related to a gym membership.

**After completing this module, you should be able to:**

<NL>

* Solve linear equations in one variable using properties of equations.
* Construct a linear equation to solve applications.
* Determine equations with no solution or infinitely many solutions.

</NL>

Solve a formula for a given variable. LO 5.2.4this module, we will study linear equations in one variable. There are several real-world scenarios that can be represented by linear equations: taxi rentals with a flat fee and a rate per mile; cell phone bills that charge a monthly fee plus a separate rate per text; gym memberships with a monthly fee plus a rate per class taken; etc. For example, if you join your local gym at $10 per month and pay $5 per class, how many classes can you take if your gym budget is $75 per month?

[H2] Linear Equations and Applications

Solving any equation is like discovering the answer to a puzzle. The purpose of solving an equation is to find the value or values of the variable that makes the equation a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle! There are many types of equations that we will learn to solve. In this module we will focus on a <term>linear equation</term>.

<DEF>[DEFINITION]

A **linear equation** is an equation in one variable that can be written as

 where *a* and *b* are real numbers and *a* ≠ 0, *a* is the coefficient of *x* and *b* is the constant.

<END>

To solve a linear equation, it is a good idea to have an overall strategy that can be used to solve any linear equation. In the Example 12, we will give the steps of a general strategy for solving any linear equation. Simplifying each side of the equation as much as possible first makes the rest of the steps easier.

<example>EXAMPLE 12—<title>Solving a Linear Equation Using a General Strategy</title>

<EXERCISE>

<PROBLEM>

Solve: 

**<SOLUTION>**

|  |  |  |
| --- | --- | --- |
| Step 1. Simplify each side of the equation as much as possible. | Use the Distributive Property.  Notice that each side of the equation is now simplified as much as possible. |  |
| Step 2. Collect all variable terms on one side of the equation. | Nothing to do – all *n*-terms are on the left side. |  |
| Step 3. Collect constant terms on the other side of the equation. | To get constants only on the right, add 29 to each side.  Simplify. |  |
| Step 4. Make the coefficient of the variable term equal to 1. | Divide each side by 7.  Simplify. |  |
| Step 5. Check the solution. | Let  Subtract. | Check: |

<END>

<BOX> 13[YOUR TURN]2

<EXERCISE>

<PROBLEM>

Solve 

**<SOLUTION>**

<ANS>

<END>

<example>EXAMPLE 13 – <title> Solving a Linear Equation Using Properties of Equations</title>

<EXERCISE>

<PROBLEM>

Solve 

**<SOLUTION>**

Step 1: Simplify each side: 

Step 2: Collect all variables on one side: 

Step 3: Collect constant terms on one side: 

Step 4: Make the coefficient of the variable 1: Already done!

Step 5: Check: 

<END>

<BOX>[YOUR TURN] 14

<EXERCISE>

<PROBLEM>

Solve 

**Answer**

<ANS>

<END>

<example>EXAMPLE 14 – <title>Constructing a Linear Equation to Solve an Application</title>

<EXERCISE>

<PROBLEM>

The Beaudrie family has two cats, Basil and Max. Together, they weigh 23 pounds. Basil weighs 16 pounds. How much does Max weigh?

**<SOLUTION>**

Let *b* = Basil’s weight and *m* = Max’s weight.



We also know that Basil weighs 16 pounds so:

Step 1 and 2: 

Since both sides are simplified, the variable is on one side of the equation, we start in step 3 and collect the constants on one side:

Step 3: 



Step 4 is already done so we go to step 5:

 ✔

Basil weighs 16 pounds and Max weighs 7 pounds.

<END>

<BOX>[YOUR TURN] 15

<EXERCISE>

<PROBLEM>

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

**Answer**

<ANS>Henry has 42 books.

<END>

<example>EXAMPLE 15 –<title> Constructing a Linear Equation to Solve an Application</title>

<EXERCISE>

<PROBLEM>

If you join your local gym at $10 per month and pay $5 per class, how many classes can you take if your gym budget is $75 per month?

**<SOLUTION>**

If we let *x* = number of classes then the equation  would represent what you pay per month if each class is $5 and there’s a $10 monthly fee per class. $10 is your constant.If you want to know how many classes you can take if you have a $75 monthly gym budget, then you set the equation equal to 75. Then you solve  for *x*.

Steps 1 and 2: 

Step 3: 



Step 4: 



Step 5: 



 ✔

The solution is 13 classes. You can take 13 classes on a $75 monthly gym budget.

<END>

<BOX>[YOUR TURN] 16

<EXERCISE>

<PROBLEM>

On June 7th, 2021 the national average price for regular gasoline was $3.053 per gallon. If Aiko fills up his car with 16 gallons, how much is the total cost? Round to the nearest cent.

**Answer**

<ANS>Total cost is $48.85

<END>

<example>EXAMPLE 16 - <title>Constructing an Application from a Linear Equation</title>

<EXERCISE>

<PROBLEM>

Write an application that can be used to solve. Then solve your application.

**<SOLUTION>**

Answers will vary. Let’s say we want to rent a snowblower for a huge winter storm coming up. If we let *x* = the number of days you rent a snowblower, then the equation would represent what you pay if, for each day it costs $50 to rent the snowblower and there is a $35 flat rental fee. $35 is your constant. If you want to know how many days you can rent a snowblower for $185, then you set the equation equal to 185. Then you solve  for *x*.

Step 1 and 2: 

Step 3: 



Step 4:  

Step 5: 





The equation is and the solution is 3 days. You can rent a snowblower for 3 days on a $185 budget.

<END>

<BOX>[YOUR TURN] 17

<EXERCISE>

<PROBLEM>

Write an application that can be used to solve . Then solve your application.

<ANS>Answers will vary. For example: You can rent a paddleboard for $25 per hour with a water shoe purchase of $75. If you spent $200, how many hours did you rent the paddle board for?

You rent the paddle board for 5 hours.

<END>

<example>EXAMPLE 17 – <title>Solving A Linear Equation with No Solution</title>

<EXERCISE>

<PROBLEM>

Solve .

**<SOLUTION>**

Step 1: Simplify each side:  

Step 2: Collect all variables to one side: 



The variable *x* disappeared! When this happens, you need to examine what remains. In this particular case, we have, which is not a true statement. When you have a false statement, then you know the equation has no solution; there does not exist a value for *x* that can be put into the equation that will make it true.

<END>

<BOX>[YOUR TURN] 18

<EXERCISE>

<PROBLEM>

Solve .

**Answer**

<ANS> , which is false; therefore this is a false statement, and the equation has no solution.

<END>

<example>EXAMPLE 18 – <title>Solving a Linear Equation with Infinitely Many Solutions </title>

<EXERCISE>

<PROBLEM>

Solve 

**<SOLUTION>**



As with the previous example, the variable disappeared. In this case, however, we have a true statement . When this occurs we say there are infinitely many solutions; any value for *x* will make this statement true.

<END>

<BOX>[YOUR TURN] 19

<EXERCISE>

<PROBLEM>

Solve 

**Answer**

<ANS> , which is true; therefore this is a true statement, and there are infinitely many solutions.

<END>

<example>EXAMPLE 19 – <title>Solving for a Given Variable with Distance, Rate, and Time</title>

<EXERCISE>

<PROBLEM>

Solve the formula  for *t*. This is the distance formula where *d* = distance, *r* = rate, and *t* = time.

**<SOLUTION>**

Divide both sides by *r:* 

**

<BOX>[YOUR TURN] 20

<EXERCISE>

<PROBLEM>

Solve the formula *I* = *Prt* for *t*. This formula is used to calculate simple interest *I*, for a principal *P*, invested at a rate *r*, for *t* years.

**Answer**

<ANS>

<END>

<example>EXAMPLE 20 – <title>Solving for a Given Variable in the Area Formula for a Triangle</title>

<EXERCISE>

<PROBLEM>

Solve the formula  *bh* for *h*. This is the area formula of a triangle where *A* = area, *b* = base, and *h* = height.

**<SOLUTION>**

Multiply both sides by 2*:* 

<END>

<BOX>[YOUR TURN] 21

<EXERCISE>

<PROBLEM>

Solve the formula for *h*. This formula is used to calculate the volume *V* of a right circular cone with radius *r* and height *h*.

**Answer**

<ANS> 

<END>

<H1>[CHECK YOUR UNDERSTANDING]

<EXERCISE>

<PROBLEM>

1. Is the solution strategy used in solving the linear equation correct? If it is correct, show the final step (check the solution). If it is not correct, explain why.













Answer: it is a correct solution strategy.

Let 





 ✔

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. Is the solution strategy used in solving the linear equation correct? If it is correct, show the final step (check the solution). If it is not correct, explain why.















Answer: it is a correct solution strategy.

Let 









 ✔

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. Is the solution strategy used in solving the linear equation correct? If it is correct, show the final step (check the solution). If it is not correct, explain why.















Answer. This is not a correct solution strategy. The negative sign is not distributed correctly in the second line of the solution strategy. The second line should read .

</SOLUTION>

</EXERCISE>

4. The Nice Cab Company charges a flat rate of $3.00 for each fare, plus $1.70 per mile.

<EXERCISE>

<PROBLEM>

a. Using the variable *x* for number of miles, write the equation which would allow you to find the total fare (T).

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

b. It is 22 miles from the airport to your hotel. What would be your total fare?

Answer: $42.10

</SOLUTION>

</EXERCISE>

5. A competing taxi service, the Enjoyable Cab Company, charges a flat rate of $5.00 for each fare, plus $1.60 per mile.

<EXERCISE>

<PROBLEM>

a. Using the variable *y* for number of miles, write the equation that would allow you to find the total fare (T).

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

b. Using the same 22 mile trip from the airport to the hotel, how much would the total fare be for using the Enjoyable Cab Company?

Answer: $40.20

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

c. Based on the cost of each cab ride, which cab company should you use for the trip from the airport to the hotel? Why?

Answer: The Enjoyable Cab Company, because the cab fare will be $ 0.2 less than what it would cost to take a taxi from the Nice Cab Company.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

6. After solving the linear equation, Nancy says there is no solution. Luis believes there are infinitely many solutions. Who is right?

Answer: Luis is; there are infinitely many solutions. If this is solved using the general strategy, it simplifies to . This is a true statement, so there are infinitely many solutions.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

7. The conversion formula between the Fahrenheit temperature scale and the Celsius temperature scale is given by this formula: , where *C* is the temperature in degrees Celsius and *F* is the temperature in degrees Fahrenheit. The correct formula when solved for *F* is:

A. 

B. 

C. 

D. 

Answer: D

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

8. To find a temperature on the Kelvin temperature scale, add 273 degrees to the temperature in Celsius. The formula that illustrates this is:

A. 

B. 

C. 

D. 

Answer: B, although D is an equivalent formula.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

9. Using the information from problems 7 and 8, write a conversion formula to find degrees Kelvin when given degrees Fahrenheit. That formula is:

A. 

B. 

C. 

D. 

Answer: A

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

10. There is a fourth temperature scale, although it is not used much today. The Rankin temperature scale varies from the Fahrenheit scale by about 460 degrees. So given a temperature in Fahrenheit, add 460 degrees to get the temperature in Rankin. Which formula below represents a formula to find degrees Rankin when given degrees Celsius?

A. 

B. 

C. 

D. 

Answer: B

</SOLUTION>

</EXERCISE>

<H1>EXERCISES

For the following exercises, solve the linear equations using a general strategy.

<EXERCISE>

<PROBLEM>

1. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

5. 

Answer: 

</SOLUTION>

</EXERCISE>

For the following exercises, solve the linear equations using properties of equations.

<EXERCISE>

<PROBLEM>

6. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

7. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

8. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

9. 

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

10. 

Answer: 

</SOLUTION>

</EXERCISE>

For the following exercises, construct a linear equation to solve an application.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

11. It costs $0.55 to mail one first class letter. Construct a linear equation and solve how much it costs to mail 13 letters.

Answer: 0.55*x* and $7.15 to mail 13 letters.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

12. Normal yearly snowfall at the local ski resort is 12 inches more than twice the amount it received last season. The normal yearly snowfall is 62 inches. Construct a linear equation and solve what the snowfall was last season.

Answer:  and  or the snowfall last season was 25 inches.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

13. Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was three more than twice the number of notebooks. He bought seven textbooks. Construct a linear equation and solve how many notebooks he bought.

Answer:  and  or he bought 2 notebooks.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

14. Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. Construct a linear equation and solve how many crossword puzzles he did.

Answer:  and  or he did 7 crossword puzzles.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

15. Laurie has $46,000 invested in stocks. The amount invested in stocks is $8,000 less than three times the amount invested in bonds. Construct a linear equation and solve how much Laurie invested in bonds.

Answer:  and  or she invested $18,000 in bonds.

</SOLUTION>

</EXERCISE>

For the following exercises, construct an application from a linear equation.

<EXERCISE>

<PROBLEM>

16. 

Answer: Answers will vary. For example: you want to rent a red corvette and the rental agency charges $1000 per day plus a $2500 non-refundable deposit. How many days can you rent the red corvette if you have $16,500?

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

17. 0.36*t* for .

Answer: Answers will vary. For example: The cost to mail a postcard is $0.36. How much would Nellie have to pay to mail 333 postcards?

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

18. .

Answer: Answers will vary. For example: An Airbnb rental charges $150 per night and a $120 cleaning fee. How many nights can you stay for $570?

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

19.  for.

Answer: Answers will vary. For example: At a local farmer’s market each cup of lavender lemonade is $4 plus a $2 deposit per cup. How much will you pay for 5 cups?

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

20. .

Answer: Answers will vary. For example: For each act of kindness a teacher awards two stickers. Each student gets 10 bonus starts at the beginning of the month. If Isabelle has 24 stickers at the end of the month, how many acts of kindness did she do?

</SOLUTION>

</EXERCISE>

For the following exercises, state whether each equation has exactly one solution, no solution, or infinitely many solutions.

<EXERCISE>

<PROBLEM>

21. 

Answer: infinitely many solutions

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

22. 

Answer: exactly one solution; 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

23. 

Answer: exactly one solution; 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

24. 

Answer: no solution

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

25. 

Answer: no solution

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

26. 

Answer: infinitely many solutions

</SOLUTION>

</EXERCISE>

For the following exercises, solve the given formula for the specified variable.

<EXERCISE>

<PROBLEM>

27. Solve the formula  for *d*.

Answer:

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

28. Solve the formula  for *L*.

Answer:

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

29. Solve the formula  for *b*.

Answer:

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

30. Solve the formula  for *d1*.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

31. Solve the formula  for *b1*.

Answer:

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

32. Solve the formula  for *a*.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

33. Solve  for *a*.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

34. Solve the formula  for *p*.

Answer: 

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

35. Solve the formula:  for *L*.

Answer: 

</SOLUTION>

</EXERCISE>

\*\*\*\*MODULE 2 END\*\*\*\*

<D>Module 3 – Linear Inequalities in One Variable with Applications

**After completing this section, you should be able to:**

<NL>

* Graph inequalities in one variable. LO 5.3.1
* Solve linear inequalities in one variable. LO 5.3.2
* Construct a linear inequality to solve applications. LO 5.3.3

</NL>

<example>EXAMPLE 21 - <title>Graphing an Inequality</title>

<EXERCISE>

<PROBLEM>

Graph the inequality  and write the solution in interval notation.

**<SOLUTION>**

Shade to the right of 3to show all the numbers greater than -3*,* and put a bracket at −3 to show that the numbers are greater than or equal to -3*:*

<FIG/>

Write in interval notation starting at –3 with a bracket to show that –3 is included in the solution and then infinity because the solution includes all the numbers greater than or equal to -3:.

<END>

<BOX>[YOUR TURN] 22

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

Graph the inequality *x* < 2.5 and write the solution in interval notation.

**Answer**

.

*.*

<END>

<example>Example 23 – Graphing a Compound Inequality

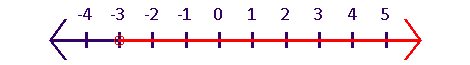
<EXERCISE>

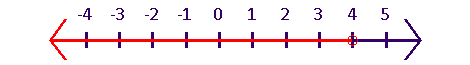
<PROBLEM>

Graph the inequality and *x* < 4 and write the solution in interval notation.

**<SOLUTION>**

.

Graph : 

Graph *x* < 4: 

Then graph both on the same number line and think of where the solutions are to BOTH inequalities. This will be where BOTH are shaded:

.

Then write your solution in interval notation:

.

<END>

<BOX>[YOUR TURN] 23

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

Graph the inequality x ≥ 0 and *x* 2.5 and write the solution in interval notation.

**Answer**

.

Graph:

<FIG/>

Interval notation:.

<END>

<example>EXAMPLE 23 - <title> Solving a Linear Inequality Using One Operation</title>

<EXERCISE>

<PROBLEM>

Solve 9*y* < 54, graph the solution on the number line, and write the solution in interval notation.

**<SOLUTION>**

9*y* < 54



*y* < 6

<FIG/>

.

<END>

<BOX>[YOUR TURN] 24

<EXERCISE>

<PROBLEM>

Solve -13*m* . Graph the solution on the number line, and write the solution in interval notation.

Answer

.

<FIG/>

.

<END>

<example>EXAMPLE 24 - <title>Solving a Linear Inequality Using Multiple Operations</title>

<EXERCISE>

<PROBLEM>

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

**<SOLUTION>**



 > 



<FIG/>



<END>

<BOX>[YOUR TURN] 25

<EXERCISE>

<PROBLEM>

Solve the inequality, graph the solution on the number line and write the solution in interval notation.

Answer

*p* ≥ 2

<FIG/>

[2, . )

<END>

<example>EXAMPLE 25 – <title>Constructing a Linear Inequality to Solve an Application with Tablet Computers</title>

<EXERCISE>

<PROBLEM>

Dawn won a mini-grant of $4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost $254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

**<SOLUTION>**

Let *t* = the number of tablets.

*t* times $254.12 has to be less than $4000, so 254.12*t* . 4000.

Solve for *t*: 

*t* 15.74

Dawn can buy 15 tablets and stay under $4000.

<END>

<BOX>[YOUR TURN] 26

<EXERCISE>

<PROBLEM>

Taleisha’s phone plan costs her $28.80 per month us $0.20 per text message. How many text messages can she send/receive and keep her monthly phone bill no more than $50?

**Answer**

Taleisha can send/receive 106 or less text messages and keep her monthly bill no more than $50.

<END>

<example>EXAMPLE 26 – <title>Constructing a Linear Inequality to Solve a Tuition Application</title>

<EXERCISE>

<PROBLEM>

The local community college charges $113 per credit hour. Your budget is $1500 for tuition this fall semester. What are the number of credit hours that you could take this fall?

**<SOLUTION>**

t *c* = the number of credit hours you could take.

*c* times $113 has to be less than $1500, so 113*c* . 1500.

Solve for *c*: 

*c* 13.27

You can take up to 13 credits and stay under $1500.

<END>

<BOX>[YOUR TURN] 27

You are awarded a $500 scholarship! So in addition to the $1500 you have saved for tuition, you now have an additional $500 to spend on credit hours for fall semester. Now, how many credit hours could you take this fall semester? Assume the cost is still $113 per credit hour.

**Answer**

You could take up to 17 credit hours and stay under $2000.

<END>

<example>EXAMPLE 27 – <title>Constructing a Linear Inequality to Solve an Application with Travel Costs</title>

<EXERCISE>

<PROBLEM>

Brenda’s best friend is having a destination wedding and the event will last three days and three nights. Brenda has $500 in savings and can earn $15 an hour babysitting. She expects to pay $350 airfare, $375 for food and entertainment and $60 a night for her share of a hotel room. How many hours must she babysit to have enough money to pay for the trip?

**<SOLUTION>**

Let *b* = number of babysitting hours.

*b* times $15 plus $500 has to be more than, so .

Solve for *b*:









Brenda must babysit at least 27 hours. do to

<END>

<BOX>[YOUR TURN] 28

<EXERCISE>

<PROBLEM>

Malik is planning a six-day summer vacation trip. He has $840 in savings, and he earns $45 per hour for tutoring. The trip will cost him $525 for airfare, $780 for food and sightseeing, and $95 per night for the hotel. How many hours must he tutor to have enough money to pay for the trip?

**Answer**

Malik must tutor at least 23 hours.

<END>

<H1>[CHECK YOUR UNDERSTANDING]

<EXERCISE>

<PROBLEM>

1. Choose the correct interval notation for the following graph:

<FIG/>

A. 

B. 

C. 

D. 

E. 

Answer: E

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. Choose the correct interval notation for the following graph:

<FIG/>

A. 

B. 

C. 

D. 

E. 

Answer: B

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

3. Choose the correct interval notation for the following graph:

<FIG/>

A. 

B. 

C. 

D. 

E. 

Answer: C

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

4. Choose the correct interval notation for the following graph:

<FIG/>

A. 

B. 

C.

D.

E. 

Answer: A

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

5.  is the solution for which inequality?

A. 4*x* > 0

B. 4*x* < 0

C. 6*x* < 24

D. 6*x* > 24

E. 6*x* > 24

Answer: E

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

6. is the solution for which inequality?

A. 

B. 

C. 

D.

E. 

Answer: D

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

7. is the solution for which inequality?

A. 

B.

C.

D.

E 

Answer: C

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

8. is the solution for which inequality?

A. 

B.

C.

D. 

E

Answer: C

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

9. Choose the equation that best models this situation: Renaldo is hauling boxes of lawn chairs. Each box is the same size, 8 cubic feet. Renaldo’s truck has a capacity of 764 cubic feet. How many boxes of lawn chairs can Renaldo put in his truck?

A. 8 < *764x*

B. *8x* < 764

C. 8 > 764*x*

D. 8*x* > 764

E. None of these

Answer: B

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

10. Choose the equation that best models this situation: Bernadette babysits the neighbor’s kids, making on average $50 a night. How many nights will she have to babysit in order to earn enough money to buy a used car, whose cost is $8120?

A. 50 < 8120*x*

B. 50*x* < 8120

C. 50 > 8120*x*

D.50*x* > 8120

E. None of these

Answer: D

</SOLUTION>

</EXERCISE>

<END>

<H1>EXERCISES

For the following exercises, graph the inequalities on a number line and write the interval notation for each.

<EXERCISE>

<PROBLEM>

1. *x* > 3

Answer:

The solution for x is greater than 3 on a number line has a left bracket 3 with shading to the right. The solution in interval notation is 3 to infinity within parentheses.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

2. 

Answer:

The solution for x is less than or equal to negative 0.5 on a number line has a right bracket at negative 0.5 with shading to the left. The solution in interval notation is negative infinity to negative 0.5 within a parenthesis and a bracket.

</SOLUTION>

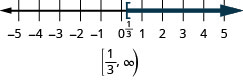
</EXERCISE>

<EXERCISE>

<PROBLEM>

3. *x* ≥

Answer:



</SOLUTION>

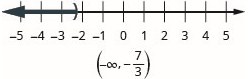
</EXERCISE>

<EXERCISE>

<PROBLEM>

4. *x* <

Answer:



</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

5. 

Answer:



</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

6. 

Answer:



</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

7. 

Answer:



</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

8. 

Answer:



</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

9. 

Answer:



</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

10. 

Answer:



</SOLUTION>

</EXERCISE>

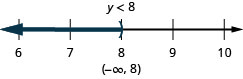
For the following exercises, solve the inequality, graph the solution on the number line, and write the solution interval notation.

<EXERCISE>

<PROBLEM>

11. 6*y* < 48

Answer:



</SOLUTION>

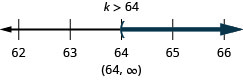
</EXERCISE>

<EXERCISE>

<PROBLEM>

12. 40 <

Answer:



</SOLUTION>

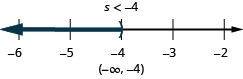
</EXERCISE>

<EXERCISE>

<PROBLEM>

13. 

Answer:



</SOLUTION>

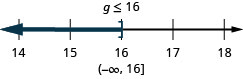
</EXERCISE>

<EXERCISE>

<PROBLEM>

14. <MISSING>

Answer:



</SOLUTION>

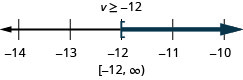
</EXERCISE>

<EXERCISE>

<PROBLEM>

15. 

Answer:



</SOLUTION>

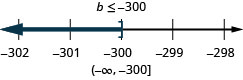
</EXERCISE>

<EXERCISE>

<PROBLEM>

16. <MISSING>

Answer:



</SOLUTION>

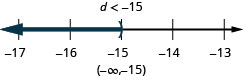
</EXERCISE>

<EXERCISE>

<PROBLEM>

17. 

Answer:



</SOLUTION>

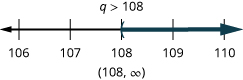
</EXERCISE>

<EXERCISE>

<PROBLEM>

18. 

Answer:



</SOLUTION>

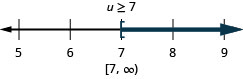
</EXERCISE>

<EXERCISE>

<PROBLEM>

19. 

Answer:



</SOLUTION>

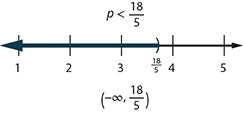
</EXERCISE>

<EXERCISE>

<PROBLEM>

20. 

Answer:



</SOLUTION>

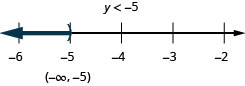
</EXERCISE>

<EXERCISE>

<PROBLEM>

21. 

Answer:



</SOLUTION>

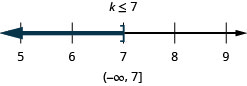
</EXERCISE>

<EXERCISE>

<PROBLEM>

22. 

Answer:



</SOLUTION>

</EXERCISE>

For the following exercises, construct a linear inequality to solve the application.

<EXERCISE>

<PROBLEM>

23. The elevator in Yehire’s apartment building has a sign that says the maximum weight is 2100 pounds. If the average weight of one person is 150 pounds, how many people can safely ride the elevator?

Answer: A maximum of 14 people can safely ride the elevator.

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

24. Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Answer: five drinks

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

25. Ryan charges his neighbors $17.50 to wash their car. How many cars must he wash next summer if his goal is to earn at least $1,500?

Answer: 86 cars

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

26. Kimuyen needs to earn $4,150 per month in order to pay all her expenses. Her job pays her $3,475 per month plus 4% of her total sales. What is the minimum Kimuyen’s total sales must be in order for her to pay all her expenses?

Answer: $16,875

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

27. Nataly is considering two job offers. The first job would pay her $83,000 per year. The second would pay her $66,500 plus 15% of her total sales. What would her total sales need to be for her salary on the second offer be higher than the first?

Answer: $110,000

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

28. Kiyoshi’s phone plan costs $17.50 per month plus $0.15 per text message. What is the maximum number of text messages Kiyoshi can use so the phone bill is no more than $56.60?

Answer: 260 messages

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

29. Kellen wants to rent a banquet room in a restaurant for her cousin’s baby shower. The restaurant charges $350 for the banquet room plus $32.50 per person for lunch. How many people can Kellen have at the shower if she wants the maximum cost to be $1,500?

Answer: 35 people

</SOLUTION>

</EXERCISE>

<EXERCISE>

<PROBLEM>

30. Noe installs and configures software on home computers. He charges $125 per job. His monthly expenses are $1,600. How many jobs must he work in order to make a profit of at least $2,400?

Answer: 32 jobs

</SOLUTION>

</EXERCISE>