Contact Tracing Coronavirus COVID-19
- Calibration Method and Proximity Accuracy -

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Abstract

Since the appearance of the Corona COVID-19 pandemic, there has been a controversy in the media about the use of smart phones to contain the pandemic. This essentially relates to two thematic areas: firstly, data protection and, secondly, the accuracy of proximity. This work concentrates on the question of accuracy.

The use of Bluetooth Low Energy Technology (BLE), which has been available in today's smart phones for many years, has proven to be the only way to determine the proximity to other smart phones. The proximity can be derived from the receiving power. Other technologies, such as location or the determination of movement profiles via cellular base stations, WLAN access points or Global Navigation Satellite Systems (GPS, Galileo, etc.) are far too inaccurate to identify possible infection contacts.

This work examines one of the problems that arise in determining the proximity. There are basically two types. Smart phones are mass-produced products that are structured very differently, what results in systematic proximity errors. This paper outlines a calibration method by which these errors can be totally removed. The other problem concerns the radio channel with multi path fading effects. If the systematic errors are not removed by the calibration method, then BLE will be completely unsuitable for the intended purpose of interrupting the infection chains.

1. Introduction

Since the appearance of the Corona COVID-19 pandemic, the use of smart phones has been the subject of controversy in the media daily. This relates essentially to two thematic areas: firstly, data protection and, secondly, the accuracy of proximity.

The data protection issue is not considered further here. Pan-European Privacy-Preserving Proximity Tracing [PEPP-PT], a European organization of about 130 scientists, has found a solution to the data protection problem, especially for the European Union. This work only concentrates on the question of the accuracy of proximity. The proximity of people with smart phones is used to determine whether infection is possible. If the proximity data (also known as contact data or transmission pairs) are collected, it is possible to determine the infection chains backwards very quickly if necessary. It is well known that the incubation period lasts several days, but that a person is already infectious, even though symptoms do not appear until several days later. If, for example, a person is tested positive, all possible infection contacts can be determined backwards, and corresponding infection chains can be interrupted.

To solve the proximity problem, the technology Bluetooth Low Energy (BLE) can be used. BLE has been standardized since 2009 [BLE 4.0] and implemented in almost all smart phones in the field. BLE from time to time allows each smart phone to send information as a beacon, which is received by smart phones located nearby (proximity). In principle, the proximity can be determined by the receiving power. If a certain value of the proximity is below a certain value and may exceed a certain amount of time, these possible infection contacts are stored.

The use of a pandemic app can help to significantly reduce the so-called reproduction rate. This was proved in [Frazer]. It is assumed that the determination of the infection contacts is precise enough. The accuracy of the proximity plays a crucial role in the use of such an app. If too many possible infection contacts were stored, too many people might be quarantined, which would reduce public acceptance in the long run. If too few infection contacts were detected, the reproduction rate could not be significantly reduced below one. This could call into question the effectiveness of the pandemic app. In practice, a BLE-based app was used for the first time in Singapore. However, the effectiveness of this app has not been proven to date, maybe because it is used by only a small part of the population, about 20% [Singapore].

This work examines the problems that arise in determining the proximity. There are two types. On the one hand, smart phones are mass-produced products that are structured in very different ways, resulting in systematic deviations in power measurement. They occur in both the transmitting and receiving paths of the smart phone. On the other hand, when transmitting the BLE signal from one smart phone to another via the radio channel, electromagnetic phenomena, such as attenuation, different propagation routes (fading), temporal changes in the locations of transmitter and receiver must also be considered. In modeling, the two problem areas can be separated. The systematic errors resulting from different smart phone types can be eliminated completely with the help of the calibration method presented in this work. If this calibration is not performed, the remaining errors for the proximity determination can be several meters. This is too inaccurate and could call into question the success of an app, as already mentioned above.

Finally, the possibilities and their limits for determining the proximity are analyzed in short: "What works and what is not possible".

2. Model of the transmission system

The reception power via a radio link can be described in the simplest form as follows [Meckelburg]:

$$P_R = H(R, \lambda) \cdot C_R \cdot C_T \cdot P_T$$

Here all parameters are scalar values and mean:

P_R receiving power
P_T transmitting power
R Distance of transmitter and receiver (proximity)
λ wavelength in free space
C_R characteristics of receiving path of a smart phone type
C_T characteristics of transmitting path of a smart phone type

$H(R,\lambda)$ describes the transmission function of the radio channel. In the simplest case, if the smart phones are in free space, the following applies (undisturbed free space transmission channel)

$$H_{FR}(R,\lambda) = \frac{\lambda^2}{(4\pi)^2}\frac{1}{R^2}$$

Since the smart phones change their roles as transmitters and receivers from time to time, the connection can be represented in matrix form. Here, index X and Y stand for smart phone X or smart phone Y, respectively.

$$\begin{pmatrix} P_{R,X} \\ P_{R,Y} \end{pmatrix} = \begin{pmatrix} 0 & H_{X,Y}(R,\lambda) \\ H_{Y,X}(R,\lambda) & 0 \end{pmatrix} \begin{pmatrix} C_{R,Y} \cdot C_{T,X} \cdot P_{T,X} \\ C_{R,X} \cdot C_{T,Y} \cdot P_{T,Y} \end{pmatrix}$$

Due to the reciprocity theorem applies, $H_{X,Y}(R,\lambda) = H_{Y,X}(R,\lambda) = H(R,\lambda)$ follows, i.e. for the radio channel it is irrelevant which smart phone transmits and which receives. This does not apply to the smart phones themselves! When smart phone X (SP_X) transmits and smart phone Y (SP_X) receives, the differences in the internal structures usually do not result in the same receiving power when swapping the transmitting and receiving direction.

For further consideration, the following representation can be chosen:

$$\begin{pmatrix} P_{R,X} \\ P_{R,Y} \end{pmatrix} = H(R,\lambda) \begin{pmatrix} C_{R,X} \cdot CP_{T,Y}) \\ C_{R,Y} \cdot CP_{T,X}) \end{pmatrix}$$

$CP_{T,X} = C_{T,X} \cdot P_{T,X}$ describes the transmitting path of SP_X and is characterized by $C_{T,X}$ and $P_{T,X}$. The receiving properties are described by $C_{R,X}$. The same statements apply, if the index X and index Y are swapped.

The smart phones are Consumer Product (mass-produced products), i.e. the product-specific characteristics in the combinations $C_{R,Y} \cdot CP_{T,X}$ and $C_{R,X} \cdot CP_{T,Y}$ can vary very much, so that the proximity determination becomes inaccurate correspondingly.

3. Receiving Power and Proximity

As already indicated, there are product-specific characteristic properties that can have a major influence on the determination of the proximity. They are systematically characteristic and, depending on the types that communicate with each other, may lead to deviations in the receiving power in the range of 5 - 10 dB or may be even greater, according to the author's experience. The reasons for this are the antenna design, the internal circuit technology, the adaptation of the antennas in transmitting and receiving path, the Bluetooth chip sets used, the measuring accuracy of the receiving power, the accuracy of the transmitting power, etc. Fortunately, these deterministic influences are identical for each **smart phone type** and can be eliminated totally. The method is presented in Chapter 4. In [Singapore, GTA], it has been reported that the transmitting power can differ between different smart phone types up to 30 dB. This applies only to the characteristic properties for the transmitting path $CP_{T,X}$. The receiving path $C_{R,X}$has not been considered in [Opentrace Community], but it can also lead to significant discrepancies between different smart phone types. This issue has already been mentioned in [Bluetooth Blog]. Finally, $C_{R,X} \cdot CP_{T,Y}$ or $C_{R,Y} \cdot CP_{T,X}$ are the relevant combinations of the transmitting and receiving paths, as already shown in the previous chapter. This chapter examines how the impact of the accuracy of receiving power affects the determination of proximity.

The statistical fluctuations within a smart phone type, such as certain manufacturing tolerances, accuracy of transmitting and receiving power, etc., can also be described with mean and standard deviation. These influences are to be low according to the current knowledge and can be considered in principle.

The formula $P_R = \frac{\lambda^2}{(4\pi)^2} \frac{1}{R^2} \cdot C_R \cdot C_T \cdot P_T$ for free space will be evaluated, where $C_R = C_T = 1$, $P_T = 1$ mW (BLE default value for beacons) and the working frequency 2.4 GHz with λ=0.125 m are selected. The receiving power depends inversely proportionally on the square of the distance R and is shown in Figure 1.

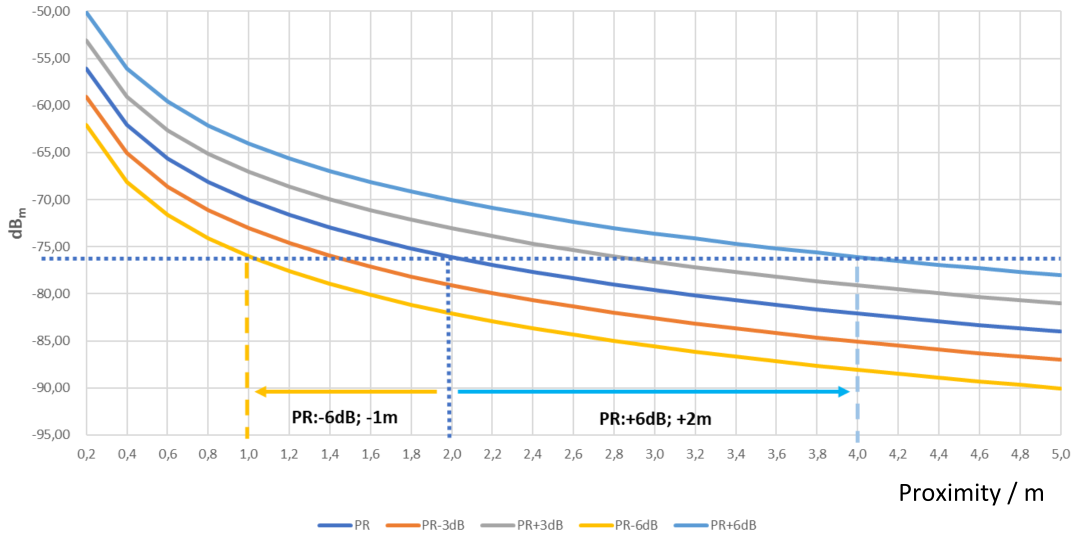

Figure 1: *Receiving power depending on the distance between transmitter and receiver*

The necessity of calibration is shown in the following example. Two smart phones have e.g. a distance of 2 m, which corresponds to a receiving power of $P_R = -76{,}1$ dB_m (horizontally dotted line). If the error in the power measurement was now 6 dB, the proximity determination could result in an error of 3 dB, i.e. half of the error compared to the power measurement. At a true distance of 2 m, this means: 2 m - 3dB= 1 m and 2 m +3 dB = 4 m. This means that the true value could be somewhere between 1 m and 4 m.

Although it is somewhat unusual to correct distance measures in dB, since the power measures in dB_m and deviations in dB are considered, a simple general context emerges. If the deviation of the power measurement is $E(P_R) = \pm x$ [dB], it follows that the deviation of the proximity $E(Proximity)[\text{dB}] = \pm x/2$ [dB]in meters:

$$E(proximity)[\text{m}] = 10^{\left\{\frac{E(P_R)\,[dB]}{2*10}\right\}} \cdot (true\ proximity)[\text{m}]$$

You can also determine the proximity R directly depending on the receiving power P_R, just as the formula in the pandemic app is needed. To do this, the formula $P_R = \frac{\lambda^2}{(4\pi)^2} \frac{1}{R^2} \cdot C_R \cdot C_T \cdot P_T$ must be rearranged to R, hence

$$R = \frac{\lambda}{4\pi} \cdot \sqrt{\frac{C_R \cdot CP_T}{P_R}}$$

follows.

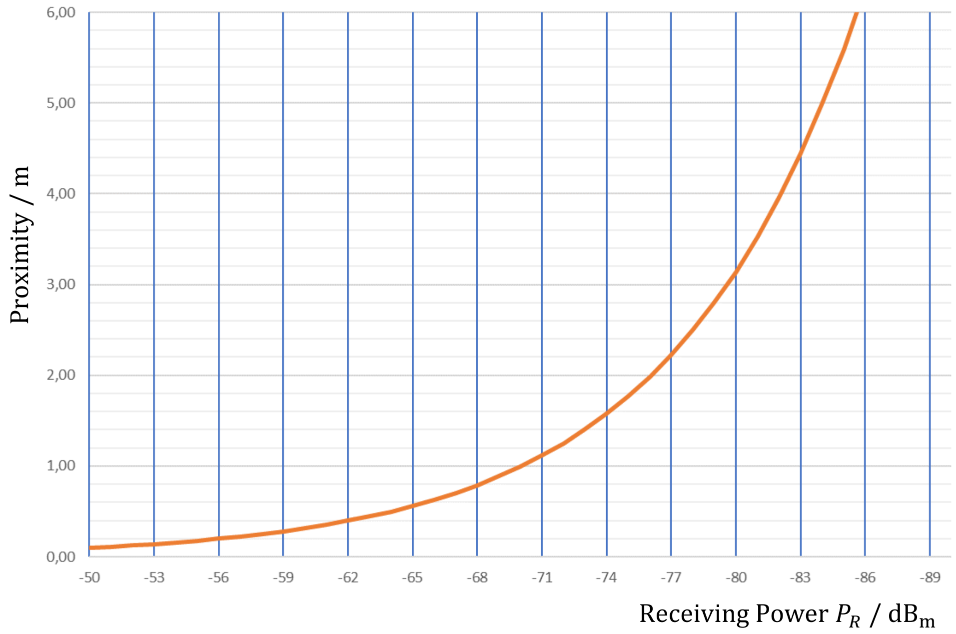

Figure 2: *Proximity depending on receiving power P_R in dB_m*

In Figure 2, the result is shown graphically depending on the receiving power P_R in dB_m. Assuming again that $C_R = 1$ and $CP_T = 1$ mW for simplification. In the case of a receiving power of e.g. $P_R = -76{,}1\ dB_m$ the result is 2 m proximity.

Table 1 shows deviations that can occur between different smart phone types. From that one can see that, for example, with a receiving power of $P_{R,X} = -76.1\ dB_m$, two ideal smart phones would result in a proximity of 2.0 m. A deviation of -12.0 dB results in 0.5 m and a deviation of +12 dB in 7.96 m. The differences arise from the combination $C_{R,X} \cdot CP_{T,Y}$, i.e. the characteristic paths of the transmitting and receiving smart phones.

Deviation/dB	**-12,0**	**-9,0**	**-6,0**	**-3,0**	**0,0**	**3,0**	**6,0**	**9,0**	**12,0**
PR/dBm	-58,0	-61,0	-64,0	-67,0	-70,0	-73,0	-76,0	-79,0	-82,0
Proximity/m	**0,25**	**0,35**	**0,50**	**0,71**	**1,0**	**1,41**	**2,00**	**2,82**	**3,98**
PR/dBm	-64,1	-67,1	-70,1	-73,1	-76,1	-79,1	-82,1	-85,1	-88,1
Proximity/m	**0,50**	**0,71**	**1,00**	**1,42**	**2,0**	**2,83**	**3,99**	**5,64**	**7,96**
PR/dBm	-67,6	-70,6	-73,6	-76,6	-79,6	-82,6	-85,6	-88,6	-91,6
Proximity/m	**0,75**	**1,06**	**1,50**	**2,12**	**3,0**	**4,24**	**5,99**	**8,46**	**11,94**
PR/dBm	-70,1	-73,1	-76,1	-79,1	-82,1	-85,1	-88,1	-91,1	-94,1
Proximity/m	**1,00**	**1,42**	**2,00**	**2,83**	**4,0**	**5,65**	**7,98**	**11,27**	**15,92**
PR/dBm	-72,0	-75,0	-78,0	-81,0	-84,0	-87,0	-90,0	-93,0	-96,0
Proximity/m	**1,26**	**1,77**	**2,51**	**3,54**	**5,0**	**7,06**	**9,98**	**14,09**	**19,91**

Table 1: *Impact of product-specific deviations of receiving power on determining proximity*

It is also worth noting that the proximity span increases linearly with the distance. If the range at a proximity of 1 m ranges from 0.25 m to 3.98 m, the span at 5 m is between 1.26 m and 19.91 m. If these deviations are not removed by the calibration method, then BLE will be completely unsuitable for the intended purpose of interrupting the infection chains.

4. Calibration method for correcting type-specific deviations

As already mentioned in Chapter 2, for each smart phone type, the product-specific characteristic deviations for the determination of the proximity can be calibrated. In the first step, two identical smart phones of the same type are required, defined as reference smart phones SP_{REF}. In the second step, two calibration factors Δ_R^X and Δ_T^X can be determined for any other smart phone X. We need to consider two factors because the characteristic influencing factors for SP_X in transmitting path are different from those in the receiving part. The index R or index T denotes the receiving or transmitting calibration value, respectively. Here the reciprocity theorem is not applicable. Fortunately, these factors are the same for each smart phone type.

Definition of Smart Phone Type:
"A smart phone type includes all devices that have the same TAC code".

The TAC code means Type Allocation Code (before 2003 Type Approval Code) and is part of IMEI, the International Mobile Equipment Identifier. The IMEI is a unique code for each smart phone and is assigned to a maximum of one million products of the same type with identical TAC code. If more than one million products are produced, the next million will get a new TAC code. After one million products are assembled there is sometimes a change of some production steps or even of the product itself for the next million, in order to further optimize the production. However, it could also be the case that without any change in production or the product, a new type is assembled, even though the products are still identical to the first million. This can be evaluated from a database of the GSM association, the so-called device map.

This means that two smart phones with the same trade name, e.g. "iPhone 11", could belong to different types and differ in terms of transmitting and receiving characteristics. Of course, this also applies to other mass-produced products such as "Samsung Galaxy S10", etc.

In any case, the correction method requires knowledge of the TAC in the pandemic app. The calibration could then be conducted directly in the app or on the pandemic server. For each TAC code contained in the IMEI, i.e. recognition of the type, there are then two calibration factors $\Delta_R^X$and $\Delta_T^X$with which all product-specific characteristic deviations for type X can be eliminated.

The correction algorithm is very simple. If two smart phones type X and type Y are considered, SP_Y transmits and SP_Xreceives, then the result is:

$$P_R^{\mathrm{REF}} = P_R^{\mathrm{X}} + \Delta_R^X + \Delta_T^Y$$

Here, P_R^{REF} means the corrected receiving power of SP_X. That means that after calibration the receiving power is identical to that of the reference phone, whose properties are known to determine the proximity. When SP_X is receiving, it needs for correction the value Δ_T^Y, which is initially known only to SP_Y. However, this can be transmitted directly during the advertising of SP_Y, so that it is also known to SP_X. The correction value $\Delta_R^X$is already known to the receiving smart phone. All power values and correction factors are determined in dB. If the Smart Phones change transmitting and receiving mode, then the following equation is valid accordingly

$$P_R^{REF} = P_R^{\mathrm{Y}} + \Delta_R^Y + \Delta_T^X$$

This removes the product-specific characteristic deviations. The calculated proximity from type X transmitting to type Y and that of type Y transmitting to type X should be almost identical when measured in an absorber hall (FAC), as it is determined after correction from the receiving power of a reference phone. Small deviations mainly depend on the calibration accuracy. In the field further deviations arise, which have nothing to do with these product-specific properties of the smart phone types calibrated here. The properties of the radio channel and the directivity effects of the transmitting and receiving antennas remain. The chosen approach means, that the characteristic values C_R and CP_T can be considered as constants for different smart phone types. The directivity characteristics of the smart phones are assigned to the radio channel, as described in Chapter 5. These are purely practical considerations. During the calibration measurements the smart phones must always be oriented in the same direction. Otherwise the directivity can influence the calibration method.

In order to prove the correctness of the calibration method, two "smart phone types" type X and type Y are considered. The following equation applies:

$$\begin{pmatrix} P_{R,X} \\ P_{R,Y} \end{pmatrix} = H(R,\lambda) \begin{pmatrix} C_{R,X} \cdot CP_{T,Y}) \\ C_{R,Y} \cdot CP_{T,X}) \end{pmatrix}$$

With the transmission function of the free space

$$H_{FR}(R,\lambda) = \frac{\lambda^2}{(4\pi)^2} \frac{1}{R^2}$$

the following two equation result:

$$P_{R,X} = H_{FR} \cdot C_{R,X} \cdot CP_{T,Y}$$

$$P_{R,Y} = H_{FR} \cdot C_{R,Y} \cdot CP_{T,X}$$

If one considers the quantities in dB, additions can be used instead of multiplications, i.e.

$$P_{R,X}[\mathrm{dB_m}] = H_{FR}[\mathrm{dB}] + C_{R,X}[\mathrm{dB}] \cdot CP_{T,Y}[\mathrm{dB_m}]$$

$$P_{R,Y}[\mathrm{dB_m}] = H_{FR}[\mathrm{dB}] + C_{R,Y}[\mathrm{dB}] \cdot CP_{T,X}[\mathrm{dB_m}]$$

In order not to have to write constantly $[\mathrm{dB_m}]$or $[\mathrm{dB}]$, the equations are presented as follows:

$$P_R^{X,Y} = H^{FR} + C_R^X + CP_T^{\mathrm{Y}}$$

$$P_R^{Y,X} = H^{FR} + C_R^Y + CP_T^{\mathrm{X}}$$

Which smart phone transmits, and which one receives is described by the raised index. In the first equation SP_Ytransmits and SP_X receives and in the second equation, in reverse. $H^{FR} = 10 \cdot \log{(H_{FR})}$. The first calibration is now performed between two smart phones belonging to the type REF (reference). Then the equations look like this:

$$P_R^{\mathrm{REF_1,REF_2}} = H^{FR} + C_R^{REF_1} + CP_T^{\mathrm{REF_2}}$$

$$P_R^{\mathrm{REF_2,REF_1}} = H^{FR} + C_R^{REF_2} + CP_T^{\mathrm{REF_1}}$$

The received powers $P_R^{\mathrm{REF_1,REF_2}}$ und $P_R^{\mathrm{REF_2,REF_1}}$ should be almost identical, because they belong to the same type (identical FAC-code), i.e. $P_R^{\mathrm{REF_1,REF_2}} \approx P_R^{\mathrm{REF_2,REF_1}}$. This means there is only one equation left.

$$P_R^{\mathrm{REF,REF}} = H^{FR} + C_R^{REF} + CP_T^{\mathrm{REF}}$$

In order to verify this claim, the two reference phones should be arranged in an absorber hall in such a way that at a distance of 1 m to 5 m, in 1 m increments, the receiving power $P_R^{\mathrm{REF_1,REF_2}}$ and afterwards $P_R^{\mathrm{REF_2,REF_1}}$ should be measured by swapping the transmitting and receiving direction. This makes it possible to check the measurement arrangement again.

The absorber hall is required because the function of the free space

$$H^{FR} = 20 \cdot \log\left(\frac{\lambda}{4\pi}\right) + 20 \cdot \log\left(\frac{1}{R}\right)$$

is precisely simulated there. Measurements in the open field or in a Semi Anechoic Chamber (SAC) should be avoided, as this results in further difficulties due to the ground reflections. Thus, it is possible to determine

$$CP_{RT}^{\mathrm{REF}} = C_R^{REF} + CP_T^{\mathrm{REF}} = P_R^{\mathrm{REF,REF}} - H^{FR}$$

for the reference phone.

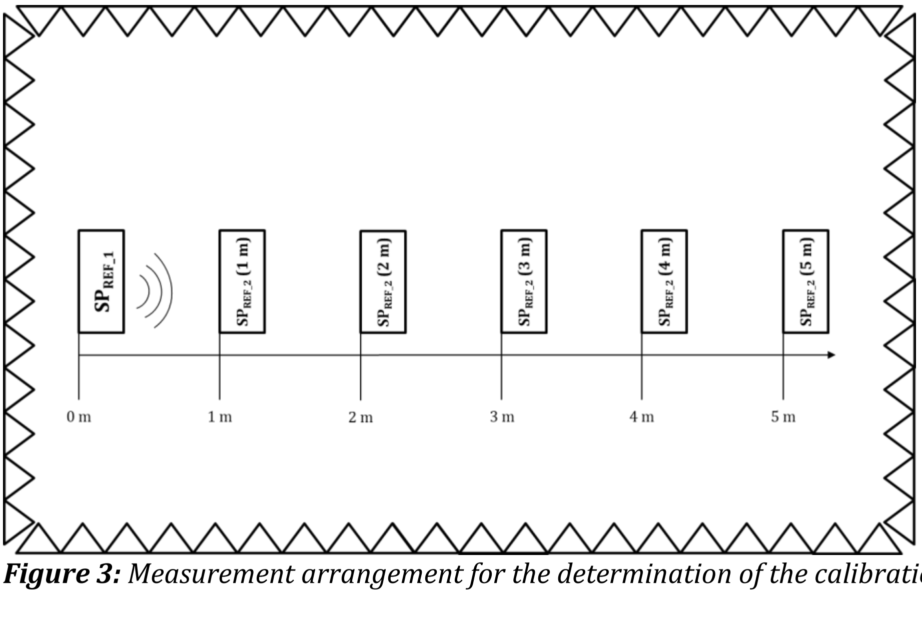

Figure 3: *Measurement arrangement for the determination of the calibration factors*

With

$$R = 10^{-\left(\frac{P_R^{\mathrm{REF,REF}} - 20 \cdot \log\left(\frac{\lambda}{4\pi}\right) - CP_{RT}^{\mathrm{REF}}}{20}\right)} \approx \frac{1}{100} \cdot 10^{-\left(\frac{P_R^{\mathrm{REF,REF}} - CP_{RT}^{\mathrm{REF}}}{20}\right)}$$

the proximity can be determined. This can easily be verified with the values in Table 1, where $CP_{RT}^{\mathrm{REF}} = -30\ \mathrm{dB_m}$ is assumed. With the correction values presented above Δ_R^X and Δ_T^Yor Δ_R^Y and Δ_T^X the receiving power can always be calculated back to the reference phone. The formula for determining proximity is essential. All remaining deviations for the receiving power occur in the field and are only related to the effects of the radio channel and the remaining directivities of the smart phones (see Chapter 5).

If you now look at two different types X and Y and now perform the measurements against a reference phone, then 5 equations result, the equation for the reference phone is listed again:

$$P_R^{\mathrm{REF,REF}} = H^{FR} + C_R^{REF} + CP_T^{\mathrm{REF}}$$

$$P_R^{\mathrm{REF,X}} = H^{FR} + C_R^{REF} + CP_T^{\mathrm{X}}$$

$$P_R^{\mathrm{X,REF}} = H^{FR} + C_R^{X} + CP_T^{\mathrm{REF}}$$

$$P_R^{\mathrm{REF,Y}} = H^{FR} + C_R^{REF} + CP_T^{\mathrm{Y}}$$

$$P_R^{\mathrm{Y,REF}} = H^{FR} + C_R^{Y} + CP_T^{\mathrm{REF}}$$

If you now create the differences between equations 1 minus 2, 1 minus 3, 1 minus 4 and 1 minus 5, four equations are derived that describe the corrections correction factors

$$\Delta_T^X = CP_T^{\mathrm{REF}} - GP_T^{\mathrm{X}}$$

$$\Delta_R^X = C_R^{\mathrm{REF}} - C_R^{\mathrm{X}}$$

$$\Delta_T^Y = CP_T^{\mathrm{REF}} - CP_T^{\mathrm{Y}}$$

$$\Delta_R^Y = C_R^{\mathrm{REF}} - C_R^{\mathrm{Y}}$$

The relationship between two types X and Y can be described as follows, type X in receiving mode and type Y in transmitting mode

$$P_R^{\mathrm{X,Y}} = H^{FR} + C_R^{X} + CP_T^{\mathrm{Y}}$$

and type Y in receiving mode and type X in transmitting mode

$$P_R^{\mathrm{Y,X}} = H^{FR} + C_R^{Y} + CP_T^{\mathrm{X}}$$

These are the receiving powers measured in the receiving phones, which are affected by the product-specific characteristic deviations. If you now include the four correction factors in these two equations, the corresponding corrections result, although types X and Y have never been measured against each other.

$$P_R^{\mathrm{X,Y}} + \Delta_R^X + \Delta_T^Y = H^{FR} + C_R^{\mathrm{REF}} + CP_T^{\mathrm{REF}} = P_R^{REF}$$

$$P_R^{\mathrm{Y,X}} + \Delta_R^Y + \Delta_T^X = H^{FR} + G_R^{\mathrm{REF}} + GP_T^{\mathrm{REF}} = P_R^{REF}$$

The receiving power $P_R^{\mathrm{X,Y}}$ is corrected by the receiving correction factor Δ_R^X of the receiving smart phone and by the transmitting correction factor Δ_T^Y of the transmitting smart phone and vice versa. As a result you get the receiving power of the reference phone and you can apply the formula for the above-derived calculation of the proximity. In free space the determination of the proximity is very precise. All other inaccuracies now depend exclusively from the radio channel and directivity effects.

For each smart phone type, only two calibration measurements are necessary with a reference phone and not, as one might suspect, each type against any other type. This makes the calibration effort manageable.

There are other ways to perform the required calibration measurements. As mentioned above, [Opentrace Community] performed calibration measurements for the transmitting path. Here, in an absorber hall FAC with the help of a spectrum analyzer (SA) and a calibrated receiving antenna (A), the transmitting power of various smart phones was measured $P_R^{\mathrm{SA+A,X}} = H^{FR} + C_R^{SA+A} + CP_T^{\mathrm{X}}$. There are two points of criticism. First, the transmission power was determined by averaging 20 measurements. The averaging was performed because the peak values vary greatly when received with the spectrum analyzer. Improvement: You can switch the smart phone into a so-called Bluetooth test mode. The smart phone transmits then continuously. Thus, the transmission signal can be determined exactly and there are no fluctuations that require mean estimates. The second point of criticism concerns the receive path that was not considered in [Opentrace Community]. A solution to this problem is already outlined above.

Here, further possibilities for determining the characteristic values CP_T^X and C_T^X are to be shown.

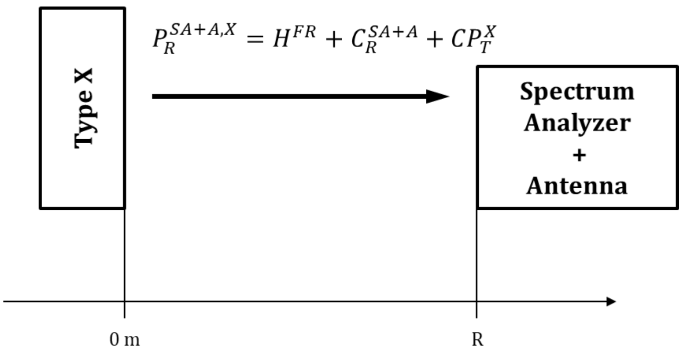

Figure 4: *Measurement arrangement for determining the product-specific characteristic* CP_T^X

Figure 4 shows the measuring arrangement as it is carried out in an absorber hall FAC. The receiving power $P_R^{\mathrm{SA+A,X}} = H^{FR} + C_R^{SA+A} + CP_T^{\mathrm{X}}$ is measured with a spectrum analyzer and an calibrated antenna, which is very precise. From this result the characteristic for the transmitting path CP_T^X can be deduced.

Accordingly, a calibrated transmission path measurement can also be established with a signal generator and a calibrated transmitting antenna (see Figure 5).

The receiving power $P_R^{\mathrm{X,SG+A}} = H^{FR} + C_R^X + CP_T^{\mathrm{SG+A}}$ is measured by the smart phone. The transmitting path can be built with appropriate precision $CP_T^{\mathrm{SG+A}}$ and thus the characteristic value for the reception path C_R^X of the smart phone can be deduced. This method can also be used to determine the correction values Δ_R^X and Δ_T^X, which means that the same formula for the proximity can always be used in each smart phone type.

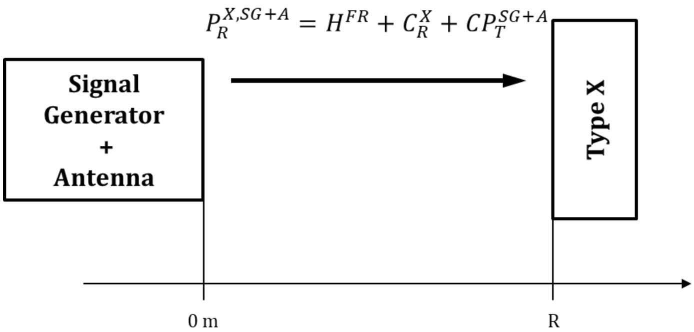

Figure 5: *Measurement arrangement for determining the product-specific characteristic* C_R^X

There is a third way. Here, the measurement of the transmitting power is carried out, as in Fig. 4. Since this measurement is very precise, reference smart phones could be qualified in a reference laboratory. This procedure can also be used for the qualification of a reference phone. You can measure several smart phones of the same type to find a type that is very uniformly produced in the series, at least in the transmission path. According to the equations from above for two reference phones

$$P_R^{\mathrm{REF}_1} = H^{FR} + C_R^{REF_1} + CP_T^{\mathrm{REF}_2}$$

$$P_R^{\mathrm{REF}_2} = H^{FR} + C_R^{REF_2} + CP_T^{\mathrm{REF}_1}$$

The assumption $P_R^{\mathrm{REF}_1} \approx P_R^{\mathrm{REF}_2}$ can be verified in the easily in the laboratory. Such a qualified reference phone could then be delivered with the value

$$CP_{RT}^{REF} = C_R^{REF} + CP_T^{REF}$$

to other laboratories, which can then determine the characteristic correction values of other smart phones.

$$P_R^{\mathrm{REF,REF}} = H^{FR} + CP_{RT}^{REF}$$

$$P_R^{\mathrm{REF,X}} = H^{FR} + C_R^{REF} + CP_T^{\mathrm{X}}$$

$$P_R^{\mathrm{X,REF}} = H^{FR} + C_R^{X} + CP_T^{\mathrm{REF}}$$

From this, the correction values

$$\Delta_T^X = P_R^{\mathrm{REF,REF}} - P_R^{\mathrm{REF,X}} = CP_T^{\mathrm{REF}} - GP_T^{\mathrm{X}}$$

$$\Delta_R^X = P_R^{\mathrm{REF,REF}} - P_R^{\mathrm{X,REF}} = C_R^{\mathrm{REF}} - C_R^{\mathrm{X}}$$

can then be calculated.

All three methods presented are in principle equivalent. However, if you want to ensure that several laboratories can participate in the measurement campaign and that a database is maintained internationally, the third method offers advantages. Based on this work, you can start immediately without having to write a special calibration specification. One can also introduce the variant that each laboratory could qualify its own reference phone according to the method described here. To produce confidence in the results of the involved laboratories Round Robin tests are highly recommended.

The database then returns a simple table as a result (Table 2). It must be mentioned, that the reference phone characteristic CP_{RT}^{REF} of a reference phone is necessary for the calculation of the proximity in the pandemic app and the correction factors Δ_R^{TAC} and Δ_T^{TAC} are only valid in combination with that factor.

No	Brand Name	Type Allocation Code	CP_{RT}^{REF}	Δ_R^{TAC}	Δ_T^{TAC}
1					
2					

Table 2: *Calibration factors for smart phone types, receive path, and transmitting path*

The question that still arises is how many smart phone types exists. For this purpose, the data of the network operators is required. They have information in their databases about all smart phones currently active in their networks, i.e. all active IMEIs. From this, all TACs can be grouped and the frequencies of the types in a region or country can be determined. In conjunction with the GSM Association database, the number of types can be reduced again. Calibration should then be started with the most common types, so that a large coverage is achieved within a short time (after a few days).

However, the pandemic app that was announced to be introduced soon is to be prepared so that the incoming results can be gradually integrated.

For modern smart phones, statistical deviations due to the high manufacturing quality can be expected to be very small compared to the product-specific characteristic deviations between the types and are therefore negligible. In principle, statistical deviations could also be recorded with mean and standard deviation. However, this would require multiple smart phones of the same type for calibration measurements.

5. Channel model and directivity characteristics

This chapter outlines the separation between the elements that can be calibrated and the remaining problems of the radio channel and the directivity of the smart phones.

Until now, it has always been assumed that the transmission function $H(R,\lambda)$is that of free space. This is useful and necessary for the calibration process. If all persons in the field were to wear a helmet on which the smart phone would stand vertically, the free space formula would be a quite good approximation for determining the proximity. However, the situation in the field may differ considerably from free space, e.g. if one person carries the smart phone in his pocket and another person keeps the smart phone in his handbag. In addition, the persons move constantly, and the receiving power fluctuates very much due to multi-path fading. Additionally, the directivities of the smart phones have also to be taken into account.

A model for the radio channel can be described as follows

$$H(R,\lambda) = \frac{\lambda^2}{(4\pi)^2}\left|\frac{\underline{a}_0(\vartheta,\varphi)}{R} + \sum_{n=1}^{N}\frac{\underline{a}_n(\vartheta,\varphi)}{R+\Delta R_n}e^{-jk\Delta R_n}\right|^2$$

The value $\underline{a}_0(\vartheta,\varphi) = a_0(\vartheta,\varphi)\cdot e^{j\Phi_0}$describes an additional attenuation that could be caused by body proximity, the material of a handbag, etc. In addition, with the dependency of $(\vartheta,\varphi)$the directivity characteristics of the smart phones is considered. There could also be a phase shift, i.e. $\underline{a}_0(\vartheta,\varphi)$ is a complex function. The terms in the sum describe further ways of wave propagation and reflections in the surrounding area. The parameters $\underline{a}_n(\vartheta,\varphi) = a_n(\vartheta,\varphi)\cdot e^{j\Phi_n}$ deviate from $\underline{a}_0(\vartheta,\varphi)$. The absolute values $a_n(\vartheta,\varphi)$include, among other things, the reflection factors that occur on floors, walls and other objects. The phase $\Phi_n$also includes the phase shift, as it occurs during reflections, e.g. on a conducting surface the shift is π. The sum is limited at N because no significant contribution is paid to $P_R$from a certain number of reflection paths.

The direct path R is very short, but reflections on floors, walls and other objects will often contribute to the receiving power P_R. Fading effects also occur, in which the direct signal overlaps with the reflected signals and partially adds or subtracts. When two people move towards or away from each other, the receiving power fluctuates greatly, because the phases change continuously as described with $e^{-jk\Delta R_n}$.

For further considerations the function H_N is introduced. $H(R,\lambda)$ is normalized by $\frac{\lambda^2}{(4\pi)^2}$ so that only the total square of the channel model is left

$$H_N = \frac{H(R,\lambda)}{\frac{\lambda^2}{(4\pi)^2}} = \left|\frac{\underline{a}_0(\vartheta,\varphi)}{R} + \sum_{n=1}^{N}\frac{\underline{a}_n(\vartheta,\varphi)}{R+\Delta R_n}e^{-jk\Delta R_n}\right|^2 = \left|A_0 + \sum_{n=1}^{N} A_n\cdot e^{-j\Theta_n}\right|^2$$

with the abbreviations

$$A_0 = \frac{a_0(\vartheta,\varphi)}{R}$$

$$A_n = \frac{a_n(\vartheta,\varphi)}{R+\Delta R_n}$$

$$\Theta_n = k\Delta R_n + \Phi_0 + \Phi_n$$

After rearranging the formula, it follows

$$\begin{aligned} H_N = &\sum_{n=0}^{N} A_n^2 \\ &+ 2A_0\sum_{n=1}^{N} A_n\cos\Theta_n \\ &+ 2A_1\sum_{n=2}^{N} A_n\cos(\Theta_1-\Theta_n) \end{aligned}$$

$$+ 2A_2 \sum_{n=3}^{N} A_n \cos(\Theta_2 - \Theta_n)$$

$$+ \cdots$$

$$+ 2A_{N-1} A_N \cos(\Theta_{N-1} - \Theta_N)$$

The result can be interpreted as follows. The first term states that in addition to the free space contribution all reflection paths provide a phase-independent contribution to the receiving power.

$$H_{N,Term1} = \sum_{n=0}^{N} A_n^2 = \frac{a_0^2}{R^2} + \sum_{n=1}^{N} \frac{a_n^2}{(R + \Delta R_n)^2}$$

Essentially, these contributions are characterized by the reflection factors and the lengths of the reflection paths. All other terms describe fading effects.

If one considers only one additional path of reflection, which is the case in practice when two people pass each other in an open field. For N=1 follows:

$$H_{N,1} = \frac{a_0^2}{R^2} + \frac{a_1^2}{(R + \Delta R_1)^2} + \frac{2a_0 a_1}{R\ (R + \Delta R_{1)}} \cos\left(\frac{2\pi}{\lambda} \Delta R_n + \Phi_0 + \Phi_n\right)$$

The fading effect cannot be neglected, at least in the case of moist ground, where the reflection factor is close to one (see Figure 6).

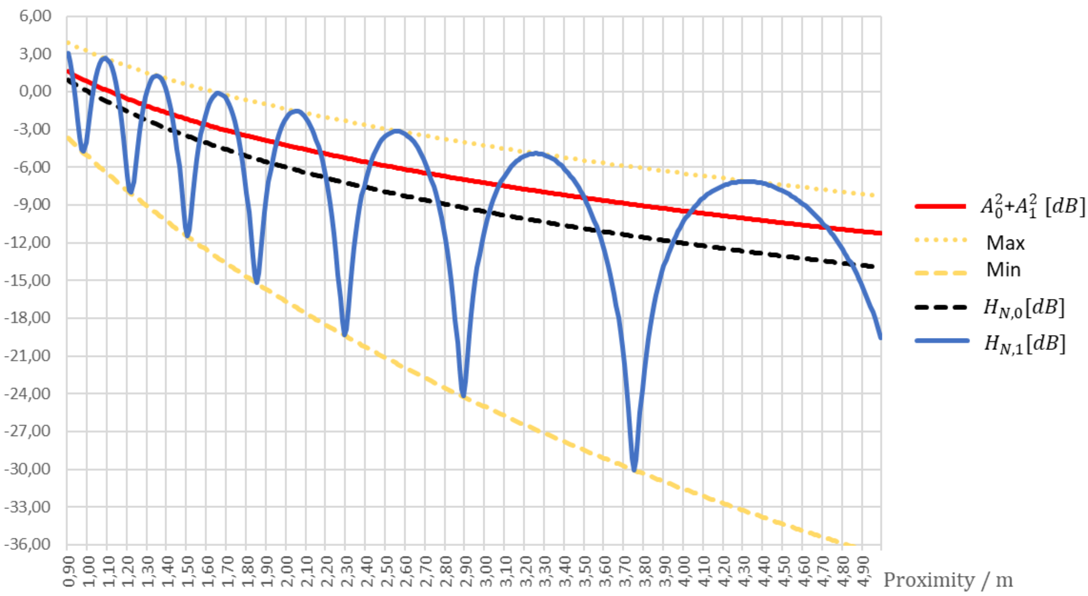

Figure 6: *Diagram showing the fading effect depending on the proximity from 0.9 to 5.0 m*

The graph $H_{N,1}[dB]$ shows the fading effect depending on the proximity, $H_{N,0}[dB]$ the course in free space and $A_0^2 + A_1^2$ the contribution to the receiving power via the direct and the reflection path.

This chapter has shown that there is still enough research work to be left to determine the real proximity very precisely from the fading profiles in the field.

6. Basic comments on runtime evaluations (time of flight)

Runtime measurements could in principle improve the accuracy for determining the proximity. However, only scalar receiving power values are available here. The bandwidth of the Bluetooth signal is so low that in principle no time of flight measurements are possible. This can be easily seen from the uncertainty principle of communications

$$\Delta B \cdot \Delta T \geq K,$$

where ΔB means the bandwidth, ΔT the signal duration and K is a factor between 0.2 and 1 and depends substantially on how bandwidth and signal duration are defined.

The flight time of light for one meter is 10/3 10^{-9} sec. For example, in order to determine the distance of one meter over the time of flight of a signal, the signal duration should not be greater than 1/10 of the time that light takes for a distance of 10 cm. This results in a bandwidth of $\Delta B \geq \frac{1}{\Delta T} = 3\ GHz$. Bluetooth has a carrier frequency of 2.4 GHz and is very narrowband.

Even if the baseband signal of the Bluetooth chip could be evaluated, there would be no prospect of improving the proximity estimate.

7. Outlook for future technologies and developments

Smart phones already have technologies implemented, unfortunately they are not yet available to the app developer. The interfaces to be used by app developers (API, objects or methods) will be made available in the near future with new releases of the operating systems of the smart phones.

The most interesting technologies in this context are Bluetooth 5.1 and UWB (Ultra-wide Band).

With Bluetooth 5.1, a new standard was released in 2019, which will make it possible to carry out a so-called direction finding. Two antennas will be controlled phase-shifted, so that a mini-phased array can be realized. This will improve the proximity estimation. However, these chips are not yet implemented in the current smart phones and are therefore not available.

In the meantime, a UWB technology with a bandwidth of 2.5 GHz has been installed in Apple's iPhone 11 and Google's Pixel Phone 4. This technology cannot be used at this time via the operating system. This technology will work similarly to Bluetooth's Beacon technology, but will allow "time off light" measurements with a spatial resolution in the range of 10 cm.

Unfortunately, so far smart phones on the market today do not have a more appropriate technology than Bluetooth BLE for determining proximity.

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