OpenTURNS Developer training: first steps

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Developers training

OpenTURNS: first steps

1. Navigation in the source code
2. Library development
3. Module development

Navigation in the source code

The Uniform distribution

- Locate the class within the library source code;
- Follow its inheritance graph in order to explore the Bridge pattern;
- Locate the associated regression test;
- Execute the test;
- Locate its SWIG interface file and its associated Python module;
- Execute the associated python test.

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Projects

1. (*) `InverseDistanceWeightingInterpolation` as a specialization of `EvaluationImplementation` (see `lib/src/Base/Func`). Given a set of data $(x_i, y_i)_{i=1,\dots,N}$ in $\mathbb{R}^n \times \mathbb{R}^p$, the IDW interpolation is defined by:

$$\forall x \in \mathbb{R}^n, u(x) = \begin{cases} \frac{\sum_{i=1}^N w_i(x) y_i}{\sum_{i=1}^N w_i(x)} & \text{if } \forall i, d(x, x_i) > 0 \\ y_i & \text{if } \exists i, d(x, x_i) = 0 \end{cases} \quad (1)$$

   where $w_i(x) = \frac{1}{d(x, x_i)^p}$ and $p > 0$ a given smoothness parameter.
   The distance d can be the Euclidean distance, the 1-norm or the sup norm.

2. (**) `DiscreteIntegralCompound` as a specialization of `DiscreteDistribution` (see `lib/src/Uncertainty/Distribution`). Given the discrete distribution of a random variable N and the common discrete distribution of a sequence of iid integral valued random variables (X_i), compute the distribution of the integral valued discrete random variable Y defined by:

$$Y = \sum_{i=1}^N X_i \quad (2)$$

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Projects

Its generating function $\phi_Y(z) = \mathrm{E}\left[z^Y\right]$ is given by:

$$\forall z \in \mathbb{C}, \phi_Y(z) = \phi_N(\phi_X(z)) \tag{3}$$

and thanks to Poisson's summation formula for discrete distributions, we have for $0 < r < 1$ and $m \in \mathbb{N}^*$:

$$\forall n \in \{0, \dots, m-1\}, p_Y(n) = \frac{1}{mr^n} \sum_{k=0}^{m-1} \phi_Y\left(re^{\frac{2i\pi k}{m}}\right) e^{-\frac{2i\pi kn}{m}} - e_d \tag{4}$$

where $0 \leq e_d \leq r^m$ is the approximation error. For a given $\epsilon > 0$ and $m \in \mathbb{N}^*$, set $r = \sqrt[m]{\epsilon}$ and compute the FFT $(\omega_0, \dots, \omega_{m-1})$ of the complex vector $\left(\phi_Y\left(re^{\frac{2i\pi k}{m}}\right), \dots, \phi_Y\left(re^{\frac{2i\pi k}{m}}\right)\right)$. Then, the distribution is equal to the `UserDefined` distribution with locations $\{0, \dots, m-1\}$ and probabilities $\left(p_i = \dfrac{\Re(\omega_i)}{mr^i}\right)_{i=0,\dots,m-1}$.

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Projects

3. (*) ClenshawCurtis integration algorithm as a specialization of IntegrationAlgorithmImplementation (see lib/src/Base/Algo). This integration algorithm allows to compute integrals of the form:

$$\begin{aligned} I(f) &= \int_a^b f(t)\,dt \\ &= \frac{b-a}{2}\int_{-1}^{1} f\left(a+\frac{b-a}{2}(1+x)\right)\,dx \\ &\simeq \frac{b-a}{2}\sum_{k=0}^{n} w_k f(a+\frac{b-a}{2}(1+x_k)) \end{aligned}$$

where $x_k = \cos\theta_k$, $\theta_k = \frac{k\pi}{n}$ and w_k is given by:

$$w_k = \frac{c_k}{n}\left(1-\sum_{j=1}\lfloor n/2\rfloor \frac{b_j}{4j^2-1}\cos(2j\theta_k)\right) \tag{5}$$

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Projects

where the coefficients b_j and c_k are given by:

$$b_j = \begin{cases} 1 & j = n/2 \\ 2 & j < n/2 \end{cases} \qquad c_k = \begin{cases} 1 & k = 0[n] \\ 2 & k \neq 0[n] \end{cases} \tag{6}$$

for $k = 0, \ldots, n$. An efficient FFT-based implementation of the computation of the weights and nodes is given in `fclencurt.m`, another one (**) in `1311.0445.pdf`.

4. (*) `Fejer1` integration algorithm as a specialization of `IntegrationAlgorithmImplementation` (see `lib/src/Base/Algo`). This integration algorithm is based on the nodes $x_k = \cos\theta_{k+1/2}$ and weights:

$$w_k^{f1} = \frac{2}{n}\left(1 - 2\sum_{j=1}^{\lfloor n/2 \rfloor} \frac{1}{4j^2 - 1}\cos\left(j\theta_{2k+1}\right)\right) \tag{7}$$

for $k = 0, \ldots, n-1$. There also exist fast implementations based on FFT or modified moments, see the references for Clenshaw Curtis.

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Projects

5. (*) Fejer2 integration algorithm as a specialization of IntegrationAlgorithmImplementation (see lib/src/Base/Algo). This integration algorithm is based on the nodes $x_k = \cos\theta_k$ and weights:

$$w_k^{f2} = \frac{4}{n}\sin\theta_k \sum_{j=1}\lfloor n/2\rfloor \frac{\sin\left((2j-1)\theta_k\right)}{2j-1} \tag{8}$$

   for $k = 0, \ldots, n$. There also exist fast implementations based on FFT or modified moments, see the references for Clenshaw Curtis.

6. (**) ClenshawCurtisProductExperiment as a specialization of WeightedExperiment: same algorithm as for ClenshawCurtis but with adaptation to any weight function.

7. (*) MarshallOlkinCopula as a specialization of CopulaImplementation (see lib/src/Uncertainty/Distribution). This copula is defined by:

$$\forall (u,v) \in [0,1]^2, C(u,v) = \begin{cases} u^{1-\alpha}v & \text{for } u^\alpha \geq v^\beta \\ uv^{1-\beta} & \text{for } u^\alpha < v^\beta \end{cases} \quad // \tag{9}$$

   where $0 < \alpha, \beta < 1$.

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Projects

8. (*) GumbelCopula as a specialization of `ExtremeValueCopula` (see `lib/src/Uncertainty/Distribution`). This copula already exists, but not as an extreme value copula. It is defined by its Pickand function:

$$\forall t \in [0,1], A(t) = \left[t^\theta + (1-t)^\theta\right]^{1/\theta} \tag{10}$$

   where $\theta \geq 1$.

9. (*) GalambosCopula as a specialization of `ExtremeValueCopula` (see `lib/src/Uncertainty/Distribution`). This copula is defined by its Pickand function:

$$\forall t \in [0,1], A(t) = 1 - \left[t^{-\theta} + (1-t)^{-\theta}\right]^{-1/\theta} \tag{11}$$

   where $\theta \geq 0$.

10. (*) TawnCopula as a specialization of `ExtremeValueCopula` (see `lib/src/Uncertainty/Distribution`). This copula is defined by its Pickand function:

$$\forall t \in [0,1], A(t) = (1-\psi_1)(1-t) + (1-\psi_2)t + \left[\{\psi_1 t\}^{1/\theta} + \{\psi_2(1-t)\}^{1/\theta}\right]^\theta \tag{12}$$

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Projects

11. (*) JoeCopula as a specialization of `ExtremeValueCopula` (see `lib/src/Uncertainty/Distribution`). This copula is defined by its Pickand function:

$$\forall t \in [0,1], A(t) = 1 - \left[\{\psi_1(1-t)\}^{-1/\theta} + \{\psi_2 t\}^{-1/\theta}\right]^{-\theta} \tag{13}$$

where $\theta > 0$ and $0 \leq \psi_1, \psi_2 \leq 1$.

12. (**) ArchiMaxCopula as a specialization of `CopulaImplementation` (see `lib/src/Uncertainty/Distribution`). Given an Archimedean copula with generator ψ and an extreme value copula with Pickand function A, an archimax copula C is defined by:

$$\forall (u,v) \in [0,1]^2, C(u,v) = \psi^{-1}\left(\min\left(\psi(0), [\psi(u)+\psi(v)]A\left(\frac{\psi(u)}{\psi(u)+\psi(v)}\right)\right)\right) \tag{14}$$

It becomes (***) if one wants to implement an efficient sampling algorithm.

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Projects

13. (*) SquaredNormal as a specialization of ContinuousDistribution (see lib/src/Uncertainty/Distribution). If X is distributed according to the $\mathcal{N}(\mu, \sigma)$ distribution, $Y = X^2$ is distributed according to the squared normal distribution with parameters μ and $\sigma > 0$. This distribution has already been implemented in Python, see SquaredNormal.py.

14. (**) ConditionalEventDistribution as a specialization of ContinuousDistribution (see lib/src/Uncertainty/Distribution). Given the joint distribution of an $(m+n)$ dimensional random vector (X, Y) and an m dimensional interval I such that $\mathbb{P}(X \in I) > 0$, it is the distribution of Y knowing that $X \in I$. This distribution has already been implemented in Python, see ConditionalEventDistribution.py. It becomes (***) if one wants to implement an efficient simplification mechanism.

15. (***) Extend archimedian copulas from 2-d to n-d. Given a 2-d Archimedean copula with generator ψ, implement its n-d counterpart using:

$$\forall (u_1, \dots, u_n) \in [0,1]^n, C(u_1, \dots, u_n) = \psi^{-1}\left(\psi(u_1) + \cdots + \psi(u_n)\right) \tag{15}$$

The main difficulties are the architecture of this extension and the implementation of an efficient sampling algorithm.

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Projects

16. (**) `BlockComposedDistribution` as a specialization of `DistributionImplementation` (see `lib/src/Uncertainty/Distribution`). Given a collection of distributions $D_1, \dots, D_n$ of dimensions $d_1, \dots, d_n$, it is the distribution of the random vector $(X_1, \dots, X_n)$ of dimension $d_1 + \dots + d_n$ where X_i is distributed as D_i and $X_1, \dots, X_n$ are independent.
    It becomes (****) if one wants to propagate this new distribution in every places it could go within the library.
17. (*) Extend `SolverImplementation` and `Solver` to the resolution of systems of nonlinear equations and provide a generic implementation using the `LeastSquaresProblem` class. The solutions x^* of a nonlinear system of equations $f_1(x) = 0, \dots, f_n(x) = 0$ where $x = (x_1, \dots, x_n)$, if they exist, have to be found in the set of solutions of the following least-squares problem:

$$x^* = \arg\min \sum_{j=1}^{n} f_j^2(x) \tag{16}$$

for which many solvers are available in OpenTURNS.

Module development 1/2

Projects

18. (*) or (**) `CloudMesher`: mesh generation over a cloud of points using kernel mixture, pca, rotation, then levelset mesher on an interval
19. (*) `UniformSphereRandomVector` as a specialization of `RandomVectorImplementation` (see `lib/src/Uncertainty/Model`). This random vector is distributed uniformly on the sphere of center $c \in \mathbb{R}^n$ and radius $r > 0$. The sampling is done using the fact that $Y/\|Y\|$ is uniformly distributed over $\mathcal{S}_{n-1}$, the unit sphere in $\mathbb{R}^n$, if Y is an n dimensional random vector with independent $\mathcal{N}(0,1)$ components.
20. (*) `UniformBallRandomVector` as a specialization of `RandomVectorImplementation` (see `lib/src/Uncertainty/Model`). This random vector is distributed uniformly on the ball of center $c \in \mathbb{R}^n$ and radius $r > 0$. The sampling is done using the fact that $Y/\sqrt{\|Y\|^2 + Z}$ is uniformly distributed over $\mathcal{B}_{n-1}$, the unit ball in $\mathbb{R}^n$, if Y is an n dimensional random vector with independent $\mathcal{N}(0,1)$ components, and Z is $\mathcal{E}(1)$ independent from Y.
21. (*) `UniformSimplexRandomVector` as a specialization of `RandomVectorImplementation` (see `lib/src/Uncertainty/Model`). This random vector is distributed uniformly in the simplex given by $n+1$ points in $\mathbb{R}^n$. The sampling is done using the fact that Y is uniformly distributed over the standard simplex in $\mathbb{R}^n$ if it follows the Dirichlet distribution with parameter $(\theta_1 = 1, \dots, \theta_n = 1)$.

Module development 2/2

Projects

22. (**) SmoliakExperiment as a specialization of WeightedExperiment (see lib/src/Uncertainty/Algorithm/WeightedExperiment). This design of experiment is obtained by interfacing the smolpack C library. A possible name for the module is OTSmolpack.
23. (**) CubaIntegration as a specialization of IntegrationAlgorithmImplementation (see lib/src/Base/algo). This algorithm is obtained by interfacing the cuba C library. A possible name for the module is OTCuba.
24. (**) HIntLibIntegration as a specialization of IntegrationAlgorithmImplementation (see lib/src/Base/algo). This algorithm is obtained by interfacing the HIntLib C++ library, see https://github.com/JohannesBuchner/HIntLib. A possible name for the module is OTHIntLib.