OpenTURNS Users Day #18

Focus on

« The conditional distributions

of OpenTURNS v1.25 »

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June 2025

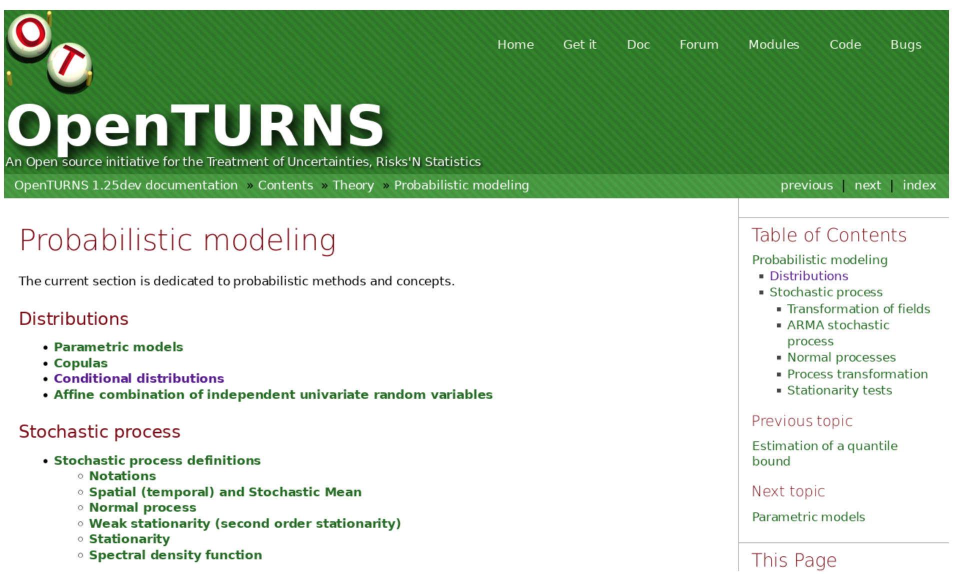
The conditional distributions of OpenTURNS v1.25

In the past releases:

- **BayesDistribution**: distribution of (Y,X) from the given X|Y and Y distributions
- **PosteriorDistribution:** Bayesian posterior distribution of Y from from the given X|Y and Y distributions and a sample of observations on X

Since the reease 1.25:

- **Case 1: Create a joint distribution using conditioning**
- **Case 2: Condition a joint distribution by some values of its marginals**
- **Case 3: Create a distribution whose parameters are random**
- **Case 4: Create a Bayesian posterior distribution**



Conditional distributions

The library offers some modelisation capacities on conditional distributions:

- Case 1: Create a joint distribution using conditioning,
- Case 2: Condition a joint distribution by some values of its marginals,
- Case 3: Create a distribution whose parameters are random,
- Case 4: Create a Bayesian posterior distribution.

The conditional distributions of OpenTURNS v1.25

❖Case 1: Create a joint distribution using conditioning

Case 1: Create a joint distribution using conditioning

The objective is to create the joint distribution of the random vector $(\boldsymbol{Y}, \boldsymbol{X})$ where $\boldsymbol{Y}$ follows the distribution $\mathcal{L}_{\boldsymbol{Y}}$ and $\boldsymbol{X}|\boldsymbol{\Theta}$ follows the distribution $\mathcal{L}_{\boldsymbol{X}|\boldsymbol{\Theta}}$ where $\boldsymbol{\Theta} = g(\boldsymbol{Y})$ with g a link function of input dimension the dimension of $\mathcal{L}_{\boldsymbol{Y}}$ and output dimension the dimension of $\boldsymbol{\Theta}$.

This distribution is limited to the continuous case, ie when both the conditioning and the conditioned distributions are continuous. Its probability density function is defined as:

$$f_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{y}, \boldsymbol{x}) = f_{\boldsymbol{X}|\boldsymbol{\theta}=g(\boldsymbol{y})}(\boldsymbol{x}|g(\boldsymbol{y}))f_{\boldsymbol{Y}}(\boldsymbol{y})$$

with $f_{\boldsymbol{X}|\boldsymbol{\theta}=g(\boldsymbol{y})}$ the PDF of the distribution of $\boldsymbol{X}|\boldsymbol{\Theta}$ where $\boldsymbol{\Theta}$ has been replaced by $g(\boldsymbol{y})$, $f_{\boldsymbol{Y}}$ the PDF of $\boldsymbol{Y}$.

See the class `JointByConditioningDistribution`.

The conditional distributions of OpenTURNS v1.25

❖Case 1: Create a joint distribution using conditioning

Example using the class:

Create a Joint by Conditioning distribution

In this example we are going to build the distribution of the random vector:

$$(Y, \boldsymbol{X} | \boldsymbol{\Theta})$$

with $\boldsymbol{X}$ conditioned by the random vector $\boldsymbol{\Theta}$ obtained with the random variable Y through a function f

$$\boldsymbol{\Theta} = f(\boldsymbol{Y})$$

```
import openturns as ot
import openturns.viewer as viewer

ot.Log.Show(ot.Log.NONE)
```

We consider the following case: $X|\boldsymbol{\Theta} \sim \mathcal{N}(\boldsymbol{\Theta})$ with $\boldsymbol{\Theta} = (Y, 0.1 + Y^2)$ and $Y \sim \mathcal{N}(0, 1)$.

We first create the Y distribution:

```
YDist = ot.Normal(0.0, 1.0)
```

Then we create the link function $f : y \rightarrow (y, 0.1 + y^2)$:

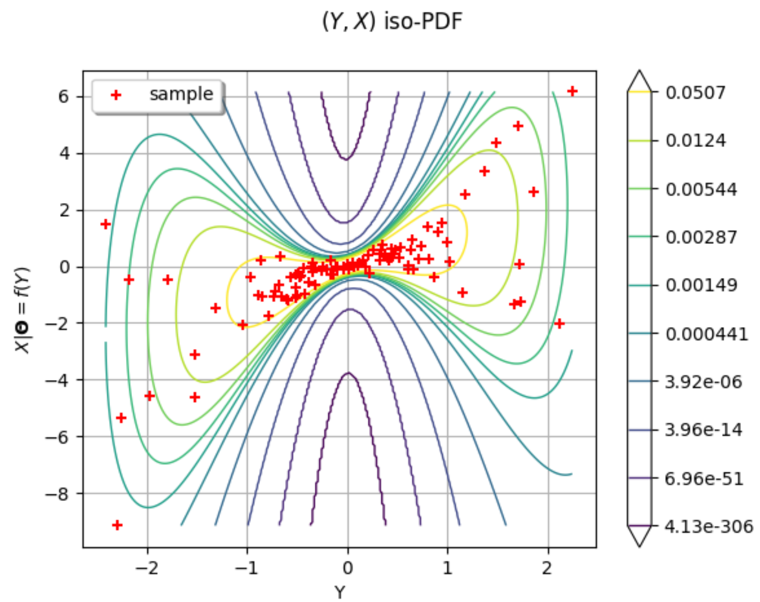
```
f = ot.SymbolicFunction(["y"], ["y", "0.1 + y^2"])
```

Then, we create the $\boldsymbol{X}|\boldsymbol{\Theta}$ distribution:

```
XgivenThetaDist = ot.Normal()
```

At last, we create the distribution of (Y, X):

```
XDist = ot.JointByConditioningDistribution(XgivenThetaDist, YDist, f)
XDist.setDescription(["Y", r"$X|\mathbf{\boldsymbol{\Theta}} = f(Y)$"])
XDist
```



The conditional distributions of OpenTURNS v1.25

❖Case 2: Condition a joint distribution by some values of its marginals

Case 2: Condition a joint distribution to some values of its marginals

Let $\boldsymbol{X}$ be a random vector of dimension d. Let $\mathcal{I} \subset \{1, \dots, d\}$ be a set of indices of components of $\boldsymbol{X}$, $\overline{\mathcal{I}}$ its complementary in $\{1, \dots, d\}$ and $\boldsymbol{x}_{\mathcal{I}}$ a real vector of dimension equal to the cardinal of $\mathcal{I}$. The objective is to create the distribution of:

$$\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}$$

See the class `PointConditionalDistribution`

This class requires the following features:

- each component X_i is continuous or discrete: e.g., it can not be a `Mixture` of discrete and continuous distributions,
- the copula of $\boldsymbol{X}$ is continuous: e.g., it can not be the `MinCopula`,
- the random vector $\boldsymbol{X}_{\overline{\mathcal{I}}}$ is continuous or discrete: all its components must be discrete or all its components must be continuous,
- the random vector $\boldsymbol{X}_{\mathcal{I}}$ may have some discrete components and some continuous components.

Then, the pdf (probability density function if $\boldsymbol{X}_{\overline{\mathcal{I}}}$ is continuous or probability distribution function if $\boldsymbol{X}_{\overline{\mathcal{I}}}$ is discrete) of $\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}$ is defined by (in the following expression, we assumed a particular order of the conditioned components among the whole set of components for easy reading):

$$p_{\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}}(x_{\overline{\mathcal{I}}}) = \frac{p_{\boldsymbol{X}}(\boldsymbol{x}_{\overline{\mathcal{I}}}, \boldsymbol{x}_{\mathcal{I}})}{p_{\boldsymbol{X}_{\mathcal{I}}}(\boldsymbol{x}_{\mathcal{I}})} \quad (1)$$

where:

$$p_{\boldsymbol{X}}(\boldsymbol{x}) = \left(\prod_{i=1}^{d} p_i(x_i) \right) c(F_1(x_1), \dots, F_d(x_d))$$

with:

- c is the probability density copula of $\boldsymbol{X}$,
- if X_i is a continuous component, p_i is its probability density function,
- if X_i is a discrete component, $p_i = \sum_{x_k^i \in \mathcal{S}^i} \mathbb{P}(X_i = x_k^i) \delta_{x_k^i}$ where $\mathcal{S}^i = \{x_k^i\}$ is its support and $\delta_{x_k^i}$ the Dirac distribution centered on x_k^i.

Then, if $\boldsymbol{X}_{\overline{\mathcal{I}}}$ is continuous, we have:

$$p_{\boldsymbol{X}_{\mathcal{I}}}(\boldsymbol{x}_{\mathcal{I}}) = \int p_{\boldsymbol{X}}(\boldsymbol{x}_{\overline{\mathcal{I}}}, \boldsymbol{x}_{\mathcal{I}}) \, \mathrm{d}\boldsymbol{x}_{\overline{\mathcal{I}}}$$

and if $\boldsymbol{X}_{\overline{\mathcal{I}}}$ is discrete with its support denoted by $\mathcal{S}(\boldsymbol{X}_{\overline{\mathcal{I}}}) = \prod_{i \in \overline{\mathcal{I}}} \mathcal{S}^i$, we have:

$$p_{\boldsymbol{X}_{\mathcal{I}}}(\boldsymbol{x}_{\mathcal{I}}) = \sum_{\boldsymbol{x}_{\overline{\mathcal{I}}} \in \mathcal{S}(\boldsymbol{X}_{\overline{\mathcal{I}}})} p_{\boldsymbol{X}}(\boldsymbol{x}_{\overline{\mathcal{I}}}, \boldsymbol{x}_{\mathcal{I}})$$

The conditional distributions of OpenTURNS v1.25

❖Case 2: Condition a joint distribution by some values of its marginals

❖ Simplification mecanisms

Mixture distributions Let $\boldsymbol{X}$ be a random vector of dimension d which distribution is defined by a `Mixture` of N discrete or continuous atoms. Let denote by $(p_1, \dots, p_N)$ the PDF (Probability Density Function for continuous atoms and Probability Distribution Function for discrete one) of each atom, with respective weights $(w_1, \dots, w_N)$. Then we get:

$$p_{\boldsymbol{X}}(\boldsymbol{x}) = \sum_{k=1}^{N} w_k p_k(\boldsymbol{x})$$

Conclusion: The conditional distribution of a Mixture is a Mixture of conditional distributions.

Kernel Mixture distributions: The Kernel Mixture distribution is a particular `Mixture`: all the weights are identical and all the kernels of the combination are of the same discrete or continuous family. The kernels are centered on the sample points. The multivariate kernel is a tensorized product of the same univariate kernel.

Let $\boldsymbol{X}$ be a random vector of dimension d defined by a Kernel Mixture distribution based on the sample $(\boldsymbol{s}_1, \dots, \boldsymbol{s}_n)$ and the kernel K. In the continuous case, k is the kernel PDF and we have:

$$p_{\boldsymbol{X}}(\boldsymbol{x}) = \sum_{q=1}^{n} \frac{1}{n} p_q(\boldsymbol{x})$$

the kernel on the remaining components $\boldsymbol{x}_{\mathcal{I}}$ and which weights α_q are proportional to:

$$\alpha_q \propto p_{q,\mathcal{I}}(\boldsymbol{x}_{\mathcal{I}}) = \prod_{j \in \mathcal{I}} \frac{1}{h^j} k\left(\frac{x^j - s_q^j}{h^j}\right)$$

The conditional distributions of OpenTURNS v1.25

❖Case 2: Condition a joint distribution by some values of its marginals

Simplification mecanisms

Truncated distributions: Let $\boldsymbol{X}$ be a random vector of dimension d which PDF is $p_{\boldsymbol{X}}$. Let $\mathcal{D}$ be a domain of $\mathbb{R}^d$ and let $\boldsymbol{X}_{\mathcal{T}} = \boldsymbol{X} | \boldsymbol{X} \in \mathcal{D}$ be the random vector $\boldsymbol{X}$ truncated to the domain $\mathcal{D}$. It has the following PDF:

$$p_{\boldsymbol{X}_{\mathcal{T}}}(\boldsymbol{x}) = \frac{1}{\alpha} p_{\boldsymbol{X}}(\boldsymbol{x}) 1_{\mathcal{D}}(\boldsymbol{x})$$

Conclusion: The conditional distribution of a truncated distribution is the truncated distribution of the conditional distribution. Care: the truncation domains are not exactly the same.

Elliptical distributions: This is the case for normal and Student distributions. If $\boldsymbol{X}$ follows a normal or a Student distribution, then $\boldsymbol{X}_{\overline{\mathcal{I}}}$ respectively follows a normal or a Student distribution with modified parameters. See **Conditional Normal** and **Conditional Student** for the formulas of the conditional distributions.

The conditional distributions of OpenTURNS v1.25

❖Case 2: Condition a joint distribution by some values of its marginals

Numerical range

Note that the numerical range of the conditional distribution might be different from the range of the numerical range of the non conditioned distribution. For example, consider a bivariate distribution (X_0, X_1) following a normal distribution with zero mean, unit variance and a correlation $R(0,1) = 0.4$. Then consider $X_1 | X_0 = 10.0$. The numerical range of $X_1 | X_0 = 10$ is $[-3.01, 11.0]$ where as the numerical range of X_1 is $[-7.65, 7.65]$. See **Create a Point Conditional Distribution** to get some more examples.

The computation of the numerical range is important to make possible the integration of the PDF on some domains. The library implements 3 strategies to compute it. We detail these strategies.

Strategy None: The numerical range of $\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}$ is the same as the numerical range of $\boldsymbol{X}_{\overline{\mathcal{I}}}$. This range is exact for all distributions with bounded support. For distributions with unbounded support, it is potentially false when the conditional values are very close to the bounds of the initial numerical support.

Strategy Normal: Let $\boldsymbol{Y}$ be the Gaussian vector of dimension d, which mean vector is defined by $\boldsymbol{\mu} = \mathbb{E}\left[\boldsymbol{X}\right]$ and covariance matrix is defined by $\boldsymbol{C} = \mathrm{Cov}\left(\boldsymbol{X}\right)$. Then, we build the conditioned Gaussian vector:

$$\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}$$

The numerical range $\mathcal{D}\left(\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}\right)$ of $\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}$ is known exactly thanks to the simplification mechanism implemented for Gaussian vectors. We assign to $\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}$ the range $\mathcal{D}\left(\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}\right)$:

$$\mathcal{D}\left(\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}\right) = \mathcal{D}\left(\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}\right)$$

The conditional distributions of OpenTURNS v1.25

❖Case 2: Condition a joint distribution by some values of its marginals

Numerical range

Strategy NormalCopula: Let $\boldsymbol{Y}$ be the Gaussian vector of dimension d, with zero mean, unit variance and which correlation matrix $\boldsymbol{R}$ is defined from the Spearman correlation matrix of $\boldsymbol{X}$: $(\rho_S(X_i, X_j))_{1 \leq i,j \leq d}$. Thus, $\boldsymbol{Y}$ is the standard representant of the normal copula having the same correlation as $\boldsymbol{X}$.

For each conditioning value x_i, we define the quantile q_i of the normal distribution with zero mean and unit variance associated to the same order as x_i, for $i \in \mathcal{I}$:

$$q_i = \Phi^{-1} \circ F_i\left(x_i\right)$$

where Φ is the CDF of the normal distribution with zero mean and unit variance. Then, we build the conditioned Gaussian vector:

$$\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{q}_{\mathcal{I}}$$

which numerical range $\mathcal{D}\left(\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{q}_{\mathcal{I}}\right)$ can be exactly computed. Let it be:

$$\mathcal{D}\left(\boldsymbol{Y}_{\overline{\mathcal{I}}} | \boldsymbol{Y}_{\mathcal{I}} = \boldsymbol{q}_{\mathcal{I}}\right) = \prod_{i \in \overline{\mathcal{I}}} \left[y_i^{min}, y_i^{max}\right]$$

Then, inversely, we compute the quantiles of each F_i for $i \in \mathcal{I}$ which have the same order as the bounds y_i^{min} and y_i^{max} with respect Φ:

$$\begin{aligned} x_i^{min} &= F_i^{-1} \circ \Phi\left(y_i^{min}\right) \\ x_i^{max} &= F_i^{-1} \circ \Phi\left(y_i^{max}\right) \end{aligned}$$

We assign to $\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}$ the numerical range defined by:

$$\mathcal{D}\left(\boldsymbol{X}_{\overline{\mathcal{I}}} | \boldsymbol{X}_{\mathcal{I}} = \boldsymbol{x}_{\mathcal{I}}\right) = \prod_{i \in \overline{\mathcal{I}}} \left[x_i^{min}, x_i^{max}\right]$$

The conditional distributions of OpenTURNS v1.25

❖Case 2: Condition a joint distribution by some values of its marginals

Example using the class:

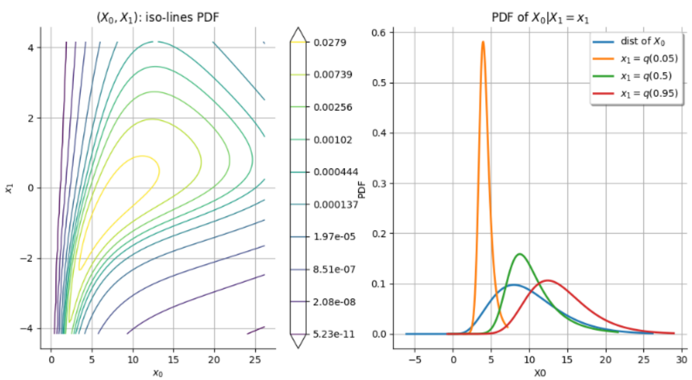
Create a Point Conditional Distribution

In this example, we create distributions conditionned by a particular value of their marginals.

We consider the following cases:

- Case 1: Bivariate distribution with continuous marginals,
- Case 2: Bivariate distribution with continuous and discrete marginals,
- Case 3: Trivariate distribution with continuous and discrete marginals,
- Case 4: Extreme values conditioning.

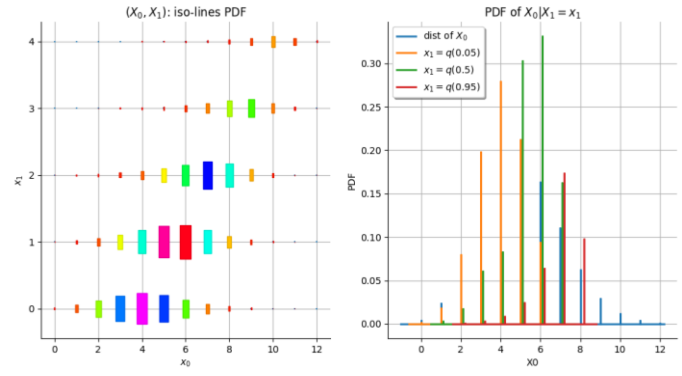
We illustrate the fact that the range of the conditioned distribution is updated.



Variable	Distribution	Parameter
X_0	Gamma (k, λ)	$(k, \lambda) = (5, 0.5)$
X_1	Student (ν)	$\nu = 5$
Copula	ClaytonCopula (θ)	$\theta = 2.5$

$X_0 \mid X_1 = x_1$

X_1 = quantile 0,05, 0,5 and0,95



Variable	Distribution	Parameter
X_0	Binomial (n, p)	$(n, p) = (25, 0.2)$
X_1	Poisson (λ)	$\lambda = 1$
Copula	GumbelCopula (θ)	$\theta = 2$

The conditional distributions of OpenTURNS v1.25

❖Case 2: Condition a joint distribution by some values of its marginals

Example using the class:

Create a Point Conditional Distribution

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- Case 1: Bivariate distribution with continuous marginals,
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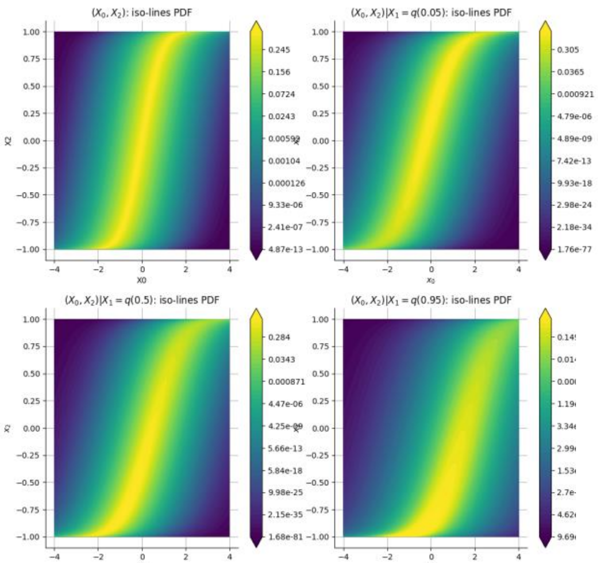
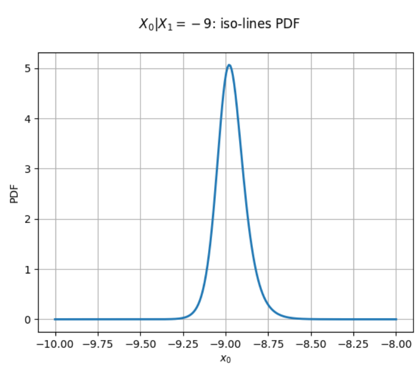


Variable	Distribution	Parameter
X_0	Normal (μ, σ)	$(\mu, \sigma) = (0, 1)$
X_1	Poisson (λ)	$\lambda = 1$
X_2	Uniform (a, b)	$(a, b) = (-1, 1)$
Copula	NormalCopula (R)	see below

$X_0 | X_1 = x_1$
X_1 = quantile 0,05, 0,5 and 0,95



$X_0 | X_1$ = -9,0

Variable	Distribution	Parameter
X_0	Normal (μ, σ)	$(\mu, \sigma) = (0, 1)$
X_1	Normal (μ, σ)	$(\mu, \sigma) = (0, 1)$
Copula	ClaytonCopula (θ)	$\theta = 2$

The numerical range has moved: [-7,65, 7,65] ➔ [-11,77, -0,82]

The conditional distributions of OpenTURNS v1.25

❖Case 3: Create a distribution whose parameters are random

Case 3: Create a distribution whose parameters are random

The objective is to create the marginal distribution of $\boldsymbol{X}$ in Case 1.

See the class `DeconditionedDistribution`.

This class requires the following features:

- the $\boldsymbol{X}$ may be continuous, discrete or neither: e.g., it can be a `Mixture` of discrete and continuous distributions. In that case, its parameters set is the union of the parameters set of each of its atoms (the weights of the mixture are not considered as parameters).
- each component Y_i is continuous or discrete: e.g., it can not be a `Mixture` of discrete and continuous distributions, (so that the random vector $\boldsymbol{Y}$ may have some discrete components and some continuous components),
- the copula of $\boldsymbol{Y}$ is continuous: e.g., it can not be the `MinCopula`,
- if $\boldsymbol{Y}$ has both discrete components and continuous components, its copula must be the independent copula. The general case has not been implemented yet.

We define:

$$p_{\boldsymbol{Y}}(\boldsymbol{y}) = \left(\prod_{i=1}^{d} p_i(y_i)\right) c(F_1(x_1), \dots, F_d(x_d))$$

where:

- c is the probability density copula of $\boldsymbol{Y}$,
- if Y_i is a continuous component, p_i is its probability density function,
- if Y_i is a discrete component, $p_i = \sum_{y_k^i \in \mathcal{S}^i} \mathbb{P}\left(Y_i = y_k^i\right) \delta_{y_k^i}$ where $\mathcal{S}^i = \{y_k^i\}$ is its support and $\delta_{y_k^i}$ the Dirac distribution centered on y_k^i.

Then, the PDF of $\boldsymbol{X}$ is defined by:

$$p_{\boldsymbol{X}}(\boldsymbol{x}) = \int p_{\boldsymbol{X}|\boldsymbol{\Theta}=g(\boldsymbol{y})}(\boldsymbol{x}|g(\boldsymbol{y})) p_{\boldsymbol{Y}}(\boldsymbol{y}) \, \mathrm{d}\boldsymbol{y}$$

with the same convention as for $\boldsymbol{Y}$.

Note that this is always possible to create the random vector $\boldsymbol{X}$ whatever the distribution of $\boldsymbol{\Theta}$: see the class `DeconditionedRandomVector`. But remember that a `DeconditionedRandomVector` (and more generally a `RandomVector`) can only be sampled.

The conditional distributions of OpenTURNS v1.25

❖Case 3: Create a distribution whose parameters are random

Create a deconditioned distribution

Example using the class:

In this example we are going to build the distribution of the random vector $\boldsymbol{X}$ defined by the conditional distribution of:

$$\boldsymbol{X}|\boldsymbol{\Theta}$$

where $\boldsymbol{\Theta}$ is the output of the random variable $\boldsymbol{Y}$ through the link function f:

$$\boldsymbol{\Theta} = f(\boldsymbol{Y})$$
$$\boldsymbol{Y} \sim \mathcal{L}_{\boldsymbol{Y}}$$

This example creates a `DeconditionedDistribution` which offers all the methods attached to the distributions.

We consider the case where X is of dimension 1 and follows a uniform distribution defined by:

Variable	Distribution	Parameter
X	`Uniform` (a, b)	$(a, b) = (Y, 1 + Y^2)$
Y	`Uniform` (c, d)	$(c, d) = (-1, 1)$

```
import openturns as ot
import openturns.viewer as otv
```

Create the Y distribution.

```
YDist = ot.Uniform(-1.0, 1.0)
```

Create the link function $f: y \rightarrow (y, 1 + y^2)$.

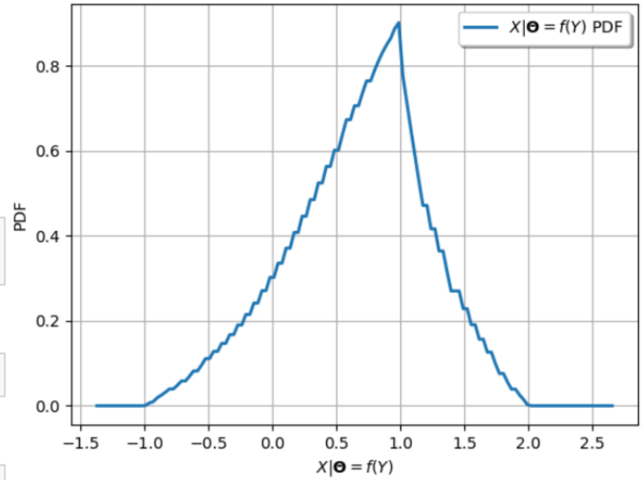
```
f = ot.SymbolicFunction(["y"], ["y", "1+y^2"])
```

Create the conditional distribution of $\boldsymbol{X}|\boldsymbol{\Theta}$: as the parameters have no importance, we use the default distribution.

```
XgivenThetaDist = ot.Uniform()
```

Create the deconditioned distribution of X.

```
XDist = ot.DeconditionedDistribution(XgivenThetaDist, YDist, f)
XDist.setDescription([r"$X|\mathbf{\boldsymbol{\Theta}} = f(Y)$"])
XDist
```



❖Case 4: Create a Bayesian posterior distribution

Case 4: Create a Bayesian posterior distribution

Consider the random vector $\boldsymbol{X}$ where $\boldsymbol{X}|\boldsymbol{\Theta}$ follows the distribution $\mathcal{L}_{\boldsymbol{X}|\boldsymbol{\Theta}}$, with $\boldsymbol{\Theta} = g(\boldsymbol{Y})$ and $\boldsymbol{Y}$ following the prior distribution $\mathcal{L}_{\boldsymbol{Y}}$. The function g is a link function which input dimension is the dimension of $\mathcal{L}_{\boldsymbol{Y}}$ and which output dimension the dimension of $\boldsymbol{\Theta}$.

The objective is to create the posterior distribution of $\boldsymbol{Y}$ given that we have a sample $(\boldsymbol{x}_1, \dots, \boldsymbol{x}_n)$ of $\boldsymbol{X}$.

See the class `PosteriorDistribution`.

This class requires the following features:

- the $\boldsymbol{X}$ may be continuous, discrete or neither: e.g., it can be a `Mixture` of discrete and continuous distributions. In that case, its parameters set is the union of the parameters set of each of its atoms (the weights of the mixture are not considered as parameters).
- each component Y_i is continuous or discrete: e.g., it can not be a `Mixture` of discrete and continuous distributions, (the random vector $\boldsymbol{Y}$ may have some discrete components and some continuous components),
- the copula of $\boldsymbol{Y}$ is continuous: e.g., it can not be the `MinCopula`.

If $\boldsymbol{Y}$ and $\boldsymbol{X}$ are continuous random vector, then the posterior PDF of $\boldsymbol{Y}$ is defined by:

$$f_{\boldsymbol{Y}|\boldsymbol{X}_1=\boldsymbol{x}_1,\dots,\boldsymbol{X}_n=\boldsymbol{x}_n}(\boldsymbol{y}) = \frac{f_{\boldsymbol{Y}}(\boldsymbol{y})\prod_{i=1}^n f_{\boldsymbol{X}|\boldsymbol{\theta}=g(\boldsymbol{y})}(\boldsymbol{x}_i)}{\int f_{\boldsymbol{Y}}(\boldsymbol{y})\prod_{i=1}^n f_{\boldsymbol{X}|\boldsymbol{\theta}=g(\boldsymbol{y})}(\boldsymbol{x}_i)d\boldsymbol{y}} \quad (5)$$

with $f_{\boldsymbol{X}|\boldsymbol{\theta}=g(\boldsymbol{y})}$ the PDF of the distribution of $\boldsymbol{X}|\boldsymbol{\Theta}$ where $\boldsymbol{\Theta}$ has been replaced by $g(\boldsymbol{y})$ and $f_{\boldsymbol{Y}}$ the PDF of the prior distribution of $\boldsymbol{Y}$.

Note that the denominator of (5) is the PDF of the deconditioned distribution of $\boldsymbol{X}|\boldsymbol{\Theta} = g(\boldsymbol{Y})$ with respect to the prior distribution of $\boldsymbol{Y}$.

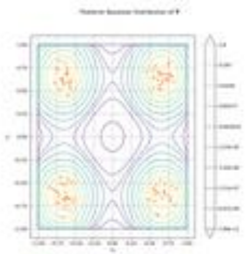
In the other cases, the PDF is the probability distribution function for the discrete components and the $\int$ are replaced by some $\sum$.

The conditional distributions of OpenTURNS v1.25

❖Case 4: Create a Bayesian posterior distribution

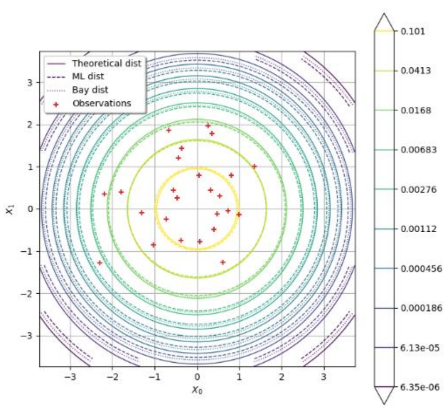
Example using the class:

Examples using the class



Compare frequentist and Bayesian estimation

Initial distribution, ML estimated dist and Bayesian estimated dist.



Compare frequentist and Bayesian estimation

In this example, we want to estimate of the parameter $\boldsymbol{\Theta}$ of the distribution of a random vector $\boldsymbol{X}$ from which we have some data. We compare the frequentist and the Bayesian approaches to estimate $\boldsymbol{\theta}$.

Let $\boldsymbol{X} = (X_0, X_1)$ be a random vector following a bivariate normal distribution with zero mean, unit variance and independent components: $\mathcal{N}_2(\boldsymbol{\theta})$ where the parameter $\boldsymbol{\theta} = (\mu_0, \sigma_0, \mu_1, \sigma_1, \rho) = (0, 1, 0, 1, 0)$. We assume to have a sample generated from $\boldsymbol{X}$ denoted by $(\boldsymbol{x}_1, \dots, \boldsymbol{x}_n)$ where $n = 25$.

We assume to know the parameters (μ_0, μ_1, ρ) and we want to estimate the parameters (σ_0, σ_1). In the Bayesian approach, we assume that $\boldsymbol{\Theta} = (0, \Sigma_0, 0, \Sigma_1, 0)$ is a random vector and we define a link function $g : \mathbb{R}^2 \to \mathbb{R}^5$ such that:

$$\boldsymbol{\Theta} = g(\boldsymbol{Y})$$

where $\boldsymbol{Y}$ follows the prior distribution denoted by $\pi^0_{\boldsymbol{Y}}$.

Note that the link function g already contains the information that the two components of $\boldsymbol{X}$ are independent (as $\rho = 0$) and centered (as $\mu_i = 0$).

We consider two interesting link functions:

$$g_1(\boldsymbol{y}) = (0, y_0, 0, y_1, 0)$$
$$g_2(\boldsymbol{y}) = (0, 0.5 + y_0^2, 0, 0.5 + y_1^2, 0)$$

each one being associated to the respective prior distributions:

$$\pi^{0,1}_{\boldsymbol{Y}} = \mathcal{T}(0, 1, 2) \times \mathcal{T}(0, 1, 2)$$
$$\pi^{0,2}_{\boldsymbol{Y}} = \mathcal{T}(-1, 0, 1) \times \mathcal{T}(-1, 0, 1)$$

The second case is such that the link function g_2 is not bijective on the range of $\pi^{0,2}_{\boldsymbol{Y}}$.

```
conditioned = ot.Normal(2)
conditioned.setDescription(["X0", "X1"])
```

```
linkFunction = ot.SymbolicFunction(["u0", "u1"], ["0.0", "u0", "0.0", "u1", "0.0"])
```

```
deconditioned = ot.DeconditionedDistribution(conditioned, conditioning, linkFunction)
```

```
posterior_Y = otexp.PosteriorDistribution(deconditioned, observations)
```

The conditional distributions of OpenTURNS v1.25

- Coming soon: interval conditioning

$$X_I \mid X_J \subset D$$