

Generalized Pareto distribution

Generalized Pareto

parameters:	$\mu \in (-\infty, \infty)$ location (real) $\sigma \in (0, \infty)$ scale (real) $\xi \in (-\infty, \infty)$ shape (real)
support:	$x \geq \mu$ ($\xi \geq 0$) $\mu \leq x \leq \mu - \sigma/\xi$ ($\xi < 0$)
pdf:	$\frac{1}{\sigma} (1 + \xi z)^{-(1/\xi+1)}$ where $z = \frac{x - \mu}{\sigma}$
cdf:	$1 - (1 + \xi z)^{-1/\xi}$
mean:	$\mu + \frac{\sigma}{1 - \xi}$ ($\xi < 1$)
median:	$\mu + \frac{\sigma(2^\xi - 1)}{\xi}$
variance:	$\frac{\sigma^2}{(1 - \xi)^2(1 - 2\xi)}$ ($\xi < 1/2$)

The family of **generalized Pareto distributions (GPD)** has three parameters μ , σ and ξ .

The cumulative distribution function is

$$F_{(\xi, \mu, \sigma)}(x) = \begin{cases} 1 - \left(1 + \frac{\xi(x - \mu)}{\sigma}\right)^{-1/\xi} & \text{for } \xi \neq 0, \\ 1 - \exp\left(-\frac{x - \mu}{\sigma}\right) & \text{for } \xi = 0. \end{cases}$$

for $x \geq \mu$ when $\xi \geq 0$, and $\mu \leq x \leq \mu - \sigma/\xi$ when $\xi < 0$, where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ the scale parameter and $\xi \in \mathbb{R}$ the shape parameter. Note that some references give the "shape parameter" as $\kappa = -\xi$.

The probability density function is:

$$f_{(\xi, \mu, \sigma)}(x) = \frac{1}{\sigma} \left(1 + \frac{\xi(x - \mu)}{\sigma}\right)^{(-\frac{1}{\xi}-1)}.$$

or

$$f_{(\xi, \mu, \sigma)}(x) = \frac{\sigma^{\frac{1}{\xi}}}{(\sigma + \xi(x - \mu))^{\frac{1}{\xi}+1}}.$$

again, for $x \geq \mu$, and $x \leq \mu - \sigma/\xi$ when $\xi < 0$.

Generating generalized Pareto random variables

If U is uniformly distributed on $(0,1]$, then

$$X = \mu + \frac{\sigma(U^{-\xi} - 1)}{\xi} \sim \text{GPD}(\mu, \sigma, \xi).$$

In Matlab Statistics Toolbox, you can easily use "gprnd" command to generate generalized Pareto random numbers.

With GNU R you can use the packages POT or evd with the "rgpd" command (see for exact usage: <http://rss.acs.unt.edu/Rdoc/library/POT/html/simGPD.html>)

References

- Balkema, A., and Laurens de Haan (1974). "Residual life time at great age", *Annals of Probability*, **2**, 792€804.
- Pickands, J. (1975). "Statistical inference using extreme order statistics", *Annals of Statistics*, **3**, 119€131.

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