Airbus Group Innovations

OTRobOpt - Robust Optimization toolkit

21st of June, Journée OT, EDF Lab Chatou

Nabil RACHDI

acknowledgments to:
IRT SystemX, R. Lebrun, V. Srithammavanh, J-C. Fort

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Outline

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Context

The present work and development was done in the frame of the **IRT SystemX**, ROM project (2013-2016), where Airbus Group is involved in.

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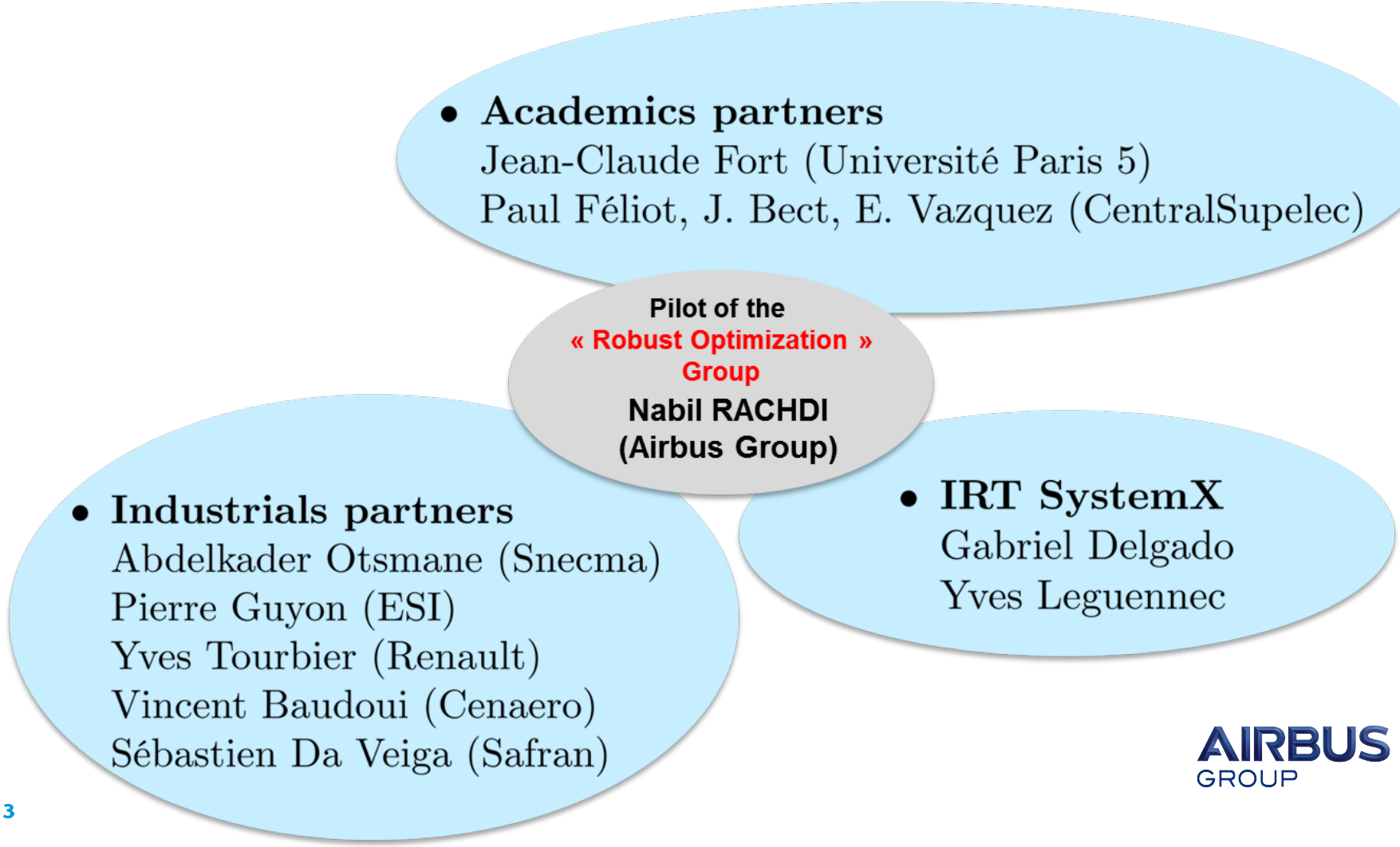

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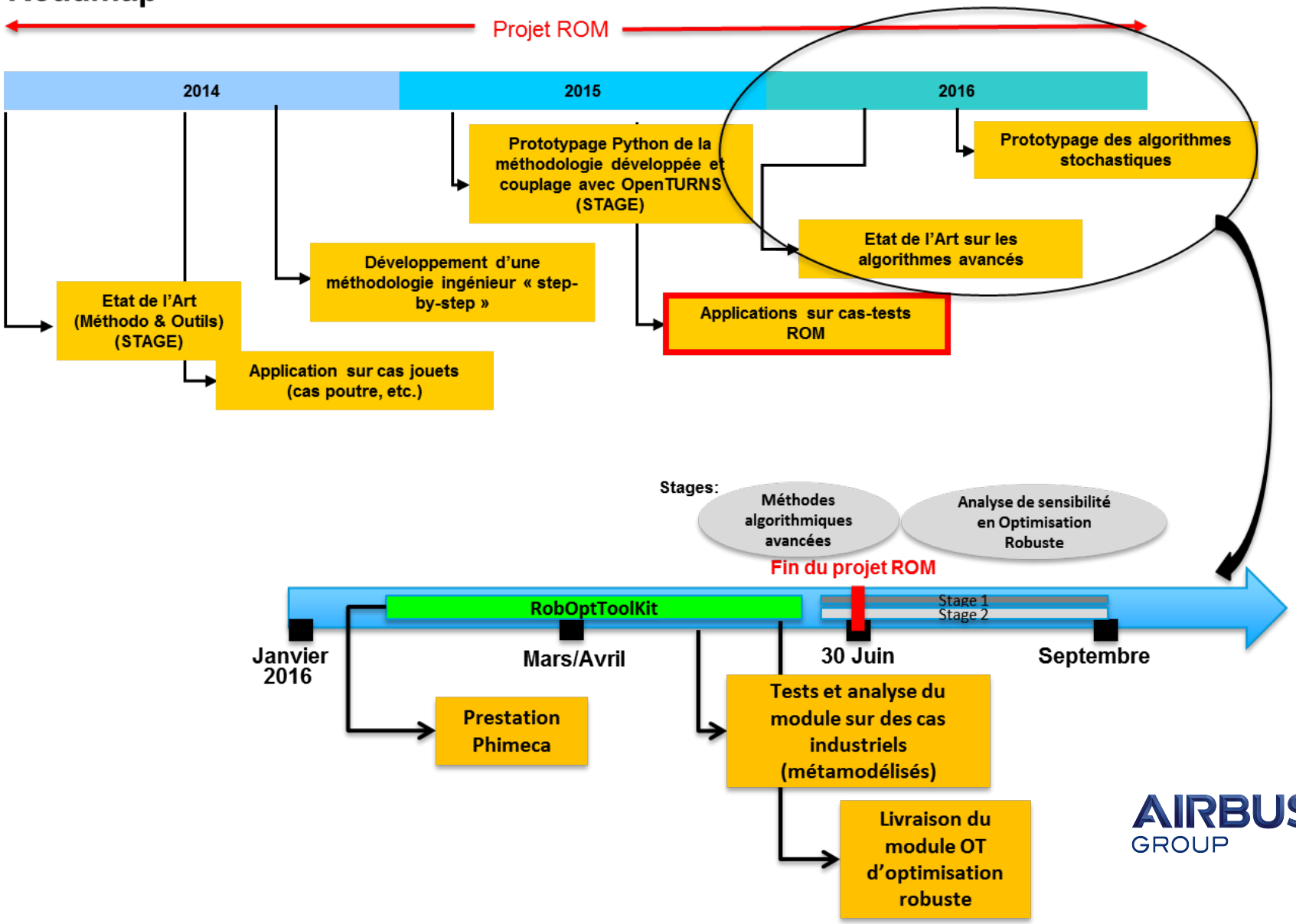
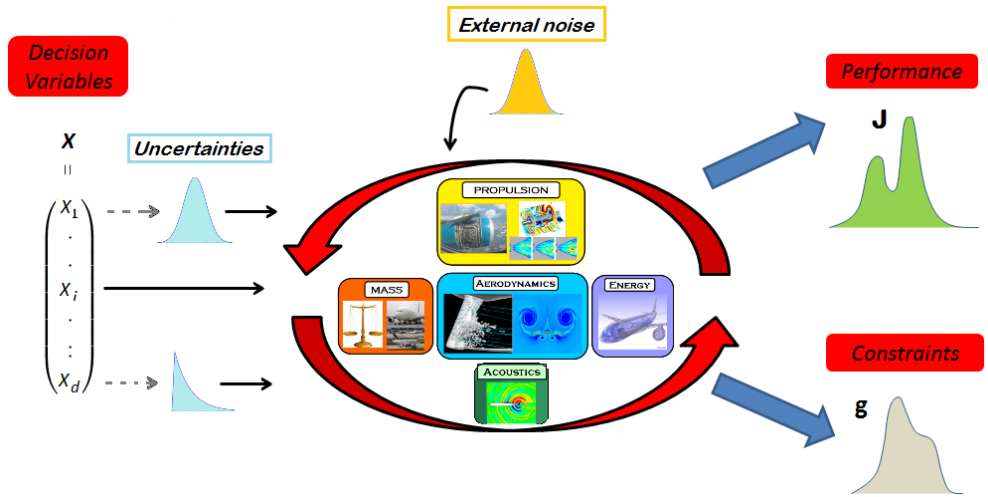
Context

Roadmap



Context of the Robust Optimization



- Cost function J is derived mainly from "functional/system" consideration (mass, consumption, range of action, etc.)
- Constraints $\mathbf{g}$ are very often "complex physical simulation" (no analytical expression) : temperature, aerodynamic coeff., displacement, etc.
- The interaction between the objective function J and constraints $\mathbf{g}$ induces couplings "physics-system" (e.g thermal regulation : system = air conditioning pack / physics = fluid behavior in the cabin)

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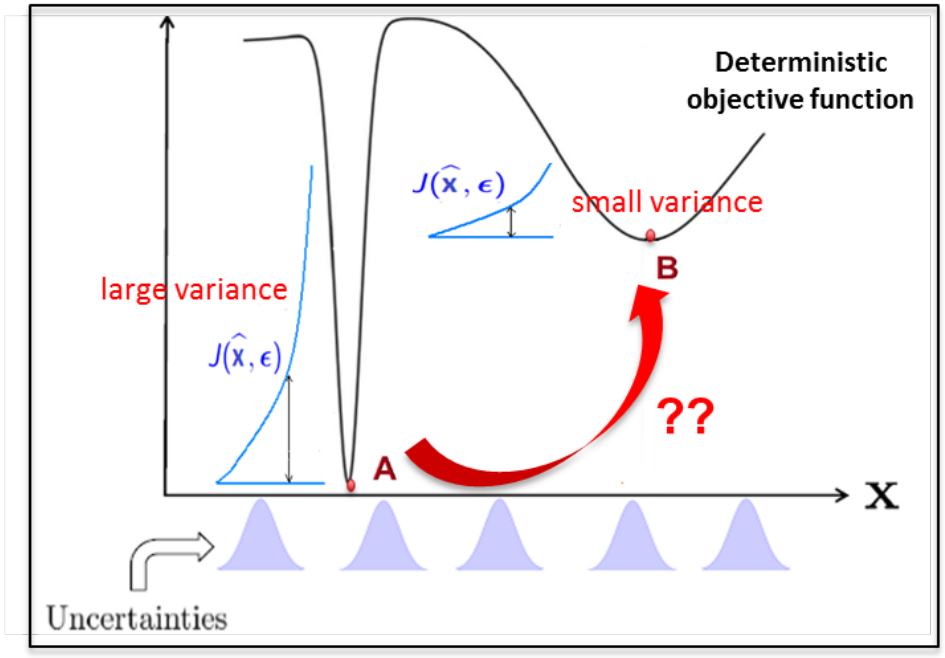
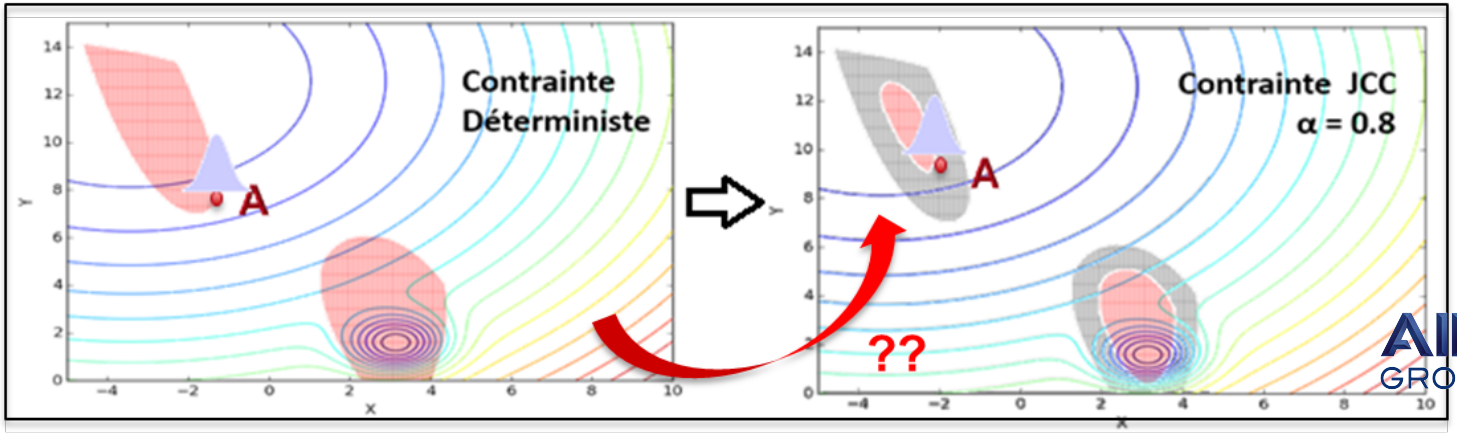
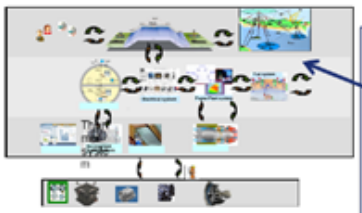
"Engineer practice"





Different contexts

VARIOUS CONTEXTS

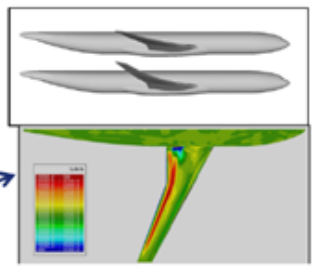


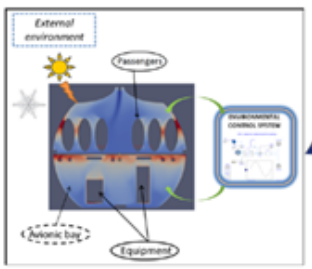
Architecture of Systems: *complexity & trade-offs*

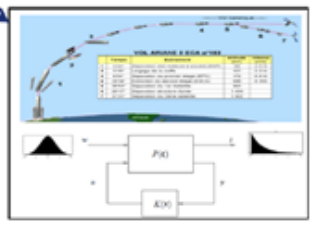
Aeroelasticity: *multiphysics & strong couplings*

Optimal Control / command: *real time on top of*

Thermal regulation: *Systems & Physics loops*







ISSUES

- **Margin Quantification**
  and its impact on the design
- **Understand / Interpret**
  the uncertainties impact on optimization process
- **Provide acceptable design(s)**
  insensitive to conditions changing

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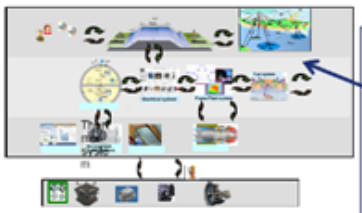
Different contexts

VARIOUS CONTEXTS



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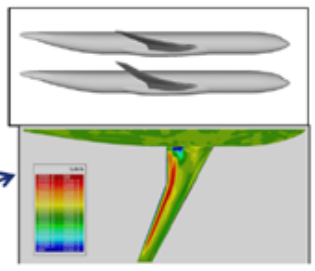

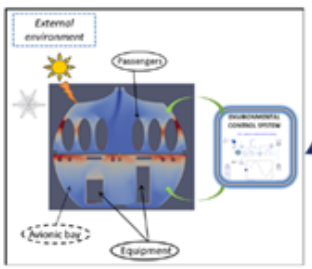

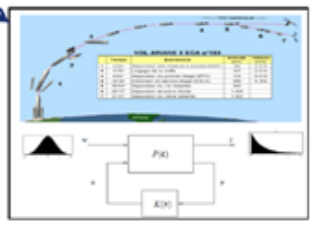

ISSUES

- **Margin Quantification** and its impact on the design
- **Understand / Interpret** the uncertainties impact on optimization process
- **Provide acceptable design(s)** insensitive to conditions changing

Robust Optimization $\Longrightarrow$ Optimization under Uncertainties

- **Robustness** = refer to uncertainties on **objective(s) function(s)**
- **Reliability** = refer to uncertainties on **constraints functions**

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Challenges

- The main industrial challenge in this context is to exhibit the (lack of) knowledge the engineers have hidden in the modeling phase
- It would provide more action levels to decision makers
- Example of action : Margin Allocation
  - "Inverse problem" : uncertainty characterization
  - "Optimization problem" : design optimization under uncertainties

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Challenges

- From a classical (parametrized) optimization problem ...

$$\min_{\mathbf{x},\, \mathbf{g}(\mathbf{x},\mathbf{u}) \leq \mathbf{0}} J(\mathbf{x}, \mathbf{u})$$

where $\mathbf{u}$ is some vector of values used in the problem (often hidden !)

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Challenges

- From a classical (parametrized) optimization problem ...

$$\min_{\mathbf{x},\, \mathbf{g}(\mathbf{x},\mathbf{u})\leq \mathbf{0}} J(\mathbf{x},\mathbf{u})$$

where $\mathbf{u}$ is some vector of values used in the problem (often hidden !)

how to take into account uncertainties on $\mathbf{u}$ and/or on the design $\mathbf{x}$?

$\rightarrow$ For simplicity, data $\boldsymbol{\xi}$ will aggregate all sources of uncertainties (on design, on environmental constants, etc.)
$\rightarrow$ $\boldsymbol{\xi}$ is modelled as a random vector with distribution $\mathbb{P}_{\boldsymbol{\xi}}$

- In practice, the engineer solves a parametrized optimization problem

$$\min_{\mathbf{x},\, \mathbf{g}(\mathbf{x},\boldsymbol{\xi}(\omega))\leq \mathbf{0}} J(\mathbf{x},\boldsymbol{\xi}(\omega))$$

for some scenario $\boldsymbol{\xi}(\omega)$ of the uncertainties that he has fixed before

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Outline

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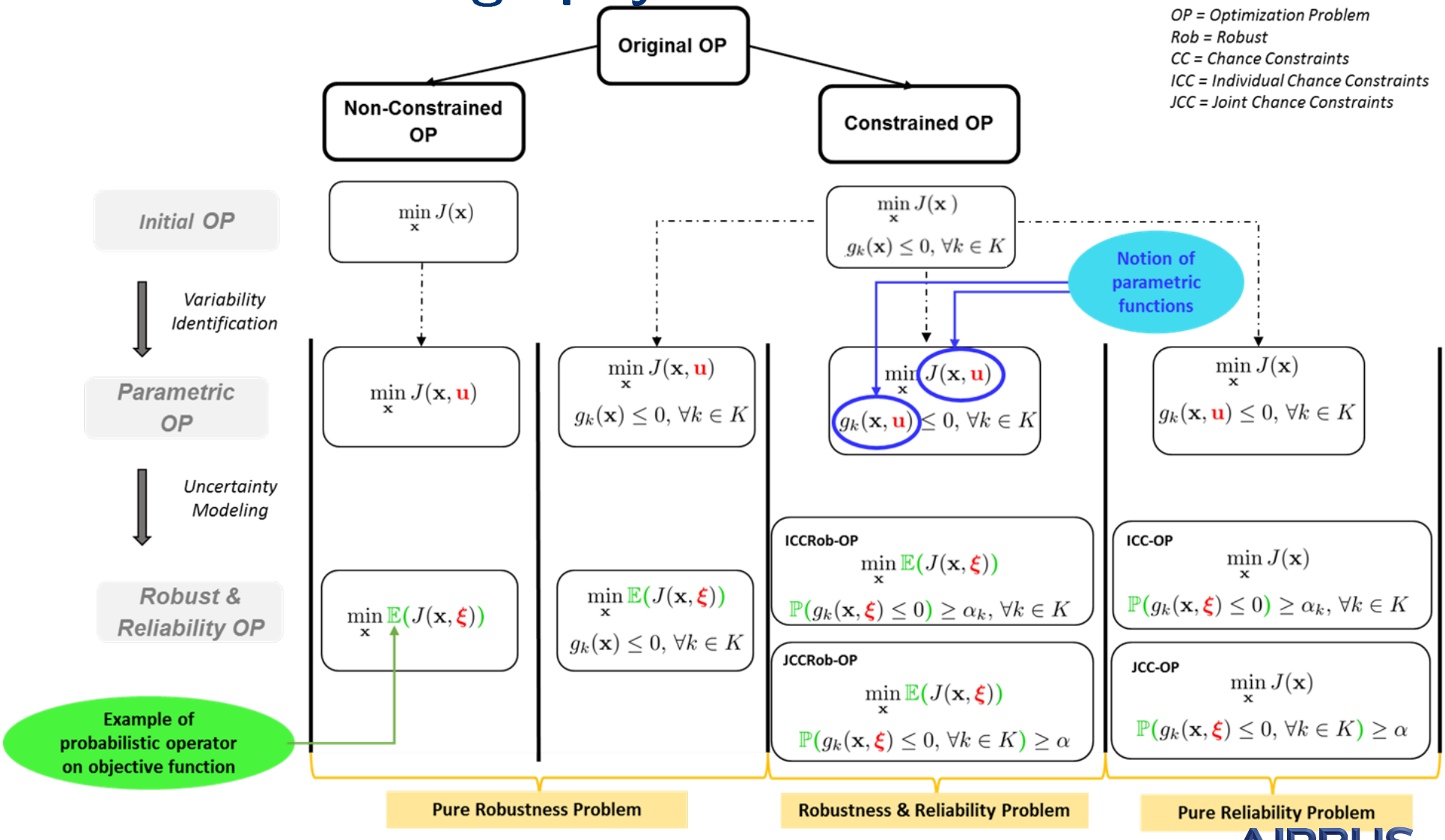
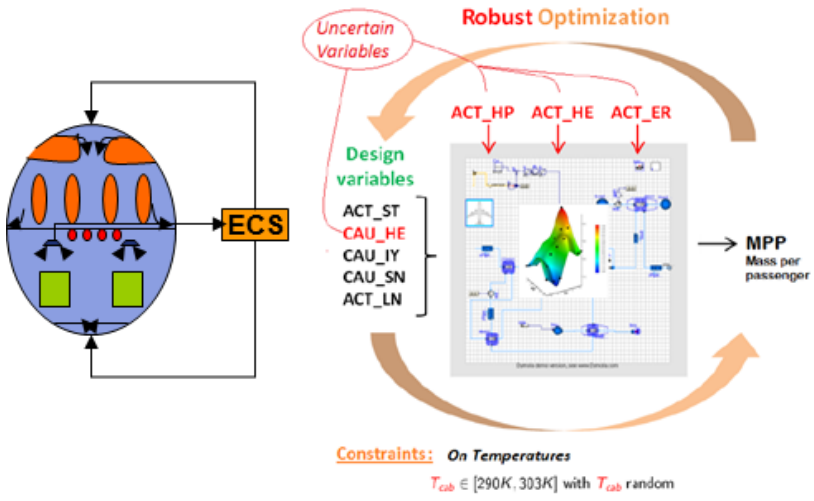
Problems cartography



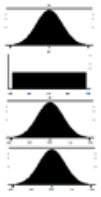
4-Steps methodology

Step RO-1 « Problem Specification »



Step RO-2 « Robustness & Reliability Formulation »

- **Probabilistic Modeling:**
  - ACT_{HP} = *Truncated Normal* $[125, 8.33, 100, 150]$
  - ACT_{HE} = *Uniform* $[450, 550]$
  - ACT_{ER} = *Normal* $[0, 1]$
  - $CAUerr_{HE}$ = *Normal* $[0, 0.03]$



- **Formulation adopted:**

Robustness

Mean value of the objective function

$$MPP(\mathbf{x}, \xi)$$

$$\rho(MPP(\mathbf{x}, \xi)) = \mathbb{E}(MPP(\mathbf{x}, \xi))$$

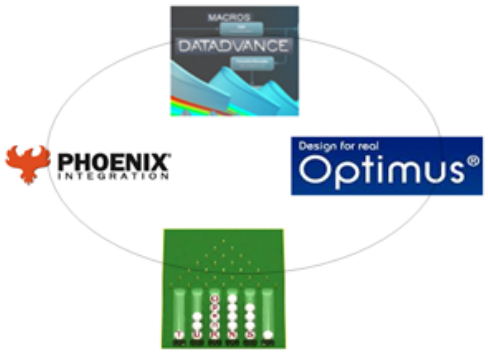
Reliability

Individual Chance Constraints

$$\begin{cases} 290 \leq PS_{TM(N,X)}(\mathbf{x}, \xi) \leq 303 \\ 10 \leq CB_{TIN}(\mathbf{x}, \xi) \leq 30 \end{cases}$$

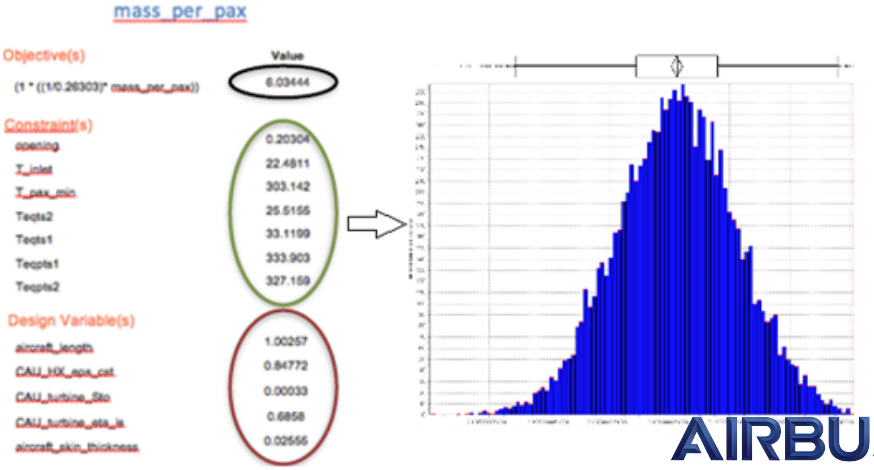
$$\begin{cases} \mathbb{P}\,(290 \leq PS_{TM(N,X)}(\mathbf{x}, \xi) \leq 303) \geq 95\% \\ \mathbb{P}\,(10 \leq CB_{TIN}(\mathbf{x}, \xi) \leq 30) \geq 95\% \end{cases}$$

Step RO-3 « Problem Resolution »



OBJECTIVE	CONVERGED X					CONVERGED SOLUTION		FEASIBILITY
	ACT_ST	CAU_HE	CAU_IY	CAU_SN	ACT_LN	F	Fe	
MPP	0.025000	0.720000	0.900000	3.300E-04	1.982143	0.273206	0.001536	8.876E-06

Step RO-4 « Feedback Analysis »



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Operators

The uncertainties are classically treated by operators

$$\min_{\mathbf{x}} \quad \rho_J(J(\mathbf{x}, \boldsymbol{\xi}))$$

$$s.c$$

$$\rho_{\mathbf{g}}(\mathbf{g}(\mathbf{x}, \boldsymbol{\xi}) \leq 0) \geq \boldsymbol{\alpha}$$

Objective operators

- *Mean* : $\rho_J(J(\mathbf{x}, \boldsymbol{\xi})) = \mathbb{E}(J(\mathbf{x}, \boldsymbol{\xi}))$
- *Variance* : $\rho_J(J(\mathbf{x}, \boldsymbol{\xi})) = \mathrm{Var}(J(\mathbf{x}, \boldsymbol{\xi}))$
- *Worst-case* : $\rho_J(J(\mathbf{x}, \boldsymbol{\xi})) = \sup_{\mathbf{u} \in supp(\xi)} J(\mathbf{x}, \mathbf{u})$
- τ-*Quantile* : $\rho_J(J(\mathbf{x}, \boldsymbol{\xi})) = inf\{q \in \mathbb{R}, \mathbb{P}(J(\mathbf{x}, \boldsymbol{\xi}) \leq q) \geq \tau\}$
  ($\tau = 1 \Leftrightarrow$ worst-case)
- *Mean-Variance tradeoff* : $\rho_J^{\lambda}(J(\mathbf{x}, \boldsymbol{\xi})) = \mathbb{E}(J(\mathbf{x}, \boldsymbol{\xi})) + \lambda\, \mathrm{Var}^{1/2}(J(\mathbf{x}, \boldsymbol{\xi}))$
- *Etc.*

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Operators

The uncertainties are classically treated by operators

$$\min_{\mathbf{x}} \quad \rho_J(J(\mathbf{x}, \boldsymbol{\xi}))$$

$$s.c$$

$$\rho_{\mathbf{g}}(\mathbf{g}(\mathbf{x}, \boldsymbol{\xi}) \leq 0) \geq \boldsymbol{\alpha}$$

Constraints operators

- Individual Chance-Constraints (ICC) : given **reliability levels** $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_p)$

$$\rho_{\mathbf{g}}(\mathbf{g}(\mathbf{x}, \boldsymbol{\xi}) \leq 0) = (\mathbb{P}(g_1(\mathbf{x}, \boldsymbol{\xi}) \leq 0), \cdots, \mathbb{P}(g_p(\mathbf{x}, \boldsymbol{\xi}) \leq 0))$$

$$\mathcal{D}^{ICC}_{\mathbb{P}^{\xi}, \mathbf{g}, \boldsymbol{\alpha}} = \bigcap_{j=1}^{p} \{\mathbf{x} \in \mathcal{X}, \ \mathbb{P}(g_j(\mathbf{x}, \boldsymbol{\xi}) \leq 0) \geq \alpha_j\}$$

- Joint Chance-Constraints (JCC) : given **the (global) reliability level** $\boldsymbol{\alpha} = \alpha$

$$\rho_{\mathbf{g}}(\mathbf{g}(\mathbf{x}, \boldsymbol{\xi}) \leq 0) = \mathbb{P}\left(\bigcap_{j=1}^{p} g_j(\mathbf{x}, \boldsymbol{\xi}) \leq 0\right)$$

$$\mathcal{D}^{JCC}_{\mathbb{P}^{\xi}, \mathbf{g}, \boldsymbol{\alpha}} = \left\{\mathbf{x} \in \mathcal{X}, \ \mathbb{P}\left(\bigcap_{j=1}^{p} g_j(\mathbf{x}, \boldsymbol{\xi}) \leq 0\right) \geq \alpha\right\}$$

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Operators

The uncertainties are classically treated by operators

$$\min_{\mathbf{x}} \quad \rho_J(J(\mathbf{x}, \boldsymbol{\xi}))$$

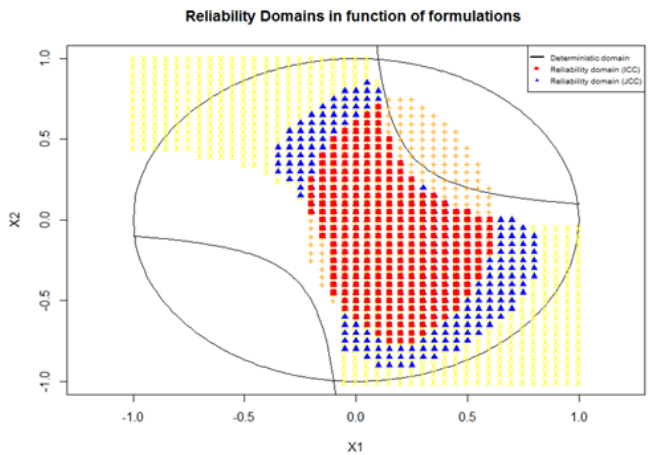
$$s.c$$

$$\rho_{\mathbf{g}}(\mathbf{g}(\mathbf{x}, \boldsymbol{\xi}) \leq 0) \geq \boldsymbol{\alpha}$$

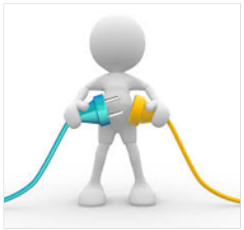
Constraints operators

- Reliability domains depend on the formulation adopted

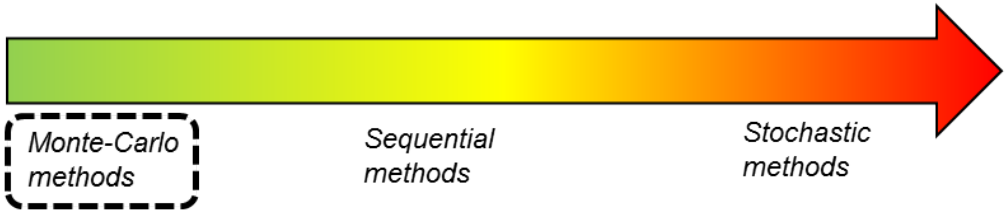
$$\mathcal{D}^{ICC}_{\mathbb{P}^{\xi},\mathbf{g},\alpha} \neq \mathcal{D}^{JCC}_{\mathbb{P}^{\xi},\mathbf{g},\boldsymbol{\alpha}}$$



Resolution methods we investigate



Plug-In



Build-In

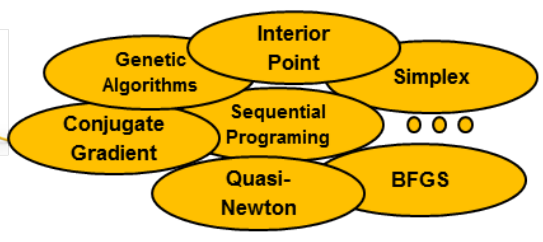
Monte-Carlo methods:

- **Principle**: *Discretize all probabilistic operators (integrals) by crude Monte-Carlo sampling*

$$\mathbb{E}(J(\mathbf{x},\Xi)) \longleftrightarrow \frac{1}{N}\sum_{i=1}^{N} J(\mathbf{x},\Xi_i) \qquad \mathbb{P}\left(g(\mathbf{x},\Xi)\leq 0\right) \longleftrightarrow \frac{1}{M}\sum_{j=1}^{M} \mathbf{1}_{g(\mathbf{x},\Xi_j)\leq 0}$$

- **Resolution**: *Plug your preffered optimizer*

$$\min_{\mathbf{x}} \frac{1}{N}\sum_{i=1}^{N} J(\mathbf{x},\Xi_i) \quad s.t. \quad \frac{1}{M}\sum_{j=1}^{M} \mathbf{1}_{g(\mathbf{x},\Xi_j)\leq 0} \geq \alpha$$



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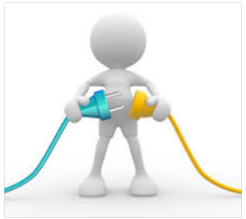
Resolution methods we investigate



Plug-In

Monte-Carlo methods — *Sequential methods* — *Stochastic methods*

Build-In

Sequential methods:

- **Principle:** *Solve sequentially uncertainty-free optimisation problems and process stochastic analysis (to update constraints)*
- **Resolution:**

 # Example

$START \quad s = 0$

$$\min_{\mathbf{x}} J(\mathbf{x}) \quad s.t. \quad g(\mathbf{x}) \leq s$$

Genetic Algorithms, Interior Point, Simplex, Conjugate Gradient, Sequential Programing, Quasi-Newton, BFGS

$\mathbf{x}_{opt}$

$\mathbb{P}\left(g(\mathbf{x}_{opt}, \Xi) \leq 0\right) \geq \alpha\,?$

No: $Update \quad s = s - \gamma$

Yes: $STOP$

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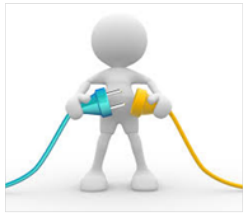
Resolution methods we investigate



Plug-In

Monte-Carlo methods — *Sequential methods* — *Stochastic methods*

Build-In

Stochastic methods:

- **Principle:** Iterative methods avoiding stochastic simulation at each optimization iteration (i.e no or few integrals computations)
- **Resolution:**

$(\Xi_k)_{k\geq 1}$ is a i.i.d sequence drawn from Ξ

From Gradient Descent ...

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \gamma_k \nabla \mathbb{E}(J(\mathbf{x}_k, \Xi))$$

Deterministic path

... To Stochastic Gradient

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \gamma_k \nabla J(\mathbf{x}_k, \Xi_{k+1})$$

Stochastic path *(Markov Chain)*

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Outline

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OTRobOpt - context

- OTRobOpt is a software development specified by the Robust Optimization Group of the ROM project, and funded by this project
- Phimeca has developed this tool (thanks Julien !)
- A great support of Régis !

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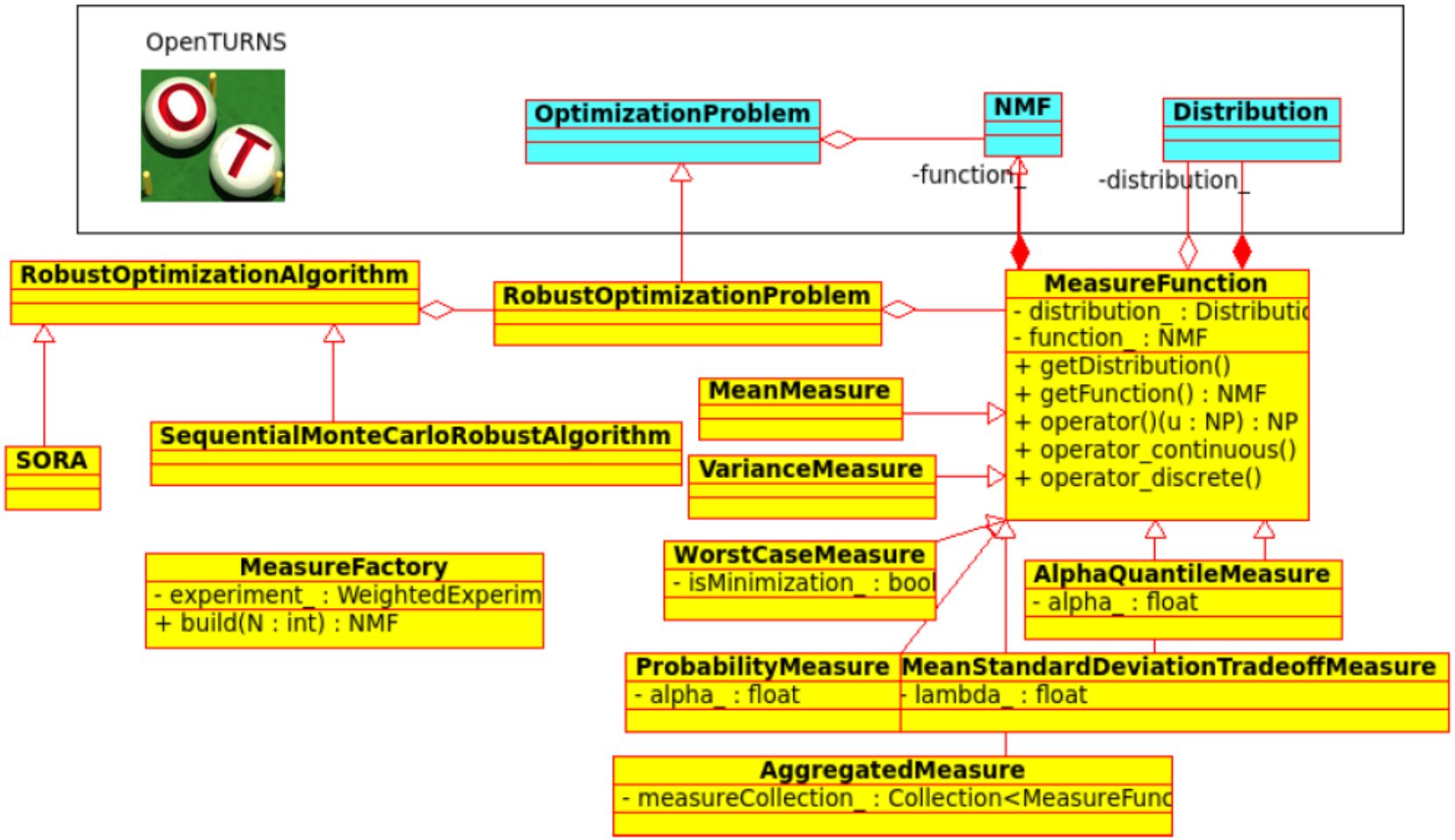
OTRobOpt - architecture

Diagramme de classe



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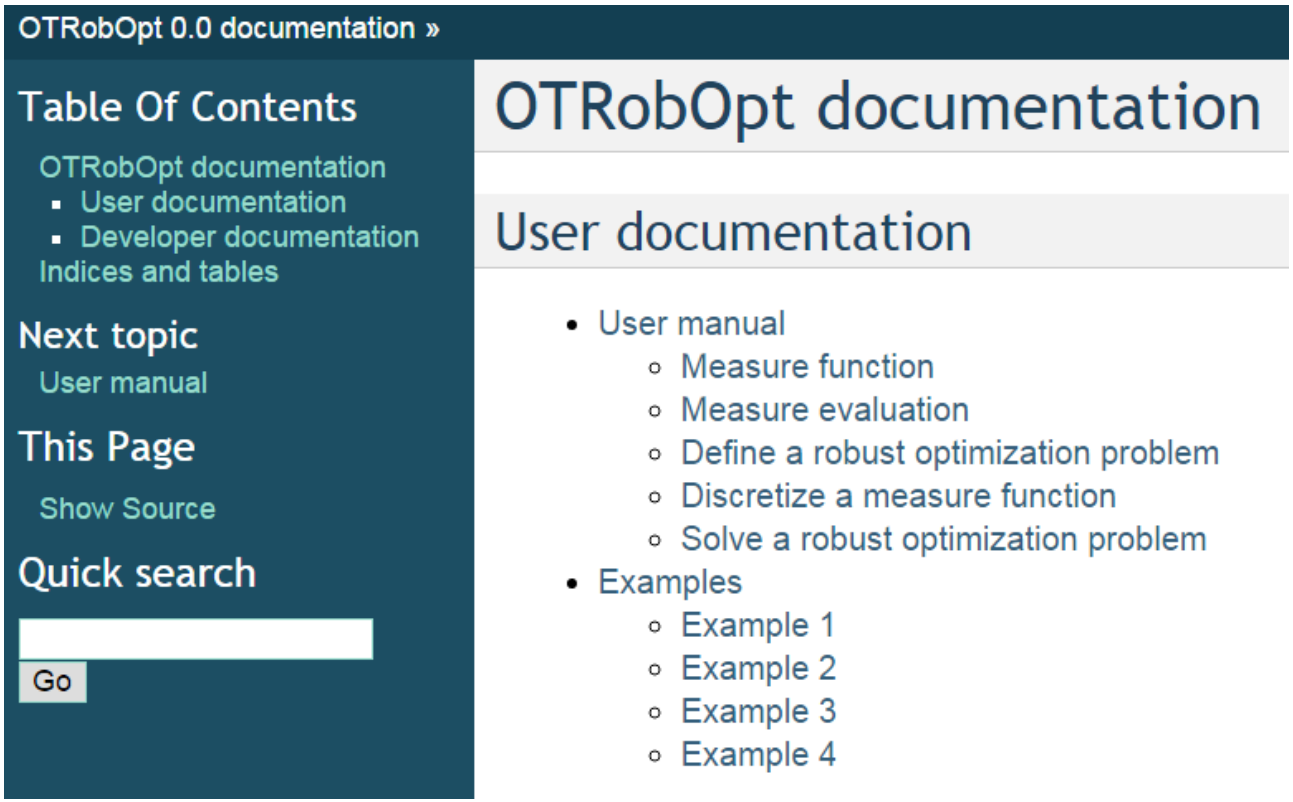
OTRobOpt - in practice

http ://openturns.github.io/otrobopt/master



OTRobOpt - in practice

OTRobOpt 0.0 documentation »

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User manual

The goal is to formulate and solve robust optimization problem.

A robust optimization problem consists of a parametric objective objective $J(x, \theta)$ and/or a parametric inequality constraint $G(x, \theta)$ where x is a design variable and θ a parameter.

$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & J(x, \theta) \\ \text{subject to} \quad & G(x, \theta) \geq 0 \end{aligned}$$

The problem is made robust by:

- modelling the parameter θ by the the random vector Θ with given distribution $\mathcal{D}$.
- choosing measure functions $\rho_{J,\mathcal{D}}$ and $\lambda_{G,\mathcal{D}}$ for the objective and constraint functions.

The the robust optimization problem reads:

$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & \rho_{J,\mathcal{D}}(x) \\ \text{subject to} \quad & \lambda_{G,\mathcal{D}}(x) \geq 0 \end{aligned}$$

The definition of the measure functions is associated to the concept of `MeasureEvaluation`.

A measure evaluation can be used through `MeasureFunction` to expose generic function services.

A robust optimization problem can be defined with `RobustOptimizationProblem`, and then solved using a `RobustOptimizationAlgorithm`.

Note that this measure evaluation can be discretized over θ so as to define a deterministic optimization problem using `MeasureFactory`.

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OTRobOpt - in practice

Measure function

`MeasureFunction`(*args)	Measure function.

Measure evaluation

`MeasureEvaluation`(*args)	Measure evaluation base class.
`MeanMeasure`(*args)	Mean measure function.
`MeanStandardDeviationTradeoffMeasure`(*args)	Mean/variance tradeoff measure function.
`QuantileMeasure`(*args)	Quantile measure function.
`WorstCaseMeasure`(*args)	Worst case measure function.
`VarianceMeasure`(*args)	Variance measure function.
`JointChanceMeasure`(*args)	Joint chance measure function.
`IndividualChanceMeasure`(*args)	Individual chance measure function.
`AggregatedMeasure`(*args)	Aggregated measure function.

Define a robust optimization problem

`RobustOptimizationProblem`(*args)	Robust optimization problem.

Discretize a measure function

`MeasureFactory`(*args)	Discretize a measure function.

Solve a robust optimization problem

`RobustOptimizationAlgorithm`(*args)	Robust optimization algorithm base class.
`SequentialMonteCarloRobustAlgorithm`(*args)	Sequential Monte Carlo robust optimization algorithm.

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OTRobOpt - in practice

Example 4

1- Problem statement

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \mathbb{E}_{\mathcal{D}}(\cos(x)\sin(\Theta)) \\ \text{subject to} \quad & \mathbb{P}_{\mathcal{D}}(-2 - x + \Theta \leq 0) \geq 0.9 \\ & x - 4 \leq 0 \\ & \Theta \sim \mathcal{U}(0, 2) \end{aligned}$$

2- Solution

$$x^* = \pi$$

OTRobOpt - in practice

3- Resolution

```
#!/usr/bin/env python

from __future__ import print_function
import openturns as ot
import otrobopt

#ot.Log.Show(ot.Log.Info)


calJ = ot.NumericalMathFunction(['x', 'theta'], ['J'], ['cos(x) * sin(theta)'])
calG = ot.NumericalMathFunction(['x', 'theta'], ['g1', 'g2'], ['-(-2 - x + theta)', '-(x - 4)'])
J = ot.NumericalMathFunction(calJ, [1], [1.0])
g = ot.NumericalMathFunction(calG, [1], [1.0])

dim = J.getInputDimension()


solver = ot.Cobyla()
solver.setMaximumIterationNumber(1000)
solver.setStartingPoint([0.0] * dim)

thetaDist = ot.Uniform(0.0, 2.0)
robustnessMeasure = otrobopt.MeanMeasure(J, thetaDist)
reliabilityMeasure = otrobopt.JointChanceMeasure(g, thetaDist, ot.Greater(), 0.9)
problem = otrobopt.RobustOptimizationProblem(robustnessMeasure, reliabilityMeasure)

algo = otrobopt.SequentialMonteCarloRobustAlgorithm(problem, solver)
algo.setMaximumIterationNumber(10)
algo.setMaximumAbsoluteError(1e-3)
algo.setInitialSamplingSize(10)
algo.run()
result = algo.getResult()
print ('x*=', result.getOptimalPoint(), 'J(x*)=', result.getOptimalValue(), 'iteration=', result.getIterationNumber())
```

AIRBUS GROUP

OTRobOpt - Some users already !

http://trac.openturns.org/attachment/blog/slides%209th%20users%20day/Pres_otrobopt_juin2016.pdf

Exemple : conception déterministe

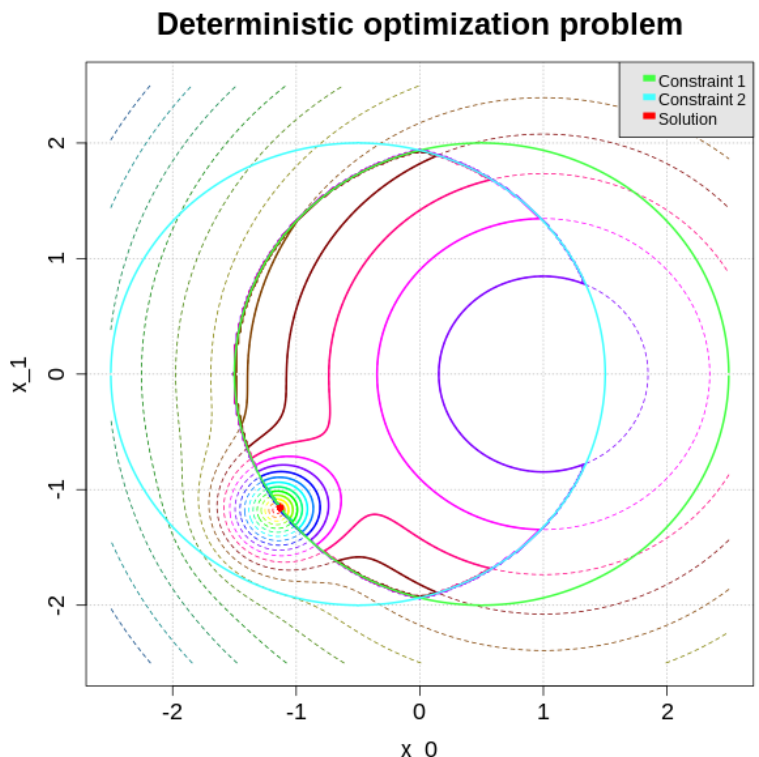
Design optimal d'une pièce de dimension (x_0, x_1) optimisant le critère $h(x_0, x_1)$ sous les contraintes $g_1^{det} \geq 0$ et $g_2^{det} \geq 0$:

- Fonction objectif : $h(\underline{x}) = 15\left[(x_0 - 1)^2 + x_1^2\right] - 160e^{-5\left[(x_0+1.2)^2+(x_1+1.2)^2\right]}$
- Contraintes : $g_1^{det}(\underline{x}) = 4 - \left[(x_0 - 0.5)^2 + x_1^2\right] \geq 0$ et $g_2^{det}(\underline{x}) = 4 - \left[(x_0 + 0.5)^2 + x_1^2\right] \geq 0$.

$$(S_{cl}) : \quad x_{cl}^* = \underset{\substack{g_1^{det}(\underline{x}) \geq 0 \\ g_2^{det}(\underline{x}) \geq 0}}{\arg\min} \; h(\underline{x})$$

$\Longrightarrow \underline{x}_{cl}^* = (-1.02, -1.08)$

. . .mais la réalisation de la pièce a une précision $\underline{\theta}$!



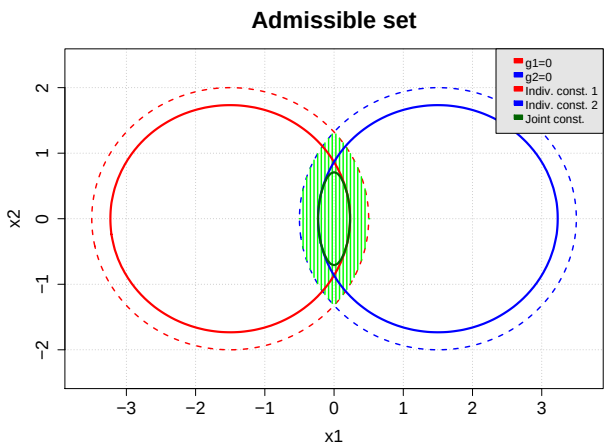
Exemple : conception robuste

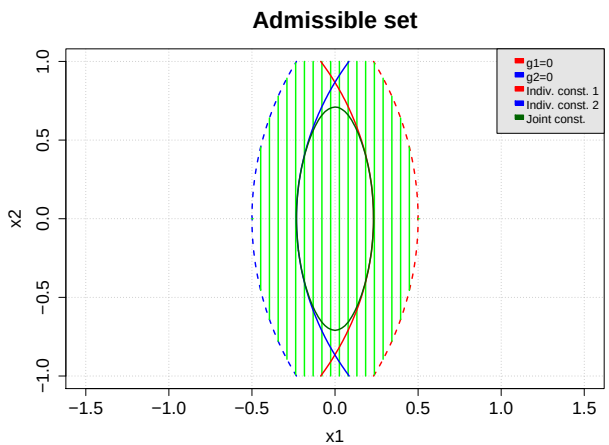
Modélisation robuste : la réalisation réelle = réalisation spécifiée $\underline{x}$ + un bruit centré $\underline{\theta} \sim \mathcal{N}(\underline{0}, \sigma I_2)$:

- Fonction objectif : $J(\underline{x}, \underline{\theta}) = h(\underline{x} + \underline{\theta}) \Longrightarrow$ **variable aléatoire !**
- Contraintes : $g_1(\underline{x}, \underline{\theta}) = g_1^{det}(\underline{x} + \underline{\theta})$ et $g_2(\underline{x}, \underline{\theta}) = g_2^{det}(\underline{x} + \underline{\theta}) \Longrightarrow$ **idem !**

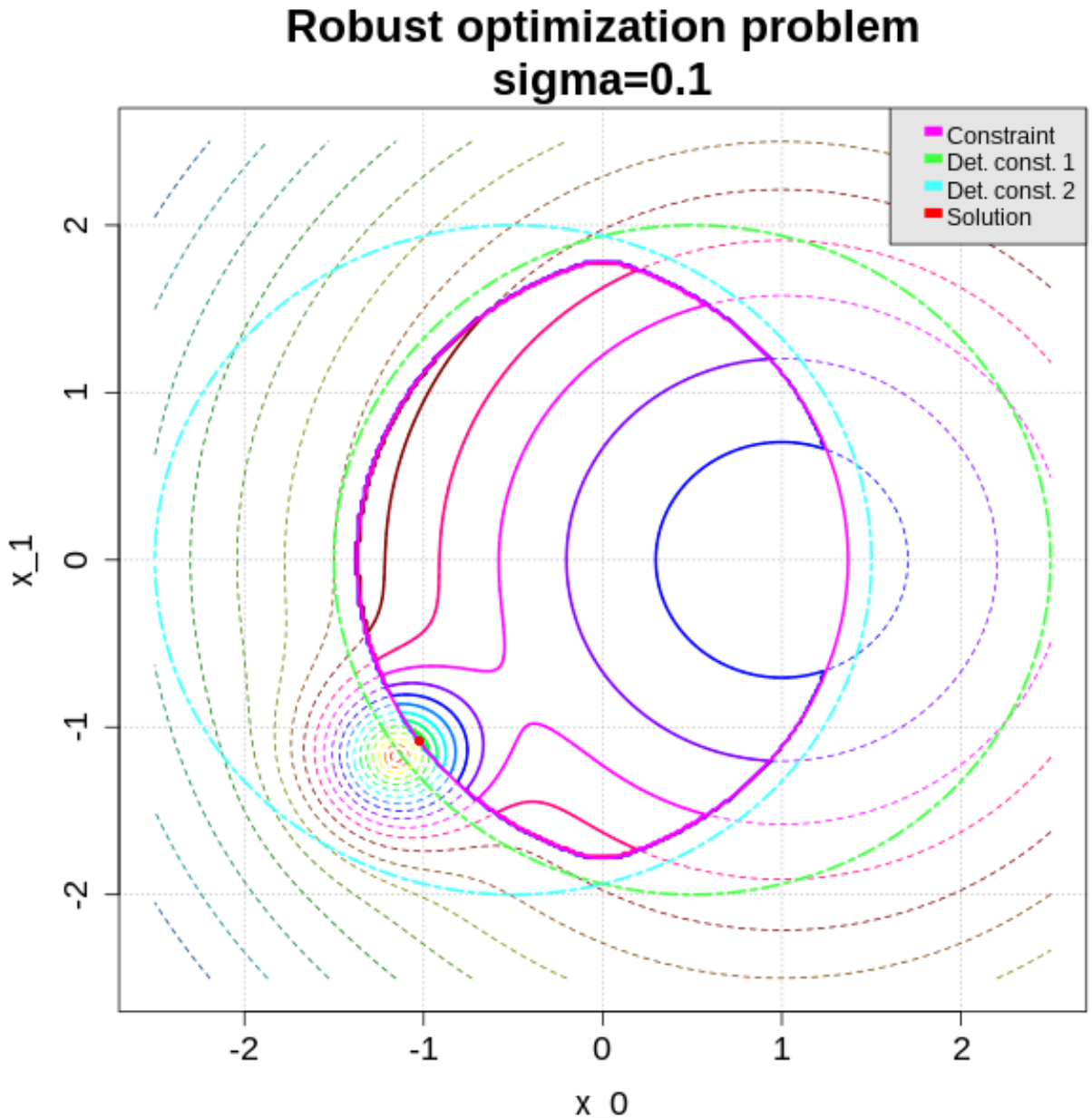
$$(S_{rob}) : \quad x^*_{rob} = \underset{\mathbb{P}(g_1(\underline{x}, \underline{\theta}) \geq 0 \cap g_2(\underline{x}, \underline{\theta}) \geq 0) \geq 0.9}{\arg\min} \mathbb{E}_{\underline{\theta}}[J(\underline{x}, \underline{\theta})]$$

Domaine admissible :

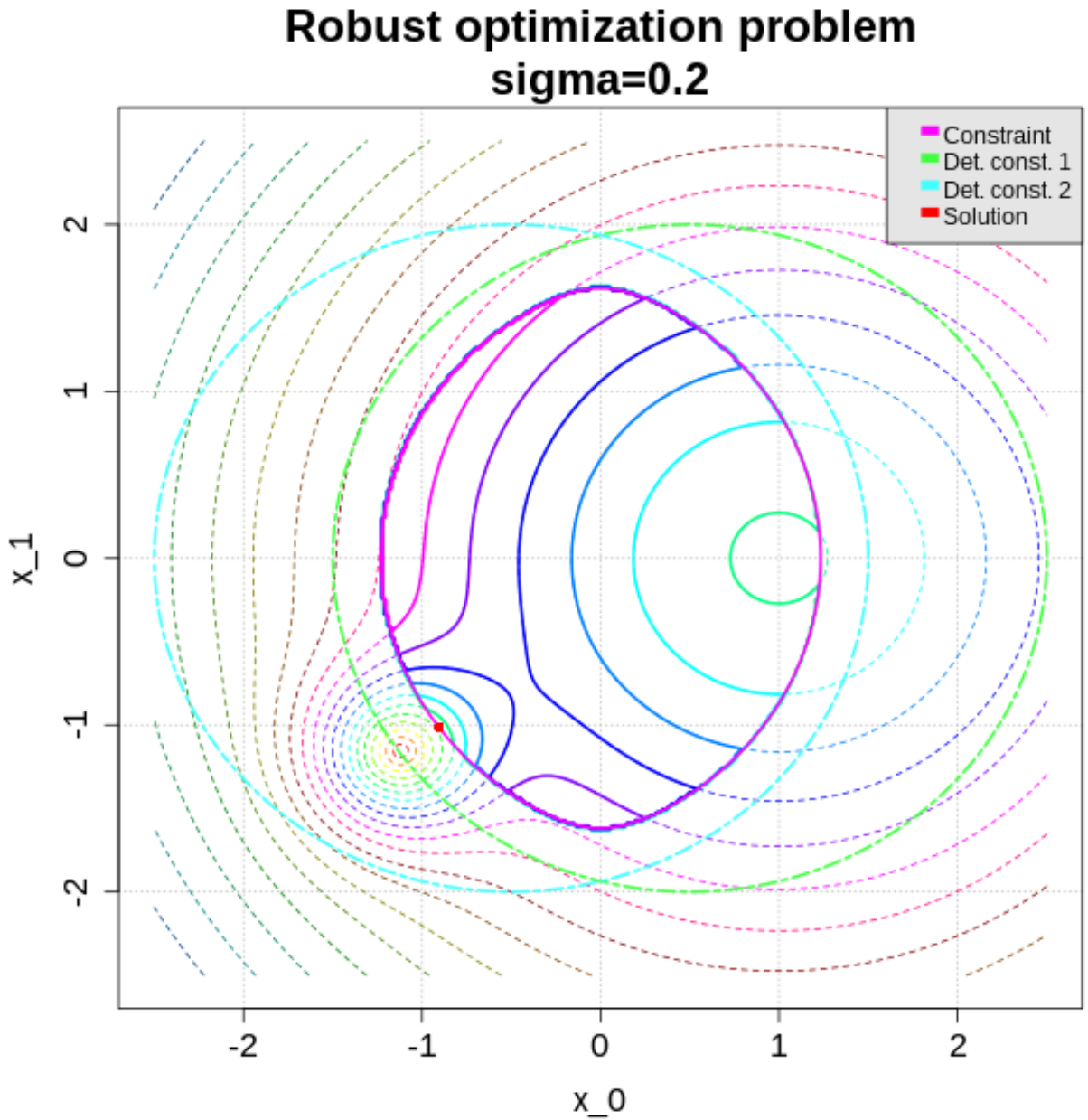




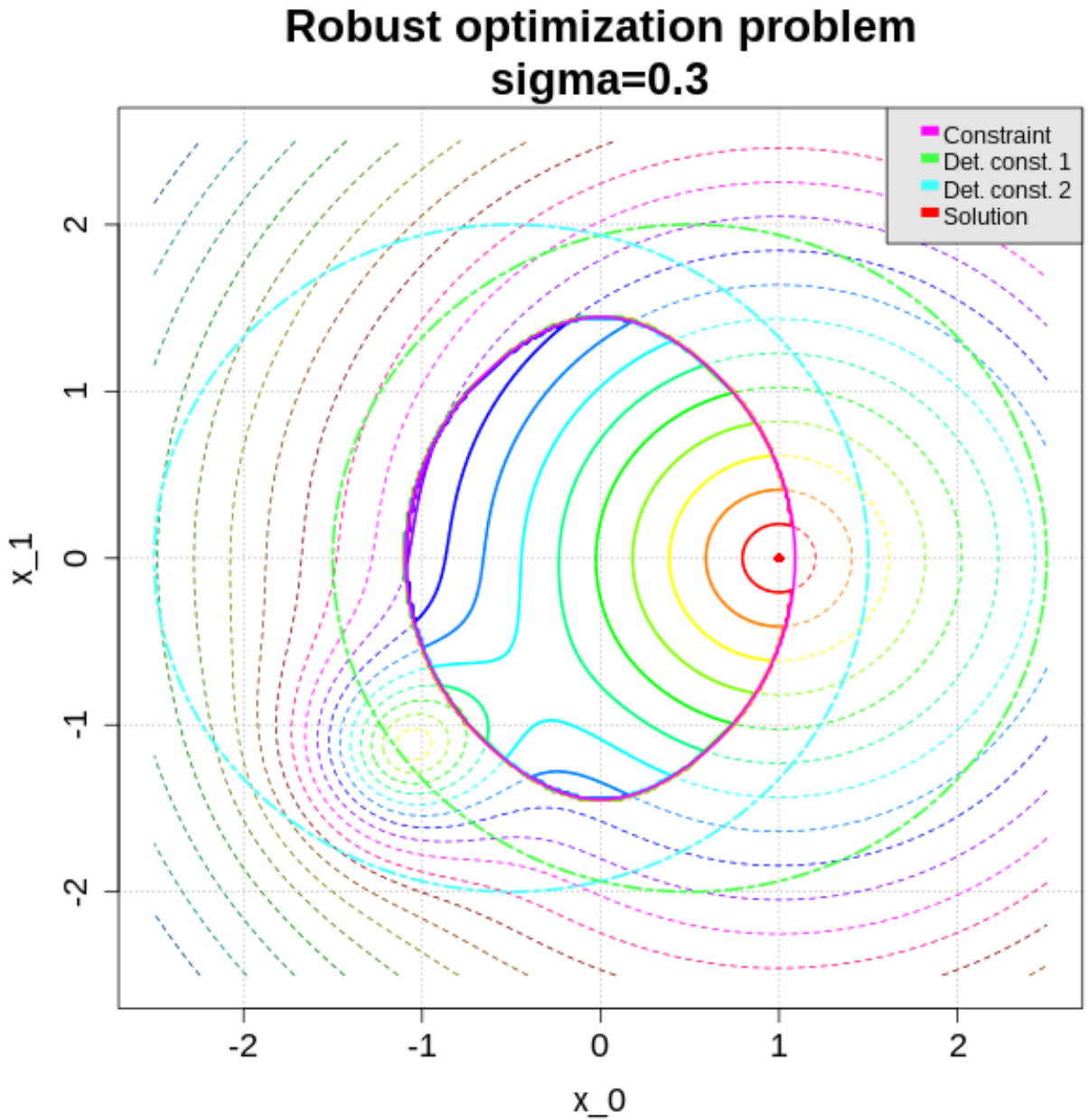
Exemple : conception robuste



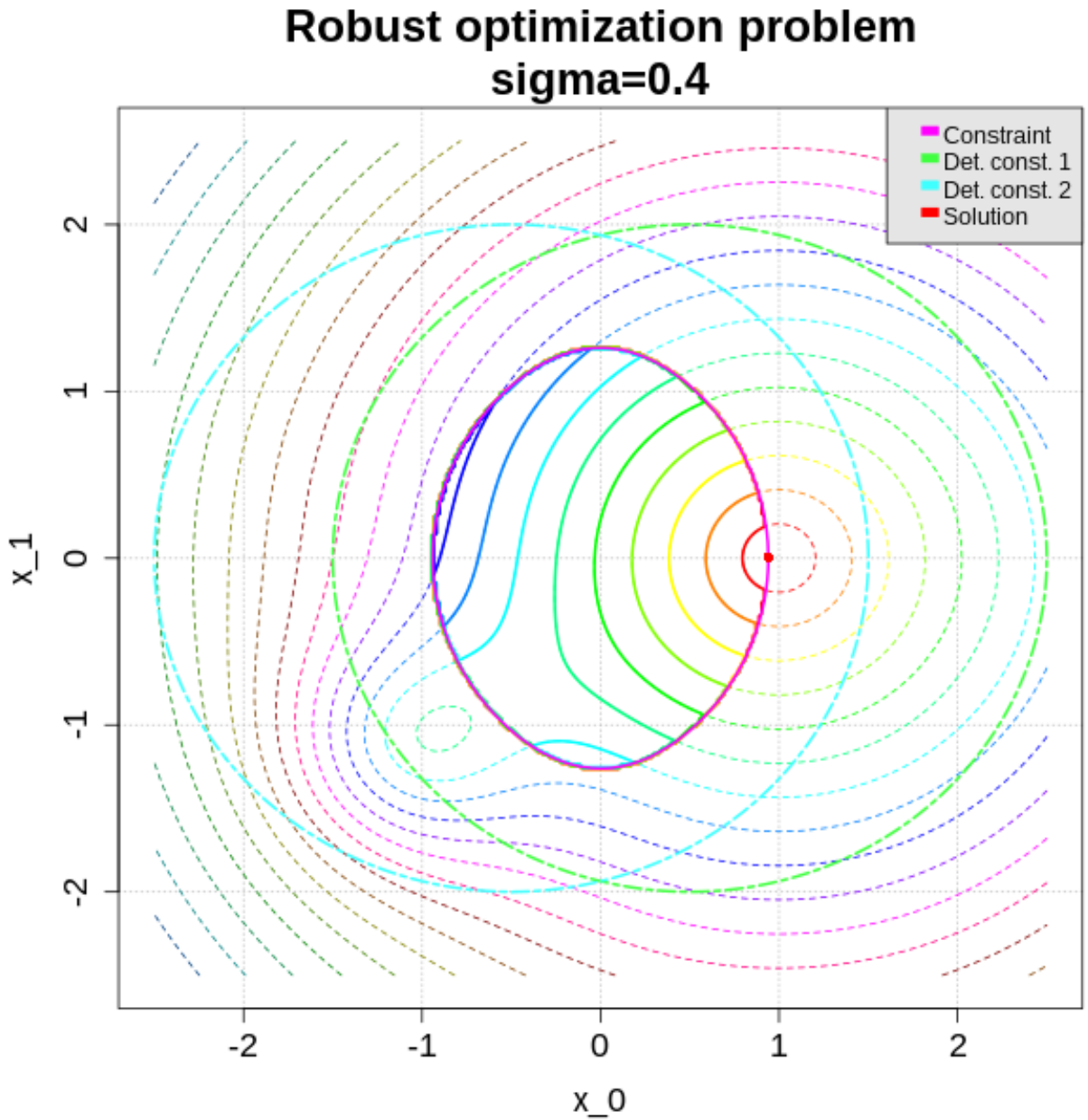
Exemple : conception robuste



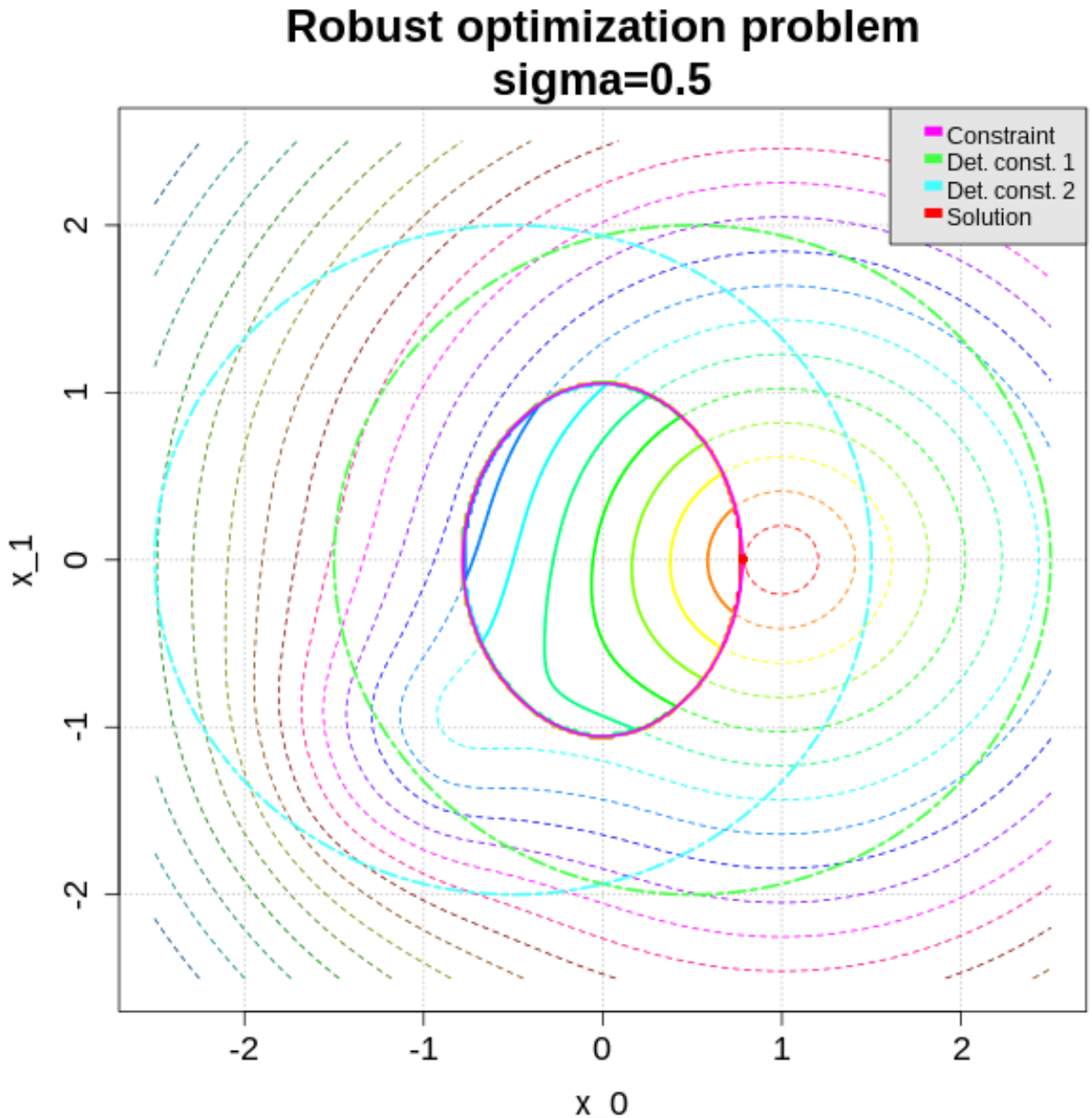
Exemple : conception robuste



Exemple : conception robuste



Exemple : conception robuste



Exemple : conception robuste