Adjustment Computation

14-08-19

* The true value of any measurement is never known.
* The differences between the estimate value and the true value is the error.

$$\varepsilon = x - X$$

x = Estimate value

X = true value.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Arithmetic mean = $\bar{x}$

Sample = n

Error

21-08-2019

Measurement ~~are~~ series of organised observations carried out to determine the magnitude of an unknown quantities using or adopting experimental procedures with the aid of technical instruments.

Numerical description of measurements are mean, mode, median, range, dispersion.

$$\sum_{i=1}^{n} \frac{\varepsilon_i}{n} = 0 \qquad n \to \infty$$

$\varepsilon_1 = x_1 - X$

$\varepsilon_2 = x_2 - X$

$\varepsilon_3 = x_3 - X$

$\varepsilon_n = x_n - X$

i.e true value is constant.

i.e the summation of all the measurement must be equal to zero which means the measurements are not ~~without~~ biegas.

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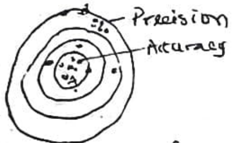
The degree of closeness or nearness of the estimate value to ~~the~~ true value is Accuracy. It can only be determined for one measurement.

Precision & Accuracy 1a?.

Precision is ~~a~~ the degree of consistency of repeated values or measurement of the same quantity ~~to~~ one another given the same standard conditions. It requires series of measurement. Precision is equal to accuracy when there is no error.



Direct measurements carries random measurements because they are not free from errors (errors are either minute or big ones). They are measured...

Measurements are either direct ~~or~~, indirect measurement.

Indirect measurements are computed and they are functions of direct measurements.

The magnitude of error determine the quality of precision of any measurements.

Before any measurement is carried out, the purpose must be known and the precision.

The set of measurement of errors is called population.

$$\varepsilon_i = x_i - X \qquad \text{mean } (\bar{x}) = \sum_{i=1}^{n} x/n$$

$i = 1, 2, 3 \ldots \infty$

where $n \to \infty$

The no of positive error is equal to the nos of negative error.

22-08-19

Sources of Error

Errors are caused by environmental instabilities referred to as natural errors.

- Natural Errors
- Accidental Errors
- Systematic Errors

* Gross Errors: It happens sometimes when carrying out observations or measurements under an unsuitable condition and it is easily recognisable because of the large margins in observed data.

* Systematic Error is also caused by the varying temperature in atmospheric pressures from one medium to the other.

Others sources are refraction error, collimation error, curvilinear error (i.e errors that deals with the curvature of the earth.)

$e = 2.7183$

* Traversing

03-09-19

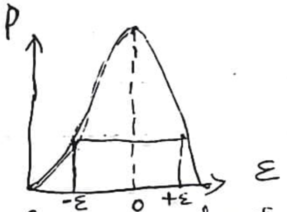
Indicator of Precision and Accuracy are mean, range, median, mode and variance

$|\varepsilon_i|$ - magnitude of Error

$$\frac{[|\varepsilon_i|]}{n} = \frac{\Sigma|\varepsilon_i|}{n} \rightarrow \text{Average error}$$

Range = Maximum Error - Minimum Error

Dispersion is used to analyze error



Gaius normal probability curve $[x_i]$ -> Gaius symbol

$$P = \frac{1}{m\sqrt{2\pi}} \times e^{-\frac{\varepsilon^2}{2m^2}}$$

Number of positive error is equal to number of negative error. The greater the number of measurements, the greater you get the best curve.

$y = x^2$

Equation of a line

standard error is used to determine variance from population

Standard deviation is used when the variance is determined from sample.

$x_1, x_2, x_3 \dots x_n$

$\varepsilon_1, \varepsilon_2, \varepsilon_3 \dots \varepsilon_n$

$\varepsilon_1^2, \varepsilon_2^2, \varepsilon_3^2 \dots \varepsilon_n^2$

standard / variance

$$\frac{[\varepsilon^2]}{n} = \frac{\varepsilon_1^2 + \varepsilon_2^2 + \dots \varepsilon_n^2}{n} = \sigma^2$$

$$\sum_{a=1}^{n} \varepsilon_a = [\varepsilon] = \frac{0}{n}$$

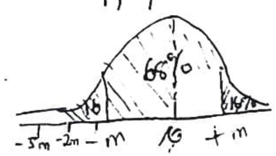
σ = variance

σ^2 = standard error

$V = \bar{x} - x_i$

$$\bar{x} = \frac{[x]}{n}$$

$$\frac{[v^2]}{n-1} = \sqrt{m^2} \rightarrow \text{Standard deviation}$$



i.e -3m is the limit, the error must not exceed one after the -3m.

Outlien - rejected error
Redundant measurement

Indices of Precision & Accuracy

Example

04-09-19 suppose we have 22 plane triangles for which their true errors for the sum of internal angles are given as follows: (i) 0.36s (ii) 0.93s (iii) −0.51s (iv) −1.46s (v) −0.95s (vi) −1.40s (vii) 1.76s (viii) 0.92s (ix) 0.56s (x) 0.00s (xi) −0.59s (xii) 0.00s (xiii) −1.36s (xiv) 1.86" (xv) −0.42" (xvi) 1.68" (xvii) 1.62" (xviii) 1.62" (xix) 1.67" (xx) −0.72", −1.35", −0.98"

Analyse the errors and determine m, Mm, u, t

Standard error = m

Standard Error of the standard error = Mm

Probable Error = u

Average Error = t

soln

$$\text{standard error } m = \sqrt{\frac{[\varepsilon^2]}{n}} = \left(\frac{[\varepsilon^2]}{22}\right)^{1/2}$$

$\beta_1 + \beta_2 + \beta_3 = [\beta]$

$\varepsilon_i = [\beta]_i - 180°$

$i = 1, 2, 3, \dots 22.$

$$M_m = \frac{m}{\sqrt{(2n-1)}} = \frac{m}{\sqrt{2n}}$$

S/N	ε_i	ε^2	Arrangement
1	0.36"	0.1290	0.00"
2	0.93"	0.8649	0.00"
3	−0.51"	0.2601	0.36"
4	−1.46"	2.1316	−0.42"
5	−0.95"	0.9025	0.51"
6	−1.40"	1.96	0.56"
7	1.76"	3.0976	0.59"
8	0.92"	0.8464	0.72"
9	0.56"	0.3136	0.92"
10	−0.00"	0	0.93"
11	−0.59"	0.3481	0.95"
12	0.00"	0	0.98"
13	−1.36"	1.8496	1.35"
14	1.86"	3.4596	1.36"
15	−0.42"	0.1764	1.40"
16	1.68"	2.8224	1.46"
17	1.62"	2.6244	1.62"
18	1.62"	2.6244	1.62"
19	1.67"	2.7889	1.67"
20	−0.72"	0.5184	1.68"
21	−1.35"	1.8225	1.76"
22	−0.98"	0.9604	1.86"
		30.5008	

standard error $(m) = \sqrt{\frac{[\varepsilon^2]}{n}}$

$= \sqrt{\frac{30.50}{22}} = 1.18''$

standard error of standard error $(M_m) = \frac{m}{\sqrt{2n}}$

$= \frac{1.18}{\sqrt{44}} = \frac{1.18}{6.63} = 0.18''$

Probable error $(u) = 0.674\,m$
[determined from theory]

$= 0.674 \times 1.18''$

$= 0.795''$

Average error $(t) = 0.798\,m$
Median

$= 0.798 \times 1.18''$

$= 0.942''$

Probable error $= \frac{[|\varepsilon|]}{n}$

NB: determined from observation

Middle or median error is obtained by arranging all error in order of magnitudes, disregarding the signs and using the one in the middle as the measure

Redundant - this is the observation taken than what is ~~required~~ actually required they are characterized by similarity or repetition

(2) An area of land which was defined on a topographical plan was measured 12 times with a polar planimeter yielding the following results in cm^2 41.1, 41.0, 40.8, 41.0, 40.5, 40.2, 40.4, 41.0, 41.2, 40.6, 40.7, 41.3. cal (i) of
- most probable value of the measurement
- standard deviation (m)
- standard deviation of the mean $(m_{\bar{x}})$
- standard dev of the standard error (M_m)
- ~~Standard~~ error from record measurements
- Probable error determined from standard dev(4)

S/N	A (cm²)	$V_2 = A - \bar{A}$	v^2
1	41.1	0.3	0.09
2	41.0	0.2	0.04
3	40.8	0	0
4	41.0	0.2	0.04
5	40.5	-0.3	0.09
6	40.2	-0.6	0.36
7	40.4	-0.4	0.16
8	41.0	0.2	0.04
9	41.2	0.4	0.16
10	40.6	-0.2	0.04
11	40.7	-0.1	0.01
12	41.3	0.5	0.25
	489.8		1.28

(i) Mean $(\bar{A}) = \frac{[A]}{n}$

(ii) standard deviation of the mean $= \sqrt{\frac{[(A-\bar{A})^2]}{n-1}}$

standard error of the mean $= \frac{m}{\sqrt{n}}$

Standard Variance = $\sqrt{\frac{[v^2]}{n-1}}$

Residual = $\sqrt{\frac{[v^2]}{n-1}} = \sqrt{\frac{[(x_i, \bar{x})^2]}{n-1}}$

(iii) $M_{\bar{x}}$

(iv) $M_m = \frac{M}{\sqrt{2n}}$

(v) $\psi = \frac{[|A|]}{n}$

(vi) $\psi = 0.674m$

(vii) SI

(1) $\bar{A} = \frac{[\bar{A}]}{n} = \frac{489.8}{12} = 40.8$

(2) standard deviation of the mean = $\sqrt{\frac{(A-\bar{A})^2}{n-1}}$

$= \sqrt{\quad}$

redundant observation
important to surveying

Significant Computation

- In order to increase precision & reliability
- To check for discrepancy between two values
- It allows adjustment to be made to obtain a final value for observation.

Weight and Weight of Measurement

The quality of the measurement depends on the magnitude of the error.
The lower the error, the higher the quality and vice versa.
Weight runs in the direction of the quality, the higher the weight, the higher the quality and vice versa.
Error and weight are inversely or directly proportional to one another ~~[illegible]~~

$$W \propto \frac{1}{m^2}$$

$$w = \frac{k}{m^2}$$

$$k = wm^2$$

Weight is a numerical value attached to any survey measurements to express the level or degree of reliability in ~~of the~~ measurement.
i.e measurements do not have the same reliability

- Equally weighted measurements [the same instrument, observer under the same condition.
- Unequally weighted measurements [diff instruments, observer under diff condition.

$W_1 \propto \frac{1}{m_1^2}$ $\quad$ W = Weight

m^2 = Error

$W_1 = \frac{K}{m_1^2}$ $\quad$ K = constant

$K = W_1 m_1^2$

Let

$i = 1, 2, 3, 4 \ldots n$

x_1, x_2, x_3, x_4

W_1, W_2, W_3, W_4

$m_1^2, m_2^2, m_3^2, m_4^2$

$\therefore K = W_2 m_2^2 = W_3 m_3^2 = W_4 m_4^2$

Assume $K = 1$

$K = W_1 m_i^2$

$W_1 = \frac{1}{m^2}$

If $W_0 = 1$ then

$K = W_0 m_0^2$

$K = 1 m_0^2 \equiv K = m_0^2$

$K = m_0^2$ = variance of unit weight

since $K = m_0^2$

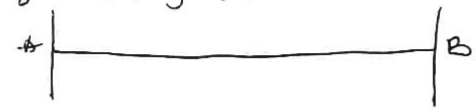
Recall,

$W_1 = \frac{K}{m_0^2}$ and

$\boxed{W_i = \frac{m_0^2}{m_i^2}}$ — Formula for calculating weig

$m_i = \sqrt{\frac{[v^2]}{n-1}}$ — variance

E.g Assuming a line AB is measured with diff instrument (tape, EDM, tacheometer)



$x_t \quad x_E \quad x_{ta}$

$W_t \quad x_E \quad x_{ta}$

$x_w = \frac{x_t W_t + x_E W_E + x_{ta} W_{ta}}{W_t + W_E + W_{ta}}$

$x_1\ x_2\ x_3 \ldots x_n$ [This measurements are not equally weighted]

$W_1\ W_2\ W_3 \ldots W_n$

Weighted mean $(\bar{x}_w) = \frac{\sum x_i W_i}{\sum W_i} = \frac{[x_w]}{[w]}$

Variance of unit weight

$m_0^2 = V_i^2 = \frac{[(\bar{x}_w - x_i)^2]}{n-1}$

Standard error of unit weight

$\sqrt{m_0^2} = \sqrt{\frac{V_i^2 [(\bar{x}_w - x_i)^2]}{n-1}}$

Residual

$$V_i^2 = (\bar{x} - x_i)^2$$

$$\frac{[v^2]}{n-1} = \sqrt{\frac{(\bar{x} - x_i)^2}{n-1}}$$

standard error of the weight mean.

$$M\bar{x}_w = \sqrt{\frac{M_0^2}{[w]}}$$

weighted mean $\bar{x}_w = \frac{[xw]}{[w]}$

variance of unit weight $M_0^2 = \frac{[(\bar{x}_w - x_i)^2]}{n-1} = \frac{[V^2]}{n-1}$

$$\sqrt{M\bar{x}_w} = \sqrt{\frac{[(\bar{x}_w - x_i)^2]}{[w](n-1)}} = \sqrt{\frac{M_0^2}{[w]}} = M\bar{x}_w$$

Ex: A distance was measured repeatedly on three different occasions. The mean values recorded and their standard errors are listed below

measurement 1 $(x_1) = 112.1125m \pm 0.007m = 7mm$

$x_2 = 112.130m \pm 0.002m = 2mm$

$x_3 = 112.128m \pm 0.005m = 5mm$

Determine the weighted mean with the three sets of measurement and the standard deviation of the weighted mean.

soln.

s/n	x_i observed mean	w_i weights	$x_i \cdot w_i$	Residual V_i	V_i^2
1	112.1125 x_1	$(1/7)^2$		$x_w - x_1$	v_1^2
2	x_2	$(1/2)^2$		$x_w - x_2$	v_2^2
3	x_3	$(1/5)^2$		$x_w - x_3$	v_3^2
		$[w]$	$[xw]$		$[v^2]$

soln

s/n	observed values	weights	$x_i \cdot w_i$
1	112.125m	$1/49 = \frac{1}{7^2}$	
2	112.130m	$1/4 = \frac{1}{2^2}$	
3	112.128m	$1/25 = \frac{1}{5^2}$	

$w_1 = \frac{1}{m_1^2}$

$w_1 = \frac{K}{m_1^2}$

$n = 3$

S/n	observed values $\bar{x}$	weight w_i		$x_i \cdot w_i$	$v_i(\bar{x}_w - x_i)$	v_i^2
1	112.125 x_1	$1/7^2$	$1/49$	2.29	0.165	0.027
2	112.130 x_2	$1/2^2$	$1/4$	28.03	0.16	0.026
3	112.128 x_3	$1/5^2$	$1/25$	4.48	0.162	0.026
		0.310 [w]		34.81 [xw]	0.487	0.079 [v^2]

1. $\bar{x}_w = \frac{[xw]}{[w]} = \frac{34.81}{0.310} = 112.290$

2. $\sqrt{m^2_{\bar{x}w}} = \sqrt{\frac{[(\bar{x}_w - x_i)^2]}{[w](n-1)}} = \frac{m_0^2}{[w]}$

$m_0^2 = \frac{(\bar{x}_w - x_i)^2}{n-1} = \frac{0.079}{2} = 0.0395$

$\frac{0.0395}{0.310} = 0.356$

② The following values were recorded for a triangle ABC, the individual measurements being uniformly precised. $\angle A = 62^\circ 28' 16''$, 6 obs.

$B = 56^\circ 44' 36''$, 8 obs.

$C = 60^\circ 46' 56''$, 6 obs.

Given that the weights of an angle varies directly as the number of observations made for the measurements of that angle, determine the correct values of the angle.

NB: The higher the weight the errors low.

$\triangle$ $w = 6, 8, 6$

Error = 12''

Error per unit weight $= \frac{12''}{20} \times 6$, $\frac{12''}{20} \times 8$, $\frac{12''}{20} \times 6$

3.6'' 4.8'' 3.6''

Soln

$\angle A = 62^\circ 28' 16''$ $n = 6 \simeq W_A$

$\angle B = 56^\circ 44' 36''$ $n = 8 \simeq W_B$

$\angle C = 60^\circ 46' 56''$ $n = 6 \simeq W_C$

$179^\circ 59' 48''$ Total number of weight = 20

$+ 12''$

$180^\circ 00' 00''$

$\frac{1}{8} + \frac{1}{6} + \frac{1}{6} = \frac{11}{24} \times 12 = 5.5$

$\frac{12 \times 24}{11} = \frac{288}{11} = 26.182$

$26.18 \times \frac{1}{8} = 3.3''$ $\qquad 3.27 \simeq 3.28$

$26.18 \times \frac{1}{6} = 4.4''$ $\qquad 4.36''$

$26.18 \times \frac{1}{6} = 4.4''$ $\qquad \underline{4.36''}$

$12''$

$A = 62^\circ\ 28'\ 16'' + 4.36'' = 62^\circ\ 28'\ 20.36''$
$B = 56^\circ\ 44'\ 36'' + 3.28'' = 56^\circ\ 44'\ 39.28''$
$C = 60^\circ\ 46'\ 56'' + 4.36'' = 66^\circ\ 47'\ 0.36$

$\underline{180^\circ\ 00'\ 00''}$

17-9-19 The internal angles of a triangle were measured different number of times as follows:

$\angle A = 45^\circ\ 15'\ 25''$, $n = 4$
$\angle B = 83^\circ\ 37'\ 2''$, $n = 8$
$\angle C = 51^\circ\ 07'\ 39''$, $n = 6$

Find out the correct values of each of the angles and the error on each of them.

Soln

$\Sigma \angle A \angle B \angle C = 180^\circ\ 00'\ 26''$

Error is 26'' which will be shared inversely proportion to their weight and is negative.

Total number of weight = 18

$\frac{1}{4} + \frac{1}{8} + \frac{1}{6} = \frac{6+3+4}{24} = \frac{13}{24}$

$\frac{24}{13} \times 26 = \frac{624}{13} = 48''$

(i) $48 \times 1/4 = 12$
(ii) $48 \times 1/8 = 6$
(iii) $48 \times 1/6 = 8$

$\underline{26''}$

subtract error from measured angle.

$A = 45^\circ \; 15' \; 13''$
$B = 83^\circ \; 39' \; 16''$
$C = 51^\circ \; 07' \; 31''$
$180^\circ \; 00' \; 00''$

Standard error of unit weight $(m) = \dfrac{[w \cdot v^2]}{\sqrt{[n-1][w]}}$

Weighted mean $M_{\bar{w}} = \dfrac{m}{\sqrt{[w]}}$ $\dfrac{[w x]}{[w]}$ $\dfrac{w_1 x_1 + w_2 x_2 + \cdots}{w_1 + w_2}$

Standard error of individual measurement

$\dfrac{m}{\sqrt{w}}$

② Two different electronic distance measuring devices were used to make repeated determination of the length of a line. The readings are given in metres as follows:

Instrument A = 556.429m , 556.432
556.435 m , 556.433m
556.440m 556.436 m

Instrument B = 556.425, 556.436
556.435
556.432, 556.434
556.436, 556.435
556.435

Assuming identical conditions for each set of determination, cal the relative precision of the two instrument. What is the most probable length of the line based upon these readings.
Determine the standard deviation of the value.

soln

Mean $S_A = \dfrac{\Sigma}{n}$

$S_A = \dfrac{556.429 + 556.435 + 556.440 + 556.432 + 556.433 + 556.436}{6}$

$S_A = 556.434$

B = 556.

Horizontal Angular measurements
The propagation of Errors

$S_B = \frac{\Sigma}{n}$

$S_B = \frac{556.435 + 556.435 + 556.432 + 556.436 + 556.435 + 556.436 + 556.431 + 556.435}{8}$

$S_B = 556.4347 \approx 556.435$

$M_{SA} = \sqrt{\frac{[v^2]}{n-1}} = \frac{[(S_{\bar{A}} - S_i)^2]}{n-1} = *$

NB: S_i = individual value.

$M_{SB} = \sqrt{\frac{[v^2]}{n-1}} = \frac{[(S_{\bar{B}} - S_i)^2]}{n-1}$

$M_{\bar{S}_A} = \frac{M_{SA}}{\sqrt{n}}, \frac{M_{SB}}{\sqrt{n}} = w$

Weighted mean

$\frac{\bar{S}_A \cdot w_A + S_B w_B}{w_A + w_B}$

$m_{\bar{S}} = \frac{[w v_{\bar{s}}^2]}{n-1}$

n: number of Instrument

$M_{SA} = \sqrt{\frac{[v^2]}{n-1}} = \frac{[(S_{\bar{A}} - S_i)^2]}{n-1}$

$= \frac{[(556.434 - 556.429)^2]}{5} = \cancel{0.000}\ 5 \times 10^{-6}$

-0.001

$\frac{1}{M_{\bar{S}_A}}$

v^2

(1) standard error relative precision of the two Instrument.

$M_{SA} = \sqrt{\frac{[v^2]}{n-1}} = \sqrt{\frac{(S_{\bar{A}} - S_i)^2}{n-1}}$

s/n	$(S_{\bar{A}} - S_i)$	v^2	$(S_{\bar{B}} - S_i)$	v^2	s/n
1	0.005	0.000025	0	0	1
2	-0.001	0.000001	0	0	2
3	-0.006	0.000036	0.003	0.000009	3
4	0.002	0.000004	-0.001	0.000001	4
5	0.001	0.000001	0	0	5
6	-0.002	0.000004	-0.001	0.000001	6
			0.001	0.000001	7
			0	0	8
				0.000012	

$M_{SA} = \sqrt{\frac{0.000071}{6-1}} = 0.0000142$

$M_{\bar{S}_A} = \frac{M_{SA}}{\sqrt{n}} = \frac{0.0000142}{\sqrt{6}}$

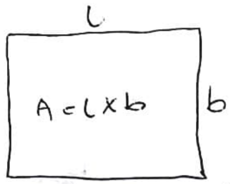
$= 0.0000058.$

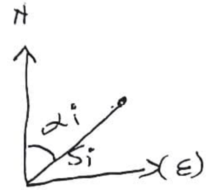
$M_{SB} = \sqrt{\frac{0.000012}{8-1}}$

$= \sqrt{0.0000017}$

$= 0.0013.$

18-09-19 Propagation of Random Errors of Measurements

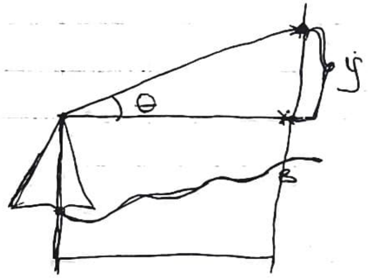




$\Delta N = S_i \cos \alpha_i$
$\Delta E = S_i \sin \alpha_i$

Area $= f(l, b)$
$h = f(a, b)$
$\delta = f(m, v)$ -> Density
$y = f(x_1, x_2 \dots x_n)$
i.e. $A = L \times b$

Dependent Measurement : The values does correlate with one another. It requires other measurement to determine its value.
Independent measurement : It depends on other measurements to get it value. It does not require other measurements to determine its own value.



$\tan\theta = \frac{y}{S}$

$S = \frac{y}{\tan\theta}$

$y = S \tan\theta$

$Y = f(x_1, x_2 \dots x_n)$

$Y = y + \Delta y = f\left((x_1 + \Delta x_1), (x_2 + \Delta x_2), \dots (x_n + \Delta x_n)\right)$

$$Y = y + \Delta y = \underbrace{f(x_1, x_2 \dots x_n)}_{y} + \underbrace{\frac{df}{dx_1}\Delta x_1 + \frac{df}{dx_2}\Delta x_2 + \dots \frac{df}{dx_n}}_{\Delta y}$$

$$\Delta y = \frac{df}{dx_1}\Delta x_1 + \dots + \frac{df}{dx_n}\Delta x_n$$

$\frac{df}{dx_1}$ = Partial derivatives of y with respect to x_1
dx_1, Δx_1 = value of error.

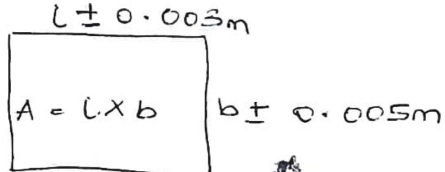
General law of the Propagation of Random error.

$$m_y^2 = \left(\frac{df}{dx_1}\right)^2 \underset{\downarrow \text{ variance}}{m^2_{x_1}} + \left(\frac{df}{dx_2}\right)^2 m^2_{x_2} + \dots + \left(\frac{df}{dx_n}\right)^2 m^2_{x_n}$$

$$m^2_A = \left(\frac{dA}{dl}\right)^2 m^2_l + \left(\frac{dA}{db}\right)^2 m^2_b.$$

$$m^2_h = \left(\frac{dh}{da}\right)^2 m^2_a + \left(\frac{dh}{db}\right)^2 m^2_b$$

Example: Determine the standard error of A and its magnitude.



$l = 25.00m$

$b = 25.05m$

$A = l \cdot b = f(l, b)$

$$M_A^2 = \left(\frac{dA}{dl}\right)^2 m_l^2 + \left(\frac{dA}{dl}\right)^2 m_b^2$$

$$M^2_A = b^2 m^2 l + l^2 m^2 b$$

$$= (\pm 0.005)^2 m^2 l + (\pm 0.003)^2 m^2 b$$

$$= \sqrt{35.03^2 \times \pm 0.005 + 25.00^2 \times \pm 0.005}$$

$$m^2 A = \sqrt{3.6813 + 8.125}$$

$$m^2 A = 2.609$$

$$A = l \times b$$

$$= 25.00 m \times 35.03 m$$

$$= 875.75 m^2$$

$$y = cx$$

$$my^2 = \left(\frac{dy}{dx}\right)^2 \cdot m_x^2$$

$$my^2 = c^2 m_x^2$$

$$my = \sqrt{c^2 m_x^2} = c \cdot m_x$$

$$x_1\ x_2\ x_3 \cdots x_n$$

$$\bar{x} = \frac{[x]}{n} = \frac{x_1}{n} + \frac{x_2}{n} + \cdots + \frac{x_n}{n}$$

$$= \frac{1}{n}x_1 + \frac{1}{n}x_2 + \cdots + \frac{1}{n}x_n = \frac{1}{n}(x_1 + x_2 \cdots + x_n)$$

$$\sqrt{m^2_{\bar{x}}} = \left(\frac{1}{n}\right)^2 m^2_{x_1} + \left(\frac{1}{n}\right)^2 m^2_{x_2} + \cdots + \left(\frac{1}{n}\right)^2 m^2_{x_n}$$

$$\left(\frac{1}{n}\right)^2 \left[m^2_{x_1} + m^2_{x_2} + \cdots + m^2_{x_n}\right]$$

$$\frac{1}{n}^2 \times n \cdot m^2 x = \sqrt{\frac{m^2 x}{n}} = \frac{m_x}{\sqrt{n}}$$

$$\sqrt{m^2_A} = \left(\frac{dA}{dr}\right)^2 \cdot m^2_r$$

$$\sqrt{(2\pi r)^2 \cdot M_r^2}$$

standard Error ~~when it~~ is differentiated.

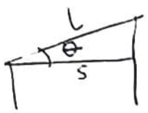
$$M_A = 2\pi r \cdot M_r$$

$2\pi r$ = Circumference

6. A slope length of 178.741m was measured with a standard error of ± 5mm to determine the corresponding horizontal length, an angle of depression of 5° 14' 25" was measured with a standard error of ± 10". calculate or compute the horizontal length and its standard error.

Sol:

$m_\theta \rightarrow$ Readiation
$\div$ by $\rho = 206265$



$m_l = \pm 5mm$
$m_\theta = \pm 10''$

$\cos\theta = \frac{S}{L}$
$S = l\cos\theta$
$S = f(l, \theta)$
$S = l \times \cos\theta$
$S = 178.741 \times \cos 5^\circ 14' 25''$
$S = 177.994$

$$m_s = \sqrt{\left(\frac{ds}{dl}\right)^2 m_l^2 + \left(\frac{ds}{d\theta}\right)^2 m_\theta^2 \text{ (Rad)}}$$

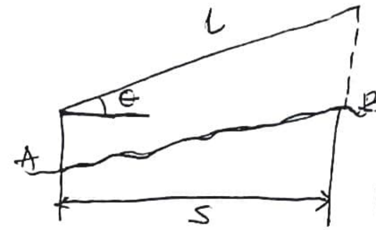
$$\cos\theta^2 \times 0.005^2 + (l \cdot \sin\theta)^2 \times \frac{m_\theta^2}{\rho}$$

$$M_s = \sqrt{\cos 5^\circ 14' 25''^2 \times 0.005^2 + (178.741 \times \sin 5^\circ 14' 25'')^2 \times \left(\frac{10''}{206265}\right)^2}$$

$$M_s = \sqrt{0.9309 \times 0.005^2 + 265.502 \times 2.35\times10^{-9}}$$
$$M_s = \sqrt{\cancel{2.3898\times10^{-5}}}$$
$$M_s = \sqrt{2.218\times10^{-5} + 6.264\times10^{-7}}$$
$$M_s = 0.0048$$

$$\boxed{M_s = 0.0050}$$

04-09-19 Principle of Balanced Accuracies



$S = f(l, \theta)$
m_l, m_θ

$$m_s^2 = \left(\frac{df}{dl}\right)^2 m_l^2 + \left(\frac{df}{d\theta}\right)^2 \cdot m_\theta^2$$

$S = l\cos\theta$

$\frac{df}{dl} = \cos\theta$

$\frac{df}{d\theta} = -l\sin\theta$

$$m_s^2 = \cos\theta^2 \cdot m_l^2 + (l\sin\theta)^2 \cdot \frac{m_\theta^2}{\rho^2}$$

$$m_s = \sqrt{\cos\theta^2 \cdot m_l^2 + (l\sin\theta)^2 \cdot \frac{m_\theta^2}{\rho^2}}$$

Application of principle of Balanced Accuracies.

$$m_s^2 = 2\left(\frac{df}{dl}\right)^2 \cdot m_l^2$$

OR

$$m_s^2 = 2\left(\frac{df}{d\theta}\right)^2 \cdot \left(\frac{m_\theta}{\rho}\right)^2$$

$$m_l = \sqrt{\frac{m_s^2}{2\left(\frac{df}{dl}\right)^2}}$$

$m_e = \sqrt{\frac{m_s^2}{2\left(\frac{df}{dl}\right)} \times \rho^2}$

Given $y = f(a, b, c)$

Find $m_a, m_b, m_c \ldots$ If m_y is known

The principle of balance accuracy states that either of the two $\left(\frac{df}{dl}\right)^2 \cdot m_l^2$ and $\left(\frac{df}{d\theta}\right)^2$ are equal to one another, multiply by the number of arragment.

$A = f(a, h)$

$M_A^2 = \left(\frac{df}{da}\right)^2 \cdot m_a^2 + \left(\frac{df}{dh}\right) m_h^2$

$m_a = \frac{M_A}{\sqrt{2} \times \frac{df}{da}}$

$m_h = \frac{M_A}{\sqrt{2} \times \frac{df}{dh}}$

$\sqrt{2 \times \cos\theta}$

$2 \times \cos 6^\circ 00' 00''$

Ex: $M_s = 0.15m$

$l = 220$

$\theta = 6^\circ 00' 00''$

Find: m_l, m_e

$S = l \cos\theta$

$S = 220 \cos 6^\circ$

$S = 218.80m$

By Principle of Ballanced Accuracies

$m_l = \frac{m_s}{\sqrt{2} \cdot \left|\frac{df}{dl}\right|}$, $m_e = \frac{m_s}{\sqrt{2} \cdot \left|\frac{df}{d\theta}\right|} \cdot \rho$

$m_l = \frac{0.15}{\sqrt{2} \times 0.994522}$

$m_\theta = \frac{0.15}{\sqrt{2} \times |l \sin\theta|} \cdot 206265$

$m_l = 0.10m$

$m_e = \frac{0.15}{\sqrt{2} \times 22.99} \cdot 206265$

$m_e = \frac{3039.75}{32.521}$

$m_e = 15.87$

$m_e = 15.9' \times 60$

$m_e = 951.3''$

Law of Propagation of Error

$y = f(a, b, c)$, determine the weight of y

$$M_y^2 = \left(\frac{df}{da}\right)^2 \cdot M_a^2 + \left(\frac{df}{db}\right)^2 \cdot M_b^2 + \left(\frac{df}{dc}\right)^2 \cdot M_c^2$$

$w_a \qquad w_b \qquad w_c$

$$y = \frac{a.w_a + b.w_b + c.w_c}{w_a + w_b + w_c} = \frac{[k.w]}{[w]}$$

$$\frac{M_y^2}{M_0^2} = \left(\frac{df}{da}\right)^2 \times \left(\frac{M_a}{M_0}\right)^2 + \left(\frac{df}{db}\right)^2 \times \left(\frac{M_b}{M_c}\right)^2 + \left(\frac{df}{dc}\right)^2 \cdot \left(\frac{M_c}{M_c}\right)^2$$

Inverse weight

$$\frac{1}{w_i} = \frac{M_i^2}{M_0^2}$$

$$\frac{M_0^2}{M_i^2} = w_i$$

$$q_i = \frac{1}{w_i} = \frac{M_i^2}{M_0^2}$$

Weight:

$$\frac{1}{w_y} = \underbrace{\left(\frac{df}{da}\right)^2}_{f_a} \cdot \frac{1}{w_a} + \underbrace{\left(\frac{df}{db}\right)^2}_{f_b} \cdot \frac{1}{w_b} + \underbrace{\left(\frac{df}{dc}\right)^2}_{f_c} \cdot \frac{1}{w_c}$$

$$\frac{1}{w_y} = \frac{f_a^2}{w_a} + \frac{f_b^2}{w_b} + \frac{f_c^2}{w_c} = \left[\frac{f^2}{w}\right]$$

$$q_y = \frac{1}{w_y} = \frac{f^2 . q}{[w]} = f_a^2 q_a + f_b^2 q_b + f_c^2 q_c$$

* Increase of variance will always give the weight of any quantity

NB: D measurement are equally weighted

$$\left.\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \cdots & x_n \\ x_1' & x_2' & x_3' & x_4' & x_5' & \cdots & x_n' \end{matrix}\right\} M_x$$

$\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \varepsilon_4 \quad \cdots \quad \varepsilon_n$

$$\varepsilon_i = x_i - x_i'$$

$$\varepsilon_1 = x_1 - x_1'$$

$$\varepsilon_2 = x_2 - x_2'$$

$$\varepsilon_n = x_n - x_n'$$

Standard dev

$$(M_x) = \sqrt{\frac{[\varepsilon_i^2]}{n_1}}$$

$$\varepsilon_n = x_n - x_n'$$

$$M_x = \sqrt{\frac{[\varepsilon^2]}{n}} \quad \text{< For from measurements >}$$

Stand error

$$= \sqrt{\frac{[v^2]}{n-1}}$$

$$\bar{x} = \frac{x_i + x_i'}{2}$$

$$\frac{1}{2}x_i + \frac{1}{2}x_i'$$

standard error of the mean:

$$M_{\bar{x}}^2 = \left(\frac{1}{2}\right)^2 M_{x_i}^2 + \left(\frac{1}{2}\right)^2 \cdot M_{x_i'}^2$$

$$= \left(\frac{1}{2}\right)^2 \left[m_{x_i}^2 + M_{x_i'}^2\right] = \sqrt{\frac{1}{\cancel{4}_2} \times \cancel{2} m^2 x} = \sqrt{\frac{m_x^2}{2}} =$$

$$\frac{M_x}{\sqrt{2}} = \sqrt{\frac{[\varepsilon^2]}{2n}} = m_{\bar{x}}$$

n	ε_i $x_i - x_i'$	ε_i^2 $(x_i - x_i')^2$
1		
2		
3		
4		
		$[\varepsilon_i^2]$

$$m_{\varepsilon_i}^2 = m_{x_i}^2 + m_{x_i'}^2 = 2m^2_{x_i}$$

$$\sqrt{m^2_{x_i}} = \sqrt{\frac{m^2_{\varepsilon_i}}{2}} = \frac{m_{\varepsilon_i}}{\sqrt{2}} \qquad m_\varepsilon = \sqrt{\frac{[\varepsilon_i^2]}{n}}$$

$$m_{x_i} = \frac{m_{\varepsilon_i}}{\sqrt{2}} = \frac{[\varepsilon^2]}{\sqrt{2n}}$$

$$m_{x_i} = \frac{m_\varepsilon}{\sqrt{2}} = \sqrt{\frac{[\varepsilon_i^2]}{n}} \times \frac{1}{\sqrt{2}} = \sqrt{\frac{[\varepsilon_i^2]}{2n}}$$ [Standard error of each measurement]

Applying propagation of error

$$\frac{x_i + x_i'}{2} = \bar{x}_i$$

$$\frac{1}{2} x_i + \frac{1}{2} x_i' = \bar{x}_i$$

$$\left(\frac{1}{2}\right)^2 m^2 x_1 + \left(\frac{1}{2}\right)^2 m^2 x_i' = 2 \times \frac{1}{4} m x_i^2 = \frac{1}{2} m x_i^2$$

$$= m_{\bar{x}}^2$$ [variance of the mean]

$$\frac{1}{4} m^2 x_2 + \frac{1}{4} m' x_i$$

$$2 \times \frac{1}{4} m^2 x_1 \quad \sqrt{\frac{m x_i}{2}}$$

$$= \frac{m_x}{\sqrt{2}} = m_{\bar{x}}$$

standard error of the mean

$$m_{\bar{x}} = \frac{m_{x_i}}{\sqrt{2}}$$

Example:

NB: These measurements are equally weighted

s/n	S_i	S_i (m)	ε_i (cm)	ε_i^2
1	81.62	81.57	+5	25
2	79.37	78.40	−3	9
			−3	9
3	80.88	80.91		
4	82.06	82.00	+6	36
5	81.36	81.29	−7	49
n = 5			−2	$[\varepsilon^2] = 128$

$$m_s = \sqrt{\frac{[\varepsilon_i^2]}{2n}} = \sqrt{\frac{128}{2 \times 5}} = \sqrt{\frac{128}{10}} = 3.6\,cm$$

NB: If the relative error is small, i.e close to zero (0), you will leave it

i.e $\frac{-2\,cm}{8088 \times 100} = \frac{-2}{8088} = 0.00025m$ (very insignificant)

When measurements are equally weighted.

$x_1 \quad x_2 \quad x_3 \cdots x_n$

$x_1' \quad x_2' \quad x_3' \cdots x_n'$

$w_1 \quad w_2 \quad w_3 \cdots w_n$

$x_i = x_i'$

$$\frac{x_i \cdot w_i + x_i' \cdot w_i}{w_i + w_i} = \frac{\cancel{w}(x_i + x_i')}{2\cancel{w}} = \frac{x_i + x_i'}{2}$$

...dard ...r of ...t weight $= \sqrt{\frac{[w_i v_i^2]}{n-1}}$

...dard ... of a ...le measurement $M x_i = \frac{M_0^2}{\sqrt{w_i}}$

...dard error ...the mean $= M\bar{x}_i = \frac{M x_i}{\sqrt{w_i}}$

$\ldots i = x_i - x_i'$

$= x_1 - x_1'$

$= x_2 - x_2'$

$\ldots = x_n - x_n'$

Applying propagation of error

$$M_{\varepsilon_i}^2 = \frac{M_{x_i}^2 + M_{x_i'}^2}{M_0^2}$$

$$\frac{1}{W_{\varepsilon_i}} = \frac{1}{W_{x_i}} + \frac{1}{W_{x_i'}} = \frac{2}{W_{x_i}}$$

$$\frac{1}{W_{\varepsilon_i}} = \frac{2}{W_{x_i}}\,; \quad 2W_{\varepsilon_i} = \frac{W_{x_i}}{2}$$

$\varepsilon_1 = x_1 - x_1' = W\varepsilon_1 = Wx_1/2$

$\varepsilon_2 = x_2 - x_2' = W\varepsilon_2 = Wx_2/2$

$\vdots$

$\varepsilon_n = x_n - x_n' = W\varepsilon_n = Wx_n/2$

Standard error of unit weight $(M_c) = \sqrt{\frac{[W \cdot \varepsilon^2]}{n}}$

$\varepsilon_i^2 \times W\varepsilon_i = \frac{1}{2} W_i \times \varepsilon_i^2$

$\varepsilon_1^2 \times W\varepsilon_1 = \frac{1}{2} W_1 \times \varepsilon_1^2$

$\varepsilon_2^2 \times W\varepsilon_2 = \frac{1}{2} W_2 \times \varepsilon_2^2$

$\vdots$

$\varepsilon_n^2 \times W\varepsilon_n = \frac{1}{2} W_n \times \varepsilon_n^2$

Standard error of variance of unit weight $(M_0) = \frac{1}{2}\frac{[W \cdot \varepsilon^2]}{n} = \sqrt{\frac{W \cdot \varepsilon^2}{2n}}$

$M x_i = \frac{M_0}{\sqrt{W_i}}$ → Standard error for any of the measurement but it is now the mean for all the measurement

$\bar{x}_i = \frac{1}{2} x_i + \frac{1}{2} x_i'$ → Mean

$$\frac{M_{\bar{x}_i}^2}{M_0^2} = \frac{\frac{1}{4} M_{x_i}^2 + \frac{1}{4} x_i'^2}{M_0^2} \rightarrow$$

$$\frac{1}{W_{\bar{x}}} = \frac{1}{4W_{x_i}} + \frac{1}{4W_{x_i'}} = \cancel{2} \times \frac{1}{\cancel{4}_2 W_x} = \frac{1}{2W_x}$$

$\frac{1}{W_{\bar{x}}} = \frac{1}{2W_x} = W_{\bar{x}} = 2Wx_i$ → Weight of the mean

Standard error of the mean $= M_{\bar{x}_1} = \frac{M_o}{\sqrt{W_{\bar{x}}}} = \frac{M_o}{\sqrt{2w_x}}$

$= \frac{1}{\sqrt{2}} \cdot M_{x_i}$ //

Example: (mm)

S/N	$\varepsilon_i = x_i - x_i'$	W_{x_i}	$w \cdot \varepsilon^2$	ε_i^2	$w \cdot \varepsilon_i$
1	3.8	0.67	9.675	14.44	2.55
2	-2.9	0.56	4.710	8.41	-1.62
3	-3.2	0.40	4.096	10.24	1.28
4	1.9	0.62	2.238	3.61	1.18
5	-4.0	0.45	7.20	16	-1.80
6	3.9	0.34	5.17	15.21	1.33
7	-3.2	0.42	4.30	10.24	-1.34
8	6.7	0.33	14.81	44.89	2.21
9	1.3	0.83	1.40	1.69	1.08
10	-4.0	0.37	5.92	16	-1.48
n=10	0.3	[w]=4.99	$\Sigma w\varepsilon_i$ = 59.52	~~[illegible]~~	0.83

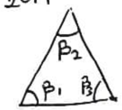
$$M_o = \sqrt{\frac{[w \cdot \varepsilon^2]}{2n}} = \sqrt{\frac{59.52}{2 \times 10}} = 1.73\,mm$$

$$M_{\bar{x}_1} = \frac{M_o}{\sqrt{2w_x}} = \frac{1.73}{\sqrt{2 \times 0.67}} = \frac{1.73}{1.156} = 1.49$$

$$\text{Weight error } (t) = \frac{[w\varepsilon]}{[w]} = \frac{0.83}{4.99} = 0.17$$

Summary

02-10-2019



$\beta_1 + \beta_2 + \beta_3 = 180° + \varepsilon$

* The mean of a set of measurements is different from its true value.
* The mean of a set of

The ~~least~~ least square Principle
$[v^2] = Min$ - Equally weight
$[wv^2] = Min$ - Unequally weight

$v_1 \quad v_2 \quad v_3 \quad \cdots \quad v_s$

$x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_n$

$\bar{x} = \frac{[x]}{n}$

$v_i^2 = (\bar{x} - x_i)^2 = (\bar{x} - x_i)(\bar{x} - x_i) = \bar{x}^2 - x_i\bar{x} - x_i\bar{x} + x_i^2$
$= \bar{x}^2 - 2x_i\bar{x} + x_i^2$

$i = 1, 2, 3, \ldots n$

$v_1^2 = (\bar{x} - x_1)^2 = \bar{x}^2 - 2x_1\bar{x} + x_1^2$

$v_2^2 = (\bar{x} - x_2)^2 = x^2 - 2x_2\bar{x} + x_2^2$

$\vdots$

$v_n^2 = (\bar{x} - x_n)^2 = \bar{x}^2 - 2x_n\bar{x} + x_n^2$

$[v^2] = [\bar{x}^2] - 2\bar{x}[x] + [x^2]$

* The higher the precision, the lower the error.

$$M_x = \left(\frac{[v^2]}{n-1}\right)^{1/2}$$

m_x to get the small value of

* To reduce m_x, $[v^2]$, you differentiate with respect to $\bar{x}$

$$\frac{d[v^2]}{d\bar{x}} = 2n\bar{x} - 2[x] = 0$$

$$2n\bar{x} - 2[x] = 0$$

$$2n\bar{x} = 2[x]$$

$$\bar{x} = \frac{[x]}{n}$$

The best estimate of our measurement when the sum of the square $[v^2]$ of residual is brought to minimum. i.e $[vv] = [v^2]min.$ the sum of the square of the residual is equal to minimum.

$l_1\ l_2\ l_3\ l_4$ $\qquad$ $V_i = \bar{l} - l_i$

$\qquad\qquad\qquad$ $i = 1, 2, 3, 4$

$V_1^2\ V_2^2\ V_3^2\ V_4^2$

$$F = V_1^2 + V_2^2 + V_3^2 + V_4^2 > [v^2] = \sum_{i=1}^{4} v^2 = \text{Minimum}.$$

$\bar{l}$ = Estimate

$$F = (\bar{l} - l_1)^2 + (\bar{l} - l_2)^2 + (\bar{l} - l_3)^2 + (\bar{l} - l_4)^2 = \text{Minimum}$$

$$\frac{dF}{d\bar{l}} = 2(\bar{l} - l_1)(1) + 2(\bar{l} - l_2)(1) + 2(\bar{l} - l_3)(1) + 2(\bar{l} - l_4)(1) = 0$$

Eliminate 2 by dividing through by 1/2

$$\frac{1}{2}\frac{dF}{d\bar{l}} = (\bar{l} - l_1) + (\bar{l} - l_2) + (\bar{l} - l_3) + (\bar{l} - l_4) = 0$$

$$4\bar{l} - (l_1 + l_2 + l_3 + l_4) = 0$$

$$4\bar{l} - [l] = 0$$

$$\bar{l} = \frac{[l]}{4}$$

~~Adjust~~ : ~~Method of Adjustment~~

- Methods of Adjustments. (General methods of Adjustment)

(i) Adjustment by Variation of Parameters {Method of Indirect Obs. equ}

(ii) Adjustment by Correlates or Adjustment by Method of Condition Equation

(i) Adjustment by variation of Parameters.

$l_1\ l_2 \ldots l_n$

$\beta_1\ \beta_2 \ldots \beta_n$

$x_1\ x_2 \ldots x_n$

Finding the parameter, we find V_n and the adjusted value of the measurements $(x_1\ x_2 \ldots x_n)$

$$f(x_i) \neq F(x)$$

$x_1 + V_1$

$x_n + V_n$

~~Ask~~

Total number of measurements and total numbers of n_o parameters are very important

n_o = Number of parameters

② Adjustment by Correlates

$x_i + v_i = X$

Condition Equation → $X = x_1 + v_1 = x_2 + v_2 = x_3 + v_3 = \dots x_n + v_n = r$

r = the redundant → It is the excess measurement that may not be needed to the determine the value of a measurement

n_0 = Number of unknown in any quantity, it is required to determine the value of measurement which is always one

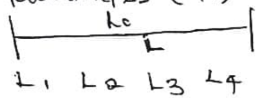
$n = n_0 - r$

$n - n_0 = r$

$r = n - n_0$

Numbers of condition equation must be equal to r redundant

Parameters (n_0)



$n = 4$

$n_0 = 1$

$r = n - n_0 = 3$

Lo = Approximate value of L

$L = L_0 + \Delta L$

Adjusted measurement must result to L

$L_i + V_i = L$

$L_i + V_i = L_0 + \Delta L$

$V_i - \Delta L = L_0 - L_i$

$V_1 - \Delta L = L_0 - L_1$

$V_2 - \Delta L = L_0 - L_2$

$V_3 - \Delta L = L_0 - L_3$

$V_4 - \Delta L = L_0 - L_4$

Model, Redundant Consistency

15-10-19 n = Total number of measurements.

n_0 = Numbers of unknown

computed measurements differs from measured measurements.

r = Total numbers of measurement to be known (either computed or measured) is ~~known~~ as redundant.

$\beta_1 = 65° 40' 20''$

$\beta_2 = 70° 45' 18''$

$\beta_1 + \beta_2 = 136° 25' 38''$

$\beta_3 = 180° - 136° 25' 38'' = 43° 34' 22''$

The β_3 here was computed.

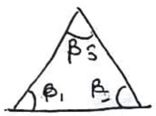
The measured β_3 is $43° 34' 30''$

* Redundancy provides a way of liquidizing the guessing of measurements.

① Approach to adjustments by variation of Parameters (Method of indirect observation equation).

⇒ To apply or use this method, you determine the numbers of parameters you want to use.

n, no, r



$$\sum_{i=1}^{3} \beta = 180^\circ \quad \text{<Mathematical model>}$$

n = 3

nc = 2

r = 1

⇒ Chose your unknown independent parameters which must be equal to no

⇒ Develop your residual observation equation. i.e develop (n) residual observation equation where by each measurement has its own equation. i.e all the measurement has its own observation equation and it must be written in terms of their parameters.

$\beta_1, \beta_2, \beta_3$ are measurements.

$n = 3$, $no = 2$, $r = n - no = 3 - 2 = 1$

$$\sum_{i=1}^{3} \beta = 180^\circ \quad \text{<Mathematical model>}$$

unknown independent parameters are $\bar{\beta}_1$ & $\bar{\beta}_2$

If $\bar{\beta}$ is true and β_1 is without error than

$\bar{\beta}_1 = \beta_1$

$\bar{\beta}_1 = \beta_1 + V_1$

The V_1 is added to β_1 to determine the true value of β_1 ($\bar{\beta}_1$)

$$\boxed{\bar{\beta}_1 = \beta_1 + V_1 = \tilde{\beta}_1 + \Delta\beta_1} \quad \text{<Residual observation Equation for } \beta_1\text{>}$$

Separate the unknown from the known

$V_1 - \Delta\beta_1 = \tilde{\beta}_1 - \beta_1$

$\boxed{\tilde{\beta}_1 - \beta_1} \rightarrow$ Free constant of the equation.

$V_1 = (\tilde{\beta}_1 - \beta_1) + \Delta\beta_1$

1st $\boxed{V_1 = \Delta\beta_1 + (\tilde{\beta}_1 - \beta_1)} \leftarrow$ Residual obs. Equation.

2nd $V_2 = \Delta\beta_2 + (\tilde{\beta}_2 - \beta_2)$

3rd $\beta_3 + V_3 = 180^\circ - (\bar{\beta}_1 + \bar{\beta}_2)$

$\beta_3 + V_3 = 180^\circ - (\tilde{\beta}_1 + \Delta\beta_1 + \tilde{\beta}_2 + \Delta\beta_2)$

$\beta_3 + V_3 = 180^\circ - \tilde{\beta}_1 - \Delta\beta_1 - \tilde{\beta}_2 - \Delta\beta_2$

$V_3 = -\Delta\beta_1 - \Delta\beta_2 - (\tilde{\beta}_1 + \tilde{\beta}_2 + \beta_3) + 180^\circ$

$V_3 = -\Delta\beta_1 - \Delta\beta_2 - \underbrace{(\tilde{\beta}_1 + \tilde{\beta}_2 + \beta_3)}_{180^\circ\ 00'\ 08''} + 180^\circ$

$$\left.\begin{aligned} V_1 &= \Delta\beta_1 + 0 \\ V_2 &= \Delta\beta_2 + 0 \\ V_3 &= -\Delta\beta_1 - \Delta\beta_2 - 08'' \end{aligned}\right\} \text{Residual Observation Equation.}$$

NB: $\tilde{\beta} - \beta = 0$

NB: $\tilde{\beta}$ is an approximate value which is then the value that is used to find the mean which is β_1 i.e. it could be $\beta', \beta'', \beta'''$ etc

* Since there is $\Delta\beta_2$, the coefficient of $\Delta\beta_2 = 0$
$[V_1 = \Delta\beta_1 + 0]$

⇒ Impose the mean square criteria or introduce the mean square criteria.

The sum of the square of the residual ⇒ minimum

$$\sum_{i=1}^{n} v^2 = [V] = \text{Min} \text{ which can be written as}$$

$V^T V$ in matrix form [When it is equally weighted]

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_n \end{bmatrix} \quad 3 \times 1 \text{ matrix}$$

$n=3$; 3 ↓ column, 1 ↓ Row

$$\underset{V_{n,1}}{\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}} = \underset{B_{n,n_0}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}} \underset{\Delta_{n_0,1}}{\begin{bmatrix} \Delta\beta_1 \\ \Delta\beta_2 \end{bmatrix}}$$

Coefficient matrix to the correction of the parameters.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \Delta\beta_1 \\ \Delta\beta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -8'' \end{bmatrix}$$

$V_{n,1} \quad B_{n,n_0} \quad \Delta_{n_0,1} + f_{n,1}$

So, the Residual Observational Equation is

$$\boxed{V_{n,1} = B_{n,n_0} \Delta_{n_0,1} + f_{n,1}}$$

For unequally weighted

$$\sum_{i=1}^{n} W_i v_i^2 = [W v v] = \text{Min} \text{ which can be written as}$$

$V^T W V = \text{Min}$ in matrix form.

16-10-19

Imposition of mean square criteria to derive normal equation

$V_1^2 = (\Delta\beta_1 + 0)^2$ ...... ①

$V_2^2 = (\Delta\beta_2 + 0)^2$ ...... ②

$V_3^2 = (-\Delta\beta_1 - 1\Delta\beta_2 - 8'')^2$ ... ③

$[V^2] = V_1^2 + V_2^2 + V_3^2 = \Delta\beta_1^2 + \Delta\beta_2^2 + (-\Delta\beta_1 - \Delta\beta_2 - 8'')^2 = \text{Min}$

To minimize, you partially differentiate the function with respect to the parameters.

$$\frac{d[v^2]}{d\Delta\beta_1} = 2\Delta\beta_1 - 2(-\Delta\beta_1 - \Delta\beta_2 - 8'') = 0$$

Multiply both sides by 1/2

$$\frac{1}{2}\frac{d[v^2]}{d\Delta\beta_1} = \left[\cancel{2}\Delta\beta_1 - \cancel{2}(-\Delta\beta_1 - \Delta\beta_2 - 8'')\right] \times \frac{1}{2} = 0$$

$$\frac{d[v^2]}{d\Delta\beta_1} = \Delta\beta_1 + \Delta\beta_1 + \Delta\beta_2 + 8'' = 0$$
$$= 2\Delta\beta_1 + \Delta\beta_2 + 8'' = 0$$

$$\frac{d[v^2]}{d\Delta\beta_2} = \cancel{2}\Delta\beta_2 - \cancel{2}(-\Delta\beta_1 - \Delta\beta_2 - 8'') = 0$$

$$= \Delta\beta_2 + \Delta\beta_1 + \Delta\beta_2 + 8'' = 0$$
$$= \Delta\beta_1 + 2\beta_2 + 8'' = 0$$

$$\boxed{\begin{matrix} 2\Delta\beta_1 + 1\Delta\beta_2 + 8'' = 0 \\ 1\Delta\beta_1 + 2\beta_2 + 8'' = 0 \end{matrix}} \rightarrow \text{Normal Equation}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \Delta\beta_1 \\ \Delta\beta_2 \end{bmatrix} + \begin{bmatrix} 8'' \\ 8'' \end{bmatrix} = 0$$