SUG 405 -

ADJUSTMENT COMPUTATION

1a MATRIX METHODS IN LEAST-SQUARES ADJUSTMENT

Least squares adjustments method is based upon the mathematical theory of probability, it is the most rigorous of adjustment procedures

The basic methods are employed in least-squares adjustments: (1) The observation equation method and

(2) The condition equation method

I. BY APPLICATION OF METHOD OF PARAMETERS OR METHOD OF OBSERVATION EQUATION

Equations that relates observed quantities to both observational residuals and independent, unknown parameters are called observation equation. For a unique solution, the number of equations must equal the number of unknowns. The evaluation of the observation equations yields a set of normal equations, which are equal in number to the number of unknowns. The normal equations are solved to obtain the most probable values ~~of~~ for the unknowns.

Any group of observation equations may be represented in matrix form as;

$$V + B\Delta = f$$

where;

V = is the matrix of the residuals.

B = is the matrix of coefficients for the unknowns
Δ = is the matrix of the unknowns
f = is the matrix of the numerical constants /misclosures.

~~The detailed structures of the matrices are.~~

STEPS:

I Determine the number of measurements (n), the number of uniquely required measurements (n_0) and the number of redundant measurements (r), which is always $(n-n_0)$

II Formulate /Derive the necessary residual observation equations, which must be equal to (n).

III Represent the observation equations in matrix form;
$V+B\Delta=f$.
The detailed structures of these matrices are;

$$\underset{n,1}{V}\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} + \underset{n,n_0}{B}\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \underset{n_0,1}{\Delta}\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix} = \underset{n,1}{f}\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

IV Derive the normal equation /constitute the normal equation matrix and solve it. $[N\Delta = t]$
where $N = B^TWB$ and $t = B^TWf$.

N:B; The number of normal equations must be equal to

$$\underset{n_0,n_0}{N} = \underset{n_0,n}{B^T}\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{bmatrix} \underset{n,n}{W}\begin{bmatrix} w_1 & & 0 \\ & w_2 & \\ 0 & & \ddots \\ & & & w_n \end{bmatrix} \times \underset{n,n_0}{B}\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$t = B^T\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{bmatrix} \underset{n,n}{W}\begin{bmatrix} w_1 & & 0 \\ & w_2 & \\ 0 & & \ddots \\ & & & w_3 \end{bmatrix} \times f\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_3 \end{bmatrix}$$

V Determine the unknowns: $\Delta = N^{-1}t$ and evaluate the residuals: $V = f - B\Delta$. Then carry out the necessary adjustments

II. BY METHOD OF CORELLATES OR METHOD OF CONDITION EQUATIONS

Observations can also be adjusted using conditional equations. In a conditional adjustment, the most probable set of residuals are found that satisfy a given functional condition. Example of conditional adjustment are: (1) The sum of angles in a triangles is 180° (2) The latitudes and departures of a ~~plo~~ polygon traverse sum to zero (3) The sum of the angles in the horizon equals 360°. etc.

Any group of conditional equations may be represented in matrix form as; ~~Av~~ $AV + f = 0$.

Where; A = is the coefficient matrix of the residuals
V = is the matrix of the residuals.
f = is the matrix of the free / numerical constants / misclosure.

STEPS:

I Determine the number of measurements (n), the number of the uniquely required measurements (n_0) and the number of the redundant measurements (r) which is $(n-n_0)$.

II Formulate the necessary residual condition equations, which must be equal to (r).

III Represent the condition equations in matrix form;
$AV + f = 0$
The detailed structures of these matrix are;

$$\underset{r,n}{A}\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \underset{n,1}{V}\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} + \underset{n,1}{f}\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_3 \end{bmatrix} = 0$$

IV Constitute the normal equations matrix and solve it.
$Q_e K + f = 0$. where;
$Q_e = AQA^T$ and $K = Q_e^{-1} f$
$Q = 1/w$ and $V = QA^T K$

$$Q_e = \underset{r,r}{} A \begin{bmatrix} q_{11} & q_{12} & - & q_{1n} \\ q_{21} & q_{22} & - & q_{2n} \\ | & | & | & | \\ q_{m1} & q_{m2} & - & q_{mn} \end{bmatrix} \times \underset{n,n}{Q} \begin{bmatrix} 1/w_1 & & 0 \\ & 1/w_2 & \\ 0 & & 1/w_3 \end{bmatrix} A^T \begin{bmatrix} q_{11} & q_{21} & - & q_{m1} \\ q_{12} & q_{22} & - & q_{m2} \\ | & | & | & | \\ q_{1n} & q_{2n} & - & q_{mn} \end{bmatrix}$$

$K = Q_e^{-1} f$:- The matrix of the unknowns, which can be solved using either matrix method or substitution & elimination method.

V After obtaining the values of 'K', solve for residuals: $V = QA^T K$. Then compute the necessary adjustments.

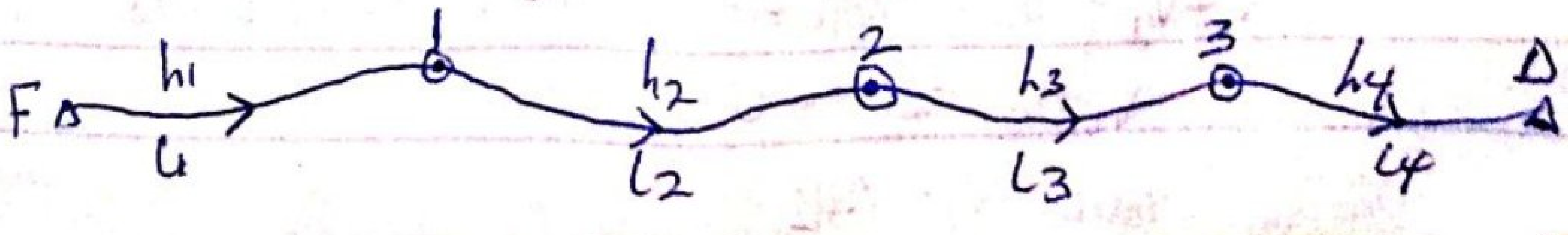
1b

Level Section	F-1	1-2	2-3	3-D
Obs. Height diff	1.825	6.745	1.590	2.224
Length (km)	1.7	2.4	2.6	1.2

$H_D = 128.444m$
$H_F = 115.048m$

Applying method of correlates or parameters, compute
(1) Adjusted height differences
(2) Adjusted elevations of marks 1, 2, and 3 and their standard errors (3) The standard error of the weakest point.

Solution $n = 4$
$n_0 = 3$ and
$r = (n - n_0) = (4-3) = 1$



SOLUTION USING PARAMETER METHOD

$\tilde{H}_1 = H_F + h_1$

$\tilde{H}_1 = 115.048 + 1.825 = 116.868m$

$\tilde{H}_2 = H_F + h_1 + h_2$

$\tilde{H}_2 = 115.048 + 1.825 + 6.745 = 123.618m$

$\tilde{H}_3 = H_F + h_1 + h_2 + h_3$

$\tilde{H}_3 = 115.048 + 1.825 + 6.745 + 1.590 = 125.208m$

$\tilde{H}_3 = H_D - h_4$

$\tilde{H}_3 = 128.444 - 3.224 = 125.220m$

* Write out the residual observation equations

$\tilde{H}_1 + \Delta H_1 = H_F + h_1 + V_1$

$\tilde{H}_1 - H_F - h_1 = V_1 - \Delta H_1$

$V_1 - \Delta H_1 = 116.868 - 115.048 - 1.825$

$V_1 - \Delta H_1 = 0 \quad \Rightarrow V_1 = \Delta H_1$ — 1st ROE

$\tilde{H}_2 + \Delta H_2 = H_F + h_1 + V_1 + h_2 + V_2$

$\tilde{H}_2 - H_F - h_1 - h_2 = V_2 - \Delta H_2$

$V_1 + V_2 - \Delta H_2 = 123.618 - 1.825 - 6.745 - 115.048$

$V_1 + V_2 - \Delta H_2 = 0$

But $V_1 = \Delta H_1$

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$\therefore\ V_2 + \Delta H_1 - \Delta H_2 = 0 \quad \Rightarrow V_2 = \Delta H_2 - \Delta H_1 \cdot$ — 2nd ROE

$\hat{H}_3 + \Delta H_3 = HF + h_1 + V_1 + h_2 + V_2 + h_3 + V_3$

$\hat{H}_3 - HF - h_1 - h_2 - h_3 = V_2 + V_3 - \Delta H_3 + V_1$

$V_2 + V_3 - \Delta H_3 = 125.208 - 115.048 - 1.825 - 6.745 - 1.590$

$V_1 + V_2 + V_3 - \Delta H_3 = 0$

But $V_2 = \Delta H_2 - \Delta H_1$ and $V_1 = \Delta H_1$

$\therefore\ \Delta H_1 + \Delta H_2 - \Delta H_1 - \Delta H_3 + V_3 = 0$

$V_3 + \Delta H_2 - \Delta H_3 = 0 \quad \Rightarrow V_3 = \Delta H_3 - \Delta H_2$ — 3rd ROE

$\hat{H}_3 + \Delta H_3 = HD - (h_4 + V_4) \Rightarrow \hat{H}_3 + \Delta H_3 = HD - h_4 - V_4.$

$\hat{H}_3 - HD + h_4 = -V_4 - \Delta H_3$

$-V_4 - \Delta H_3 = 125.208 - 128.444 + 3.224$

$-V_4 - \Delta H_3 = -0.012m$

$V_4 + \Delta H_3 = 0.012m$ — 4th ROE

$\underline{V} + \underline{B\Delta} = \underline{f}$

$V_1 - \Delta H_1 = 0$ — (1)

$V_2 + \Delta H_1 - \Delta H_2 = 0$ — (2)

$V_3 + \Delta H_2 - \Delta H_3 = 0$ — (3)

$V_4 + \Delta H_3 = 0.012m$ — (4)

$$V\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} + B\begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \Delta\begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta H_3 \end{bmatrix} = f\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.012 \end{bmatrix}$$

$$B^T_{3,4} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad W_{4,4} = \begin{bmatrix} \frac{1}{1.7} & & & \\ & \frac{1}{2.4} & & \\ & & \frac{1}{2.6} & \\ & & & \frac{1}{1.2} \end{bmatrix}$$

$$M = B^T W B \Rightarrow B^T W_{3,4} = \begin{bmatrix} -\frac{10}{17} & \frac{5}{12} & 0 & 0 \\ 0 & -\frac{5}{12} & \frac{5}{13} & 0 \\ 0 & 0 & -\frac{5}{13} & \frac{5}{6} \end{bmatrix}$$

$$B^T W B_{3,3} = \begin{bmatrix} \frac{205}{204} & -\frac{5}{12} & 0 \\ -\frac{5}{12} & \frac{125}{156} & -\frac{5}{13} \\ 0 & -\frac{5}{13} & \frac{95}{78} \end{bmatrix} \Bigg/\!\!\Bigg/ \begin{bmatrix} 1.0049 & -0.4167 & 0 \\ -0.4167 & 0.8013 & -0.3846 \\ 0 & -0.3846 & 1.2179 \end{bmatrix}$$

$$t = B^T W f_{3,1} = \begin{bmatrix} 0 \\ 0 \\ 0.01 \end{bmatrix} \qquad M\Delta = t$$

$$M \begin{bmatrix} 1.005 & -0.417 & 0 \\ -0.417 & 0.801 & -0.385 \\ 0 & -0.385 & 1.218 \end{bmatrix} \Delta \begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta H_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ 0 \\ 0.01 \end{bmatrix}$$

$$1.005\Delta H_1 - 0.417\Delta H_2 + 0 = 0$$

$$-0.417\Delta H_1 + 0.801\Delta H_2 - 0.385\Delta H_3 = 0$$

$$0 - 0.385\Delta H_2 + 1.218\Delta H_3 = 0.01$$

$$\Delta H_1 = 0.00269 \simeq 0.0026\,m.$$

$$\Delta H_2 = 0.00624 \simeq 0.0062\,m$$

$$\Delta H_3 = 0.01018 \simeq 0.0102\,m$$

(iv) ~~Compute~~ the ~~adjusted~~ ~~height~~

$V_1 = \Delta H_1 = 0.0026m$

$V_2 = \Delta H_2 - \Delta H_1 = 0.0062 - 0.0026 = 0.0036m$

$V_3 = \Delta H_3 - \Delta H_2 = 0.0102 - 0.0062 = 0.0040m.$

$V_4 = 0.012 - \Delta H_3 = 0.012 - 0.0102 = 0.0018m$

(i) Compute the adjusted height differences.

$h_1^T = h_1^{obs} + V_1 = 1.825 + 0.0026$

$h_1^T = 1.8276m$

$h_2^T = h_2^{obs} + V_2 = 6.745 + 0.0036$

$h_2^T = 6.7486m$

$h_3^T = h_3^{obs} + V_3 = 1.590 + 0.0040$

$h_3^T = 1.5940m$

$h_4^T = h_4^{obs} + V_4 = 3.224 + 0.0018$

$h_4^T = \cancel{3.2} \; 3.2258m$

(ii) Compute the adjusted elevations of marks 1, 2 & 3. and their standard error.

$H_1 = H_F + h_1^T = 115.048 + 1.8276 = 116.876m$

$H_2 = H_1 + h_2^T = 116.876 + 6.7486 = 123.624m$

$H_3 = H_2 + h_3^T = 123.624 + 1.5940 = 125.218m$

check $\Rightarrow H_3 = H_D - h_f^T = 128.444 - 3.2258$

$H_3 = 125.218m.$

Standard Errors

$w_i = \frac{m_0^2}{m_i^2} \Rightarrow m_i = \frac{m_0}{\sqrt{w_i}}$

$m_0 = \sqrt{\frac{[w_i v_i^2]}{r}} \Rightarrow w_i = \frac{1}{L_i}$

$w_1 = 1 \div 1.7 = 0.588m$

$w_2 = 1 \div 2.4 = 0.417m$

$w_3 = 1 \div 2.6 = 0.385m$

$w_4 = 1 \div 1.2 = 0.833m$

$v_1^2 = 0.0026^2 = 0.00000676m$

$v_2^2 = 0.0036^2 = 0.00001296m$

$v_3^2 = 0.0040^2 = 0.0000160m$

$v_4^2 = 0.0018^2 = 0.00000324m$

$w_1v_1^2 = 0.588 \times 0.00000676 = 0.00000397m$

$w_2v_2^2 = 0.417 \times 0.00001296 = 0.00000540m$

$w_3v_3^2 = 0.385 \times 0.0000160 = 0.00000616m$

$w_4v_4^2 = 0.833 \times 0.00000324 = 0.00000270m$

$\Sigma wv^2 = 0.00001823m$

$m_0 = \sqrt{\frac{0.00001823}{1}} = 0.00427m$

$m_1 = \frac{m_0}{\sqrt{w_1}} = \frac{0.00427}{\sqrt{0.588}} = 0.006m$

$$m_2 = \frac{m_0}{\sqrt{w_2}} = \frac{0.00427}{\sqrt{0.417}} = 0.007m$$

$$m_3 = \frac{m_0}{\sqrt{w_3}} = \frac{0.00427}{\sqrt{0.385}} = 0.007m$$

(iii) Standard Error of the weakest point.

$$W_{Emin} = \frac{4}{[L]} \text{ or } \frac{4}{n}. \quad [L] = 7.9.$$

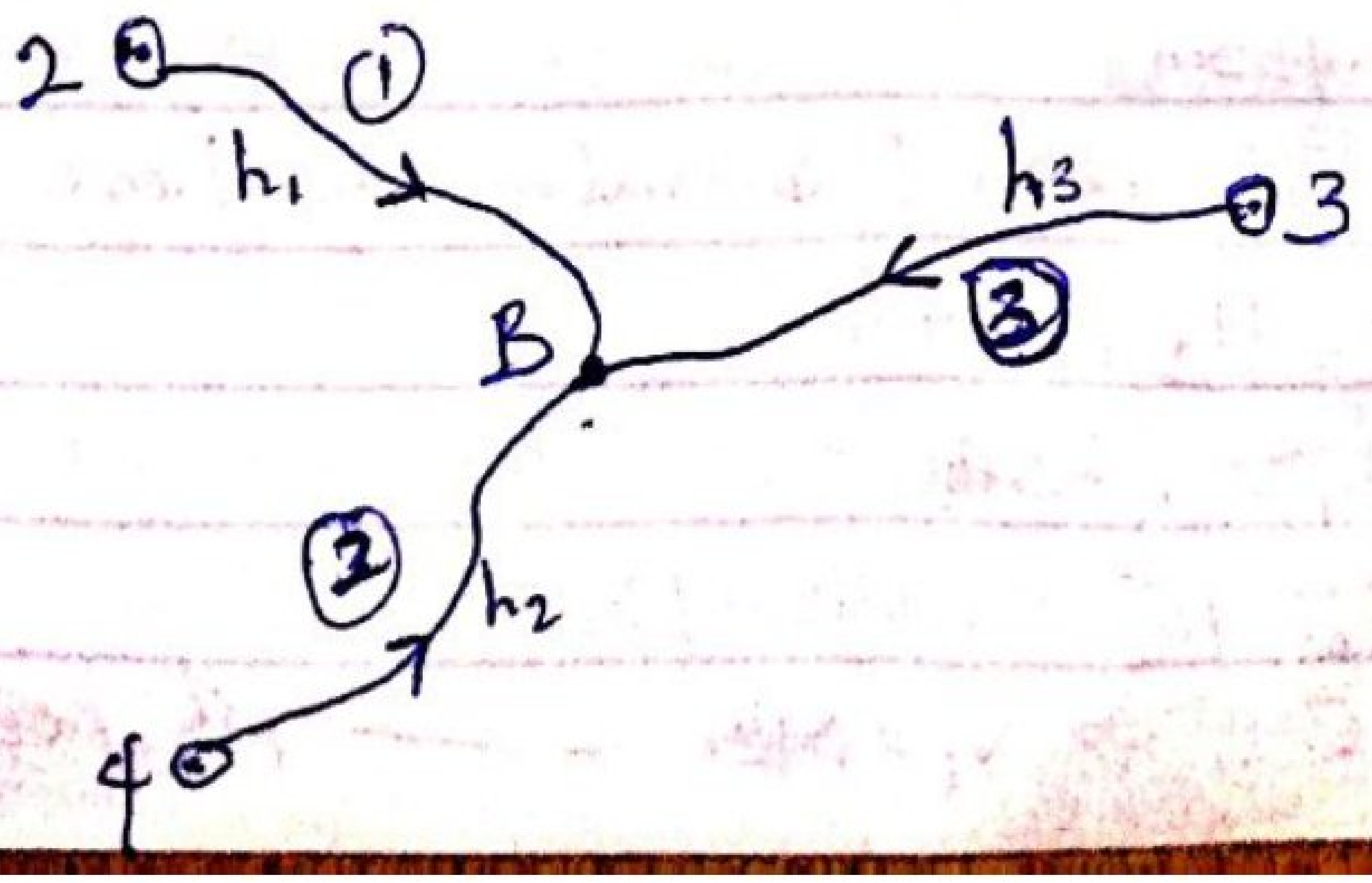
$$= \frac{4}{7.9} \text{ or } \frac{4}{4} \Rightarrow 0.506. \text{ or } 1$$

$$M_{HEmin} = \frac{m_0}{\sqrt{W_{Emin}}} = \frac{0.00427}{\sqrt{0.506}} \text{ or } \frac{0.00427}{\sqrt{1}}$$

$$= 0.006m \text{ or } 0.00427m$$

3.

Level lines	Obs. Elev. Diff.	line Length	Weight
1	13.800	20	0.50
2	-43.420	52.6	0.19
3	16.053	33.5	0.30



$H_2 = 56.655m$
$H_3 = 54.400m$
$H_4 = 113.880m$

* Using Parameter method

$n = 3$, $n_o = 1$ and $r = 2$.

* Compute the approximate elevation of point B.

$\tilde{H}_{B_2} = H_2 + h_1$

$\tilde{H}_{B_2} = 56.655 + 13.800 = 70.455m$

~~$\tilde{H}_{B3} = H_4 + h_B$~~ $\tilde{H}_{B_3} = H_3 + h_3$

~~$\tilde{H}_{B4} = 113.880$~~ $\tilde{H}_{B_3} = 54.400 + 16.053m = 70.453m$

$\tilde{H}_{B_4} = H_4 + h_2$

$\tilde{H}_{B_4} = 113.880 - 43.420 = 70.460m$

$$H_{BW} = \frac{\tilde{H}_{B_2} \cdot w_1 + \tilde{H}_{B_3} \cdot w_3 + \tilde{H}_{B_4} \cdot w_2}{w_1 + w_2 + w_3}$$

$$= \frac{(70.455 \times 0.50) + (70.453 \times 0.30) + (70.460 \times 0.19)}{0.50 + 0.19 + 0.20}$$

$$= \frac{32.2275 + 21.1359 + 13.3874}{0.99} = \frac{69.7508}{0.99}$$

$\overline{H}_{BW} = 70.455m$

* Write out the residual observation equations.

$H_B + \Delta H_B = H_2 + h_1 + v_1$

$H_B - H_2 - h_1 = v_1 - \Delta H_B$

$v_1 - \Delta H_B = 70.455 - 56.655 - 13.800$

$v_1 - \Delta H_B = 0 \Rightarrow v_1 = \Delta H_B$ —— 1^{st} ROE.

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$$H_B + \Delta H_B = H_4 + h_2 + V_2$$
$$H_B - H_4 - h_2 = V_2 - \Delta H_B.$$
$$V_2 - \Delta H_B = 70.455 - 113.880 - (-43.420)$$
$$V_2 - \Delta H_B = -0.005 \quad \text{——— 2nd ROE.}$$

$$H_B + \Delta H_B = H_3 + h_3 + V_3$$
$$H_B - H_3 - h_3 = V_3 - \Delta H_B$$
$$V_3 - \Delta H_B = 70.455 - 54.400 - 16.053$$
$$V_3 - \Delta H_B = 0.002 \quad \text{——— 3rd ROE.}$$

$$V + B\Delta = f$$
$$V_1 - \Delta H_B = 0 \quad - (1)$$
$$V_2 - \Delta H_B = -0.005 \quad - (2)$$
$$V_3 - \Delta H_B = 0.002 \quad - (3)$$

$$V\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + B\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \Delta\begin{bmatrix} \Delta H_B \end{bmatrix} = f\begin{bmatrix} 0 \\ -0.005 \\ 0.002 \end{bmatrix}, \quad W = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.19 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}$$

$$B^T\begin{bmatrix} -1 & -1 & -1 \end{bmatrix}$$

$$\cancel{N = B^T W} \; \cancel{B^T} \quad N = B^T W B. \Rightarrow B^T W_{1,3} \begin{bmatrix} -0.5 & -0.19 & -0.3 \end{bmatrix}$$

$$B^T W B \begin{bmatrix} 0.5 + 0.19 + 0.3 \end{bmatrix} = \begin{bmatrix} 0.99 \end{bmatrix}.$$

$$t = B^T W f \begin{bmatrix} 0.00095 - 0.006 \end{bmatrix} = \begin{bmatrix} 0.00035 \end{bmatrix}.$$

$$N\Delta = t \Rightarrow 0.99 \Delta H_B = 0.00035$$
$$\Delta H_B = \frac{0.00035}{0.99} = 0.000354\,m$$

$V_1 = 0.000354m$

$V_2 = \Delta HB - 0.005 = 0.000354 - 0.005 = -0.004646m$

$V_3 = \Delta HB + 0.002 = 0.000354 + 0.002 = 0.002354m$

$h_1^T = h_1 + V_1 = 13.800 + 0.000354 = 13.800m$

$h_2^T = h_2 + V_2 = -43.420 - 0.004646 = -43.425m$

$h_3^T = h_3 + V_3 = 16.053 + 0.002354 = 16.055m$

To check. $H_B = H_2 + h_1^T = 70.455m$

$H_B = H_3 + h_3^T = 70.455m$

$H_B = H_4 + h_2^T = 70.455m$

4a. Given

$S = 250.000m$ $\quad m_S = 0.050m$

$\alpha = 30°45'00"$ $\quad m_\alpha = 20.0"$

Calculate;

$(E, N)_K = ?$

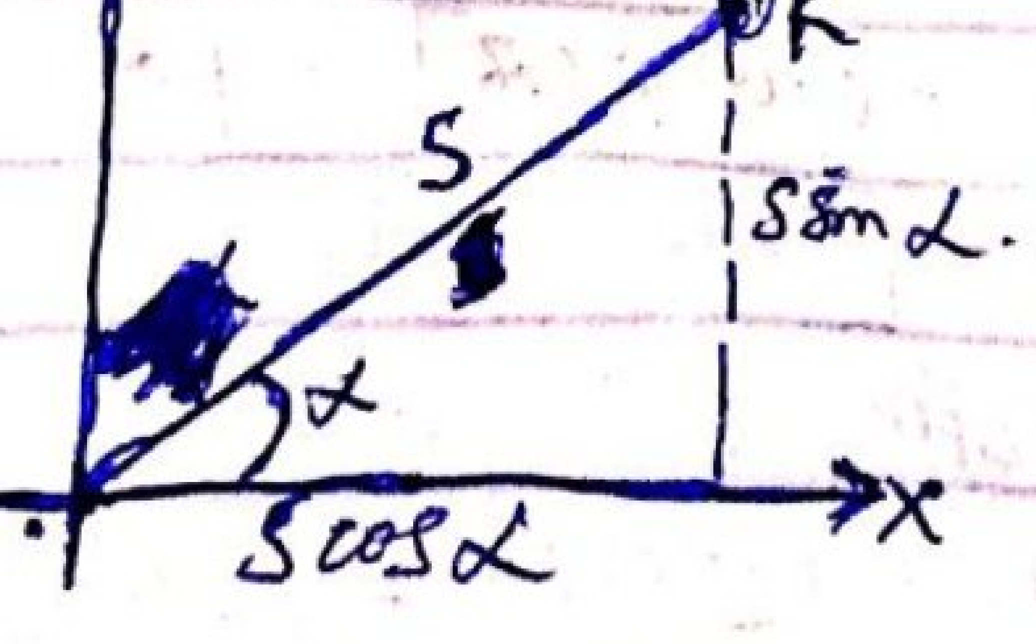
m_E & $m_N = ?$

~~m_{EN}~~ = ?, ?con = ?

For E_K & N_K.

$E_K = S\sin\alpha = 250 \times \sin 30°45' = \cancel{127.823m}$ $-214.852m$.

$N_K = S\cdot\cos\alpha = 250 \times \cos 30°45' = \cancel{214.852m}$ $127.823m$



For $m_{E,N}$ {m_E & m_N}

Using the general laws of propagation of variances for linear and non linear equations, standard error of E and N can be estimated.

$N = S\cos\alpha$

$E = S\sin\alpha$

$\frac{\delta N}{\delta S} = \cos\alpha$, $\frac{\delta N}{\delta\alpha} = -S\sin\alpha$

$\frac{\delta E}{\delta S} = \sin\alpha$, $\frac{\delta E}{\delta\alpha} = S\cos\alpha$

~~Their estimate~~ The estimated errors in the value of E & N are solved using $\Sigma_{E,N} = A\Sigma A^T$

$$A\Sigma A^T = A\begin{bmatrix} \frac{\delta N}{\delta S} & \frac{\delta N}{\delta\alpha} \\ \frac{\delta E}{\delta S} & \frac{\delta E}{\delta S} \end{bmatrix} \Sigma \begin{bmatrix} m_S^2 & 0 \\ 0 & m_\alpha^2 \end{bmatrix} \times A^T \begin{bmatrix} \frac{\delta N}{\delta S} & \frac{\delta E}{\delta S} \\ \frac{\delta N}{\delta\alpha} & \frac{\delta E}{\delta\alpha} \end{bmatrix}$$

$$\Sigma_{E,N} = \begin{bmatrix} \cos\alpha & -S\sin\alpha \\ \sin\alpha & S\cos\alpha \end{bmatrix}\begin{bmatrix} 0.05^2 & 0 \\ 0 & \left(\frac{20''}{\rho}\right)^2 \end{bmatrix} \times \begin{bmatrix} \cos\alpha & \sin\alpha \\ -S\sin\alpha & S\cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 0.5113 & 214. \\ \cancel{0.8594} & -127.8233 \\ -0.8594 & 214.8516 \end{bmatrix}\begin{bmatrix} 0.0025 & 0 \\ 0 & 9.4\times10^{-9} \end{bmatrix} \times \begin{bmatrix} 0.8594 & 0.5113 \\ -127.8233 & 214.8516 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0021485 & -0.000001120154 \\ 0.00127825 & 0.00000201961 \end{bmatrix} \times \begin{bmatrix} 0.8594 & 0.5113 \\ -127.8233 & 214.8516 \end{bmatrix}$$

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$$= \begin{bmatrix} 0.5113 & 214.852 \\ -0.8594 & 127.823 \end{bmatrix} \begin{bmatrix} 0.0025 & 0 \\ 0 & 9.4\times10^{-9} \end{bmatrix} \times \begin{bmatrix} 0.5113 & -0.8594 \\ 214.852 & 127.823 \end{bmatrix}$$

$$= \begin{bmatrix} 0.00127825 & 0.0000020196088 \\ -0.0021485 & 0.0000012015362 \end{bmatrix} \times \begin{bmatrix} 0.5113 & -0.8594 \\ 214.852 & 127.823 \end{bmatrix}$$

$$\Sigma_{E,N} = \begin{bmatrix} 0.0010874821489276 & -0.0008403755594 \\ -0.0008403755594 & 0.00200000048616926 \end{bmatrix}.$$

$$\Sigma_{EN} = \begin{bmatrix} 0.00108748 & -0.00084038 \\ -0.00084038 & 0.00200000 \end{bmatrix} \Rightarrow \begin{bmatrix} Q_N & Q_{E,N} \\ Q_{E,N} & Q_E \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$$

In the above result, q_{11} is the variance of latitude (N) and q_{22} is the variance of departure (E) while q_{12} and q_{21} are the covariances. Thus the standard errors are

$$M_N = \sqrt{q_{11}} = \sqrt{0.00108748}$$

$$M_N = 0.03\text{m}.$$

$$M_E = \sqrt{q_{22}} = \sqrt{0.00200000}$$

$$M_E = 0.04\text{m}$$

Correlation co-efficient for E and N is

$$r_{E,N} = \frac{Q_{E,N}}{Q_E \times Q_N} = \frac{-0.00084038}{0.04 \times 0.03}$$

$$r_{E,N} = -0.7010\text{m}$$

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* For $\rho_{E,N}$

$$\rho_{E,N} = \frac{Q_{EN}}{Q_N \cdot Q_E} = \frac{Q_{E,N}}{m_N \cdot m_E}$$

$$\rho_{E,N} = \frac{-0.00084038}{0.03 \times 0.04} = \frac{-0.00084038}{0.0012}$$

$$\rho_{E,N} = -0.700m$$

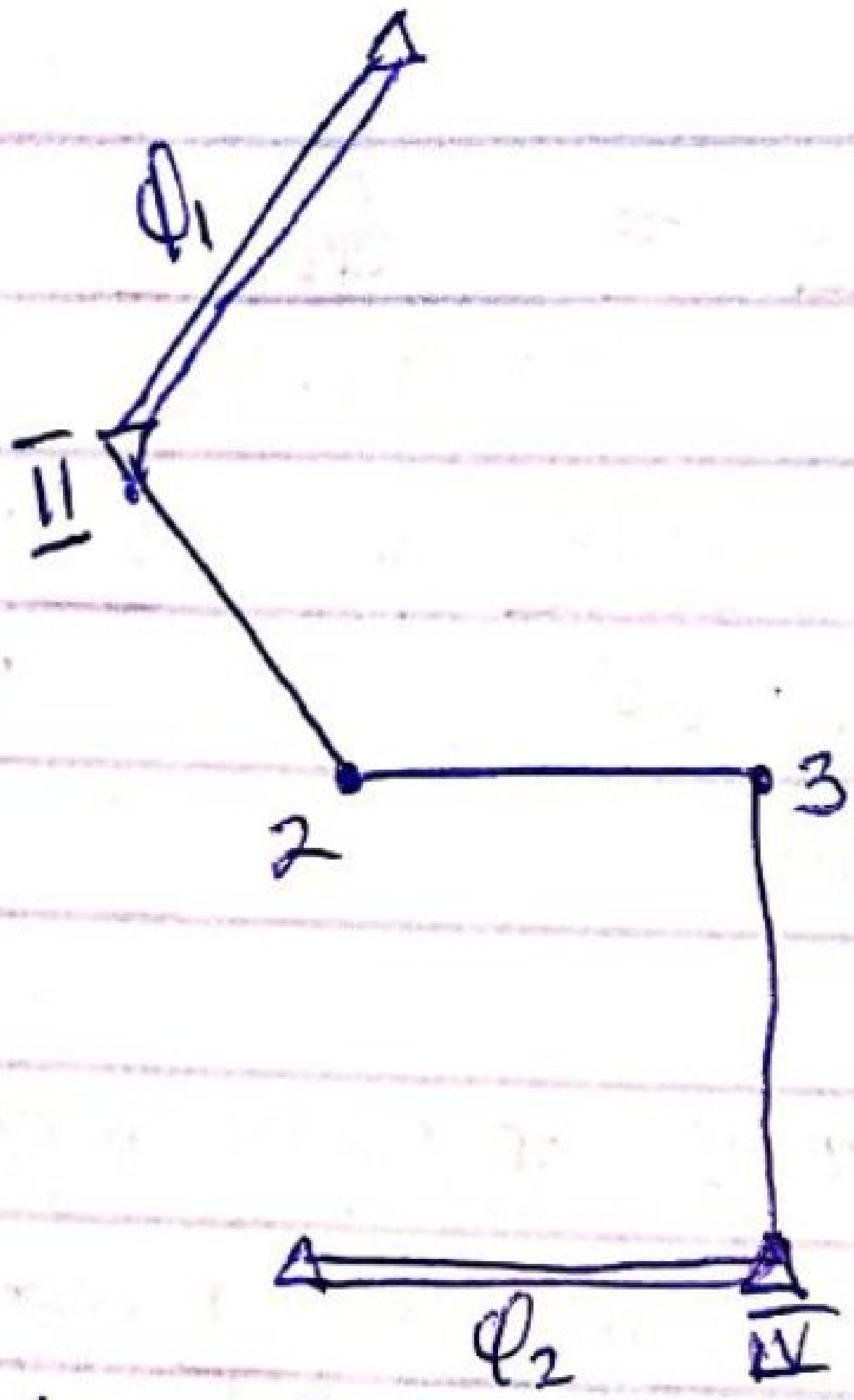
4b. ESSENCE OF PRE ANALYSIS OF SURVEY MEASUREMENT TO SURVEY PROJECT PLANNING AND EXECUTION.

Preanalysis of survey measurements is analysis of the component measurements of a survey project before the project is actually undertaken.

* Pre-analysis is very helpful in the overall design of the survey project because;
- it provides a basis for evaluation of the accuracies of the survey measurements, for meeting tolerances that may be imposed on these measurements and for selection of suitable instrumentation and measurement procedures.

5.

Traverse lines	Line length	WCB	Partial ΔN	Partial ΔE
II – 2	5544.47	172° 44' 47.4	-5500.10	700.04
2 – 3	5393.52	100° 41' 06.3	-1000.02	5300.00
3 – IV	5500.12	180° 00' 00.2	-5500.12	-0.01



$\phi_1 = 67^\circ 09' 58.3''$
$\phi_2 = 270^\circ 00' 00.1''$

$\overline{II}(E,N) = 22000.00E, 24000.00N$
$\overline{IV}(E,N) = 28000.00E, 12000.00N$

- Write out the condition equations

$[V\Delta E] + f_E = 0 \quad -(1)$
$[V\Delta N] + f_N = 0 \quad -(2)$

$V\Delta E_1 + V\Delta E_2 + V\Delta E_3 + f_E = 0 \quad -(3)$
$V\Delta N_1 + V\Delta N_2 + V\Delta N_3 + f_N = 0 \quad -(4)$

Recall : $V\Delta E_i = VS_i \sin\alpha_i$
$V\Delta N_i = VS_i \cos\alpha_i$

$VS_1\sin\alpha_1 + VS_2\sin\alpha_2 + VS_3\sin\alpha_3 + f_E = 0 \quad -(5)$
$VS_1\cos\alpha_1 + VS_2\cos\alpha_2 + VS_3\cos\alpha_3 + f_N = 0 \quad -(6)$

$f_E = [\Delta E] - [E_{\overline{IV}} - E_{\overline{II}}]$
$= [700.04 + 5300 - 0.01] - [28000 - 22000]$
$f_E = 6000.03 - 6000.$
$f_E = 0.03m$

$$f_N = [\Delta N] - [N_{IV} - N_{II}]$$
$$= [-5500.10 - 1000.02 - 5500.12] - [12000 - 24000]$$
$$= -12000.24 - (-12000)$$
$$f_N = -0.24\text{m}.$$

$$\sin\alpha_1 = \sin 172°44'47.4'' = 0.1263$$
$$\sin\alpha_2 = \sin 100°41'06.3'' = 0.9827$$
$$\sin\alpha_3 = \sin 180°00'00.2'' = 0.0000$$

$$\cos\alpha_1 = \cos 172°44'47.4'' = -0.9920$$
$$\cos\alpha_2 = \cos 100°41'06.3'' = -0.1854$$
$$\cos\alpha_3 = \cos 180°00'00.2'' = -1.0000$$

$$V_{S1}\sin\alpha_1 + V_{S2}\sin\alpha_2 + V_{S3}\sin\alpha_3 + 0.03\text{m} = 0$$
$$V_{S1}\cos\alpha_1 + V_{S2}\cos\alpha_2 + V_{S3}\cos\alpha_3 - 0.24\text{m} = 0$$

$$AV + f = 0$$

$$A\begin{bmatrix}\sin\alpha_1 & \sin\alpha_2 & \sin\alpha_3 \\ \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3\end{bmatrix} \quad V\begin{bmatrix}V_{S1} \\ V_{S2} \\ V_{S3}\end{bmatrix} + f\begin{bmatrix}0.03 \\ -0.24\end{bmatrix}$$

$$A^T\begin{bmatrix}\sin\alpha_1 & \cos\alpha_1 \\ \sin\alpha_2 & \cos\alpha_2 \\ \sin\alpha_3 & \cos\alpha_3\end{bmatrix} \quad Q\begin{bmatrix}5544.47 & 0 & 0 \\ 0 & 5393.52 & 0 \\ 0 & 0 & 5500.12\end{bmatrix}$$

$$Q_e.$$

$$\underset{3,2}{QA^T}\begin{bmatrix} 700.2666 & -5500.1142 \\ 5300.2121 & -999.9586 \\ 0 & -5500.12 \end{bmatrix} \quad \underset{2,2}{AQA^T}\begin{bmatrix} 5296.9621 & -1677.3238 \\ -1677.3238 & 1141.6257 \end{bmatrix}$$

$$Q_e K = f.$$

$$\begin{bmatrix} 5296.9621 & -1677.3238 \\ -1677.3238 & 1141.6257 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0.03 \\ -0.24 \end{bmatrix}$$

$$5296.9621 k_1 - 1677.3238 k_2 = 0.03$$

$$-1677.3238 k_1 + 1141.6257 k_2 = -0.24.$$

$$k_1 = -0.00000121539$$

$$k_2 = -0.00002172381.$$

$$V = QA^T K$$

$$V\begin{bmatrix} 0.1186 \\ 0.0153 \\ 0.1195 \end{bmatrix}\begin{matrix} V_{S_1} \\ V_{S_2} \\ V_{S_3} \end{matrix}$$

$$V\Delta E_1 = V_{S_1} \sin\alpha_1 = 0.1186 \times 0.1263 = 0.01498 \simeq 0.0150 m$$

$$V\Delta E_2 = V_{S_2} \sin\alpha_2 = 0.0153 \times 0.9827 = 0.0150 m$$

$$V\Delta E_3 = V_{S_3} \sin\alpha_2 = 0.1195 \times 0.0000 = 0 m$$

$$V\Delta N_1 = V_{S_1} \cos\alpha_1 = 0.1186 \times -0.9920 = -0.118 m$$

$$V\Delta N_2 = V_{S_2} \cos\alpha_2 = 0.0153 \times -0.1854 = -0.003 m$$

$$V\Delta N_3 = V_{S_3} \cos\alpha_3 = 0.1195 \times -1.0000 = -0.120 m$$

$A\Delta E_1 = P\Delta E_1 + V\Delta E_1$

$A\Delta E_1 = +700.040 + 0.015 = 700.055$

$A\Delta E_2 = P\Delta E_2 + V\Delta E_2$

$A\Delta E_2 = +5300.00 + 0.015 = +5300.015m.$

$A\Delta E_3 = P\Delta E_3 + V\Delta E_3.$

$A\Delta E_3 = -0.01 + 0 = -0.01m$

$A\Delta N_1 = P\Delta N_1 + V\Delta N_1$

$A\Delta N_1 = -5500.10 - 0.118m = -5500.218m$

$A\Delta N_2 = P\Delta N_2 + V\Delta N_2$

$A\Delta N_2 = -1000.02 + -0.003 = -1000.023m$

$A\Delta N_3 = P\Delta N_3 + V\Delta N_3$

$A\Delta N_3 = -5500.12 + -0.120 = 5500.240m$

$F_E = [A\Delta E] - [E_{IV} - E_{II}]$
$= 700.055 + 5300.015 + [-0.01] - [28000 - 22000]$

F_E

6b x is a component of the random vector f.

$Q_{ff} = B Q_{xx} B^T$

Q_{ff} is the cofactor ~~g~~ matrix of f.

Q_{xx} is the cofactor matrix of x

How to get the B matrix

$f_1 = x_1 + x_2$

$f_2 = x_1 + x_2 + x_3$

'B' represents the co-efficient of x

$$\underset{2,3}{B}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \underset{3,3}{Q_{xx}}\begin{bmatrix} 0.50 & -0.25 & 0 \\ -0.25 & 0.50 & -0.25 \\ 0 & -0.25 & 0.50 \end{bmatrix}$$

$$\underset{3,2}{B^T}\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \underset{3,2}{Q_{xx}B^T}\begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0 \\ -0.25 & 0.25 \end{bmatrix}$$

(i) $$Q_{ff} = \underset{2,2}{B Q_{xx} B^T}\begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \Rightarrow \begin{bmatrix} q_{11}^2 & q_{12} \\ q_{21} & q_{22}^2 \end{bmatrix}$$

(ii) $$e_{f_1f_2} = \frac{q_{12}}{q_{11} \cdot q_{22}}$$

~~$q_{11}^2 = 0.25$~~

$q_{11}^2 = 0.5$

$q_{11} = \sqrt{0.5} = 0.707m$

$$e_{f_1f_2} = \frac{0.25}{0.707 \times 0.707}$$

$$e_{f_1f_2} = \frac{0.25}{0.5}$$

$q_{22}^2 = 0.5$

$q_{22} = \sqrt{0.5} = 0.707m$

$e_{f_1f_2} = \underline{0.5}$

6c. $\underset{2,5}{B^T} = \begin{bmatrix}1 & 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1 & 1\end{bmatrix}$ $B = \begin{bmatrix}1 & 0\\ 1 & 0\\ -1 & 1\\ 0 & 1\\ 0 & 1\end{bmatrix}$ $\underset{2,2}{Q_{\Delta\Delta}} = \frac{1}{8}\begin{bmatrix}4 & 2\\ 2 & 3\end{bmatrix}$

$\cancel{h = \begin{bmatrix}h_1\\ h_2\\ h_3\\ h_4\\ h_5\end{bmatrix}}$ $h = [h_1\ h_2\ h_3\ h_4\ h_5]$

Solution

$v + B\Delta = f$

$Q_{hh} = BN^{-1}B^T$

where $N^{-1} = Q_{\Delta\Delta} = Q_{\Delta}$.

$Q_{hh} = BQ_{\Delta\Delta}B^T$..

$\underset{2,6}{Q_{\Delta\Delta}B^T} = \frac{1}{8}\begin{bmatrix}4 & 4 & -2 & 2 & 2\\ 2 & 2 & 1 & 3 & 3\end{bmatrix}$ $\cancel{Q_{hh} = BQ}$

(i) $\underset{5,5}{Q_{hh}} = BQ_{\Delta\Delta}B^T = \frac{1}{8}\begin{bmatrix}4 & 4 & -2 & 2 & 2\\ 4 & 4 & -2 & 2 & 2\\ -2 & -2 & 3 & 1 & 1\\ 2 & 2 & 1 & 3 & 3\\ 2 & 2 & 1 & 3 & 3\end{bmatrix} \Rightarrow \begin{bmatrix}q_{h1} & & & & \\ & q_{h2} & & & \\ & & q_{h3} & & \\ & & & q_{h4} & \\ & & & & q_{h5}\end{bmatrix}$

$q_{h1} = 4/8$, $q_{h2} = 4/8$, $q_{h3} = 3/8$, $q_{h4} = 3/8$ and $q_{h5} = 3/8$

(ii) $\Sigma_{hh} = m_0^2 \cdot Q_{hh}$.

$m^2_{h3} = m_0^2 \cdot q_{h3} = 0.015^2 \times 3/8$

$m_{h3} = \sqrt{0.015^2} \times \sqrt{3/8} \Rightarrow 0.015 \times 0.612 = \underline{0.009}\,m$

$m^2_{h5} = m_0^2 \times q_{h5} = 0.015^2 \times 3/8$

$m_{h5} = \sqrt{0.015^2} \times \sqrt{3/8} \Rightarrow 0.015 \times 0.612 = \underline{0.009}\,m$

6a. 1. ~~CO~~ VARIANCE – COVARIANCE MATRIX

A variance – covariance matrix is a square matrix that contains the variances and covariances associated with several variables. The diagonal elements of the matrix contain the variances of the variables and the off-diagonal elements contain the covariances between all possible pairs of variables. Covariance is basically a relationship between two randome variables only.

variance

covariance

ii ~~WEIGHT MATRIX~~ eg ⇒ $\begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$

covariance — variance

ii.