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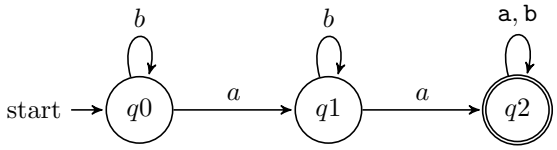
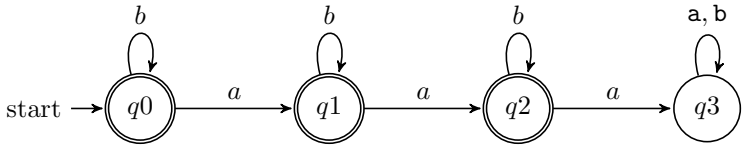
Homework 1–CSC 320 Spring 2020

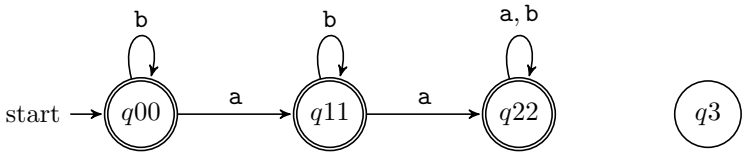
1. (a) DFA for L_1:



(b) DFA for L_2:



(c) DFA for $L_1 \cup L_2$:

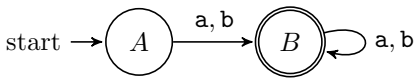


2. Transition table for the DFA:

$E(1) = \{1, 3\} = A$
$\delta(A, a) = E(\{2\}) = \{1, 2, 3\} = B$
$\delta(A, b) = E(\{2, 3\}) = \{1, 2, 3\} = B$
$\delta(B, a) = E(\{2, 3\}) = \{1, 2, 3\} = B$
$\delta(B, b) = E(\{2, 3\}) = \{1, 2, 3\} = B$

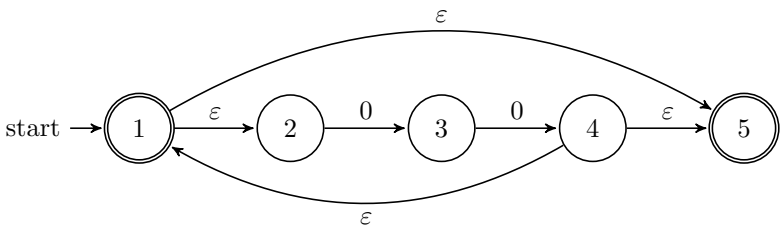
	a	b
A	B	B
B	B	B

Transition diagram for the DFA.

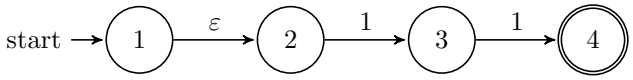


3. Convert the following regular expression to an NFA: $(((\texttt{00})^*(\texttt{11})) \cup \texttt{01})^*$.

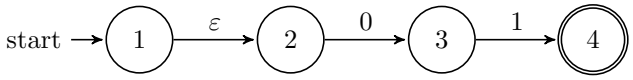
$R_1 = (00)^*$:



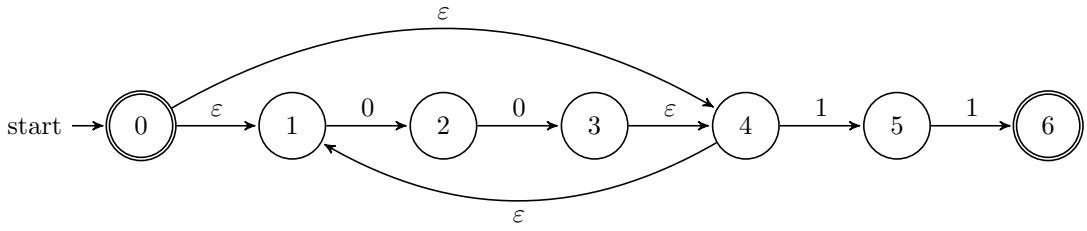
$R_2 = 11$:



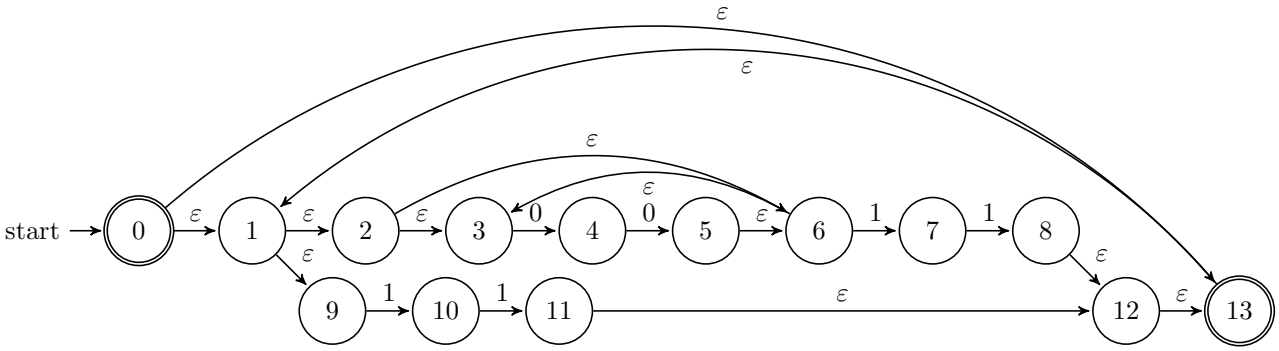
$R_3 = 01$:



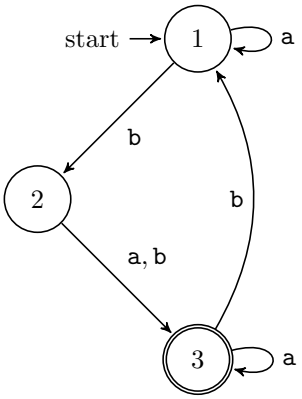
$R_{12} = R_1 R_2$:



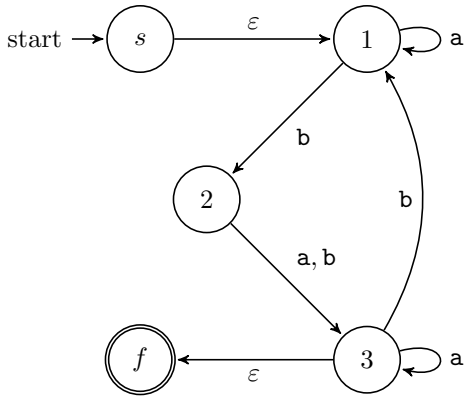
$R = (R_{12} \cup R_3)^*$:



4. (20 Marks) Use the procedure given in class to convert the following DFA to a regular expression

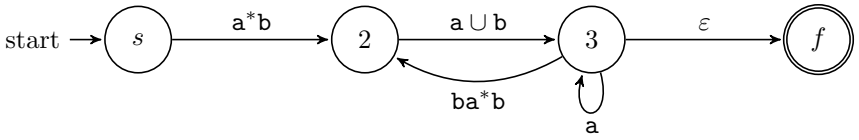


(a) Start and end states have no incomings and outgoings respectively

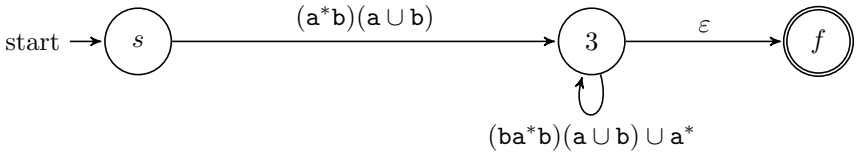


(b) Remove intermediate states

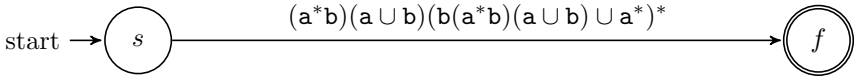
i. Remove state 1



ii. Remove state 2



iii. Remove state 3



The regular expression of the given DFA is: $(\mathtt{a}^*\mathtt{b})(\mathtt{a} \cup \mathtt{b})(\mathtt{b}(\mathtt{a}^*\mathtt{b})(\mathtt{a} \cup \mathtt{b}) \cup \mathtt{a}^*)^*$.

5. For languages A and B, define the *interleave* of A and B to be the language

$$\{w \mid w = a_1 b_1 \dots a_k b_k \text{ where } a_1 \dots a_k \in A,\ b_1 \dots b_k \in B,\ \text{and } a_i, b_i \in \Sigma, 1 \leq i \leq k\}$$

Give a construction that shows that the regular languages are closed under the mix operation.
Construction: Let $M_A = (Q_A, \Sigma_A, \delta_A, q_A, F_A)$, $M_B = (Q_B, \Sigma_B, \delta_B, q_B, F_B)$, and $M = (Q, \Sigma, \delta, q_0, F)$:

(a) $Q = Q_A \times Q_B \cup q_0$: all possible states of M_A and M_B;

(b) $\Sigma = \Sigma_A \times \Sigma_B$: M accepts the string of the *interleave* of M_A and M_B and the empty string;

(c) δ, or the transition function, follows the manner below:

i. $\delta((a, b, A), x) = (\delta(a, x), b, B)$: If the current state of M_A is a, the current state of M_B is b, and the next character should be in M_A, then when x is read, we change the current state of M_A to $\delta(a, x)$, and keep the current state of M_B. The next character at this point will be in M_B;

ii. Similarly for $\delta((a, b, B), x) = (\delta(b, x), a, A)$;

iii. $\delta(q_0, \varepsilon) = q_0$

(d) $q_0 \in Q \cap F$: q_0 is the *startstate* and one of *acceptstates*;

(e) $F = F_A \times F_B \cup q_0$: M accepts a string if after processing the whole string, both DFAs are in accept states, or in the start state q_0 (the string is empty).