Homework 3

Problem 1

For a 2-link arm, assume

- Upper arm length: $l_1 = 0.34\ \mathrm{m}$
- Forearm length: $l_2 = 0.46\ \mathrm{m}$
- Hand position: $x = (0.36, 0.65)$ in m
- Hand velocity: $\dot{x} = (-3.89, 1.30)$ in $\mathrm{m/s}$
- Hand acceleration: $\ddot{x} = (-7.79, -26.18)$ in $\mathrm{m/s^2}$

a: The Jacobian

Work through the derivaitves by hand to find $J(\theta)$ and $\dot{J}(\theta)$.

b: Inverse kinematics

Use Python (or other program) to calculate numerically the joint angle in degrees, angular velocity in deg/s, and angular acceleration in deg/s^2.

c: Forward kinematics

Starting with your answers in part (b), calculate the forward kinematics to get hand position, velocity and acceleration. Compare your results to the original values.

Problem 2

a: Simulate dynamics of two joint arm

Following the equations of motion defined in class, simulate the dynamics a two-link arm of 4 sec. Plot the trajectory in the x/y plane and the x and y velocities as functions of time.

Use a time step of $1\ \mathrm{msec}$ and a total simulation time of $4\ \mathrm{sec}$.

Use the following constants for the arm:

Upper arm

- Mass: $m_1 = 2.1\ \mathrm{kg}$
- Moment of inertia: $I_1 = 0.025\ \mathrm{N\ s^2}$
- Length: $l_1 = 0.3384\ \mathrm{m}$
- Center of mass: $r_1 = 0.1692\ \mathrm{m}$

Lower arm

- Mass: $m_2 = 1.65\ \mathrm{kg}$
- Moment of inertia: $I_2 = 0.075\ \mathrm{N\ s^2}$
- Length: $l_2 = 0.4554\ \mathrm{m}$
- Center of mass: $r_2 = 0.2277\ \mathrm{m}$

- Gravitational constant: $g = 9.81\ \mathrm{m/s^2}$

- Shoulder muscle torque: $\tau_s = 0$
- Elbow mucle torque: $\tau_e = 0$

And initial conditions:

- $\theta_s = 180^\circ$
- $\theta_e = 1^\circ$ = $\dot{\theta}_s = 0^\circ/s$ = $\dot{\theta}_e = 0^\circ/s$

b: Show energies

Plot the total kinetic energy (T), total potential energy (U) and grand total energy (T+U) over the course of the simulation.