Homework 3

Question 1:

Implement the grid method for the coin flip problem using a beta prior with $\alpha = \beta = 20$.

Assume that we flipped the coin 13 times and got 3 heads.

Compare the results to those of the analytical calculation.

Question 2:

Implement the Metropolis-Hastings method for the coin flip problem.

1. Use the same prior and data as Question 1.
2. Use a normal distribution as the proposal distribution. Use a standard deviation of 0.05.

a. Show the chain for 100 steps and for 10,000 steps.
b. Show the resulting samples of the posterior distribution for each of these number of steps. Plot the comparison to the analytical posterior.

Question 3:

a. Load and present the chemical shift data you saw in the lecture.
   You can use this line to load the chemical shift data:

```
data =
np.loadtxt("https://github.com/aloctavodia/BAP3/raw/refs/heads/main/code/data/chemical_shifts.csv")
```

b. Model the data using the Student's t-distribution, as in Question 3 in Homework 2.
c. Present the following diagnostics:
   a. Trace plots
   b. Rank plots
   c. Rhat
   d. Effective sample size
   e. Monte Carlo Standard Error
d. Did the sampler appear to work well? Explain.

Question 4:

We will use the howell dataset (available at https://github.com/aloctavodia/BAP3), which provides data about the weight and height.

You can download the data from the moodle and load it using the following code:

data = pd.read_csv(io.BytesIO(uploaded['howell.csv']), delimiter=";")

a. Plot the data.
b. Create a linear model of the weight (x) against the height (y). Exclude subjects that are younger than 18. Explain the results.
c. For four subjects, we get the weights (45.73, 65.8, 54.2, 32.59), but not their heights. Using the model, predict the height for each subject.
d. Repeat a-b, this time including those below 18 years old. Explain the results.
e. It is known for many species that weight does not scale with height, but with the logarithm of the weight. Additionally, we will not assume that the variance is constant for all weights. Use this information to fit the howell data (including subjects from all ages).

Question 5:

A colleague of yours, who now lives on an unknown planet, ran experiment to test the basic laws of physics. She dropped a ball of a cliff and registers the position for 20 seconds. The data is available in the gravity_measurements.csv file.
You know that from Newton's Laws of physics if the acceleration is g and the time t then:

$$velocity = gt$$
$$position = \frac{1}{2}gt^2$$

Your friend asks you to estimate the following quantities.

a. The gravitational constant of the planet.
b. A characterization of the noise of her measurement device.
c. The velocity of the ball at each point during her measurements.
d. The estimated position of the ball from time 20 to time 30.