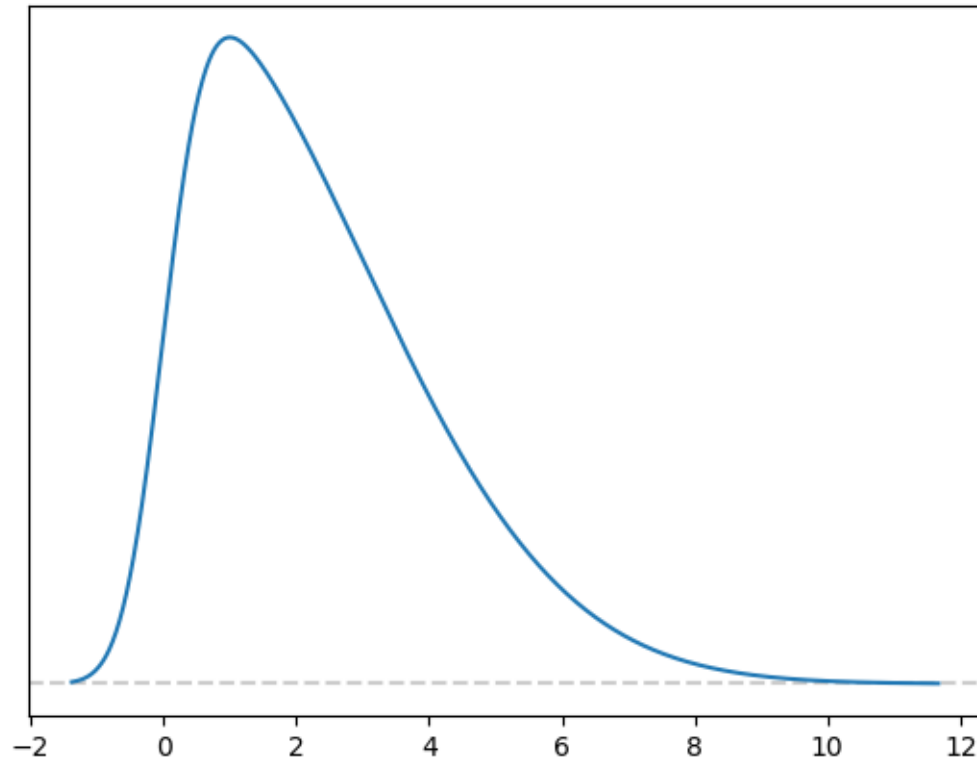


## Homework 1 Solutions

### Question 1:

a.

**SkewNormal**(mu=0, sigma=3, alpha=6)  
 $\mu=2.36, \sigma=1.85, \gamma=0.891, \kappa=0.75$



b.

Sample size 10: mean = 1.91, sigma = 1.64

Sample size 100: mean = 2.58, sigma = 1.79

Sample size 1000: mean = 2.44, sigma = 1.97

As the sample size increases, the values of the parameters are generally closer to the correct values.

c. Similar and answers are in the notebook.

Question 2:

Coin 1 – fair

Coin 2 – loaded

**Probabilities of choosing each coin:**

$$P(\text{coin1}) = P(\text{coin2}) = 0.5$$

**Probabilities of flips within each coin:**

$$P(\text{heads}|\text{coin1}) = P(\text{tails}|\text{coin1}) = 0.5$$

$$P(\text{heads}|\text{coin2}) = 1$$

$$P(\text{tails}|\text{coin2}) = 0$$

**Bayes:**

$$P(\text{coin2}|\text{heads}) = \frac{P(\text{heads}|\text{coin2})P(\text{coin2})}{\sum_{i=1}^2 P(\text{heads}|\text{coin}_i)P(\text{coin}_i)} = \frac{1 \cdot 0.5}{0.5 \cdot 0.5 + 1 \cdot 0.5} = \frac{2}{3}$$

Question 3:

- a. The likelihood is the normal distribution with parameters  $\mu, \sigma$ .  
 $\mu$  has a normal prior with  $(N(0,2))$  and  $\sigma$  has a half normal prior.
- b. The posterior will have two parameters – one for each  $\mu$  and  $\sigma$ .

c. 
$$P(\mu, \sigma|Y) = \frac{P(Y|\mu, \sigma)P(\mu)P(\sigma)}{P(Y)}$$

$$P(Y|\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(Y-\mu)^2}{2\sigma^2}}$$

$$P(\mu) = \frac{1}{\sqrt{8\pi}} e^{-\frac{\mu^2}{8}}$$

$$P(\sigma) = \frac{1}{\sqrt{2\pi} \cdot 0.75} e^{-\frac{\sigma^2}{2 \cdot 0.75^2}}$$

d.

