

A Additional figures and tables

Figure A.1: Relationship between minimum sentenced incarceration length and actual time served

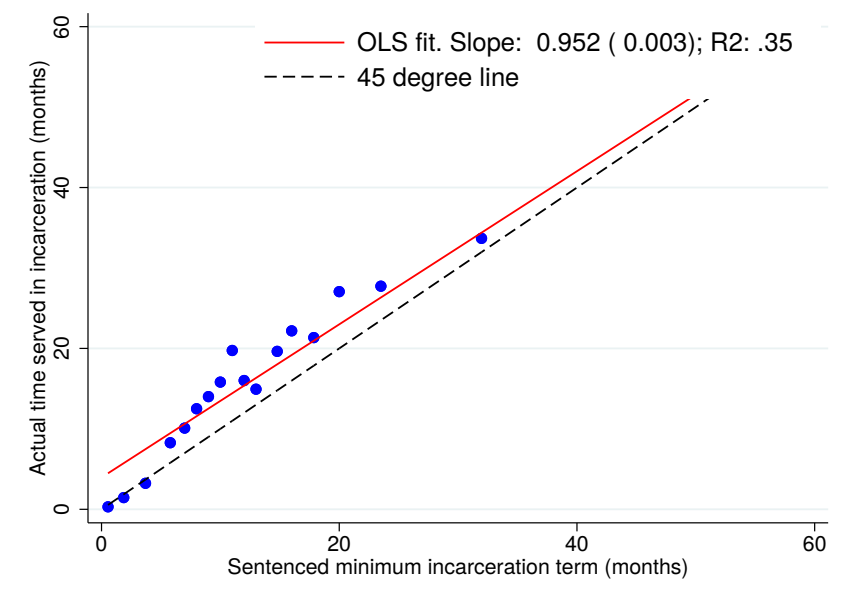
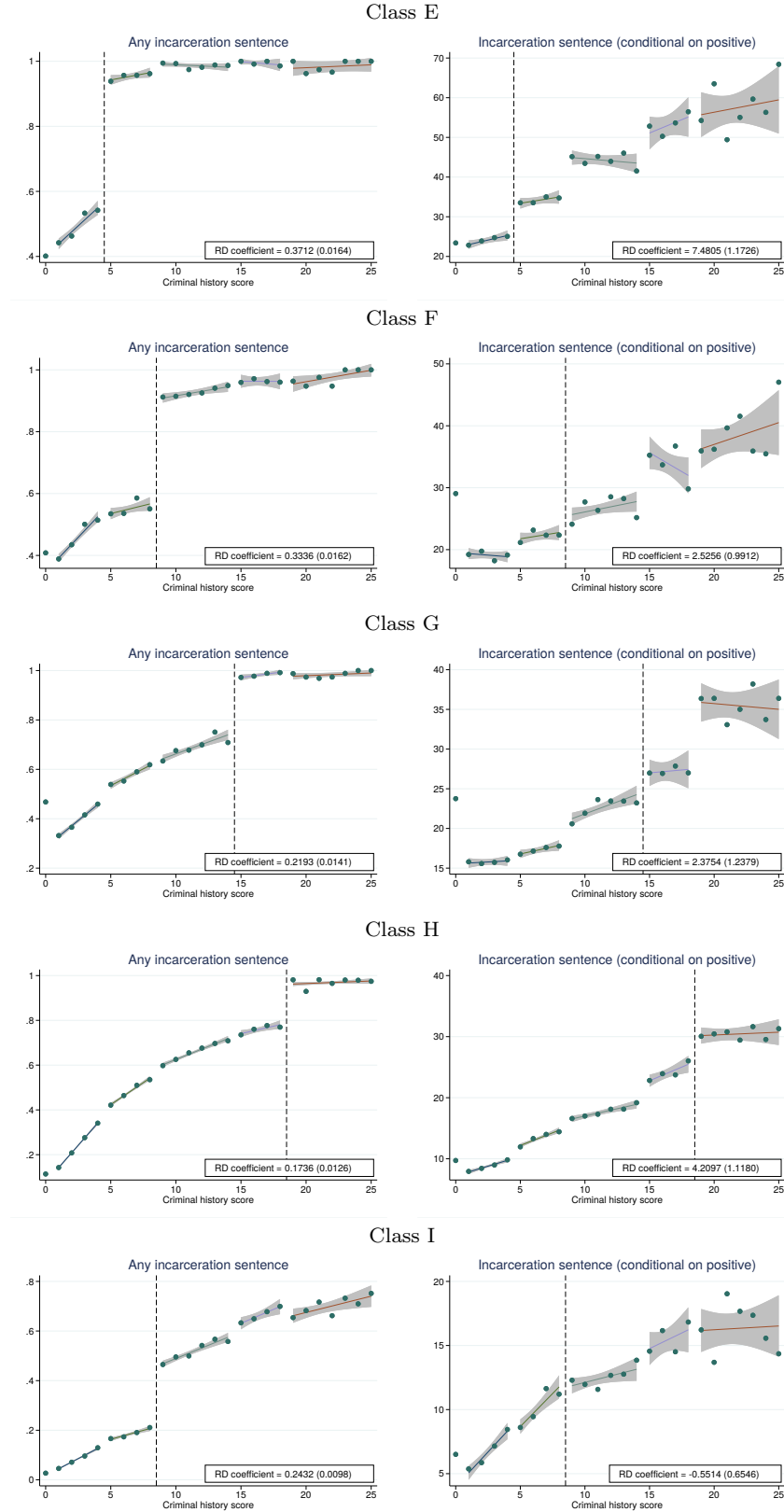


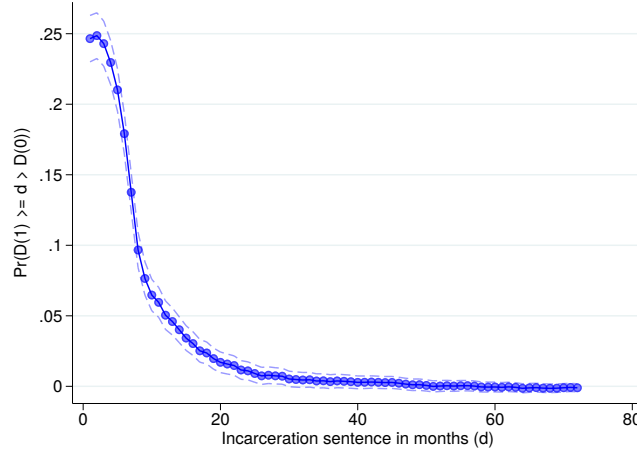
Figure A.2: Sentencing outcomes by felony class and prior points



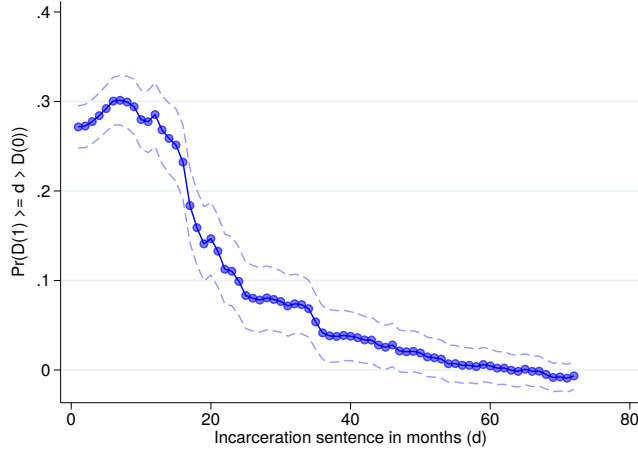
Notes: The x-axis in all plots is the number of prior record points. The y-axis is the share of offenders who are sentenced to incarceration (left plots) or the number of months incarcerated conditional on a positive sentence (right plots). The figures only include offenses sentenced under the sentencing grid that applied to offenses committed between 1995 to 2009. In 2009 the guidelines changed and the discontinuities shifted by one prior points either to the left or to the right. All official grids are in Appendix B.

Figure A.3: Average causal response (ACR) weights across punishment type discontinuities

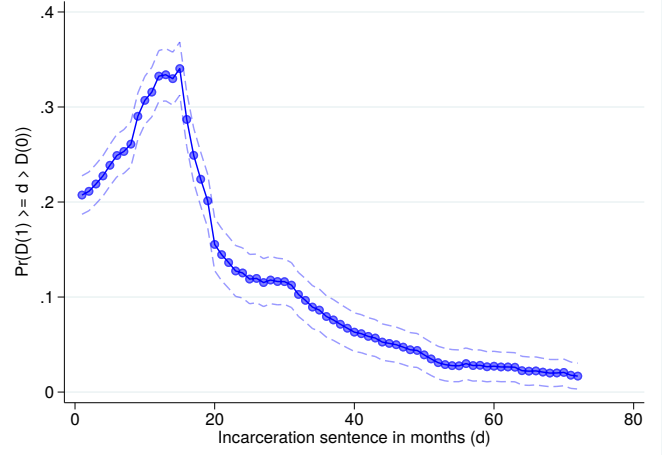
Class I



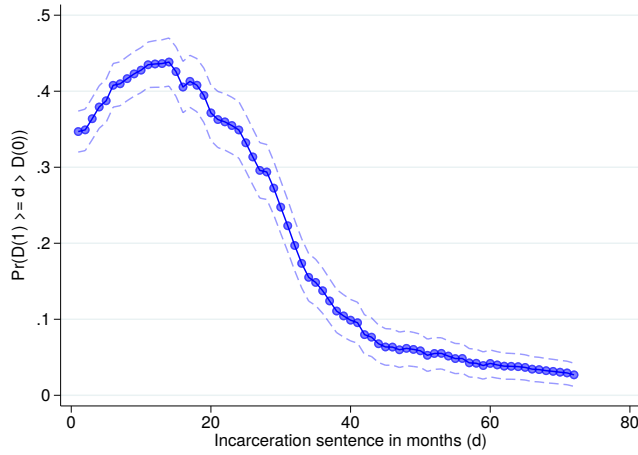
Class G



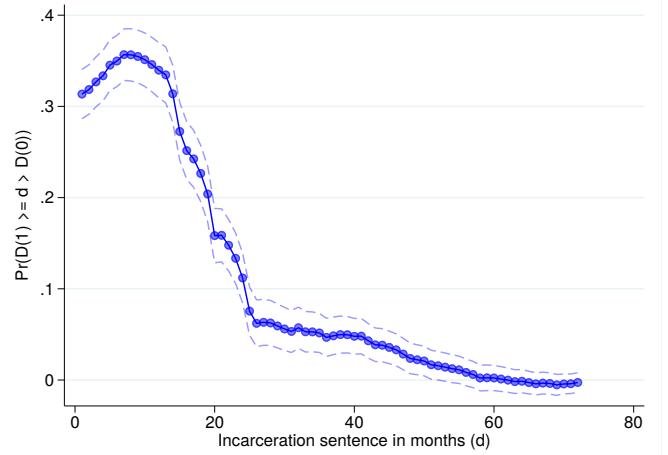
Class H



Class E

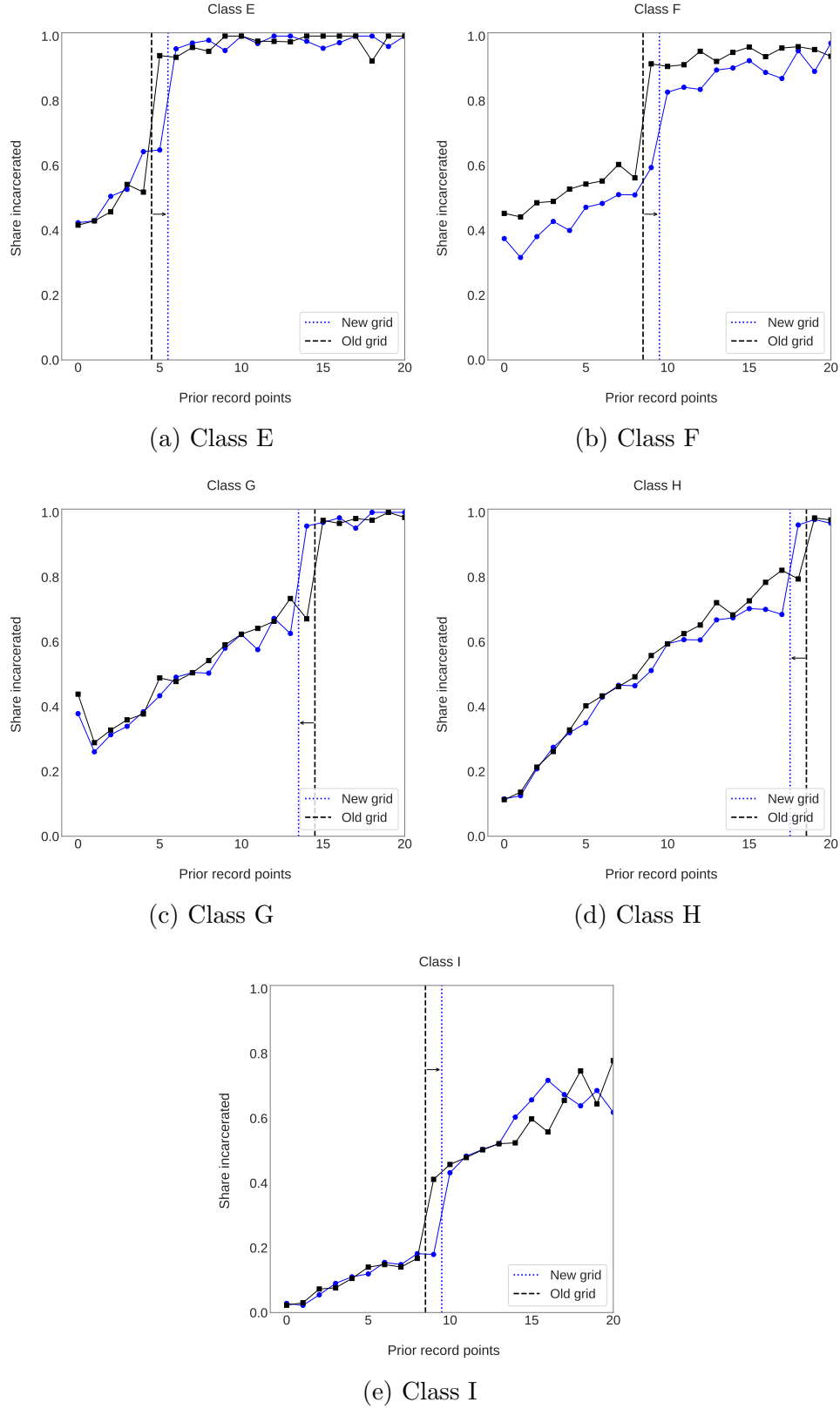


Class F



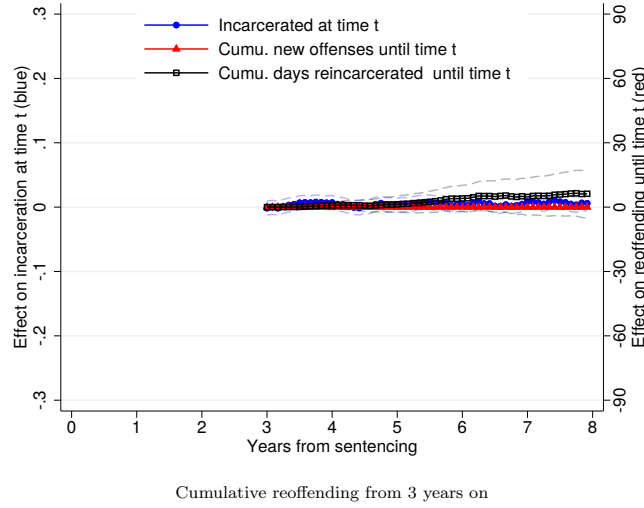
Notes: Each figure plots estimates of the shifts in incarceration exposure generated by each instrument, which correspond to the weights in the average causal response. These shifts reflect the probability an offender would spend less than d months incarcerated if assigned $Z_i = 0$, but at least d months if assigned $Z_i = 1$. This probability can be estimated non-parametrically as $\mathbb{E}[1(D_i \geq d)|Z_i = 1] - \mathbb{E}[1(D_i \geq d)|Z_i = 0]$, which corresponds to the coefficient on Z_i in our first stage specification when $1(D_i \geq d)$ is the outcome.

Figure A.4: Shifts in incarceration exposure as a result of 2009 grid changes



Notes: The x-axis in all plots is the number of prior record points. The y-axis is the share of offenders who are sentenced to an incarceration punishment. The black line represents the share of offenders sentenced to incarceration prior to the 2009 reform, with the blue line plotting the share afterwards. The plots demonstrate how the discontinuities in the sentencing grid, and thus exposure to incarceration, changed following the 2009 change in sentencing guidelines. The old grid refers to the sentencing in place between 1995 to 2009; the new grid refers to the sentencing in place from 2009 to 2011 (see Appendix B). The location of the discontinuities in the punishment type and severity did not change since the 2009 reform to the present, although sentence lengths within each cell have been adjusted slightly.

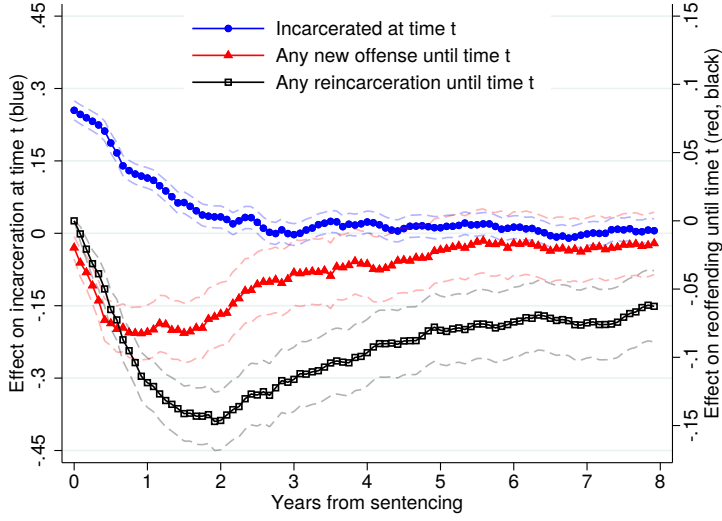
Figure A.5: Reduced form estimates on *cumulative* reoffending three years and beyond sentencing



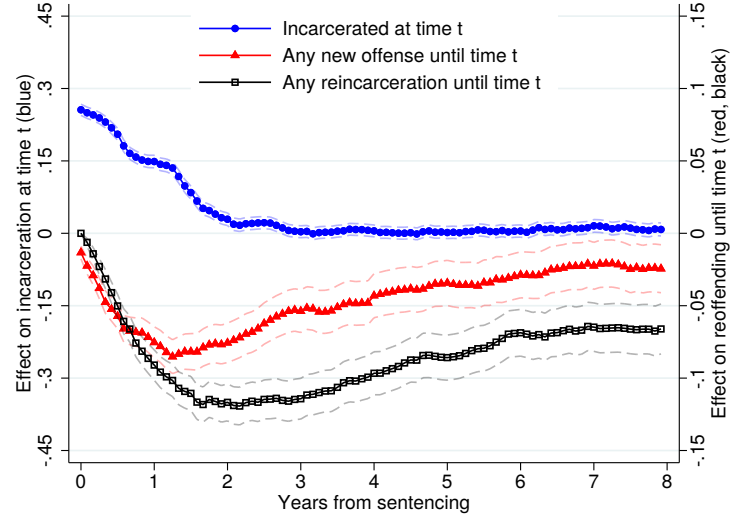
Notes: This figure shows the reduced form effects of being to the right of a punishment type discontinuity on several key outcomes. The blue line with circle shaped markers (left y-axis) in all panels shows effects on an indicator for being incarcerated at any point *in* month *t* from sentencing. The red line with triangle shaped markers (right y-axis) reports effects on the cumulative number of new offenses committed from month 36 until month *t* after sentencing, with the black line with hollow square shaped markers also including probation revocations. Standard errors are clustered by individual. Each point in each figure is an estimate of γ^{RF} for the relevant outcome for month *t*. This estimate is a constrained version of Equation 1 that requires the coefficients on all instruments to be the same (i.e., $\gamma_{E,4}^2 = \gamma_{F,9}^2 = \gamma_{G,14}^2 = \gamma_{H,19}^2 = \gamma_{I,9}^2 = \gamma^{RF}$). This strategy averages across all five offense classes and instruments, but collapses our variation into a single coefficient. γ^{RF} can therefore be thought of as the average reduced form effect across the five punishment type discontinuities (taking the actual average of the individual reduced forms yields highly similar results). The notation used is based on the guidelines in place prior to the 2009 reform, although all observations are used in estimation. The regression specifications include as controls demographics (e.g., race, gender, age FEs), FEs for the duration of time previously incarcerated, the number of past incarceration spells and the number of past convictions, county FEs, and year FEs. Estimates without controls yield similar results (see Table 2).

Figure A.6: Heterogeneity in reduced form effects by offenders' previous incarceration history

Effect on any reoffending up to time t

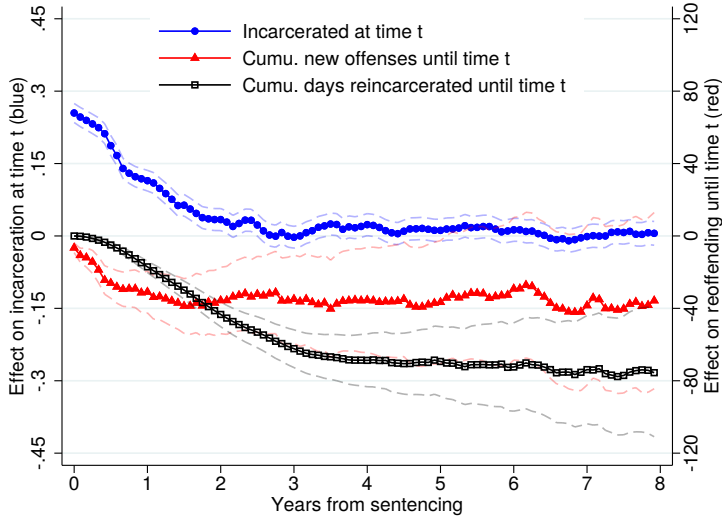


(a) Not previously incarcerated

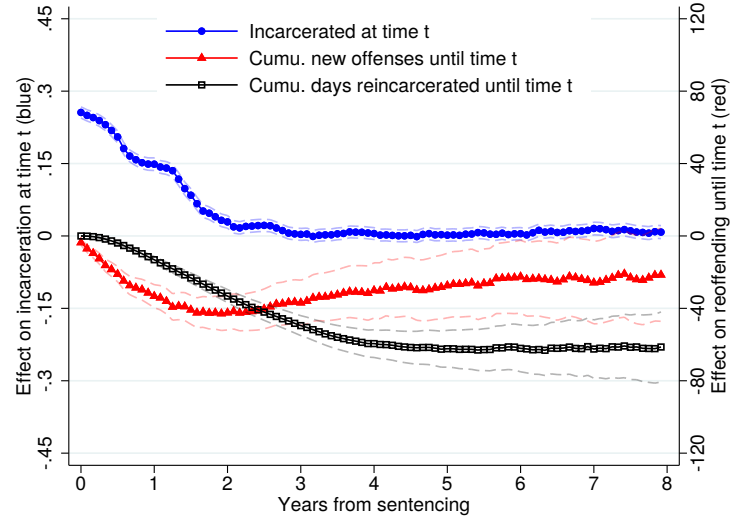


(b) Previously incarcerated

Effect on cumulative reoffending up to time t



(a) Not previously incarcerated

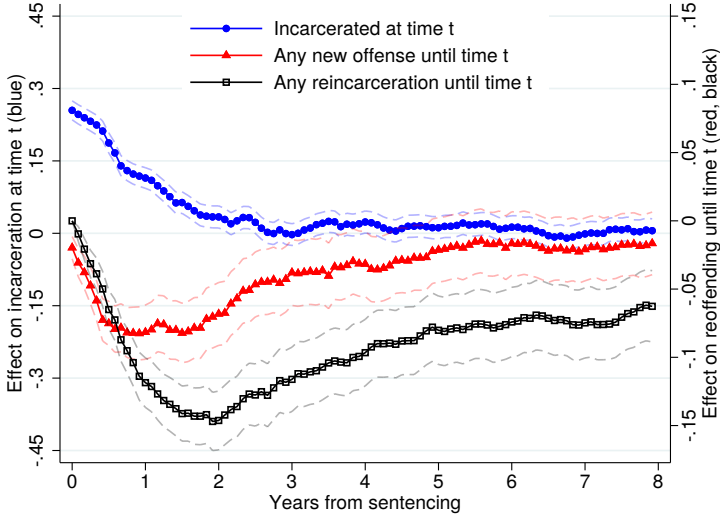


(b) Previously incarcerated

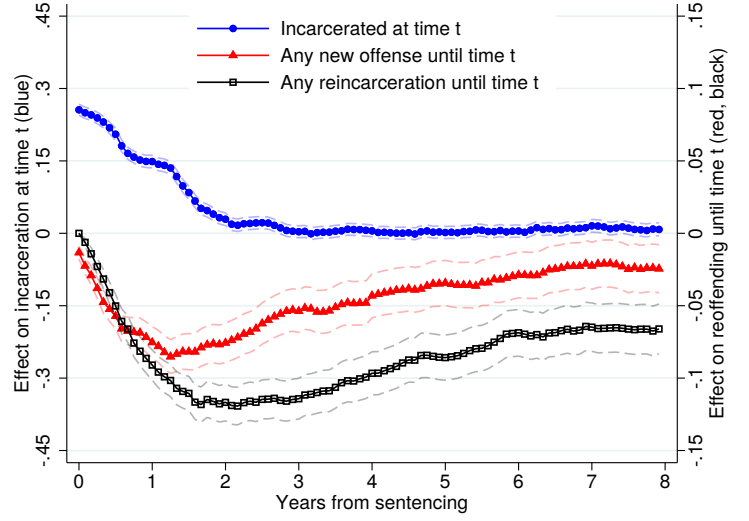
Notes: See the notes in Figure 4.

Figure A.7: Heterogeneity in reduced form effects by offenders' age

Effect on any reoffending up to time t

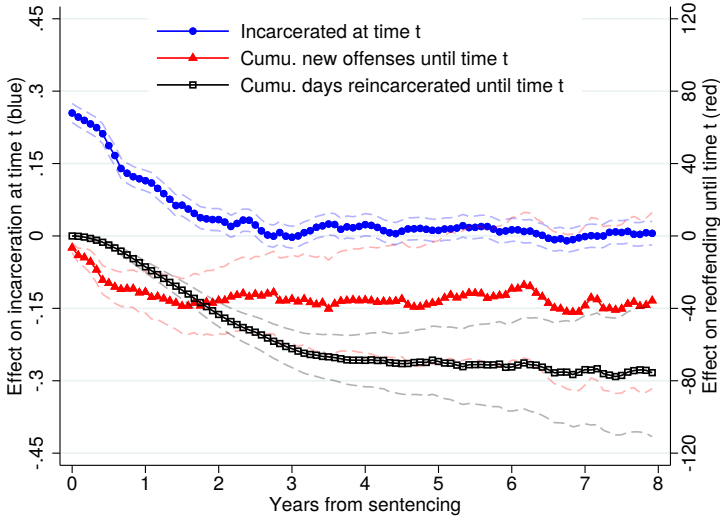


(a) 28 or younger

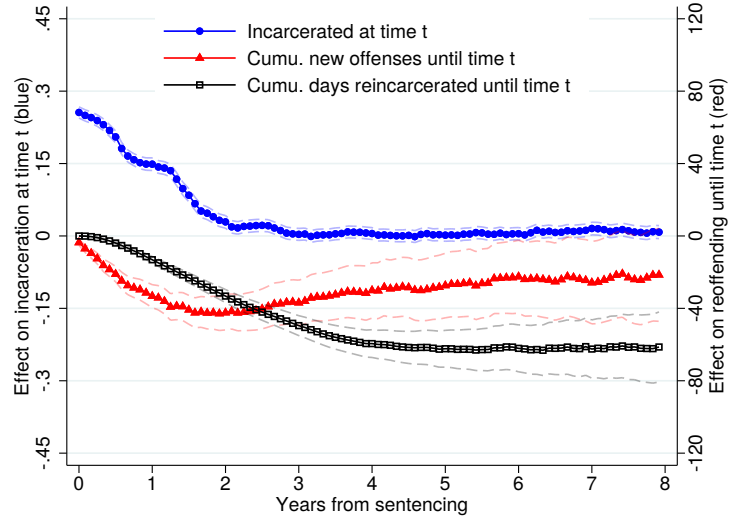


Older than 28 (b)

Effect on cumulative reoffending up to time t



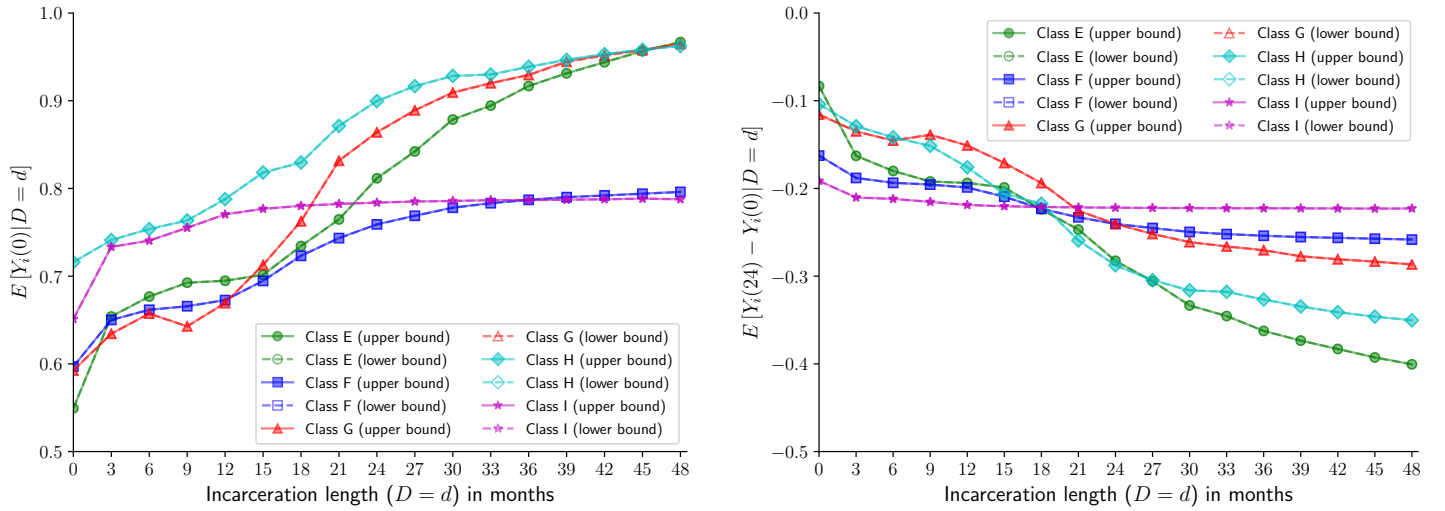
(c) 28 or younger



Older than 28 (d)

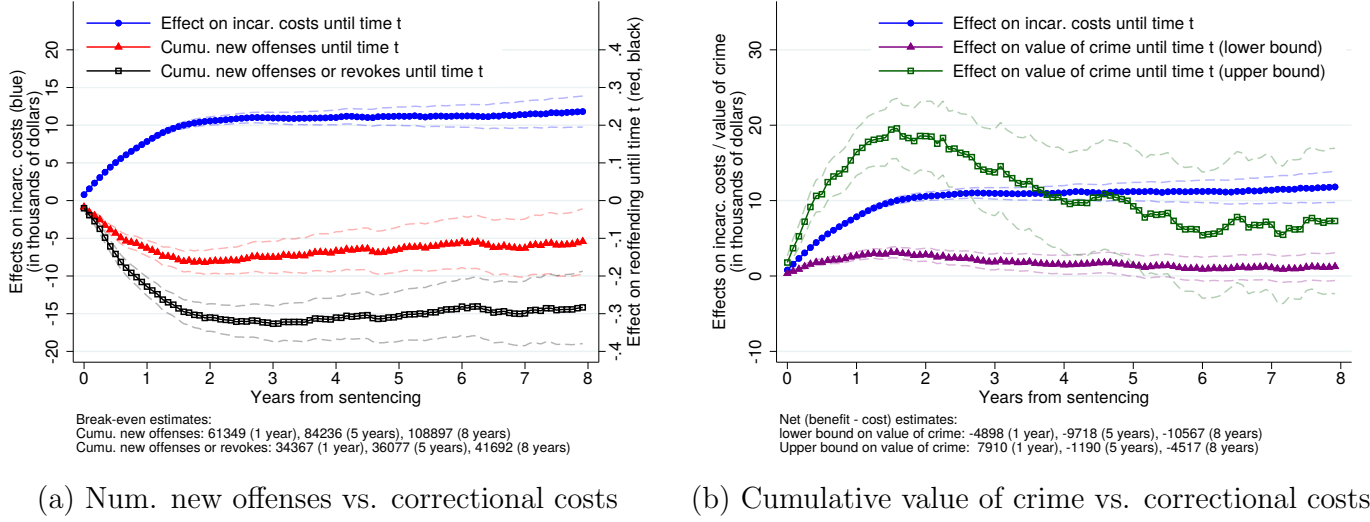
Notes: See the notes in Figure 4.

Figure A.8: Selection on levels and gains in the just-identified model by felony class



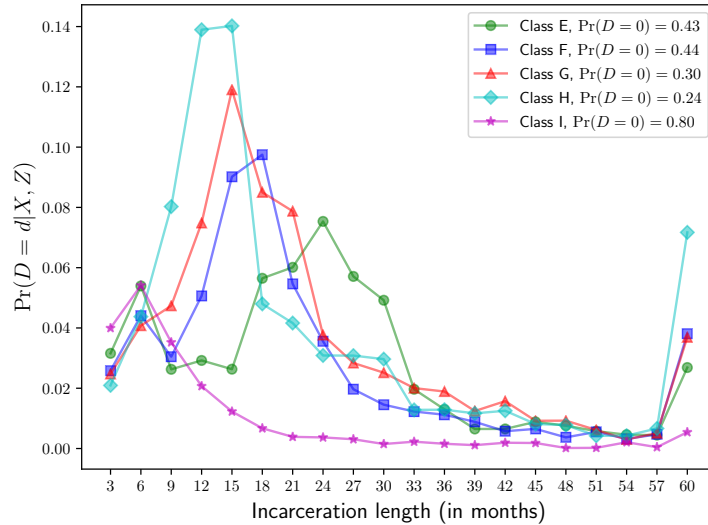
Notes: Panel a plots bounds on selection into incarceration in levels, i.e., $E[Y_i(0)|D_i = d]$, for different values of incarceration lengths (d) by discontinuity. This measure captures how much individuals who are sentenced to longer terms in prison have higher reoffending propensities. Panel b plots bounds on selection into incarceration based on gains, $E[Y_i(24) - Y_i(0)|D_i = d]$, where 24 is number of months, i.e., two years. This estimand reflects the degree to which the individuals who are assigned to longer incarceration ($D_i = d$) spells have a higher (or lower) impacts from the treatment. The MTRs are approximated using Bernstein polynomials of degree 1. This is the “just-identified” specification of the MTRs in the sense that the number of parameters exactly equals the number of reduced form moments. In Panels a we do not impose any shape restriction (e.g., monotonicity, additive separability) and in Panel b we impose monotonicity of the MTR functions w.r.t u .

Figure A.9: Reduced form effects on cumulative number of new offense and costs of incarceration



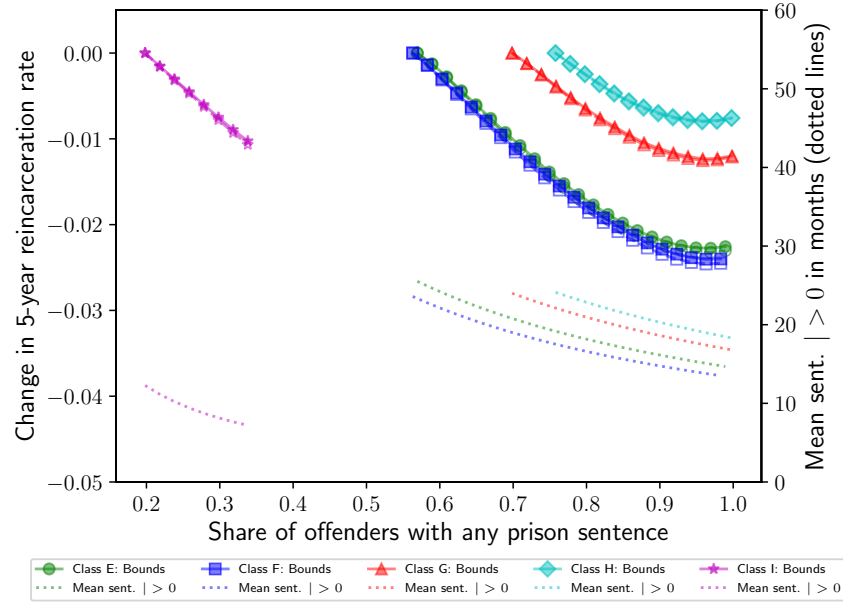
Notes: This figure shows reduced form estimates of being to the right of a punishment type discontinuity on several cumulative measures. Panel a shows effects on the cumulative number of new offenses (red line) and also the cumulative number of new offense or revokes committed *up to* month t from sentencing. The blue line (in both panels) is the effect on cumulative costs of incarceration, which is the cumulative months incarcerated up to period t multiplied by the average additional costs incurred by incarcerating an offender for a month. Panel b shows effects on the cumulative dollar value of crime averted. The value of crime averted is calculated by multiplying each criminal event with the appropriate dollar value reported in Appendix Table A.11. The table includes upper and lower bounds; effects on the value of crime are calculated for each of these bounds. All outcomes/measures are with respect to the sentencing date. In Panel b, the dark green line reports the reduced form effect on the upper bound of the cumulative value of crime averted and the purple line the lower bound. For details on the exact regression specification see the main text or the notes in Figure 4.

Figure A.10: Distribution of incarceration sentences to the left of punishment type discontinuities



Notes: This figure shows the distribution of incarceration terms just before the discontinuities. The probabilities are calculated from the $\pi_d(x, z)$ estimates, i.e., $\Pr(D_i = d|Z = 0) = \pi_d(x, 0) - \pi_{d+1}(x, 0)$.

Figure A.11: Impacts of budget-neutral shifts in sentences imposing separability in observables



Notes: This figure reports the results of budget-neutral counterfactual exercises that reduce longer prison sentences and use the additional resources to incarcerate more offenders for short prison sentences. In each counterfactual, we reduce average sentence length among those sent to prison (the dotted lines labeled “Mean sent. > 0 ”, measured on right y-axis) and increase the share of offenders sent to a short prison sentence (x-axis) in each offense class while holding total incarceration spending constant. The lines demarcated with symbols bound the impact on five-year reincarceration rates. The leftmost points, where the estimated impact is zero, reflect current sentencing policy in each offense class. The bounds stop when it is no longer feasible to continue budget-neutral reallocations, for example because 100% of offenders are imprisoned. The MTRs are approximated using Bernstein polynomials of degree 5. We impose the shape constraint that the MTR functions are monotonic with respect to u . In addition, we also impose that the MTE functions are the same across the five discontinuities. This constraint is the only difference between this figure and Figure 7.

Table A.1: Most frequent offenses committed by offenders in each felony class

	Most common offenses
Class E	ASSAULT W/DEADLY WEAPON, KIDNAPPING 2ND DEGREE, DISCHG FIREARM-OCC PROPERTY, ROBBERY W/DANGEROUS WEAPON, ASSAULT ISI
Class F	INDECENT LIBERTY W/CHILD, FAIL TO REGISTER (SEX OFFENDER, HABITUAL IMPAIRED DRIVING, ASSAULT INFLECT SERI BODY INJ, ASSAULT ISI
Class G	POSSESSION OF FIREARM BY FELON, SELL SCHEDULE II, COMMON LAW ROBBERY, BURGLARY 2ND DEGREE, IDENTITY FRAUD/THEFT
Class H	FELONY B&E, POSSESS WITS SCHEDULE II, OBT PROP BY FALSE PR/CHTS/SER, LARCENY OVER \$1000, POSSESSING STOLEN GOODS
Class I	POSSESS SCHEDULE II, POSSESS WITS SCHEDULE VI, FORGERY, B & E VEHICLES, UTTERING FORGEDPAPER/INST/END

Table A.2: Tests of change in covariates after introduction of 2009 changes in guidelines

	F-statistic	P-value
Any prison	8.819202	2.23e-08
Prison length	11.76384	2.16e-11
Predicted recidivism (from at-risk)	1.425866	.211214
Predicted recidivism (from conviction)	1.835459	.102214
Black	1.239866	.2873173
Male	1.254622	.2805579
Age at offense	1.370458	.2318747
Any previous incarceration	1.771596	.1148808
# previous cases	.5391755	.746748
Previous incar. duration	1.857827	.0980884

Notes: This table shows the F-statistic and p-value of the Wald test of whether imbalances in punishment and covariates at each of the five discontinuities change after the introduction of the 2009 sentencing grid. The test comes from estimating Equation 1 with the location of each discontinuity defined using the *old* grid in the two years before and after the change. We then interact the indicators for being to the right of each discontinuity with an indicator for being sentenced under the new grid and test for their joint significance. The F-statistics has five degrees of freedom since there are five instruments. Standard errors are clustered by individual.

Table A.3: Effect of incarceration on additional reoffending measures within three years of sentencing when probation revocations are treated as random censoring

	Measure of crime					
	(1) Re-incarceration	(2) Any new offense	(3) Felony	(4) Violent	(5) Property	(6) Drug
Length of incarceration (months)	-0.0118*** (0.000864)	-0.0115*** (0.000952)	-0.00884*** (0.000893)	-0.00388*** (0.000659)	-0.00486*** (0.000692)	-0.00409*** (0.000645)
One year effect in percentages	-48.26	-31.94	-34.20	-48.27	-37.21	-28.30
Dep. var. mean among non-incarcerated	0.294	0.434	0.310	0.0965	0.157	0.173
F-statistic (excluded-instruments)	133.1	133.1	133.1	133.1	133.1	133.1
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	419442	419442	419442	419442	419442	419442

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The dependent variable is an indicator for any charges (or conviction) recorded in the AOC (or DPS) data between zero and three years of the individual's sentencing date. The difference between this table and Table 3 is that we drop from the sample offenders who had a probation revocation prior to committing a new offense. This implies that all reincarceration events are the result of arrests for a new offense and not technical violations on probation. This sample restriction can be interpreted as assuming that the risks of probation revocation and criminal offending are independent. Standard errors (in parentheses) are clustered by individual. Each column represents a different type of new offense. For example, the estimates in Column 2 are the same as the estimates in Column 4 of Appendix Table 2. The F-statistics show the strength of the first stage and are based only on the excluded-instruments. The F-statistics are at the bottom of each column are all well above the rule-of-thumb thresholds proposed by [Stock et al. \(2002\)](#) and [Andrews et al. \(2018\)](#). Due to clustering, the F statistic reported is cluster-robust. Effective and non-robust F statistics are similar. The number of observations is smaller than in Table 1 because the sample in the regressions is restricted to individuals that are observed at least three years after the date of sentencing.

Table A.4: Effect of incarceration on additional reoffending measures within eight years

	Measure of crime					
	(1) Re-incarceration	(2) Any new offense	(3) Felony	(4) Violent	(5) Property	(6) Drug
Length of incarceration (months)	-0.00931*** (0.000951)	-0.00406*** (0.000911)	-0.00306** (0.000955)	-0.00241** (0.000888)	-0.00176* (0.000872)	-0.000849 (0.000863)
One year effect in percentages	-20.89	-8.471	-8.394	-16.84	-8.979	-3.659
Dep. var. mean among non-incarcerated	0.535	0.575	0.437	0.171	0.235	0.279
F-statistic (excluded-instruments)	117.5	117.5	117.5	117.5	117.5	117.5
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	376204	376204	376204	376204	376204	376204

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The dependent variable is an indicator for any charges (or convictions) recorded in the AOC (or DPS) data between zero and eighth years of the individual's sentencing date. The only difference between this table and Table 3 is that the time span for measuring reoffending is eighth years here relative to three years in Table 3. Standard errors (in parentheses) are clustered by individual. Each column represents a different type of new offense. For example, the estimates in Column 2 are the same as the estimates in Column 4 of Appendix Table 2. The F-statistics show the strength of the first stage and are based only on the excluded-instruments. The F-statistics are at the bottom of each column are all well above the rule-of-thumb thresholds proposed by [Stock et al. \(2002\)](#) and [Andrews et al. \(2018\)](#). Due to clustering, the F statistic reported is cluster-robust. Effective and non-robust F statistics are similar. The number of observations is smaller than in Table 1 because the sample in the regressions is restricted to individuals that are observed at least three years after the date of sentencing.

Table A.5: Evidence for non-linearity and heterogeneity in treatment effects when probation revocations are treated as random censoring

	Only length of incarceration			Plus indicator for any sentence	Plus polynomial square term
	(1) All	(2) 5 punishment type	(3) 15 primarily intensive	(4) All	(5) All
Linear effects:					
1 year	-0.109*** (0.0113)	-0.101*** (0.0118)	-0.179*** (0.0251)		
Non-linear effects:					
0 to 1 year				-0.137*** (0.0293)	-0.147*** (0.0321)
1 to 2 years				-0.0949*** (0.0176)	-0.166*** (0.0289)
2 to 3 years				-0.0949*** (0.0176)	-0.0703** (0.0231)
3 to 4 years				-0.0949*** (0.0176)	0.0256 (0.0421)
Dep. var. mean among non-incarcerated	0.352	0.352	0.352	0.352	0.352
J stat	43.36	9.125	23.18	42.58	20.54
J stat p-value	0.00116	0.0581	0.0573	0.000918	0.247
Weak instruments tests:					
Length of incarceration p-value	4.96e-132	2.62e-122	1.07e-30	2.72e-68	1.80e-15
Any incarceration p-value	.	.	.	1.37e-124	1.04e-42
Length of incarceration square p-value	3.29e-09
Controls	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The dependent variable is an indicator for any reincarceration recorded in the DPS data between zero and five years of the individual's sentencing date. The difference between this table and Table 5 is that we drop from the sample offenders who had a probation revocation prior to committing a new offense. This implies that all reincarceration events are the result of arrests for a new offense and not technical violations on probation. This sample restriction can be interpreted as assuming that the risks of probation revocation and criminal offending are independent. Standard errors (in parentheses) are clustered by individual. The F-statistics show the strength of the first stage and are based only on the excluded-instruments. The F-statistics are at the bottom of each column are all well above the rule-of-thumb thresholds proposed by [Stock et al. \(2002\)](#) and [Andrews et al. \(2018\)](#). Due to clustering, the F statistic reported is cluster-robust. Effective and non-robust F statistics are similar. The number of observations is smaller than in Table 1 because the sample in the regressions is restricted to individuals that are observed at least five years after the date of sentencing.

Table A.6: Estimates by offender and reoffending category

	Measure of crime					
	(1) Re-incarceration	(2) Any new offense	(3) Felony	(4) Violent	(5) Property	(6) Drug
All offenders	-0.0162*** (0.000822)	-0.00875*** (0.000824)	-0.00678*** (0.000771)	-0.00299*** (0.000568)	-0.00393*** (0.000599)	-0.00318*** (0.000553)
Assault offenders	-0.0159*** (0.00128)	-0.00815*** (0.00129)	-0.00528*** (0.00117)	-0.00399*** (0.000927)	-0.00337*** (0.000835)	-0.00234** (0.000850)
Drug offenders	-0.0191*** (0.00179)	-0.0105*** (0.00172)	-0.00896*** (0.00167)	-0.00167 (0.00113)	-0.00683*** (0.00146)	-0.00303* (0.00118)
Property offenders	-0.0205*** (0.00187)	-0.0104*** (0.00176)	-0.00958*** (0.00168)	-0.00214* (0.00104)	-0.00318** (0.00122)	-0.00562*** (0.00141)
Other offenders	-0.0133*** (0.00128)	-0.00612*** (0.00133)	-0.00507*** (0.00125)	-0.00192* (0.000974)	-0.00315** (0.000976)	-0.00175* (0.000741)

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The dependent variable is an indicator for any charges (or conviction) recorded in the AOC (or DPS) data for each type of offense between zero and three years of the individual's sentencing date. Standard errors are clustered by individual. Offender categorization refers to the focal offense for which the individual is being sentenced.

Table A.7: 95% confidence sets for bounds on average treatment effects of incarceration on five-year reincarceration rates

	Class I (1)	Class H (2)	Class G (3)	Class F (4)	Class E (5)	Avg. (6)	Avg. & same MTEs (7)
Marginal effects							
0 to 1 year	[-0.54, -0.14]	[-0.36, -0.12]	[-0.38, -0.01]	[-0.45, -0.08]	[-0.39, -0.03]	[-0.43, -0.15]	[-0.27, -0.16]
1 to 2 year	[-0.41, 0.23]	[-0.21, -0.07]	[-0.39, -0.06]	[-0.37, -0.05]	[-0.31, -0.05]	[-0.30, 0.04]	[-0.19, -0.12]
2 to 3 year	[-0.33, 0.20]	[-0.34, -0.06]	[-0.33, 0.01]	[-0.25, 0.07]	[-0.30, -0.03]	[-0.27, 0.03]	[-0.23, -0.10]
3 to 4 year	[-0.33, 0.07]	[-0.30, -0.06]	[-0.26, -0.03]	[-0.22, 0.04]	[-0.21, -0.03]	[-0.25, -0.02]	[-0.14, -0.04]
Total effects							
0 to 2 year	[-0.64, -0.23]	[-0.50, -0.24]	[-0.61, -0.21]	[-0.65, -0.27]	[-0.54, -0.19]	[-0.55, -0.28]	[-0.43, -0.29]
0 to 3 year	[-0.67, -0.30]	[-0.77, -0.36]	[-0.77, -0.36]	[-0.75, -0.34]	[-0.72, -0.34]	[-0.66, -0.39]	[-0.60, -0.44]
0 to 4 year	[-0.70, -0.48]	[-0.88, -0.55]	[-0.84, -0.53]	[-0.79, -0.44]	[-0.79, -0.48]	[-0.75, -0.56]	[-0.66, -0.55]

Notes: This table reports bootstrap confidence sets/intervals, based on 1000 repetitions, on the average treatment effects in Table 6. We follow [Shea and Torgovitsky \(2020\)](#) and use the forward and reverse bootstrap procedures described by [Andrews and Han \(2009\)](#) to construct confidence intervals for the identified set. Although this approach does not provide valid inference, nonetheless, it provides a sense of how variability due to sampling error impacts our estimates.

Table A.8: Robustness of bounds on average treatment effects with respect to flexibility of MTR approximation

	Ave. ATE			Ave. ATE under same MTEs		
	$deg = 5$	$deg = 10$	$deg = 15$	$deg = 5$	$deg = 10$	$deg = 15$
	(1)	(2)	(3)	(4)	(5)	(6)
Marginal effects						
0 to 1 year	[-0.40, -0.18]	[-0.44, -0.14]	[-0.47, -0.12]	[-0.21, -0.21]	[-0.23, -0.22]	[-0.22, -0.21]
1 to 2 year	[-0.21, 0.02]	[-0.29, 0.06]	[-0.31, 0.08]	[-0.13, -0.13]	[-0.14, -0.14]	[-0.14, -0.14]
2 to 3 year	[-0.24, -0.04]	[-0.28, 0.02]	[-0.29, 0.04]	[-0.16, -0.15]	[-0.16, -0.13]	[-0.17, -0.13]
3 to 4 year	[-0.22, -0.07]	[-0.24, -0.03]	[-0.25, -0.01]	[-0.12, -0.10]	[-0.13, -0.11]	[-0.13, -0.10]
Total effects						
0 to 2 year	[-0.46, -0.31]	[-0.53, -0.29]	[-0.55, -0.28]	[-0.34, -0.34]	[-0.36, -0.36]	[-0.36, -0.35]
0 to 3 year	[-0.61, -0.43]	[-0.66, -0.40]	[-0.68, -0.39]	[-0.50, -0.48]	[-0.52, -0.49]	[-0.53, -0.49]

Notes:

Table A.9: Bounds on average treatment effects of incarceration when probation revocations are treated as random censoring

	Class I	Class H	Class G	Class F	Class E	Ave.	Ave. & same MTEs
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Marginal effects							
0 to 1 year	[-0.14, -0.02]	[-0.30, -0.17]	[-0.32, -0.04]	[-0.28, -0.06]	[-0.21, -0.07]	[-0.22, -0.06]	[-0.09, -0.09]
1 to 2 year	[-0.25, -0.02]	[-0.11, -0.11]	[-0.17, -0.06]	[-0.14, -0.07]	[-0.16, -0.10]	[-0.19, -0.06]	[-0.12, -0.12]
2 to 3 year	[-0.28, -0.02]	[-0.14, -0.13]	[-0.28, -0.17]	[-0.15, -0.10]	[-0.21, -0.14]	[-0.23, -0.09]	[-0.20, -0.20]
3 to 4 year	[-0.21, -0.06]	[-0.27, -0.26]	[-0.22, -0.10]	[-0.14, -0.06]	[-0.05, -0.04]	[-0.19, -0.10]	[-0.05, -0.05]
Total effects							
0 to 2 year	[-0.31, -0.13]	[-0.42, -0.28]	[-0.38, -0.21]	[-0.35, -0.20]	[-0.37, -0.17]	[-0.35, -0.18]	[-0.21, -0.21]
0 to 3 year	[-0.45, -0.29]	[-0.55, -0.41]	[-0.66, -0.38]	[-0.49, -0.30]	[-0.53, -0.38]	[-0.51, -0.33]	[-0.42, -0.42]

Notes: The difference between this table and Table 6 is that we drop from the sample offenders who had a probation revocation prior to committing a new offense. This implies that all reincarceration events are the result of arrests for a new offense and not technical violations on probation. This sample restriction can be interpreted as assuming that the risks of probation revocation and criminal offending are independent. The outcome is an indicator for any reincarceration within five years of sentencing. Each bound is the minimum or maximum value of the ATE associated with all possible marginal treatment response (MTR) functions that a) rationalize the quasi-experimental moments generated by our instruments, and b) satisfy certain shape constraints. In the first six columns, MTRs are approximated using Bernstein polynomials of degree 5 and are constrained to be decreasing in u , the unobserved resistance to treatment. Each bound corresponds to the marginal or total effect listed in the row for the punishment type discontinuity listed in the column header. Column 6 bounds the average of effects across each discontinuity, weighted by the sample frequency of offenders in adjacent prior record levels. In column 7, MTRs are constrained to produce the same marginal treatment effects (MTEs) at each u for each discontinuity, implying ATEs are the same for each. Note that bounds on marginal effects do not sum to bounds on total effects because the MTR functions overlap between marginal effects (e.g., zero to one year and one to two years both depend on the MTR for one year of incarceration), implying that the lower bounds across marginal effects are not necessarily consistent. See Section 4 for full details on the approach.

Table A.10: Effect of incarceration on cumulative reoffending measures within five years of sentencing

	Measure of crime						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Re-incarceration	Any new offense or revoke	Violent	Property	Drug	Prob. revoke	Other crimes
Months incap	-10.09*** (0.577)	-0.0380*** (0.00798)	-0.00297 (0.00207)	-0.00939 (0.00590)	-0.00109 (0.00330)	-0.0216*** (0.00106)	-0.00460* (0.00202)
One year effect in percentages	-62.06	-14.38	-12.22	-9.687	-1.692	-46.08	-12.81
Dep. var. mean among non-incarcerated	195.0	3.170	0.291	1.163	0.776	0.563	0.431
F-statistic (excluded-instruments)	151.1	151.1	151.1	151.1	151.1	151.1	151.1
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	451547	451547	451547	451547	451547	451547	451547

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The dependent variable is a cumulative measure of reoffending that counts the number of events in the column header that occurred within five years of sentencing. New offenses (both overall and by crime type) are measured using either arrests recorded in the AOC data or convictions recorded in the DPS data. We use the date at which the offense occurred rather than the date an individuals was arrested or convicted to date the offense. Controls include indicators for gender, age, race, ethnicity, number of previous cases, number of previous incarceration spells, months of previous incarceration, number of previous convictions, year of offense, county of conviction, and the offense code of the convicted offense. Standard errors (in parentheses) are clustered by individual. The F-statistics test the joint hypothesis that the coefficients on the excluded instruments are all equal to zero. Due to clustering, the F statistic reported is cluster-robust. Effective and non-robust F statistics are similar. The number of observations is smaller than in Table 1 because the sample in the regressions is restricted to individuals that are observed at least three years after the date of sentencing.

Table A.11: Estimates of lower and upper bounds of the costs/value of crime

Offense category	Lower bound \$			Upper bound \$		
	Raw estimate	Including discounting	Reference	Raw estimate	Including discounting	Reference
Homicide	7,000,000	7,350,000	Chalfin and McCrary (2017)	9,700,000	19,205,337	Cohen et al. (2004)
Rape	142,020	149,121	Chalfin and McCrary (2017)	237,000	469,243.8	Cohen et al. (2004)
Assault	38,924	40,870.2	Chalfin and McCrary (2017)	70,000	138,595.2	Cohen et al. (2004)
Robbery	12,624	13,255.2	Chalfin and McCrary (2017)	232,000	459,344.1	Cohen et al. (2004)
Arson	38,000	128,681	Miller et al. (1996)	38,000	128,681	Miller et al. (1996)
Burglary	2,104	2,209.2	Chalfin and McCrary (2017)	25,000	49,498.29	Cohen et al. (2004)
Larceny	473	497	Chalfin and McCrary (2017)	370	1,253	Miller et al. (1996)
Theft	473	497	Chalfin and McCrary (2017)	370	1,253	Miller et al. (1996)
Drug	500	990		2,544	2,945	Mueller-Smith (2015)
DWI	500	990		25,842	29,915	Mueller-Smith (2015)
Other	500	990	Cohen et al. (2004)	500	990	Cohen et al. (2004)

Notes: “Discounting” means updating the cost estimate to 2018 \$, using a rate of 5% as in [Mueller-Smith \(2015\)](#). Offenses without a relevant cost estimate are assigned a value of \$990 (in 2018 \$) as was suggested by [Cohen et al. \(2004\)](#). The lower bounds for drug and DWI offenses were assigned in this way. Note that only [Miller et al. \(1996\)](#) and [Cohen et al. \(2004\)](#) calculated value of crime estimates the other studies used estimates from various other studies including from [Miller et al. \(1996\)](#) and [Cohen et al. \(2004\)](#).

B Sentencing grids in North Carolina

*** Effective for Offenses Committed on or after 12/1/95 ***

FELONY PUNISHMENT CHART PRIOR RECORD LEVEL

	I 0 Pts	II 1-4 Pts	III 5-8 Pts	IV 9-14 Pts	V 15-18 Pts	VI 19+ Pts	
A	Death or Life Without Parole						
B1	A	A	A	A	A <i>Life Without Parole</i>	A <i>Life Without Parole</i>	DISPOSITION
	240 - 300	288 - 360	336 - 420	384 - 480			Aggravated Range
	192 - 240	230 - 288	269 - 336	307 - 384	346 - 433	384 - 480	PRESUMPTIVE RANGE
	144 - 192	173 - 230	202 - 269	230 - 307	260 - 346	288 - 384	Mitigated Range
B2	A	A	A	A	A	A	
	157 - 196	189 - 237	220 - 276	251 - 313	282 - 353	313 - 392	
	125 - 157	151 - 189	176 - 220	201 - 251	225 - 282	251 - 313	
	94 - 125	114 - 151	132 - 176	151 - 201	169 - 225	188 - 251	
C	A	A	A	A	A	A	
	73 - 92	100 - 125	116 - 145	133 - 167	151 - 188	168 - 210	
	58 - 73	80 - 100	93 - 116	107 - 133	121 - 151	135 - 168	
	44 - 58	60 - 80	70 - 93	80 - 107	90 - 121	101 - 135	
D	A	A	A	A	A	A	
	64 - 80	77 - 95	103 - 129	117 - 146	133 - 167	146 - 183	
	51 - 64	61 - 77	82 - 103	94 - 117	107 - 133	117 - 146	
	38 - 51	46 - 61	61 - 82	71 - 94	80 - 107	88 - 117	
E	I/A	I/A	A	A	A	A	
	25 - 31	29 - 36	34 - 42	46 - 58	53 - 66	59 - 74	
	20 - 25	23 - 29	27 - 34	37 - 46	42 - 53	47 - 59	
	15 - 20	17 - 23	20 - 27	28 - 37	32 - 42	35 - 47	
F	I/A	I/A	I/A	A	A	A	
	16 - 20	19 - 24	21 - 26	25 - 31	34 - 42	39 - 49	
	13 - 16	15 - 19	17 - 21	20 - 25	27 - 34	31 - 39	
	10 - 13	11 - 15	13 - 17	15 - 20	20 - 27	23 - 31	
G	I/A	I/A	I/A	I/A	A	A	
	13 - 16	15 - 19	16 - 20	20 - 25	21 - 26	29 - 36	
	10 - 13	12 - 15	13 - 16	16 - 20	17 - 21	23 - 29	
	8 - 10	9 - 12	10 - 13	12 - 16	13 - 17	17 - 23	
H	C/I/A	I/A	I/A	I/A	I/A	A	
	6 - 8	8 - 10	10 - 12	11 - 14	15 - 19	20 - 25	
	5 - 6	6 - 8	8 - 10	9 - 11	12 - 15	16 - 20	
	4 - 5	4 - 6	6 - 8	7 - 9	9 - 12	12 - 16	
I	C	C/I	I	I/A	I/A	I/A	
	6 - 8	6 - 8	6 - 8	8 - 10	9 - 11	10 - 12	
	4 - 6	4 - 6	5 - 6	6 - 8	7 - 9	8 - 10	
	3 - 4	3 - 4	4 - 5	4 - 6	5 - 7	6 - 8	

A – Active Punishment I – Intermediate Punishment C – Community Punishment
Numbers shown are in months and represent the range of minimum sentences

Revised: 08-04-95

*** Effective for Offenses Committed on or after 12/1/09 ***

FELONY PUNISHMENT CHART
PRIOR RECORD LEVEL

OFFENSE CLASS		I 0-1 Pt	II 2-5 Pts	III 6-9 Pts	IV 10-13 Pts	V 14-17 Pts	VI 18+ Pts	
	A	Death or Life Without Parole						
	B1	A	A	A	A	A	A	DISPOSITION
		<i>240 - 300</i>	<i>276 - 345</i>	<i>317 - 397</i>	<i>365 - 456</i>	<i>Life Without Parole</i>	<i>Life Without Parole</i>	<i>Aggravated Range</i>
		192 - 240	221 - 276	254 - 317	292 - 365	336 - 420	386 - 483	PRESUMPTIVE RANGE
		<i>144 - 192</i>	<i>166 - 221</i>	<i>190 - 254</i>	<i>219 - 292</i>	<i>252 - 336</i>	<i>290 - 386</i>	<i>Mitigated Range</i>
	B2	A	A	A	A	A	A	
		<i>157 - 196</i>	<i>180 - 225</i>	<i>207 - 258</i>	<i>238 - 297</i>	<i>273 - 342</i>	<i>314 - 393</i>	
		125 - 157	144 - 180	165 - 207	190 - 238	219 - 273	251 - 314	
		<i>94 - 125</i>	<i>108 - 144</i>	<i>124 - 165</i>	<i>143 - 190</i>	<i>164 - 219</i>	<i>189 - 251</i>	
	C	A	A	A	A	A	A	
		<i>73 - 92</i>	<i>83 - 104</i>	<i>96 - 120</i>	<i>110 - 138</i>	<i>127 - 159</i>	<i>146 - 182</i>	
		58 - 73	67 - 83	77 - 96	88 - 110	101 - 127	117 - 146	
		<i>44 - 58</i>	<i>50 - 67</i>	<i>58 - 77</i>	<i>66 - 88</i>	<i>76 - 101</i>	<i>87 - 117</i>	
	D	A	A	A	A	A	A	
		<i>64 - 80</i>	<i>73 - 92</i>	<i>84 - 105</i>	<i>97 - 121</i>	<i>111 - 139</i>	<i>128 - 160</i>	
		51 - 64	59 - 73	67 - 84	78 - 97	89 - 111	103 - 128	
		<i>38 - 51</i>	<i>44 - 59</i>	<i>51 - 67</i>	<i>58 - 78</i>	<i>67 - 89</i>	<i>77 - 103</i>	
	E	I/A	I/A	A	A	A	A	
		<i>25 - 31</i>	<i>29 - 36</i>	<i>33 - 41</i>	<i>38 - 48</i>	<i>44 - 55</i>	<i>50 - 63</i>	
		20 - 25	23 - 29	26 - 33	30 - 38	35 - 44	40 - 50	
		<i>15 - 20</i>	<i>17 - 23</i>	<i>20 - 26</i>	<i>23 - 30</i>	<i>26 - 35</i>	<i>30 - 40</i>	
	F	I/A	I/A	I/A	A	A	A	
		<i>16 - 20</i>	<i>19 - 23</i>	<i>21 - 27</i>	<i>25 - 31</i>	<i>28 - 36</i>	<i>33 - 41</i>	
		13 - 16	15 - 19	17 - 21	20 - 25	23 - 28	26 - 33	
		<i>10 - 13</i>	<i>11 - 15</i>	<i>13 - 17</i>	<i>15 - 20</i>	<i>17 - 23</i>	<i>20 - 26</i>	
	G	I/A	I/A	I/A	I/A	A	A	
		<i>13 - 16</i>	<i>14 - 18</i>	<i>17 - 21</i>	<i>19 - 24</i>	<i>22 - 27</i>	<i>25 - 31</i>	
		10 - 13	12 - 14	13 - 17	15 - 19	17 - 22	20 - 25	
		<i>8 - 10</i>	<i>9 - 12</i>	<i>10 - 13</i>	<i>11 - 15</i>	<i>13 - 17</i>	<i>15 - 20</i>	
	H	C/I/A	I/A	I/A	I/A	I/A	A	
		<i>6 - 8</i>	<i>8 - 10</i>	<i>10 - 12</i>	<i>11 - 14</i>	<i>15 - 19</i>	<i>20 - 25</i>	
		5 - 6	6 - 8	8 - 10	9 - 11	12 - 15	16 - 20	
		<i>4 - 5</i>	<i>4 - 6</i>	<i>6 - 8</i>	<i>7 - 9</i>	<i>9 - 12</i>	<i>12 - 16</i>	
	I	C	C/I	I	I/A	I/A	I/A	
		<i>6 - 8</i>	<i>6 - 8</i>	<i>6 - 8</i>	<i>8 - 10</i>	<i>9 - 11</i>	<i>10 - 12</i>	
		4 - 6	4 - 6	5 - 6	6 - 8	7 - 9	8 - 10	
		<i>3 - 4</i>	<i>3 - 4</i>	<i>4 - 5</i>	<i>4 - 6</i>	<i>5 - 7</i>	<i>6 - 8</i>	

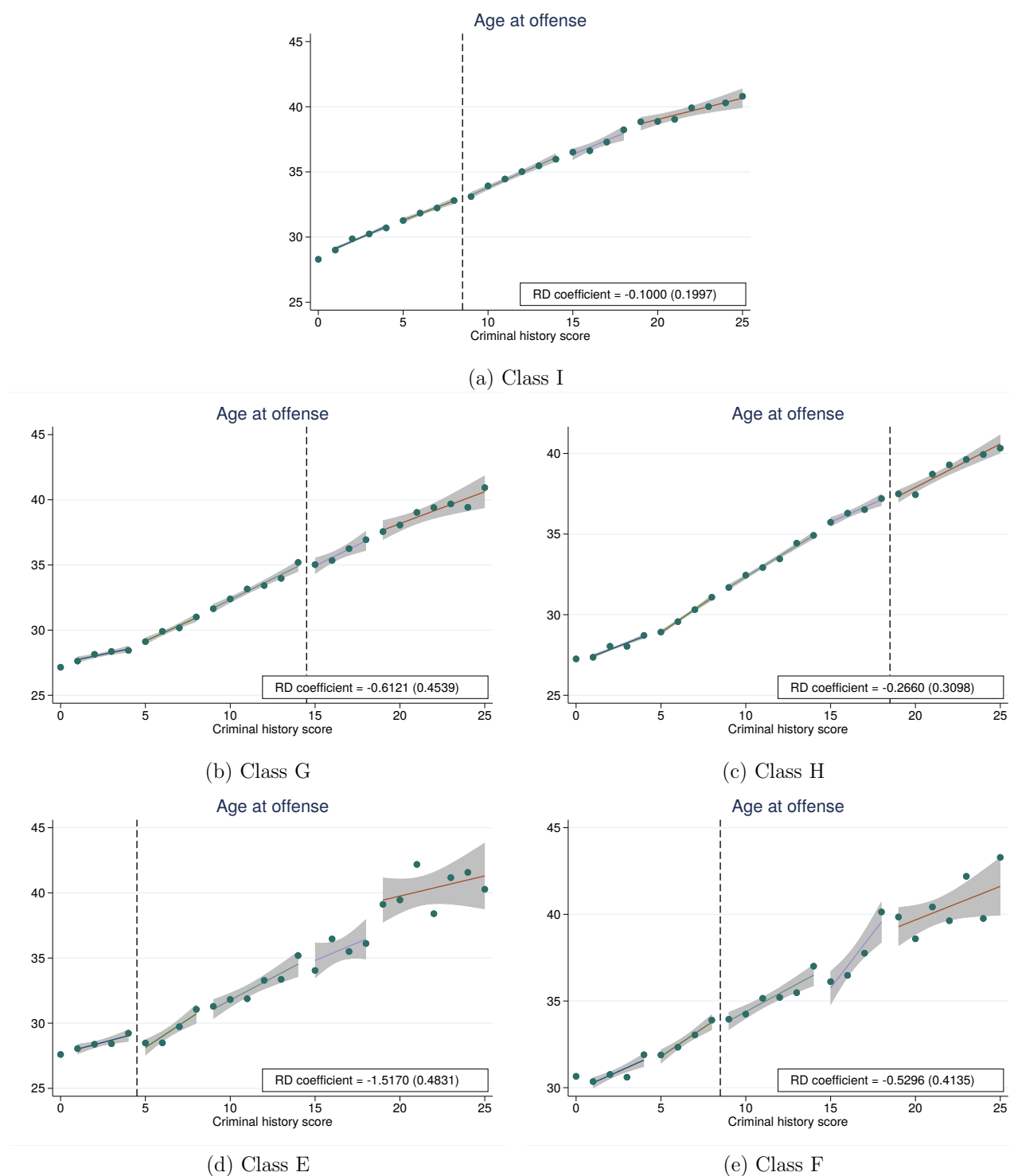
A – Active Punishment I – Intermediate Punishment C – Community Punishment
Numbers shown are in months and represent the range of minimum sentences

Revised: 08-31-09

C Tests of instrument validity

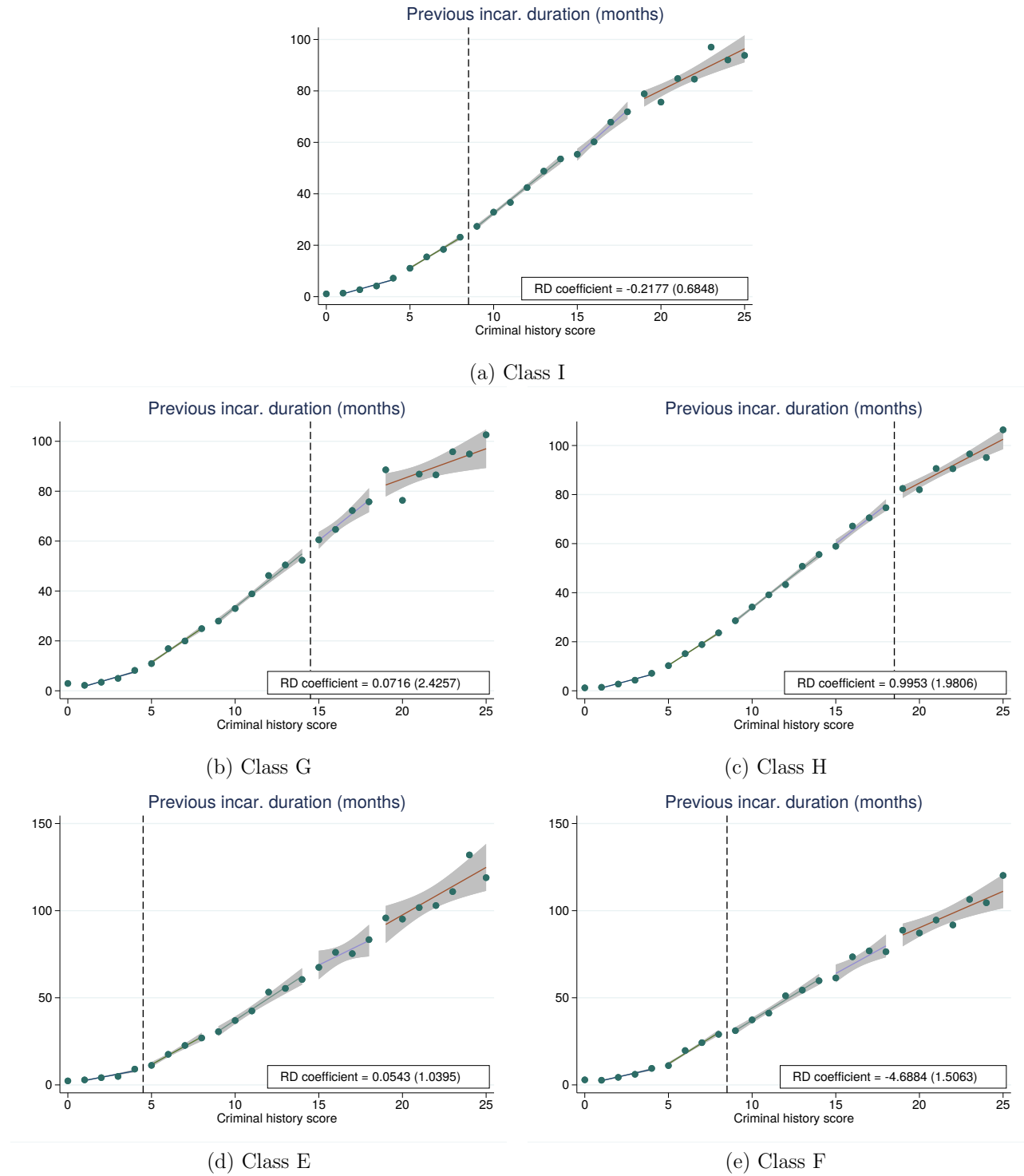
This appendix includes additional figures and tables that present evidence in support of the validity of the instrumental variables. The figures and tables are discussed in the main text of the paper.

Figure C.1: Age at the time of offense by offense severity class and prior points



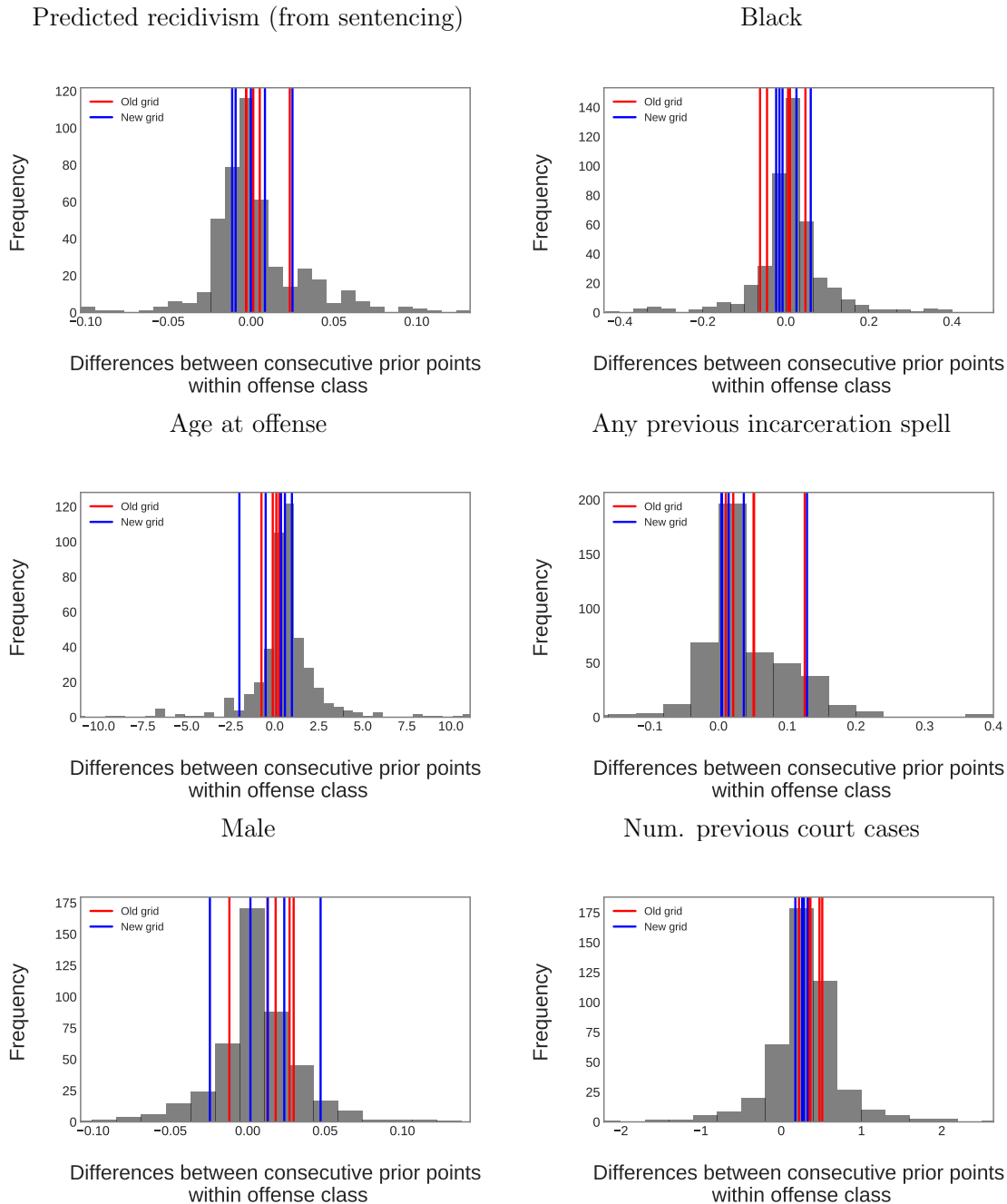
Notes: This figure demonstrates that the offender's age at the time the offense took place varies smoothly across the punishment type discontinuities in each offense class. The x-axis in all plots reports the number of prior record points. The y-axis shows mean age at offense of offenders in each bin. Standard errors are clustered at the individual level. Only offenses sentenced under the sentencing grid that applied to offenses committed between 1995 to 2009 are plotted.

Figure C.2: Previous incarceration duration by offense severity class and prior points



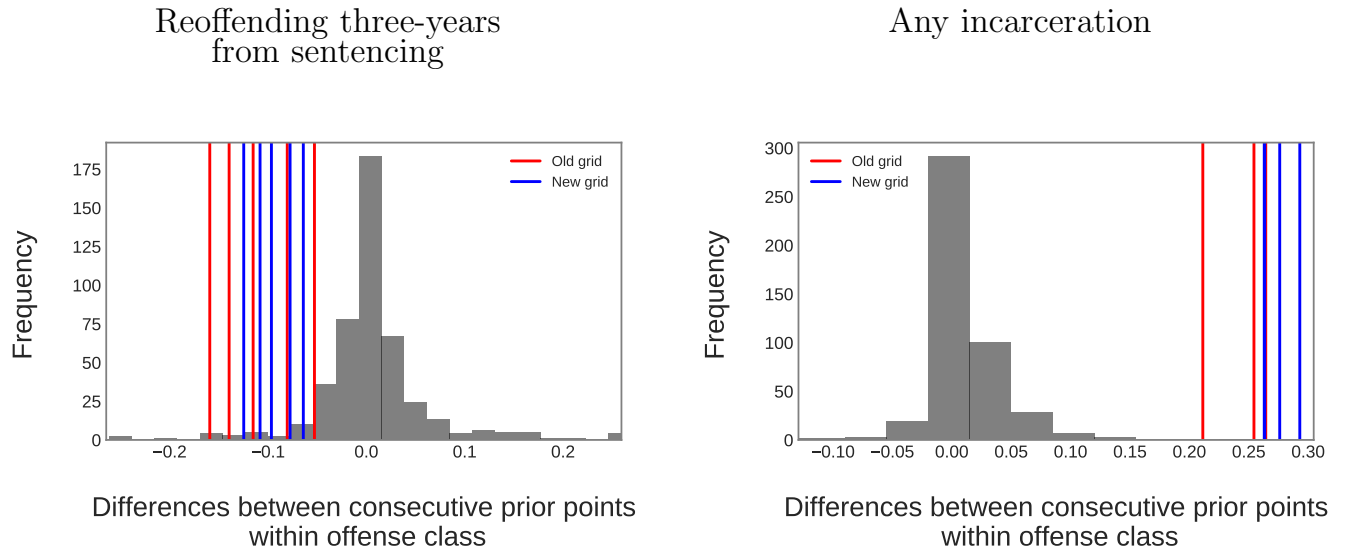
Notes: This figure demonstrates that an offender's previous incarceration duration (a pre-treatment covariate) varies smoothly across the punishment type discontinuities in each offense class. The x-axis in all plots reports the number of prior record points. The y-axis shows mean previous incarceration duration of offenders in each bin. Standard errors are clustered at the individual level. Only offenses sentenced under the sentencing grid that applied to offenses committed between 1995 to 2009 are plotted.

Figure C.3: Difference in covariates before and after punishment type discontinuities relative to differences between consecutive prior points without a punishment type change



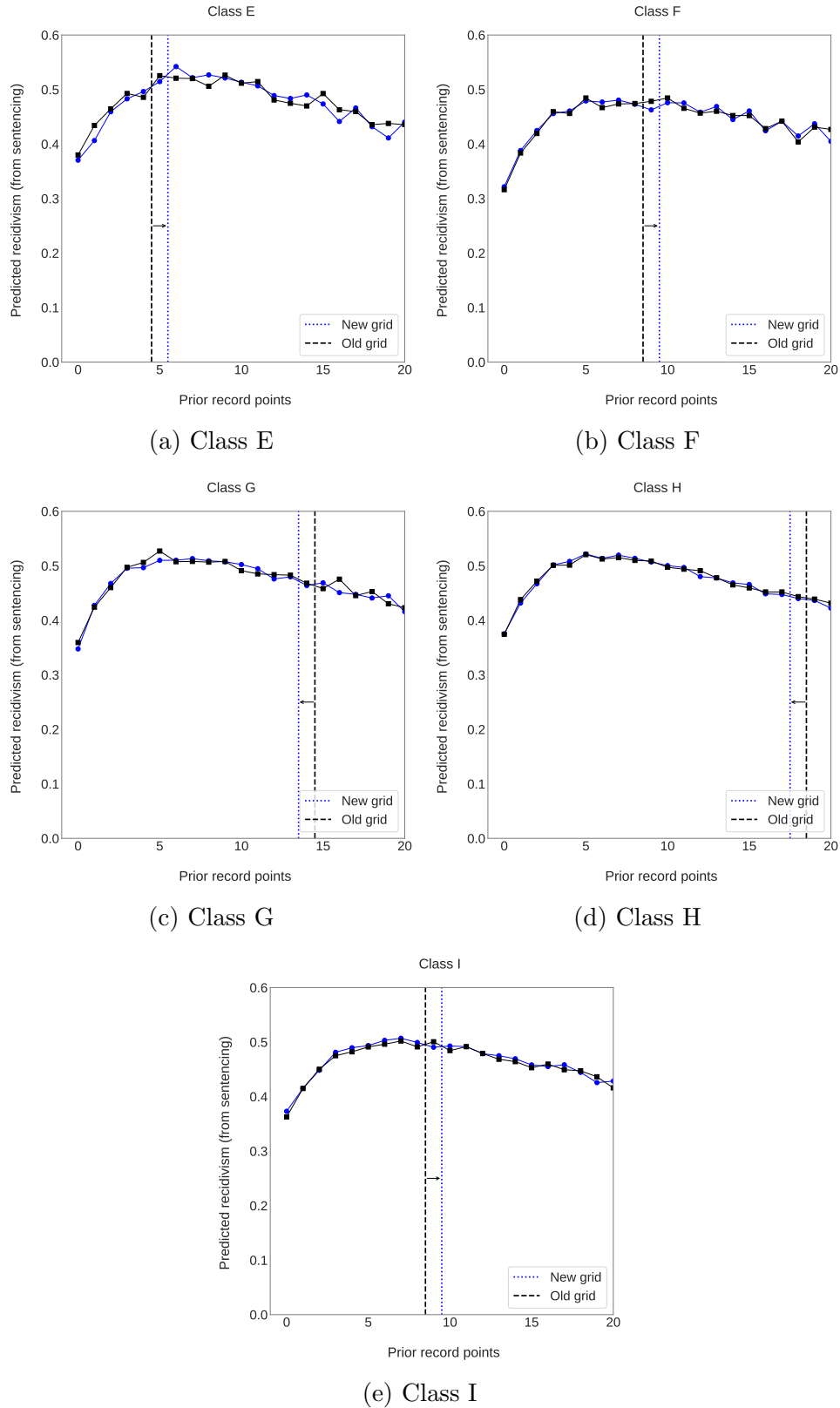
Notes: This figure tests for imbalances in covariates (pre-conviction characteristics) at the discontinuities in punishment relative to any transition across prior points in which there is no change in punishment type. The figure plots the distribution of the difference in the mean values of a given covariate (e.g., male, black) between two consecutive prior points by felony class and before and after the grid changes in 2009. The red (or blue) lines indicate the differences at prior points transitions with a punishment type discontinuity using date before (after) the 2009 grid changes. The figure includes four different covariates, the distribution of each is plotted separately. The covariates in the figure are an indicator for whether the offender is black, the age at the time the offense took place, the predicted recidivism (i.e., reoffending) risk from at-risk and from conviction. Since there are many important pre-treatment covariates, we make use of this predicted reoffending (risk) score that is calculated by regressing reoffending on all the pre-treatment covariates (using only non-incarcerated offenders) and fitting predicted values to all offenders. Summarizing imbalance by the covariates' relationship to the outcome surface is a common methodology in the literature [Bowers and Hansen \(2009\)](#) and [Card et al. \(2015\)](#).

Figure C.4: Difference in incarceration and reoffending before and after punishment type discontinuities relative to differences between any two consecutive prior points without a punishment type discontinuity between them



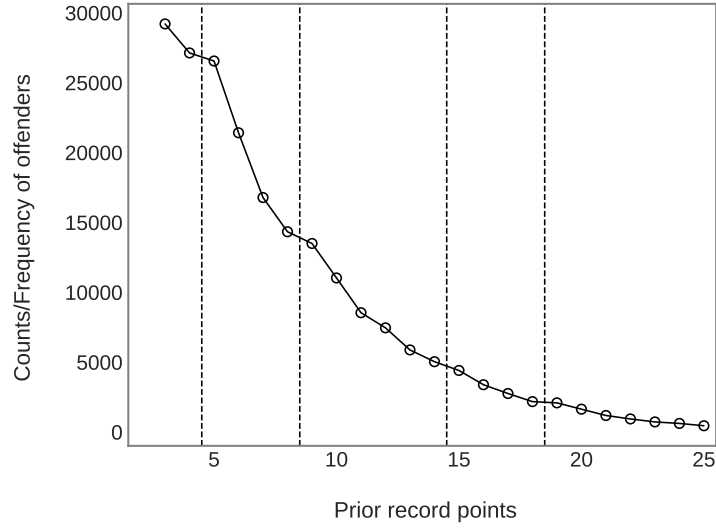
Notes: This figure illustrates the variation caused by the discontinuities in incarceration exposure (first-stage) and reoffending. The figure plots the distribution of the difference in the mean values of a given outcome (e.g., any initial incarceration, any reoffending within 3 years) between two consecutive prior points by felony class and before and after the grid changes in 2009. The red (or blue) lines indicate the differences at prior points transitions with a punishment type discontinuity using date before (after) the 2009 grid changes. The reoffending measure in the figure is any new offense or probation revocation.

Figure C.5: Predicted recidivism score does not vary due to 2009 changes in the location of discontinuities in sentencing guidelines



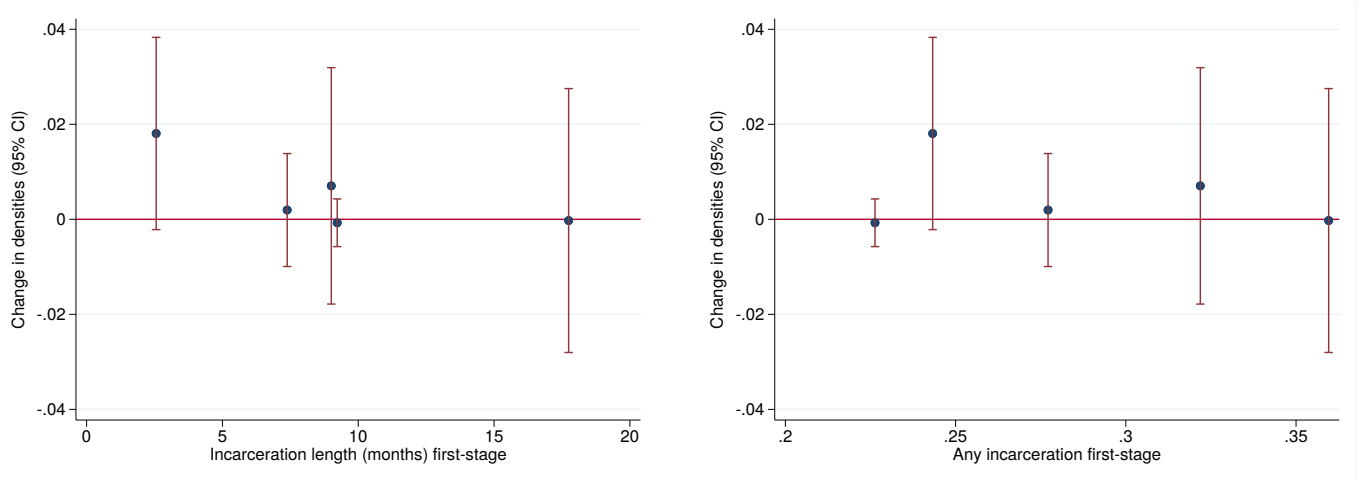
Notes: The x-axis in all plots is the number of prior record points. The y-axis reports the offender's average predicted recidivism score. The black line represents the average predicted recidivism score prior to the 2009 reform and the blue line the predicted recidivism score after the reform. The plots demonstrate how the 2009 changes in the location of discontinuities in the sentencing grid do not lead to any discontinuities in the predicted recidivism score. The old grid refers to the sentencing grid between 1996 to 2013 (see Appendix B), and the new grid refers to the sentencing from 2009 to 2011 (see Appendix A). The location of the discontinuities in the punishment type and severity did not change since the 2009 reform to the present, although changes within the grid have been made.

Figure C.6: Distribution of offenders across prior record points



Notes: The x-axis in all plots is the number of prior record points. The y-axis show the mean age of offenders at the time the offense was committed. The figure present only offenses that took place between 1995 and 2009 and have been sentenced under the sentencing grid that applied for offenses committed between 1995 to 2009, see Appendix B for the official grid. In 2009 the guidelines changes and the discontinuities shifted by one prior points either to the left or to the right, see Appendix B. The figure for offenses that took place after 2009 looks very similar and the density of individuals also varies smoothly across between prior record levels.

Figure C.7: VIV of punishment severity first-stage and reduced-form coefficients of changes in density (count of observations) at the different discontinuities



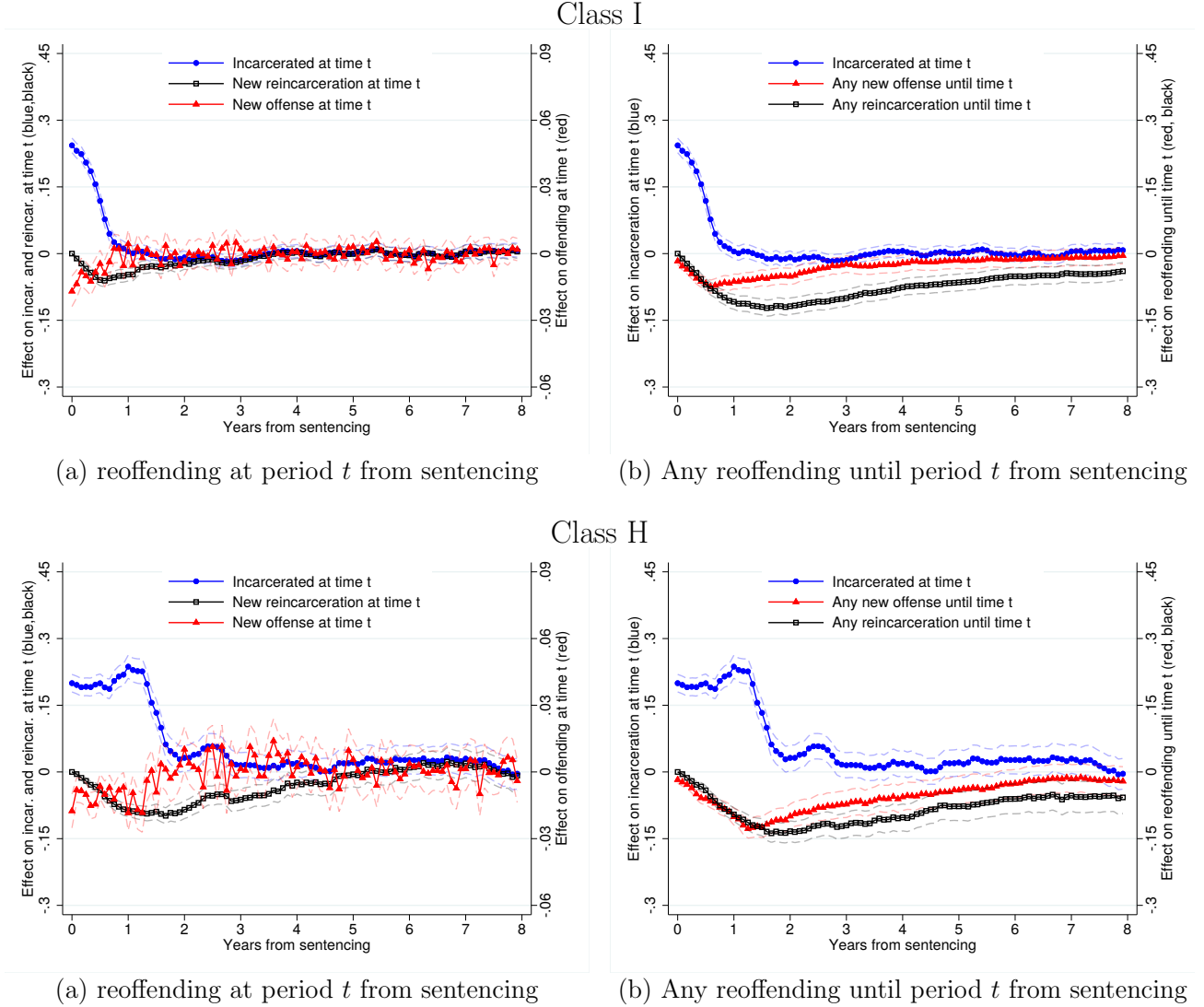
D Heterogeneity by discontinuity

In this appendix, we explore heterogeneity by felony class and report estimates of reduced form figures that are analogous to Figure 4 for each felony class separately. As noted in the main text, the reduced form results combine and average the effects of crossing multiple discontinuities. Because each discontinuity applies to different offenders, has a different first stage, and has different mean compliance rates to the left and the right of the threshold, each may also capture treatment effects for different complier populations. Because each instrument also shifts exposure to different amounts of incarceration, the reduced forms may also vary because they capture different weighted averages of the same incremental treatment effects (see Equation 3).

Appendix Figures D.1 and D.2 show the main reduced form estimates by felony class. Panel a plots documents effects on incarceration and reoffending at the monthly level. The patterns in all the classes look similar, although there is substantial variation in duration of incapacitation. For example, in class I, the instruments stop being predictive of incarceration status one year from sentencing; however, in class E it takes over four years. Nevertheless, in all classes there is a reduction in the period-by-period offending rates while the instruments are predictive of incarceration status and afterwards no visible differences in monthly reoffending rates.

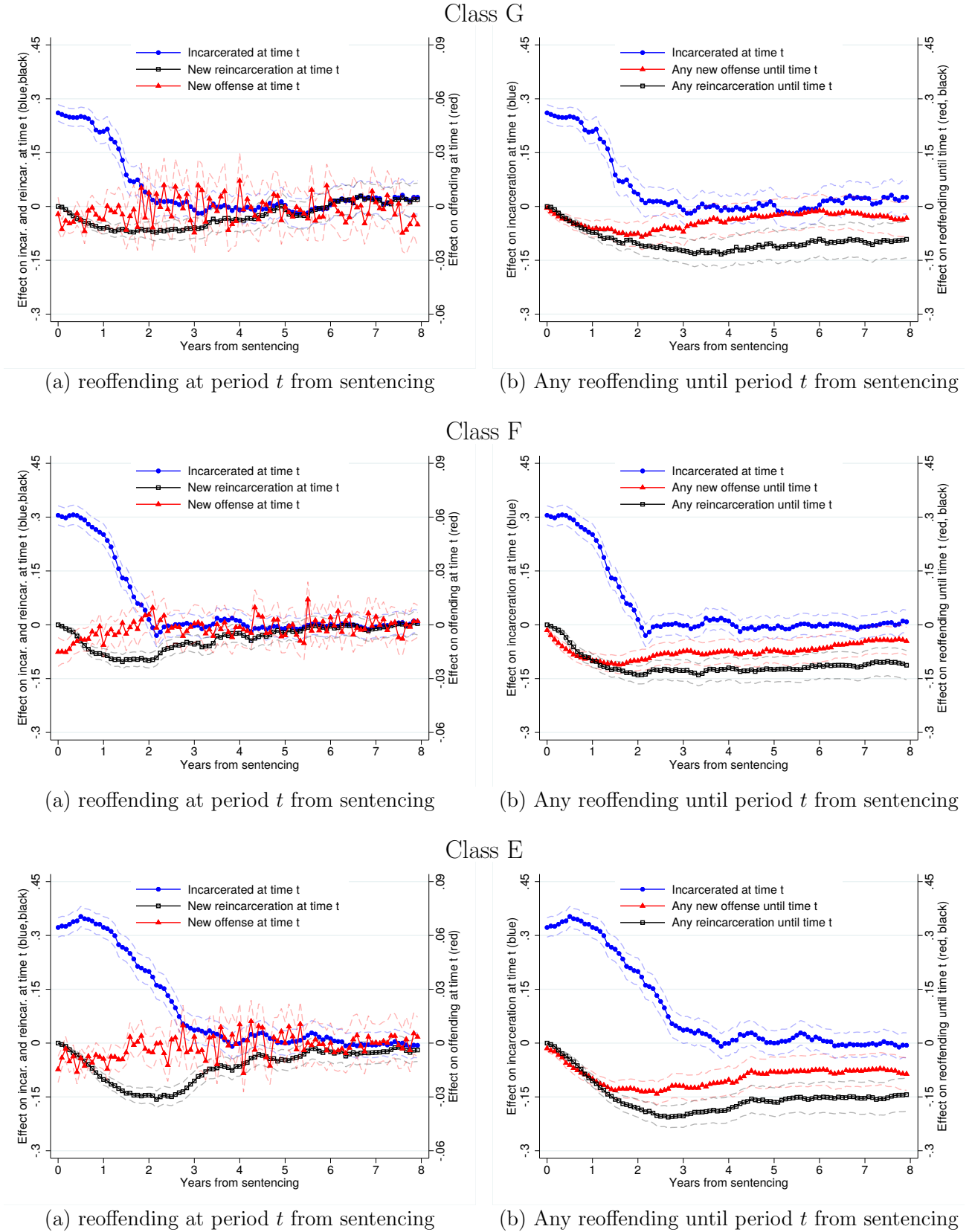
Panel b plots show that although there is substantial heterogeneity in the magnitude of the incapacitation effects, the impacts on any reoffending in the long term show either a zero effect (e.g., Class I) or permanent reduction in some classes (e.g., E or F). It is interesting to note that the reduced forms with the largest permanent reductions in offending also have the longest incarceration treatments. Thus while no class shows incarceration ever increases offending post-release, there is some suggestive evidence that longer sentences persistently reduce it.

Figure D.1: Reduced form estimates of reoffending *at* period t from sentencing and also estimates of *any* reoffending up to period t from sentencing



Notes: This figures shows reduced form estimates of being to the right of a punishment type discontinuity on several different outcomes of interest. All outcomes/measures are with respect to the sentencing date. The blue line (left y-axis) on both panels represents the the reduced form effect on an indicator for spending any positive amount of time behind bars *at* month t from sentencing. In Panel a, the red color line with triangle shaped markers (right y-axis) reports the reduced form effects on committing a new offense *at* month t , and the black color line with hollow square shaped markers (right y-axis) the estimates when also including probation revocations as offending. In Panel b, the red color line with triangle shaped markers (right y-axis) reports the reduced form effects on committing *any* new offense *until* month t , and the black color line with hollow square shaped markers (right y-axis) the estimates when also including probation revocations as offending. The reduced form coefficients are estimated using Equation 1, when the dependent variable is various outcomes of interest. Standard errors are clustered by individual. The regression specifications include as controls demographics (e.g., race, gender, age FEs), criminal history FEs for the duration of time previously incarcerated, the number of past incarceration spells and the number of past convictions, county FEs, and year FEs. Estimates without controls yield similar results (see for example Table 2).

Figure D.2: Reduced form estimates of reoffending *at* period t from sentencing and also estimates of *any* reoffending up to period t from sentencing



Notes: See notes of above Figure D.1.

E Additional robustness and validity tests

Table E.1: 2SLS estimates of the effect of length of incarceration on reincarceration within three years of sentencing by the time period in which the offense took place

	Offense committed within time period			
	(1) 1994-1999	(2) 2000-2004	(3) 2005-2009	(4) 2010-2014
Length of incarceration (months)	-0.0141*** (0.00146)	-0.0132*** (0.00120)	-0.0142*** (0.00135)	-0.0110*** (0.00143)
One year effect in percentages	-0.427	-0.409	-0.458	-0.364
Dep. var. mean among non-incarcerated	0.456	0.448	0.447	0.415
F-statistic (excluded-instruments)	39.13	62.15	71.30	93.12
Controls	Yes	Yes	Yes	Yes
Number of observations	115261	127091	135264	128359
Standard errors in parentheses				
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$				

E.1 Plea bargaining does not impact our results

In this appendix, we present another way of testing that plea bargaining does not bias our estimates (in addition to the estimates in Table E.2 that are discussed in the main text). We examine whether plea bargainers are selected by taking all individuals convicted in a given offense class and prior record points value and comparing those who were initially charged in that offense class to those who plead down from more severe offenses. Since the key concern for our research design is that this type of sorting *increases* at the discontinuity, we compare these two groups of offenders just before and just after a major discontinuity.

We document that both groups also face the same punishment regime and similar exposure to incarceration. According to Appendix Figure E.1 there is no evidence that individuals initially charged with a more severe offense are incarcerated more. This result holds for both individuals just before or just after a punishment type discontinuity. Given that the two groups receive similar levels of punishment, any observed differences in reoffending should arise through selection. Appendix Figure E.2 shows that the two groups—those “Charged same felony class” and those “Charged higher felony class”—have the same likelihood of reoffending within three years after being released and also within three years from the sentencing date. To conclude, we find no evidence that our results are influenced by plea bargaining.

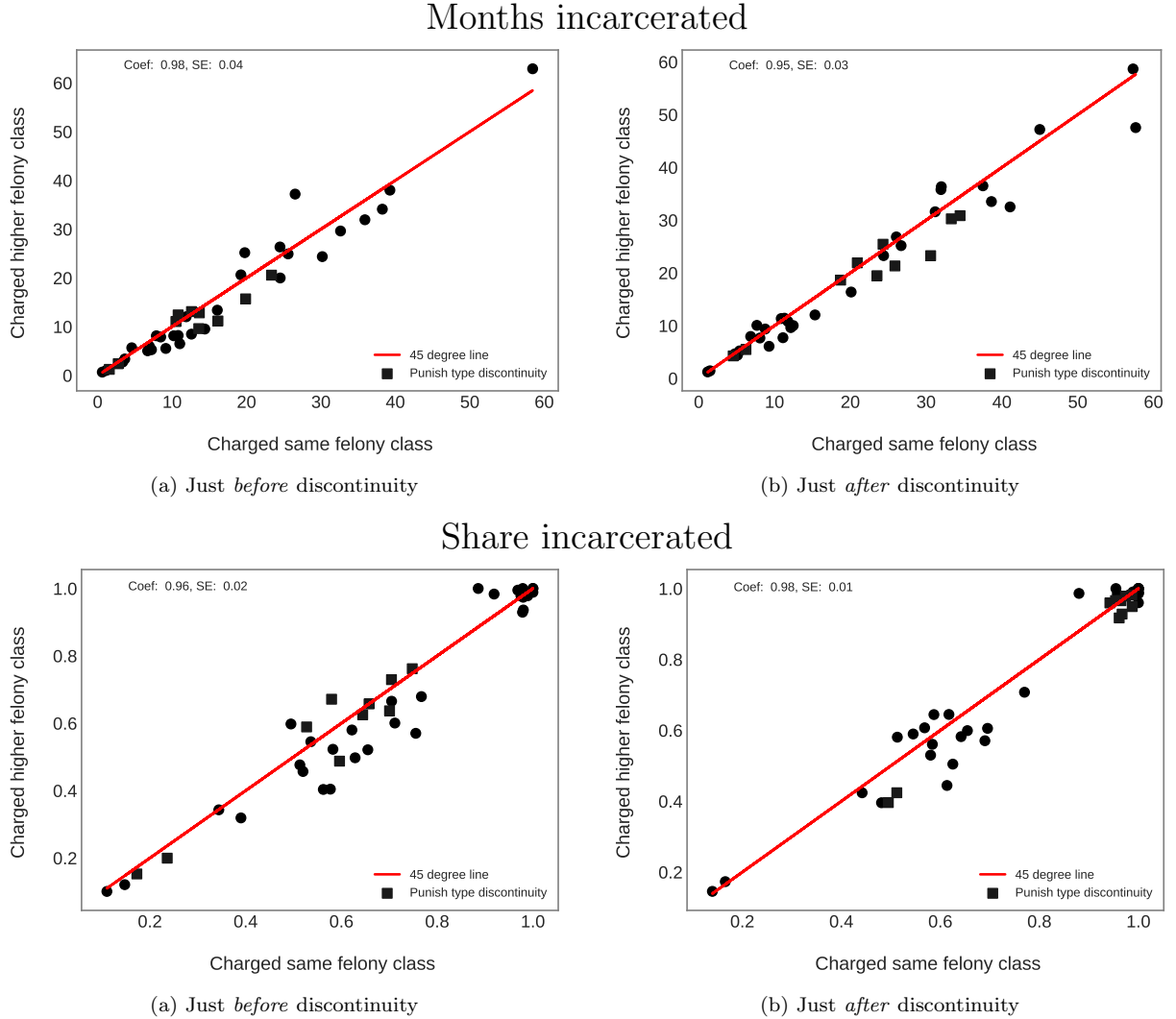
Table E.2: Estimates of the effect of incarceration on reoffending from sentencing using charged vs. convicted offense class

	New offense			New offense of revoke			Re-incarceration		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Arraigned	Charged	Convicted	Arraigned	Charged	Convicted	Arraigned	Charged	Convicted
Months incarcerated	-0.0104*** (0.00174)	-0.0105*** (0.00174)	-0.0102*** (0.00112)	-0.0161*** (0.00177)	-0.0162*** (0.00178)	-0.0159*** (0.00116)	-0.0185*** (0.00178)	-0.0185*** (0.00179)	-0.0173*** (0.00113)
N	358701	358701	358701	358701	358701	358701	358701	358701	358701
Dep. var. mean	0.418	0.418	0.418	0.530	0.530	0.530	0.399	0.399	0.399

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

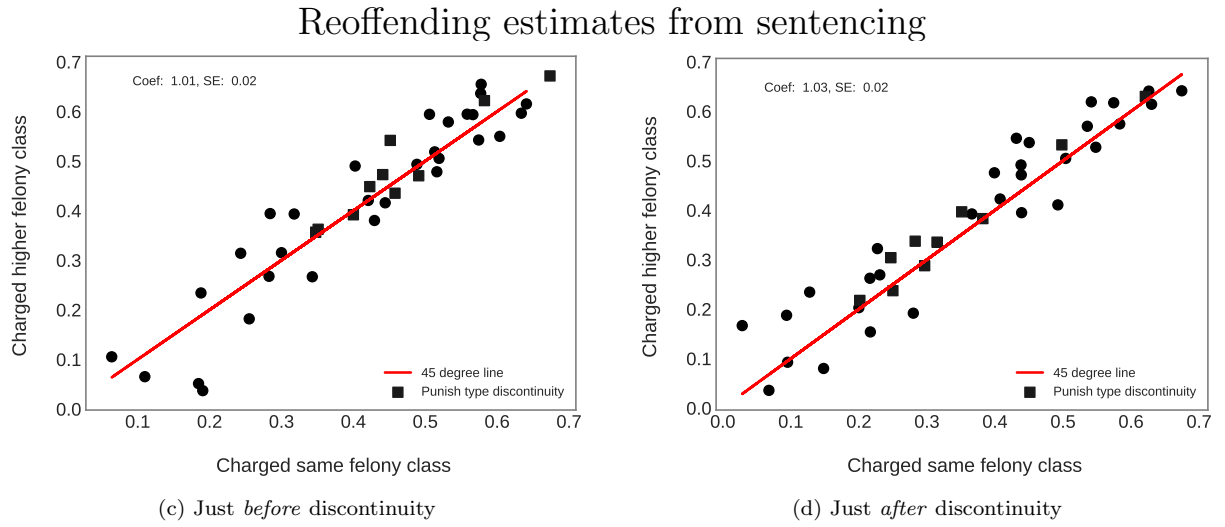
Notes: This table reports 2SLS estimates of incarceration length (D_i) on reoffending within three years of sentencing according to three different measures of reoffending. For each measure of reoffending (e.g., New offense), three estimates are reported. Each column shows the estimated effect when calculating the instruments using a different classification of offenses felony severity classes. The first column uses the offense that the individual was arrested for, The second column the offense that she was arraigned for, and lastly the third column the offense she got convicted of. In our main analysis we use the third column. It is clear that the estimates in all columns are similar, however, the standard errors in the third column are substantially lower. Standard errors are clustered at the individual level.

Figure E.1: Difference in punishment between plead down and same charged offender



Notes: See the notes in Figure E.2.

Figure E.2: Reoffending rates between plead down and same charged offenders

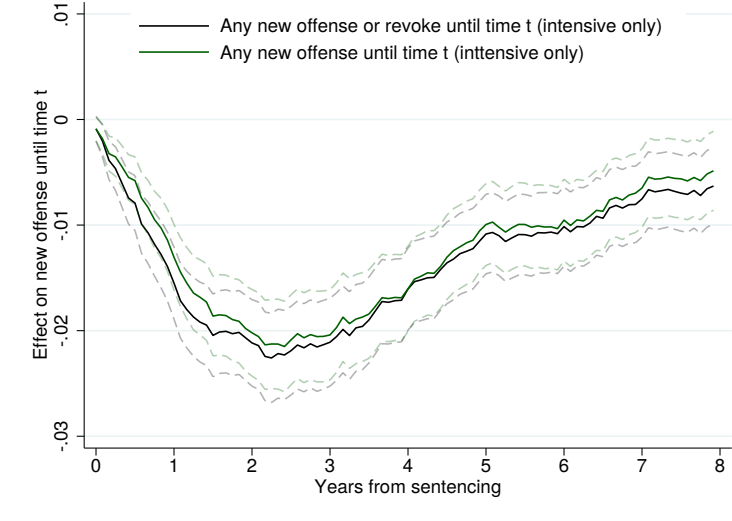


Notes: This figure splits all individuals convicted in a given offense class and prior record points value and compares those who were initially charged in that offense class (x-axis) to those who plead down from more severe offenses (y-axis). Since the key concern for our research design is that this type of sorting increases at the discontinuity, we compare these two groups of offenders just before (left panel plots) and just after (right panel plots) a major discontinuity.

E.2 No evidence of differences in detection

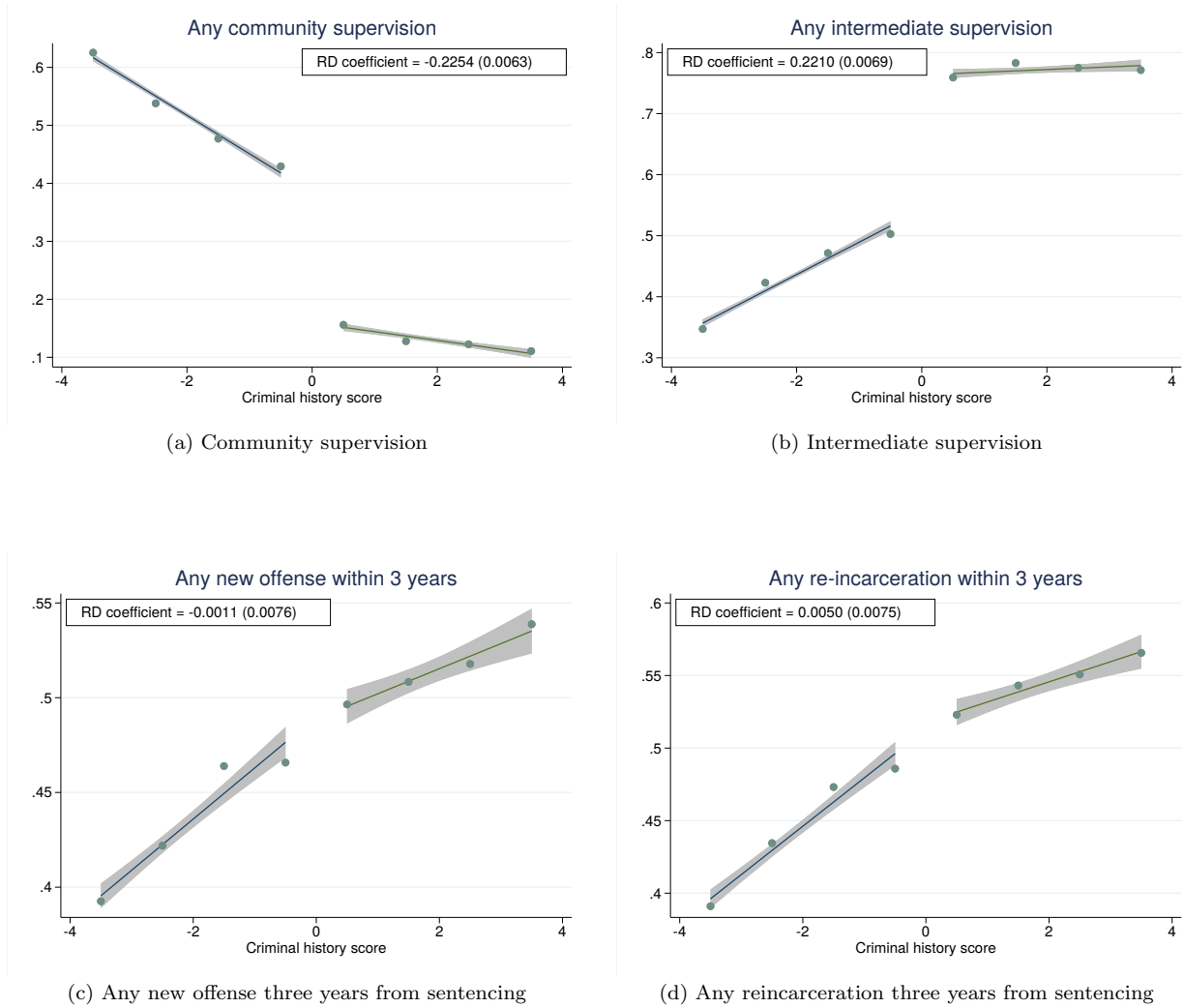
The figures below are discussed in the main text.

Figure E.3: The effect of length of incarceration on re-offending using only intensive margin variation



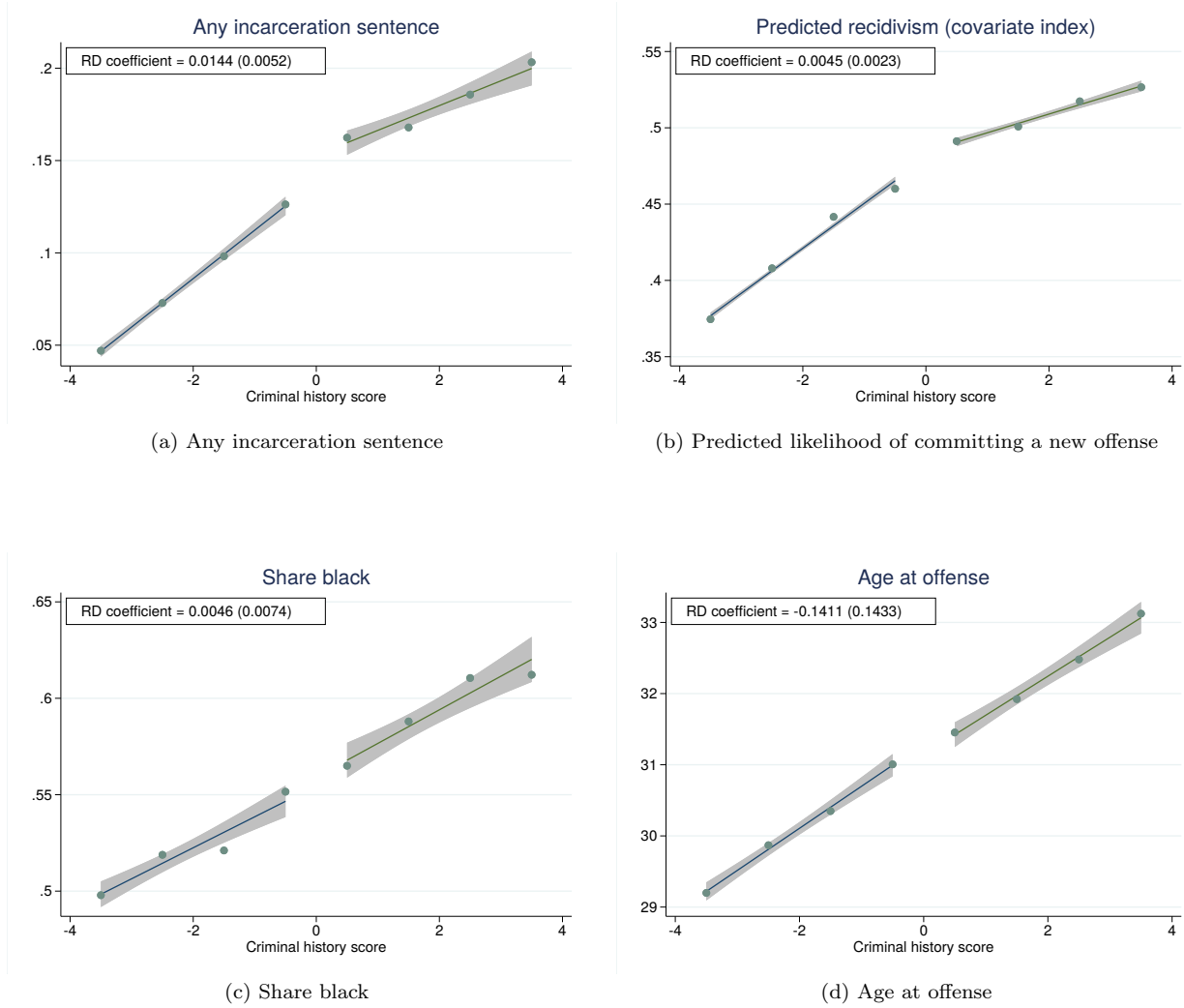
Notes: This figure reports 2SLS estimates of incarceration length (D_i) on reoffending within t months from the sentencing date. Two measures of reoffending are used. The first is an indicator for whether the individual committed any new offense until month t from sentencing (green line). The second includes also probation revocations in the reoffending indicator. All estimates are from a 2SLS that uses *only* the 15 discontinuities that shift only the intensive margin of the length of incarceration and do not impact the type of punishment (incarceration vs. probation). Standard errors are clustered at the individual level.

Figure E.4: Effects on the type of punishment (community vs. intermediate supervision) and future re-offending and re-incarceration within three years of sentencing



Notes: This figure shows the impacts of the discontinuity in the type of probation supervision (community vs. intermediate) in felony offense class I, when moving between prior record levels II and III, on the type of probation supervision. The plots in the first row show that the transition between prior record levels has a salient effect on the type of supervision that offenders are assigned. The plots in the second row show that the discontinuity does not have an influence on re-offending outcomes such as committing a new offense or being re-incarcerated within three years of the time of sentencing.

Figure E.5: Validity checks that incarceration exposure and pre-conviction controls vary smoothly at discontinuity



Notes: This figure shows the impacts of the discontinuity in the type of probation supervision (community vs. intermediate) in felony offense class I, when moving between prior record levels II and III, on outcomes that are not supposed to be influenced by the discontinuity. This figure presents validity checks that support a causal interpretation to the estimates effects in Figure E.4.

F Implementation of Treatment Effect Bounds

This appendix provides additional details on the bounding procedure described in Section 4 of the main text. The approach adapts Mogstad et al. (2018) to the ordered treatment case and accounts for empirical details specific to our research design. The procedure requires three separate steps:

1. Estimation of the $\pi_d(X_i, Z_i)$ in Equation 5.
2. Estimation of the conditional moments $E[Y_i|D_i, Z_i, X_i]$.
3. Estimation of bounds on desired target parameters.

In what follows, we use X_i to refer to the covariates that determine individuals' location in North Carolina's sentencing grid: their criminal history score (prior points) and the offense severity class of their convicted offense. In the notation of our primary reduced form specification Equation (2), $X_i = [p_i, class_i]'$. Z_i is an indicator for whether an individual falls to the right or left of the punishment type discontinuity in her class, or $1\{p_i \geq l_k\}1\{class_i = k\}$ for each $k \in classes$ and for the class-specific prior points threshold l_k .

(1) When considering the five punishment type discontinuities, we need to estimate $5 \cdot \bar{D} \cdot 2$ total π s. We do so using an ordered Probit specification:

$$\begin{aligned} D_i &= d \quad \text{if} \quad C_d(X_i, Z_i) \leq \nu_i < C_{d+1}(X_i, Z_i) \\ C_{d-1}(X_i, Z_i) &\leq C_d(X_i, Z_i) \quad \forall X_i, Z_i, l \\ C_0(X_i, Z_i) &= -\infty, \quad C_{\bar{D}+1}(X_i, Z_i) = \infty \quad \forall X_i, Z_i \\ \nu_i &\sim N(0, 1) \end{aligned}$$

The thresholds $C_d(X_i, Z_i)$ depend on observables and instruments in the same form as in our reduced form analysis. However, in order to ensure the thresholds are increasing, we add an exp transform and sum thresholds for $d > 1$. Specifically:

$$\begin{aligned} f_d(X_i, Z_i) &= \eta_{class_i}^d + \sum_{k \in classes} 1\{class_i = k\} \left[\sum_{l \in thresh} \beta_{lk}^d 1\{p_i \geq l\} (p_i - l + 0.5) + \psi_k^d p_i \right] \\ &+ \sum_{k \in classes} \sum_{l \in thresh \neq 0} \xi_{kl}^d 1\{p_i \geq l\} 1\{class_i = k\} + \sum_{k \in classes} \gamma_k^d 1\{p_i \geq thresh_0\} 1\{class_i = k\} \\ C_1(X_i, Z_i) &= f_1(X_i, Z_i) \\ C_d(X_i, Z_i) &= C_1(X_i, Z_i) + \sum_{m=2}^d \exp(f_m(X_i, Z_i)) \quad \text{if } d > 1 \end{aligned}$$

We fit this model via maximum likelihood. We then use the fitted values of $Pr(D_i \geq d|X_i, Z_i)$ as estimates of each $\pi_d(X_i, Z_i)$. We take the fits at each punishment type discontinuity. For

example in Class I, we use the fits at $p_i = 8.5$, adding and subtracting the relevant ξ_{kl}^d to get values with $Z_i = 0$ and $Z_i = 1$.⁵⁰ Intuitively, these fits measure the probability of receiving a sentence of at least d just to the left and just to the right of the punishment type discontinuity in each class. Note that because no other observable characteristics (e.g., age, gender) enter the model, individuals not in sentencing grid cells directly adjacent to each discontinuity do not contribute to the estimation of the π s. Hence for computational speed we drop all such observations when estimating the model.

(2) Estimation of the conditional moments $E[Y_i|D_i, Z_i, X_i]$ requires estimating the mean Y_i just to the left and just to the right of each discontinuity and for each $d \in \{0, \dots, \bar{D}\}$. We do so by estimating the following linear specification, which interacts our primary reduced form regressors with a third order polynomial in d :

$$\begin{aligned} g_w(X_i, Z_i) = & \eta_{class_i}^w d^w + \sum_{k \in classes} 1\{class_i = k\} \left[\sum_{l \in thresh} \beta_{lk}^w d^w 1\{p_i \geq l\} (p_i - l + 0.5) + \psi_k^w d^w p_i \right] \\ & + \sum_{k \in classes} \sum_{l \in thresh \neq 0} \xi_{kl}^w d^w 1\{p_i \geq l\} 1\{class_i = k\} + \sum_{k \in classes} \gamma_k^w d^w \{p_i \geq thresh_0\} 1\{class_i = k\} \\ Y_i = & g_0(X_i, Z_i) + g_1(X_i, Z_i) + g_2(X_i, Z_i) + g_3(X_i, Z_i) + e_i \end{aligned}$$

We fit the model using ordinary least squares and continue to drop all observations not in grid cells adjacent to each discontinuity. As in step (1), we then use fitted values at each discontinuity to estimate conditional moments for each d , x , and z . We use values of d at the mid point of the discrete units considered. For example, in our main analysis where we consider three-month doses of incarceration, we take the fits at $d = 0$, $d = 1.5$, $d = 4.5$, etc. for doses of zero months, 0-3 months, 3-6 months, etc.

(3) With estimates of the π s and conditional moments in hand, we are now prepared to estimate bounds on treatment effects of interest. To do so, we approximate the MTRs $m_d(x, u)$ using Bernstein polynomials of fixed degree and compute bounds as the solution to a linear programming problem.⁵¹ A Bernstein polynomials of degree n is defined recursively as the sum of $n+1$ Bernstein basis polynomials:

$$\begin{aligned} B_n(u) &= \sum_{v=0}^n \theta_v^d(x) b_{v,n}(u) \\ b_{v,n}(u) &= \binom{n}{v} u^v (1-u)^{n-v} \end{aligned}$$

⁵⁰Or $p_i = 9.5$ if the individual was sentenced under the post-2009 grid.

⁵¹We follow [Shea and Torgovitsky \(2020\)](#) and [Mogstad et al. \(2018\)](#) in using Bernstein polynomials to estimate MTR functions.

Bernstein polynomials are convenient analytically because many shape constraints can be expressed as constraints on the θ s. For example, imposing $0 \leq m_d(x, u) \leq 1$ requires that $0 \leq \theta_v^d(x) \leq 1$. In addition, each basis polynomial has a monomial representation of the form:

$$b_{v,n}(u) = \sum_{l=v}^n \binom{n}{l} \binom{l}{v} (-1)^{l-v} u^l$$

Thus the definite integral of $m_d(x, u)$ over the range $[a, r]$ can be computed as:

$$\begin{aligned} \int_a^r m_d(x, u) du &= \sum_{v=0}^n \theta_v^d(x) \tilde{b}_{v,n}(r) - \sum_{v=0}^n \theta_v^d(x) \tilde{b}_{v,n}(a) \\ \tilde{b}_{v,n}(u) &= \sum_{l=v}^n \binom{n}{l} \binom{l}{v} \frac{(-1)^{l-v}}{l+1} u^{l+1} \end{aligned}$$

This integral is linear in the parameters $\theta_v^d(x)$. This allows us to write target parameters such as the ATE as linear functions of the Bernstein polynomial coefficients. Specifically, let θ collect the set of $\theta_v^d(x)$ that define $m_d(x, u)$ for each of our five discontinuities and dosages d . Then there exists a vector $C_{x,d,d'}$ such that $C'_{x,d,d'} \theta$ yields the ATE for a given d, d' and x . The entries of $C_{x,d,d'}$ are either zero for MTRs that do not contribute to the given ATE, or reflect the appropriate $\tilde{b}_{v,n}(\cdot)$ multiplied by 1 or -1 .

The conditional moments can also be expressed as linear functions of θ . For example, $E[Y_i | D_i = d, Z_i = z, X_i = x]$ is simply:

$$\frac{\int_{\pi_{d+1}(x,z)}^{\pi_d(x,z)} m_d(x, u) du}{\pi_d(x, z) - \pi_{d+1}(x, z)} = \frac{1}{\pi_d(x, z) - \pi_{d+1}(x, z)} \left(\sum_{v=0}^n \theta_v^d(x) \tilde{b}_{v,n}(\pi_d(x, z)) - \sum_{v=0}^n \theta_v^d(x) \tilde{b}_{v,n}(\pi_{d+1}(x, z)) \right)$$

Hence, for each moment there exists a vector $A_{d,z,x}$ such that $A'_{d,z,x} \theta$ yields the conditional moment. The entries of $A_{d,z,x}$ are either zero for MTRs irrelevant to the particular moment or reflect the appropriate $\tilde{b}_{v,n}(\cdot)$. Stacking all such $A_{d,z,x}$ into a single matrix A allows us to express the constraint that candidate MTRs reproduce all conditional moments as requiring that $A\theta = M$, where M is the vector of moments.

A practical consideration is that due to sampling error, it may not be possible to find a θ such that $A\theta = M$ exactly. Hence in practice, we follow [Mogstad et al. \(2018\)](#) and require that $|A\theta - M| \leq Q$, where $|\cdot|$ is the L1 norm and Q is a tuning parameter that ensures a solution is always feasible (discussed further below). We use the L1 norm so that the problem remains linear.⁵² This requires defining a positive and negative component of $e = M - A\theta$ as $e = u - v$ with $u, v \geq 0$. The constraint is then that $\text{sum}(u + v) \leq Q$.

⁵²The constraint using the L1 norm can be reformulated as a linear programming problem similarly to the case of a quantile regression.

Most shape constraints, such as requiring that $0 \leq m_d(x, u) \leq 1$, can be expressed as linear functions of θ . Let $S'\theta \leq 0$ represent these constraints. Other constraints, however, can only be expressed as linear functions of the MTRs themselves. These constraints include, for example, requiring that MTEs are the same in the support of X_i , or that $m_d(x, u) - m_{d'}(x, u) = m_d(x', u) - m_{d'}(x', u) \forall d', x, x'$. To enforce these constraints, we define a new matrix E such that $E\theta$ evaluates each MTR at many values of u . The resulting vector is length $(\bar{D} + 1) \cdot 5 \cdot n_{points}$, reflecting the values of the MTRs for each dosage d , for each discontinuity x , and for each of the n_{points} in u (e.g., $[0, 0.1, 0.2, \dots, 1]$). Enforcing constraints on MTRs at each of the n_{points} in u considered can be expressed as linear functions W of this vector, so that the total constraint is $WE\theta \leq 0$.

Bounding the ATE for dosages d and d' at discontinuity x therefore requires solving:

$$\begin{aligned} \min / \max_{\theta} C'_{x,d,d'}\theta & \\ \text{s.t. } |A\theta - M| &\leq Q \\ S\theta &\leq 0 \\ WE\theta &\leq 0 \end{aligned} \tag{F.1}$$

Computing bounds on alternative parameters requires simply adjusting $C_{x,d,d'}$, while changing the shape constraints applied requires adjusting S and W .

A final technical issue arises in that constraints encoded by $WE\theta \leq 0$ are only enforced at the chosen n_{points} . Thus it is not guaranteed that the constraint holds at all $u \in [0, 1]$. To account for this, we follow [Shea and Torgovitsky \(2020\)](#) and first solve the problem using a relatively low n_{points} (e.g., 100). After a solution has been found, we then evaluate the constraints on a much finer grid (e.g., with $n_{points} = 1,000$), add any points where the constraint is violated to the constraint matrices W and E , and then recompute the solution. We repeat this procedure until we find no more violations on the finer grid. We find our results are insensitive to the quantity of points in this finer grid.

We pick Q by finding the minimal value such that a solution is feasible. Formally, this amounts to first solving an auxiliary problem:

$$\begin{aligned} \hat{Q} &= \min_{\theta} |A\theta - M| \\ \text{s.t. } S\theta &\leq 0 \\ WE\theta &\leq 0 \end{aligned} \tag{F.2}$$

and then computing the min / max in F.1 using \hat{Q} in the constraints. This ensures a solution is always feasible. We estimate bounds in Python using the Gurobi Solver. In practice we find that each bound requires only 10-20 seconds to compute.