

# Evaluating Statistical Methods for Nuclear Forensics Analysis

Preliminary Examination

Arrielle Opotowsky

University of Wisconsin-Madison

29 January 2018



**WISCONSIN**  
UNIVERSITY OF WISCONSIN-MADISON

# Outline



## 1 Introduction

Motivation

Methodology

## 2 Literature Review

Nuclear Forensics

Statistical Models

Algorithms for Prediction

ML Model Assessment

ML Model Validation

Computational Tools

Previous Work

## 3 Demonstration

Training Data

Reactor Parameter Prediction

ML Model Validation

## 4 Research Proposal

Experiment 1

Experiment 2

Experiment 3

Method Comparison

## 5 Summary

# Research Overview



How does the ability to determine forensic-relevant spent nuclear fuel attributes using machine learning techniques degrade as less information is available?

## Determine

The inverse problem: given end measurements, calculate the model parameters that created them

## Information

Nuclide vectors, measurements of isotope ratios

## Forensic-relevant Attributes

Reactor type, enrichment, cooling time, burnup

## Machine Learning Techniques

Creating statistical models (not physical)

## Degrade

Model prediction performance

## Less Information

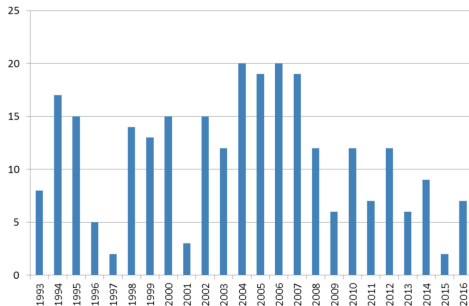
Error in nuclide vectors, fewer measurements, etc

Figure 1: Definitions of terms within the main research question



# Nuclear Security and Forensics

Incidents related to trafficking or malicious use, 1993–2016



- FY2016 DHS DNDO budget : 0.3 bill
- FY2016 DOE NNSA nonpro budget : 1.6 bill

Figure 2: 24 years of incidents: HEU (12), Pu (2), Pu-Be neutron sources (4) [Obtained from: <https://www.iaea.org/sites/default/files/17/12/itdb-factsheet-2017.pdf>]



# Needs in Nuclear Forensics

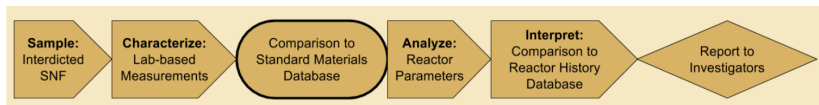


Figure 3: Typical technical nuclear forensics workflow

## Material-specific:

- Measurement needs
- Measurement techniques
- Forensic signatures

## Challenges:

- Rapid characterization
- Forensics databases
  - Multidimensional
  - Inconsistent uncertainties
  - International cooperation



# Computational Methods

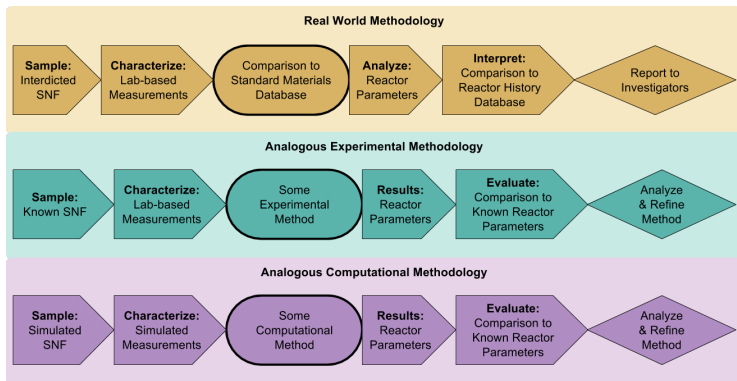


Figure 4: Nuclear forensics research: physical, experimental, and computational

# Computational Methods

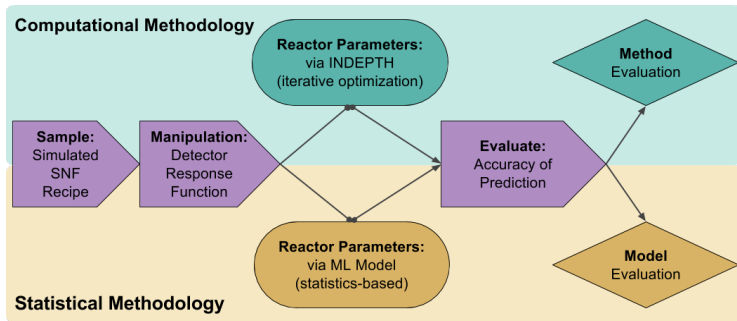


Figure 5: Comparison of two different computational approaches



# Statistical Methods

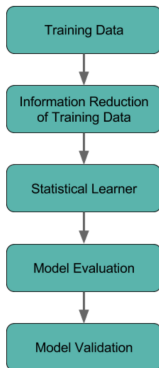


Figure 6: Workflow of a methodology using statistical models

- Training data: large set of SNF measurements
  - Labels (e.g., burnup)
  - Features (e.g., nuclide concs)
  - Instances (individual SNF recipe)
- Statistical learner
  - Machine learning algorithms
  - Algorithm parameters
  - Predict label of new instance
- Model evaluation
  - Diagnostic curves
    - Learning curves
    - Validation curves
  - Prediction error
    - Bias versus variance
    - Generalizability



# Statistical Methods



|                                     | TRAINING DATA                       | TESTING DATA                        |
|-------------------------------------|-------------------------------------|-------------------------------------|
| <b>Physical Motivation</b>          |                                     |                                     |
| Ideal World                         | <i>Lab-Measured Mass Spectra</i>    | <i>Lab-Measured Mass Spectra</i>    |
| Real World                          | <i>Lab-Measured Gamma Spectra</i>   | <i>Field-Measured Gamma Spectra</i> |
| <b>Computational Representation</b> |                                     |                                     |
| Ideal World                         | <i>Simulation-Created Isotopics</i> | <i>Simulation-Created Isotopics</i> |
| Real World                          | <i>DRF-Derived Gamma Spectra</i>    | <i>DRF-Derived Gamma Spectra</i>    |

Figure 7: Illustration of data set modularity

# Outline



## 1 Introduction

Motivation

Methodology

## 2 Literature Review

Nuclear Forensics

Statistical Models

Algorithms for Prediction

ML Model Assessment

ML Model Validation

Computational Tools

Previous Work

## 3 Demonstration

Training Data

Reactor Parameter Prediction

ML Model Validation

## 4 Research Proposal

Experiment 1

Experiment 2

Experiment 3

Method Comparison

## 5 Summary

# Nuclear Forensics Investigations



## Post-detonation

- Collection: debris, swipe samples
- Characterization: rapid analysis of isotope ratios
- Goals
  - Inverse problem: reconstruct weapon design/yield
  - Safety: informing disaster response
- Data evaluation



# Nuclear Forensics Investigations

## Post-detonation

- Collection: debris, swipe samples
- Characterization: rapid analysis of isotope ratios
- Goals
  - Inverse problem: reconstruct weapon design/yield
  - Safety: informing disaster response
- Data evaluation

## Pre-detonation

- Collection: depends on intercepted material
- Characterization: non-destructive and destructive
- Goals:
  - Inverse problem: material chain of custody
  - Safety: material handling and security
- Data evaluation



# Nuclear Forensics as an Inverse Problem

Necessary to determine the quality of prediction  
Use Bayes' Framework:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

# Machine Learning



machine vs. statistical (domain knowledge-¿none)  
supervised and unsupervised  
clustering, dimensionality reduction  
classification, regression – discrete and continuous variables

# Supervised Regression

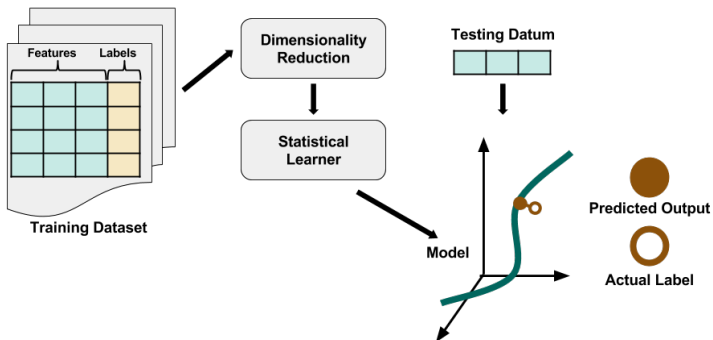


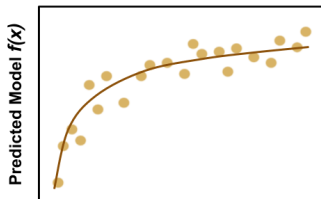
Figure 8: Schematic of a representative prediction workflow



# Linear Models

Objective: minimize error over all training data wrt their labels

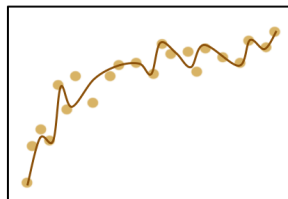
$$F(\mathbf{X}) = \beta_0 + \sum_{j=1}^p x_j \beta_j$$



Input Feature  $(x)$ ,  $\lambda > 0$

Regularization using  $\lambda$

$$F(\mathbf{X}) = \beta_0 + \sum_{j=1}^p x_j \beta_j + \lambda \sum_{j=1}^p \beta_j^2$$



Input Feature  $(x)$ ,  $\lambda = 0$

Figure 9: How regularization might affect the generalizability of an ML model





# Nearest Neighbor Methods

Objective: minimum distance between test sample and training instance(s)

$$Y(\mathbf{x}) = \frac{1}{k} \sum_{x_i \in N_k(\mathbf{x})} y_i$$

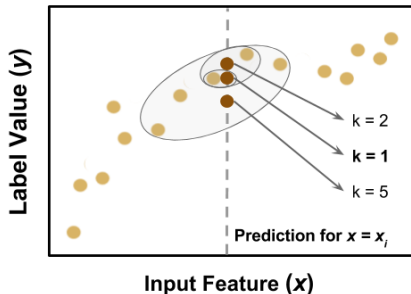


Figure 10: Illustration of the regularization effects by choosing  $k$



# Support Vector Machines

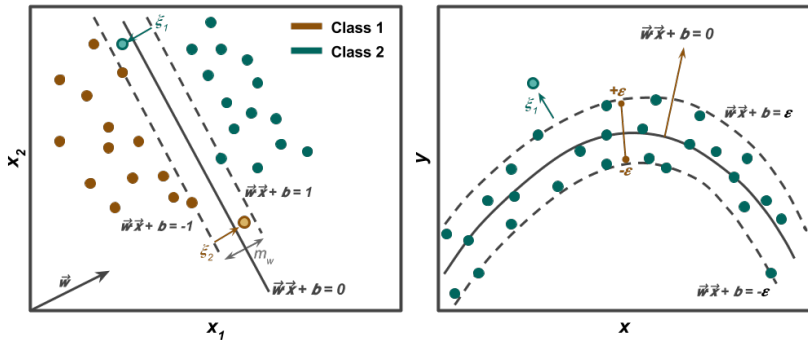


Figure 11: Classification with SVM and regression with SVR

# Support Vector Regression with Many Dimensions



Objective: minimize margin width and outliers

$$\min \frac{1}{2} \|w\|^2 + C \sum_i \xi_i$$

$$\text{subject to : } |y_i - (w\phi(x_i) + b)| \leq \varepsilon + \xi_i$$

$$\text{where : } w = \sum_i \alpha_i y_i \phi(x_i)$$

$$\text{and : } K(x_i, x_j) = \phi(x_i)\phi(x_j) = e^{\gamma \|x_i - x_j\|^2}$$

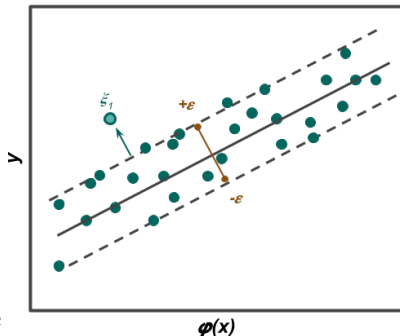


Figure 12: Diagram showing the use of the kernel trick with SVR

# Dimensionality Reduction



Manual via domain knowledge or some measure

PCA

Factor Analysis

ICA

# Sources of Error

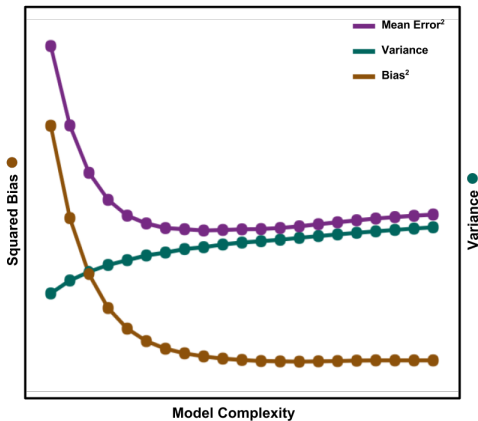


Figure 13: Bias and variance comprise the prediction error

# Types of Error

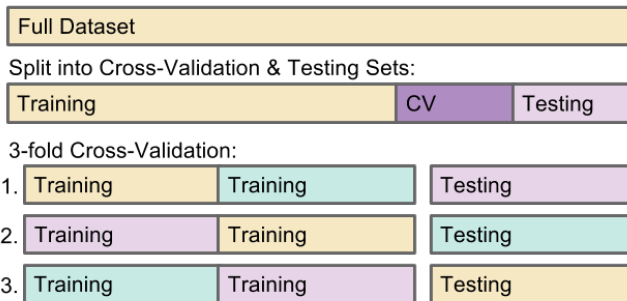


Figure 14: Diagram explaining the concept of  $k$ -fold cross-validation

# Error Metrics



L1, L2: absolute error and squared error

Others:  $r^2$  score, percent error

Used for model prediction error and optimization of algs in obj funcs



# Training Set Size: Learning Curves

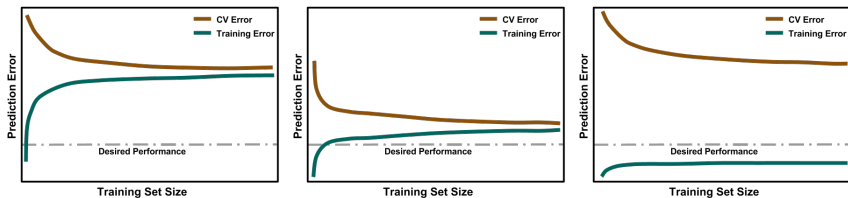


Figure 15: Learning curves for three training scenarios: high bias, balanced bias and variance, and high variance





# Model Complexity: Validation Curves

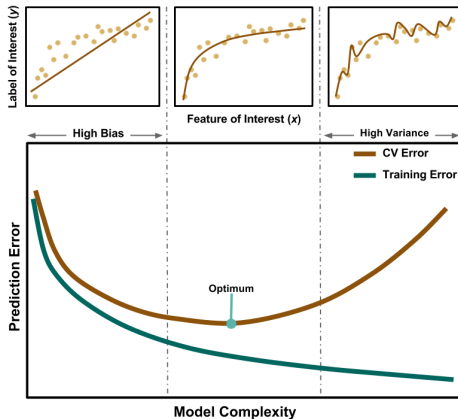


Figure 16: Validation curve showing different fitness of models



# Model Comparison

| Probabilities              | Calculation Method and Example   |
|----------------------------|--|
| <b>P(D M)</b><br>Prior     | Given: M = BWR U-oxide with burnup = $x$ GWd/MTU<br>This is true: D = nuclide vector with Pu-239 = $y\%$<br>With: $z\%$ probability<br>Calc'd from: ORIGEN simulations in training set |
| <b>P(M)</b><br>Likelihood  | Given: No direct information on D<br>This is true: M = BWR U-oxide with burnup = $x$ GWd/MTU<br>With: $z\%$ probability<br>Calc'd from: Machine-learned model prediction               |
| <b>P(D)</b><br>Marginal L. | Given: No direct information on M<br>This is true: D = nuclide vector with Pu-239 = $y\%$<br>With: $z\%$ probability<br>Calc'd from: Summation of training set instances               |
| <b>P(M D)</b><br>Posterior | Given: D = nuclide vector with Pu-239 = $y\%$<br>This is true: M = BWR U-Ox with burnup = $x$ GWd/MTU<br>With: $z\%$ probability<br>Calc'd from: All quantities above                  |

$$\text{Posterior} = \frac{\text{Likelihood} * \text{Prior}}{\text{Marginal Likelihood}}$$

Table 1: Bayes

# Computational Tools



cite stuff

- Training Data : SCALE/ORIGEN-ARP
- Statistics Toolkit : scikit-learn (python)
- Information Reduction
  - Gamma energies: ORIGEN
  - Computational gamma spectra: GADRAS

# Pre-detonation Materials of Interest



UOC

UOX powder

SNF

Reprocessed SNF

# Statistical Methods Employed



get images

Factor Analysis

SFCOMPO extension

Dayman paper on prediction ability wrt info reduction



# Outline

## 1 Introduction

- Motivation
- Methodology

## 2 Literature Review

- Nuclear Forensics
- Statistical Models
  - Algorithms for Prediction
  - ML Model Assessment
  - ML Model Validation
- Computational Tools
- Previous Work

## 3 Demonstration

- Training Data
- Reactor Parameter Prediction
- ML Model Validation

## 4 Research Proposal

- Experiment 1
- Experiment 2
- Experiment 3
- Method Comparison

## 5 Summary



# Proposed Experiment Methodology

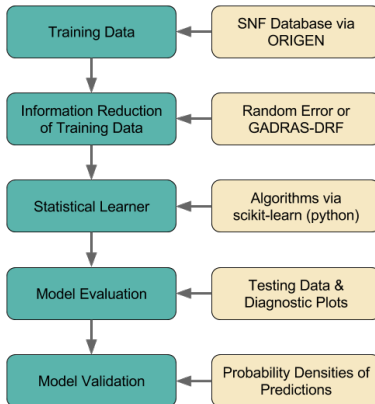


Figure 17: Workflow of the experiments with tools used for each step

# Training Set



| ORIGEN Rxtr  | Rxtr Type | Enrichment |
|--------------|-----------|------------|
| CE14x14      | PWR       | 2.8        |
| CE16x16      | PWR       | 2.8        |
| W14x14       | PWR       | 2.8        |
| W15x15       | PWR       | 2.8        |
| W17x17       | PWR       | 2.8        |
| S14x14       | PWR       | 2.8        |
| VVER440      | PWR       | 3.60       |
| VVER440_3.82 | PWR       | 3.82       |
| VVER440_4.25 | PWR       | 4.25       |
| VVER440_4.38 | PWR       | 4.38       |
| VVER1000     | PWR       | 2.8        |
| GE7x7-0      | BWR       | 2.9        |
| GE8x8-1      | BWR       | 2.9        |
| GE9x9-2      | BWR       | 2.9        |
| GE10x10-8    | BWR       | 2.9        |
| Abb8x8-1     | BWR       | 2.9        |
| Atrium9x9-9  | BWR       | 2.9        |
| SVEA64-1     | BWR       | 2.9        |
| SVEA100      | BWR       | 2.9        |
| CANDU28      | PHWR      | 0.711      |
| CANDU37      | PHWR      | 0.711      |





# Independent Testing Set

| Reactor | Type | Enrichment | Cooling Time      |
|---------|------|------------|-------------------|
| CANDU28 | PHWR | 0.711      | {1m, 7d, 30d, 1y} |
| CANDU28 | PHWR | 0.711      | {3m, 9d, 2y}      |
| CE16x16 | PWR  | 2.8        | {1m, 7d, 30d, 1y} |
| CE16x16 | PWR  | 2.8        | {3m, 9d, 2y}      |
| CE16x16 | PWR  | 3.1        | {7d, 9d}          |
| GE7x7-0 | BWR  | 2.9        | {1m, 7d, 30d, 1y} |
| GE7x7-0 | BWR  | 2.9        | {3m, 9d, 2y}      |
| GE7x7-0 | BWR  | 3.2        | {7d, 9d}          |

# Information Reduction



Random error here  
gamma not implemented here



# Algorithm Parameters

| Algorithm                   | Parameter                | Value                  |
|-----------------------------|--------------------------|------------------------|
| Nearest Neighbor Regression | $n$ -neighbors           | 1                      |
|                             | Weights                  | uniform                |
|                             | Distance Metric          | L2: Euclidian Distance |
| Ridge Regression            | Regularization, $\alpha$ | 1.0                    |
|                             | Normalization            | False                  |
|                             | Stopping Tolerance       | 0.001                  |
| Support Vector Regression   | Kernel                   | Radial Basis Function  |
|                             | Gamma, $\gamma$          | 0.001                  |
|                             | $C$                      | 1000                   |
|                             | Epsilon, $\epsilon$      | 0.1                    |
|                             | Stopping Tolerance       | 0.001                  |

Table 5: caption



# Initial Results

| Algorithm                      | Error Origin            | MAPE  | RMSE<br>[MWd/MTU] |
|--------------------------------|-------------------------|-------|-------------------|
| Nearest Neighbor<br>Regression | Testing Set             | 9.82  | 812.43            |
|                                | 5-fold Cross-Validation | 2.24  | 421.41            |
| Ridge<br>Regression            | Testing Set             | 15.68 | 1049.66           |
|                                | 5-fold Cross-Validation | 0.08  | 13.08             |
| Support Vector<br>Regression   | Testing Set             | 12.28 | 769.97            |
|                                | 5-fold Cross-Validation | 2.08  | 188.07            |

Table 6: caption

# ML Model Prediction with Reduced Information

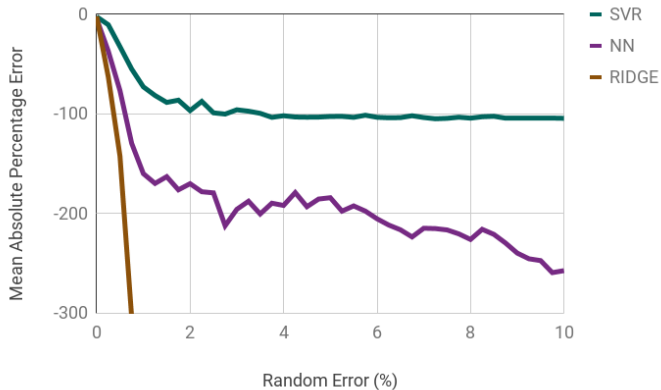
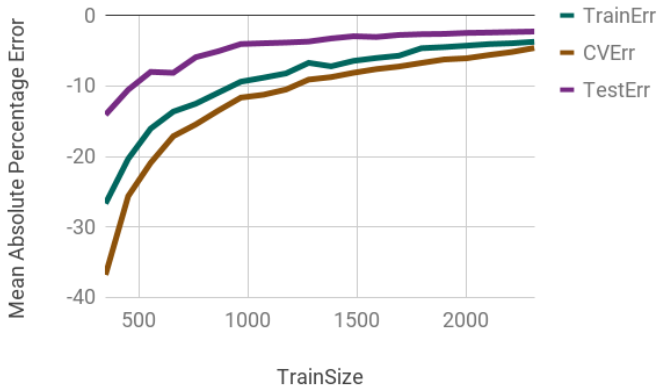


Figure 18: caption



# SVR Learning Curve

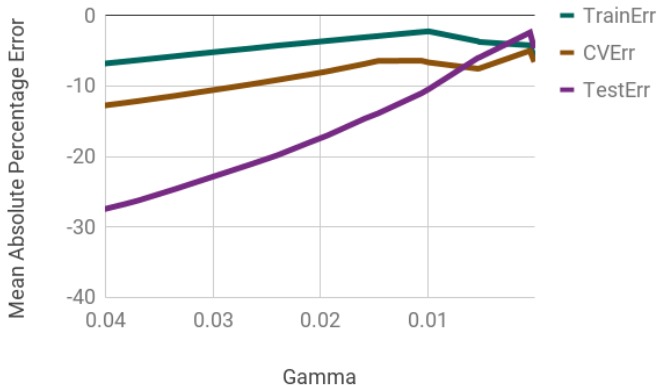
add in example LCs for comparison add in NN or Ridge LC (They look the same)





# SVR Validation Curve

add in example VCs for comparison add in NN or Ridge VC (They look the same)



# Outline



## 1 Introduction

Motivation  
Methodology

## 2 Literature Review

Nuclear Forensics  
Statistical Models  
    Algorithms for Prediction  
    ML Model Assessment  
    ML Model Validation  
Computational Tools  
Previous Work

## 3 Demonstration

Training Data  
Reactor Parameter Prediction  
ML Model Validation

## 4 Research Proposal

Experiment 1  
Experiment 2  
Experiment 3  
Method Comparison

## 5 Summary



# Research Proposal Preparations



Previous Work -  $\lambda$  SFCOMPO-based  
Finalizing set of algorithms  
computational resources

# Statistical Learning with Direct Isotopics



**Goals** : Understand limits of simplest scenario

- ① Usefulness of statistical methods for reactor parameter prediction
- ② Best performing methods

**Variables**

- ① the complexity of the ML algorithm used,
- ② feature reduction, and
- ③ different subsets of the decision space.



# Statistical Learning with Direct Isotopics

## Qualitative Hypotheses

- Complex algorithm will provide best behavior
- Manual preprocessing (feature reduction): speed, accuracy
- Reduction of decision space should help: PWR vs. BWR?

## Risk Mitigation

- New algorithms: tree-based, neural nets, Bayesian MLE
- Statistical preprocessing: PCA, ICA
- New materials: Pu, UOC, Post-detonation (urban canyon [1])

# Statistical Learning with Gamma Spectra



**Goals** : Understand limits of real-world scenario

- ① Level of reduction in reactor parameter prediction
- ② Best performing methods

**Variables**

- ① the complexity of the ML algorithm used,
- ② feature reduction (implicit), and
- ③ quality of training and/or testing data set.

# Statistical Learning with Gamma Spectra



## Qualitative Hypotheses

- Complex algorithm will provide best behavior
- Indirect isotopics = implicit feature reduction: less accurate
- Higher quality gamma spectra will yield better results

## Risk Mitigation

- New algorithms: tree-based, neural nets, Bayesian MLE
- Further manual or statistical preprocessing
- Add isotope identification step

# Statistical Learning with Reprocessed Fuel



**Goals :** Probe prediction performance in reprocessing scenario

- ① Experiment with both direct and indirect isotopics
- ② Fresh evaluation of preprocessing
- ③ Best performing methods for materials with multiple sources

## **Variables**

- ① the complexity of the ML algorithm used,
- ② quality of training data set, and
- ③ type of preprocessing for feature reduction.

# Statistical Learning with Reprocessed Fuel



## Qualitative Hypotheses

- Complex algorithm will provide best behavior
- Reduced information will provide less accurate results
- ICA may outperform PCA, but factor analysis may outperform components analysis [4, 5, 6, 8, 7, 2, 3]

## Risk Mitigation

- New algorithms: tree-based, neural nets, Bayesian MLE
- Manual preprocessing
- Results may be interesting even if prediction fails
- Ensemble methods or other creative solutions [8, 7]



# Probability Distributions

Include uncertainty for measures of confidence, posterior probs become prob distrib

$C$  : constant given by marginal likelihood

$\mathbf{d}$  : training data set

$\mathbf{m}$  : model parameters

$P(\mathbf{d}|\mathbf{m})$  : likelihood distribution function

$P(\mathbf{m})$  : prior probability distribution

$P(\mathbf{m}|\mathbf{d})$  : posterior probability distribution

$$P(\mathbf{m}|\mathbf{d}) = C * P(\mathbf{d}|\mathbf{m}) * P(\mathbf{m})$$

Integrate over prob densities to get prob distrib

$\mathbf{m}$  : range of predicted model parameters

$\mathbf{d}$  is a set of nuclide vectors

$$\rho(\mathbf{x}) = \prod_i \rho(x_i)$$

$$P(\mathbf{m}) = \int_{\mathbf{m}} \rho(\mathbf{d}) d\mathbf{d}$$

Likelihood distribution function:

$$P(\mathbf{d}|\mathbf{m}) = \int_{\mathbf{d}, \mathbf{m}} \rho(\mathbf{d}|\mathbf{m}) d\mathbf{m}$$

But, we infer them...





# Estimating Density Functions

estimate  $\rho$ , have a 'sense' or try different  
prior probability distributions are given by the model space, e.g., reactor  
parameters as predicted from the ML models. [?] Note: This implies the  
posterior is now only dependent on the likelihood.

likelihood function: the training phase provides the maximum likelihood  
distribution through the use of CV, since the results are reported as a  
mean error with a standard deviation (which can be converted to  
accuracy for likelihood) [?]

MLE is not this simple for other methods that do not employ CV [?, ?]



# Posterior Odds

citations plz

calc a non-normalized posterior probability distribution,  $P(m_i|d)$  then do it for a model obtained from a different algorithm,  $P(m_j|d)$

relative posterior probability distribution : *posterior odds*  $B_{ij} = \frac{\rho(d|m_i)}{\rho(d|m_j)}$  : *Bayes factor*.

$$\frac{P(m_i|d)}{P(m_j|d)} = B_{ij} \frac{P(m_i)}{P(m_j)}$$

| $ \ln B_{ij} $ | Probability | Likelihood Strength |
|----------------|-------------|---------------------|
| < 1.0          | < 0.750     | Inconclusive        |
| 1.0            | 0.750       | Weak                |
| 2.5            | 0.923       | Moderate            |
| 5.0            | 0.993       | Strong              |

**Table 9:** Model comparison using likelihood strength

posterior probabilities calculated from  $|\ln B_{ij}|$

Summarize:

Given a mean-squared error and its standard deviation from using CV with any alg, get MLE

compare two models : MLE to MLE



# Outline

## 1 Introduction

- Motivation
- Methodology

## 2 Literature Review

- Nuclear Forensics
- Statistical Models
  - Algorithms for Prediction
  - ML Model Assessment
  - ML Model Validation
- Computational Tools
- Previous Work

## 3 Demonstration

- Training Data
- Reactor Parameter Prediction
- ML Model Validation

## 4 Research Proposal

- Experiment 1
- Experiment 2
- Experiment 3
- Method Comparison

## 5 Summary

# Summary



Summarize



# References I

- [1] Kenneth G.W. Inn, Jacqueline Mann, Jeffrey Leggitt, JoAnne Buscaglia, Simon Jerome, John Molloy, and William Pramenko.  
Nuclear forensic reference materials for attribution of urban nuclear terrorism, 2015.  
*Presentation for NIST.*
- [2] Andrew Jones, Phillip Turner, Colin Zimmerman, and J.Y. Goulermas.  
Machine learning for classification and visualisation of radioactive substances for nuclear forensics.  
*In Techniques and Methods for Safeguards, Nonproliferation and Arms Control Verification Workshop, Portland, Oregon, May 2014.*
- [3] Andrew E. Jones, Phillip Turner, Colin Zimmerman, and John Y. Goulermas.  
Classification of spent reactor fuel for nuclear forensics.  
*Analytical Chemistry*, 86:5399–5405, 2014.



## References II

- [4] G. Nicolaou.  
Determination of the origin of unknown irradiated nuclear fuel.  
*Journal of Environmental Radioactivity*, 86:313–318, 2006.
- [5] G. Nicolaou.  
Identification of unknown irradiated nuclear fuel through its fission product content.  
*Journal of Radioanalytical and Nuclear Chemistry*, 279(2):503–508, 2009.
- [6] G. Nicolaou.  
Discrimination of spent nuclear fuels in nuclear forensics through isotopic fingerprinting.  
*Annals of Nuclear Energy*, 72:130–133, Oct 2014.  
Technical Note.



## References III

[7] Martin Robel and Michael J. Kristo.

Discrimination of source reactor type by multivariate statistical analysis of uranium and plutonium isotopic concentrations in unknown irradiated nuclear fuel material.

*Journal of Environmental Radioactivity*, 99(11):1789–1797, November 2008.

[8] Martin Robel, Michael J. Kristo, and Martin A. Heller.

Nuclear forensic inferences using iterative multidimensional statistics.

In *Proceedings of the Institute of Nuclear Materials Management 50th Annual Meeting*, Tuscon, AZ, USA, Jul 2009. Institute of Nuclear Materials Management. LLNL-CONF-414001.