

trajectory_utils: Mathematical background

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1. Cartpoles

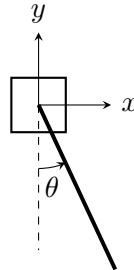


Figure 1: Cart–pole system. The cart is actuated - by various means - and is constrained to move along $\pm x$. Objectives are to swing the pole up to vertical - and/or stabilize it there. The physical cart is actuated by a belt drive, and the pole is a steel rod.

1.1 Equations of motion

By inspection, figure 1 offers:

$$x_r(t) = r\sin(\theta(t)), r \in [0, L] \quad (1)$$

$$y_r(t) = -r\cos(\theta(t)), r \in [0, L] \quad (2)$$

$$\dot{x}_r(t) = r\cos(\theta(t))\dot{\theta}(t) + \dot{x}(t), r \in [0, L] \quad (3)$$

$$\dot{y}_r(t) = r\sin(\theta(t))\dot{\theta}(t), r \in [0, L] \quad (4)$$

where r parameterizes the location of infinitesimal mass along the pole. It is straightforward to compute the Lagrangian $\mathcal{L} = T - V$ - the first step in deriving the equations of motion.

1.2 Cart force control

1.3 Cart velocity servo control

1.4 The cvxpy experience

2. Differential Drive Control trajectories

2.1 Signed Distance Function (SDF) for obstacle avoidance

2.2 Differentiable SDF in pyTorch

2.3 The cvxpy experience

References