

# trajectory\_utils: Mathematical background

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## 1. Cartpoles

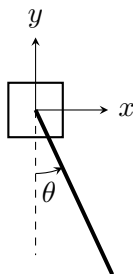


Figure 1: Cart-pole system. The cart is actuated - by various means - and is constrained to move along  $\pm x$ . Objectives are to swing the pole up to vertical - and/or stabilize it there. The physical cart is actuated by a belt drive, and the pole is a steel rod.

### 1.1 Equations of motion

By inspection, figure 1 offers:

$$x_r(t) = r \sin(\theta(t)), r \in [0, L] \quad (1)$$

$$y_r(t) = -r \cos(\theta(t)), r \in [0, L] \quad (2)$$

$$\dot{x}_r(t) = r \cos(\theta(t)) \dot{\theta}(t) + \dot{x}(t), r \in [0, L] \quad (3)$$

$$\dot{y}_r(t) = r \sin(\theta(t)) \dot{\theta}(t), r \in [0, L] \quad (4)$$

where  $r$  parameterizes the location of infinitesimal mass along the pole. It is straightforward to compute the Lagrangian  $\mathcal{L} = T - V$  - the first step in deriving the equations of motion.

## **1.2 Cart force control**

## **1.3 Cart velocity servo control**

## **1.4 The cvxpy experience**

# **2. Differential Drive Control trajectories**

## **2.1 Signed Distance Function (SDF) for obstacle avoidance**

## **2.2 Differentiable SDF in pyTorch**

## **2.3 The cvxpy experience**

## **References**