TAKORADI TECHNICAL UNIVERSITY
FACULTY OF APPLIED SCIENCE
DEPARTMENT OF MATHEMATICS, ACTUARIAL AND STATISTICS
END OF FIRST SEMESTER EXAMINATION
2018/2019 ACADEMIC YEAR

DECEMBER, 2018 | **PROBABILITY III**
STA 211 | **TIME:3 HOURS**

ANSWER ALL QUESTIONS IN SECTION "A" AND TWO QUESTIONS IN SECTION "B"

SECTION A

Answer all questions in this section

1) (a) An electric station services an area with 12,000 bulbs. The probability of switching on each of these bulbs every evening is 0.9.what is the lower bound for the probability that the number of bulbs switched on in the area in one particular evening is different from its expected value in absolute terms by:

   (i) Less than 100? **(10Marks)**

   (ii) At least 120 **(6Marks)**

   (b) The mean lifetime of certain electrical device is 4 years. Find the lower bounds for the probability that a randomly selected device from a consignment of such devices will not exceed 20 years. **(4Marks)**

2) Given that the function: $p(\chi, y) = k(3\chi + 2y)$, $\chi = 0,1;\ y = 0,1,2$

   (i)Find the constant $k \succ 0$ such that the $p(\chi, y)$ is a joint probability mass function. **(5Marks)**

   (ii) Present in a tabular form for the probabilities associate with the sample points (χ, y). Find the marginal probability mass functions of X and Y. **(6Marks)**

3) The binomail distribution is defined as ${}^{n}c_{x} p^{x} q^{n-x}$, where $x = 0,1,2 \ldots n$

   Using the moment generating function, calculate the

   (i) Mean $E(X)$ **(5Marks)**

   (ii) Variance $Var(X)$ **(9Marks)**

4) The uniform distribution of a continuous random variable X is given by

$$f(x) = \begin{cases} 1, 0 \le x \le 1 \\ 0, otherwise \end{cases}$$

Find the moment generating function of X and use it to find the mean and variance of the distribution **(15Marks)**

SECTION B

Answer **two** questions **Only** from this section

5) Suppose that (X,Y) is two-dimensional continuous random variable with joint probability density function

$$f(x)=\begin{cases} k(x+y-2xy), 0\le x\le 1; 0\le y\le 1 \\ 0, otherwise \end{cases}$$

where k is a constant,

a) Find the value of k

b) Find the marginal probability distribution of X and Y.

c) Find the conditional probability;

(i) X given $Y=y$

(ii) Y given $X=x$

d) Verify whether X and Y are independent or not

(20Marks)

6) The joint probability distribution of X and Y is given by

$$f(X,y)=\frac{X+y}{21}, \quad X=1,2,3; \quad y=1,2.$$

Find:

(a) $P(X=3)$

(b) $P(Y=2)$

(20Marks)

7) (a) the joint probabilities of two random variables X and Y are given below

		Y	
		3	6
X	-2	0.28	0.12
	4	0.42	0.18