Office of the Dean
Faculty of Applied Sciences
Takoradi Technical University
Takoradi

TAKORADI TECHNICAL UNIVERSITY
FACULTY OF APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES
END OF FIRST SEMESTER EXAMINATION (2020/2021)

~~CALCULUS AND MATRIX ALGEBRA II: STA 210 TIME: 3 HRS~~

ATTEMPT ANY FOUR QUESTIONS (EACH QUESTION CARRIES 25 MARKS)

QUESTION 1

a. Find $\int x^3 e^{2x}\, dx$ (12 MARKS)

b. Evaluate $\int \frac{(x+1)}{x^3+x^2-6x}\, dx$ (13 MARKS)

QUESTION 2

Consider the matrix A = $\begin{vmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{vmatrix}$

a. Compute $|A|$ (8 MARKS)

b. Find the adjoint of A (9 MARKS)

c. Find A^{-1} (8 MARKS)

QUESTION 3

a. If $z = \ln\sqrt{x^2+y^2}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 1$ (7 MARKS)

b. For $f(x,y) = \cos(xy) - x^3 + y^4$, compute f_{xyy} and f_{xyyy} (9 MARKS)

c. Suppose that $f(x,y) = e^{xy}, x(u,v) = 3u\sin v$ and $y(u,v) = 4v^2u$.

   i. Write out the chain rule for the derivative $\frac{\partial f}{\partial u}$ (2 MARKS)

   ii. Find the derivative $\frac{\partial f}{\partial v}$ (7 MARKS)

QUESTION 4

a. Find x, y, z, t where

$-\frac{1}{6}\ln($

$$3\begin{vmatrix} x & y \\ z & t \end{vmatrix} = \begin{vmatrix} x & 6 \\ -1 & 2t \end{vmatrix} + \begin{vmatrix} 4 & x+y \\ z+t & 3 \end{vmatrix}$$ (10 MARKS)

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b. Let $A = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 0 & -4 \\ 3 & -2 & 6 \end{vmatrix}$ Find i) AB and ii) BA (9 MARKS)

c. Let $(r \times s)$ denote the dimensions of a matrix. Find the dimensions of the matrix product of the matrices with the stated dimensions in each case.

1. $(2 \times 3)(3 \times 4)$ 2. $(1 \times 2)(3 \times 1)$ 3. $(4 \times 4)(3 \times 3)$

4. $(4 \times 1)(1 \times 2)$ 5. $(5 \times 2)(2 \times 3)$ 6. $(2 \times 2)(2 \times 4)$ (6 MARKS)

QUESTION 5

a. Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$ (12 MARKS)

b. Find the critical numbers of $f(x) = 5 - 2x + x^2$ and determine whether they yield relative maxima, relative minima or inflexion points. (8 MARKS)

c. State the law of Mean Value Theorem. (5 MARKs)

Final answer for partial fraction

QUESTION 6

a. i. Suppose an equation $F(x, y) = 0$ determines implicitly a differentiable function, f of one variable x such that $y = f(x)$. Prove that $\frac{dy}{dx} = -\frac{F_x}{F_y}$ (7 MARKS)

ii. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 6xy$ (6 MARKS)

b. Find $\int \frac{\sqrt{x^2-1}}{x} dx$ (12 MARKS)

$15 \times (6B + 6(-2)) = 15$

$90B - 12 = 15$

$90 = 15 + 12$

$\frac{90B}{90} = \frac{27}{90}$

$B = 3/10$

$f_x = -y \sin xy - 3x^2$

$f_{xy} = -yx \cos xy - \sin xy$

$f_{xyy} = -yx^2 - \sin xy - x \cos xy - x \cos xy$

$yx^2 \sin xy - 2x \cos xy$

$f_{xyyy} = yx^2 (x \cos xy) + x^2 \sin xy + 2x^2 \sin xy$

$yx^3 \cos xy + 3x^2 \sin xy$