Минимизация выражений булевой алгебры

$$f(a,b,c) = \overline{a}\overline{c} \vee ab \vee a\overline{b}\overline{c} \vee a\overline{b}c$$

$$f(a,b,c) = \overline{a}\overline{c} \lor ab \lor \overline{ab}\overline{c} \lor \overline{ab}c$$

$$f(a,b,c) = \overline{a}\overline{c} \vee ab \vee (ab)\overline{c} \vee (ab)c = \overline{a}\overline{c} \vee ab \vee (ab)(\overline{c} \vee c) =$$

$$f(a,b,c) = \overline{a}\overline{c} \lor ab \lor a\overline{b}\overline{c} \lor a\overline{b}c = \overline{a}\overline{c} \lor ab \lor a\overline{b}(\overline{c} \lor c) =$$

$$= \overline{a}\overline{c} \lor ab \lor a\overline{b}$$

$$f(a,b,c) = \overline{a}\overline{c} \vee ab \vee a\overline{b}\overline{c} \vee a\overline{b}c = \overline{a}\overline{c} \vee ab \vee a\overline{b}(\overline{c} \vee c) =$$

$$= \overline{a}\overline{c} \vee ab \vee a\overline{b}$$

$$f(a,b,c) = \overline{a} \, \overline{c} \vee ab \vee a\overline{b} \, \overline{c} \vee a\overline{b} \, c = \overline{a} \, \overline{c} \vee ab \vee a\overline{b} \, (\overline{c} \vee c) =$$

$$= \overline{a} \, \overline{c} \vee ab \vee a\overline{b} = \overline{a} \, \overline{c} \vee a(b \vee \overline{b})$$

$$f(a,b,c) = \overline{a} \, \overline{c} \vee ab \vee a\overline{b} \, \overline{c} \vee a\overline{b} \, c = \overline{a} \, \overline{c} \vee ab \vee a\overline{b} \, (\overline{c} \vee c) =$$

$$= \overline{a} \, \overline{c} \vee ab \vee a\overline{b} = \overline{a} \, \overline{c} \vee a \, (b \vee \overline{b}) = \overline{a} \, \overline{c} \vee a$$

$$f(a,b,c) = \overline{a}\overline{c} \vee ab \vee a\overline{b}\overline{c} \vee a\overline{b}c = \overline{a}\overline{c} \vee ab \vee a\overline{b}(\overline{c} \vee c) =$$

$$= \overline{a}\overline{c} \vee ab \vee a\overline{b} = \overline{a}\overline{c} \vee a(b \vee \overline{b}) = \overline{a}\overline{c} \vee a$$

$$\overline{a}\overline{c} \vee a = \overline{a}\overline{c} \vee a(c \vee \overline{c})$$

$$f(a,b,c) = \overline{a} \, \overline{c} \vee ab \vee ab \, \overline{c} \vee ab \, c = \overline{a} \, \overline{c} \vee ab \vee ab \, (\overline{c} \vee c) =$$

$$= \overline{a} \, \overline{c} \vee ab \vee ab = \overline{a} \, \overline{c} \vee a(b \vee b) = \overline{a} \, \overline{c} \vee a$$

$$\overline{a}\overline{c} \lor a = \overline{a}\overline{c} \lor a(c \lor \overline{c}) = \overline{a}\overline{c} \lor ac \lor a\overline{c}$$

$$f(a,b,c) = \overline{a}\overline{c} \vee ab \vee a\overline{b}\overline{c} \vee a\overline{b}c = \overline{a}\overline{c} \vee ab \vee a\overline{b}(\overline{c} \vee c) =$$

$$= \overline{a}\overline{c} \vee ab \vee a\overline{b} = \overline{a}\overline{c} \vee a(b \vee \overline{b}) = \overline{a}\overline{c} \vee a$$

$$\overline{a}\overline{c} \lor a = \overline{a}\overline{c} \lor a(c \lor \overline{c}) = \overline{a}\overline{c} \lor ac \lor a\overline{c} = \overline{a}\overline{c} \lor ac \lor a\overline{c} \lor a\overline{c} = \overline{a}\overline{c} \lor ac \lor a\overline{c} \lor a\overline{c} \lor a\overline{c} = \overline{a}\overline{c} \lor ac \lor a\overline{c} \lor a\overline{c} \lor a\overline{c} = \overline{a}\overline{c} \lor ac \lor a\overline{c} \lor a\overline{c} \lor a\overline{c} \lor a\overline{c} \lor a\overline{c} \to a\overline{c} \lor a\overline{c$$

$$f(a,b,c) = \overline{a}\overline{c} \vee ab \vee a\overline{b}\overline{c} \vee a\overline{b}c = \overline{a}\overline{c} \vee ab \vee a\overline{b}(\overline{c} \vee c) =$$

$$= \overline{a}\overline{c} \vee ab \vee a\overline{b} = \overline{a}\overline{c} \vee a(b \vee \overline{b}) = \overline{a}\overline{c} \vee a$$

$$\overline{a}\,\overline{c} \vee a = \overline{a}\,\overline{c} \vee a(c \vee \overline{c}) = \overline{a}\,\overline{c} \vee ac \vee a\overline{c} = \overline{a}\,\overline{c} \vee ac \vee a\overline{c} = \overline{a}\,\overline{c} \vee a\overline{c} \vee$$

$$f(a,b,c) = \overline{a} \, \overline{c} \vee ab \vee a\overline{b} \, \overline{c} \vee a\overline{b} \, c = \overline{a} \, \overline{c} \vee ab \vee a\overline{b} \, (\overline{c} \vee c) =$$

$$= \overline{a} \, \overline{c} \vee ab \vee a\overline{b} = \overline{a} \, \overline{c} \vee a \, (b \vee \overline{b}) = \overline{a} \, \overline{c} \vee a$$

$$\overline{a} \, \overline{c} \vee a = \overline{a} \, \overline{c} \vee a \, (c \vee \overline{c}) = \overline{a} \, \overline{c} \vee ac \vee a\overline{c} = \overline{a} \, \overline{c} \vee ac \vee a\overline{c} \vee \overline{a} \, \overline{c} =$$

$$= a(c \vee \overline{c}) \vee \overline{c}(a \vee \overline{a}) =$$

$$f(a,b,c) = \overline{a} \, \overline{c} \vee ab \vee ab \, \overline{c} \vee ab \, c = \overline{a} \, \overline{c} \vee ab \vee ab \, (\overline{c} \vee c) =$$

$$= \overline{a} \, \overline{c} \vee ab \vee ab = \overline{a} \, \overline{c} \vee a(b \vee b) = \overline{a} \, \overline{c} \vee a$$

$$\overline{a} \, \overline{c} \vee a = \overline{a} \, \overline{c} \vee a(c \vee \overline{c}) = \overline{a} \, \overline{c} \vee ac \vee a\overline{c} = \overline{a} \, \overline{c} \vee ac \vee a\overline{c} \vee a\overline{c} =$$

$$= a(c \vee \overline{c}) \vee \overline{c}(a \vee \overline{a}) = a \vee \overline{c}$$

Импликанта^{*)}

Импликантой g(a,b,...) функции f(a,b,...) называется такая функция, для которой справедливо:

$$(g \rightarrow f) = 1$$
 или $(\overline{g} \lor f) = 1$

^{*)} лат. *implicire* – входить во что-то, впутываться

а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
С	0	1	0	1	0	1	0	1
f(a,b,c)								

а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
C	0	1	0	1	0	1	0	1
f(a,b,c)								

$egin{array}{ c c c c c c c c c c c c c c c c c c c$	а	0	0	0	0	1	1	1	1
	b	0	0	1	1	0	0	1	1
f(a,b,c) 1 1	С	0	1	0	1	0	1	0	1
	f(a,b,c)	1		1					

$$f(a,b,c) = \overline{a}\overline{c} \lor ab \lor a\overline{b}\overline{c} \lor a\overline{b}c$$

а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
c	0	1	0	1	0	1	0	1
f(a,b,c)	1		1					

$$f(a,b,c) = \overline{a}\overline{c} \vee ab \vee a\overline{b}\overline{c} \vee a\overline{b}c$$

а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
С	0	1	0	1	0	1	0	1
f(a,b,c)	1		1				1	1

$$f(a,b,c) = \overline{a}\overline{c} \lor ab \lor \overline{ab}\overline{c} \lor a\overline{b}c$$

$egin{array}{c ccccccccccccccccccccccccccccccccccc$	а	0	0	0	0	1	1	1	1
	b	0	0	1	1	0	0	1	1
$f\left(a,b,c ight)$ 1 1 1	c	0	1	0	1	0	1	0	1
	f(a,b,c)	1		1				1	1

$$f(a,b,c) = \overline{ac} \lor ab \lor \overline{abc} \lor a\overline{bc}$$

$egin{array}{c ccccccccccccccccccccccccccccccccccc$	a	0	0	0	0	1	1	1	1
	b	0	0	1	1	0	0	1	1
$f\left(a,b,c ight)$ 1 1 1 1	c	0	1	0	1	0	1	0	1
	f(a,b,c)	1		1		1		1	1

$$f(a,b,c) = \overline{ac} \lor ab \lor a\overline{bc} \lor \overline{abc}$$

$egin{array}{ c c c c c c c c c c c c c c c c c c c$	a	0	0	0	0	1	1	1	1	
	b	0	0	1	1	0	0	1	1	
$f\left(a,b,c ight)$ 1 1 1 1 1	c	0	1	0	1	0	1	0	1	
	f(a,b,c)	1		1		1		1	1	

$$f(a,b,c) = \overline{ac} \lor ab \lor a\overline{bc} \lor a\overline{bc}$$

$egin{array}{c ccccccccccccccccccccccccccccccccccc$	а	0	0	0	0	1	1	1	1	
	b	0	0	1	1	0	0	1	1	
$f\left(a,b,c ight)$ 1 1 1 1 1	c	0	1	0	1	0	1	0	1	
	f(a,b,c)	1		1		1	1	1	1	

a	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
С	0	1	0	1	0	1	0	1
f(a,b,c)	1	0	1	0	1	1	1	1

a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
С	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}c \vee a\overline$

a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
c	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{abc} - \overline{c} \vee \overline{abc} \vee a\overline{bc} \vee a$
<i>g</i> ₁	0	0	0	0	1	0	0	1	

a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
С	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{abc} - \overline{c} \vee \overline{abc} \vee a\overline{bc} \vee a$
<i>g</i> ₁	0	0	0	0	1	0	0	1	$g_1 = a\overline{b}\overline{c} \vee abc$

а	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
С	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{abc} - \overline{c} \vee \overline{abc} \vee a\overline{bc} \vee a$
<i>g</i> ₁	0	0	0	0	1	0	0	1	$g_1 = a\overline{b}\overline{c} \vee abc$
<i>g</i> ₂	1	0	1	0	0	0	0	0	

а	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
С	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{abc} - \overline{c} \vee \overline{abc} \vee a\overline{bc} \vee a$
<i>g</i> ₁	0	0	0	0	1	0	0	1	$g_1 = a\overline{b}\overline{c} \vee abc$
<i>g</i> ₂	1	0	1	0	0	0	0	0	$g_2 = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} = \overline{a}\overline{c}$

а	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
c	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}c \vee a\overline$
<i>g</i> ₁	0	0	0	0	1	0	0	1	$g_1 = a\overline{b}\overline{c} \vee abc$
<i>g</i> ₂	1	0	1	0	0	0	0	0	$g_2 = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} = \overline{a}\overline{c}$
<i>g</i> ₃	0	0	0	0	0	1	1	1	

a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
c	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{abc} - \overline{c} \vee \overline{abc} \vee a\overline{bc} \vee a$
<i>g</i> ₁	0	0	0	0	1	0	0	1	$g_1 = a\overline{b}\overline{c} \vee abc$
<i>g</i> ₂	1	0	1	0	0	0	0	0	$g_2 = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} = \overline{a}\overline{c}$
<i>g</i> ₃	0	0	0	0	0	1	1	1	$g_3 = a\overline{b} c \lor ab\overline{c} \lor abc = ac \lor ab$

a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
c	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{abc} - \overline{c} \vee \overline{abc} \vee a\overline{bc} \vee a$
<i>g</i> ₁	0	0	0	0	1	0	0	1	$g_1 = a\overline{b}\overline{c} \vee abc$
<i>g</i> ₂	1	0	1	0	0	0	0	0	$g_2 = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} = \overline{a}\overline{c}$
<i>g</i> ₃	0	0	0	0	0	1	1	1	$g_3 = a\overline{b} c \vee ab\overline{c} \vee abc = ac \vee ab$
g ₄	0	0	0	0	1	1	1	1	

a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
c	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}c \vee a\overline$
<i>g</i> ₁	0	0	0	0	1	0	0	1	$g_1 = a\overline{b}\overline{c} \vee abc$
8 2	1	0	1	0	0	0	0	0	$g_2 = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} = \overline{a}\overline{c}$
<i>g</i> ₃	0	0	0	0	0	1	1	1	$g_3 = a\overline{b} c \vee ab\overline{c} \vee abc = ac \vee ab$
g_{4}	0	0	0	0	1	1	1	1	$g_4 = a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee ab\overline{c} \vee ab\overline{c} = a$

Общая идея методов минимизации

$$f = g_1 \vee g_2 \vee ... \vee g_k$$

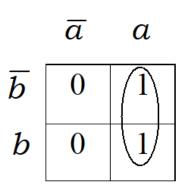
Основные этапы работы:

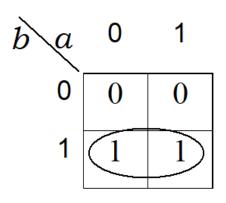
- а) подходящее представление исходных данных;
- б) выявление простых импликант;
- в) сокращение множества импликант до необходимого минимума;
- г) представление найденной минимальной формы в виде, требуемом для дальнейшей работы.

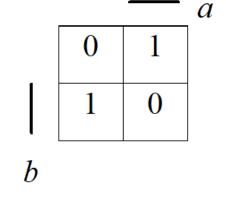
Минимизация методом карт Карно



Мо́рис Ка́рно (англ. *Maurice Karnaugh*, род. 4 октября 1924 года, Нью-Йорк) — американский физик, создатель метода минимизации булевых функций, известного как «карта Карно».





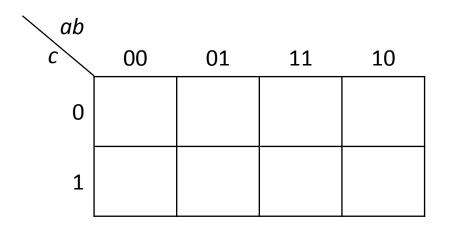


$$ab \vee a\overline{b} = a$$

$$\overline{a}b \lor ab = b$$

$$\overline{a}b \vee a\overline{b}$$

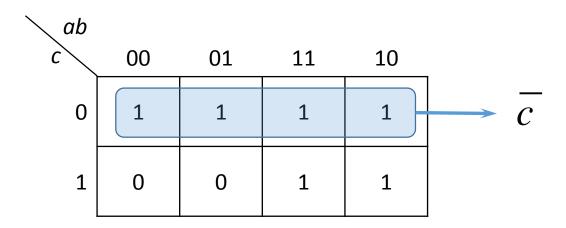
a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
C	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}c \vee a\overline$



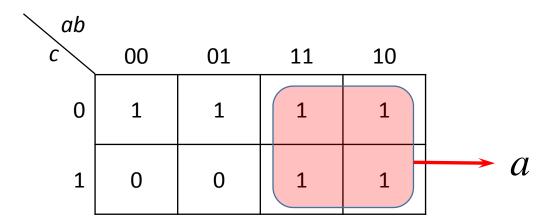
a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
C	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}c \vee a\overline$

ab				
c	00	01	11	10
0	1	1	1	1
1	0	0	1	1

a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
c	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}c \vee a\overline{b}\overline{c} \vee a\overline{b}$



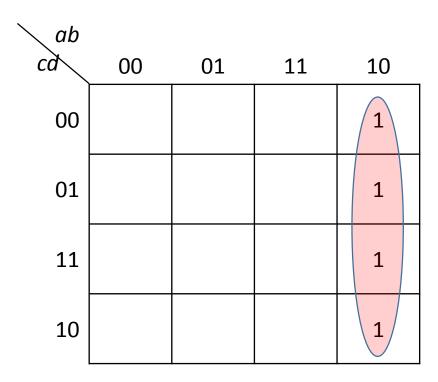
a	0	0	0	0	1	1	1	1	
b	0	0	1	1	0	0	1	1	
С	0	1	0	1	0	1	0	1	
f(a,b,c)	1	0	1	0	1	1	1	1	$f(a,b,c) = \overline{a}\overline{b}\overline{c} \vee \overline{a}b\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}\overline{c} \vee a\overline{b}c \vee a\overline$



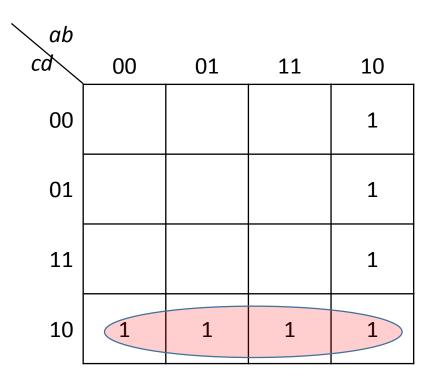
 $f(a,b,c,d) = a\overline{b} \vee c\overline{d} \vee abcd$

ab cd	00	01	11	10
00				
01				
11				
10				

$$f(a,b,c,d) = ab \lor cd \lor abcd$$



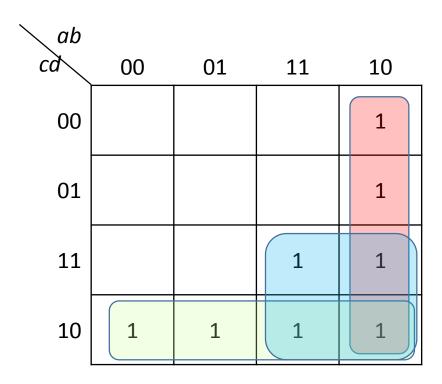
$$f(a,b,c,d) = a\overline{b} \vee \overline{cd} \vee abcd$$



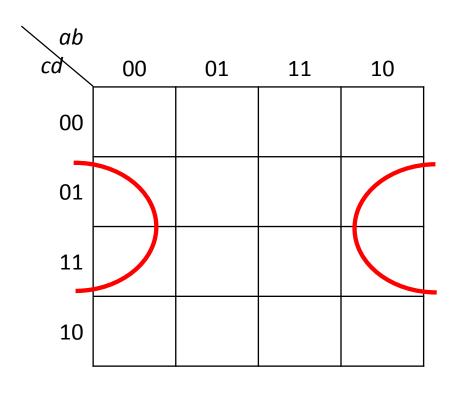
$$f(a,b,c,d) = a\overline{b} \vee c\overline{d} \vee abcd$$

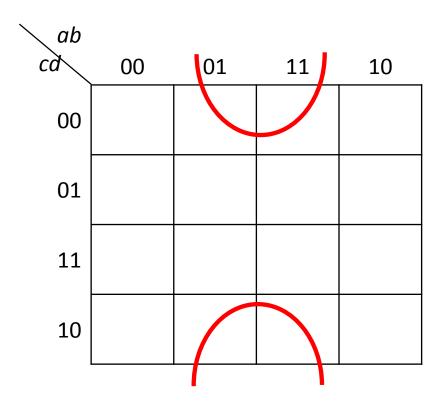
ab cd	00	01	11	10
00				1
01				1
11			1	1
10	1	1	1	1

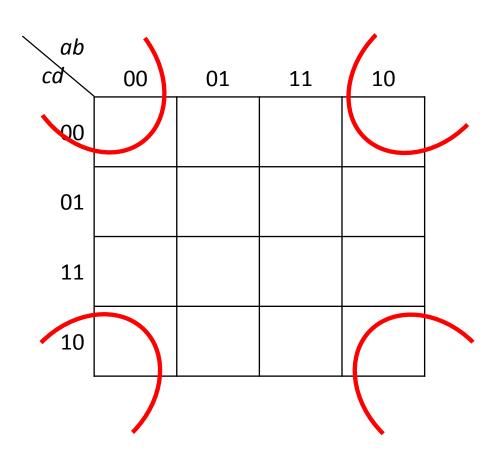
$$f(a,b,c,d) = a\overline{b} \vee c\overline{d} \vee abcd$$

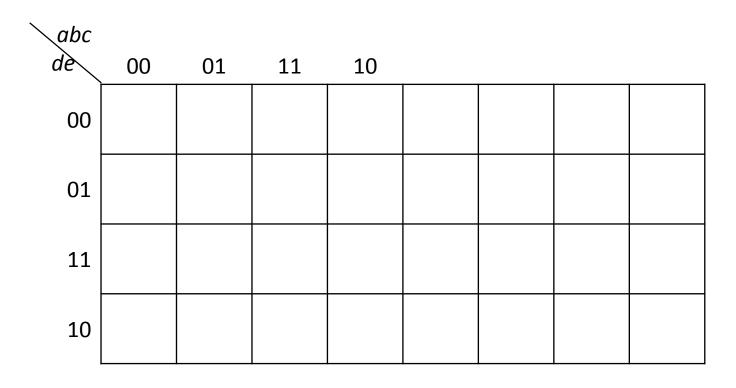


$$f(a,b,c,d) = a\overline{b} \vee c\overline{d} \vee ac$$





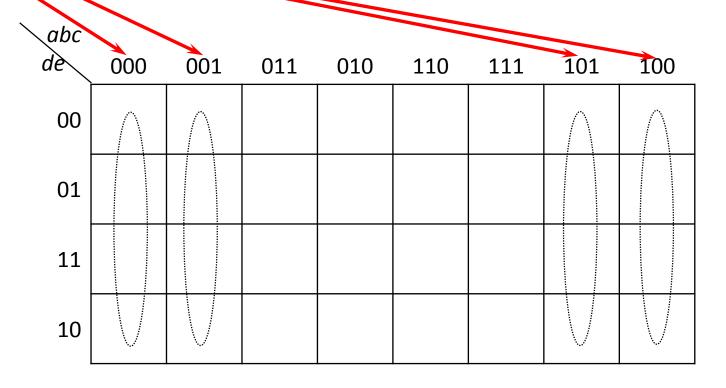


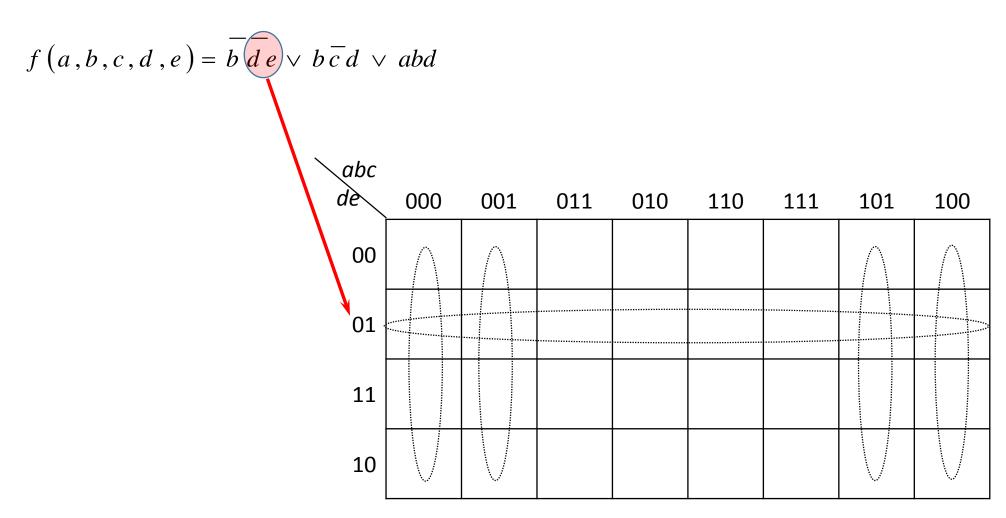


abc de	00	01	11	10	10	11	01	00
00								
01								
11								
10								

abc de								
de	000	001	011	010	110	111	101	100
00								
01								
11								
10	_							

 $f(a,b,c,d,e) = b\overline{d}e \vee b\overline{c}d \vee abd$





abc de	000	001	011	010	110	111	101	100
00								
01	1	1					1	1
11								
10								

$$f(a,b,c,d,e) = \overline{b} \overline{d} e \vee \overline{b} \overline{c} d \vee abd$$

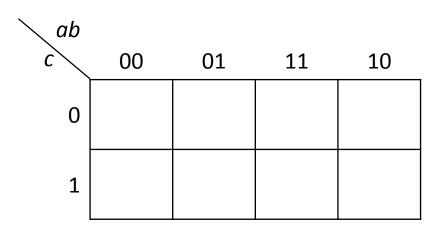
abc de	000	001	011	010	110	111	101	100
00								
01	1	1					1	1
11				1	1			
10				1	1			

$$f(a,b,c,d,e) = \overline{b} \overline{d} e \vee b \overline{c} d \vee abd$$

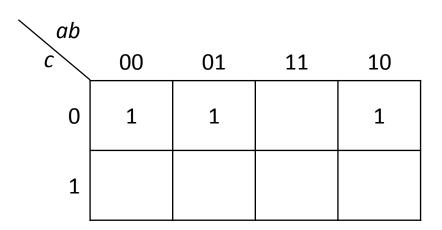
abc de	000	001	011	010	110	111	101	100
00								
01	1	1					1	1
11				1	1	1		
10				1	1	1		

abc de	000	001	011	010	110	111	101	100
00	0	0	0	0	0	0	0	0
01	1	1	0	0	0	0	1	1
11	0	0	0	1	1	1	0	0
10	0	0	0	1	1	1	0	0

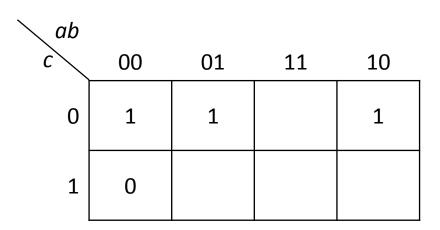
а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
c	0	1	0	1	0	1	0	1
f(a,b,c)	1	0	1		1			



а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
c	0	1	0	1	0	1	0	1
f(a,b,c)	1	0	1		1			



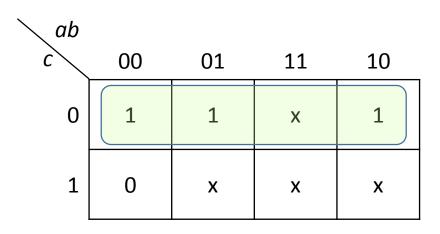
а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
c	0	1	0	1	0	1	0	1
f(a,b,c)	1	0	1		1			



а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
c	0	1	0	1	0	1	0	1
f(a,b,c)	1	0	1		1			

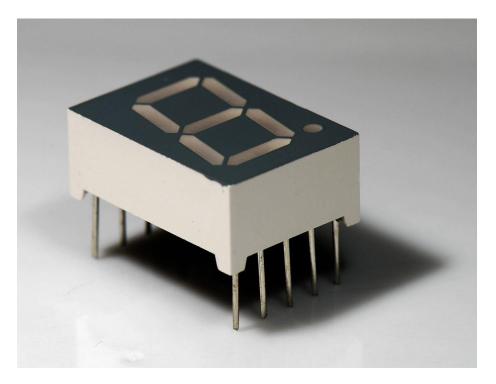
ab c	00	01	11	10
0	1	1	x	1
1	0	х	х	х

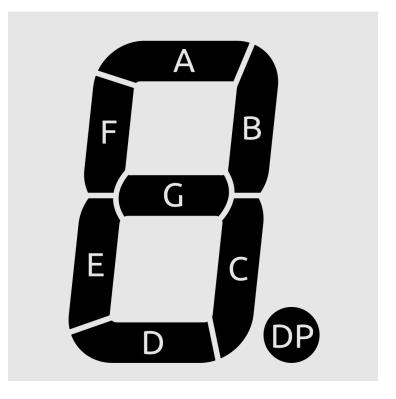
а	0	0	0	0	1	1	1	1
b	0	0	1	1	0	0	1	1
c	0	1	0	1	0	1	0	1
f(a,b,c)	1	0	1		1			



$$f(a,b,c) = \overline{c}$$

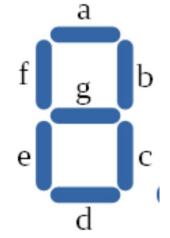
Семисегментный индикатор



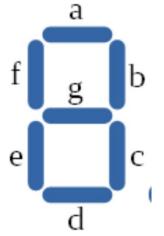




<i>X</i> ₃	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
<i>X</i> ₂	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
X ₁	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x ₀	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1



<i>X</i> ₃	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
<i>X</i> ₂	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
X ₁	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x ₀	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
сегмент а	1	0	1	1	0	1	1	1	1	1	х	х	х	х	х	х



<i>X</i> ₃	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
<i>x</i> ₂	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
<i>X</i> ₁	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
<i>x</i> ₀	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
сегмент а	1	0	1	1	0	1	1	1	1	1	Х	Х	Х	Х	Х	х
сегмент <i>b</i>	1	1	1	1	1	0	0	1	1	1	Х	Х	х	х	х	х
сегмент <i>с</i>	1	1	0	1	1	1	1	1	1	1	Х	Х	Х	Х	Х	Х
сегмент <i>d</i>	1	0	1	1	0	1	1	0	1	1	Х	Х	х	х	х	х
сегмент <i>е</i>	1	0	1	0	0	0	1	0	1	0	Х	Х	х	х	х	х
сегмент <i>f</i>	1	0	0	0	1	1	1	0	1	1	Х	Х	Х	х	Х	х
сегмент <i>g</i>	0	0	1	1	1	1	1	0	1	1	Х	Х	Х	х	Х	х

