

Immune Algorithm

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Outline

- General Immune Algorithm
- Negative Selection
- Clonal Selection Algorithm
- Immune Network Theory
- Artificial Immune Network Model
- Hybrid Immune

Immune Algorithm

General Immune Algorithm

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Biointelligent algorithm, 2018

Biological principles of immune algorithm

The working model of the immune system

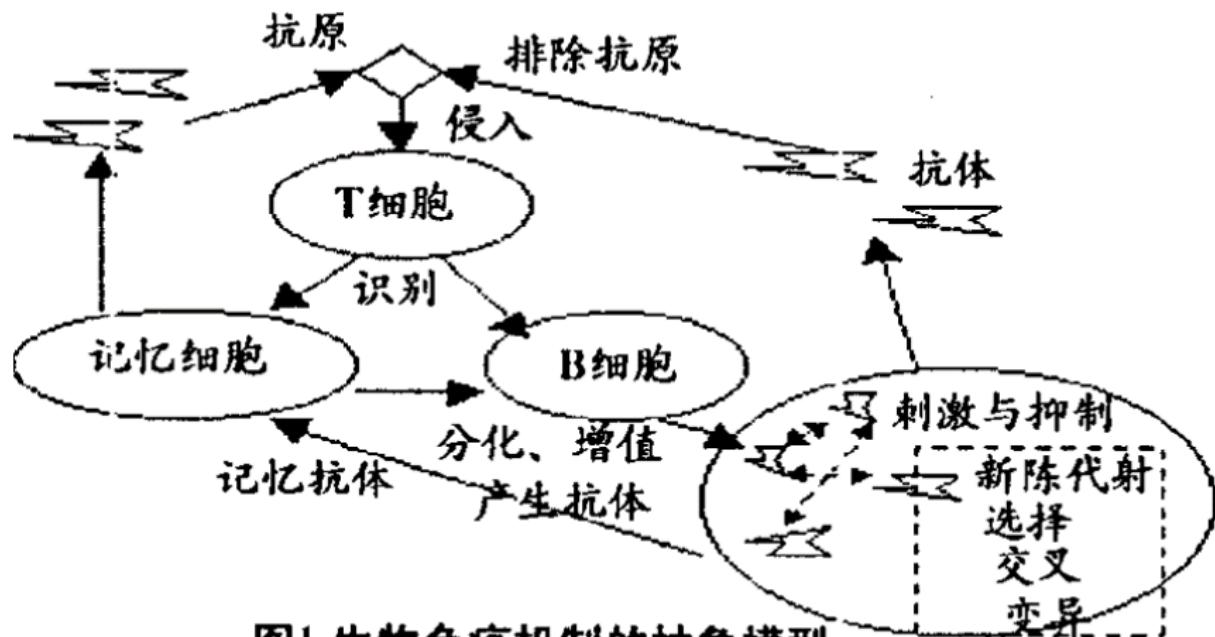


图1 生物免疫机制的抽象模型

Comparison of immune system and immune algorithm

免疫系统	免疫算法
抗原	要解决的问题
抗体	最佳解向量
抗原识别	问题识别
从记忆细胞产生抗体	联想过去的成功
淋巴细胞分化	优良解(记忆)的保持
细胞抑制	剩余候选解的消除
抗体增加(细胞克隆)	利用遗传算子产生新抗体

Comparison of immune system and immune algorithm

Immune Operator:

1. Full Immunity
2. Target Immunity

Immune Operator

Full Immunity

Full immunity refers to the type of immunity that each group of individuals in a group has an immune operation on every link after the action of genetic operators. It is mainly applied to the initial stage of individual evolution, but not in the process of evolution.

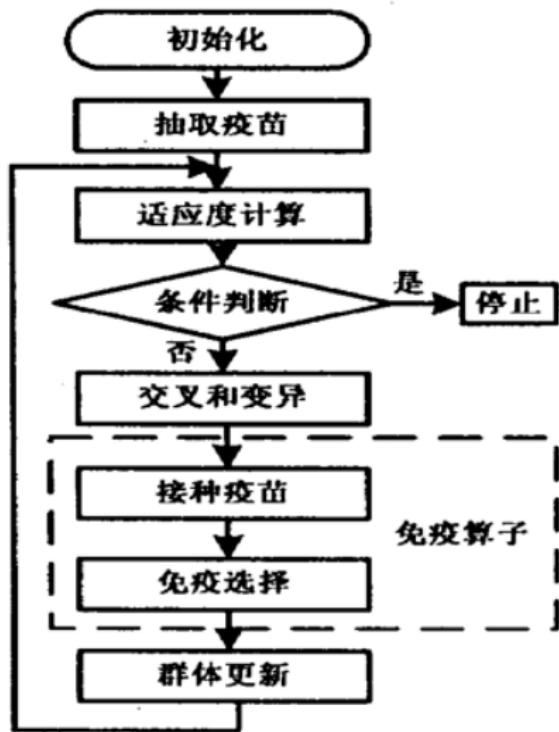
Target Immunity

The target immunity refers to a type of immune response at the point of action only after a genetic operation. The target immunization is generally associated with the whole process of population evolution, and it is a basic operator of the immune operation.

Immune Algorithm

1. The specific analysis of the solved problems (regarded as an antigen)
2. This feature information is processed to transform it into a solution to the problem The set of solutions obtained according to the scheme is called the antibody based on the vaccine above
3. Transform this scheme into an immune operator in an appropriate form

Immune algorithm based on genetic algorithm



Immune Operator

1.Vaccination According to the prior knowledge, some genes in X are modified to make the individual have higher fitness for greater probability.

2.Immune selection

(1)Immunoassay The detection of individuals vaccinated is not as good as the parent, indicating that the phenomenon of severe degeneration has occurred during the process of crossover and mutation, and this individual will be replaced by the corresponding individuals in the parent.

(2)Annealing selection: In the current progeny group $E_k = (x_1, \dots, x_{n_0})$,
the individual X_i is selected by the probability $P(x_i)$ to enter the new parent group:

$$P(x_i) = e^{f(x_i)/T_k} \sum_{i=1}^{n_0} e^{f(x_i)/T_k}$$

$f(x_i)$ is the adaptive degree of individual,

T_k is a temperature control sequence approaching 0

Immune Operator

$a_{H,k}^i$ is the antibody obtained after vaccinating the k generation the i individual a_k^i . P_I is the probability of vaccinating individuals, and P_V is the probability of updating the vaccine. $V(a_k^i, h_j)$ is an inoculation operation modifying the gene on an individual a_k^i according to the pattern h_j , and N and m are the size of the population and the vaccine, respectively.

The execution algorithm of immune operator

Begin:

Extraction of vaccine

Analyze the problem to be asked and collect the characteristic information

Estimation of patterns on specific gene sites based on characteristic information $H=h_j | j = 1, 2, \dots, m$

$k=0$ and $j=0$;

while(Conditions=true)

 if $P_v = true$, then $j = j + 1$;

$i=0$;

 for($i=j$)

 Vaccination $a_{H,k}^j = V_{p_l}(a_k^i, h_j);$

 Immunoassay if

$a_{H,k}^i < a_{k-1}^i$, then $a_k^i = a_{k-1}^i$; else $a_k^i = a_{H,k}^j$;
 $i=i+1$;

 Annealing selection $A_{k+1} = S(A_k);$

$k=k+1$;

Negative Selection Algorithm

Li, Meizhen

Inspiration-Biological immune system

- The antigen-antibody reaction is an exclusive process.
- The primary role of immune system is to distinguish self from non-self.
- Immune cells are tolerant to self antigens but activate defense mechanisms when recognize non-self antigens.
- The negative selection of T cells happened in the thymus during maturation.

Biological immune system

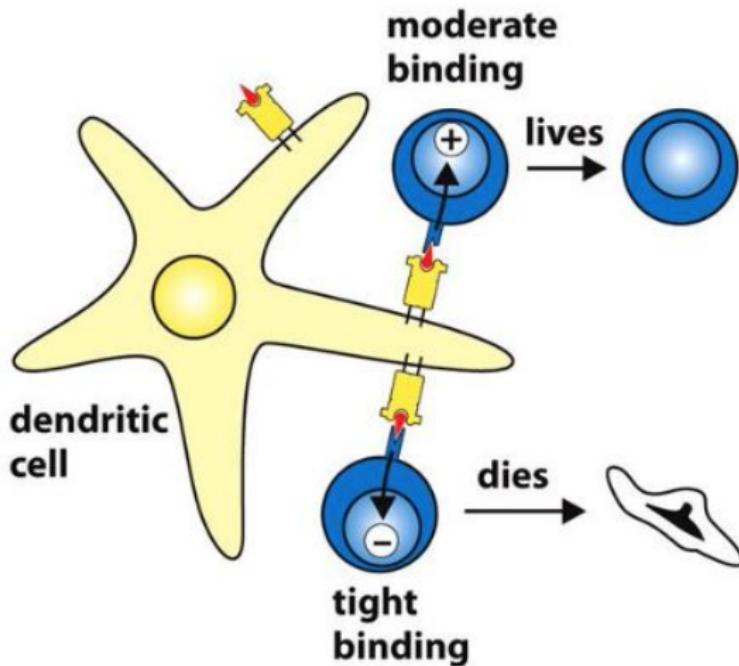


Figure: Negative selection of T cells

Biological immune system

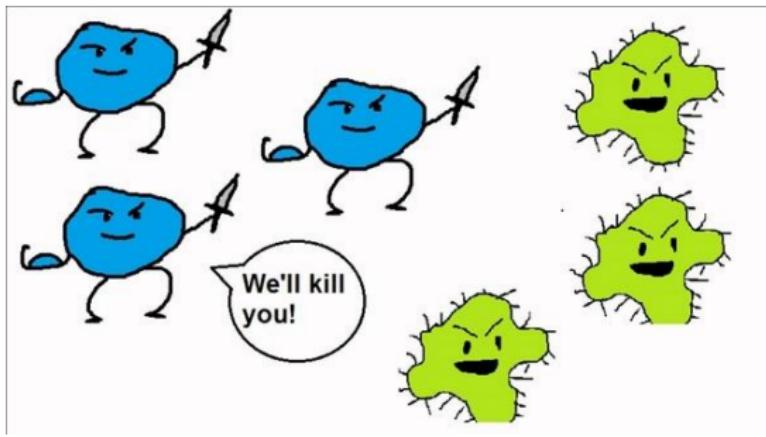


Figure: Antigen-antibody interaction

Defining the NSA

- Define Self as a normal pattern of activity or stable behavior of a system/process.
 - Represent the collection as multiset of S of strings of length l over a finite alphabet.

Defining the NSA

- Define Self as a normal pattern of activity or stable behavior of a system/process.
 - Represent the collection as multiset of S of strings of length l over a finite alphabet.
- Generate a set R of detectors, each of which fails to match any string in S .

Flowchart

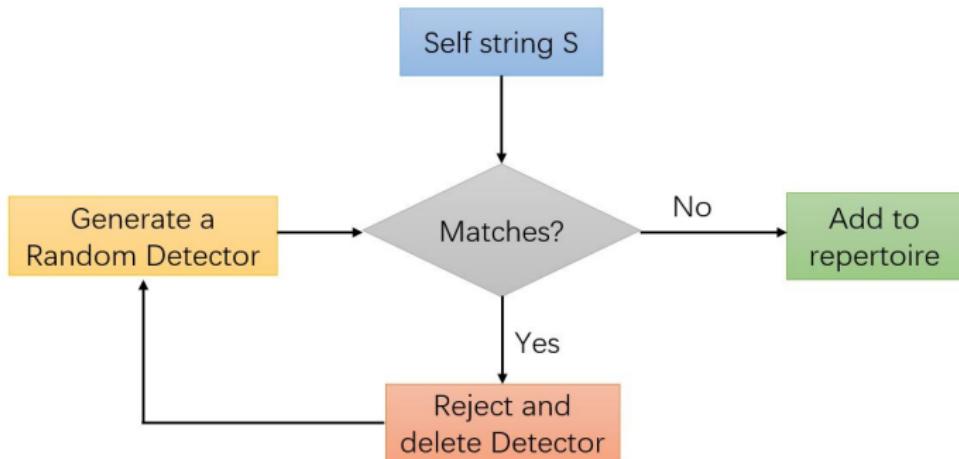


Figure: Generation of effective detector

Pseudocode

Algorithm 1 Detector Generation

Input:SelfData

Output:Repertoire

Repertoire \leftarrow emptyset

while !StopCondition() **do**

 Detectors \leftarrow GenerateRandomDetectors()

for Detector[i] \in Repertoire **do**

if not Matches(Detector[i],SelfData) **then**

 Repertoire \leftarrow Detector[i]

end if

end for

end while

Return (Repertoire)

Defining the NSA

- Monitor new observations for changes by continually testing the detectors matching against representatives of S . If any detector ever matches, a change must have occurred in system behavior.

Flowchart

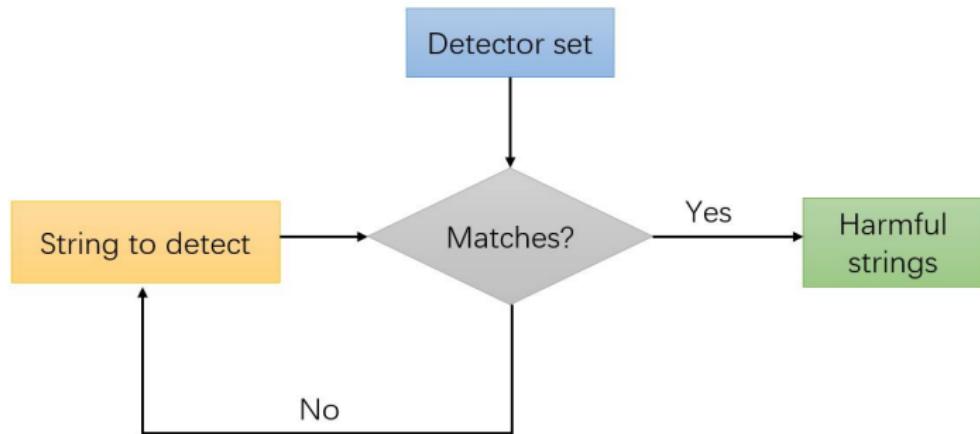


Figure: Detector application

Algorithm 2 Detector Monitoring

```
Input:InputSamples,Repertoire
for Input[i] ∈ InputSamples do
    Inputiclass ← nonself
    for Detector[j] ∈ Repertoire do
        if Matches(Detector[j],SelfData) then
            Inputiclass ← self
        end if
    end for
end for
```

Mathematical description

Algorithm 3 Define parameters

N_{R_0} :number of initial detectors

N_R :number of mature detectors

N_S :number of self elements

P_M :random match probability at specific condition

P_f :failed match probability

f:the possibility of a random string not matching Selfset

Mathematical description

$$P_{Detectors} : f = (1 - P_M)^{N_s} \quad (1)$$

$$FailedMatch : P_f = (1 - P_M)^{N_R} \quad (2)$$

$$Detector : N_R = \frac{\ln P_f}{\ln(1 - P_M)} \quad (3)$$

$$Iterations : D = \frac{\ln P_f}{(1 - P_M)^{N_s} \ln(1 - P_M)} \quad (4)$$

Optimization-detector generation

- Linear time detector generating algorithm

$$TimeComplexity = O((I - r) \times N_S) + O((I - r) \times 2^r) + O(I \times N_R) \quad (5)$$

- Greedy detector generating algorithm

$$TimeComplexity = O((I - r) \times N_S) + O((I - r) \times 2^r \times N_R) + O(I \times N_R) \quad (6)$$

Optimization-detection time complexity

- Detection time complexity has linear corelation with N_R

$$Detector : N_R = \frac{\ln P_f}{\ln(1 - P_M)} \quad (7)$$

$$P_M \text{continuous} \approx \frac{1}{2^r} \left[\frac{l-r}{2} + 1 \right] \quad (8)$$

$$P_M \text{hamming} = \frac{1}{2^r} \sum_{i=r}^n C_i^j \quad (9)$$

Optimization-detection time complexity

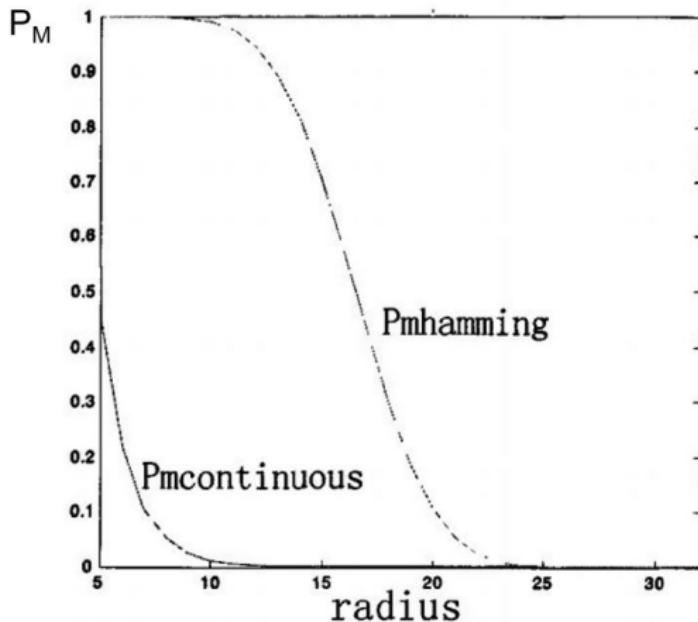


Figure: With a specified r , $P_{M\text{continuous}} < P_{M\text{hamming}}$, $N_{R\text{continuous}} > N_{R\text{hamming}}$

Define black hole

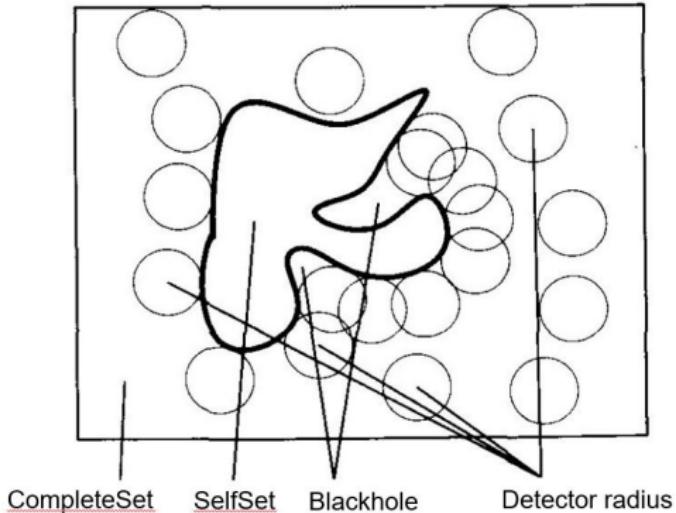


Figure: Definition of black hole

Reduce black hole

Algorithm 4 r-adjustable negative selection algorithm

Step1: Define SelfSet S (length=l) r_1, r_2, \dots, r_c
Step2: $a \leftarrow \text{GenerateRandomString} (\text{length}=l)$ $r=r_1$
Step3: Matches ($a, S[i]$)
if not Matches($a, S[i]$) **then**
 go to step2
else if $r > r_c$ **then**
 go to step2
else
 $r=r_{++}$
 go to step3
end if

Reduce black hole

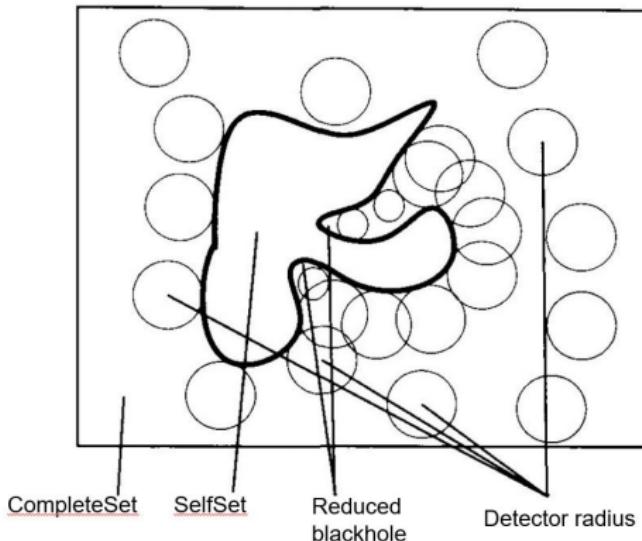


Figure: the number of blackhole and iteration to generate mature detector have reduced

Thanks

Li, Meizhen

Clonal Selection Algorithm

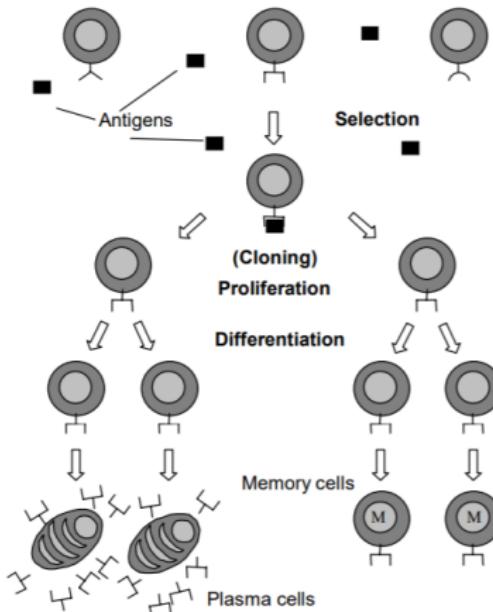
Cao,Jing

The Clonal Selection Theory

- 1. B cell products antibodies(Ab) when an animal is exposed to an antigen;
- 2. Each B cell secrets only one kind of antibody, which is relatively specific for antigen;
- 3. The antigen stimulates the B cell to proliferate(divide) and mature into terminal (non-dividing) antibody secreting cells, called plasma cells;

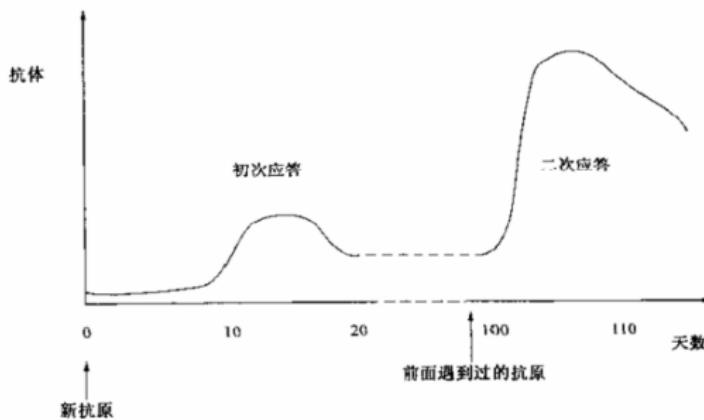
The Clonal Selection Theory

- 4. Lymphocytes, in addition to proliferating and/or differentiating into plasma cells, can differentiate into long-lived B memory cells;
- 5. When exposed to the same antigen again, the B memory cells can differentiate into large lymphocytes quickly to produce high affinity antibodies;



Reinforcement Learning and Memory

- 1. The effectiveness of the immune response to secondary encounters is considerably enhanced by storing some high affinity antibody producing cells from the first infection;
- 2. Such a strategy ensures that both the speed and accuracy of the immune response becomes successively greater after each infection;
- This scheme is intrinsic of a reinforcement learning strategy

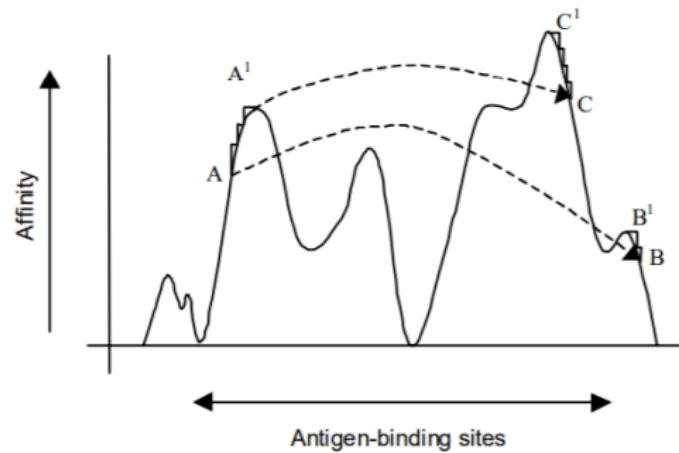


Reinforcement Learning and Memory

- One important characteristic of the immune memory is that it is associative: B cells adapted to a certain type of antigen A1 presents a faster and more efficient secondary response not only to A1, **but also to any structurally related antigen A2. This phenomenon is called immunological cross-reaction, or cross-reactive response.**
- Antibodies present in a memory response have, **on average, a higher affinity than those of the early primary response.** This phenomenon, is referred to as the maturation of the immune response

Somatic Hypermutation, Receptor Editing and Repertoire Diversity

- 1. Point mutations allow the immune system to explore local areas around A by making **small steps** towards an antibody with higher affinity;
- 2. Receptor editing allows an antibody to take **large steps** through the landscape, landing in a locale where the affinity might be lower;



The Regulation of the Hypermutation Mechanism

- 1. The majority of the mutations will lead to poorer or non-functional antibodies;
- 2. If a cell mutate at a fixed rate, the accumulation of deleterious changes may cause the loss of the advantageous mutation;
- **So there should be a selection mechanism**

The Regulation of the Hypermutation Mechanism

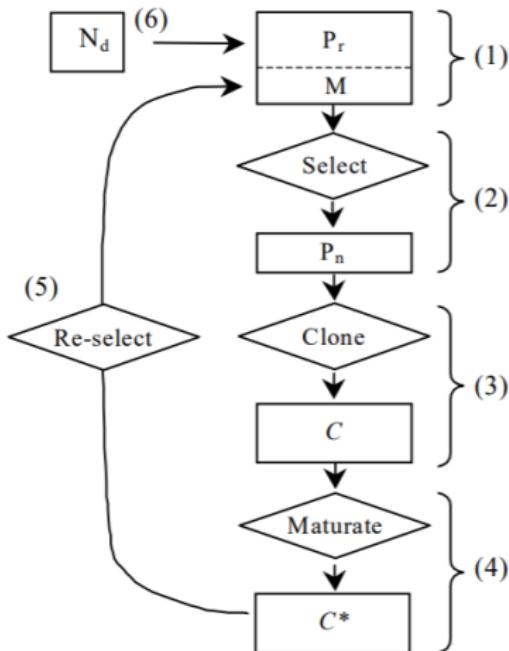
- Cells with low affinity receptors may be further mutated and, as a rule, **die** if they do not become higher affinity cells. In cells with high-affinity antibody receptors however, hypermutation may be **inactivated**.

The Shape-Space Model

- 1. The shape-space model (S) aims at quantitatively describing the interactions among antigens and antibodies (be Ag-Ab);
- 2. The set of features that characterize a molecule is called its **generalized shape**;
- 3. Mathematically, the generalized shape of a molecule (m), either an antibody or an antigen, can be represented by a set of coordinates $m = \langle m_1, m_2, \dots, m_L \rangle$, which can be regarded as a point in an L -dimensional real-valued shape-space ($m \in S^L$);

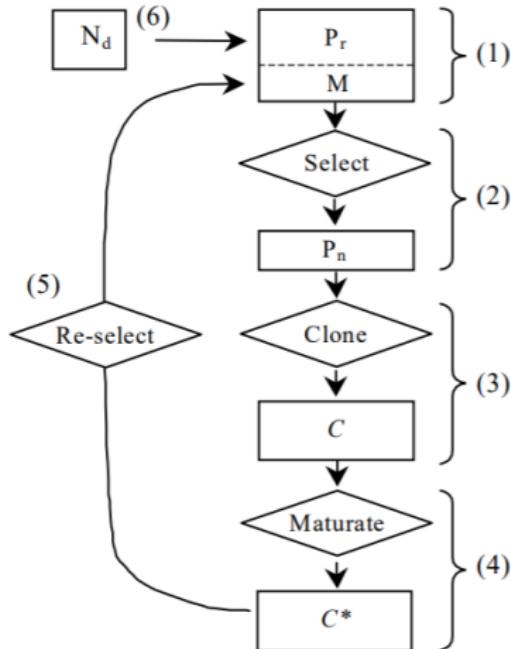
The Proposed Algorithm

- After each six steps we have one cell generation
- (1) Generate a set of (P) of candidate solutions, composed of the subset of memory cells (M) added to the remaining (P_r) population ($P = P_r + M$);
- (2) Determine (Select) the n best individuals of population (P_n), based on affinity measure;



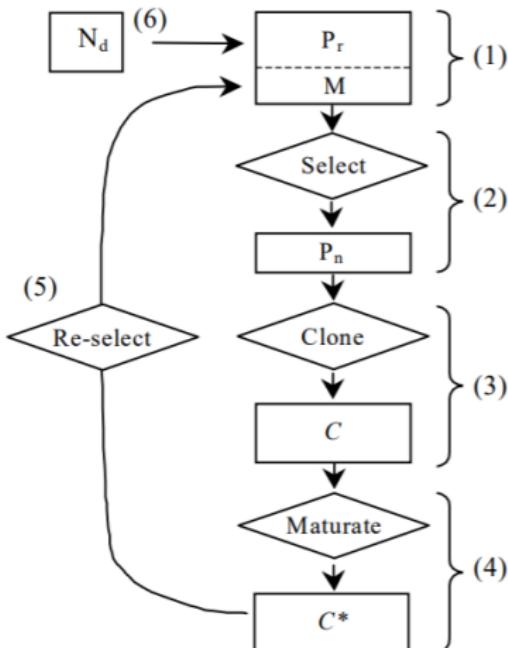
The Proposed Algorithm

- (3) Reproduce (Clone) these n best individuals of the population, giving rise to a temporary population of clones (C). The clone size is an increasing function of the affinity with the antigen;
- (4) Submit the population of clones to a hypermutation scheme, where the hypermutation is proportional to the affinity of the antibody with the antigen. A matured antibody population is generated (C^*);



The Proposed Algorithm

- (5) Re-select the improved individuals from C^* to compose the memory set M . Some members of P can be replaced by other improved members of C^* ;
- (6) Replace d antibodies by novel ones (diversity introduction). The lower affinity cells have higher probabilities of being replaced;



Engineering Applications

Binary Character Recognition

- The goal is to demonstrate that a cumulative blind variation together with selection can **produce individuals with increasing affinities** (maturation of the immune response);
- In this case, we assume that the antigen population is represented by a set of eight binary characters to be learned. ;
- Each character is represented by a bitstring of length $L = 120$;



Engineering Applications

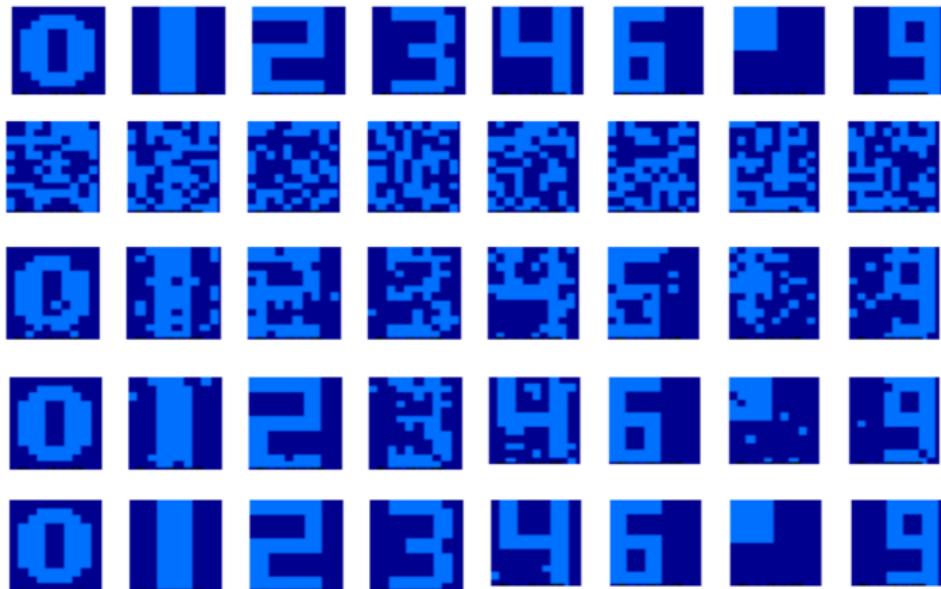
Binary Character Recognition

- The affinity measure takes into account the Hamming distance (D) between antigens and antibodies, according to Equation (1):

$$D = \sum_{i=1}^L \delta \quad \text{where } \delta = \begin{cases} 1 & \text{if } ab_i \neq ag_i \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Engineering Applications

Binary Character Recognition



(a) Input patterns

(b) 0 generations

(c) 50 generations

(d) 100 generations

(e) 200 generations

Engineering Applications

Multi-Modal Optimization

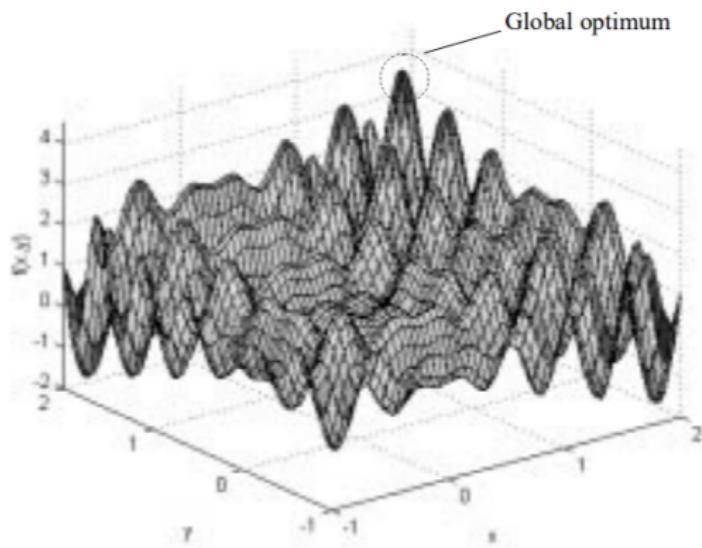
- The CSA reproduces those individuals with higher affinities and selects their improved matured progenies;
- This strategy suggests that the algorithm performs a greedy search, where single members will be locally optimized (exploitation of the surrounding space), and the newcomers yield a broader exploration of the searchspace;
- This characteristic makes the CSA **very suitable for solving multi-modal optimization tasks**

Engineering Applications

Multi-Modal Optimization

- Consider the maximizing the function:

$$f(x, y) = x \cdot \sin(4\pi x) - y \cdot \sin(4\pi y + \pi) + 1 \quad (11)$$



Engineering Applications

Multi-Modal Optimization

- We employed the Hamming shape-space, with binary strings representing real values for x and y ;
- The chosen bitstring length was $L = 22$, corresponding to a precision of six decimal places
- The variables x and y are defined over the range $[-1, 2]$, and the mapping from a binary string $m = \langle m_L, \dots, m_2, m_1 \rangle$ into a real number z is completed in two steps:
 - convert the binary string $m = \langle m_L, \dots, m_2, m_1 \rangle$ from base 2 to base 10:

$$(\langle m_L, \dots, m_2, m_1 \rangle)_2 = \left(\sum_{i=0}^{21} m_i \cdot 2^i \right)_{10} = z' \quad (12)$$

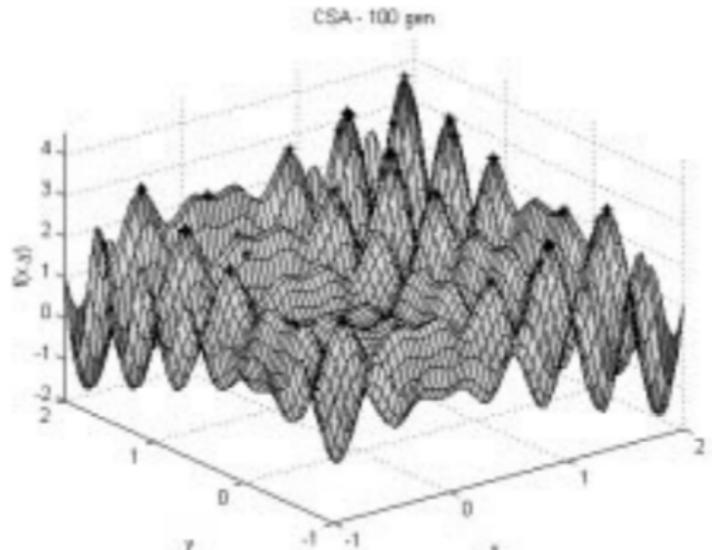
- find the corresponding real value for z :

$$z = z_{\min} + z' \cdot \frac{z_{\max} - z_{\min}}{2^{22} - 1}, \text{ where } z_{\max} = 2 \text{ and } z_{\min} = -1 \quad (13)$$

Engineering Applications

Multi-Modal Optimization

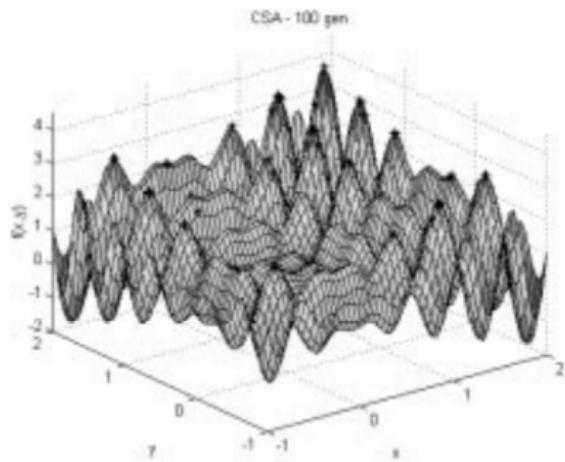
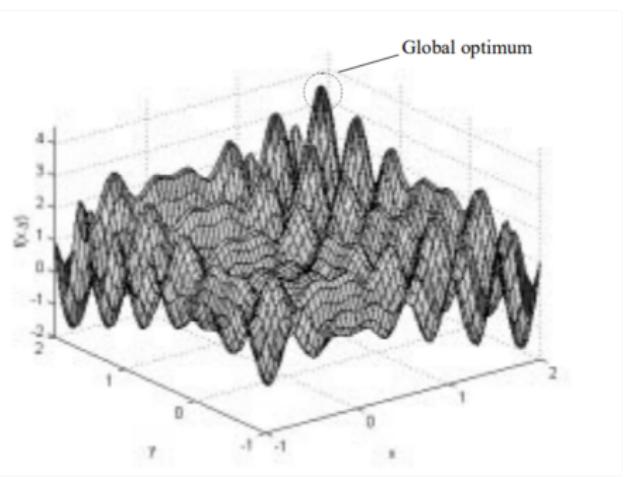
- The affinity measure corresponds to the evaluation of the function $f(x, y)$ after decoding x and y , as described above;
- The figure below present the optimized population after 100 generations:



Engineering Applications

Multi-Modal Optimization

- Notice that the solutions (stars) covers most of the peaks, including the global optimum.



Conclusion

- The algorithm was verified to be capable of performing learning and maintenance of high quality memory and, it was also capable of solving complex problems, like multi-modal and combinatorial optimization.

Immune Network Theory

Shi, Haixin

Immune Network Theory

Jerne's idiotypic network hypothesis

- Immunologists in the early 1970's were in the process of discovering a wealth of information about how the immune system worked.
- Jerne's network hypothesis was a radical innovation, which states that the regulation of the adaptive immune system involves interactions between V regions.

Immune Network Theory

Jerne's idiotypic network hypothesis

- Jerne introduced three terms:
 - **epitope** (antigenic determinant): the part of an antigen that is recognized by the immune system.
 - **idiotope**: the unique set of antigenic determinants (epitopes) of the variable portion of an antibody.
 - **paratope** (antigen-binding site): is a part of an antibody which recognizes and binds to an antigen.

Immune Network Theory

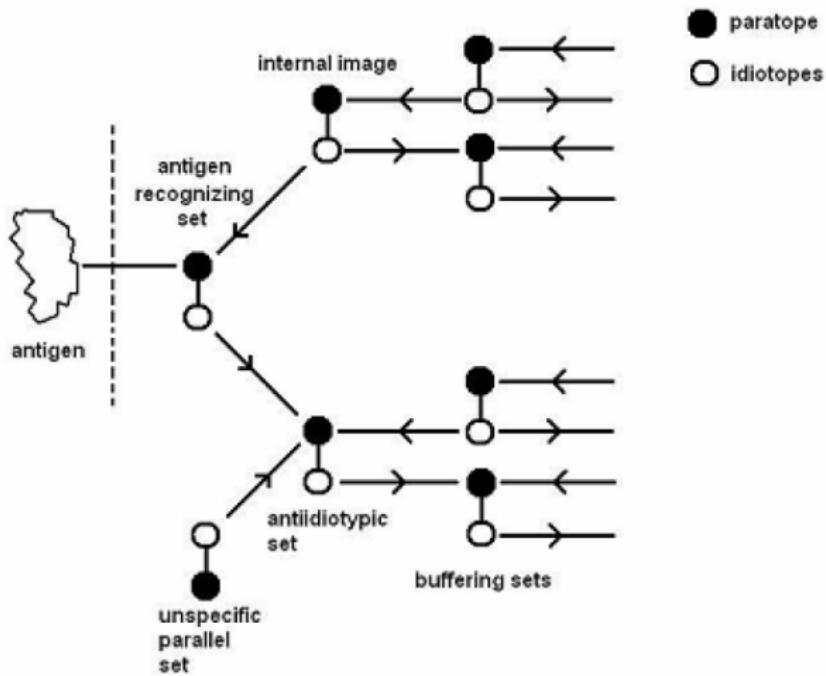
Dualisms

- two main kinds of cells:
 - T cells
 - B cells
- two main kinds of interactions:
 - stimulation
 - suppression

Immune Network Theory

Jerne's model of the network

- The network was proposed by Jerne in 1973.



Immune Network Theory

The first mathematical model

- The formula describes the dynamics of a typical clone consisting of L cells (lymphocytes):

$$\frac{dL}{dT} = \alpha - \beta L + \sum_{i=1}^N \varphi(E_i, K_i, t) - L \sum_{j=1}^n \psi(I_j, K_j, t) \quad (14)$$

Immune Network Theory

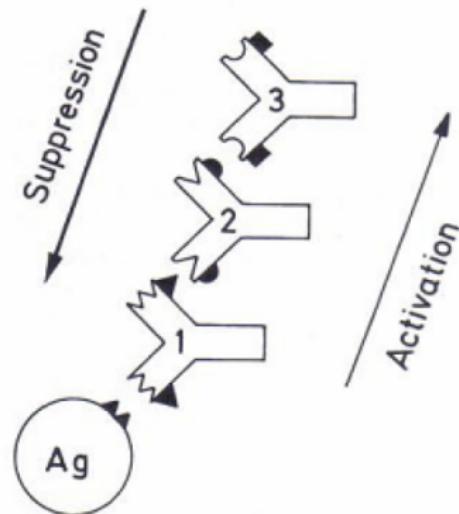
Limitations of the Jerne model

- While Jerne's model was a huge conceptual advance, he candidly recognized it's limitations:
 - (a)Simplicity
 - (b)Scope
 - (c)Predictions
 - (d)Resolution of Paradoxes
 - (e)Mechanistic basis
 - (f)Rigour
 - (g)Robustness
 - (h)Aesthetics

Immune Network Theory

The Richter theory

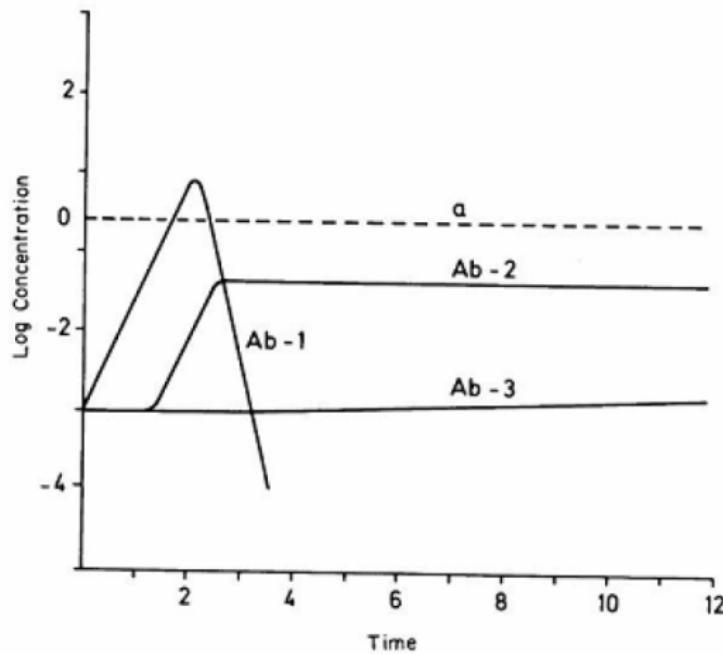
- The network was thus simplified from Jerne's two-dimensional network to a one dimensional chain.



Immune Network Theory

Modes of response

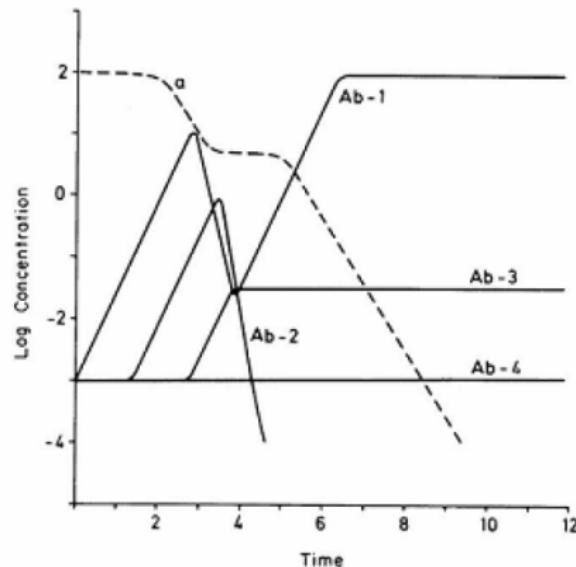
- Low dose tolerance in the Richter model.



Immune Network Theory

Modes of response

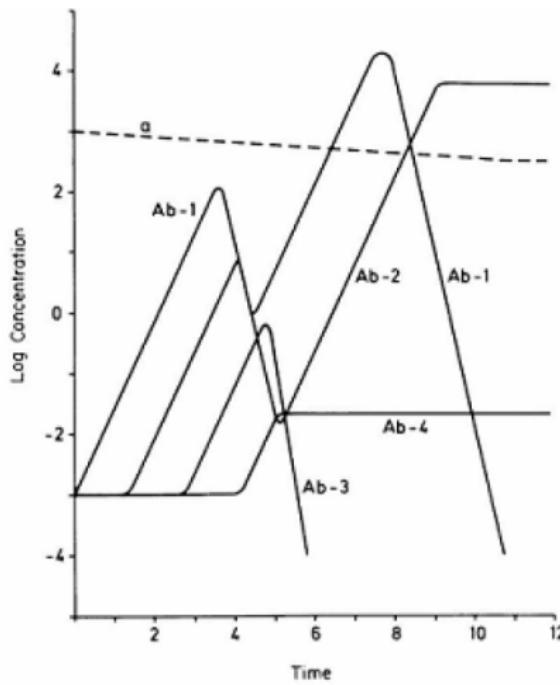
- The immune response in the Richter model.



Immune Network Theory

Modes of response

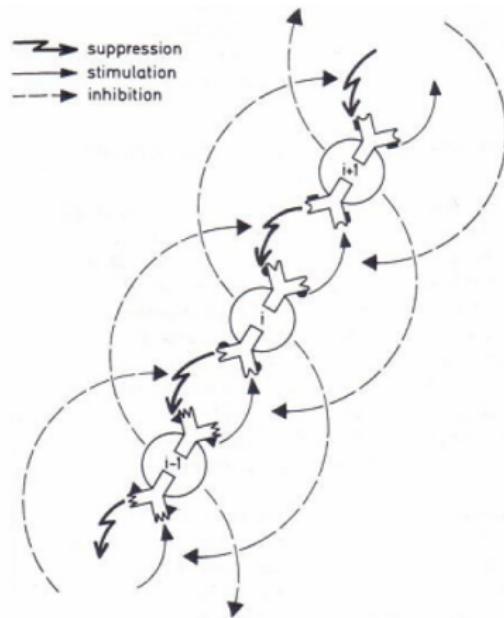
- High dose tolerance in the Richter model.



Immune Network Theory

Modes of response

- The improved network model which adds inhibitory interactions.



Immune Network Theory

Richter's mathematical model

- Richter translated the above ideas into differential equations, and pictures above can be obtained by integrating his equations:

$$\frac{dS_i}{dt} = \frac{1}{\tau_b} f(S_{i-1}, S_i, S_{i+1}) S_i - \frac{1}{\tau_d} g(S_{i-1}, S_i, S_{i+1}) S_i \quad (15)$$

Immune Network Theory

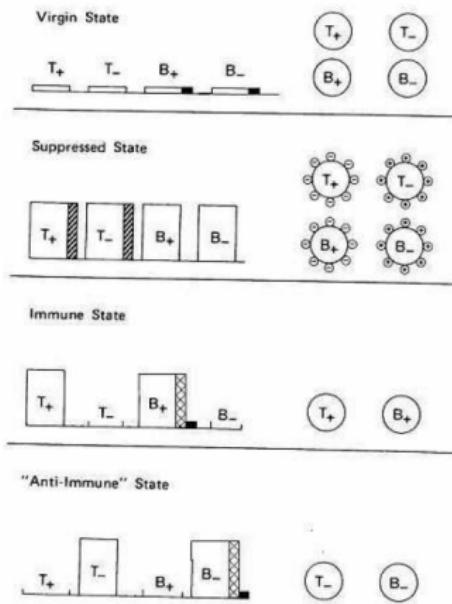
Achievements of the Richter theory

- It showed that Jerne's network concept could be reduced to manageable proportions.
- The Richter theory showed that there are three basic types of specific interactions which are important for such models - stimulation, inhibition (blocking) and elimination (killing).
- It illustrated a potential importance of thresholds in stabilizing the immune system.

Immune Network Theory

The symmetrical network theory

- The symmetrical network theory incorporates symmetric interactions between idiotypes and antiidiotypes.



Immune Network Theory

The environmental detection algorithm

- The movement direction of robot:

1	2	3
8		4
7	6	5

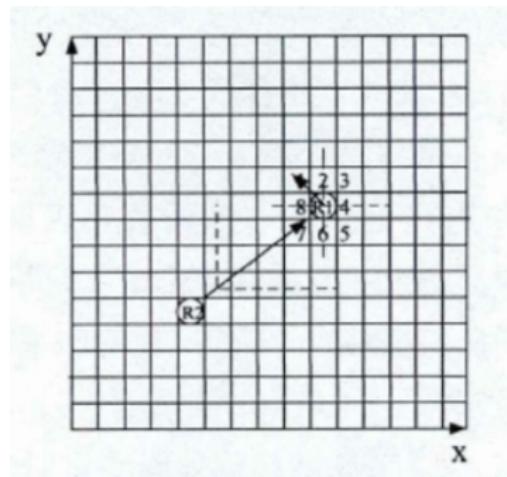
- The antigen information of robot:

$$g_i = \frac{1}{(k_1 \cdot expense + k_2 \cdot occupy + k_3 \cdot gain + 1)} \quad (16)$$

Immune Network Theory

The environmental detection algorithm

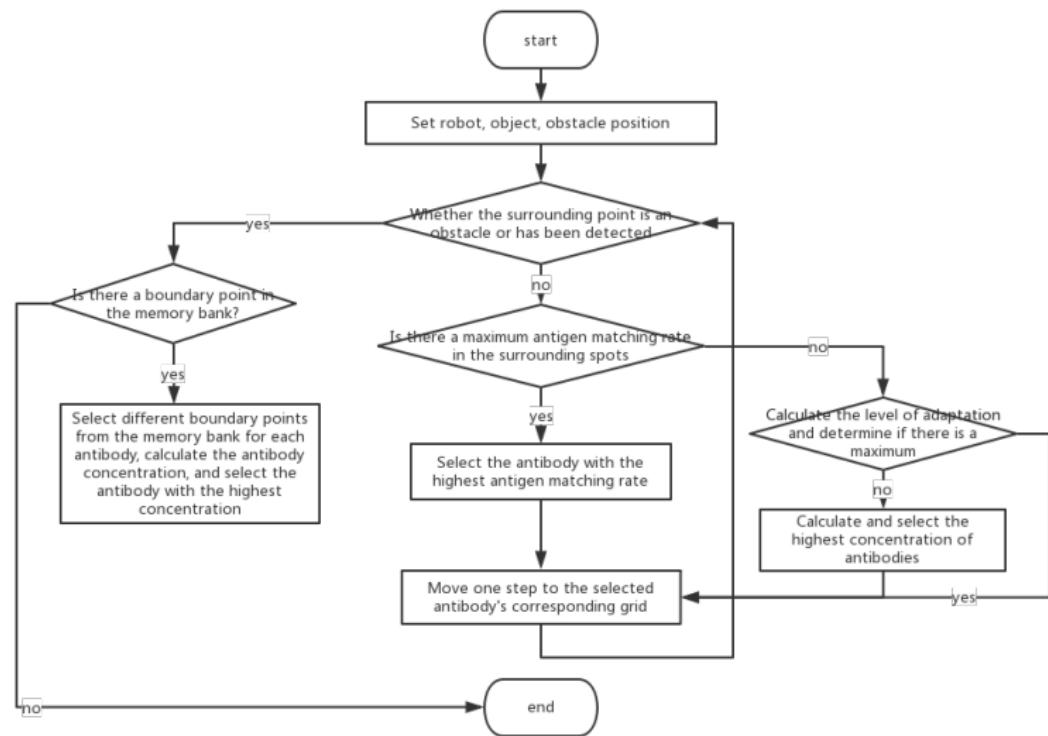
- The interaction between robotic antibodies:



$$\overline{R_2 R_1} = \overline{(x_2 - x_1)(y_2 - y_1)} \quad (17)$$

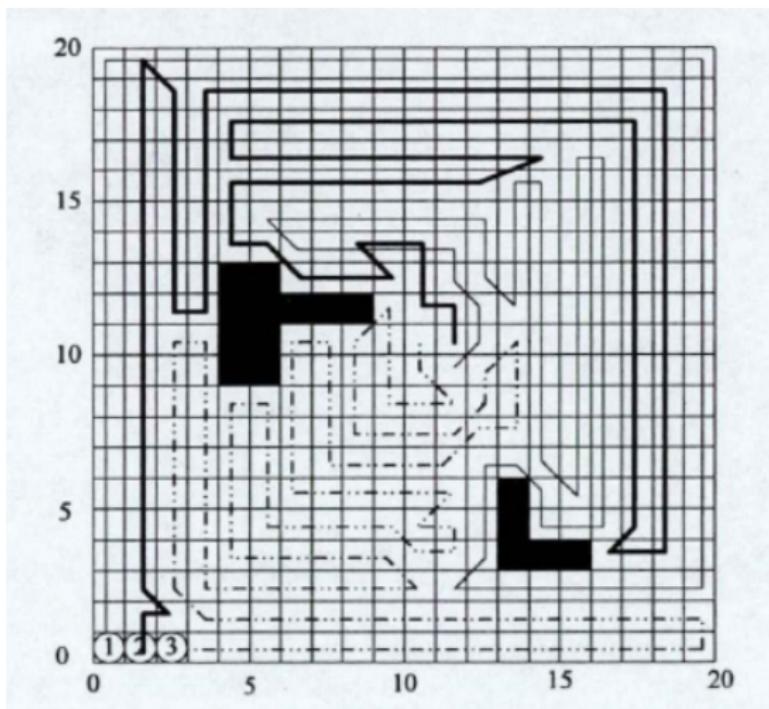
Immune Network Theory

The environmental detection algorithm



Immune Network Theory

The environmental detection algorithm



Artificial Immune Network(AIN)

Zhong,Wenfeng

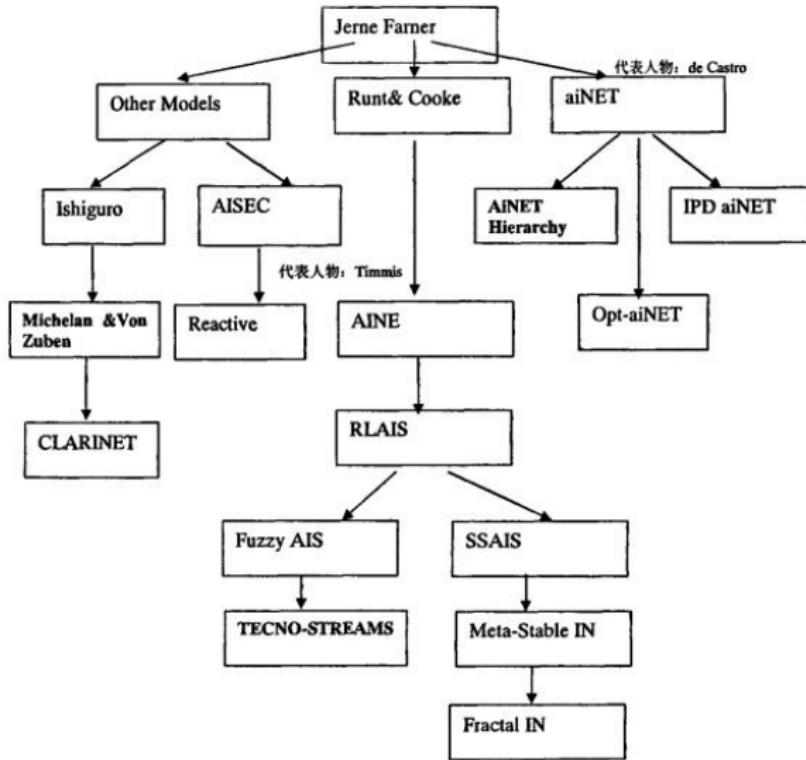
What is AIN?

- **The artificial immune network model(AIN)** regards artificial immune system(AIS) as a network structure composed of nodes (lymphocytes). Through the information transfer and interaction between nodes, the immune system functions such as recognition, response, and memory are achieved.
- **applied to** data mining , time series prediction, pattern recognition, optimization, fault detection ...

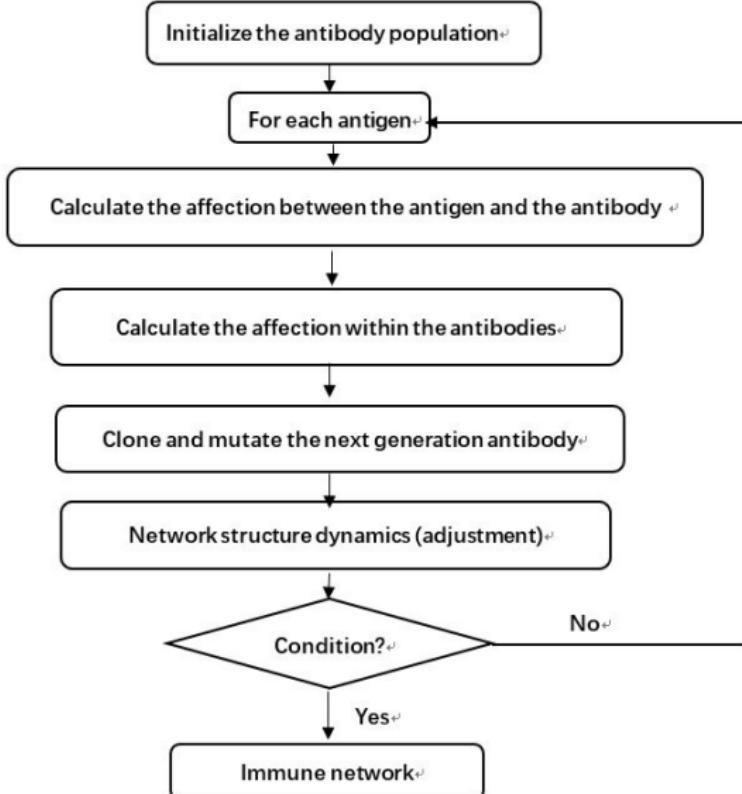
Features

- Realize and express the results of data information processing in the form of a network
- Deal with large amounts of data in decentralized organizations;
- Achieve sustainable learning in the form of a dynamic network;
- Threshold-based information processing;
- Quickly process data information in a parallel, distributed manner.

Relationship of AIN Models

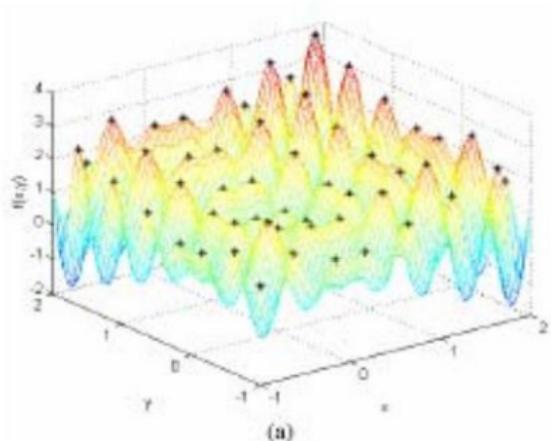


Basic framework of AIN

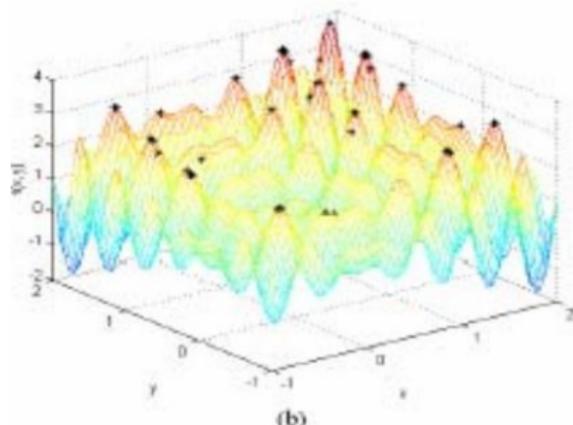


Applications of AIN

multimodal function optimization



(a)



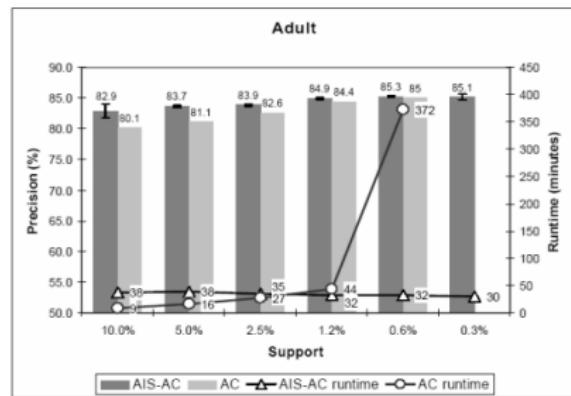
(b)

Applications of AIN

associative classification

Support threshold	AIS-AC		AC	
	No. of rules	No. of frequent sets	No. of rules	No. of rules
10.0%	138	10794	174	
5.0%	227	34344	575	
2.5%	202	102722	1333	
1.2%	290	280452	2689	
0.6%	384	663276	4690	
0.3%	327	1462940	7737	

Numbers of rules and frequent item sets for Adult data set



Associative classification performance of Adult data set

Thank You!

Hybrid Immune Algorithm

Han,Shiqi

Why Hybrid?

- **No Free Lunch Theorem**

— David Wolpert & William Macready

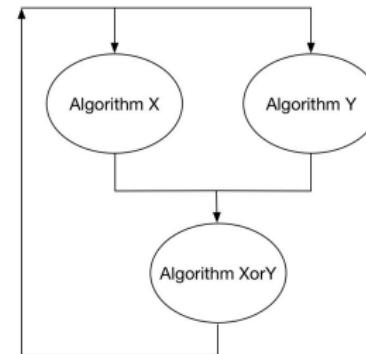
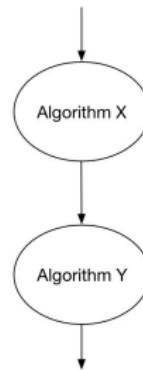
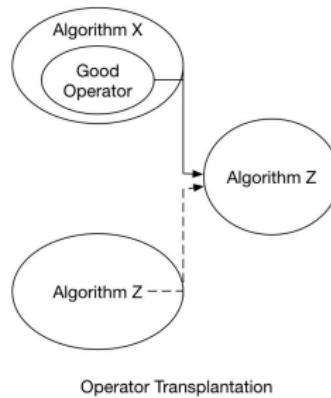
- **Hybridize Where Possible**

— L. D. Davis

In the limited search space, for the iterative optimization algorithm, there is no algorithm that is best for all problems. Different optimization algorithms have different application advantages and disadvantages, and there is a complementarity between the algorithms.

Hybrid Ways

- Operator Transplantation
- Operator Series
- Operator Competition



Operator Transplantation

Operator Series

Operator Competition

Hybrid Immune Algorithm

- Immune Genetic Algorithm
 - Immune Particle Swarm Optimization
 - Synergetic Evolutionary Immune Algorithm
 - Immune Ant colony algorithm
 - Quantum Immune Algorithm
 - Chaotic Immune Algorithm
 - Fuzzy Immune System
 - Immune Neural Network Algorithm
-

Ant Colony Algorithm

Ants leave a **pheromone** in the path that it travels when foraging, and can feel the intensity of it during the movement and tend to move in the direction of **high concentration**, so the group behavior will show **positive feedback** with pheromone information.

Immune Ant Colony Algorithm

Problems and Solving

Ant Colony Weakness

Lack of information at first, convergence slows

Easy to fall into local extreme point and premature convergence occurs

Immune Algorithms Advantages

Clone mutation mechanism, use affinity describe the matching between antibody and antigen, fast global search capabilities

Concentration inhibition mechanism, influencing road path selection probability to maintain the ant colony diversity

Traveling Salesman Problem

TSP is a **shortest path problem**. For n cities, choose any cities as the starting point and traverse all the cities back to the starting point. When the sum of the paths is the **shortest**, the path is optimal.

$$T_d = \sum_{i=1}^{n-1} d(v_i, v_{i+1}) + d(v_1, v_n) \quad (18)$$

Immune Ant Colony Algorithm

Term Definition

Term Definition

- **Antigen**
- **Antibody**

$$D = \{\langle s, T_d \rangle \mid s \in P, T_d \in N\} \quad (19)$$

- **Affinity**

$$f_{dist}(A_b, A_g) = 1 / (A_b.dist - A_g.dist) \quad (20)$$

$$T = \left(\sum_{i=1}^n \sum_{j=1}^n d(i,j) \right) / (2n) \quad (21)$$

$$f_{dist}(A_b, A_g) = 1 / (A_b.dist - T) \quad (22)$$

Immune Ant Colony Algorithm

Term Definition

Term Definition

- **Memory Cells**

$$M = \{x | f_{dist}(A_b, A_g) \geq \theta, x \in D\} \quad (23)$$

- **Path Transition Probability**

$$p_{ij}^k = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha \cdot [\eta_{ij}]^\beta}{\sum_{s \in allowed_k} [\tau_{is}(t)]^\alpha \cdot [\eta_{is}(t)]^\beta} & j \in allowed_k \\ 0 & else \end{cases} \quad (24)$$

Immune Ant Colony Algorithm

Algorithms Step

Step1 Calculate a feasible solution using immune algorithm

- Enter question and determine the antibody encoding
- Calculate antibody affinity f_{dist} , Memory cell $M(f_{dist} \geq \theta)$
- Antibody clone variation $A_{bj} \rightarrow C_j \rightarrow C_j^*$

$$i = INT[num \times \beta \div con] \quad (25)$$

$$con_i = \frac{M_{bi}}{M_b} \quad (26)$$

- Update group $f_{dist}(A_{bj}^*, A_{gj}) > f_{dist}(A_{bj}, A_{gj})?$
- Terminal condition

Immune Ant Colony Algorithm

Algorithms Step

Step2 Initialize the pheromone based on feasible solution

$$\tau_{ij}(0) = \begin{cases} \tau_C + \tau_G & e_{ij} \in L^* \\ \tau_C & \text{else} \end{cases} \quad (27)$$

$$\tau_G = \frac{Q}{L^*} \quad (28)$$

τ : pheromone amount

Q : a cycle pheromone amount

L^* : optimal path length

Immune Ant Colony Algorithm

Algorithms Step

Step3 Construct probability formula using immune inhibition operators

Path Concentration C_{ij} : the amount of ants that choose path e_{ij}

$$C_{ij}(t) = \frac{1}{M} \sum_{k=1}^M iif(e_{ij} \in \text{Tour}^k(t), 1, 0) \quad (29)$$

$$C_{ij}^k(t) = \frac{1}{M} \sum_{k=1}^{k-1} iif(e_{ij} \in \text{Tour}^k(t), 1, 0) \quad (30)$$

Immune Ant Colony Algorithm

Algorithms Step

Step3 Construct probability formula using immune inhibition operators

Update pheromone amount τ_{ij} considering path concentration c_{ij}

$$\tau_{ij}^k(t) = \begin{cases} \tau_{ij} \cdot (\lambda \cdot c_{ij}^k(t)) & \text{if } c_{ij}^k(t) > c_0 \\ \tau_{ij} & \text{else} \end{cases} \quad (31)$$

$$p_{ij}^k = \begin{cases} \frac{[\tau_{ij}^k(t)]^\alpha \cdot [\eta_{ij}]^\beta}{\sum_{\mu \in N_i^k(t)} [\tau_{i\mu}^k(t)]^\alpha \cdot [\eta_{i\mu}(t)]^\beta} & \text{if } j \in N_i^k(t) \\ 0 & \text{else} \end{cases} \quad (32)$$

Immune Ant Colony Algorithm

Algorithms Step

Step4 Update pheromones

$$\tau_{ij}(t + n) = (1 - \omega) \cdot \tau_{ij}(t) + \omega \cdot C \quad (33)$$

Update the pheromone of shortest path and longest path

$$\tau_{ij}(t + n) = (1 - \rho) \cdot \tau_{ij}(t) + \rho \cdot \Delta\tau'_{ij}(t) \quad (34)$$

$$\Delta\tau'_{ij}(t) = \begin{cases} \frac{Q}{L_{best}} & e_{ij} \in L_{best} \\ -\frac{Q}{L_{worst}} & e_{ij} \in L_{worst} \\ 0 & \text{else} \end{cases} \quad (35)$$

Two types of Hybrid Immune Algorithms

- Introduce the immune concept and mechanism into other algorithms, overcome local extreme points and improve convergence accuracy
- Introduce other algorithms into immune algorithm, increase the convergence speed of algorithm

Thank You!