

Localization in Wireless Rechargeable Sensor Networks Using Mobile Directional Charger

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Abstract—Existing work on localization in wireless rechargeable sensor networks (WRSNs) assumes that the charger is equipped with an omnidirectional antenna. We notice that the current available COTS (Commercial off-the-shelf) energy harvesting kit such as the one manufactured by Powercast includes wireless charger with directional antenna. We consider the localization problem in a wireless rechargeable sensor network with a mobile wireless charger. We propose two efficient region dividing methods, *Angle Division* and *Grid Division*. Both exploit the field angle of the directional charger to localize individual sensor nodes. The *Angle Division* is more suitable for general convex polygons. *Grid Division* algorithm is efficient when the nodes are distributed uniformly and the area is approximate to a rectangle. Further, we extend the two methods by combining the charging time to improve the localization precision. To verify the design, we have extensively evaluated our design by both field experiments and large-scale simulations. The experiment and simulation results show that our algorithm can achieve a localization distance error of less than a quarter meter by as less as four stops.

I. INTRODUCTION

Recent advance in wireless energy transfer has fomented the emergence of wireless rechargeable sensor networks (WRSNs) with a longer lifetime compared to the traditional wireless sensor networks. In wireless sensor networks, it is usually important for the nodes to know their positions. The problem of nodes localization in traditional wireless sensor networks has been investigated. For traditional wireless sensor networks, the existing localization schemes can be classified as range-based methods and range-free methods. The range-based methods include Angle of Arrival (AOA), Time of Arrival (TOA), Time Difference of Arrival (TDOA), Received Signal Strength (RSSI), measuring point-point distances or angles among sensor nodes and/or anchor nodes to compute per-node's localization. The range-free methods (e.g. [1]) locate the nodes based on their connectivity information. In wireless rechargeable sensor network, Shu et al. [2] proposed a novel method utilizing the Time of Charge (TOC) to estimate the nodes positions. The advantage of this method is that TOC is more robust to transient signal fluctuations. But the algorithms in [2] are designed based on the assumption that the charger is equipped with omnidirectional antenna. We notice that the current available COTS device for wireless charging, the Powercast energy harvesting kit, is equipped with a directional antenna since directional antennas can provide more reliable and uniform power distribution in their magnetic field [3].

In this paper, we investigate the problem of localizing sensor nodes in WRSNs with a mobile directional charger. Methods based on omnidirectional antenna such as the one proposed in [2] cannot work in this scenario. For example, the charger can not obtain the charging time of those sensors out of charging range. We two efficient localization methods for multiple sensor nodes when charged by directional chargers. The two methods for localizing sensor nodes are *AngleDivision* and *GridDivision*. The *AngleDivision* is suitable for general convex polygons. *GridDivision* algorithm is efficient when the nodes are distributed uniformly and the area is approximate to a rectangle. Finally, we incorporate the TOC into our algorithms to improve the localization precision. We perform field experiments and simulations to evaluate the performance of the proposed algorithms.

The remainder of the paper is organized as follows. The charging model of directional charger is introduced in Section II. We discuss the localization problem and propose *AngleDivision* and *GridDivision* algorithms in Section III. In Section IV, we propose an optimal *AngleDivision* for single point and then extend it for multiple sensor nodes. We evaluate our algorithms through testbed experiments and simulations in Section V. The related work is discussed in Section VI. Finally, we conclude the paper in Section VII.

II. CHARGING MODEL

We use the wireless charger TX91501 of the energy harvesting kit developed by Powercast. Its charging range form a sector of 60° width. We first conduct a series of experiments based on TX91501 to verify how the charging power varies with the charging distances and the angles. In the experiment, we draw 19 rays rooted at the position of the charger as the sampling-lines in the half-plane (from 0° to 180°) with the angle interval of 10° and define the direction the charger faces to as 90° . In each sampling-line, we draw sampling-points from the distance away from the charger 40cm to 100cm every 5cm. A sensor node is placed at each sampling-point successively to be charged several times to obtain the average charging power. The distribution of the charging power is illustrated in Fig. 1. In the figure the rays from the circle of the center present the angles that the sensors towards the charger to the base line. The concentric circles represent the charging power from 0mW to 50mW. The colored contour lines show how the actual charging power varies with the angle at the

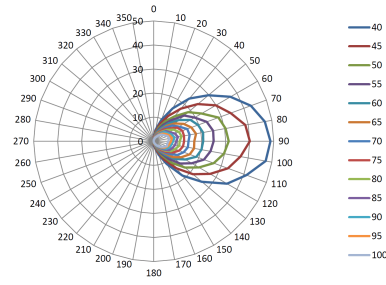


Fig. 1. Illustration of the distribution of the directional charging

same distance. For example, the outermost curve shows the actual charging power varies with the angle when the sensor is away from the charger 40cm. In the figure, when the angle is between 60° and 120° , the power measured is strong. While the power in the two sides decrease quickly with the angle tends to 0° or 180° . Namely, most of the electromagnetic signal is distributed among the charger's front region forming a sector with 60° width.

Based on the above observation, our work in this paper adopts the sector model, namely, the magnetic field region of the directional charger is a sector with a certain angle (60°). The experiment also shows that the charging power is affected by the angle.

III. BASIC LOCALIZATION ALGORITHM DESIGN

In this section, we first present our basic angle localization algorithm *AngleDivision* and *GridDivision*, and then combines the algorithms with the charging time to improve the localization performance. Angle is an inherent property of the directional charger that can be utilized to discover the sensors since the coverage of the directional charger forms a shape of sector.

A. Settings

Consider the scenario where there is a convex polygon region and N wireless rechargeable sensor nodes are randomly deployed in it. We first define that the feasible region of the nodes as the minimum region covering all nodes. Obviously, at the beginning the feasible region of each target sensor node is the whole region. The center of the gravity of the feasible region that the target sensor belongs to is estimated as the position of the target node. An accurate localization result can be obtained if we can effectively narrow down the feasible region.

Generally, when the directional charger stops at different positions and faces to different orientations, the regions it cover are different. The position of one node can only have the following two cases: (1) in the intersection of previous region and current region; (2) in the difference region of previous region and current region. Accordingly, we can narrow down the feasible region of every node gradually base on this observation. We let the node covered by the charger send back a response once it is charged to a certain threshold, thus the charger knows it is locating in the magnetic field. Based on experiments, we can calculate the maximal charging time T .

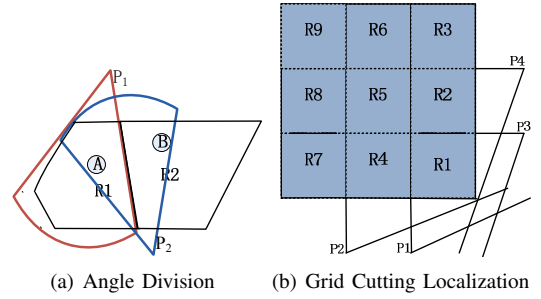


Fig. 2. Localization Algorithm

Therefore, the charger receives a response within T if it covers the sensor node. In addition, since the energy leaked outside the magnetic field is weak enough, the outside sensors will not be able to respond to the charger within T . In the following sections, we will show how to reduce the feasible region with a directional charger.

B. Angle Division

We first illustrate the idea of how to narrow down the feasible region of each node individually with the angle of the charger by random stops.

Based on the Section II, we know that the magnetic field of the directional charger is not distributed uniformly around all its orientation, but usually in a sector region with a fixed angle in the front. A sensor node in the sector will be charged and out of the sector will not. For example in Fig. 2(a), target nodes A and B in the region are to be localized. The charger moves freely around the region and stops at two different positions P_1 and P_2 with certain orientation. At the position P_1 , since node A locates in the magnetic region, the charger will receive its response within the time of T and will determine it to the intersection of the original region and the sector of its magnetic field, i.e. the region R_1 . For node B , the charger will never receive its response since it is not in the magnetic field and never be charged. Then the charger will determines node B lying in the region R_2 . For the second stop position, P_2 in Fig. 2(a), its two chords can divide the region R_1 and R_2 at the same time. Iteratively doing this, the feasible region per node can be reduced gradually.

Notice that the two chords of the sector can divide the region and result in concave regions. It is more complicated to deal with concave shapes. We thus restrict that one region can only be divided by one chord. This ensures that every node is in a convex region at any time. Since the coordinates of the regions and the magnetic field when stopping at some point is known, the charger can detect if some region is across the magnetic field, such as the cases shown in Fig.3(a) and Fig.3(b) when the charger stops at position p_1 . If so, the charge uses the following two translation methods to fix the stop position. The first case is shown in Fig.3(a), position p_1 in the region is selected to stop. Among the vertices of the region on the clockwise side of its two chords l_1 and l_2 , enumerate the distances between the vertices and the chord l_2 to obtain the maximal distance. Since the field angle α is known, we can compute the distance of p_1p_2 , which the charger needs to move along l_1 . Fig.3(b)

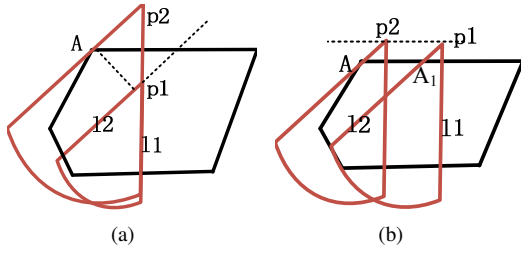


Fig. 3. Translation Transformation

illustrates the second case, which the position p_1 selected is outside the region. Similarly, we can obtain the maximal distance among the vertices on the clockwise side of l_1 and l_2 . Suppose the node is A , and A_1 is the intersection point on l_2 satisfying that the line AA_1 is parallel with the boundary. Thus the vector $\overrightarrow{A_1A}$ gives the distance and the orientation the charger should move. Applying the two translation methods, the charger can always divide the region into two convex parts.

C. Grid Localization

We have a more effective localization method, *GridDivision*, for densely deployed WRSNs in a rectangle. Fig. 2(b) demonstrates a simple example on how this localization method carries out. To elaborate clearly, we divide the region into grids beforehand. In the figure, P_1, P_2, P_3 and P_4 are four stop positions. The two rays at each point represent the boundaries of the magnetic field region when the charger stops at this position. For example, when the charger stops at position P_1 , grids R_1, R_2, R_3 are in the magnetic field and the nodes in this region will give the corresponding response to the charger. When at position P_2 , grids (R_1 to R_6) are in the magnetic field and the nodes in the region will give corresponding response to the charger. According to this, the nodes whose responses are not received by the charger at position both P_1 and P_2 are determined to be in the grids R_7 to R_9 . The nodes whose response can be received at the position P_2 but not at P_1 are determined to be in the grids R_4 to R_6 . The nodes whose responses are received at the position P_1 are determined to be in the grids R_1 to R_3 . Similar divisions can be done at the positions P_3 and P_4 . And at last the charger can determine per node to the corresponding square grid.

For an irregular polygon, its minimum bounding rectangle can be easily obtained. For a given rectangle with the length of its two adjacent sides being L and aL ($a > 1$), each node can be determined to a corresponding grid with side length of l . If we use the center point of the grid to estimate the actual position of the node, the maximal localization distance error is half of the diagonal length, $\frac{\sqrt{2}}{2}l$. Given the required localization distance error ϵ , the side length l of the grid should meet Eq.(1).

$$\frac{\sqrt{2}}{2}l \leq \epsilon \quad (1)$$

Then, the number of the stop positions in the vertical direction and the horizontal direction n_v and n_h can be

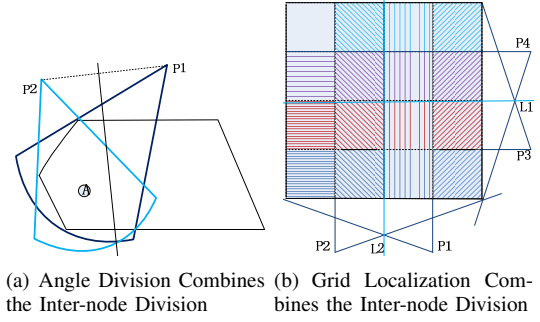


Fig. 4. Localization Combines the Inter-node Division

obtained though Eq.(2).

$$\frac{L}{n_v + 1} = \frac{aL}{n_h + 1} = l \quad (2)$$

Knowing the number of the stop positions n on one direction, first find n point dividing the side into $n + 1$ equal parts. Computing the maximal distance away from the boundary, the stop positions can be fixed in the line parallel to the boundary. To examine the performance of the algorithm, we conduct simulation where 100 sensor nodes are randomly distributed in an area of 100×100 unit and a mobile directional charger stops at the positions calculated by the above algorithm to localize the nodes. The simulation result shows that the area of the feasible region (the grid) per node can be narrowed down to 6.25% of the original region and the localization distance error can be decreased to about 9 unit with 6 stop positions.

D. Combining with TOC

For the above two localization technique, the charger only uses the information at each position and does not take the relative information between different positions into account. In this section, we combine the above two cutting algorithms with the observation proposed in [2] to further improve the localization result. The observation can be presented as follows: there is an arbitrary pair of charger stop positions, p_1 and p_2 , and one node A . Under the assumption of the positive relationship between the charging time and the charging distance, if the charging time $t(A, p_1) \leq t(A, p_2)$, then node A is closer to p_1 than to p_2 , namely, we can determine node A locating closer side of the perpendicular bisector.

1) *Angle Division*: Suppose the charger stops at two different positions, denoted by p_1 and p_2 as in Fig.4(a). At each position the charger can reduce the feasible region. In addition, if the node locates in two magnetic fields, the perpendicular bisector of the corresponding two positions will further reduce the feasible region. As shown in Fig.4(a), after two angle division, since node A is closer to p_2 , it will have a shorter charging time for position p_2 than p_1 . Therefore, we can infer that node A must lie on the left of the perpendicular bisector of the two positions.

2) *Grid Division*: For the grid division localization, the perpendicular bisector between a pair of positions still can

be used to reduce the region. In order to figure out how it works clearly, we suppose 4 charging positions selected as shown in Fig.4(b). On each direction, the charging positions in the line parallel to the side. For each charging position, the cutting line is vertical to the side and stabs the quadrant of the side. In addition, the charger's orientation is opposite at the two charging positions so that the regions between them will be in the intersection of the magnetic fields. The nodes in the magnetic field of p_1 but not in that of p_2 are locating in the region denoted as horizontal lines. And the nodes in magnetic field of p_2 but not in that of p_1 are locating in the region denoted by the left slashes. Then we draw the perpendicular bisector L_2 between p_1 and p_2 and the nodes both in the two magnetic fields can be determined to lie on which side of L_2 according to the charging time. The same steps can be done on the adjacent side. Calculate the intersection between the regions divided horizontally and vertically, all the nodes will be reduced to a small grid. It is obviously to see that the grid divided in this section is much smaller than that in Section III-C.

In the following we will describe how to divide the region into grids for a general case according to the number of the charging positions n on each side and the length of the side L . When n is equal to 1, it is obvious that the cutting line stabs the middle of the side and the case of $n = 2$ has already presented above. For the case of $n \geq 3$, we consider two cases of n being even and odd. When n is even, denote $t = \frac{n}{2}$, and we can divide the positions into t pairs ($t \geq 2$). For each pair, the orientations of the charger at the two positions are opposite to each other. And the positions can be the ranked as Fig.4(b). Between the pairs, the distance is l and in each pair, the two positions are apart by $2l$. Thus the number of $2l$ is t and the number of interval distance l is $t + 1$, then l can be calculated by Eq.(3). While n is odd, denote $t = \frac{n-1}{2}$, and the positions can be split into t pairs ($t \geq 1$) and one single position. Similarly, the adjacent pairs are apart l and the positions in the pair are apart $2l$. Thus l can be calculated by Eq.(4).

$$t \cdot 2l + (t + 1)l = L \quad (3)$$

$$t \cdot 2l + (t + 2)l = L \quad (4)$$

The performance of the algorithm will be shown in Section V-C. From the simulation, we can see it performs well when the nodes are distributed normally in the region. However, if there exist some subregions without nodes lying in it, the small grids divided by the algorithm will unavailing.

IV. EXTENDED DESIGN FOR ANGLE DIVISION

Based on the ideas of the basic design, in this section we will show how to localize the target nodes more effectively by planning the charger stop positions for the Angle Division. First, we propose an optimal solution for the scenario where single node is required to be localized. Then we extend the solutions to solve the general multi-sensor scenario.

A. Optimal Localization Algorithm for One Node

Obviously, the smaller is the feasible region per node (strictly, the maximal diameter of the feasible region), the more accurate is the estimated position. With Angle Division, at each step, the target node must lie in one of the two sub-regions divided by the cutting line. [2] presents that if we can minimize the maximal diameter of the two sub-regions, then the maximal localization error is certainly minimized. And it also gives a simple algorithm to optimally cut the region with the computation complexity of $O(n^4)$ where n is the vertex number of the polygon. On the problem of node localization both for the scenario of an omni-directional charger and the scenario of a directional charger, the algorithm is helpful to decrease the localization error. Following we will show how the algorithm obtaining the optimal cutting line.

First, we use the optimal cutting algorithm (used in [2]) to get the line optimally divided the region and the charger will divide the region along this line. At each position, initialize the charging time per node by ∞ . During the charging process, the charger records the corresponding charging time if its response is received. For the nodes whose charging time is not ∞ when the charger stops at this position and the previous positions, the charger determines the node lying in which side of the perpendicular bisector of two positions. Iterate the above steps until the number of stop positions reaches the given number and then estimate the center of the gravity of the feasible region as the actual position of the target node.

B. Extension for the Multiple Nodes Scenario

Taking the multiple sensors scenario into consideration, focusing on the feasible region of one node is impracticable and the cutting line optimal for all the feasible regions of the nodes may not exist at all. [2] provides an approximate solution to handle this problem for the scenario of the omni-directional charger. To minimize the maximal distance of the regions, they tend to cut more longer diagonal line of the original polygons. Base on this, in this section, we give an approximate algorithm suitable for the multiple nodes scenario. Before localization, we assume that we know the total number of the sensor nodes to be localized. Actually this also can be simply calculated by the charger through selecting a stop position and a fixed orientation satisfying that the whole original region can be covered and will receive all the responses of the sensor nodes to count the number of the sensors. And in this paper, we assume that the total number of the sensors is already known.

For the multiple sensors scenario, to locate efficiently, we need to let the charger cut larger regions every time. A line stabbing through the longest diagonal of all the convex polygons will certainly divide the largest region. If we compute all the diagonals of the original regions and sort them in the non-increasing order of their length, a line stabbing more diagonals will stab more polygons in the general case. This is a well studied topic in computational geometry [4], [5]. We iterate through all the sorted lines and check whether there exists a line that intersects with all previous ones. Not like [2] just cuts the previous longest $|L|$ diagonals, we try to continuously cut

longest diagonal lines by adding the lines one by one. If we cannot find a line crossing all the diagonals, remove the line added last and then add the next line continue to cut until all the diagonals are added.

V. PERFORMANCE EVALUATION

In this section, we first evaluate the performance of the extended cutting algorithm for multiple-sensors through experiments. Then large-scale simulations are further proposed in Section V-C to evaluate the performance of the Optimal Cutting algorithm for single node and Grid Division and the Extended Cutting algorithm for multiple nodes.

A. Experimental Evaluation

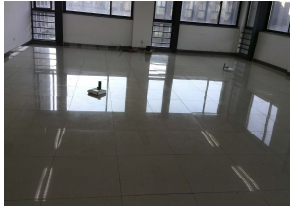


Fig. 5. Experiment Area

We firstly evaluate the Angle Division method with extended design for multiple sensor localization with Powercast devices. In our experiment, two rechargeable sensor nodes are randomly distributed in an area of $5m \times 5m$ (shown in Fig.5) where a wireless charger can stop at different positions and turn to different orientations to charge the nodes. Based on our history experiment data, the maximal charging time T , is estimated as 60s. Firstly, a line optimally cutting the original region is obtained and the first stop position is fixed in the line through the method in Section IV-A. Then the following stop positions are calculated by the Extended Angle algorithm in Section IV-B. In our experiment, when the total number of the stop positions reach 4, the process of the localization will terminate.

Fig. 6 shows how the feasible regions of all the nodes evolves along with each stop of the charger and the localization results per node at each step are listed in Table I. Table I shows that 80% original area can be excluded and the localization error per node decreases to less than 1m after 2 stops. And after only 4 stops, the localization error of both nodes is reduced to a quarter meter.

TABLE I
EXPERIMENTAL RESULTS

	Node 1		Node 2	
	Distance (m)	Area(%)	Distance (m)	Area (%)
Stop 1	2.23	59.80	1.92	40.20
Stop 2	0.92	16.68	0.83	15.28
Stop 3	0.37	6.01	0.32	9.67
Stop 4	0.20	3.97	0.13	4.89

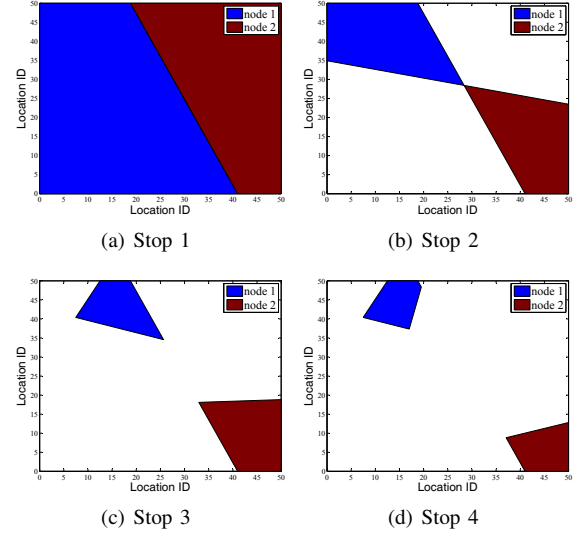


Fig. 6. The process of feasible regions narrowing down.

B. Simulation Settings

TABLE II
SIMULATION PARAMETERS

Parameters	Description
Field Area	100 × 100 (Grid Unit)
Number of Stops	6
Number of Target Nodes	100
Target Node Distribution	Uniform Distribution
Statistics	Feasible Region; Mean Distance Error
Random Seed	100 runs

In addition to further evaluate the performance of the optimal cutting algorithm for single node and the angle algorithm and the extended algorithm for multiple nodes, large-scale simulations are conducted. Default simulation are shown in Table II (the Number of Target Nodes only for the multiple scenario). We adopt percentage area of the feasible region and the localization distance error as two metric to evaluate the localization performance. The percentage of the feasible region refers to the ratio of the segmented area after each stop over the original feasible region. The localization distance error is defined as the distance between the estimated position and the real location of the sensor node.

C. Simulations

In this section we conduct large-scale simulations to illustrate the performance of the Optimal Angle Division over the Optimal TOC as well as the Grid localization and the Extended Angle algorithm over the Extended TOC algorithm in [2].

1) *Algorithm Performance for Single Node:* To evaluate the performance of our algorithm, we conduct the simulation that the directional charger selects charging positions by the optimal cutting algorithm. And then we compare the performance of ours with the TOC Optimal Cutting algorithm in [2].

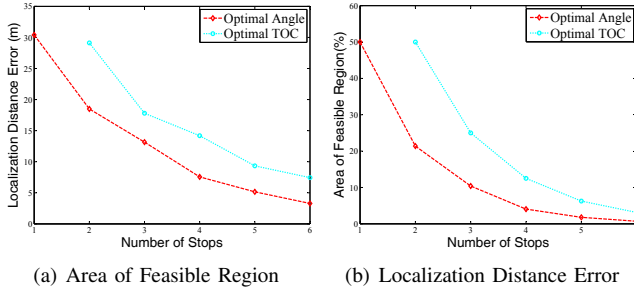


Fig. 7. Our Algorithm vs. TOC Optimal Cutting

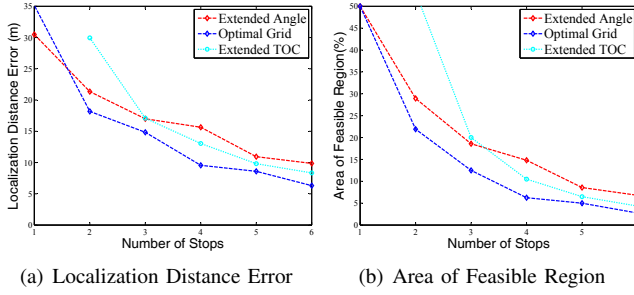


Fig. 8. Performance Comparison (Number of Nodes = 100)

At first stop position, the TOC design based on the perpendicular bisector cannot divide the region since the perpendicular bisector does not come up yet. However, our algorithm based on the Angle Division still can carry out the division on the region. We run the two algorithms iteratively for 100 times and the mean localization error (including the distance error and the percentage of area) at each stop position is illustrated in Fig. 7. From the figure we can see that our algorithm outperforms the TOC design both in the terms of percentage of the feasible region area and the localization distance error.

2) *Algorithm Performance for Multiple Nodes:* In Fig.8(a) and Fig.8(b), by optimizing the charging positions, the Grid Localization and the Extended Angle algorithm are both able to reduce the mean localization distance error and the area of the feasible region. For the curve of the Grid Localization algorithm, it shows the localization results by selecting 1,2,3,4,5,6 charging positions to divide to region into grids respectively. Combining with the Inter-node division, the Grid localization algorithm performs better than the Extended TOC. After 6 stop positions, the Grid Division algorithm decreases the localization distance error to 6.31 unit, which is 24.34% less than the Extended TOC algorithm (6.31 vs 8.34).

VI. RELATED WORK

Based on the underlying localization techniques, the most significant localization methods can be mainly classified into two categories: range-based localization methods and range-free localization methods. Range-based techniques use distance or angle to estimate per-node's location, such as Received Signal Strength indication (RSSI) [6], Time of Arrival (TOA) [7], Time Difference of Arrival (TDOA) [8], [9], Angle of Arrival (AOA) [10], [11]. Although they tend to offer precise localizations of nodes, they usually incur costly additional hardware or environment profiling. Range-free

localization methods, such as [1], do not rely on the distance or angle measured directly by hardware to calculate the position. Instead they are based on the connectivity information to determine relative positions using only knowledge of their neighbours. Since much useful information may be thrown away, the method may be not suitable for sparse networks and most of them are suffering from localization inaccuracy.

One recent interesting work of localization in WRSNs is [2]. They use the time of charge to iteratively narrow down per-node's feasible region. It is assumed in their work that the charger is equipped with omnidirectional antenna. In this paper, we adopt a directional charger to localize the sensor nodes using the angle of itself. Our methods get rid of the requirement of the additional anchors.

VII. CONCLUSION

We study the problem of localization in WRSNs using a mobile charger with directional antenna. We propose two localization methods, i.e., Angle Division and Grid Division. We combine the TOC (Time of Charge) with the two proposed methods since it is a very precise indicator of the distance between the charger and the sensor. We further extend the Angle Division algorithm with line cutting methods, considering both single node and multiple nodes localization. We conduct both field experiment and simulation to evaluate our proposed methods.

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