Artificial-Noise-Aided Coordinated Secure Transmission in Multi-Cell Multi-Antenna Systems with Limited Feedback

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***Abstract*— In this paper, we consider the secure communications in a multi-cell downlink multi-antenna system, where each multi-antenna base station (BS) sends confidential messages to one intended legitimate user (LU) with a passive multi-antenna eavesdropper trying to intercept the massages in this cell. To enhance the secure communications of the cell-edge users, the multiple BSs employ coordinated beamforming with the aid of artificial-noise (AN) beamforming. We conduct a mathematically rigorous secrecy performance analysis of the considered system and obtain new accurate closed-form expressions of a lower bound on the ergodic rate of each LU and an upper bound of the ergodic rate of each eavesdropper without assuming asymptotes for any system parameter. Then, a lower bound on the ergodic secrecy rate (ESR) of each LU follows. Based on the derived analytical results, we propose a low-complexity algorithm to optimize the CSI feedback bits allocation for the channels of target signal link and inter-cell interference links. Furthermore, we develop explicit sufficient conditions on the set of system parameters under which there are at least two BSs to coordinated secure transmission. Through both theoretical analysis and numerical results, we can verify that the obtained analytical lower bound on ESR of each LU as a function of the set of power allocation coefficients is generally *neither convex nor concave*. Thus, we propose to employ some numerical method to find a sub-optimal power allocation solution. Finally, numerical results are also provided to validate our theoretical results and the proposed feedback bits and power allocation methods.**

***Index Terms*— Physical layer security, coordinated multi-point, secrecy rate, artificial noise, limited feedback.**

I. Introduction

Security becomes a very critical concern in wireless networks as the amount of private and sensitive data transmitted over wireless channels grows very rapidly. Physical layer security (PLS) is regarded as an alternative to the traditional cryptography-based techniques and has drawn much attentions from the research community in the past decade [1–12]. It was shown in the pioneer works of [1, 13] that a positive perfect information rate (secrecy rate, SR) can be achieved when the legitimate channel is “more capable” than the eavesdropping channel.

Secure communications in multi-antenna systems assuming perfect CSI at transmitter (CSIT) have been widely studied in the previous literature [3–6, 14, 15]. To assume perfect CSI of legitimate channels or even illegitimate channels is generally impractical, especially for the cases with passive eavesdroppers. Moreover, for the widely-deployed frequency division duplex (FDD) systems, CSIT of the main channels is typically obtained at each receiver through pilot training, and conveyed to transmitter through digital feedback. When the CSIT of illegitimate channels is not available, artificial-noise (AN)-aided secure transmission is commonly employed to degrade any illegitimate channel by transmitting AN in the null space of the main channel [4, 5, 7–9, 12]. It has been shown in our previous works [12] and [16] that, when there is only imperfect CSIT of main channels, a secure transmission scheme without AN is generally *not optimal*.

It is well known that the potential capacity gains of point-to-point and multiuser MIMO wireless systems are very limited in cellular networks due to intra- and inter-cell interference (ICI). Moreover, the challenge to achieve confidential message broadcasting in the multi-cell networks is to deal with not only the *intra-cell information leakage* but also the *inter-cell information leakage*. Thus, the existing secure transmission schemes cannot be directly applied to multi-cell scenarios. Coordinated multi-point (CoMP) is a very promising technique to handle ICI in cellular networks since it improves the cell-edge and average data rates [17, 18]. To the best of our knowledge, while the secure communications in a single cell have been widely studied, there have been very few works studying the solution to distribute confidential messages by CoMP in multi-cell networks. [19–21] are the very few exceptions. However, the design and analysis in [19] and [20] were for the special setting of two cells and based on the perfect CSIT of legitimate channels.

With only incomplete CSIT of legitimate channels, [21] analyzed the secrecy performance of a coordinated beamforming (CBF) scheme based on linear ZFBF, and proposed an iterative CSI feedback bits allocation algorithm for the multiple downlink channels of each LU. However, there are some drawbacks with [21]. Firstly, in [21], the eavesdropper in each cell treated the received ICI created by confidential message-bearing signals sent by $K-1$ neighbor cells as useless interference, which underestimated the capability of an eavesdropper[1]. Secondly, a minor problem with [21] is that it did not employ the well recognized practical CSI model for FDD systems as in many previous works[2], such as [7, 8, 11, 12, 22, 23]. Despite of the importance of AN for the system without perfect CSIT as stated above, as far as we know, except for [20] there has been no other work studying this issue for secure CoMP transmission in

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[1]It is obvious that the optimal processing method for any eavesdropper in a cell to do is the joint decoding of the confidential message-bearing signals from the K coordinated BSs, which leads to higher rate of eavesdropping each LU's messages than that can be achieved by the scheme considered in [21].

[2]Specifically, the CSI model therein replaced the time-varying terms $\sin^2\theta_{k,j}$ and $\cos^2\theta_{k,j}$ of the channel model in [22, 23] (see (II-A) in this paper) by the constant terms $2^{-\frac{B_{k,j}}{M-1}}$ and $1-2^{-\frac{B_{k,j}}{M-1}}$ respectively.

multi-cell systems in the related literature. Moreover, [19–21] all considered systems with only single-antenna eavesdropper(s). However, our previous studies in [12, 16] showed that it is *very likely* that the system design for *single-antenna* eavesdropper even can not work for the systems with a *multi-antenna* eavesdropper. All aforementioned limitations with the previous related works strongly motivate this work.

The contributions of this work are summarized as follows:

- To the best of our knowledge, this is the first work to investigate the AN-aided secure CoMP transmission in FDD multi-cell systems with only quantized CSIT of each LU, including the optimization of the quantized CSI feedback bits allocation and power allocation. Existing secure CoMP transmission works mainly assume perfect full CSIT of each LU, or do not employ AN-aided secure transmission technique.
- For a given set of power allocation coefficients of the confidential message-bearing signals and the CSI feedback bits allocation to the downlink channels of all LUs in the CoMP cells, we conduct a mathematically rigorous ESR analysis of each LU for the considered system, which has never been done before. We derive a new accurate closed-form expression of a lower bound on the ergodic rate of each LU, and also an accurate upper bound on the maximum ergodic rate of each LU's messages that can be supported by the channel of the corresponding eavesdropper. Then, a closed-form expression of a lower bound on the ESR of each LU follows. All above analytical results are obtained without assuming asymptote for any system parameter.
- For a given set of power allocation coefficients of the coordinated cells, we propose a closed-form expression for the optimal feedback bits allocation (without integer constraint) to the target signal link and the ICI links of each LU which minimizes an upper bound on the ESR loss due to quantized CSI. Then, we further devise a low-complexity dynamic programming algorithm to find the optimal integer bits allocation solution. We also develop some explicit sufficient conditions on the set of the system parameters, under which there are at least two BSs to coordinate transmission, which has been rarely studied before.
- To optimize the power allocation between the message-bearing signals and AN of each cell, we also study the properties of the analytical lower bound on the ergodic rate of each LU and the analytical upper bound on the ergodic rate of the eavesdropper obtained above. Both theoretical analysis and numerical results verify that the analytical ergodic rate bounds of each LU and eavesdropper obtained above and also the ESR lower bound of each LU are *neither convex nor concave* as the functions of the set of power allocation coefficients of all LUs. Therefore, the global optimal solution of power allocation can not be guaranteed. Then, we propose to employ some numerical method to obtain sub-optimal solution to optimize the power coefficients allocation with the above CSI feedback bits allocation.

Notations: $\mathcal{C}$, $\mathcal{R}$ and $\mathcal{N}$ denote the sets of complex numbers, real numbers and natural number respectively. $\mathbb{E}_X\{\cdot\}$ represents expectation with respect to random variable X. $\|\cdot\|$ denotes the L_2-norm of a vector, $|\cdot|$ denotes the absolute value of a scalar. The ceiling operation is denoted by $\lceil\cdot\rceil$.

II. System and Signal Models

We consider secure communications in a downlink multi-cell multiple-input-single-output (MISO) system with $K > 1$ cells, where each base station (BS) is equipped with $M > K$ antennas sending confidential messages to a single-antenna LU associated with the cell. A passive multiple-antenna eavesdropper (Eve) equipped with $N_e > 1$ antennas attempts to eavesdrop on the confidential messages of the LU in this cell. The system operates in FDD mode with frequency reuse of one. We assume each LU is associated with the nearest BS. In this multi-cell scenario, the quality of the received signals of both LU and Eve may be severely degraded by ICI, especially at the cell edge. To guarantee the information security of the system with PLS, we propose a secure transmission scheme that combines CBF and AN.

We assume all channels in the system undergo block frequency-flat fading. Then, the received signals at LU and Eve in the k-th cell ($1 \le k \le K$) can be written as in (1) and (2) respectively at the top of the next page, where $d_{i,j}$ and $l_{i,j}$ are respectively the distances from the j-th cell's BS to the LU and Eve in the i-th cell (denoted as BS j, LU i and Eve i afterward). $\mathbf{h}_{i,j} \in \mathcal{C}^{1\times M}$ and $\mathbf{G}_{i,j} \in \mathcal{C}^{N_e\times M}$ are respectively the fast-fading channels from the BS j to the LU i and Eve i. $\hat{\mathbf{w}}_i$ and $\hat{\mathbf{\Gamma}}_i$ respectively denote the BF vector of useful signals and the precoding matrix of AN at BS i which will be described later. x_i is the confidential-message bearing signal for LU i with unit variance, and $\mathbf{z}_i \in \mathcal{C}^{(M-K)\times 1}$ is the AN vector generated by BS i which is distributed as $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{M-K})$. P is the maximum total transmit power of each BS, and ϕ_k is the power allocation coefficient that denotes the fraction of P allocated to the useful signals at BS k. $n_{u,k} \sim \mathcal{CN}(0, \sigma_u^2)$ and $\mathbf{n}_{e,k} \sim \mathcal{CN}(\mathbf{0}, \sigma_e^2\mathbf{I}_{N_e})$ are the additive white Gaussian noise (AWGN) received at LU k and Eve k respectively. α is the path-loss exponential coefficient. We assume each LU is always served by the nearest BS such that $d_{k,k} \le d_{k,j}$ $\forall j \ne k$ holds.

A. Channel and CSI Feedback Models

For the considered FDD systems, to focus on the effect of limited CSI feedback, we follow many previous works (e.g. [18, 21–24]) to assume that each LU k ($1 \le k \le K$) can estimate the downlink CSI from all BSs (i.e., $\mathbf{h}_{k,j}$ $\forall j \in \{1, 2, \ldots, K\}$) perfectly, and then the quantized versions of the channel direction vectors (CDVs) $\tilde{\mathbf{h}}_{k,j} = \mathbf{h}_{k,j}/\|\mathbf{h}_{k,j}\|$ $\forall j$ are fed back to BS k through an error-free feedback channel with a constraint of total B bits, i.e.,

$$B = \sum_{j=1}^{K} B_{k,j}, \tag{3}$$

where $B_{k,j}$ is the number of feedback bits to quantize $\tilde{\mathbf{h}}_{k,j}$. BS k and LU k maintain the codebooks $\mathcal{C}_{k,j}$ with $2^{B_{k,j}}$ codewords for each $j \in \{1, 2, \cdots, K\}$. Since the optimal CSI quantization strategy is generally unknown and is out of the scope of this paper, we employ the random vector quantization codebooks the same as those in [22, 23]. The rule for LU k to quantize $\tilde{\mathbf{h}}_{k,j}$ is given by $\hat{\mathbf{h}}_{k,j} = \arg\max_{\mathbf{c}\in\mathcal{C}_{k,j}} \left|\tilde{\mathbf{h}}_{k,j}\mathbf{c}^H\right|$. Then, $\tilde{\mathbf{h}}_{k,j}$ can be decomposed as [22]

$$\tilde{\mathbf{h}}_{k,j} = \cos\theta_{k,j}\hat{\mathbf{h}}_{k,j} + \sin\theta_{k,j}\tilde{\mathbf{e}}_{k,j}, \tag{4}$$

$$y_{u,k} = d_{k,k}^{-\frac{\alpha}{2}} \sqrt{\phi_k P}\ \mathbf{h}_{k,k}\hat{\mathbf{w}}_k x_k + \sum_{j=1,j\neq k}^{K} d_{k,j}^{-\frac{\alpha}{2}} \sqrt{\phi_j P}\ \mathbf{h}_{k,j}\hat{\mathbf{w}}_j x_j + \sum_{j=1}^{K} d_{k,j}^{-\frac{\alpha}{2}} \sqrt{\frac{(1-\phi_j)P}{M-K}}\ \mathbf{h}_{k,j}\hat{\mathbf{\Gamma}}_j \mathbf{z}_j + n_{u,k}, \quad (1)$$

$$\mathbf{y}_{e,k} = l_{k,k}^{-\frac{\alpha}{2}} \sqrt{\phi_k P}\ \mathbf{G}_{k,k}\hat{\mathbf{w}}_k x_k + \sum_{j=1,j\neq k}^{K} l_{k,j}^{-\frac{\alpha}{2}} \sqrt{\phi_j P}\ \mathbf{G}_{k,j}\hat{\mathbf{w}}_j x_j + \sum_{j=1}^{K} l_{k,j}^{-\frac{\alpha}{2}} \sqrt{\frac{(1-\phi_j)P}{M-K}}\ \mathbf{G}_{k,j}\hat{\mathbf{\Gamma}}_j \mathbf{z}_j + \mathbf{n}_{e,k}, \quad (2)$$

where $\theta_{k,j} = \angle(\tilde{\mathbf{h}}_{k,j}, \hat{\mathbf{h}}_{k,j})$ and $\tilde{\mathbf{e}}_{k,j}$ is the normalized quantization error vector that is isotropically distributed in the nullspace of $\hat{\mathbf{h}}_{k,j}$ [22]. The neighboring BSs can exchange the quantized CSI of the interference links of each active LU, but do not share confidential message-bearing data.

B. Coordinated ZFBF and AN Precoding

To mitigate the ICI caused by the useful signals and AN at each LU and to enhance the received signal-to-interference-plus-noise ratio (SINR) of the desired signal, we employ the coordinated ZFBF with AN precoding at each BS based on the quantized CSI of the downlink channels to the target LU and to the LUs of the other cells, where the BF vector $\hat{\mathbf{w}}_k$ and the AN precoding matrix $\hat{\mathbf{\Gamma}}_k$ used by BS k respectively satisfy the constraints that

$$\hat{\mathbf{h}}_{j,k}\hat{\mathbf{w}}_k = 0, \quad 1 \le j \le K, j \neq k; \quad (5)$$
$$\hat{\mathbf{h}}_{j,k}\hat{\mathbf{\Gamma}}_k = 0, \quad 1 \le j \le K. \quad (6)$$

Then, the ICI can be completely canceled, if each BS has perfect knowledge of CDV of all interference links. We note that, for (5) to be satisfied, $\hat{\mathbf{w}}_k$ can be arbitrarily chosen in the null space of the channel vectors of all LUs other than LU k which is the same as the scheme in [21]. However, this choice is generally *not* the optimal in term of achieving the maximum channel gain (or the power of the useful signal). Thus, we employ the optimal linear ZFBF scheme as that in [22, 23]. Specifically, $\hat{\mathbf{w}}_k$ is obtained by normalizing the k-th column of the matrix $\hat{\mathbf{W}}_k = \bar{\mathbf{H}}_k^H(\bar{\mathbf{H}}_k\bar{\mathbf{H}}_k^H)^{-1}$, where the composite matrix $\bar{\mathbf{H}}_k = \left[\hat{\mathbf{h}}_{1,k}^H, \ldots, \hat{\mathbf{h}}_{K,k}^H\right]^H$ consists of the quantized CDVs of the links from BS k to all LUs.

For $\hat{\mathbf{\Gamma}}_k$, let the singular value decomposition of $\bar{\mathbf{H}}_k$ be $\bar{\mathbf{H}}_k = \bar{\mathbf{U}}_k\bar{\mathbf{\Sigma}}_k[\bar{\mathbf{V}}_k^{(1)}, \bar{\mathbf{V}}_k^{(0)}]$, where $\bar{\mathbf{V}}_k^{(1)} \in \mathcal{C}^{M\times \bar{r}_k}$ and $\bar{\mathbf{V}}_k^{(0)} \in \mathcal{C}^{M\times (M-\bar{r}_k)}$ are respectively the right-singular vectors corresponding to the non-zero and zero singular values. $\bar{r}_k \triangleq \text{rank}(\bar{\mathbf{H}}_k) = K$ with probability of 1 when the channel fading is continuously distributed. Then, $\hat{\mathbf{\Gamma}}_k$ can be obtained as $\hat{\mathbf{\Gamma}}_k = \bar{\mathbf{V}}_k^{(0)}\mathbf{p}_k$, where $\mathbf{p}_k$ is an arbitrary vector uniformly distributed on the surface of the $(M-\bar{r}_k)$-dimensional unit sphere. Moreover, it is easy to see that the condition $M > K$ needs to be satisfied for this AN-aided ZFBF scheme to work properly, which is also required for AN-aided transmission in single-cell systems [5, 8, 11, 12, 16]. With the above processing at each BS given by (5) (6) and (4), we have $\mathbf{h}_{k,j}\hat{\mathbf{w}}_j = \|\mathbf{h}_{k,j}\| \sin\theta_{k,j}\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j\ \forall j \neq k$ and $\mathbf{h}_{k,j}\hat{\mathbf{\Gamma}}_j = \mathbf{h}_{k,j}\sin\theta_{k,j}\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{\Gamma}}_j\ \forall j$. Then, the received signal at LU k can be re-written as

$$\begin{aligned} y_{u,k} &= d_{k,k}^{-\frac{\alpha}{2}} \sqrt{\phi_k P}\ \mathbf{h}_{k,k}\hat{\mathbf{w}}_k x_k \\ &+ \sum_{j=1,j\neq k}^{K} d_{k,j}^{-\frac{\alpha}{2}} \sqrt{\phi_j P}\ ||\mathbf{h}_{k,j}|| \sin\theta_{k,j}\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j x_j \\ &+ \sum_{i=1}^{K} d_{k,i}^{-\frac{\alpha}{2}} \sqrt{\frac{(1-\phi_i)P}{M-K}}\ ||\mathbf{h}_{k,i}|| \sin\theta_{k,i}\tilde{\mathbf{e}}_{k,i}\hat{\mathbf{\Gamma}}_i \mathbf{z}_i + n_{u,k}. \end{aligned}$$

The corresponding SINR of LU k can be written as (7) at the top of the next page.

In addition, as we have explained in Footnote 1, to guarantee the secrecy of the confidential messages of all LUs, in contrast to the scheme in [21] which treated the ICI of the confidential message-bearing signals as useless interference at each eavesdropper, we follow [25] to assume the ICI created by the message-bearing signals can be perfectly canceled by the powerful Eves, but the ICI created by the AN can not. Moreover, since the noise variance at any Eve is generally unknown to the transmitters due to Eve being passive, we will follow many previous papers (e.g. [5, 8]) to facilitate a robust design and consider a "worst-case" scenario with $\sigma_e^2 \to 0$. Then, the received signals after ICI cancellation at Eve k can be written as

$$\mathbf{y}_{e,k} = l_{k,k}^{-\frac{\alpha}{2}} \sqrt{\phi_k P}\ \mathbf{G}_{k,k}\hat{\mathbf{w}}_k x_k + \mathbf{I}_{\text{AN},k}, \quad (8)$$

where $\mathbf{I}_{\text{AN},k} = \sum_{i=1}^{K} l_{k,i}^{-\frac{\alpha}{2}} \sqrt{\frac{(1-\phi_i)P}{M-K}}\ \mathbf{G}_{k,i}\hat{\mathbf{\Gamma}}_i\mathbf{z}_i$ is the sum of the AN leakage from all coordinated BSs. The above assumptions with each Eve k and the signal model given by (8) lead to an upper bound on the maximal achievable ergodic rate of x_k at Eve.

III. The Ergodic Secrecy Rate Analysis

For the ESR analysis, we consider all small-scale fading channels follow spatially uncorrelated Rayleigh fading, i.e., independent and identically distributed (i.i.d.) with zero-mean and unit-variance complex Gaussian elements, which has been considered in many previous PLS works [5] – [11] and [19] – [21]. The ESR of LU k and the ergodic sum secrecy rate (ESSR) of the coordinated K cells can be respectively expressed as[3]

$$R_{sec,k} = [R_{u,k} - R_{e,k}]^+, \quad R_{sec,sum} = \sum_{k=1}^{K} R_{sec,k}, \quad (9)$$

where $R_{u,k} = \mathbb{E}[\log_2(1+\gamma_{u,k})]$, and $R_{e,k}$ is the maximum achievable ergodic rate of LU k's messages at Eve k *by any possible method*. We first derive an analytical expression of a lower bound on $R_{u,k}$. The corresponding result of $R_{e,k}$ will be studied afterwards.

We note that the main difficulty in obtaining $R_{u,k}$ lies in the co-existence of multi-cell confidential message-bearing signals and AN leakage signals in $y_{u,k}$, and it has been illustrated in [24] that there is substantial statistical dependence (correlation) among the received

[3]Note that, when full CSI of the downlink legitimate channels and the channels of the interference links is known by each coordinated transmitter, the transmitters can vary the transmission rates in every channel fading block. Then, there can be an alternative definition of ESR/ESSR as in [1]. However, this is impossible for the settings considered here, since only quantized CDVs are available at each BS.

$$\gamma_{u,k}=\frac{d_{k,k}^{-\alpha}\phi_k P|\mathbf{h}_{k,k}\hat{\mathbf{w}}_k|^2}{\sum_{j=1,j\neq k}^{K} d_{k,j}^{-\alpha}\phi_j P||\mathbf{h}_{k,j}||^2\sin^2\theta_{k,j}|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j|^2+\sum_{i=1}^{K}\frac{d_{k,i}^{-\alpha}(1-\phi_i)P}{M-K}||\mathbf{h}_{k,i}||^2\sin^2\theta_{k,i}|\tilde{\mathbf{e}}_{k,i}\hat{\mathbf{\Gamma}}_i|^2+\sigma_u^2}. \tag{7}$$

information-bearing signal (i.e., $|\mathbf{h}_{k,k}\hat{\mathbf{w}}_k|^2$), the ICI (i.e., $||\mathbf{h}_{k,j}||^2\sin^2\theta_{k,j}|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j|^2$ for $j\neq k$) and the AN leakage signals (i.e., $\|\mathbf{h}_{k,j}\|^2\sin^2\theta_{k,j}|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{\Gamma}}_j|^2\ \forall j$) at each LU k. This is quite different from the scheme in [21] without employing AN, where the received information-bearing signal and the ICI are independent. For illustrative purposes, we refer the readers to Fig. 1 of [24], which showed that the correlation coefficients can be substantially large for a practical system. All above facts make the exact theoretical analysis intractable. Therefore, we instead develop as accurate a closed-form lower bound on $R_{u,k}$ as possible, which is given in the following theorem.

Theorem 1: $R_{u,k}\ \forall k$ can be lower-bounded as $R_{u,k}\geq R_{u,k}^{lb}$ with $R_{u,k}^{lb}$ given by

$$\begin{aligned}R_{u,k}^{lb}&=\log_2(e)C_1\sum_{k=0}^{\infty}\eta_k\sum_{m=0}^{\nu+k}\frac{1}{(\nu+k-m)!}\\&\times\left[\frac{(-1)^{\nu+k-m-1}}{(\gamma b_{min})^{\nu+k-m}}\exp\left(\frac{1}{\gamma b_{min}}\right)E_i\left(-\frac{1}{\gamma b_{min}}\right)\right.\\&\left.+\sum_{j=1}^{\nu+k-m}\frac{(j-1)!}{(-\gamma b_{min})^{\nu+k-m-j}}\right]-\log_2(e)C_2\\&\times\sum_{k=0}^{\infty}\rho_k\sum_{m=0}^{\nu+k-1}\frac{1}{(\nu+k-m-1)!}\\&\times\left[\frac{(-1)^{\nu+k-m-2}}{(\gamma c_{min})^{\nu+k-m-1}}\exp\left(\frac{1}{\gamma c_{min}}\right)E_i\left(-\frac{1}{\gamma c_{min}}\right)\right.\\&\left.+\sum_{j=1}^{\nu+k-m-1}\frac{(j-1)!}{(-\gamma c_{min})^{\nu+k-m-j-1}}\right],\end{aligned}\tag{10}$$

where $\nu=K(M-1)$, $\gamma=\frac{P}{(M-1)\sigma_u^2}$, $C_1=\prod_{i=1}^{K+1}\left(\frac{b_{min}}{b_i}\right)^{a_i}$, $C_2=\prod_{i=1}^{K}\left(\frac{c_{min}}{c_i}\right)^{M-1}$. Here, a_i, b_i and c_i are the i-th element of vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, and $b_{min}=\min\{\mathbf{b}\}$ and $c_{min}=\min\{\mathbf{c}\}$, where $\mathbf{a}=\Big[1,\underbrace{M-1,\ldots,M-1}_{\text{length of } K}\Big]\in\mathbb{R}^{1\times(K+1)}$, $\mathbf{b}=\Big[(M-K)\phi_k d_{k,k}^{-\alpha},\ d_{k,k}^{-\alpha}\left[(M-K)\phi_k(1-\delta_{k,k})+(1-\phi_k)\delta_{k,k}\right],\ \underbrace{d_{k,1}^{-\alpha}\delta_{k,1},\ldots,d_{k,(k-1)}^{-\alpha}\delta_{k,(k-1)},d_{k,(k+1)}^{-\alpha}\delta_{k,(k+1)},\ldots,d_{k,K}^{-\alpha}\delta_{k,K}}_{K-1\text{ terms}}\Big]\in\mathbb{R}^{1\times(K+1)}$ and $\mathbf{c}=\Big[d_{k,k}^{-\alpha}(1-\phi_k)\delta_{k,k},\ \underbrace{d_{k,1}^{-\alpha}\delta_{k,1},\ldots,d_{k,(k-1)}^{-\alpha}\delta_{k,(k-1)},d_{k,(k+1)}^{-\alpha}\delta_{k,(k+1)},\ldots,d_{k,K}^{-\alpha}\delta_{k,K}}_{K-1\text{ terms}}\Big]\in\mathbb{R}^{1\times K}$, where $\delta_{k,j}=2^{-\frac{B_{k,j}}{M-1}}$, $\eta_k=\frac{1}{k}\sum_{i=1}^{k}r_i\eta_{k-i}$ with $\eta_0=1$ and $r_i=\sum_{j=1}^{K+1}a_j\left(1-\frac{b_{min}}{b_j}\right)^i$; $\rho_k=\frac{1}{k}\sum_{i=1}^{k}s_i\rho_{k-i}$ with $\rho_0=1$ and $s_i=(M-1)\sum_{j=1}^{K}\left(1-\frac{c_{min}}{c_j}\right)^i$.

Proof: See Appendix A. □

Remark 1: It can be observed from the expressions of $R_{u,k}=\mathbb{E}\left[\log_2\left(1+\gamma_{u,k}\right)\right]$ and $\gamma_{u,k}$ in (7) that the channel model with limited feedback used in [21] actually equivalently further employs Jensen's inequality on the random terms $\sin^2\theta_{k,j}$ with the denominator in (7) considering that log function is convex, thus results in lower ergodic rate performance than that of any practical system.

We next derive an upper bound on the ergodic rate of x_k achievable at Eve k. According to (8), we have

$$\begin{aligned}R_{e,k}\leq R_{e,k}^{up}&=I\left(\mathbf{y}_{e,k};x_k\right)=H\left(\mathbf{y}_{e,k}\right)-H\left(\mathbf{I}_{\mathrm{AN},k}\right)\\&=\mathbb{E}\left[\log_2\det\left(\mathbf{W}_{SI,k}\right)\right]-\mathbb{E}\left[\log_2\det\left(\mathbf{W}_{I,k}\right)\right],\end{aligned}\tag{11}$$

where $\mathbf{W}_{SI,k}=\tilde{\mathbf{G}}_{SI,k}\mathbf{\Sigma}_{SI,k}\tilde{\mathbf{G}}_{SI,k}^H$ and $\mathbf{W}_{I,k}=\tilde{\mathbf{G}}_{I,k}\mathbf{\Sigma}_{I,k}\tilde{\mathbf{G}}_{I,k}^H$ with $\tilde{\mathbf{G}}_{SI,k}=[\mathbf{G}_{k,k}\hat{\mathbf{w}}_k,\mathbf{G}_{k,1}\hat{\mathbf{\Gamma}}_1,\ldots,\mathbf{G}_{k,K}\hat{\mathbf{\Gamma}}_K]$, $\tilde{\mathbf{G}}_{I,k}=[\mathbf{G}_{k,1}\hat{\mathbf{\Gamma}}_1,\ldots,\mathbf{G}_{k,K}\hat{\mathbf{\Gamma}}_K]$, and $\mathbf{\Sigma}_{SI,k}=\mathrm{Blockdiag}\Big\{l_{k,k}^{-\alpha}\phi_k,\frac{l_{k,1}^{-\alpha}(1-\phi_1)}{M-K}\mathbf{I}_{M-K},\frac{l_{k,2}^{-\alpha}(1-\phi_2)}{M-K}\mathbf{I}_{M-K},\ldots,\frac{l_{k,K}^{-\alpha}(1-\phi_K)}{M-K}\mathbf{I}_{M-K}\Big\}\in\mathbb{R}^{[K(M-K)+1]\times[K(M-K)+1]}$, $\mathbf{\Sigma}_{I,k}=\mathrm{Blockdiag}\Big\{\frac{l_{k,1}^{-\alpha}(1-\phi_1)}{M-K}\mathbf{I}_{M-K},\frac{l_{k,2}^{-\alpha}(1-\phi_2)}{M-K}\mathbf{I}_{M-K},\ldots,\frac{l_{k,K}^{-\alpha}(1-\phi_K)}{M-K}\mathbf{I}_{M-K}\Big\}\in\mathbb{R}^{[K(M-K)]\times[K(M-K)]}$. It is easy to see that $\tilde{\mathbf{G}}_{SI,k}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_{N_e}\otimes\mathbf{I}_{[K(M-K)+1]})$ and $\tilde{\mathbf{G}}_{I,k}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_{N_e}\otimes\mathbf{I}_{K(M-K)})$. Thus, $\mathbf{W}_{SI,k}$ and $\mathbf{W}_{I,k}$ have complex central Wishart distributions, i.e., $\mathbf{W}_{SI,k}\sim\mathcal{CW}_{N_e}(K(M-K)+1,\mathbf{\Sigma}_{SI,k})$ and $\mathbf{W}_{I,k}\sim\mathcal{CW}_{N_e}(K(M-K),\mathbf{\Sigma}_{I,k})$. It is easy to see from (11) that, if $K(M-K)<N_e$, the matrix $\mathbf{W}_{I,k}$ is rank-deficient with probability of 1. Then, $R_{e,k}^{up}$ will be infinite, and it is impossible to achieve positive ESR for LU k. Thus, it is required that $K(M-K)\geq N_e$. To obtain $R_{e,k}^{up}$, we first need the following result.

Lemma 1: Let the random matrix $\mathbf{W}\sim\mathcal{CW}_{N_e}(n,\mathbf{\Sigma})$ with $n\geq N_e$, where the $n\times n$ covariance matrix $\mathbf{\Sigma}\succeq\mathbf{0}$ is with L different eigenvalues. Let $\boldsymbol{\mu}=[\mu_{(1)},\mu_{(2)},\ldots,\mu_{(L)}]$ with $\mu_{(1)}>\mu_{(2)}>\cdots>\mu_{(L)}$ be the L different eigenvalues of $\mathbf{\Sigma}^{-1}$, where $\mu_{(i)}$ has m_i multiplicities such that $\sum_{i=1}^{L}m_i=n$. Then, $\mathbb{E}[\log_2(\mathbf{W})]$ can by obtained as

$$\mathbb{E}[\log_2(\mathbf{W})]=\Upsilon(n,\mathbf{\Sigma})\triangleq K_1\sum_{p=1}^{N_e}\det(\mathbf{\Theta}_{(p)}),\tag{12}$$

where $K_1=\frac{(-1)^{N_e(n-N_e)}\prod_{i=1}^{L}\mu_{(i)}^{N_e m_i}}{\Gamma_{(N_e)}(N_e)\prod_{i=1}^{L}\Gamma_{(m_i)}(m_i)\prod_{i<j}(\mu_{(i)}-\mu_{(j)})^{m_i m_j}}$. The (i,j)-th element of $\mathbf{\Theta}_{(p)}\in\mathbb{R}^{n\times n}$ is given by

$$[\mathbf{\Theta}_{(p)}]_{i,j}=\begin{cases}\frac{(-1)^{u_i}\Gamma(j+u_i)}{\mu_{(e_i)}^{(j+u_i)}}, & j=1,\cdots,N_e;j\neq p\\ \frac{(-1)^{u_i}\Gamma(j+u_i)}{\ln(2)\mu_{(e_i)}^{(j+u_i)}}[\psi(j+u_i)-\ln(\mu_{(e_i)})], & j=1,\cdots,N_e;j=p\\ [n-j]_{u_i}\,\mu_{(e_i)}^{n-j-u_i}, & j=N_e+1,\cdots,n\end{cases},$$

where $[a]_k=a(a-1)\ldots(a-k+1)$ with$[a]_0=1$, and e_i denotes the unique integer that satisfies $m_1+\cdots+m_{e_i-1}<i<m_1+\cdots+m_{e_i}$ and $u_i=\sum_{j=1}^{e_i}m_j-i$. $\psi(x)$ is the Euler psi function [26, 8.36].

Proof: We provide here a brief proof of this lemma due to space limit. The joint probability density function (PDF) of the ordered eigenvalues of $\mathbf{W}$ can be readily obtained using [27, *Lemma* 6]. Then, (12) can be easily obtained by employing [27, *Theorem* 2] with this joint PDF and the formulas [26, 3.351] and [26, 4.352]. □

Applying *Lemma* 1 to (11), $R_{e,k}^{up}$ can be easily obtained and given in the following theorem.

Theorem 2: The exact value of $R_{e,k}^{up}$ is given by

$$\begin{aligned} R_{e,k}^{up} &= \Upsilon\Big(K(M-K)+1, \mathbf{\Sigma}_{SI,k}\Big) \\ &\quad -\Upsilon\Big(K(M-K), \mathbf{\Sigma}_{I,k}\Big), \end{aligned} \tag{13}$$

where the function $\Upsilon(n,\mathbf{\Sigma})$ is as defined in *Lemma* 1.

Then, by combining *Theorem* 1 and *Theorem* 2, we can obtain a lower bound on $R_{sec,sum}$ as

$$R_{sec,sum} \geq R_{sec,sum}^{lb} \triangleq \sum_{k=1}^{K}\Big(R_{u,k}^{lb} - R_{e,k}^{up}\Big)^{+}, \tag{14}$$

where $R_{u,k}^{lb}$ and $R_{e,k}^{up}$ are respectively given by (10) and (13).

Remark 2: The effects of the power allocation coefficients, the number of feedback bits for each link and the other system parameters are mostly captured in the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ of the expression of LU k's rate lower bound $R_{u,k}^{lb}$ of *Theorem* and also mostly captured in the functions of $\Upsilon(n,\mathbf{\Sigma})$ and elements of matrices $\mathbf{\Sigma}_{SI,k}$ and $\mathbf{\Sigma}_{I,k}$ of the expression of Eve k's rate $R_{e,k}^{up}$ of *Theorem* 2. Thus, using the two theorems, the ESR of each LU in the system considered can be analyzed, although the results presented in *Theorem* 1 and *Theorem* 2 are given in complicated form. In spite of this, the theoretical results obtained are able to avoid the time-consuming computer simulations in evaluating system performance.

IV. Feedback Design Given the Power Allocation Coefficients

In this section, we will consider optimizing the ESR by feedback bits allocation for each LU with the feedback constraint given by (3). It is not difficult to see from the signal model (8) and *Theorem* 2 that the feedback bits allocation does not affect $R_{e,k}^{up}$ (and also $R_{e,k}$), but only affects the $R_{u,k}$. Therefore, the problem of optimizing ESR $R_{sec,k}$ given power allocation coefficients is equivalent to optimizing $R_{u,k}$. Denote the ergodic rate loss of LU k due to imperfect CSIT as $\Delta R_{u,k} \triangleq R_{u,k}^{ideal} - R_{u,k}$, where $R_{u,k}^{ideal} = \mathbb{E}\left[\log_2\left(1+\frac{d_{k,k}^{-\alpha}\phi_k P}{\sigma_u^2}|\mathbf{h}_{k,k}\mathbf{w}_k|^2\right)\right]$ is the ergodic rate of LU k with perfect CSIT of all downlink channels. Then, we can further turn the original optimization problem to minimizing the ergodic rate loss of each LU k. We first upper-bound $\Delta R_{u,k}$ in a relatively simpler closed form in the following theorem.

Theorem 3: $\Delta R_{u,k}$ can be upper-bounded as $\Delta R_{u,k} \leq \Delta R_{u,k}^{ub}$, where

$$\Delta R_{u,k}^{ub} = \log_2\left(g_k(2^{-\frac{B_{k,k}}{(K-1)(M-1)}}, \phi_k)\right) \tag{15}$$

with the function $g_k(z,\phi_k)$ being defined as

$$g_k(z,\phi_k) \triangleq \frac{C_{M,K} + (1-\phi_k)\tau_{k,k}z^{K-1} + \tilde{\tau}_k z^{-1}}{1-\vartheta_k z^{K-1}} \quad \forall k \tag{16}$$

and $C_{M,K} = \frac{(M-1)(M-K+1)}{(M-K)M}$, $\tau_{k,j} = \frac{d_{k,j}^{-\alpha}P(M-K+1)}{\sigma_u^2(M-K)}\Gamma\left(\frac{M}{M-1}\right)$ $(\forall k,j \in \{1,2,\cdots,K\})$ and $\vartheta_k = (1-Q_k)\Gamma\left(\frac{M}{M-1}\right)$, where

$$\begin{aligned} Q_k &\triangleq \mathbb{E}\left[\frac{1}{1+q_k||\mathbf{h}_{k,k}||^2}\right] = \frac{(-1)^{M-2}}{\Gamma(M)}q_k^{-M}\exp\left(q_k^{-1}\right) \\ &\quad \times E_i\left(-q_k^{-1}\right) - \sum_{m=1}^{M-1}\frac{(-1)^{M-m}\Gamma(m)}{\Gamma(M)}q_k^{m-M} < 1 \end{aligned} \tag{17}$$

with $q_k = \frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}$.

Proof: See Appendix B. □

Remark 3: It has been observed in many previous works and it is well known that the ergodic rate performance will be saturated once the transmit power exceeds a specific value in the system with imperfect/incomplete CSIT (see e.g., [21–23]). In addition, it is known that, by increasing the amount of CSI feedback bits as the transmit power increases, we can keep a constant ESR gap with respect to the system with perfect CSIT. The corresponding results are very similar to those in Section III-A 3) of [21]. And thus we do not present here for simplicity.

With *Theorem 3*, the problem of feedback bits allocation optimization can be formulated as

$$\begin{aligned} \min_{\{B_{k,j}\}} \quad & \frac{C_{M,K}+(1-\phi_k)\tau_{k,k}2^{-\frac{B_{k,k}}{M-1}}+\sum_{j=1,j\neq k}^{K}\tau_{k,j}2^{-\frac{B_{k,j}}{M-1}}}{1-\vartheta_k 2^{-\frac{B_{k,k}}{M-1}}} \\ \text{s.t.} \quad & \textstyle\sum_{j=1}^{K} B_{k,j} \leq B, \\ & B_{k,j} \in \mathcal{N},\ 1 \leq j \leq K, \end{aligned} \tag{18}$$

where $C_{M,K}$, $\tau_{k,j}$ and ϑ_k are defined in *Theorem* 3. It is easy to see that the objective function of Problem (18) is a continuous function of $\{B_{k,j}\}$ without the non-negative integer constraint on $\{B_{k,j}\}$. Moreover, it is easy to check that Problem (18) becomes a convex optimization by relaxing the constraint. Therefore, we can first obtain an approximate solution in the following theorem by relaxing the constraint on $\{B_{k,j}\}$, which will be shown later to be very close to the actual global optimal solution of (18).

Theorem 4: Without non-negative integer constraint, the solution to (18) is given by

$$B_{k,k}^{\star} = -(K-1)(M-1)\log_2(z^{\star}), \tag{19}$$

$$\begin{aligned} B_{k,j}^{\star} &= \frac{B-B_{k,k}^{\star}}{K-1} + (M-1)\log_2(d_{k,j}^{-\alpha}) \\ &\quad -\frac{M-1}{K-1}\sum_{i=1,i\neq k}^{K}\log_2(d_{k,i}^{-\alpha}), j \neq k, \end{aligned} \tag{20}$$

where

$$z^{\star} = \begin{cases} 1, & h_k(1,\phi_k) \leq 0, \\ \tilde{z}, & h_k\left(2^{-\frac{B}{(K-1)(M-1)}},\phi_k\right) < 0 \\ & \text{and } h_k(1,\phi_k) > 0, \\ 2^{-\frac{B}{(K-1)(M-1)}}, & h_k\left(2^{-\frac{B}{(K-1)(M-1)}},\phi_k\right) \geq 0 \end{cases}. \tag{21}$$

$\tilde{z}$ is the solution of Equation $h_k(z,\phi_k) = 0$ that satisfies $2^{-\frac{B}{(K-1)(M-1)}} < \tilde{z} < 1$, where the functions $h_k(z,\phi_k)$ for $k = 1,\cdots,K$ are defined as

$$h_k(z,\phi_k) = (K-1)[C_{M,K}\vartheta_k + (1-\phi_k)\tau_{k,k}]z^K$$

$$+K\vartheta_k\tilde{\tau}_k z^{K-1} - \tilde{\tau}_k, \quad (22)$$

with $\tilde{\tau}_k = (K-1)\left(\prod_{j=1,j\neq k}^{K} \tau_{k,j}^{\frac{1}{K-1}}\right) 2^{-\frac{B}{(K-1)(M-1)}}$ and $C_{M,K}$, $\tau_{k,j}$, ϑ_k are the same as defined in *Theorem* 3.

Proof: We solve the problem considered here in two steps. The first step is to optimize the feedback bits allocation to the ICI links given that for the target LU's link, which can be written as

$$\begin{aligned} \min_{\{B_{k,j}\},j\neq k} \quad & \sum_{j=1,j\neq k}^{K} \tau_{k,j} 2^{-\frac{B_{k,j}}{M-1}} \\ \text{s.t.} \quad & \sum_{j=1,j\neq k}^{K} B_{k,j} \leq B - B_{k,k}. \end{aligned} \quad (23)$$

It follows with arithmetic-geometric mean inequality that

$$\sum_{j=1,j\neq k}^{K} \tau_{k,j} 2^{-\frac{B_{k,j}}{M-1}} \geq (K-1)\left(\prod_{j=1,j\neq k}^{K} \tau_{k,j}^{\frac{1}{K-1}}\right) 2^{-\frac{B-B_{k,k}}{(K-1)(M-1)}},$$

where the equality holds if and only if $\tau_{k,j} 2^{-\frac{B_{k,j}}{M-1}}$ $\forall j \neq k$ are all equal. Then, the solution of $B_{k,j}$ in (20) follows.

Secondly, substituting (20) into (18) with the definition of function $g_k(z,\phi_k)$ in (16) and $z = 2^{-\frac{B_{k,k}}{(K-1)(M-1)}}$, the original problem in (18) can be equivalently written as

$$\begin{aligned} \min_{z} \quad & g_k(z,\phi_k) \\ \text{s.t.} \quad & 2^{-\frac{B}{(K-1)(M-1)}} \leq z \leq 1. \end{aligned} \quad (24)$$

It is easy to obtain that $\frac{\partial g_k(z,\phi_k)}{\partial z} = \frac{z^{-2} h_k(z,\phi_k)}{(1-\vartheta_k z^{K-1})^2}$ and $\frac{\partial h_k(z,\phi_k)}{\partial z} = K(K-1)z^{K-2}\{[C_{M,K}\vartheta_k + (1-\phi_k)\tau_{k,k}]z + \vartheta_k\tilde{\tau}_k\}$, where $h_k(z,\phi_k)$ is defined as (22). It is easy to see that $\frac{\partial h_k(z,\phi_k)}{\partial z} > 0$ always holds for $z > 0$. Thus, $h_k(z,\phi_k)$ is monotonically increasing of z. Notice that $h_k(0,\phi_k) = -\tilde{\tau}_k < 0$ and $\lim_{z\to+\infty} h_k(z,\phi_k) = +\infty$, $\forall \phi_k \in (0,1]$. It follows that $\frac{\partial g_k(z,\phi_k)}{\partial z} = 0$ must have a unique root of $z > 0$ $\forall \phi_k \in (0,1]$. The theorem follows by combing the above properties. □

Equation $h_k(z,\phi_k) = 0$ is a polynomial of degree K. According to *Abel-Ruffini Theorem*, there is no analytical solution to a general polynomial equations of degree five or higher with arbitrary coefficients [28]. The size of a coordinated BS cluster in practise is usually no more than 5. When $K \geq 5$, the roots of equation $h_k(z,\phi_k) = 0$ are generally obtained by numerical method.

We then propose an algorithm to find practical integer feedback bits allocation for LU k. First, let $\bar{\mathbf{B}}_k \in \mathcal{N}^{K\times 1}$ denote the integer bits allocation with the i-th element obtained as $\bar{\mathbf{B}}_k(i) = \lfloor B^\star_{k,i} \rfloor$, where $B^\star_{k,i}$ is obtained from *Theorem* 4, and $\tilde{\mathbf{B}}^\star_k \in \mathcal{N}^{K\times 1}$ denote the final feedback bits allocation obtained from **Algorithm I**. Moreover, let $\mathbf{r}_{v,k} \in \mathbb{R}^{K\times 1}$ denote the ergodic rate gain with the i-th element $\mathbf{r}_{v,k}(i) = \Delta R^{ub}_{u,k}(\bar{\mathbf{B}}_k(i)) - \Delta R^{ub}_{u,k}(\bar{\mathbf{B}}_k(i)+1)$ being the ergodic rate gain if one extra bit is added to $B_{k,i}$, where $\Delta R^{ub}_{u,k}$ is given by (15). Let $B_{r,k} = B - \sum_{i=1}^{K} \bar{\mathbf{B}}_k(i)$ denote the remaining bits that need to be re-allocated to the feedback of LU k. Then, the problem to find the optimal practical feedback bits allocation can be written as

$$\begin{aligned} \max_{\tilde{\mathbf{B}}^\star_k} \quad & \left[\tilde{\mathbf{B}}^\star_k - \bar{\mathbf{B}}_k\right]^T \mathbf{r}_{k,v} \\ \text{s.t.} \quad & \sum_{i=1}^{K} \left[\tilde{\mathbf{B}}^\star_k(i) - \bar{\mathbf{B}}_k(i)\right] \leq B_{r,k}, \\ & \left[\tilde{\mathbf{B}}^\star_k(i) - \bar{\mathbf{B}}_k(i)\right] \in \{0,1\}, \forall i. \end{aligned} \quad (25)$$

TABLE I: Dynamic Programming Algorithm to Find Practical Feedback Bits Allocation

Algorithm I. **Input:** $\bar{\mathbf{B}}_k$, $\mathbf{r}_{v,k}(i)$, $B_{r,k}$ **Output:** $\tilde{\mathbf{B}}^\star_k$
Initialize: $\mathbf{S} = \text{zeros}(K+1, B_{r,k}+1)$, $\mathbf{B}_{rc} = \text{zeros}(K, K+1, B_{r,k}+1)$.
for $i = 2$ *to* $K+1$ **do**
for $j = 2$ *to* $B_{r,k}+1$ **do**
if $\mathbf{S}(i-1,j) > \mathbf{S}(i-1,j-1) + \mathbf{r}_{v,k}(i-1)$ **then**
$\mathbf{B}_{rc}(:,i,j) = \mathbf{B}_{rc}(:,i-1,j)$;
$\mathbf{S}(i,j) = \mathbf{S}(i-1,j)$;
else
$\mathbf{B}_{rc}(:,i,j) = \mathbf{B}_{rc}(:,i-1,j-1)$;
$\mathbf{B}_{rc}(i,i,j) = 1$;
$\mathbf{S}(i,j) = \mathbf{S}(i-1,j-1) + \mathbf{r}_{v,k}(i-1)$;
end if
end for
end for
$\tilde{\mathbf{B}}^\star_k = \bar{\mathbf{B}}_k + \mathbf{B}_{rc}(:,K+1,B_{r,k}+1)$;

(25) is a classical $0/1$ Knapsack Problem [29] and can be solved by **Algorithm** I in Table I.

Remark 4: The time complexity of the **Algorithm** I is $\mathcal{O}(KB_{r,k})$. Moreover, it is easy to see that $B_{r,k} \leq \lceil K/2 \rceil$. Thus, the time complexity of the algorithm is a polynomial of the scale of a coordinated system (i.e., K), which is a very low-complexity algorithm.

The analytical results of the optimal feedback bits allocation for general system settings are a bit complex, and thus it is difficult to observe any design insights. Therefore, we provide in the following corollary the result for the very typical case of two-cell coordination (i.e., $K = 2$), which can be easily obtained by solving $h_k(z,\phi_k) = 0$ as a quadratic equation with the constraints given in the second line of (21) and substituting $\tilde{z}$ into (19) and (20).

Corollary 1: When $K = 2$ and the conditions $h_k(2^{-\frac{B}{M-1}},\phi_k) < 0$ and $h_k(1,\phi_k) > 0$ in (21) are satisfied given ϕ_k, the optimal solution to (18) becomes

$$B^\star_{k,k} = (M-1) \times \log_2\left(\sqrt{\vartheta_k^2 + \frac{1}{\tilde{\tau}_k}[C_{M,2}\vartheta_k + (1-\phi_k)\tau_{k,k}]} + \vartheta_k\right), \quad (26)$$

$$B^\star_{k,j} = B - B^\star_{k,k}, \ j \neq k, \quad (27)$$

where $C_{M,2} = 1 + \frac{1}{M(M-2)}$, and $\tau_{k,j}$, ϑ_k and $\tilde{\tau}_k$ are as defined in *Theorem* 3 and *Theorem* 4.

According to the definition of ϑ_k in *Theorem* 3, it is easy to show that $0 < \vartheta_k < \Gamma(\frac{M}{M-1}) < 1$, and ϑ_k is an increasing function of transmit signal-to-noise ratio (SNR) $\frac{P}{\sigma_u^2}$ and a decreasing function of distance $d_{k,k}$. Moreover, it is easy to obtain that $\frac{\tau_{k,k}}{\tilde{\tau}_k} = 2^{\frac{B}{M-1}} \frac{d^\alpha_{k,j}}{d^\alpha_{k,k}}$ and $\frac{\vartheta_k}{\tilde{\tau}_k} = \frac{M-2}{M} 2^{\frac{B}{M-1}} d^\alpha_{k,j}(1-Q_k)/(\frac{P}{\sigma_u^2})$. Similarly, it is easy to show that $(1-Q_k)/(\frac{P}{\sigma_u^2})$ is a decreasing function of both $\frac{P}{\sigma_u^2}$ and $d_{k,k}$. Thus, $\frac{\vartheta_k}{\tilde{\tau}_k}$ is an increasing function of $d_{k,j}$ and a decreasing function of $\frac{P}{\sigma_u^2}$, and $\frac{\tau_{k,k}}{\tilde{\tau}_k}$ is a decreasing function of $d_{k,k}$ and an increasing function of $d_{k,j}$ $(j \neq k)$. Moreover, the term $\frac{1}{\tilde{\tau}_k}[C_{M,2}\vartheta_k + (1-\phi_k)\tau_{k,k}]$ in (26) can be written as

$$\begin{aligned}&\frac{1}{\tilde{\tau}_k}[C_{M,2}\vartheta_k + (1-\phi_k)\tau_{k,k}]\\&= \left[C_{M,2}\frac{M-1}{M-2}\frac{(1-Q_k)}{\left(\frac{P}{\sigma_u^2}d_{k,k}^{-\alpha}\right)} + (1-\phi_k)\right]\left(\frac{d_{k,j}^{\alpha}}{d_{k,k}^{\alpha}}\right)2^{\frac{B}{M-1}} \quad (28)\end{aligned}$$

with $\frac{(1-Q_k)}{\left(\frac{P}{\sigma_u^2}d_{k,k}^{-\alpha}\right)} = \mathbb{E}\left[\frac{\phi_k\frac{M-1}{M}\|\mathbf{h}_{k,k}\|^2}{1+\phi_k\frac{P}{\sigma_u^2}d_{k,k}^{-\alpha}\frac{M-1}{M}\|\mathbf{h}_{k,k}\|^2}\right]$ which is a decreasing function of $\frac{P}{\sigma_u^2}$. According to user association rule, we have $d_{k,k} \leq d_{k,j}$ $\forall j \neq k$. It follows that $\frac{d_{k,j}^{\alpha}}{d_{k,k}^{\alpha}} \geq 1$. Although ϑ_k is an increasing of $\frac{P}{\sigma_u^2}$, it is very limited (< 1) compared with the term $\frac{1}{\tilde{\tau}_k}[C_{M,2}\vartheta_k + (1-\phi_k)\tau_{k,k}]$ in (26) and changes very little in the medium and high SNR regime. Thus, $\frac{1}{\tilde{\tau}_k}[C_{M,2}\vartheta_k+(1-\phi_k)\tau_{k,k}]$ dominates in (26). Then, the following observations about the optimal feedback bit design for two-cell coordination follow from the above results.

(1) With the other system parameters fixed, the optimal feedback bits $B_{k,k}^{\star}$ decreases as $d_{k,k}$ increases and increases as $d_{k,j}$ $(j \neq k)$ increases. $B_{k,j}^{\star}$ changes conversely.

(2) With the other system parameters fixed, $B_{k,k}^{\star}$ decreases as $\frac{P}{\sigma_u^2}$ increases. $B_{k,j}^{\star}$ changes conversely.

(3) With the other system parameters fixed, the change of $B_{k,k}^{\star}$ as power allocation coefficient ϕ_k is more involved. However, it is easy to see that, when ϕ_k is large enough (satisfies the sufficient condition $\phi_k\frac{P}{\sigma_u^2}d_{k,k}^{-\alpha} \geq \frac{(M-1)^3}{(M-2)^2M}$), $B_{k,k}^{\star}$ decreases as ϕ_k increases.

According to (21) in *Theorem* 4, for a given set of system parameters (except ϕ_k and $B_{k,j}$ $\forall k,j$), the sufficient and necessary conditions for there is at least one BS other than BS k to coordinate with it for the data transmission to LU k are given by $h_k\left(2^{-\frac{B}{(K-1)(M-1)}}, \phi_k\right) < 0$ *and* $h_k(1,\phi_k) > 0$. However, the conditions are given in an implicit form. It is very difficult to observe any insight on the interactions among multiple system parameters. Thus, we provide sufficient conditions for there are at least two BSs to coordinate in the following lemma.

Lemma 2: Let the following two conditions be (C1) $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2} > \chi(\kappa_k)$ and $B \geq B_{th1,k}$, and (C2) $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2} \leq \chi(\kappa_k)$ and $B \geq B_{th2,k}$, where $\chi(x) \triangleq \frac{\frac{2[(M-1)(M-K)-1]}{M(M-K+1)}}{x+\sqrt{x^2+\frac{4[(M-1)(M-K)-1]}{M}}}$ with $\kappa_k = 2 - \frac{K\Gamma\left(\frac{M}{M-1}\right)\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}}{d_{k,k}^{-\alpha}}$, and $B_{th1,k}$ and $B_{th2,k}$ are respectively given by

$$B_{th1,k} \triangleq (M-1)\times \log_2\left(\frac{\left(1+\frac{M-1}{M}\cdot\frac{\sigma_u^2}{d_{k,k}^{-\alpha}P}\right)d_{k,k}^{-\alpha}}{\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}} + K\Gamma\left(\frac{M}{M-1}\right)\right),$$

$$B_{th2,k} \triangleq (M-1)\times \log_2\left(\frac{\frac{(M-1)}{M}\frac{d_{k,k}^{-\alpha}}{\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}} + K\Gamma\left(\frac{M}{M-1}\right)\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2}}{\frac{1}{M-K+1}+\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2}}\right).$$

When either one of the conditions (C1) or (C2) is satisfied, there is at least one BS other than BS k that coordinates with it. Moreover, when $K \leq 4$, we have $\chi(\kappa_k) < 1$ for arbitrary system parameters other than K.

Proof: See Appendix C. □

Remark 5: According to this lemma, the minimum required B of each LU approximately increases linearly as $(M-1)$ and decreases logarithmically as $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2}$ increases, which look natural results.

Remark 6: In the very related paper [21], the feedback bits allocation of each LU was obtained by an intuitive greedy method whose optimality in term of ESR was not proved and could not be guaranteed. In contrast, we propose an analytical closed-form solution of feedback bits allocation (without integer constraint), which together with **Algorithm I** can be proved to be the *global optimal solution* to maximize the obtained upper bound of ESR loss. The analytical solution is also useful for further analysis to develop the other analytical results in *Lemma* 2. Moreover, the complexity of our scheme is no more than $\mathcal{O}(K^2/2)$, whilst the complexity of the scheme in [21] is $\mathcal{O}((K-1)B)$. In practise, the number of B is much larger than that of K. Thus, the complexity of our proposed scheme is much lower than that in [21]. The numerical results in Section VI will illustrate that our proposed feedback bits allocation can be overlapped with or be very close to the real optimal solution.

V. Optimization of Power Allocation Coefficients

In the previous section, we have optimized the feedback bits allocation for the target signal link and ICI links of each LU with the power allocation coefficients of all BS $\{\phi_k\}$ given. With the solution of feedback bits allocation (as functions of $\{\phi_k\}$) given by *Theorem* 4, we study in this section the optimization of power allocation coefficient at each BS to maximize the system ESSR of all coordinated cells. Since the exact analytical result of ESSR is unknown, we take the obtained lower bound on system ESSR $R_{sec,sum}^{lb}$ given by (14) as the objective. Moreover, it is very inconvenient to directly take $R_{sec,sum}^{lb}$ as the objective function. Instead, we first define $\mathcal{L}_k(\phi_1,\cdots,\phi_K) \triangleq R_{u,k}^{lb} - R_{e,k}^{up}$ and $\mathcal{L}(\phi_1,\cdots,\phi_K) \triangleq \sum_{k=1}^{K}\mathcal{L}_k(\phi_1,\cdots,\phi_K)$. Then, we can transform the original problem as

$$\begin{aligned}\max_{\{\phi_k\}} \quad & \mathcal{L}(\phi_1,\cdots,\phi_K) \\ \text{s.t.} \quad & 0 < \phi_k \leq 1, \quad \forall k.\end{aligned} \quad (29)$$

We note that, in multi-cell scenario, the global optimal power allocation for AN-aided secure transmission design (*with or without* multi-BS coordination) in general are extremely difficult to obtain due to the complicated analytical secrecy performance results (if they could be obtained). As as far as we know, there has been *no* published papers that developed *any analytical solution* for the power allocation optimization problems. Even, some existing papers did not consider analytical method of the power allocation optimization (see e.g., [25, 30]). The same thing happens to our system design. As it will be shown below, the objective function of our power allocation problem is generally *not* a concave/convex function, and thus the global optimal solution cannot be guaranteed to be obtained in theory.

$$\frac{\partial^2 R_{u,1}^{lb}}{\partial \phi_1^2} = \mathbb{E}_{D_1,D_2,D_3}\left[\frac{\log_2(e)(M-K)(D_1-D_2)(D_2+D_3)\varrho(\phi_1)}{(-D_2\phi_1+D_2+D_3)^2\{[(M-K)(D_1-D_2)-D_2]\phi_1+D_2+D_3\}^2}\right], \tag{31}$$

For illustrative purpose, in the following we will focus on the two-cell case. The two-cell case is such a typical case of multi-cell coordination that, for many published papers, the whole article *only* considered the two-cell system (see e.g., [19, 20]). When $K = 2$, $\mathcal{L}(\phi_1, \phi_2) = R_{u,1}^{lb} - R_{e,1}^{up} + R_{u,2}^{lb} - R_{e,2}^{up}$. It is easy to see $R_{u,k'}^{lb}$ is ϕ_k-independent for $k' \neq k$ and $k, k' \in \{1, 2\}$. Thus, we have

$$\frac{\partial^2 \mathcal{L}(\phi_1,\phi_2)}{\partial \phi_k^2} = \frac{\partial^2 R_{u,k}^{lb}}{\partial \phi_k^2} - \frac{\partial^2 R_{e,k}^{up}}{\partial \phi_k^2} - \frac{\partial^2 R_{e,k'}^{up}}{\partial \phi_k^2}. \tag{30}$$

To see the property of $\mathcal{L}(\phi_1, \phi_2)$, we first need to study each of $R_{u,k}^{lb}$ and $R_{e,k}^{up}$ ($k = 1, 2$) as functions of ϕ_1, ϕ_2. In the following, we first consider the property of $R_{u,k}^{lb}$.

A. Some Properties of $R_{u,k}^{lb}$ as A Function of (ϕ_1, ϕ_2)

We will present the results for the case with $k = 1$ and $k' = 2$ only. The corresponding results for the case with $k = 2$ and $k' = 1$ follow in the same way. According to (36) and using the joint distribution results in [23, *Lemma* 2] which are presented in Appendix A, we can rewrite $R_{u,1}^{lb}$ as[4] $R_{u,1}^{lb} = \mathbb{E}\left[\log_2\left(1 + \frac{(M-K)(D_1-D_2)\phi_1}{-D_2\phi_1+D_2+D_3}\right)\right]$, where the expectation is on three RVs D_1, D_2 and D_3 which are distributed as $D_1 \overset{d.}{=} d_{1,1}^{-\alpha}(X_{1,1} + Y_{1,1})$, $D_2 \overset{d.}{=} d_{1,1}^{-\alpha}\delta_{1,1}Y_{1,1}$ and $D_3 \overset{d.}{=} d_{1,2}^{-\alpha}\delta_{1,2}Y_{1,2} + \frac{(M-1)\sigma_u^2}{P}$ with independent RVs $X_{1,1} \sim \text{Gamma}(1, 1)$, $Y_{1,1} \sim \text{Gamma}(M-1, 1)$ and $Y_{1,2} \sim \text{Gamma}(M-1, 1)$. Then, $\frac{\partial^2 R_{u,1}^{lb}}{\partial \phi_1^2}$ in (30) can be obtained as (31) at the top of this page, where $\varrho(\phi_1) \triangleq 2D_2[(M-K)(D_1-D_2)-D_2]\phi_1 - (D_2+D_3)[(M-K)(D_1-D_2)-2D_2]$.

As far as we can see that the exact closed-form expression of $\frac{\partial^2 R_{u,1}^{lb}}{\partial \phi_1^2}$ for arbitrarily given set of system parameters is very difficult to obtain if not impossible. And it also seems intractable to determine the sign of $\frac{\partial^2 R_{u,1}^{lb}}{\partial \phi_1^2}$ by any *analytical* method. Thus, we will present some observations from the *instantaneous* term $\tilde{R}_{u,1}^{lb} \triangleq \log_2\left(1 + \frac{(M-K)(D_1-D_2)\phi_1}{-D_2\phi_1+D_2+D_3}\right)$ inside the expectation of $R_{u,1}^{lb}$. According to (31), the sign of $\frac{\partial^2 \tilde{R}_{u,1}^{lb}}{\partial \phi_1^2}$ depends on $\varrho(\phi_1)$ only. Denote the root of Equation $\varrho(\phi_1) = 0$ as $\tilde{\varepsilon}$, which is obtained as $\tilde{\varepsilon} = \frac{D_2+D_3}{2D_2}\left(1 - \frac{D_2}{(M-K)(D_1-D_2)-D_2}\right)$. It is easy to check that, when $B_{1,1} > (M-1)\log_2\left(1 + \frac{2}{M-K}\right)$ holds, the condition $(M-K)(D_1-D_2) > 2D_2$ is satisfied, which can be easily satisfied by a practical system. For example, the condition only requires $B_{1,1} \geq 3$ for $M \geq 4$ with $K = 2$. Then, it is easy to see that the property of $\tilde{R}_{u,1}^{lb}$ belongs to one of the following two cases:

- (a) When $(M-K)(D_1-D_2) \geq 2D_2$ and $0 < \tilde{\varepsilon} < 1$ (notice that $\tilde{\varepsilon} > 0$ when $(M-K)(D_1-D_2) > 2D_2$), if $0 < \phi_1 < \tilde{\varepsilon}$, we have $\frac{\partial^2 \tilde{R}_{u,1}^{lb}}{\partial \phi_1^2} \geq 0$ and thus $\tilde{R}_{u,1}^{lb}$ is a concave function; if $\tilde{\varepsilon} < \phi_1 < 1$, we have $\frac{\partial^2 \tilde{R}_{u,1}^{lb}}{\partial \phi_1^2} \geq 0$ and thus $\tilde{R}_{u,1}^{lb}$ is a convex function.
- (b) When $(M-K)(D_1-D_2) \geq 2D_2$ and $\tilde{\varepsilon} \geq 1$, we have $\frac{\partial^2 \tilde{R}_{u,1}^{lb}}{\partial \phi_1^2} \leq 0$, and thus $\tilde{R}_{u,1}^{lb}$ is a concave function for $0 < \phi_1 \leq 1$.

[4]To gain some insights on the properties of $R_{u,k}^{lb}$ and for the tractability of analysis, we assume the feedback bits allocation is given and fixed in this subsection.

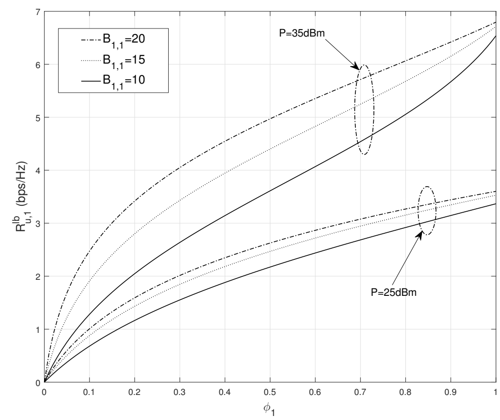

Fig. 1: $R_{u,1}^{lb}$ as a function of ϕ_1 under different values of P and $B_{1,1}$.

Although the above analysis is on the instantaneous term $\tilde{R}_{u,1}^{lb}$, since it is related to $R_{u,1}^{lb}$ by $R_{u,1}^{lb} = \mathbb{E}\big[\tilde{R}_{u,1}^{lb}\big]$, we can infer that the corresponding property of $R_{u,1}^{lb}$ is very similar. Specifically, one of the following two cases happens in a practical system: (1) $R_{u,1}^{lb}$ as a function of ϕ_1 is concave for $\phi_1 \in (0, \varepsilon)$ and convex for $\phi_1 \in (\varepsilon, 1]$ with some $\varepsilon \in (0, 1)$; or (2) $R_{u,1}^{lb}$ is concave for $\phi_1 \in (0, 1]$. Unfortunately, as far as we can see, it is intractable to obtain the closed-form expression of ε and to further analytically characterize the condition for each case to happen for an arbitrary set of system parameters. For verifications, we plot in Fig. 1 $R_{u,1}^{lb}$ versus ϕ_1 for the systems with $M = 6$, $N_e = 2$, $d_{1,1} = 40$m, $d_{1,2} = 50$m, $l_{1,1} = 40$m, $l_{1,2} = 50$m, $\sigma_u^2 = -50$dBm and different values of P and $B_{1,1}$. We can see that, when $P = 35$dBm, as $B_{1,1}$ increases as 10, 15 and 20, the property of $R_{u,1}^{lb}$ belongs to Case (a) and the boundary ε increases as $B_{1,1}$ increases. When $P = 25$dBm, the property of $R_{u,1}^{lb}$ with different values of $B_{1,1}$ belong to Case (b) or belong to Case (a) with $\varepsilon < 1$, but is very close to 1.

B. Some Properties of $R_{e,k}^{up}$ as A Function of (ϕ_1, ϕ_2)

In this subsection, we study the properties of $\frac{\partial^2 R_{e,1}^{up}}{\partial \phi_1^2}$ and $\frac{\partial^2 R_{e,2}^{up}}{\partial \phi_1^2}$. Some results are given in the following lemma.

Lemma 3: Given ϕ_k, $R_{e,k}^{up}$ ($k = 1, 2$) is a convex function of $\phi_{k'} \in (0, 1]$ with $k' \neq k$. But it is *generally neither* a convex *nor* a concave function of $\phi_k \in (0, 1]$ with a given $\phi_{k'} \in (0, 1]$. Specifically, $R_{e,k}^{up}$ is a concave function of ϕ_k for $\phi_k \in (0, \varsigma]$ and a convex function of ϕ_k for $\phi_k \in (\varsigma, 1]$ with some $\varsigma \in (0, 1)$ which depends on the concrete system parameters. Therefore, $R_{e,k}^{up}$ is generally *neither* a convex *nor* a concave function of $(\phi_1, \phi_2) \in (0, 1] \times (0, 1]$.

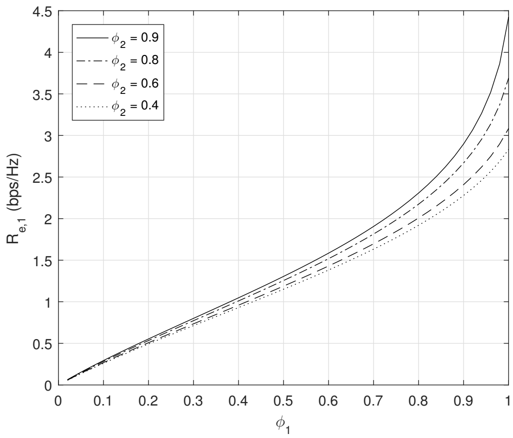

Fig. 2: $R_{e,1}^{up}$ as a function of ϕ_1 with fixed ϕ_2.

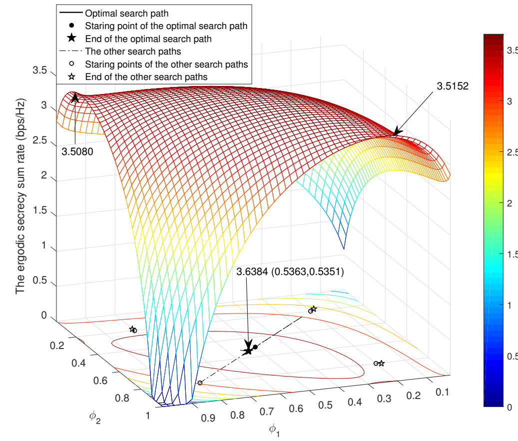

Fig. 3: The ESSR as a function of (ϕ_1, ϕ_2) with the path parameters (a) and $P = 35$ dBm.

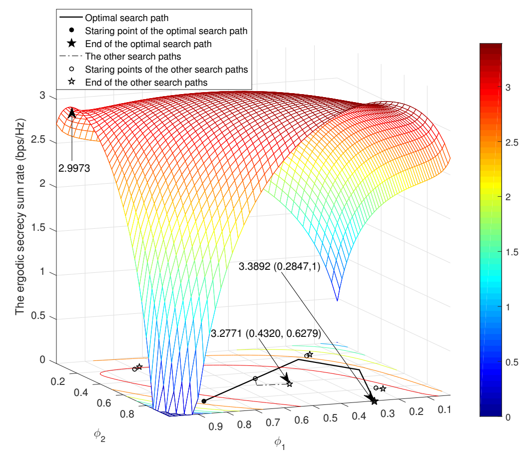

Fig. 4: The ESSR as a function of (ϕ_1, ϕ_2) with the path parameters (b) and $P = 35$ dBm.

Proof: See Appendix D. □

For verifications, we particularly plot $R_{e,1}^{up}$ as a function of ϕ_1 for some values of ϕ_2 in Fig. 2. We can see $R_{e,1}^{up}$ is concave of ϕ_1 when ϕ_1 is closer to zero and is convex when ϕ_1 is large enough.

Considering the above analytical study of the two-cell system is already very complicated, the corresponding study of the system with more coordinated cells seems to be analytically intractable. With the above properties of $R_{u,k}^{lb}$ and $R_{e,k}^{up}$ as functions of (ϕ_1, ϕ_2) for the two-cell scenario, one may expect that the objective function $\mathcal{L}(\phi_1, \phi_2, \cdots, \phi_K)$ for $K \geq 2$ is *generally neither* a convex *nor* concave function. The numerical results in the next section will verify that this fact is really true. Since the problem in (29) is generally non-convex, an optimum solution cannot be found using standard optimization methods. Therefore, we propose to employ some numerical method to obtain sub-optimal (local-optimal) solution to jointly optimize the power allocation coefficients with feedback bits allocation. Specifically, *Lemma* 2 shows that $z^\star = \tilde{z}$ in (21) holds under most practical system settings. Thus, we can substitute the optimized feedback bits allocation given by (19) and (20) into the objective function $\mathcal{L}(\phi_1, \phi_2, \cdots, \phi_K)$ in (29). Then, the problem (29) can be solved by interior point method with time complexity of $\mathcal{O}(TK^{3.5}\log(K/\varepsilon_0))$, where T is the iteration times and $\varepsilon_0 > 0$ is the preset accuracy tolerance [31].

VI. Numerical Results

For the numerical results, we consider the systems of two coordinated cells ($K = 2$) with $M = 6$, $N_e = 2$ and two different sets of path parameters, i.e., (a) $d_{1,1} = l_{1,1} = 40$ m, $d_{1,2} = l_{1,2} = 50$ m, $d_{2,1} = l_{2,1} = 50$ m and $d_{2,2} = l_{2,2} = 40$ m, and (b) $d_{1,1} = l_{1,1} = 40$ m, $d_{1,2} = l_{1,2} = 50$ m, $d_{2,1} = l_{2,1} = 45$ m and $d_{2,2} = l_{2,2} = 45$ m. The path-loss exponent $\alpha = 4$ and the power of AWGN at each LU is $\sigma_u^2 = -50$ dBm.

Fig. 3 - Fig. 5 each shows the simulated actual system ESSR as a function of (ϕ_1, ϕ_2) with the optimized feedback bits allocation given by *Theorem 4* and $B = 20$ for different path parameters and transmit power. We choose five initial points for the interior point method, i.e., $(\phi_1^{\text{ini}}, \phi_2^{\text{ini}}) = \{(0.2, 0.2), (0.2, 0.8), (0.5, 0.5), (0.8, 0.2), (0.8, 0.8)\}$ with $\varepsilon_0 = 10^{-6}$ and the maximum iteration times 1000. Moreover, the search path leading to the optimal solution is marked using solid line and the others are marked with dotted lines. Specifically, Fig. 3 shows the results for the system with $P = 35$ dBm and path parameters (a) which is symmetric, i.e., $d_{1,1} = d_{2,2}$ and $d_{1,2} = d_{2,1}$. It is natural to expect that the curve of ESSR is also symmetric with respect to the axis $\phi_1 = \phi_2$. The optimal power allocation obtained by numerical method is $\phi_1^\star = 0.5363$ and $\phi_2^\star = 0.5351$, which is approximately located on the line of $\phi_1 = \phi_2$.

Fig. 4 plots the simulated system ESSR for the system with $P = 35$ dBm and the path parameters (b) which is asymmetric. The numerical method obtains the optimal power allocation $\phi_1^\star = 0.2847$ and $\phi_2^\star = 1$. Thus, compared to the system with symmetric path parameters in Fig. 3 BS 1 reduces the power of the message-bearing signals and BS 2 allocates more (full) power to message-bearing signals. Similar to Fig. 4, Fig. 5 plots the simulated system ESSR for asymmetric path parameters (b) with the transmit power reduced to $P = 30$ dBm. The optimal power allocation obtained by the numerical method is $\phi_1^\star = 0.4537$ and $\phi_2^\star = 0.5803$. Then, we can observe an interesting result in the systems with asymmetric path parameters (b) that, BS 1 allocates larger ratio of power but BS 2 allocates smaller

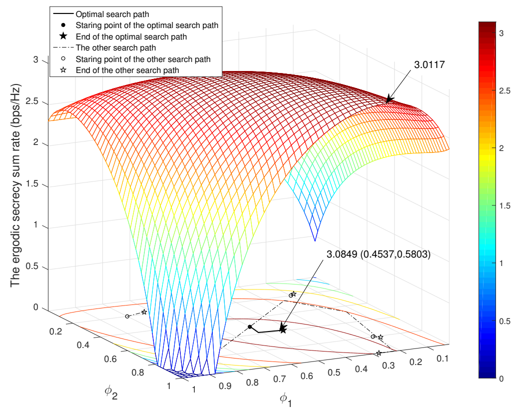

Fig. 5: The ESSR as a function of (ϕ_1, ϕ_2) with the path parameters (b) and $P = 30$ dBm.

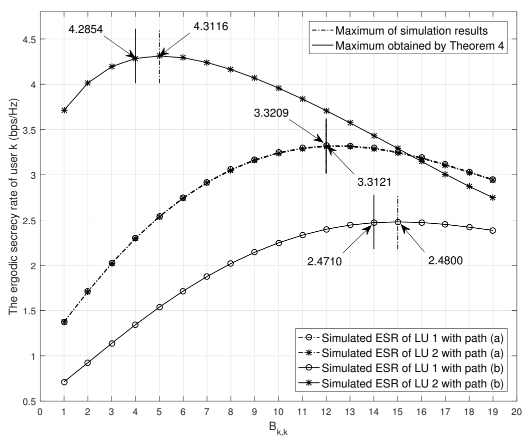

Fig. 6: The simulation result of each LU's ergodic rate as a function of $B_{k,k}$.

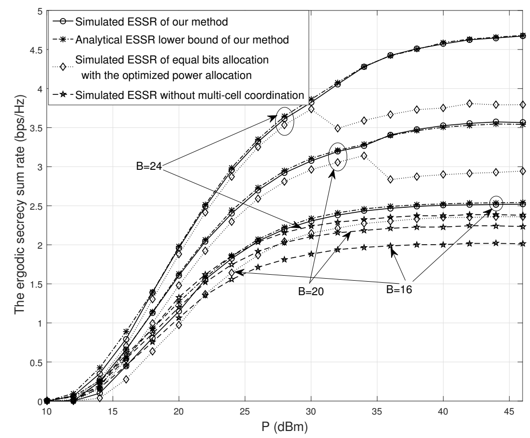

Fig. 7: Comparisons of the simulation and analytical results of the ESSR v.s. P of different power and feedback bits allocation method.

ratio of transmit power to the message-bearing signals when the transmit power is reduced. By observing these three figures, it can be easily seen that the ESSR as a function of (ϕ_1, ϕ_2) generally is neither concave nor convex at the whole region of $(0, 1] \times (0, 1]$, which can be explained by our analytical studies in Section V. Moreover, it can be easily seen the coordinated scheme without employing AN (compared with the scheme in [21]) even can not work for the systems with multi-antenna Eves (The ESR of each LU is zero).

We plot in Fig. 6 the simulation result of each LU's actual ergodic rate as a function of $B_{k,k}$ for different sets of path parameters (a) and (b), and $P = 35$ dBm. The power allocation of each BS is determined according to the optimized solutions of Fig. 3 and Fig. 4, respectively. We also marked down the maximum value of each curve and the value corresponding to our method. We find in the figure the rate loss from the optimal is within 1 percentage for all the system settings considered. These results illustrate the effectiveness of our proposed feedback bits allocation method.

In Fig. 7, we plot the simulation and analytical results of the ESSR v.s. P in dBm of our proposed joint power and feedback bits allocation method with path parameters (b) and $B = 20, 30$ bits. For comparisons, we also plot the curves of the simulation with equal feedback bits allocation and the optimized power allocation using our numerical method, and the simulation for the two-cell systems without BS-coordination where the B bits are all used for the CSI feedback of each LU's channel. We can see our proposed scheme always works better than any other schemes, and the performance of the un-coordinated transmission is the worst of all when P is large enough ($P > 22$ dBm in the figure) for all system settings considered. The performance gaps between ours and any other one will increase as B increases large. Moreover, our analytical lower bound of ESR given by (14) is very close to the real values. In addition, as the transmit power increase from low SNR to $P < 35$ dBm, $\phi_2^\star$ is smaller than 1, and the performance gap between our scheme and that of equal feedback bits allocation is relatively small. We find this is because the optimized power allocation scheme tends to result in close to the equal bits allocation. As P increases to $P > 35$ dBm, $\phi_2^\star$ reaches 1 and $\phi_1^\star$ changes very little, the optimal feedback bits allocation becomes far from the equal feedback bits allocation, and thus the performance gap increases.

VII. Conclusions

In this paper, we have considered the AN-aided secure CoMP transmission design in downlink FDD multi-cell multi-antenna systems with limited CSI feedback in the presence of an multi-antenna eavesdropper in each cell. Closed-form expression of a lower bound on the ESR of each LU has been derived without assuming asymptotes for any system parameter for the first time. A low-complexity algorithm has been proposed to optimize the CSI feedback bits allocation for the channels of the target signal link and ICI links of each LU. In addition, sufficient conditions on the set of the system parameters under which there are at least two BSs to coordinate transmission have been investigated and obtained. The analytical lower bound on the ESR of each LU as a function of the set of power allocation coefficients of all coordinated cells has been verified to be neither convex nor concave. Then, some numerical method has been proposed to find sub-optimal solution to the set of power allocation coefficients. The numerical results have

$$R_{u,k} \geq \mathbb{E}\Bigg\{ \log_2 \Bigg[1 + \Big(d_{k,k}^{-\alpha}\phi_k P||\mathbf{h}_{k,k}||^2 \cos^2\theta_{k,k}|\hat{\mathbf{h}}_{k,k}\hat{\mathbf{w}}_k|^2\Big) \Big/ \Bigg(\frac{d_{k,k}^{-\alpha}(1-\phi_k)P}{M-K}||\mathbf{h}_{k,k}||^2 \sin^2\theta_{k,k}$$
$$+ \sum_{j=1,j\neq k}^{K} \Bigg(d_{k,j}^{-\alpha}\phi_j P||\mathbf{h}_{k,j}||^2 \sin^2\theta_{k,j}\mathbb{E}\{|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j|^2\} + \frac{d_{k,i}^{-\alpha}(1-\phi_j)P}{M-K}||\mathbf{h}_{k,j}||^2 \sin^2\theta_{k,j}\mathbb{E}\{|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{\Gamma}}_j|^2\}\Bigg) + \sigma_u^2\Bigg)\Bigg]\Bigg\} \quad (32)$$

$$\geq \mathbb{E}\left[\log_2\left(1 + \frac{\frac{d_{k,k}^{-\alpha}\phi_k P}{\sigma_u^2}||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k}|\hat{\mathbf{h}}_{k,k}\hat{\mathbf{w}}_k|^2}{\frac{d_{k,k}^{-\alpha}(1-\phi_k)P}{\sigma_u^2(M-1)}||\mathbf{h}_{k,k}||^2\sin^2\theta_{k,k} + \sum_{j=1,j\neq k}^{K}\frac{d_{k,j}^{-\alpha}P}{\sigma_u^2(M-1)}||\mathbf{h}_{k,j}||^2\sin^2\theta_{k,j} + 1}\right)\right] \quad (33)$$

$$\geq \mathbb{E}\left\{\log_2\left[1 + \frac{\frac{d_{k,k}^{-\alpha}\phi_k P}{\sigma_u^2}||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k}}{\mathbb{E}\left[\frac{1}{|\hat{\mathbf{h}}_{k,k}\hat{\mathbf{w}}_k|^2}\right]\left(\frac{d_{k,k}^{-\alpha}(1-\phi_k)P}{\sigma_u^2(M-1)}||\mathbf{h}_{k,k}||^2\sin^2\theta_{k,k} + \sum_{j=1,j\neq k}^{K}\frac{d_{k,j}^{-\alpha}P}{\sigma_u^2(M-1)}||\mathbf{h}_{k,j}||^2\sin^2\theta_{k,j} + 1\right)}\right]\right\} \quad (34)$$

$$= \mathbb{E}\left[\log_2\left(1 + \frac{\frac{d_{k,k}^{-\alpha}\phi_k P}{\sigma_u^2}||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k}}{\frac{d_{k,k}^{-\alpha}(1-\phi_k)P}{\sigma_u^2(M-K)}||\mathbf{h}_{k,k}||^2\sin^2\theta_{k,k} + \sum_{j=1,j\neq k}^{K}\frac{d_{k,j}^{-\alpha}P}{\sigma_u^2(M-K)}||\mathbf{h}_{k,j}||^2\sin^2\theta_{k,j} + \frac{M-1}{M-K}}\right)\right] \triangleq R_{u,k}^{lb}, \quad (35)$$

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also been presented to validate the obtained analytical results and the our proposed algorithms.

Appendix

A. Proof of Theorem 1

By using the result of [18, Appendix B], the term $|\mathbf{h}_{k,k}\hat{\mathbf{w}}_k|^2$ can be lower-bounded as $|\mathbf{h}_{k,k}\hat{\mathbf{w}}_k|^2 = ||\mathbf{h}_{k,k}||^2|\tilde{\mathbf{h}}_{k,k}\hat{\mathbf{w}}_k|^2 \geq ||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k}|\hat{\mathbf{h}}_{k,k}\hat{\mathbf{w}}_k|^2$. Then, $R_{u,k}$ can be lower-bounded with (7) as (35) at the top of this page, which is defined as $R_{u,k}^{lb}$, where (32) and (34) are obtained respectively by applying Jensen's inequality $\mathbb{E}_x\left[\log_2\left(1 + a/(bx+c)\right)\right] \geq \log_2\left(1 + a/(b\mathbb{E}(x)+c\right)$ $(a,b,c > 0)$ to the terms $|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j|^2$, $|\tilde{\mathbf{e}}_{k,i}\hat{\mathbf{\Gamma}}_i|^2$, and applying $\mathbb{E}_x\left[\log_2(1+ax)\right] \geq \log_2\left(1 + \frac{a}{\mathbb{E}(1/x)}\right)$ to $\frac{1}{|\hat{\mathbf{h}}_{k,k}\mathbf{w}_k|^2}$ with $\mathbb{E}\left[|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j|^2\right] = \frac{1}{M-1}$, $\mathbb{E}\left[|\tilde{\mathbf{e}}_{k,i}\hat{\mathbf{\Gamma}}_i|^2\right] = \frac{M-K}{M-1}$ and $\mathbb{E}\left[\frac{1}{|\hat{\mathbf{h}}_{k,k}\mathbf{w}_k|^2}\right] = \frac{M-1}{M-K}$, since $|\tilde{\mathbf{e}}_{k,j}\hat{\mathbf{w}}_j|^2 \sim \text{Beta}(1, K-2)$, $|\tilde{\mathbf{e}}_{k,i}\hat{\mathbf{\Gamma}}_i|^2 \sim \text{Beta}(M-K, K-1)$ and $|\hat{\mathbf{h}}_{k,k}\hat{\mathbf{w}}_k|^2 \sim \text{Beta}(M-K+1, K-1)$ according to [22, Lemma 2] and [11, Lemma 1] respectively.

In addition, it was shown in [23, *Lemma* 2] that the joint distribution of the random variable (RV) pair $(||\mathbf{h}_{k,k}||^2\sin^2\theta_{k,k}, ||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k})$ is the same as that of $(I_{k,k}, S_{k,k})$, where $I_{k,k} \overset{d.}{=} \delta_{k,k}Y_{k,k}$ and $S_{k,k} \overset{d.}{=} X_{k,k} + (1-\delta_{k,k})Y_{k,k}$ with $X_{k,k} \sim \text{Gamma}(1,1)$ and $Y_{k,k} \sim \text{Gamma}(M-1,1)$ being two independent RVs for all k. And $||\mathbf{h}_{k,j}||^2\sin^2\theta_{k,j} \overset{d.}{=} \delta_{k,j}Y_{k,j}$ with $Y_{k,j} \sim \text{Gamma}(M-1,1)$ for $k \neq j$. All these RVs $X_{k,k}$ and $Y_{k,j}$ are independent with each other. Then, we can re-write $R_{u,k}^{lb}$ as

$$R_{u,k}^{lb} = \mathbb{E}\Bigg\{\log_2\Bigg[1 + \gamma\Bigg((M-K)\phi_k d_{k,k}^{-\alpha}X_{k,k}$$
$$+ \sum_{j=1,j\neq k}^{K} d_{k,j}^{-\alpha}\delta_{k,j}Y_{k,j} + d_{k,k}^{-\alpha}\big[(M-K)\phi_k(1-\delta_{k,k})$$
$$+(1-\phi_k)\delta_{k,k}\big]Y_{k,k}\Bigg)\Bigg]\Bigg\} - \mathbb{E}\Bigg\{\log_2\Bigg[1 +$$
$$\gamma\Bigg((1-\phi_k)d_{k,k}^{-\alpha}\delta_{k,k}Y_{k,k} + \sum_{j=1,j\neq k}^{K} d_{k,j}^{-\alpha}\delta_{k,j}Y_{k,j}\Bigg)\Bigg]\Bigg\} \quad (36)$$

$$= \mathbb{E}\left[\log_2\left(1+\gamma\Xi\right)\right] - \mathbb{E}\left[\log_2\left(1+\gamma\Psi\right)\right], \quad (37)$$

where $\Xi = \xi_1 + \xi_2 + \cdots + \xi_{K+1}$ with $\xi_k \sim \text{Gamma}(a_k, b_k)$ and $\Psi = \psi_1 + \psi_2 + \cdots + \psi_K$ with $\psi_k \sim \text{Gamma}(M-1, c_k)$. γ, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, a_k, b_k and c_k are as defined in *Theorem 1*. Using [32, *Theorem* 1], the PDFs of Ξ and Ψ can be obtained respectively as

$$f_\Xi(x) = C_1\sum_{k=0}^{\infty}\frac{\eta_k x^{(\nu+k)}e^{-\frac{x}{b_{min}}}}{\Gamma(\nu+k+1)b_{min}^{(K(M-1)+k+1)}}, (x \geq 0), \quad (38)$$

$$f_\Psi(x) = C_2\sum_{k=0}^{\infty}\frac{\rho_k x^{(\nu+k-1)}e^{-\frac{x}{c_{min}}}}{\Gamma(\nu+k)c_{min}^{(K(M-1)+k)}}, (x \geq 0), \quad (39)$$

where ν, b_{min}, c_{min}, C_1, C_2, η_k, ρ_k are given in *Theorem 1*. Then, $R_{u,k}^{lb}$ in (37) can be obtained by substituting (38) and (39) into (37) and applying [26, 4.222.8].

B. Proof of Theorem 3

With a lower bound of $R_{u,k}$ given in (35) and the expression of $R_{u,k}^{ideal}$, we have

$$\Delta R_{u,k} \leq \mathbb{E}\left[\log_2\left(1 + \frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2\right)\right]$$
$$-\mathbb{E}\left[\log_2\left(1 + \frac{\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k}}{\frac{(M-1)(M-K+1)}{(M-K)M} + I_k}\right)\right]$$
$$= \mathbb{E}\left[\log_2\left(1 + \frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2\right)\right]$$
$$+\mathbb{E}\left[\log_2\left(C_{M,K} + I_k\right)\right]$$
$$-\mathbb{E}\Bigg[\log_2\Bigg(1 + \frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}$$
$$\times||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k} + \frac{K-1}{(M-K)M} + I_k\Bigg)\Bigg] \quad (40)$$
$$\leq \mathbb{E}\left[\log_2\left(1 + \frac{d_{k,k}^{-\alpha}\phi_k P}{\sigma_u^2}\frac{M-K+1}{M}||\mathbf{h}_{k,k}||^2\right)\right]$$
$$+\mathbb{E}\left[\log_2\left(C_{M,K} + I_k\right)\right] - \mathbb{E}\Bigg[\log_2\Bigg(1 + \frac{d_{k,k}^{-\alpha}\phi_k P}{\sigma_u^2}$$

$$\Delta R_{u,k} \lessapprox \log_2(e)\mathbb{E}\left[1-\frac{1}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2}\right]\mathbb{E}\left[\sin^2\theta_{k,k}\right]+\mathbb{E}\left[\log_2\left(C_{M,K}+I_k\right)\right] \tag{43}$$

$$\leq \log_2(e)\vartheta_k 2^{-\frac{B_{k,k}}{M-1}}+\log_2\left(C_{M,K}+\mathbb{E}[I_k]\right) \tag{44}$$

$$= \log_2(e)\vartheta_k 2^{-\frac{B_{k,k}}{M-1}}+\log_2\left(C_{M,K}+(1-\phi_k)\tau_{k,k}2^{-\frac{B_{k,k}}{M-1}}+\sum_{j=1,j\neq k}^{K}\tau_{k,j}2^{-\frac{B_{k,j}}{M-1}}\right) \tag{45}$$

$$\simeq -\log_2\left(1-\vartheta_k 2^{-\frac{B_{k,k}}{M-1}}\right)+\log_2\left(C_{M,K}+(1-\phi_k)\tau_{k,k}2^{-\frac{B_{k,k}}{M-1}}+\sum_{j=1,j\neq k}^{K}\tau_{k,j}2^{-\frac{B_{k,j}}{M-1}}\right) \tag{46}$$

$$= \log_2\left(\frac{C_{M,K}+(1-\phi_k)\tau_{k,k}2^{-\frac{B_{k,k}}{M-1}}+\sum_{j=1,j\neq k}^{K}\tau_{k,j}2^{-\frac{B_{k,j}}{M-1}}}{1-\vartheta_k 2^{-\frac{B_{k,k}}{M-1}}}\right)\triangleq \Delta R_{u,k}^{ub}, \tag{47}$$

$$\times\frac{M-K+1}{M}||\mathbf{h}_{k,k}||^2\cos^2\theta_{k,k}\Big)\Big] \tag{41}$$

$$=\mathbb{E}\left[-\log_2\left(1-\frac{\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2\sin^2\theta_{k,k}}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2}\right)\right]$$
$$+\mathbb{E}\left[\log_2\left(C_{M,K}+I_k\right)\right], \tag{42}$$

where $I_k = \frac{d_{k,k}^{-\alpha}(1-\phi_k)P(M-K+1)}{\sigma_u^2(M-K)M}||\mathbf{h}_{k,k}||^2\sin^2\theta_{k,k} + \sum_{j=1,j\neq k}^{K}\frac{d_{k,j}^{-\alpha}P(M-K+1)}{\sigma_u^2(M-K)M}||\mathbf{h}_{k,j}||^2\sin^2\theta_{k,j}$. (41) follows by neglecting $\frac{K-1}{(M-K)M}+I_k$ in the third $\log_2$ term of (40). (42) follows by combining the first and the third terms of (41) and re-writing in the form of $-\log_2(1-x)$ with $x = \frac{\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2\sin^2\theta_{k,k}}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}||\mathbf{h}_{k,k}||^2}$. Next, by applying $-\ln(1-x)\simeq x$ for $x\to 0$ to (42), we can approximately upper-bound $\Delta R_{u,k}$ as $\Delta R_{u,k}^{ub}$ in (47) at the top of this page, where $C_{M,K}$, ϑ_k and $\tau_{k,j}$ are as given in the theorem. $Q_k \triangleq \mathbb{E}\left[\frac{1}{1+q_k||\mathbf{h}_{k,k}||^2}\right]$ can be easily obtained as (17) with the distribution of $||\mathbf{h}_{k,k}||^2 \sim \mathrm{Gamma}(M,1)$. (44) follows by applying Jensens inequality to the $\log_2$ term of (43). Similarly, $\mathbb{E}[I_k]$ can be easily obtained as $\mathbb{E}[I_k] = (1-\phi_k)\tau_{k,k}2^{-\frac{B_{k,k}}{M-1}}+\sum_{j=1,j\neq k}^{K}\tau_{k,j}2^{-\frac{B_{k,j}}{M-1}}$.

C. Proof of Lemma 2

We can upper-bound $h_k\left(2^{-\frac{B}{(K-1)(M-1)}},\phi_k\right)$ in (22) as

$$h_k\left(2^{-\frac{B}{(K-1)(M-1)}},\phi_k\right)<\frac{(K-1)(M-K+1)\Gamma\left(\frac{M}{M-1}\right)}{M-K}$$
$$\times s_k(\phi_k)2^{-\frac{B}{(K-1)(M-1)}}, \tag{48}$$

where (48) follows by Jensen's inequality with

$$s_k(\phi_k) = \frac{\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2}}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2}}\left(\frac{M-1}{M}\right.$$
$$\left.+K\Gamma\left(\frac{M}{M-1}\right)\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}\frac{P}{\sigma_u^2}\right)2^{-\frac{B}{M-1}}$$
$$+(1-\phi_k)d_{k,k}^{-\alpha}\frac{P}{\sigma_u^2}2^{-\frac{B}{M-1}}-\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}\frac{P}{\sigma_u^2}. \tag{49}$$

Thus, $h_k\left(2^{-\frac{B}{(K-1)(M-1)}},\phi_k\right) < 0$ for $\phi_k \in (0,1]$, if $s_k(\phi_k) \leq 0$. It is easy to check that $s_k(\phi_k)$ is a concave function and $\frac{\mathrm{d}s_k(\phi_k)}{\mathrm{d}\phi_k}$ is a decreasing function of ϕ_k. Moreover, it is easy to check that $\left.\frac{\mathrm{d}s_k(\phi_k)}{\mathrm{d}\phi_k}\right|_{\phi_k=0} > 0$ and $\left.\frac{\mathrm{d}s_k(\phi_k)}{\mathrm{d}\phi_k}\right|_{\phi_k\to+\infty} = -\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2}2^{-\frac{B}{M-1}} < 0$. It follows that Equation $\frac{\mathrm{d}s_k(\phi_k)}{\mathrm{d}\phi_k}=0$ has an unique root which is given by $\phi_k^o = \left[\sqrt{(M-K+1)\left(\frac{M-1}{M}+\frac{K\Gamma\left(\frac{M}{M-1}\right)\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}P}{\sigma_u^2}\right)}-1\right]\Big/\left[\frac{d_{k,k}^{-\alpha}P(M-K+1)}{\sigma_u^2}\right]$. It is obvious that $\phi_k^o > 0$. Moreover, for $\phi_k^o < 1$, $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2}$ should satisfy $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2} > \chi(\kappa_k)$, where $\chi(x)$ and κ_k are as defined in the lemma. It is easy to check that $\chi(x)$ is a decreasing function of x. Since $\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}} \leq d_{k,k}^{-\alpha}$ and $\Gamma\left(\frac{M}{M-1}\right)$ is an increasing function of M, it follows that $2-K < \kappa_k < 2$ for any set of parameters. Thus, we have $\chi(\kappa_k) < \chi(2-K) = \frac{\sqrt{(K-2)^2+\frac{4[(M-1)(M-K)-1]}{M}}}{2(M-K+1)}+\frac{K-2}{2(M-K+1)}$. Recall that, it is required that $M \geq K+N_e$ for secure communications and $N_e \geq 2$. It is easy to check that, as long as $K \leq 4$ holds, $\chi(\kappa_k) < \chi(2-K) < 1$ holds for any system parameters other than K.

Substituting the result of ϕ_k^o into (49), we can obtain $\forall\phi_k \in (0,1]$

$$s_k(\phi_k) \leq s_k(\phi_k^o) < \left(\frac{M-1}{M}+K\Gamma\left(\frac{M}{M-1}\right)\right.$$
$$\left.\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}\times\frac{P}{\sigma_u^2}+\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2}\right)2^{-\frac{B}{M-1}}$$
$$-\frac{\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}P}{\sigma_u^2}. \tag{50}$$

Then, a sufficient condition for $h_k\left(2^{-\frac{B}{(K-1)(M-1)}},\phi_k\right) < 0$ follows from (50) as $B > B_{th1,k}$ with $B_{th1,k}$ defined in the lemma. In addition, $\phi_k^o \geq 1$ when $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2} \leq \chi(\kappa_k)$ is satisfied. Then, $\forall \phi_k \in (0,1]$, $s_k(\phi_k) \leq s_k(1)$. Another sufficient condition for $h_k\left(2^{-\frac{B}{(K-1)(M-1)}}, \phi_k\right) < 0$ follows as $B > B_{th2,k}$ with $B_{th2,k}$ given in the lemma.

Next, we consider the condition for $h_k(1,\phi_k) \geq 0$. Using (22), we can rewrite $h_k(1,\phi_k)$ as $h_k(1,\phi_k) = \frac{(K-1)(M-K+1)}{M-K}\Gamma\big(\frac{M}{M-1}\big)\tilde{s}_k(\phi_k)$, where $\tilde{s}_k(\phi_k) = \left(1 - \mathbb{E}\left[\frac{1}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)||\mathbf{h}_{k,k}||^2}{\sigma_u^2 M}}\right]\right)\left(\frac{M-1}{M} + K\Gamma\left(\frac{M}{M-1}\right)\frac{\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}2^{-\frac{B}{(K-1)(M-1)}}P}{\sigma_u^2}\right) + \frac{(1-\phi_k)d_{k,k}^{-\alpha}P}{\sigma_u^2} - \frac{\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}2^{-\frac{B}{(K-1)(M-1)}}P}{\sigma_u^2}$. Similar to function $s_k(\phi_k)$, it can be easily check that $\frac{\mathrm{d}^2\tilde{s}_k(\phi_k)}{\mathrm{d}\phi_k^2} < 0$ and $\frac{\mathrm{d}\tilde{s}_k(\phi_k)}{\mathrm{d}\phi_k}\Big|_{\phi_k=0} > 0$. Thus, it follows that if $\tilde{s}_k(1) \geq 0$, $h_k(1,\phi_k) \geq 0$ for $\phi_k \in (0,1]$. Moreover, applying Jensen's inequality $\mathbb{E}[\frac{1}{X}] \geq \frac{1}{\mathbb{E}[X]}$ with $\mathbb{E}\left[\frac{1}{||\mathbf{h}_{k,k}||^2}\right] = \frac{1}{M-1}$, we can obtain $1 - \mathbb{E}\left[\frac{1}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)||\mathbf{h}_{k,k}||^2}{\sigma_u^2 M}}\right] \geq \frac{\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}}{\mathbb{E}\left[\frac{1}{||\mathbf{h}_{k,k}||^2}\right]+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)}{\sigma_u^2 M}} = 1 - \frac{1}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)(M-1)}{\sigma_u^2 M}}$. Then, we can obtain a sufficient condition for $\tilde{s}_k(1) > 0$ as $\tilde{s}_k(1) \geq \left(\frac{\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)(M-1)}{\sigma_u^2 M}}{1+\frac{d_{k,k}^{-\alpha}\phi_k P(M-K+1)(M-1)}{\sigma_u^2 M}}\right)\Big(K\Gamma\big(\frac{M}{M-1}\big)\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}} \times \frac{P}{\sigma_u^2}2^{-\frac{B}{(K-1)(M-1)}} + \frac{M-1}{M}\Big) - \prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}2^{-\frac{B}{(K-1)(M-1)}}\frac{P}{\sigma_u^2} > 0$, which can be equivalently rewritten as

$$\frac{M-1}{M}\left(1 - \frac{1}{1+\frac{d_{k,k}^{-\alpha}P(M-K+1)(M-1)}{\sigma_u^2 M}}\right) \geq \prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}} \times\frac{P}{\sigma_u^2}\left[1 - K\Gamma\left(\frac{M}{M-1}\right)\left(1 - \frac{1}{1+\frac{d_{k,k}^{-\alpha}P(M-K+1)(M-1)}{\sigma_u^2 M}}\right)\right] \times 2^{-\frac{B}{(K-1)(M-1)}}. \quad (51)$$

According to (51), it is easy to see that on the one hand, if $1-K\Gamma\left(\frac{M}{M-1}\right)\left(1-\frac{1}{1+\frac{d_{k,k}^{-\alpha}P(M-K+1)(M-1)}{\sigma_u^2 M}}\right) \leq 0$, or equivalently $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2} \geq \frac{M}{\left[K\Gamma\left(\frac{M}{M-1}\right)-1\right](M-K+1)(M-1)}$, the inequality (51) can always hold. Moreover, it is easy to check that, for the system considered with $K \geq 2$ and $N_e \geq 2$ (then $M \geq 4$), $\chi(\kappa_k) > \frac{M}{\left[K\Gamma\left(\frac{M}{M-1}\right)-1\right](M-K+1)(M-1)}$ holds. On the other hand, if $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2} < \frac{M}{\left[K\Gamma\left(\frac{M}{M-1}\right)-1\right](M-K+1)(M-1)}$, (51) requires that $B > \tilde{B}_{th,k} \triangleq (K-1)(M-1)\log_2\left(\frac{\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}M^2}{d_{k,k}^{-\alpha}(M-K+1)(M-1)^2} - \left(K\Gamma\left(\frac{M}{M-1}\right)-1\right) \times \frac{M\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}P}{(M-1)\sigma_u^2}\right)$, which is a decreasing function of $\frac{d_{k,k}^{-\alpha}P}{\sigma_u^2}$. It follows that $\tilde{B}_{th,k} < (K-1)(M-1)\log_2\left(\frac{\prod_{j=1,j\neq k}^{K}(d_{k,j}^{-\alpha})^{\frac{1}{K-1}}M^2}{d_{k,k}^{-\alpha}(M-K+1)(M-1)^2}\right)$. Then, $\tilde{B}_{th,k} < 0$ for $K \geq 2$ and $N_e \geq 2$ ($M \geq 4$). Thus, $h_k(1,\phi_k) \geq 0$, $\forall \phi_k \in (0,1]$, holds for any systems with $K \geq 2$, $N_e \geq 2$ ($M \geq 4$).

D. Proof of Lemma 3

According to (11), $R_{e,k}$ can be rewritten as $R_{e,k} = \mathbb{E}\left[\log_2\det\left(1 + l_{k,k}^{-\alpha}\mathbf{v}_k^H\mathbf{R}_k^{-1}\mathbf{v}_k\right)\right]$, where $\mathbf{v}_k = \mathbf{G}_{k,k}\hat{\mathbf{w}}_k \sim \mathcal{CN}(\mathbf{0},\mathbf{I}_{N_e})$ and $\mathbf{R}_k = \tilde{\mathbf{G}}_{I,k}\mathbf{\Lambda}_k\tilde{\mathbf{G}}_{I,k}^H$ with $\mathbf{\Lambda}_k = \frac{1}{\phi_k}\mathbf{\Sigma}_{I,k}$. Let $f_{e,k} = l_{k,k}^{-\alpha}\mathbf{v}_k^H\mathbf{R}_k^{-1}\mathbf{v}_k$. We first consider $\frac{\partial^2 R_{e,1}}{\partial\phi_1^2}$ (given ϕ_2), which is obtained as

$$\frac{\partial^2 R_{e,1}}{\partial\phi_1^2} = \mathbb{E}\left[\frac{\frac{\partial^2 f_{e,1}}{\partial\phi_1^2}(1+f_{e,1}) - \left(\frac{\partial f_{e,1}}{\partial\phi_1}\right)^2}{(1+f_{e,1})^2}\right]. \quad (52)$$

By applying the results in [33, Ch. 17], we can obtain

$$\frac{\partial f_{e,1}}{\partial\phi_1} = \frac{1}{\phi_1^2}l_{1,1}^{-\alpha}\mathbf{v}_1^H\mathbf{R}_1^{-1}\tilde{\mathbf{G}}_{I,1}\tilde{\mathbf{\Lambda}}_1\tilde{\mathbf{G}}_{I,1}^H\mathbf{R}_1^{-1}\mathbf{v}_1, \quad (53)$$

$$\frac{\partial^2 f_{e,1}}{\partial\phi_1^2} = \frac{2}{\phi_1^3}l_{1,1}^{-\alpha}\mathbf{v}_1^H\mathbf{R}_1^{-1}\Big(\frac{1}{\phi_1}\tilde{\mathbf{G}}_{I,1}\tilde{\mathbf{\Lambda}}_1\tilde{\mathbf{G}}_{I,1}^H\mathbf{R}_1^{-1} \times\tilde{\mathbf{G}}_{I,1}\tilde{\mathbf{\Lambda}}_1\tilde{\mathbf{G}}_{I,1}^H - \tilde{\mathbf{G}}_{I,1}\tilde{\mathbf{\Lambda}}_1\tilde{\mathbf{G}}_{I,1}^H\Big)\mathbf{R}_1^{-1}\mathbf{v}_1 \quad (54)$$

with $\tilde{\mathbf{\Lambda}}_1 = \text{Blockdiag}\Big\{\frac{l_{1,1}^{-\alpha}}{M-2}\mathbf{I}_{M-2}, \frac{l_{1,2}^{-\alpha}(1-\phi_2)}{M-2}\mathbf{I}_{M-2}\Big\}$. Then, by substituting (53), (54), $f_{e,1}$ into (52) and rewriting $\tilde{\mathbf{\Lambda}}_1$ as $\tilde{\mathbf{\Lambda}}_1 = \phi_1(\mathbf{\Lambda}_1 + \mathbf{F})$ with $\mathbf{F} = \text{Blockdiag}\Big\{\frac{l_{1,1}^{-\alpha}}{M-2}\mathbf{I}_{M-2}, \mathbf{0}_{M-2}\Big\}$, the numerator of $\frac{\partial^2 R_{e,1}}{\partial\phi_1^2}$ can be written as

$$\frac{1}{\phi_1^2}l_{1,1}^{-\alpha}\mathbf{v}_1^H\mathbf{R}_1^{-1}\Big\{\left[2 + l_{1,1}^{-\alpha}(\mathbf{v}_1^H\mathbf{R}_1^{-1}\mathbf{v}_1)\right]\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H\mathbf{R}_1^{-1} \tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H + 2\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H + l_{1,1}^{-\alpha}\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H\Big[(\mathbf{v}_1^H\mathbf{R}_1^{-1}\mathbf{v}_1) \mathbf{R}_1^{-1} - \mathbf{R}_1^{-1}\mathbf{v}_1\mathbf{v}_1^H\mathbf{R}_1^{-1}\Big]\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H - l_{1,1}^{-\alpha}\mathbf{v}_1\mathbf{v}_1^H\Big\}\mathbf{R}_1^{-1}\mathbf{v}_1. \quad (55)$$

It is easy to see that the first and the second terms in the brace of (55) are both positive semi-definite matrices. Moreover, by applying *Cauchy-Schwartz Inequality*, we can prove $\mathbf{x}^H\left[(\mathbf{v}_1^H\mathbf{R}_1^{-1}\mathbf{v}_1)\mathbf{R}_1^{-1} - \mathbf{R}_1^{-1}\mathbf{v}_1\mathbf{v}_1^H\mathbf{R}_1^{-1}\right]\mathbf{x} = (\mathbf{v}_1^H\mathbf{R}_1^{-1}\mathbf{v}_1)(\mathbf{x}^H\mathbf{R}_1^{-1}\mathbf{x}) - |\mathbf{x}^H\mathbf{R}_1^{-1}\mathbf{v}_1|^2 \geq 0$ for arbitrary vector $\mathbf{x}$. Thus, the third term is also a positive semi-definite matrix. It follows that, except for the last term $-l_{1,1}^{-\alpha}\mathbf{v}_1\mathbf{v}_1^H$, the other terms in (55) are positive with probability of 1 for $\phi_1 \in (0,1]$. In addition, it is easy to see that the terms $\left[2 + l_{1,1}^{-\alpha}(\mathbf{v}_1^H\mathbf{R}_1^{-1}\mathbf{v}_1)\right]\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H\mathbf{R}_1^{-1}\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H$ and $l_{1,1}^{-\alpha}\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H\left[(\mathbf{v}_1^H\mathbf{R}_1^{-1}\mathbf{v}_1)\mathbf{R}_1^{-1} - \mathbf{R}_1^{-1}\mathbf{v}_1\mathbf{v}_1^H\mathbf{R}_1^{-1}\right]\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H$ in (55) approach zero as $\phi_1 \to 0^+$. Moreover, it is also easy to check that the terms $2\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H$ and $l_{1,1}^{-\alpha}\mathbf{v}_1\mathbf{v}_1^H$ are not related to ϕ_1, and the probability of $\frac{1}{\phi_1^2}l_{1,1}^{-\alpha}\mathbf{v}_1^H\mathbf{R}_1^{-1}\left\{2\tilde{\mathbf{G}}_{I,1}\mathbf{F}\tilde{\mathbf{G}}_{I,1}^H - l_{1,1}^{-\alpha}\mathbf{v}_1\mathbf{v}_1^H\right\}\mathbf{R}_1^{-1}\mathbf{v}_1 < 0$ is larger than 0. Therefore, it is possible that $\frac{\partial^2 R_{e,1}}{\partial\phi_1^2} < 0$ when ϕ_1 is close to zero; and $\frac{\partial^2 R_{e,1}}{\partial\phi_1^2} \geq 0$ when ϕ_1 is larger than a certain value in $(0,1)$.

Next, we consider $\frac{\partial^2 R_{e,2}}{\partial \phi_1^2}$ (given ϕ_2), which is given by

$$\frac{\partial^2 R_{e,2}}{\partial \phi_1^2} = \mathbb{E}\left[\frac{\frac{\partial^2 f_{e,2}}{\partial \phi_1^2} + \left[\frac{\partial^2 f_{e,2}}{\partial \phi_1^2} f_{e,2} - \left(\frac{\partial f_{e,2}}{\partial \phi_1}\right)^2\right]}{(1+f_{e,2})^2}\right] \quad (56)$$

Similarly, it can be obtained that

$$\begin{aligned}\frac{\partial f_{e,2}}{\partial \phi_1} &= -l_{2,2}^{-\alpha}\mathbf{v}_2^H\mathbf{R}_2^{-1}\frac{\partial \mathbf{R}_2}{\partial \phi_1}\mathbf{R}_2^{-1}\mathbf{v}_2,\\ \frac{\partial^2 f_{e,2}}{\partial \phi_1^2} &= l_{2,2}^{-\alpha}\mathbf{v}_2^H\mathbf{R}_2^{-1}\left(2\frac{\partial \mathbf{R}_2}{\partial \phi_1}\mathbf{R}_2^{-1}\frac{\partial \mathbf{R}_2}{\partial \phi_1} - \frac{\partial^2 \mathbf{R}_2}{\partial \phi_1^2}\right)\mathbf{R}_2^{-1}\mathbf{v}_2.\end{aligned}$$

Moreover, it is easy to obtain $\frac{\partial \mathbf{R}_2}{\partial \phi_1} = \frac{1}{\phi_2}\tilde{\mathbf{G}}_{I,k}\tilde{\mathbf{\Lambda}}_2\tilde{\mathbf{G}}_{I,k}^H$ with $\tilde{\mathbf{\Lambda}}_2 = \text{Blockdiag}\left\{\frac{-l_{2,1}^{-\alpha}}{M-2}\mathbf{I}_{M-2}, \mathbf{0}_{M-2}\right\}$ and $\frac{\partial^2 \mathbf{R}_2}{\partial \phi_1^2} = 0$. It follows that $\frac{\partial^2 f_{e,2}}{\partial \phi_1^2} = \frac{2}{\phi_2^2}l_{2,2}^{-\alpha}\mathbf{v}_2^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\tilde{\mathbf{\Lambda}}_2\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\tilde{\mathbf{\Lambda}}_2\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\mathbf{v}_2 \geq 0$ Denote $\varpi(\phi_1) \triangleq \frac{\partial f_{e,2}}{\partial \phi_1}\Big/ f_{e,2}$. Then $\varpi'(\phi_1) = \frac{\frac{\partial^2 f_{e,2}}{\partial \phi_1^2}f_{e,2} - \left(\frac{\partial f_{e,2}}{\partial \phi_1}\right)^2}{f_{e,2}^2}$. According to (56), $\frac{\partial^2 R_{e,2}}{\partial \phi_1^2} \geq 0$ if $\varpi'(\phi_1) \geq 0$. Moreover, we can rewrite $\tilde{\mathbf{\Lambda}}_2$ as $\tilde{\mathbf{\Lambda}}_2 = \frac{\phi_2}{\phi_1}\mathbf{\Lambda}_2 - \mathbf{D}$ with $\mathbf{D} = \frac{1}{\phi_1}\text{Blockdiag}\left\{\frac{l_{2,1}^{-\alpha}}{M-2}\mathbf{I}_{M-2}, \frac{l_{2,2}^{-\alpha}(1-\phi_2)}{M-2}\mathbf{I}_{M-2}\right\}$. Then, $\varpi(\phi_1)$ can be written as

$$\begin{aligned}\varpi(\phi_1) &= \frac{-\frac{1}{\phi_2}l_{2,2}^{-\alpha}\mathbf{v}_2^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\left(\frac{\phi_2}{\phi_1}\mathbf{\Lambda}_2 - \mathbf{D}\right)\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\mathbf{v}_2}{l_{2,2}^{-\alpha}\mathbf{v}_2^H\mathbf{R}_2^{-1}\mathbf{v}_2}\\ &= \frac{\varpi_1(\phi_1)}{\phi_2\varpi_2(\phi_1)} - \frac{1}{\phi_1},\end{aligned}$$

where $\varpi_1(\phi_1) \triangleq \mathbf{v}_2^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\mathbf{v}_2 \geq 0$ and $\varpi_2(\phi_1) \triangleq \mathbf{v}_2^H\mathbf{R}_2^{-1}\mathbf{v}_2 \geq 0$. And $\varpi'(\phi_1)$ can be obtained as $\varpi'(\phi_1) = \frac{\frac{\phi_1^2}{\phi_2}\left[\varpi_1'(\phi_1)\varpi_2(\phi_1) - \varpi_1(\phi_1)\varpi_2'(\phi_1)\right] + \varpi_2^2(\phi_1)}{\phi_1^2\varpi_2^2(\phi_1)}$, where $\varpi_1'(\phi_1) = \frac{2}{\phi_2}\mathbf{v}_2^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\mathbf{v}_2 - \frac{3}{\phi_1}\mathbf{v}_2^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\mathbf{v}_2$ and $\varpi_2'(\phi_1) = \frac{1}{\phi_2}\mathbf{v}_2^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\mathbf{v}_2 - \frac{1}{\phi_1}\mathbf{v}_2^H\mathbf{R}_2^{-1}\mathbf{v}_2$. It can be easily observed that the denominator of $\varpi'(\phi_1)$ is positive with probability of 1, and the numerator can be re-written as $\frac{\phi_1^2}{\phi_2^2}\mathbf{v}_2^H\mathbf{R}_2^{-1}\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\left[(\mathbf{v}_2^H\mathbf{R}_2^{-1}\mathbf{v}_2)\mathbf{R}_2^{-1} - \mathbf{R}_2^{-1}\mathbf{v}_2\mathbf{v}_2^H\mathbf{R}_2^{-1}\right]$ $\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}\mathbf{v}_2 + \mathbf{v}_2^H\left(\frac{\phi_1}{\phi_2}\mathbf{X} - \mathbf{Y}\right)^H\left(\frac{\phi_1}{\phi_2}\mathbf{X} - \mathbf{Y}\right)\mathbf{v}_2$ $\left(\mathbf{v}_2^H\mathbf{R}_2^{-1}\mathbf{v}_2\right)$ with $\mathbf{X} = \mathbf{R}_2^{-\frac{1}{2}}\tilde{\mathbf{G}}_{I,2}\mathbf{D}\tilde{\mathbf{G}}_{I,2}^H\mathbf{R}_2^{-1}$ and $\mathbf{Y} = \mathbf{R}_2^{-\frac{1}{2}}$. It can be similarly proved that $\mathbf{x}^H\left[(\mathbf{v}_2^H\mathbf{R}_2^{-1}\mathbf{v}_2)\mathbf{R}_2^{-1} - \mathbf{R}_2^{-1}\mathbf{v}_2\mathbf{v}_2^H\mathbf{R}_2^{-1}\right]\mathbf{x} \geq 0$ for arbitrary vector $\mathbf{x}$. It follows that both the first term and the second term of the numerator are non-negative. Then, it follows that $\varpi'(\phi_1) \geq 0$ for $\phi_1 \in (0,1]$. Thus, $R_{e,2}$ is a convex function of ϕ_1 (given ϕ_2) for $\phi_1 \in (0,1]$. The corresponding properties of $R_{e,1}$ and $R_{e,2}$ as a function of ϕ_1 follows similarly. Then the proof is completed.

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