Lecture 1. Image Representation

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1. Digital image

- A function of two variables $f(x,y)$ is often depicted graphically in the form of an *image*
- An image is represented as a matrix (array of numbers) in MATLAB and is referred to as an *image array*
- Each entry of the image array represents an image element called a *pixel*
- Each pixel is assigned an intensity value to illustrate the color of the image (color, grayscale or binary [0's and 1's])

Basic MATLAB functions for reading, displaying, and writing images:

`imread, imshow, imagesc, imwrite`

1.1 Binary image

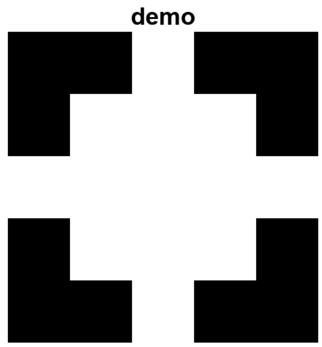
The image array that only consists of 0's and 1's.

Example 1.1

A simple binary pattern.

```
IB=[0 0 1 0 0
    0 1 1 1 0
    1 1 1 1 1
    0 1 1 1 0
    0 0 1 0 0]; % The 2-D image array

figure;
imagesc(IB);
colormap('gray');
axis image;
axis off; % remove axis from the image
axis equal; % use equal data unit lengths along each axis
title('demo');
```

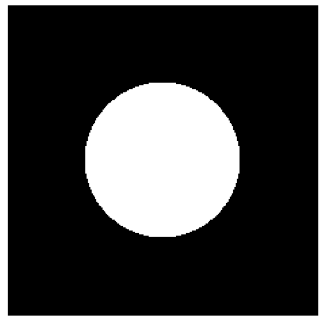
```
xlabel('x_0 axis');
set(gcf,'Position',[100 100 200 200]); % set the position and size of the image
```



Example 1.2

A circle with variable radius.

```
x = 1:256;
y = 1:256;
[X,Y] = ndgrid(x,y);

F = zeros(256,256);
r =64; % radius of the circle
Cx = 128; % center of the circle x-coordinate
Cy = 128; % center of the circle y-coordinate
F((X-Cx).^2+(Y-Cy).^2<r^2)=1; % circle equation

figure;
imagesc(F);
colormap('gray');
axis image;
axis equal;
axis off;
set(gcf,'Position',[100 100 200 200]);
```



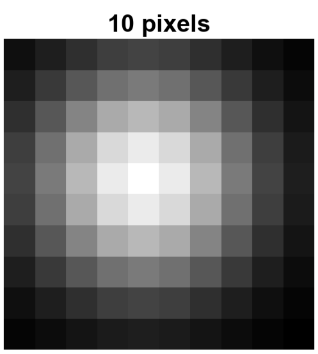
1.2 Gray-scale image

The intensity of a gray-scale image is an integer value in the range [0,255] for 8-bit images and [0,65535] for 16-bit images.

Example 1.3

Two-dimensional Guassian function (try increasing the number of pixels for finer sampling).

```
N =10; % number of pixels
x = 1:N;
y = 1:N;
[X,Y]=ndgrid(x,y);

sigma = N/4; % standard deviation
F=exp(-((X-N/2).^2+(Y-N/2).^2)./(2*sigma^2)); % Gaussian function

figure;
imagesc(F);
colormap('gray');
axis image;
axis off;
axis equal;
set(gcf,'Position',[100 100 200 200]);
title([num2str(N),' pixels']);
```



1.3 Indexed color image

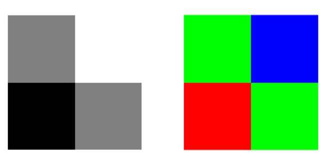
Instead of using a gray scale, indexed color image defines a colormap that converts the index to a specific color.

Example 1.4

Display a Bayer pattern by simply defining a colormap with RGB (red, green, blue) colors.

```
 x=[2,3;
   1,2];
dup=ones(32); % Making the block 32*32 pixels
X=kron(x,dup);
```

```
map=[1 0 0  % Red
     0 1 0  % Green
     0 0 1];% Blue

Color_X=zeros([size(X),3]);
[Color_X(:,:,1),Color_X(:,:,2),Color_X(:,:,3)]=ind2rgb(X,map);

figure;
subplot(121);
imagesc(X);
axis image;
axis off;
colormap(gray);
subplot(122);
imagesc(Color_X);
axis image;
axis off;
set(gcf,'Position',[100 100 200 200]);
```

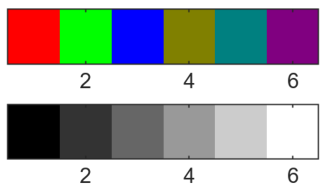


Example 1.5

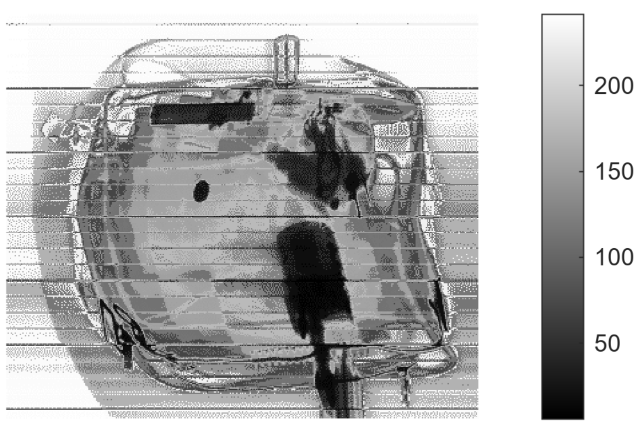
A pseudocolor display of a transmission x-ray image of a bag.

```
map=[1 0 0        % Red
     0 1 0        % Green
     0 0 1        % Blue
     0.5 0.5 0    % Yellow
     0 0.5 0.5    % Cyan
     0.5 0 0.5];  % Magenta
n_idx=size(map,1); % number of colors
bar=1:n_idx;

figure;
color_bar=ind2rgb(bar,map); % converts the matrix X and corresponding colormap map
to RGB (truecolor) format

subplot(211);
```

```
imagesc(color_bar);
set(gca,'YTick',[]); % remove the y-axis tick marks
subplot(212);
imagesc(bar);
set(gca,'YTick',[]); % remove the y-axis tick marks
colormap(gray);
set(gcf,'Position',[100 100 200 100]);
```

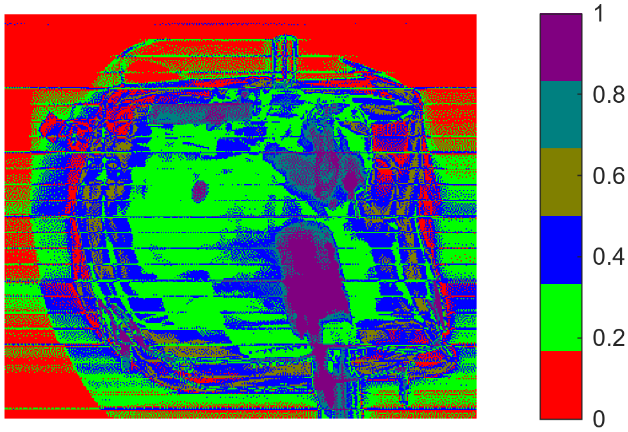



```
figure;
Img=imread('bag.png');
Img_n=255-Img;

imagesc(Img_n);
colormap('gray');
axis off;
axis equal;
colorbar;
set(gcf,'Position',[100 100 400 300]);
```




```
figure;
Img_n=round(double(Img)/double(max(Img(:))) * n_idx);
Img_clr=ind2rgb(Img_n,map);
imagesc(Img_clr);
colormap(map);
axis off;
axis equal;
colorbar;
```

```
set(gcf,'Position',[100 100 400 300]);
```



2. Image transformation

An image $f(x, y)$ can be transformed to another image $g(x, y)$ by modifying its value at each point (x, y) in accordance with a mathematical rule (mapping). Here we introduce two fundamental transformations:

1. Geometrical transformations (linear)
2. Gray-scale transformations

2.1 Coordinate transformation

Geometrical transformation, $f(x, y) \to g(x, y)$, followed by a defined coordinate transformation $(x, y) \to (u, v)$,

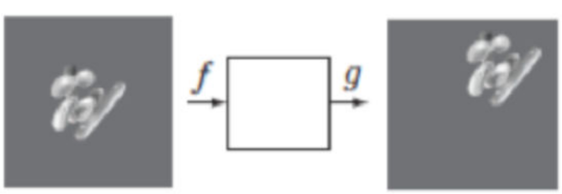
$$\begin{cases} u = u(x, y) \\ v = v(x, y) \end{cases}.$$

The geometric transformation $f(x, y) \to g(x, y)$ follows $g(u(x, y), v(x, y)) = f(x, y)$. When the coordinate transformation is a linear one, we call the corresponding image transformation **linear geometric transformation**.

2.1.1 Image translation (shift)

Image translation transformation equation

$$g(x + x_0, y + y_0) = f(x, y)$$

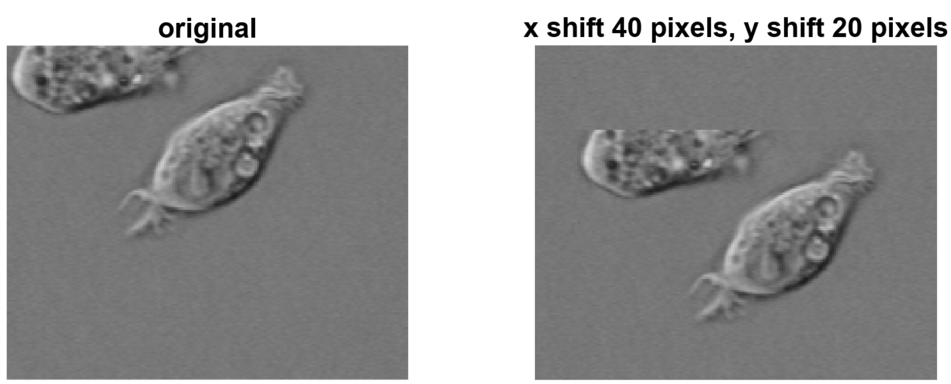


ILL. 2.1 *Image translation transformation*

Example 2.1

`Y = circshift(A,K)` circularly shifts the elements in array `A` by `K` positions.

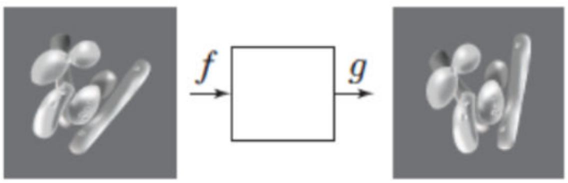
```
Img=imread('cell.tif');
figure;

x0=40;
y0=20;
Img_t=circshift(Img, [x0,y0]);
figure;
subplot(121);
imshow(Img);
title('original');
subplot(122);
imshow(Img_t);
title(['x shift ',num2str(x0), ' pixels, y shift ', num2str(y0),' pixels']);
set(gcf,'Position',[100 100 600 300]);
```



2.1.2 Image rotation

Image rotation transformation equation

$$g(x \cdot \cos\theta - y \cdot \sin\theta, x \cdot \sin\theta + y \cdot \cos\theta) = f(x, y)$$

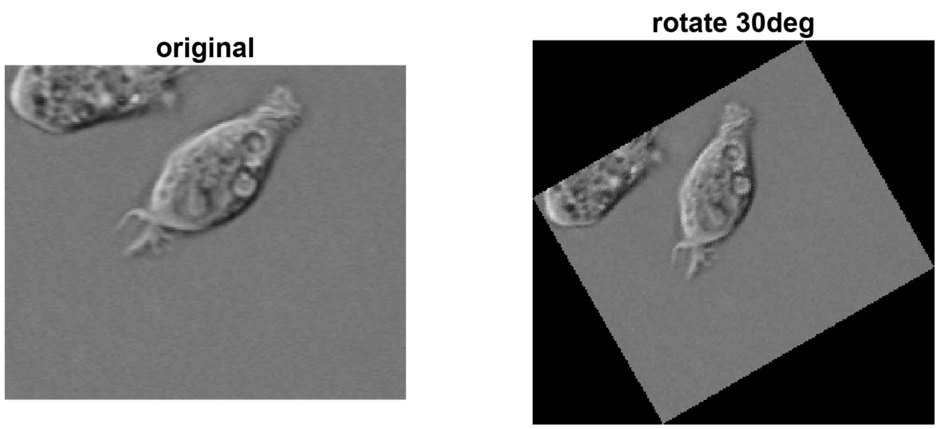


ILL. 2.2 *Image rotation transformation*

Example 2.2

`Y = imrotate(I,angle)` rotates image `I` by `angle` degrees in a counterclockwise direction around its center point.

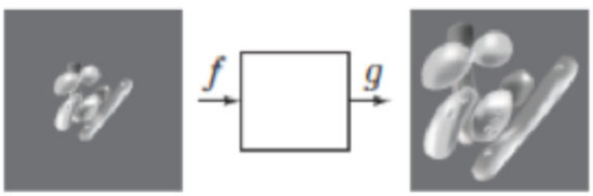
```
theta=30;
Img_r=imrotate(Img, theta);
figure;
subplot(121);
imshow(Img);
title('original');
subplot(122);
imshow(Img_r);
set(gcf,'Position',[100 100 600 300]);
title(['rotate ',num2str(theta), 'deg']);
```



2.1.3 Linear image scaling

Linear image scaling transformation equation.

$g(s_x, s_y) = f(x, y)$

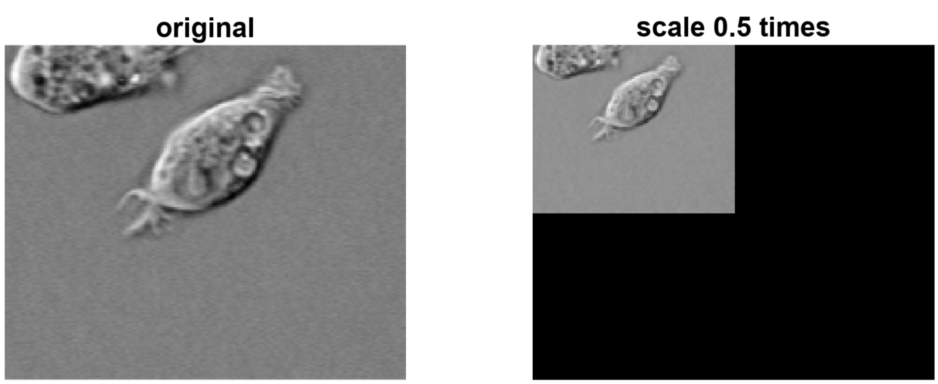


ILL. 2.3 *Linear image scaling transformation*

Example 2.3:

`Y = imresize(A,scale)` returns image Y that is `scale` times the size of image `A`.

```
s=0.5;
[m,n]=size(Img);
Img_s=zeros(m,n);
Img_s0=imresize(Img,s);

[m0,n0]=size(Img_s0);
if s>1
    Img_s=Img_s0(1:m,1:n);
else
    Img_s(1:m0,1:n0)=Img_s0;
end

figure;
subplot(121);
imagesc(Img);
axis image;
axis off;
title('original');

subplot(122);
imagesc(Img_s);
axis image;
axis off;
colormap('gray');
title(['scale ',num2str(s),' times']);
set(gcf,'Position',[100 100 600 300]);
```



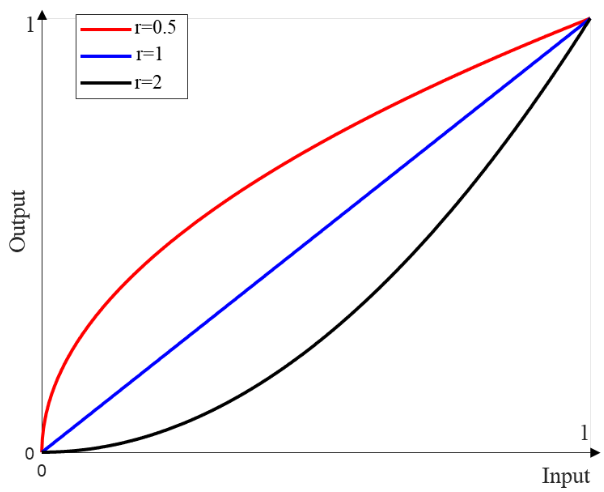
2.2 Gray-scale transformations

In contrast to geometric transformation, gray-scale transformation $f(x, y) \rightarrow g(x, y)$ does not involve coordinate transformation. More specifically, the value g at point (x, y) is determined only by the original image value f at point (x, y).

The transformation $f(x, y) \rightarrow g(x, y)$ follows $g(x, y) = \Phi\{f(x, y)\}$ or $g = \Phi\{f\}$, since the coordinate is the same. The common gray-scale transformations include Gamma transformation, inversion, and quantization.

2.2.1 Gamma transformation

The Gamma transformation $g = f^{\gamma}$ is common in photography. The coefficient � is called the *contrast index*.

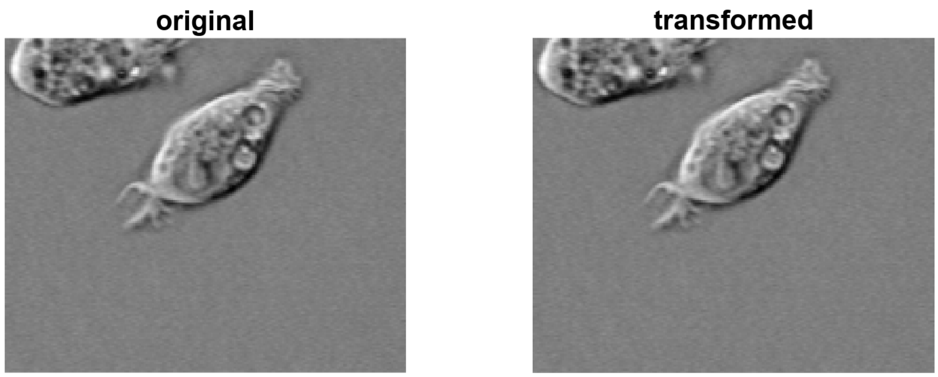


ILL. 2.4 *Gamma transformation*

Example 2.4

```
Img=imread('cell.tif');
Img2=double(Img)./255;
figure;
subplot(121);
imagesc(Img2);
colormap('gray');
axis image;
axis off;
title('original');

g =1; % gamma value
Img2_g= Img2.^g;

subplot(122);
imagesc(Img2_g);
colormap('gray');
axis image;
axis off;
title('transformed');
```

```
axis equal;
set(gcf,'Position',[100 100 600 300]);
```



Contrast

$$\text{Contrast} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Contrast is a real number between 0 and 1. Larger contrast (closer to 1) shows larger variance among pixel values. If the image does not have variance, i.e., $I_{\max} = I_{\min}$, the contrast is 0. Try different γ values and see how the contrast changes.

```
% calculate the contrast
i_max=double(max(max(Img2_g))),
```

```
i_max = 0.8314
```

```
i_min=double(min(min(Img2_g))),
```

```
i_min = 0.1098
```

```
contrast=(i_max-i_min)/(i_max+i_min),
```

```
contrast = 0.7667
```

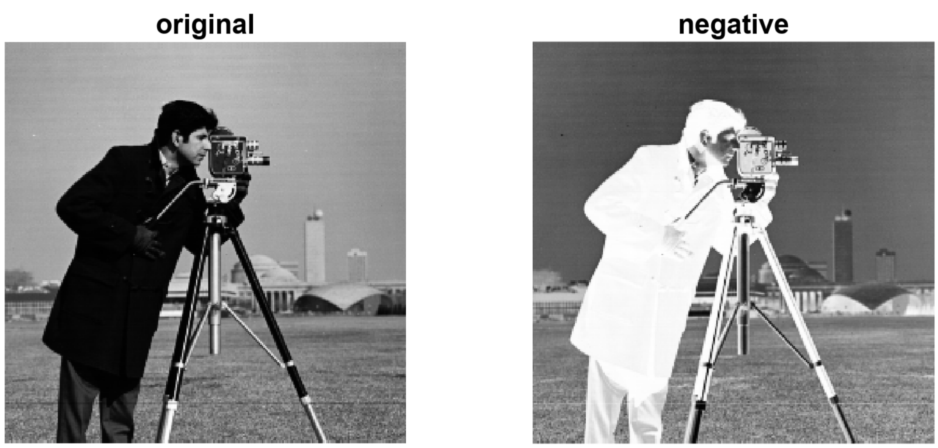
2.2.1 Negative image

Historically, in photography, the photos are developed from films, which is usually on a strip of transparent substrate. The light exposed area usually has high absorption, which makes it dark. The image on the film is usually called photo negative.

Negative image = Maximum pixel value - Positive Image

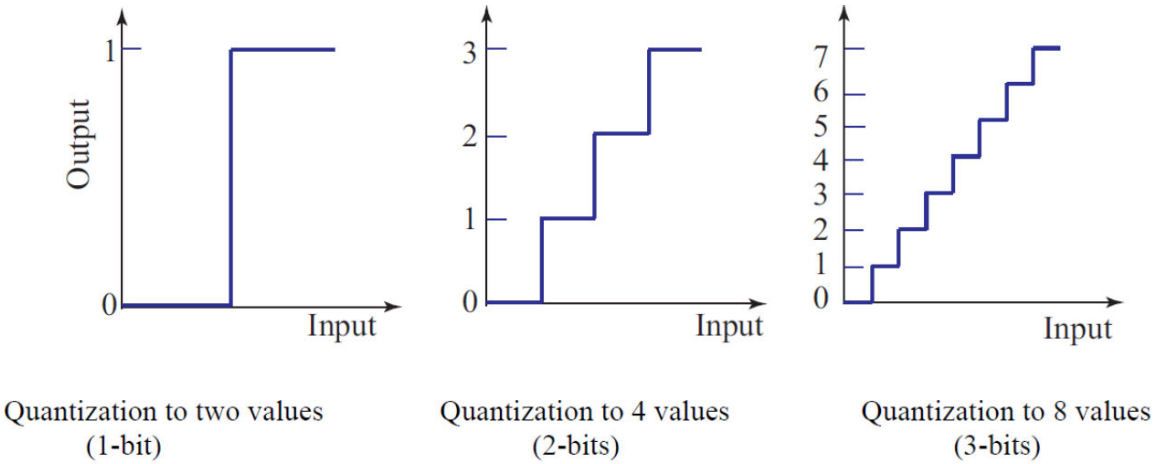
Example 2.5

```
Img=imread('cameraman.tif');
Img_n=255-Img; % 8-bit image, maxium pixel value is 255.

figure;
subplot(121);
imagesc(Img);
colormap(gray)
axis image;
axis off;
title('original');

subplot(122);
imagesc(Img_n);
colormap(gray);
axis image;
axis off;
axis equal;
title('negative');
set(gcf,'Position',[100 100 600 300]);
```



2.2.2 Quantization

Quantization is a transformation which assigns discrete values to the original range of the continous gray scale value, resulting in a fixed-point valued image.



ILL. 2.5 *Quantization 1, 2, 3 bits*

Example 2.6

`thresh = multithresh(A,N);` returns `thresh` a 1-by-N vector containing `N` threshold values using Otsu's method. You can use `thresh` as an input argument to `imquantize` to convert image `A` into an image with `N+1` discrete levels.

`quant_A = imquantize(A,levels)` quantizes image `A` using specified quantization values contained in the `N` element vector `levels`.

```
Img=imread('cameraman.tif');
levels = 10; % quantization level
thresh=multithresh(Img,levels);
Img_q=imquantize(Img,thresh);

figure;
subplot(121);
imagesc(Img);
colormap(gray);
axis image;
axis off;
title('original');
subplot(122);
imagesc(Img_q);
colormap(gray);
axis image;
axis off;
axis equal;
title('quantized');
set(gcf,'Position',[100 100 600 300]);
```

original

quantized