



# Optimization Workshop:

## Theory, Algorithms, and Applications

Universidad de Los Andes

December 9 - 13 2024 / Bogotá, Colombia

## Sponsors



We would like to thank Universidad de los Andes, the John Hopkins University, and University for Southern California for sponsoring this event.

# Contents

<b>Sponsors</b>	<b>2</b>
<b>Schedule</b>	<b>4</b>
Monday, Dec 9 . . . . .	4
Tuesday, Dec 10 . . . . .	5
Wednesday, Dec 11 . . . . .	6
Thursday, Dec 11 . . . . .	7
Friday, Dec 12 . . . . .	8
<b>Keynote Speaker Abstracts</b>	<b>9</b>
<b>Invited Speaker Abstracts</b>	<b>11</b>
Tuesday, Dec 10 . . . . .	11
Wednesday, Dec 11 . . . . .	14
Thursday, Dec 12 . . . . .	16
Friday, Dec 13 . . . . .	19
<b>Accepted Posters</b>	<b>21</b>

# Schedule

## Monday, Dec 9

Event		Location
7:30 - 7:50 AM	Check-in	Building B - Room 202
7:50 - 8:00 AM	Opening	
8:00 - 10:00 AM	<b>Mateo Díaz:</b> Introduction to Continuous Optimization	
	MINICOURSE	
10:00 - 10:30 AM	Coffee Break	
10:30 - 12:30	<b>Justin Goodson:</b> Stochastic Dynamic Programming	
	MINICOURSE	
12:30 - 2:00PM	Lunch Break	
2:00 - 4:00 PM	<b>José Walteros:</b> Introduction to Integer Optimization	
	MINICOURSE	
4:00 - 4:30 PM	Coffee Break	
4:30 AM - 5:30 PM	<b>Panel:</b> Applying to gradschool and life in academia	

Tuesday, Dec 10

Event		Location	
8:00 - 8:30 AM	Check-in	Building B - Room 202	
8:30 - 9:00 AM	Welcome remarks		
9:00 - 10:00 AM	<b>Claudia Sagastizábal:</b> Industrial Mathematics in Action: Full Wave Inversion		KEYNOTE
10:00 - 10:30 AM	Coffee Break		
10:30 - 11:10 AM	Computational Optimization at Google		Daniel Duque
11:10 - 11:50 AM	The Role of Level-Set Geometry on the Performance of PDHG for Conic Convex Optimization		Robert Freund
11:50 AM - 1:30 PM	Lunch Break		
1:30 - 2:10 PM	Data-driven bilevel optimization for inverse problems		Juan Carlos De los Reyes
2:10 - 2:50 PM	Decision Diagram based approaches for discrete bilevel programming		Leonardo Lozano
2:50 - 3:20 PM	Coffee Break		
3:20 - 4:00 PM	A Parametric Approach for Solving Convex Quadratic Optimization with Indicators Over Trees	Salar Fattahi	
4:00 - 6:00 PM	<b>Posters / Cocktail</b>	Building SD 7th Floor	

Wednesday, Dec 11

Event			Location
8:30 - 9:00 AM	Check-in		Building B - Room 202
9:00 - 10:00 AM	<b>Stephen Wright:</b> Revisiting Inexact Fixed-Point Iterations for Min-Max Problems: Stochasticity and Structured Nonconvexity	KEYNOTE	
10:00 - 10:30 AM	Coffee Break		
10:30 - 11:10 AM	TBD	Katya Scheinberg	
11:10 - 11:50 AM	On Graphs with Finite-Time Consensus	César Uribe	
11:50 AM - 1:30 PM	Lunch Break		
1:30 - 2:10 PM	Rounding the Lovász Theta Function with a Value Function Approximation	Alejandro Toriello	
2:10 - 2:50 PM	Non-SOS Polynomial Optimization via a Lifting procedure	Luis Zuluaga	
2:50 - 3:20 PM	Coffee Break		
3:20 - 4:00 PM	Partially Adaptive Multistage Stochastic Programming with Formulations, Algorithms, and Applications	Beste Basciftci	
4:00 - 4:40 PM	TBD	Jorge Sefair	Origen Bistro
6:00 - 8:00 PM	<b>Dinner</b>		

Thursday, Dec 11

Event		Location
8:30 - 9:00 AM	Check-in	Building B - Room 202
9:00 - 10:00 AM	<b>Simge Küçükyavuz:</b> Convex Mixed-Integer Optimization for Causal Discovery	
10:00 - 10:30 AM	Coffee Break	
10:30 - 11:10 AM	Contextualizing, Fast and Slow	
11:10 - 11:50 AM	Problem-based clustering and opportunity cost matrices in stochastic programming	
11:50 AM - 1:30 PM	Lunch Break	
1:30 - 2:10 PM	Variational Theory and Algorithms for a Class of Asymptotically Approachable Nonconvex Problems	
2:10 - 2:50 PM	Non-Convex Semi-Infinite Programming: From discretization to local reduction methods	
2:50 - 3:20 PM	Coffee Break	
3:20 - 4:00 PM	Some ideas for convex optimization using quantum computers	
4:00 - 4:40 PM	TBD	

## Friday, Dec 12

Time	Event	Location
8:30 - 9:00 AM	Check-in	Building B - Room 202
9:00 - 10:00 AM	<b>Andrés Medaglia:</b> Leveraging shortest-path structures for engineering solutions	
10:00 - 10:30 AM	Coffee Break	
10:30 - 11:10 AM	Outlier Robustness via Partial Optimal Transport	
11:10 - 11:50 AM	A tractable algorithm, based on optimal transport, for computing adversarial training lower bounds	
11:50 AM - 1:30 PM	Lunch Break	
1:30 - 2:10 PM	Adversarial robustness for stochastic programs through decision-dependent ambiguity sets	
2:10 - 2:50 PM	Projection onto Cones Generated by Epigraphs of Perspective Functions	
2:50 - 3:20 PM	Coffee Break	
3:20 - 4:00 PM	Proximal Splitting Methods Through the Lenses of Moreau-type Envelopes	
4:00 - 4:40 PM	TBD	
4:40 - 5:00 AM	Closing remarks	



# Keynote Speaker Abstracts

## **Industrial Mathematics in Action: Full Wave Inversion**

Claudia Sagastizábal (*Universidade Estadual de Campinas*)

*Time:* Tuesday, December 10, 9:00-10:00 am.

*Abstract:* An industrial mathematician combines analytical and problem-solving skills, built upon a background in mathematical theory, computing, and statistics. To illustrate these features, we present an industrial application involving advanced tools of mathematical optimization. We discuss how to apply the theory of optimal mass transportation to geosciences, for full-waveform inversion, a seismic imaging technique.

## **Revisiting Inexact Fixed-Point Iterations for Min-Max Problems: Stochasticity and Structured Non-convexity**

Stephen Wright (*University of Wisconsin-Madison*)

*Time:* Wednesday, December 11, 9:00-10:00 am

*Abstract:* We focus on constrained,  $L$ -smooth, nonconvex-nonconcave min-max problems either satisfying  $\rho$ -cohyponotonicity or admitting a solution to the  $\rho$ -weakly Minty Variational Inequality (MVI), where larger values of the parameter  $\rho > 0$  correspond to a greater degree of nonconvexity. Relevant problem classes include two player reinforcement learning and interaction dominant min-max problems. It has been conjectured that first-order methods can tolerate values of  $\rho$  no larger than  $1/L$ , but results until now have stagnated at the tighter requirement  $\rho < 0.5/L$ . We obtain optimal or best-known complexity guarantees with cohyponotonicity or weak MVI conditions for  $\rho < 1/L$ , using inexact variants of Halpern and Krasnoselskii-Mann (KM) iterations. We also provide algorithms and complexity guarantees in the stochastic case with the same range on  $\rho$ . Our improvements come from harnessing the recently proposed "conic nonexpansiveness" property of operators. Finally, we provide a refined analysis for inexact Halpern iteration and propose a stochastic KM iteration with a multilevel Monte Carlo estimator. This talk represents joint work with Ahmet Alacaoglu and Donghwan Kim.

## **Convex Mixed-Integer Optimization for Causal Discovery**

Simge Küçükyavuz (*Northwestern University*)

*Time:* Thursday, December 12, 9:00-10:00 am

*Abstract:* Bayesian Networks (BNs) represent conditional probability relations among a set of random variables (nodes) in the form of a directed acyclic graph (DAG), and have found diverse applications in casual discovery. We study the problem of learning the sparse DAG structure of a BN from continuous observational data. The central problem can be modeled as a mixed-integer program with an objective function composed of a convex loss function and a regularization penalty subject to linear constraints. The optimal solution to this mathematical program is known to have desirable statistical properties under certain conditions. However, the state-of-the-art optimization solvers are not able to obtain provably optimal solutions to the existing mathematical formulations for medium-size problems within reasonable computational times. To address this difficulty, we tackle the problem from both

computational and statistical perspectives. On the one hand, we propose a concrete early stopping criterion to terminate the branch-and-bound process in order to obtain a near-optimal solution to the mixed-integer program, and establish the consistency of this approximate solution. On the other hand, we improve the existing formulations by replacing the linear big-M constraints that represent the relationship between the continuous and binary indicator variables with second-order conic constraints. Our numerical results demonstrate the effectiveness of the proposed approaches. This is joint work with Tong Xu, Armeen Taeb, Ali Shojaie.

### **Leveraging shortest-path structures for engineering solutions**

Andrés Medaglia *Universidad de los Andes*

*Time:* Friday, December 13, December 9:00 - 10:00 am

*Abstract:* Shortest paths are fundamental to solving complex combinatorial problems in various application contexts, from public transportation to infrastructure resilience. This talk presents recent applications where shortest paths serve as a crucial component in the solution procedures for larger problems. By introducing additional constraints and uncertainty, shortest paths become challenging to solve. This talk explores how an effective algorithmic framework can address a wide range of difficult shortest path variants. At the end, I will share some lessons learned along the way that could be applied in other contexts.

# Invited Speaker Abstracts

**Tuesday, Dec 10**

## **Computational Optimization at Google**

Daniel Duque (Google)

*Abstract: Computational optimization at Google is used pervasively across a wide range of applications that differ in quality attributes such as latency, scalability, reliability, among others. This talk explores some of the software engineering considerations that are useful to building optimization infrastructure, as well as some of the tools that have been open-sourced or exposed via APIs for the general public. In particular, I'll talk about Google's mathematical modeling language, MathOpt, and Google's Operations Research API.*

## **The Role of Level-Set Geometry on the Performance of PDHG for Conic Convex Optimization**

Robert M. Freund (MIT)

*Abstract: In joint work with Zikai Xiong, we consider solving huge-scale instances of (convex) conic linear optimization problems, at the scale where matrix-factorization-free methods are attractive or necessary. The restarted primal-dual hybrid gradient method (rPDHG)—with heuristic enhancements and GPU implementation—has been very successful in solving these huge-scale LP problems; however, its performance can have substantial variance, and an intuitive understanding of the drivers of its performance has been lacking. We present a new theoretical analysis of rPDHG for general (convex) conic linear optimization and for LP as a special case thereof. We show a relationship between geometric measures of the primal-dual (sub-)level sets and the convergence rate of rPDHG. We then specialize our results to the case of LP with unique optima. Under unique optima, we present an iteration bound that is “accessible” in the sense that computing the bound is no more difficult than computing the optimal solution itself. This furthermore enables an analysis of the “two-stage performance” of rPDHG: we present a bound on the number of iterations of the first stage (which identifies the optimal basis), and also a bound on the second stage (which computes a nearly-optimal solution). Furthermore, computational tests mostly confirm the tightness of these iteration bounds. We also show a reciprocal relation between the iteration bounds and three equivalent types of condition measures: (i) stability under data perturbation, (ii) proximity to multiple optima, and (iii) the LP sharpness of the instance.*

## **Data-driven bilevel optimization for inverse problems**

Juan Carlos De los Reyes (MODEMAT)

*Abstract: In recent years, novel ideas have been applied to several inverse problems in combination with machine learning approaches, to improve the inversion by optimally choosing different parameters of interest. A fruitful approach in this sense is bilevel optimization, where the inverse problems are considered as lower-level constraints, while on the upper-level a loss function based on a training set is used. When confronted with inverse problems with sparsity-based regularizers, however, the bilevel optimization problem structure becomes quite involved to be analyzed, as classical nonlinear or bilevel programming results cannot be directly utilized. In this talk, I will discuss on a strategy to overcome these difficulties, leading to a reformulation of the bilevel problems as mathematical programs with complementarity constraints. This enables to obtain sharp first-order optimality conditions, but at the price of lifting the problems to a higher dimension. Some ideas on how to reduce the dimension of the problems back will also be presented, together with the different challenges that these problems pose, when dealing with large training sets.*

## **Decision Diagram based approaches for discrete bilevel programming**

Leonardo Lozano (University of Cincinnati)

*Abstract: Integer bilevel programming problems are known to be very challenging due to the lack of strong relaxations that can be efficiently computed. We propose single-level representations of integer bilevel programming problems that rely on network flow-based approximations of the follower's value function, using a decision diagram. We then show how we can derive scalable relaxations from this representation by constructing a minorizer of the follower's value function. We experimentally compare our approach with state-of-the-art bilevel programming solvers and show that we can obtain competitive results for certain problem classes.*

# A Parametric Approach for Solving Convex Quadratic Optimization with Indicators Over Trees

Salar Fattahi (University of Michigan)

*Abstract: Discriminative probabilistic modeling over a finite label space is an advantageous approach for supervised learning tasks such as multi-class classification and learning to rank. Recently, the paradigm shift from supervised to self-supervised learning has significantly advanced AI, particularly language modeling and visual representation learning. However, discriminative probabilistic modeling methods have not been fully addressed in self-supervised learning and their benefits are yet to be uncovered. We study the discriminative probabilistic modeling problem over a continuous domain for (multimodal) self-supervised representation learning to bridge this gap. To address the challenge of computing the integral in the partition function for each anchor data, we leverage the multiple importance sampling (MIS) technique for robust Monte Carlo integration. Within this probabilistic modeling framework, we conduct generalization error analysis to reveal the limitation of the current InfoNCE-based contrastive loss for self-supervised representation learning and derive insights for developing better approaches by reducing the error of Monte Carlo integration. To this end, we propose a novel non-parametric method for approximating the sum of conditional densities required by MIS through optimization, yielding a new contrastive objective for self-supervised representation learning. Moreover, we design an efficient optimization algorithm for solving the proposed objective, which can be interpreted as a contrastive learning method that progressively learns individualized additive margins for negative pairs. Finally, we benchmark the performance of our method against baselines on bimodal contrastive representation learning with CC3M and CC12M datasets. Experimental results demonstrate the overall superior performance of our approach on downstream tasks.*

**Wednesday, Dec 11**

**TBD**

Katya Scheinberg (Georgia Tech)

*Abstract: TBD*

## **On Graphs with Finite-Time Consensus**

César Uribe (Rice University)

*Abstract: In this talk, we present sequences of graphs satisfying the finite-time consensus property (i.e., iterating through such a finite sequence is equivalent to performing global or exact averaging) and their use in Gradient Tracking. We provide an explicit weight matrix representation of the studied sequences and prove its finite-time consensus property. Moreover, we incorporate the studied finite-time consensus topologies into Gradient Tracking and present a new algorithmic scheme called Gradient Tracking for Finite-Time Consensus Topologies (GT-FT). We analyze the new scheme for nonconvex problems with stochastic gradient estimates. Our analysis shows that the convergence rate of GT-FT does not depend on the heterogeneity of the agents' functions or the connectivity of any individual graph in the topology sequence. Furthermore, owing to the sparsity of the graphs, GT-FT requires lower communication costs than Gradient Tracking using the static counterpart of the topology sequence.*

## **Rounding the Lovász Theta Function with a Value Function Approximation**

Alejandro Toriello (Georgia Tech)

*Abstract: The Lovász theta function is a semidefinite programming (SDP) relaxation for the maximum weighted stable set problem, which is tight for perfect graphs. In this paper, we develop a novel rounding scheme for the theta function, i.e., a procedure that extracts a stable set from the solution of the SDP. The main idea is to construct a value function approximation from the SDP solution and then retrieve the stable set using dynamic programming. We prove that our method recovers an optimal stable set in generalized split, chordal, and co-chordal graphs; it is known that almost all perfect graphs are generalized split graphs in an asymptotic sense. To the best of our knowledge, this is the only known rounding strategy for the Lovász theta function that recovers an optimal stable set for large classes of perfect graphs. Our rounding scheme relies on simple linear algebra computations; we only solve one SDP to construct the value function approximation. In contrast, existing methods for computing an optimal stable set in perfect graphs using the theta function require solving multiple SDPs. Preliminary computational experiments show that our rounding strategy produces near-optimal solutions even on non-perfect graphs. Joint with Rui Gong and Diego Cifuentes.*

## Non-SOS Polynomial Optimization via a Lifting procedure

Luis Zuluaga (Lehigh University)

*Abstract:* Sum of square (SOS) polynomials are central in solving polynomial optimization problems; that is, problems whose objective and constraints are defined by polynomials. This is due to their global non-negativity, their representability via positive semidefinite matrices, and their role in fundamental algebraic geometric results. A drawback of using SOS polynomials in polynomial optimization (PO) is that it results in the need to solve computationally expensive semidefinite optimization problems. This fact has motivated the use of alternative classes of non-negative polynomials to address the solution of PO problems. In this talk, we show that by appropriately lifting the PO problem to a higher dimension, any class of non-negative polynomials satisfying a mild condition can be used to address the solution of PO problems. This opens the door to using alternative optimization techniques such as linear, second-order, geometric, and relative entropy optimization to approximately solve PO problems. We illustrate our results with some numerical experiments.

## Partially Adaptive Multistage Stochastic Programming with Formulations, Algorithms, and Applications

Beste Basciftci (University of Iowa)

*Abstract:* Multistage stochastic programming is a powerful tool allowing decision-makers to revise their decisions at each stage based on the realized uncertainty. However, in various practical settings, organizations are not able to be fully flexible, as decisions cannot be revised too frequently due to reasons including contractual restrictions and high organizational impact of revisions. Consequently, decision commitment becomes crucial to ensure that initially made decisions remain unchanged for a certain period. In this talk, we introduce partially adaptive multistage stochastic programming approaches that strike an optimal balance between decision flexibility and commitment by determining the best stages to revise decisions depending on the allowed level of flexibility. We introduce a novel mathematical formulation and theoretical properties eliminating certain constraint sets. We then develop a decomposition method that effectively handles mixed-integer adaptive multistage programs by utilizing the integer L-shaped method and Benders decomposition. We further provide analytical results and approximation algorithms over stylized problems depending on the revision time. Our computational experiments on stochastic lot-sizing and generation expansion planning problems show substantial advantages attained through optimal selections of revision times when flexibility is limited, while demonstrating computational efficiency of the proposed properties and solution methodology.

## TBD

Jorge Sefair (University of Florida)

*Abstract:* TBD

**Thursday, Dec 12**

### **Contextualizing, Fast and Slow**

Bernardo Pagnoncelli (SKEMA Business School)

*Abstract: The integration of machine learning within decision-making can take multiple forms. Recent research has focused on incorporating contextual information—such as weather conditions, day of the week, or unemployment rates—into these processes. The goal is to observe and consider the state of the world before making a decision. I will present two frameworks that integrate predictions and prescriptions, one using chance-constrained optimization, and the other employing simulation optimization. The former can be used in situations where there is sufficient time for decision-making, while the latter addresses situations that require quick responses without the ability to run complex algorithms in real-time, instead performing the necessary computation offline. This is joint work with Hamed Rahimian (Clemson University), Barry Nelson (Northwestern University), Gregory Keslin (Northwestern University) and Matthew Plumlee (Amazon).*

### **Problem-based clustering and opportunity cost matrices in stochastic programming**

Janosch Ortmann (University of Quebec in Montreal)

*Abstract: In stochastic programming, scenarios approximate distributions of unknown parameters, but in many applications the number of scenarios required to realistically model the uncertainty makes the optimisation problem numerically intractable. This motivates the scenario reduction problem: by finding a smaller subset of scenarios, reduce the numerical complexity while keeping the error at an acceptable level. Recently, problem-based scenario reduction methods based on opportunity cost dissimilarities have shown to be effective. In this setting, opportunity cost means the cost of wrongly predicting scenario 1 when actually scenario 2 happens and can be viewed as a measure of how different these two scenarios are with respect to the optimisation problem at hand. In this talk, I will discuss new distance matrices on the scenario set that are based on the idea of giving the decision maker some flexibility to change the first-stage decisions after the uncertainty has been revealed. We then apply clustering on the scenario set equipped with these distances. Preliminary results indicate promising bounds and reveal interesting new structures on the scenario set.*



## Variational Theory and Algorithms for a Class of Asymptotically Approachable Non-convex Problems

Ying Cui (University of California Berkeley)

*Abstract:* We investigate a class of composite nonconvex functions, where the outer function is the sum of univariate extended-real-valued convex functions and the inner function is the limit of difference-of-convex functions. A notable feature of this class is that the inner function can be merely lower semicontinuous instead of continuously differentiable. It covers a range of important yet challenging applications, including the composite value functions of nonlinear programs and the value-at-risk constraints. We propose an asymptotic decomposition of the composite function that guarantees epi-convergence to the original function, leading to necessary optimality conditions for the corresponding minimization problem. The proposed decomposition also enables us to design a numerical algorithm such that any accumulation point of the generated sequence, if exists, satisfies the newly introduced optimality conditions. These results expand on the study of so-called amenable functions introduced by Poliquin and Rockafellar in 1992, which are compositions of convex functions with smooth maps, and the prox-linear methods for their minimization.

## Non-Convex Semi-Infinite Programming: From discretization to local reduction methods

Diego Alejandro Muñoz (Universidad Nacional de Colombia)

*Abstract:* Semi-infinite programs (SIPs) are mathematical programs with finitely many variables and infinitely many constraints. This kind of problems are commonly expressed by the following formulation

$$\begin{aligned} \min_{x \in X} f(x) \\ \text{s.t. } g(x, y) \leq 0, \quad \text{for all } y \in Y \end{aligned}$$

where  $X, Y$  are nonempty compact sets,  $X \subset \mathbb{R}^n$ ,  $Y \subset \mathbb{R}^p$  and  $f: X \rightarrow \mathbb{R}$ ,  $g: X \times Y \rightarrow \mathbb{R}$  are continuous functions. A non-trivial problem is finding a constructive algorithm that provides such a solution. The fundamental idea of solving a SIP is to reduce the infinitely-constrained to a finitely-constrained problem, such that standard nonlinear programming can be applied. Numerical procedures to solve SIP include discretization/exchange and local reduction approaches. For the first procedure, a finite approximation of the original infinite problem is available when the uncertain set is replaced by its discretization or by a sequence of successively refined grids, while for local reduction methods the infinitely many constraints are reduced considering a local description for the feasible set. In this talk, theoretical results are developed for both algorithms and a discussion on remaining challenges and future research directions are presented.

## Some ideas for convex optimization using quantum computers

Giacomo Nannicini (University of Southern California)

*Abstract: Optimization is often mentioned as one of the main application areas for quantum computers, but is this claim backed up by theoretical evidence? In this talk we provide an overview of recent advances in quantum optimization, focusing on convex optimization, and discuss the remaining challenges that need to be addressed. The main results of this talk are a faster classical algorithm for the semidefinite relaxation of the MaxCut problem, an even faster quantum algorithm for the same problem, and a new idea for linear optimization on quantum computers.*

## TBD

David Bernal (Purdue University)

*Abstract: TBD.*

Friday, Dec 13

## Outlier Robustness via Partial Optimal Transport

Soorosh Shafiee (Cornell University)

*Abstract: Distributionally robust optimization (DRO) is an effective approach for data-driven decision-making in the presence of uncertainty. Geometric uncertainty due to sampling or localized perturbations of data points is captured by Wasserstein DRO (WDRO), which seeks to learn a model that performs uniformly well over a Wasserstein ball centered around the observed data distribution. However, WDRO fails to account for non-geometric perturbations such as adversarial outliers, which can greatly distort the Wasserstein distance measurement and impede the learned model. We address this gap by proposing a novel outlier-robust WDRO framework for decision-making under both geometric (Wasserstein) perturbations and non-geometric (total variation (TV)) contamination that allows an epsilon-fraction of data to be arbitrarily corrupted. We design an uncertainty set using a certain robust Wasserstein ball that accounts for both perturbation types and derive minimax optimal excess risk bounds for this procedure that explicitly capture the Wasserstein and TV risks. We prove a strong duality result that enables tractable convex reformulations and efficient computation of our outlier-robust WDRO problem. When the loss function depends only on low-dimensional features of the data, we eliminate certain dimension dependencies from the risk bounds that are unavoidable in the general setting. Finally, we present experiments validating our theory on standard regression and classification tasks.*

## A tractable algorithm, based on optimal transport, for computing adversarial training lower bounds

Nicolás García Trillos (University of Wisconsin-Madison)

*Abstract: Despite the success of deep learning-based algorithms, it is widely known that neural networks may fail to be robust to adversarial perturbations of data. In response to this, a popular paradigm that has been developed to enforce robustness of learning models is adversarial training (AT), but this paradigm introduces many computational and theoretical difficulties. Recent works have developed a connection between AT in the multiclass classification setting and multimarginal optimal transport (MOT), unlocking a new set of tools to study this problem. In this talk, I will leverage the MOT connection to discuss new computationally tractable numerical algorithms for computing universal lower bounds on the optimal adversarial risk. The key insight in the AT setting is that one can harmlessly truncate high-order interactions between classes, preventing the combinatorial run times typically encountered in MOT problems. I'll present a rigorous complexity analysis of the proposed algorithm and validate our theoretical results experimentally on the MNIST and CIFAR-10 datasets, demonstrating the tractability of our approach. This is joint work with Matt Jacobs (UCSB), Jakwang Kim (UBC), and Matt Werenski (Tufts).*

## **Adversarial robustness for stochastic programs through decision-dependent ambiguity sets**

Mauricio Junca (Universidad de Los Andes)

*Abstract: This work presents a new distributionally robust optimization formulation using Wasserstein distances, where the ambiguity set's center and radius depend on the decision variable. We show how to choose the radius to consider adversarial scenarios for the stochastic program. In addition, we numerically compare our proposed approach with the standard formulation of distributionally robust optimization in the context of two-stage linear stochastic problems.*

## **Projection onto Cones Generated by Epigraphs of Perspective Functions**

Luis Briceño Arias (Universidad Técnica Federico Santa María)

*Abstract: In this paper we provide an efficient computation of the projection onto the cone generated by the epigraph of the perspective of any convex lower semicontinuous function. Our formula requires solving only two scalar equations involving the proximity operator of the function. This enables the computation of projections, for instance, onto exponential and power cones, and extends to previously unexplored conic projections, such as the projection onto the hyperbolic cone. We compare numerically the efficiency of the proposed approach in the case of exponential cones with an open source available method in the literature, illustrating its efficiency.*

## **TBD**

Ashwin Pananjady (Georgia Tech)

*Abstract: TBD.*

## Network Reduction Based on Flow Performance

Paulina Castillo (University of Oklahoma)

*Abstract:* Transportation networks in large cities, such as Bogotá, face significant challenges due to the dynamic nature of demand across various scenarios. Traditional approaches to network design typically focus on topological analyses, which may fail to capture the complex flow patterns that emerge under changing conditions. This research introduces a new approach to optimize urban transportation networks by integrating network flow theory with scenario-based analysis. Initially, we solve the Minimum Cost Flow Problem (MCFP) for multiple demand scenarios within a fixed network. In a subsequent stage, we aim to redesign the network by maximizing the overall similarity to the flow solutions of each scenario. This is achieved by reconstructing the network using the original nodes and defining the edges and associated costs, as well as determining how much flow is sent through each edge to best match the MCFP for each scenario. By combining these methods, we seek to create a more adaptive and efficient network that responds to real-time fluctuations in demand, leading to improved service in densely populated urban areas.

## Continous Social Networks

Julián Chitiva (HEC Paris)

*Abstract:* We develop an extension of the classical model of DeGroot (1974) to a continuum of agents when they interact among them according to a DiKernel. We show that, under some regularity assumptions, the continuous model is the limit case of the discrete one. We provide some applications of this result. First, we establish a canonical way to reduce the dimensionality of matrices by comparing matrices of different dimensions in the space of DiKernels. Then, we develop a model of Lobby Competition where two lobbies compete to bias the opinion of a continuum of agents. We give sufficient conditions for the existence of a Nash Equilibrium. Furthermore, we establish the conditions under which a Nash Equilibrium of the game induce an  $\varepsilon$ -Nash Equilibrium of the discretization of the game. Finally, we put forward some elements for the characterization of equilibrium strategies.

## Deterministic Dynamic Lookahead for Energy Management in Three-Phase Unbalanced Microgrids

Pablo José Cortés (Universidad de los Andes)

*Abstract:* In recent years, the widespread adoption of renewable energy sources has grown considerably due to their inherent advantages, including environmental sustainability and economic feasibility. However, the management of electric microgrids presents significant challenges, particularly in coordinating energy production and consumption under the uncertainties posed by fluctuating meteorological conditions. Furthermore, the complexity of microgrid network models adds another layer of difficulty to their effective modeling and solution. This study proposes a set of linearization techniques to address the non-convex, nonlinear optimization model of an unbalanced three-phase microgrid, alongside a deterministic dynamic lookahead method. The modified IEEE-34 system is employed as a test case, incorporating solar panels, wind turbines, diesel generators, and energy storage systems. The analysis also considers stochastic factors such as energy pricing, load demands, and the variability in solar and wind energy generation.

## Fair and Accurate Regression: Relaxations, Heuristics and Exact Methods

Anna Deza

We consider the problem of fair regression which aims to train a machine learning model subject to fairness constraints or regularizers. While there is a large body of work addressing fair classification, the regression setting has received far less attention. Key to measuring popular fairness metrics is the use of indicators based on the intervals into which predictions fall. This poses a challenge as it makes the training problem non-convex and non-smooth. Most existing approaches avoid these hurdles by using convex proxies, approximations, and heuristics to produce fair models. We instead consider the exact fairness measures at hand and develop mixed-integer non-linear formulations for training fair models. We give a strong reformulation obtained by jointly convexifying the non-linear loss function of the model and indicators associated with each prediction. Solution times relative to the natural big-M formulation are substantially improved via the proposed reformulation. We provide an efficient coordinate descent algorithm to produce locally optimal fair models. Our numerical results show that the models produced by the relaxation alone have competitive statistical performance when compared to the state-of-the-art, achieving better accuracy-fairness trade-offs at a fraction of the time. Coordinate descent further improves upon the relaxed models. These results are consistent across synthetic and real datasets.

## A Rollout Algorithm for Dynamic Stochastic Purchasing Routing With Perfect Information Model Estimation

Daniel Hernando Cuellar (Universidad de los Andes)

*Abstract:* This paper addresses the dynamic stochastic purchasing routing problem faced by a purchaser in negotiating with multiple suppliers to meet its demand. The purchaser contacts suppliers sequentially and must decide whether to buy based on purchase prices, available quantities, and routing costs. Each supplier can be contacted only once, with uncertain price and quantity information revealed upon contact. The purchaser faces a maximum number of contacts within a planning horizon. This problem is challenging because of the dynamic stochastic components of the negotiation process. The purchaser must weigh the information revealed by the current supplier against the remaining suppliers' behaviors and route consolidation to make decisions. We propose a rollout algorithm that combines dynamic programming with a perfect information model to determine whether to purchase from the current supplier and which supplier to contact next. Our method samples the supplier behavior scenarios and solves independent deterministic problems to estimate the opportunity cost of not purchasing from remaining suppliers or the impact of purchasing from the current supplier. We introduce a reverse discount factor to reduce the optimism in perfect information model estimation. Our method is validated against benchmark policies using real procurement data from an e-commerce platform. Results confirm the superior performance of the rollout algorithm and provide insights into benchmark policy effectiveness.

## Active Set Identification and Rapid Local Convergence for Primal-Dual Degenerate Problems

Pedro Izquierdo Lehmann (Johns Hopkins University)

*We study the convergence of primal-dual algorithms for convex problems of the form*

$$\min f(x) \quad \text{s.t.} \quad g_j(x) \leq 0 \quad \text{for all } j = 1, \dots, m.$$

*The convergence often shows a two-stage behavior: first it is slow (sublinear), then it is fast (linear). Classical theory explains this behavior when converging to a non-degenerate solution (strict complementarity holds). Yet in practice, the obtained solution is often degenerate. Under mild local growth assumptions, we explain the degenerate case proving: (i) a finite time active set identification result that depends on a notion of distance to degeneracy of the non-degenerate constraints and (ii) a local linear convergence result that depends on the local conditioning of the problem around the solution.*

## Preconditioning for Overparameterized Composite Optimization

Abdel Ghani Labassi (Johns Hopkins University)

*Composite optimization problems involve minimizing a composition of a smooth map with a convex function. Such objectives arise from numerous applications in data science and signal processing, including phase retrieval, matrix sensing, and tensor recovery. Iterative methods like subgradient descent are known to converge linearly when the smooth map is well-conditioned and not overparameterized. However, when either of these conditions fails, subgradient descent exhibits much slower sublinear convergence. To overcome this limitation, we introduce a new preconditioned variant of subgradient descent that, under mild conditions, converges linearly at a rate independent of the conditioning or overparameterization of the smooth map. We further demonstrate that these conditions hold for several problems of interest, including square-variable formulations, symmetric and asymmetric low-rank matrix recovery problems, and tensor factorization.*

## An Exact Method for Reliable Shortest Path Problems with Correlation

Esteban Leiva (Universidad de los Andes)

*Abstract: This poster presents a novel approach to solving Reliable Shortest Path (RSP) problems in road networks with uncertain, correlated travel times. While traditional Least Expected Travel Time (LET) paths fail to account for the high variability and unreliability experienced in real-world scenarios, we focus on RSP objectives that prioritize reliability. Key objectives such as the  $\alpha$ -reliable shortest path ( $\alpha$ -RSP), maximum probability of on-time arrival path (MPOAP), and the shortest  $\alpha$ -reliable path (S- $\alpha$ RP) are considered. Travel times are modeled using a joint normal distribution with non-negative correlations where correlations significantly impact path reliability. This is the first study to provide an exact solution for different RSP objectives under correlated travel times. We propose an algorithm based on a recursive depth-first search procedure, known as the pulse algorithm. The algorithm uses an upper bound on partial path reliability and demonstrates competitive performance across large-scale road networks, outperforming existing state-of-the-art methods, such as dominance-based A\* algorithms by a factor of  $10^6$  in execution time. The algorithm is implemented in a fully documented Julia package, providing a practical tool for routing applications under uncertainty.*



## Agent-Based Modeling Meets Quantum Computing: A Study on Segregation

Nixon Daniel Lizcano (Universidad Nacional de Colombia - Sede Medellín)

Schelling's segregation model offers insights into hidden segregation patterns arising from mild preferences within groups. Traditionally modeled as a random process, agents select new grid locations at random to maximize their "happiness" parameter. In this work, we reinterpret Schelling's model as a probabilistic Quadratic Unconstrained Binary Optimization (QUBO) problem, solved on a Quantum Annealer. By leveraging quantum superposition and entanglement, we aim to efficiently explore the computational complexity and the state-space of possible agent configurations. We also study existing algorithms for inverting agent-based models, such as those used in certain variants of Conway's Game of Life, to determine if a given output state can be generated by a set of rules, further deepening our understanding of algorithmic complexity in these systems.

## Geometric Factor Analysis

Daniel López-Castaño (Johns Hopkins University)

**Abstract:** "Factor analysis is a key tool in statistics and data science, with applications spanning psychology, economics, climatology, chemistry, biology, marketing, social sciences, and operations research. However, the interpretability of factors is often hindered by the rotational invariance intrinsic to the method, which has been a long-standing challenge in the field. To address this, rotations like Varimax and Quartimax are commonly applied to enhance interpretability. This poster presentation introduces a new framework for factor rotations based on optimization on the Stiefel manifold using a Wasserstein distance-based energy. Our approach offers interpretability of factor analysis results. We also provide theoretical guarantees for the speed of convergence of the empirical measure in Wasserstein distance and linear convergence of gradient descent within geodesically convex neighborhoods of the local solutions. Because factor analysis has wide applications, we show the effectiveness of our algorithm in identifying latent factors for image feature extraction, climate modeling, and network analysis. This work is in collaboration with Mateo Diaz, Congwei Yang, and Nicolas Garcia Trillos

## Wasserstein Empirical Barycenter for Wasserstein Submanifolds

Ian McPherson (Johns Hopkins University)

**Abstract:** We consider the problem of fair regression which aims to train a machine learning model subject to fairness constraints or regularizers. While there is a large body of work addressing fair classification, the regression setting has received far less attention. Key to measuring popular fairness metrics is the use of indicators based on the intervals into which predictions fall. This poses a challenge as it makes the training problem non-convex and non-smooth. Most existing approaches avoid these hurdles by using convex proxies, approximations, and heuristics to produce fair models. We instead consider the exact fairness measures at hand and develop mixed-integer non-linear formulations for training fair models. We give a strong reformulation obtained by jointly convexifying the non-linear loss function of the model and indicators associated with each prediction. Solution times relative to the natural big-M formulation are substantially improved via the proposed reformulation. We provide an efficient coordinate descent algorithm to produce locally optimal fair models. Our numerical results show that the models produced by the relaxation alone have competitive statistical performance when compared to the state-of-the-art, achieving better accuracy-fairness trade-offs at a fraction of the time. Coordinate descent further improves upon the relaxed models. These results are consistent across synthetic and real datasets.



## The Semi-infinite programming problem: Brief review and Computational Complexity Analysis

Gabriel Suarez (Universidad Nacional de Colombia - Sede Medellín)

*This poster presents an in-depth analysis of the computational complexity of a specific algorithm, denoted as  $A_n^m$ , used for solving Semi-Infinite Programming (SIP) problems. The analysis focuses on the j-loop within the algorithm, which iteratively refines a "candidate of feasible local solutions" and evaluates constraints imposed on the problem. The study breaks down the computational costs of various steps within the loop, particularly emphasizing the matrix operations required to compute objective functions and constraint evaluations. The worst-case scenario analysis reveals that the computational complexity is primarily dominated by quadratic terms involving the number of constraints,  $k$ , and the number of iterations,  $p$ . Specifically, the total complexity is characterized by the expression  $pO(P) + kpO(Q) + O(k^2p^2)$ , where  $P$  and  $Q$  are the complexities of the sub-procedures used within the algorithm. The findings suggest that as the problem size grows, particularly in terms of constraints and required iterations, the algorithm's complexity becomes increasingly dominated by these quadratic terms. This analysis provides a clear understanding of the algorithm's performance limits, offering valuable insights for optimizing computational efficiency in solving SIP problems.*

## The Normalized Direct Trigonometry Model for the Two-Dimensional Irregular Strip Packing Problem

Germán Fernando Pantoja (Universidad de los Andes)

*Abstract: Background: The Irregular Strip Packing Problem (ISPP) involves packing a set of irregularly shaped items within a strip while minimizing its length. Methods: This study introduces the Normalized Direct Trigonometry Model (NDTM), an innovative enhancement of the Direct Trigonometry Model (DTM). The NDTM incorporates a distance function that supports the integration of the separation constraint, which mandates a minimum separation distance between items. Additionally, the paper proposes a new set of constraints based on the bounding boxes of the pieces aimed at improving the non-overlapping condition. Results: Comparative computational experiments were performed using a comprehensive set of 90 instances. Results show that the NDTM finds more feasible and optimal solutions than the DTM. While the NDTM allows for the implementation of the separation constraint, the number of feasible and optimal solutions tends to decrease as more separation among the items is considered, despite not increasing the number of variables or constraints. Conclusions: The NDTM outperforms the DTM. Moreover, the results indicate that the new set of non-overlapping constraints facilitates the exploration of feasible solutions at the expense of optimality in some cases.*

## Lagrangian Reformulation for Nonconvex Optimization: Tailoring Problems to Specialized Solvers

Rodolfo Quintero (Lehigh University)

*Abstract: Advancements in computing technologies, such as Quantum and Ising devices, have led to a surge of interest in studying different ways to reformulate nonconvex optimization problems, especially those involving binary variables. In this poster, I will present results that show the equivalence between a class of equality-constrained nonconvex optimization problems and their Lagrangian relaxation, bridging classical optimization theory with emerging quantum computational approaches and filling a gap in classical optimization literature.*

## Semi-Definite Optimization Approach for Calderón Inverse Problem

Julián Villalón (Universidad de los Andes)

Calderón's inverse problem deals with the recovery of  $p$  in the differential expression  $-\operatorname{div}(p \cdot \nabla u) = 0$ . It is well-known for its ill-posedness which yields a discontinuous reconstruction map taking the measurements to the conductivity  $p$ . Natural energy inequalities lead to variational constraints on the space of admissible conductivities. This implies that  $p$  can be characterized as an extreme point of a convex feasible region defined by a positive semidefinite restriction. Recently it was shown that, for a finite-dimensional restriction of the problem, there exists a linear functional having  $p$  as its unique optimizer. However, it is still unknown how to compute such a functional  $f$ , and how it depends on the setting (e.g. the underlying domain). In this work, we explore structural properties of the feasible set and some implications for the form of  $f$ . For example, we show that in the case of infinite-dimensional measurements (i.e. full knowledge of the Neumann-to-Dirichlet operator) the computation of  $f$  can be avoided. We also implement and numerically test an interior-point method for cases when certain properties of  $f$  are known. Although a (potentially large) finite number of measurements is enough to reconstruct  $p$ , in this case it is not possible to use a localized potentials result on which most of the proofs are based. This limitation makes it harder to understand the structure of the feasible set resulting in more