Homework Assignment 1
FE 621: Computational Methods in Finance
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Overview

In this Homework Assignment, we explore various numerical optimization methods through the lens of the Black-Scholes-Merton Option pricing model[1]. Using this, we calculate and explore the implied volatility of options for various assets traded on the market. Furthermore, we also explore numeric methods of differential calculation to compute the Greeks of these candidate options. Finally, we explore numeric integration and the behavior of various quadrature methods.

Unless otherwise stated, the following shorthand notation is used to distinguish between dates:

- **DATA1** - Wednesday, February 6 2019 (*2/6/19*);
- **DATA2** - Thursday, February 7 2019 (*2/7/19*).

The content of this Homework Assignment is divided into three sections; the first discusses data gathering, formatting, and a discussion of the assets being examined. The second contains data analysis, and an exploration of implied volatility through the Black-Scholes-Merton pricing framework and related computations. Finally, the third section discusses numerical integration and the convergence of various quadrature rules.

See Appendix D for specific question implementations, and the project GitHub repository[2] for full source code of the `fe621` Python package.

1. Shreve 2004
2. Weerawarana 2019

Contents

1 Data Overview

1.1 Asset Descriptions

1.1.1 *SPY* - SPDR S&P 500 ETF[3]

The S&P 500 (*Standard & Poor's 500*) is a stock market index tracking the 500 largest companies on the American Stock Exchange by Market Capitalization. In this case, the market capitalization is defined as the number of outstanding shares, multiplied by the current share price. A stock market index is designed to be a metric that can be used by market observers as a benchmark to gauge the relative health of the stock market, by analyzing the aggregate performance of its largest components.

However, this index is not the same as the *SPY* ETF. An ETF (*Exchange Traded Fund*) is a basket of stocks that is designed to track a specific index or benchmark. That is, it provides investors with exposure to a index or benchmark, without having to own all of the underlying assets that constitute a composite ETF. In addition to higher liquidity, this type of investment also provides lower transaction costs and required minimum investment to gain exposure to a given index or benchmark. It is traded on an exchange, akin to a typical traded asset.

1.1.2 *VIX* - CBOE Volatility Index[4]

The CBOE (*Chicago Board Options Exchange*) volatility index, *VIX* is an exchange traded product (*ETP*) designed to give investors exposure to the market's expectation of 30-day volatility. It is priced using a large set of implied volatility of put and call options on the S&P 500 index to gauge investor sentiment. Typically, the price of the VIX has an inverse relationship to the price of the S&P 500 index. Similar to an ETF, an ETP is also traded on an exchange as a typical traded asset.

1.2 Data Gathering

For the assignment, we downloaded monthly options on *Amazon Inc.* (ticker: AMZN) and *S&P 500 ETF* (ticker: SPY) at various strike prices for the following dates:

- *02/15/19* - Friday, February 15 2019;
- *03/15/19* - Friday, March 15 2019;
- *04/18/19* - Thursday, April 18 2019.

A wide variety of option strike prices were considered, with the following ranges:

- *AMZN* - \$1555 to \$1725 in increments of \$5 (35 strike prices);
- *SPY* - \$256 to \$284 in increments of \$1 (29 strike prices).

Intra-day minute closing price data was gathered for both put and call options with expiration dates and strike prices detailed above. This intra-day data was gathered for the trading day *2/6/19* (February 6 2019; **DATA1**). Additionally, intra-day minute closing price data was also downloaded for each of the underlying assets. This data was downloaded for both *2/6/19* (February 6 2019; **DATA1**), and *2/7/19* (February 7 2019; **DATA2**).

3. State Street Global Advisors 2019
4. CBOE (Chicago Board Options Exchange) 2019

This data detailed above was gathered utilizing *Rblpapi*[5], which provides an R interface to data on the Bloomberg Terminal[6]. The data download was automated, and corresponding intra-day prices for each of the options were output to individual files. The source code for this implementation is available in Appendix D.1.1.

Furthermore, as a proxy for the *risk-free rate*, we chose to utilize the effective Federal Funds Rate (FFR). This is the interest rate at which depository institutions in the United States lend reserve balances to other depository institutions overnight. This data was gathered for both dates, and correspond to **DATA1** and **DATA2**. The effective FFR is published daily by the US Federal Reserve Board of Governors, and are expressed as yields per annum.[7]

1.2.1 Data Cleaning

For easier programmatic access, the data was placed in a hierarchical structure, corresponding to the **DATA1**, **DATA2** data division. Each of the option and asset prices for the corresponding days were placed in the requisite sub-folders. This directory structure is reproduced below.

```
data
├── DATA1
│   ├── AMZN
│   ├── SPY
│   └── VIX
└── DATA2
    ├── AMZN
    ├── SPY
    └── VIX

8 directories
```

Option price filenames were changed to OOC format option names, discussed further below. This was done utilizing a cleaning script, written in Python. This script employs utility functions from the `fe621` Python package[8].

1.3 Option Naming Convention

A modern convention for naming option contracts was proposed by the Options Clearing Commission (OCC) in 2008[9], and adopted in 2010. The OCC is an organization that acts as both the issuer and guarantor for option and future contracts. The OCC is governed by the Securities and Exchange Commission (SEC) and the Commodities Futures Trading Commission (CFTC). The current convention for option naming is best explained by example.

Consider the option code, *AMZN190215C01960000*. This corresponds to a **Call Option** on **Amazon Inc. (AMZN)**, with a strike price of **$1960.00** and an expiration date of **2/15/19** (February 15 2019).

The methodology of this nomenclature is explained in detail below:

5. Armstrong et al. 2018
6. Bloomberg L.P. 2019
7. Board of Governors of the Federal Reserve System 2019
8. Weerawarana 2019
9. Options Symbology Initiative Working Group 2008

AMZN190215C01960000

- **AMZN** - Ticker of the company (arbitrary length; always first sequence of characters)
- **19** - Expiration year of the contract (shortened to two digits, i.e. 2019 $\rightarrow$ 19)
- **02** - Expiration month of the contract
- **15** - Expiration day of the contract
- **C** - Type of option (C for call, P for put)
- **01960** - Dollar component of strike price (in \$; always 5 digits)
- **000** - $\frac{1}{1000}^{\text{th}}$ Dollar component of strike price (in $\frac{1}{1000}$\$; always 3 digits)

Similarly, the following option code corresponds to a **Put Option** on **SPDR S&P 500 ETF (SPY)**, with a strike price of **\$287.50** and an expiration date of **3/15/19** (March 15 2019):

SPY190315P00287500

Finally, the following option code corresponds to a **Call Option** on **CBOE Volatility Index (VIX)**, with a strike price of **\$16.35** and an expiration date of **4/18/19** (February 18 2019):

VIX190418C00016350

2 Data Analysis

Note: All Python scripts reproduced in this section are a subset of the `fe621`[10] *Python package developed for this class.*

2.1 Black-Scholes Model

With the probabilities d_1 and d_2 defined as:

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$$\Phi(x) = \int_{-\infty}^{x} \phi(z)dz = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz$$

```python
from typing import Tuple

import numpy as np


def computeD1D2(current: float, volatility: float, ttm: float, strike: float,
                rf: float) -> Tuple[float, float]:
    """Helper function to compute the risk-adjusted priors of exercising the
    option contract, and keeping the underlying asset. This is used in the
    computation of both the Call and Put options in the
    Black-Scholes-Merton framework.

    Arguments:
        current {float} -- Current price of the underlying asset.
        volatility {float} -- Volatility of the underlying asset price.
        ttm {float} -- Time to expiration (in years).
        strike {float} -- Strike price of the option contract.
        rf {float} -- Risk-free rate (annual).

    Returns:
        Tuple[float, float] -- Tuple with d1, and d2 respectively.
    """

    d1 = (np.log(current / strike) + (rf + ((volatility ** 2) / 2)) * ttm) \
        / (volatility * np.sqrt(ttm))
    d2 = d1 - (volatility * np.sqrt(ttm))

    return (d1, d2)
```

../fe621/black_scholes/util.py

10. Weerawarana 2019

***Note:* The following assumes the dividend rate, $q = 0$.**

2.1.1 Put Option

The Black-Scholes Option price for a European Put ($P(S_t)$) option is defined as:

$$P(S_t) = Ke^{-r(T-t)}\Phi(-d_2) - S_t\Phi(-d_1)$$

```
from .util import computeD1D2

from scipy.stats import norm
import numpy as np


def blackScholesPut(current: float, volatility: float, ttm: float,
                    strike: float, rf: float) -> float:
    """Function to compute the Black-Scholes-Merton price of a European Put
    Option, parameterized by the current underlying asset price, volatility,
    time to expiration, strike price, and risk-free rate.

    Arguments:
        current {float} -- Current price of the underlying asset.
        volatility {float} -- Volatility of the underlying asset price.
        ttm {float} -- Time to expiration (in years).
        strike {float} -- Strike price of the option contract.
        rf {float} -- Risk-free rate (annual).

    Returns:
        float -- Price of a European Put Option contract.
    """

    d1, d2 = computeD1D2(current, volatility, ttm, strike, rf)

    put = (strike * np.exp(-1 * rf * ttm) * norm.cdf(-1 * d2)) \
        - (strike * norm.cdf(-1 * d1))

    return put
```

../fe621/black_scholes/put.py

2.1.2 Call Option

The Black-Scholes Option price for a European Call ($C(S_t)$) option is defined as:

$$C(S_t) = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

```
from .util import computeD1D2

from scipy.stats import norm
import numpy as np


def blackScholesCall(current: float, volatility: float, ttm: float,
                     strike: float, rf: float) -> float:
    """Function to compute the Black-Scholes-Merton price of a European Call
```

```
    Option, parameterized by the current underlying asset price, volatility,
    time to expiration, strike price, and risk-free rate.

    Arguments:
        current {float} -- Current price of the underlying asset.
        volatility {float} -- Volatility of the underlying asset price.
        ttm {float} -- Time to expiration (in years).
        strike {float} -- Strike price of the option contract.
        rf {float} -- Risk-free rate (annual).

    Returns:
        float -- Price of a European Call Option contract.
    """

    d1, d2 = computeD1D2(current, volatility, ttm, strike, rf)

    call = (current * norm.cdf(d1)) \
        - (strike * np.exp(-1 * rf * ttm) * norm.cdf(d2))

    return call
```

../fe621/black_scholes/call.py

2.1.3 Put-Call Parity

The relationship between the price of a Call and Put option is governed by Put-Call parity:

$$P(S_t) = C(S_t) - S_t + Ke^{-r(T-t)}$$

```
import numpy as np


def call(put: float, current: float, strike: float, ttm: float,
         rf: float) -> float:
    """Function to compute the price of a European Call option contract from a
    European Put option contract price using Put-Call parity.

    Arguments:
        put {float} -- Price of the put option.
        current {float} -- Current price of the underlying asset.
        strike {float} -- Strike price of the option contract.
        ttm {float} -- Time to expiration (in years).
        rf {float} -- Risk-free rate (annual).

    Returns:
        float -- Price of a European Call Option contract.
    """

    return put + current - (strike * np.exp(-1 * rf * ttm))


def put(call: float, current: float, strike: float, ttm: float,
        rf: float) -> float:
    """Function to compute the price of a European Put option contract from a
    European Call option contract price using Put-Call parity.

    Arguments:
        call {float} -- Price of the call option.
        current {float} -- Current price of the underlying asset.
```

```
        strike {float} -- Strike price of the option contract.
        ttm {float} -- Time to expiration (in years).
        rf {float} -- Risk-free rate (annual).

    Returns:
        float -- Price of a European Put Option contract.
    """

    return call - current + (strike * np.exp(-1 * rf * ttm))
```

../fe621/black_scholes/parity.py

2.1.4 The Greeks

The Greeks are the quantities representing the sensitivity of the price of a derivative with respect to changes in the underlying parameters. The following formulas are implemented to calculate each of the Greeks using the Black-Scholes option pricing formula. These formulas are derived in full in (Stefanica 2011) and (Weerawarana 2016).

***Note:* The following assumes the dividend rate, $q = 0$.**

Delta

The Delta (Δ) of an option is the first derivative of an option with respect to the price of the underlying asset at time t, S_t.

$$\Delta(C) = \frac{\partial C(S_t)}{\partial S_t} = \Phi(d_1)$$

Gamma

The Gamma (Γ) of an option is the second derivative of an option with respect to the price of the underlying asset at time t, S_t.

$$\Gamma(C) = \frac{\partial^2 C(S_t)}{\partial S_t^2} = \frac{\phi(d_1)}{S_t \sigma \sqrt{T-t}}$$

Vega

The Vega (ν) of an option is the first derivative of an option with respect to the volatility of the underlying asset at time t, σ.

$$\nu(C) = \nu(P) = \frac{\partial C(S_t)}{\partial \sigma} = S_t \sqrt{T-t}\, \phi(d_1)$$

```
from .util import computeD1D2

from scipy.stats import norm

import numpy as np


def callDelta(current: float, volatility: float, ttm: float, strike: float,
              rf: float) -> float:
    """Function to compute the Delta of a call option using the Black-Scholes
```

```
    formula.

    Arguments:
        current {float} -- Current price of the underlying asset.
        volatility {float} -- Volatility of the underlying asset price.
        ttm {float} -- Time to expiration (in years).
        strike {float} -- Strike price of the option contract.
        rf {float} -- Risk-free rate (annual).

    Returns:
        float -- Delta of a European Call Option contract.
    """

    d1, _ = computeD1D2(current, volatility, ttm, strike, rf)

    return norm.cdf(d1)


def callGamma(current: float, volatility: float, ttm: float, strike: float,
              rf: float) -> float:
    """Function to compute the Gamma of a Call option using the Black-Scholes
    formula.

    Arguments:
        current {float} -- Current price of the underlying asset.
        volatility {float} -- Volatility of the underlying asset price.
        ttm {float} -- Time to expiration (in years).
        strike {float} -- Strike price of the option contract.
        rf {float} -- Risk-free rate (annual).

    Returns:
        float -- Delta of a European Call Opton Option contract.
    """

    d1, _ = computeD1D2(current, volatility, ttm, strike, rf)

    return (norm.pdf(d1) / (current * volatility * np.sqrt(ttm)))


def vega(current: float, volatility: float, ttm: float, strike: float,
         rf: float) -> float:
    """Function to compute the Vega of an option using the Black-Scholes formula.

    Arguments:
        current {float} -- Current price of the underlying asset.
        volatility {float} -- Volatility of the underlying asset price.
        ttm {float} -- Time to expiration (in years).
        strike {float} -- Strike price of the option contract.
        rf {float} -- Risk-free rate (annual).

    Returns:
        float -- Vega of a European Option contract.
    """

    d1, _ = computeD1D2(current, volatility, ttm, strike, rf)

    return current * np.sqrt(ttm) * norm.pdf(d1)
```

../fe621/black_scholes/greeks.py

2.2 Numeric Optimization

2.2.1 Bisection Method

In this section, we implement the Bisection optimization method. The bisection algorithm is outlined in Algorithm 1. The algorithm is implemented recursively.

Algorithm 1: Bisection Algorithm

Input: Input function, f to be optimized; must have sign change. Search space start and stop points, a and b. Tolerance level, ϵ.
Output: Point $x^* \in [a, b]$ where $f(x^*) = 0$.
Let midpoint $= m$;
repeat
  $m = \frac{a+b}{2}$;
  if $f(a) \times f(mid) < 0$ **then**
    $b = m$
  end
  if $f(b) \times f(mid) < 0$ **then**
    $a = m$
  end
until $(b - a) < \epsilon$;
return $\frac{a+b}{2}$;

```python
from typing import Callable
import numpy as np


def bisectionSolver(f: Callable, a: float, b: float,
                    tol: float=10e-6) -> float:
    """Bisection method solver, implemented using recursion.

    Arguments:
        f {Callable} -- Function to be optimized.
        a {float} -- Lower bound.
        b {float} -- Upper bound.

    Keyword Arguments:
        tol {float} -- Solution tolerance (default: {10e-6}).

    Raises:
        Exception -- Raised if no solution is found.

    Returns:
        float -- Solution to the function s.t. f(x) = 0.
    """

    # Compute midpoint
    mid = (a + b) / 2

    # Check if estimate is within tolerance
    if (b - a) < tol:
        return mid

    # Evaluate function at midpoint
    f_mid = f(mid)
```

```
    # Check position of estimate, move point and re-evaluate
    if (f(a) * f_mid) < 0:
        return bisectionSolver(f=f, a=a, b=mid)
    elif (f(b) * f_mid) < 0:
        return bisectionSolver(f=f, a=mid, b=b)
    else:
        raise Exception("No solution found.")
```

../fe621/optimization/bisection.py

2.2.2 Newton Method

In this section, we implement the Newton optimization method. The Newton method algorithm is outlined in Algorithm 2.[11] The algorithm is implemented recursively.

Algorithm 2: Newton's Method

Input: A differentiable function $f : \mathbb{R}^a \to \mathbb{R}^b \, \forall \, a, b \in \mathbb{N}_{>0}$. Starting guess for the root x_0. Tolerance level, ϵ.
Output: $x^* \in \mathbb{R}^a$, such that $f(x^*) = 0$
$k = 1$;
repeat
  $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$;
  $k = k + 1$
until $|x_k - x_{k-1}| < \epsilon$;
return x_{k+1};

```
from typing import Callable
import numpy as np


def newtonSolver(f: Callable, f_prime: Callable, guess: float,
                 tol: float=10e-6, prev: float=0) -> float:
    """Newton method solver for 1 dimension, implemented recursively.

    Arguments:
        f {Callable} -- Objective function (must have zero root).
        f_prime {Callable} -- First derivative of objective with respect to
                              the decision variable.
        guess {float} -- Guess for the decision variable.

    Keyword Arguments:
        tol {float} -- Tolerance level (default: {10e-6}).
        prev {float} -- Guess from previous iteration (for convergence check).

    Returns:
        float -- Solution to the function s.t. f(x) = 0.
    """

    # Assigning current guess to x_old
    x_old = guess

```

11. Stefanica 2011

```
    # Checking if decision variable changed by less than tolerance level
    if np.abs(x_old - prev) < tol:
        return x_old
    else:
        # Compute new estimate for x
        x_new = x_old - (f(x_old) / f_prime(x_old))
        return newtonSolver(f=f, f_prime=f_prime, guess=x_new,
                            tol=tol, prev=x_old)
```

../fe621/optimization/newton.py

2.2.3 Convergence Comparison

Here, we compare the performance of each of the optimization methods described above, the Bisection method and Newton method. This was done by computing the average daily implied volatility on the complete SPY option chain in the dataset.

The average daily implied volatility is computed by first calculating the implied volatility by-minute. Then, the mean of these minute-level implied volatilities is computed and is treated as the average daily implied volatility of the given option. For this comparison, the tolerance level of each of the termination conditions was set to 1×10^{-4}.

	Newton Method	Bisection Method
Number of Input Options	165.0	165.0
Number of Output Options	164.0	164.0
Number of Dropped Options	1.0	1.0
Time Elapsed for Computation (s)	2423.1484701633453	2406.0394039154053
Average Time per Option (s)	14.685748304020274	14.582056993426699

Table 1: Convergence comparison of average daily implied volatility computation on the SPY option chain using the Bisection and Newton optimization methods.

The time elapsed for these computations, and other related statistics under each of the two optimization methods are presented in Table 1.

Despite having a theoretical quadratic convergence rate, Newton's method results in slower performance compared to the Bisection method. This is evident from both the total time elapsed, and the average time per operation (computed to include dropped option computations for consistency).

This can be attributed to the fact that some of the minute-level implied volatility optimizations do not have solutions. The Bisection method reaches a state of "no solution" faster than Newton's method, as it employs a technique of reducing the possible range of the solution. This converging search space would suggest it discovers a state of "no solution" faster than the unbounded search space of the Newton method. In principle - on the condition that the existence of a solution is guaranteed - the Newton method will converge faster than the Bisection method, given a reasonable initial guess.

2.3 Implied Volatility

In this section, we utilize the functions and data described above to calculate the average implied volatility of each of the option chains. This was done for the entire dataset using the Bisection Method. Additionally, we also discuss the differences in average daily implied volatility between *in-the-money* and *out-of-the-money* options.

2.3.1 Average Daily Implied Volatility

Average daily implied volatility was computed for each option, across all strike prices and expiration dates, for both SPY and AMZN option chains. This optimization on the aggregate dataset was completed using the Bisection Method.

This was done by first computing the implied volatility for each minute, solving for some σ such that $(C(S_t)|_\sigma - P = 0)$ or $(P(S_t)|_\sigma - P = 0)$ for a call or put option respectively. Then, the mean of each of these implied volatilities was computed to obtain the daily average implied volatility for an option with a given strike price and expiration date. For this comparison, the tolerance level of each of the termination conditions was set to 1×10^{-7}.

The complete dataset of average daily implied volatility is reproduced for the complete option chains on SPY in Appendix A.1 and AMZN in Appendix A.2.

2.4 Implied Volatility Analysis

We also compared the average daily implied volatility of options *in-the-money*, and *out-of-the-money*. For this comparison, we defined the ratio of *money-ness* to be $\pm 5\%$ of the current underlying asset price, where options within the range are *in-the-money*, and *out-of-the-money* otherwise. This comparison data is presented in Table 2.

	SPY	AMZN
In-the-money Options Average Daily Implied Vol	0.15739310934145576	0.29781477676343643
Out-of-the-money Options Average Daily Implied Vol	0.16631389625540363	0.3189350623328305

Table 2: Comparison of *in-the-money* and *out-of-the-money* options through the lens of their average daily implied volatility.

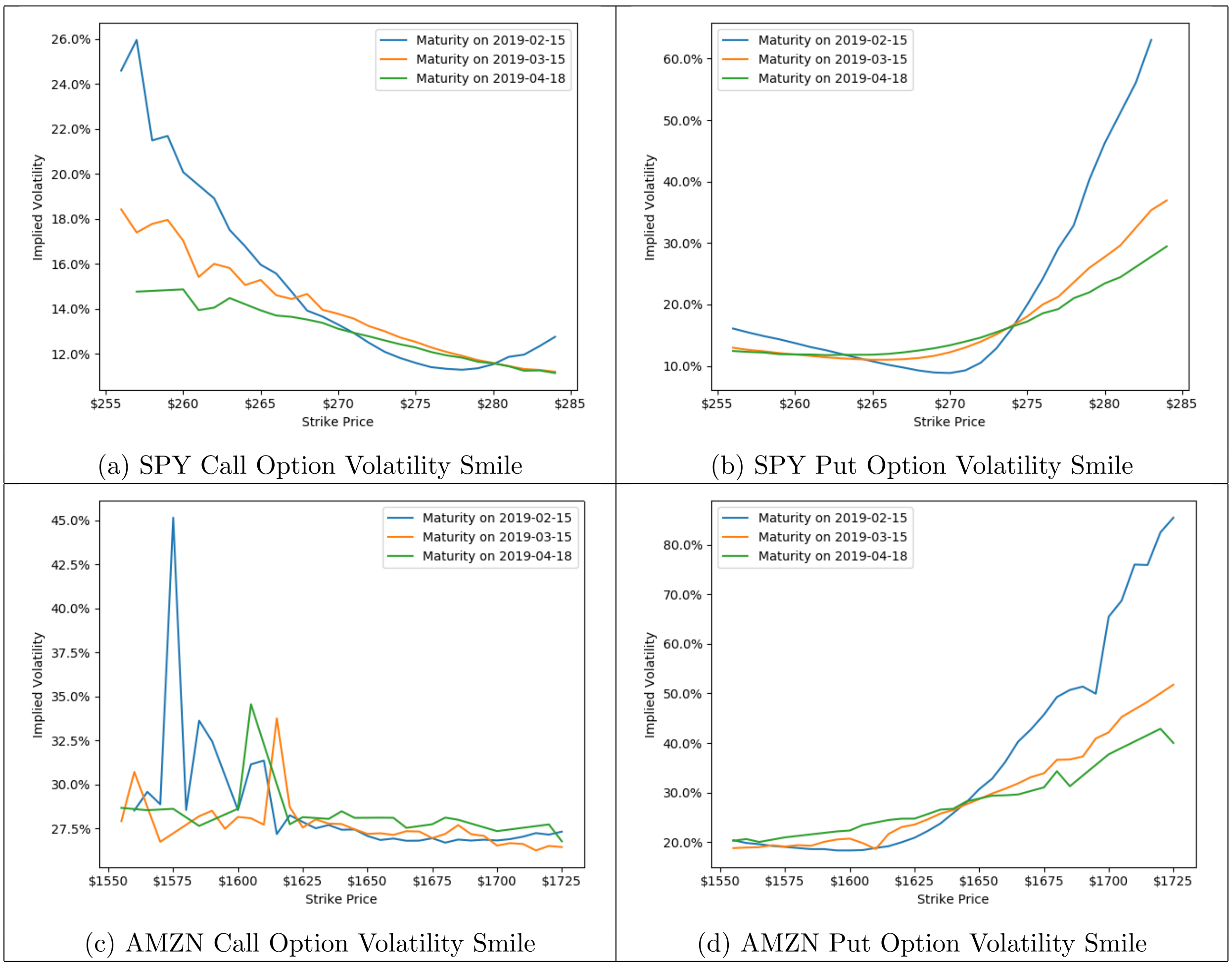
2.5 Volatility Plots

In this section, we explore the visual relationship between the computed average daily implied volatility (see Section 2.3.1), the strike price, and the expiration date of the options. The source code for this question is reproduced in Appendix D.2.4.

2.5.1 Volatility Smile

The Volatility Smile is the graph of the relationship between the strike price and the average daily implied volatility of the option:

$$\hat{\sigma} = \hat{f}(K)$$



(a) SPY Call Option Volatility Smile

(b) SPY Put Option Volatility Smile

(c) AMZN Call Option Volatility Smile

(d) AMZN Put Option Volatility Smile

Figure 1: Volatility Smiles of call and put option chains on AMZN and SPY. Plots the relationship between the strike price and implied volatility for various maturities.

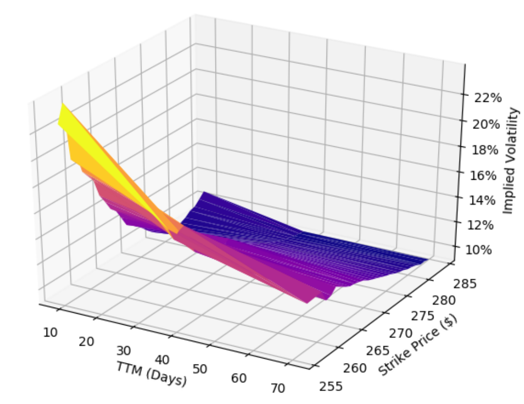
The various options (of the same type and underlying asset) are graphed on the same axes, and different expiration dates are displayed in different colors. The Volatility Smile is plotted for both put and call options on both SPY and AMZN in Figure 1.

2.5.2 Volatility Surface

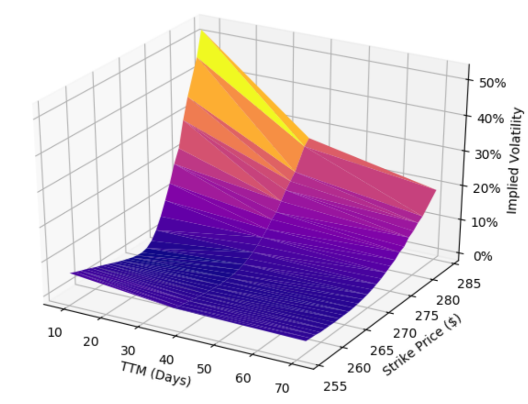
The Volatility Surface is the graph of the relationship between the strike price, the time to maturity, and the average daily implied volatility of the option:

$$\hat{\sigma} = \hat{f}(K, \sqrt{T-t})$$

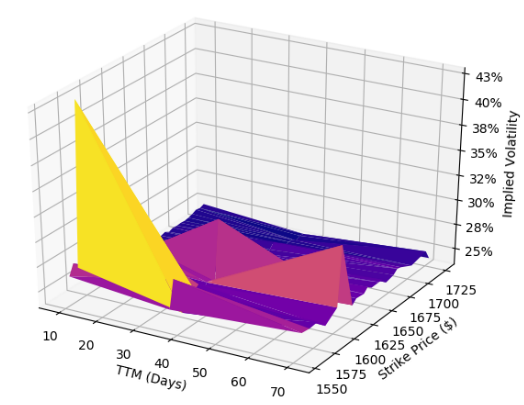
The various options (of the same type and underlying asset) are graphed on the same axes. The Volatility Surface is plotted for both put and call options on both SPY and AMZN in Figure 2.



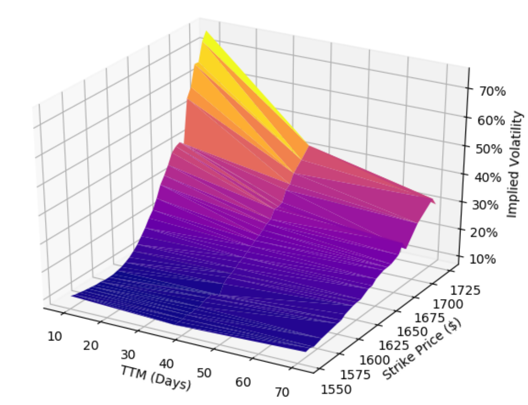
(a) SPY Call Option Volatility Surface



(b) SPY Put Option Volatility Surface



(c) AMZN Call Option Volatility Surface



(d) AMZN Put Option Volatility Surface

Figure 2: Volatility Surfaces of call and put option chains on AMZN and SPY. Plots the relationship between the strike price, time to maturity, and the implied volatility.

2.6 The Greeks

In this section, we compute the Greeks for the options. To do this, we employ the estimate of the average daily implied volatility (see Section 2.3.1). We compute the Greeks using both the analytical formula (see Section 2.1.4), and by estimation of the derivatives using the central finite difference Method.

2.6.1 Central Finite Difference Method

The central finite difference method is a framework for computing the numerical derivative of a three times differentiable function in an interval around the point a, f. Then, numerical approximations for the first and second derivatives are[12]:

$$\text{Let } h > 0$$
$$f'(a) \approx \frac{f(a+h) - f(a-h)}{2h} + O(h^2)$$
$$f''(a) \approx \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} + O(h^2)$$

2.6.2 Analytical and Estimated Greeks

The analytical and estimated Delta, Δ, Gamma, Γ, and Vega, ν, for the complete SPY and AMZN option chains are presented in Appendix B.1 and Appendix B.2, respectively. The source code for this computation is reproduced in Appendix D.2.5.

2.7 DATA2 Computed Prices

Finally, we compute option prices utilizing the closing price data for DATA2. This was accomplished using the risk-free rate for DATA2, and correspondingly computed time-to-maturities. The computed prices are presented for both the SPY and AMZN option chains in Appendix C.1 and Appendix C.1, respectively. The source code for this computation is reproduced in Appendix D.2.6.

12. Stefanica 2011

3 Numerical Integration

3.1 Quadrature Methods

In this section, we implement the *Trapezoidal Rule* and *Simpson's Rule* quadrature methods.

$$\text{Let data} = \boldsymbol{x}$$
$$\text{Let } i^{\text{th}} \text{ element of } \boldsymbol{x} = x_i$$

3.1.1 Trapezoidal Rule

$$\text{Let Trapezoidal rule approximation} = T_N(f)$$
$$\Rightarrow T_N(f) = \sum_{i=1}^{N} \left[\left(\frac{f(x_{i-1}) + f(x_i)}{2} \right) \times h \right]$$
$$\Rightarrow h \times \sum_{i=1}^{N} \left[\frac{f(x_{i-1}) + f(x_i)}{2} \right] = h \times \left(\frac{1}{2} f(x_0) + f(x_1) + \cdots + f(x_{N-1} + \frac{1}{2} f(x_N)) \right)$$
$$\therefore T_N(f) = hf(\boldsymbol{x}) - \frac{h}{2}(f(x_0) + f(x_N))$$

```python
from typing import Callable
import numpy as np


def trapezoidalRule(f: Callable, N: float, start: float=-1e6,
                    stop: float=1e6) -> float:
    """Function to approximate the numeric integral of a function, f, using
    the Trapezoidal rule.

    Arguments:
        f {Callable} -- Function who's integral is to be estimated.
        N {int} -- Number of nodes to consider.

    Keyword Arguments:
        start {float} -- Starting point (default: {-1e6}).
        stop {float} -- Stopping point (default: {1e6}).

    Returns:
        float -- Approximation of the area under the function.
    """

    # Building values for approximation, and getting step size
    x, h = np.linspace(start=start, stop=stop, num=N, retstep=True)

    # Estimating area using trapezoidal rule, return
    return np.sum((h * f(x))) - ((h / 2) * (f(start) + f(stop)))
```

../fe621/numerical_integration/trapezoidal.py

3.1.2 Simpson's Rule

The following equation is derived in full in (Florescu 2019).

$$
\begin{aligned}
&\text{Let Simpson's rule approximation} = S_N(f) \\
\Rightarrow S_N(f) &\approx \frac{h}{6} \times \sum_{i=1}^{N} \left[f(x_{i-1}) + 4f\left(\frac{x_{i-1}+x_i}{2}\right) + f(x_i) \right] \\
&= \frac{h}{6} \left(\sum_{i=1}^{N} [f(x_{i-1}) + f(x_i)] + 4 \times \sum_{i=1}^{N} \left[f\left(\frac{x_{i-1}+x_i}{2}\right) \right] \right)
\end{aligned}
$$

$$
\begin{gathered}
\text{Note that } \left(\frac{x_{i-1}+x_i}{2}\right) \text{ is the midpoint between the points in } \boldsymbol{x}. \\
\text{Let the above} = \boldsymbol{x}_{\text{mid}} \\
\therefore\ S_N(f) \approx \frac{h}{6} \left(2f(\boldsymbol{x}) - (f(x_0) + f(x_N)) + 4f(\boldsymbol{x}_{\text{mid}})\right)
\end{gathered}
$$

```python
from typing import Callable
import numpy as np


def simpsonsRule(f: Callable, N: float, start: float=-1e6,
                 stop: float=1e6) -> float:
    """Function to approximate the numeric integral of a function, f, using
    Simpson's rule.

    Arguments:
        f {Callable} -- Function for which the integral is to be estimated.
        N {float} -- Number of nodes to consider.

    Keyword Arguments:
        start {float} -- Starting point (default: {-1e6}).
        stop {float} -- Stopping point (default: {1e6}).

    Returns:
        float -- Approximation of the area under the function.
    """

    # Building values for approximation, and getting step size
    x, h = np.linspace(start=start, stop=stop, num=N, retstep=True)

    # Computing midpoints
    x_mid = np.array([(x[i - 1] + x[i]) / 2 for i in range(1, N)])

    # Estimating using Simpson's rule
    area = np.sum(2 * f(x)) - (f(start) + f(stop)) + (4 * np.sum(f(x_mid)))

    # Scaling area
    area *= (h / 6)

    return area
```

../fe621/numerical_integration/simpsons.py

3.2 Truncation Error Analysis

To examine the behavior of each of the quadrature methods described above, we approximate the integral of the following function:

$$f(x) = \begin{cases} \frac{\sin(x)}{x}, & \text{for } x \neq 0, \\ 1, & \text{for } x = 0. \end{cases}$$

We parameterize the start and stop points of the quadrature methods with a variable a, such that start $= -a$ and stop $= a$. Furthermore, we define the number of segments with variable N.

$$\text{Let approximation with parameters } a \text{ and } N = I_{N,a}$$

It is know analytically that the value of the integral $\int_{\infty}^{\infty} f(x)dx = \pi$. We evaluate the performance of each of the quadrature methods with various values of a and N. Then, we compute the *truncation error* of the approximation, defined as:

$$\text{Truncation error for approximation with parameters } a \text{ and } N = |I_{N,a} - \pi|$$

	N = 1000	N = 10000	N = 100000	N = 1000000	N = 10000000
a = 100	1.70837393e-02	7.23666391e-04	6.28030676e+00	6.28753820e+00	8.02007982e-04
a = 1000	1.71411414e-02	1.12266273e-03	1.22268346e-04	6.28287768e+00	6.28285876e+00
a = 10000	1.71417140e-02	1.12637221e-03	1.89801996e-04	1.28334824e-05	6.28321420e+00
a = 100000	1.71417198e-02	1.12640928e-03	1.90430835e-04	1.99205411e-05	1.20297055e-06
a = 1000000	1.71417198e-02	1.12640965e-03	1.90437119e-04	1.99865427e-05	1.86725626e-06

Table 3: Trapezoidal quadrature rule truncation error for varying values of a and N.

	N = 1000	N = 10000	N = 100000	N = 1000000	N = 10000000
a = 100	1.71417296e-02	1.13348528e-03	1.04706334e+01	1.27753456e+02	1.33203419e+03
a = 1000	1.71417198e-02	1.12641028e-03	1.91636193e-04	1.04718335e+01	1.27758461e+02
a = 10000	1.71417198e-02	1.12640965e-03	1.90437288e-04	2.01130216e-05	1.04719888e+01
a = 100000	1.71417198e-02	1.12640965e-03	1.90437182e-04	1.99872209e-05	1.88530816e-06
a = 1000000	1.71417198e-02	1.12640965e-03	1.90437182e-04	1.99872089e-05	1.87350648e-06

Table 4: Simpsons quadrature rule truncation error for varying values of a and N.

Table 3 and Table 4 report the truncation error for the Trapezoidal and Simpson's quadrature rules, respectively. The script used to produce this table is reproduced in Appendix D.3.1. Variations of N and a are explores in increasing powers of 10, with a progressing from 100 to 1,000,000, and N from 1,000 to 10,000,000.

It is evident from Table 3 that the Trapezoidal quadrature rule performs relatively well across all values of a, even at relatively low values of N. Compared to Simpson's quadrature rule truncation error (Table 4), the Trapezoidal quadrature rule also performs relatively better with larger values of N, and small values of a.

A potential explanation of this may be the interpolating behavior of the Simpson's quadrature rule. The function $\frac{sin(x)}{x}$ is significantly more linear than quadratic in small intervals, and thus the quadratic interpolating behavior of the Simpson's quadrature rule is a poor approximation heuristic for the function with low values of a.

Finally, it can be observed that both quadrature rule approximations converge commensurately as the values of a and N increase. However, it is clear that the Trapezoidal quadrature rule approximation converges at a faster rate than the Simpson's quadrature rule approximation with increasing values of a and N.

3.3 Convergence Analysis

Typically, the true value of the objective integral is unknown. In this case, we would evaluate the rate of change of the objective function (i.e. convergence) computation with respect to the number of segments, N. We assign an arbitrary convergence criteria, ϵ to test the convergence with progressively increasing (in powers of 10) values of N.

$$\text{Let approximation with parameter } N = I_N$$
$$\text{Repeat while: } |I_N - I_{N_{\text{old}}}| > \epsilon$$

We evaluate the number of iterations required for a convergence level of $\epsilon = 10^{-3}$ for the Trapezoidal and Simpson's quadrature rules. The output of this evaluation is reproduced in Table 5. The solution source code for this analysis is reproduced in Appendix D.3.2. The `fe621` package[13] sub-module used in this analysis is presented below.

```python
from typing import Callable, Tuple
import numpy as np


def convergenceApproximation(f: Callable, rule: Callable, epsilon: float=1e-3,
                             start: float=-1e6, stop: float=1e6) \
                            -> Tuple[float, int]:
    """Function to approximate the numeric integral of a function, f, using
    a given quadrature rule and a tolerance level epsilon.

    Arguments:
        f {Callable} -- Function for which the integral is to be estimated.
        rule {Callable} -- Function to be used to approximate area. Must take
                           positional arguments f, N, start and stop.

    Keyword Arguments:
        epsilon {float} -- Tolerance level (default: {1e-3}).
        start {float} -- Starting point (default: {-1e6}).
        stop {float} -- Stopping point (default: {1e6}).

    Returns:
        Tuple[float, int] -- Approximation of the area under the function
                             and the number of segments (area, segments).
    """

    # Flags
    area_old = 0
    area_new = 1
    N = 1

    while (np.abs(area_new - area_old) > epsilon):
        # Set new area to old area
        area_old = area_new
```

13. Weerawarana 2019

```python

        # Increase N by powers of 10
        N *= 10

        # Computing area with given parameters
        area_new = rule(f=f, N=N, start=start, stop=stop)

        # Log
        print('On iteration {0} method {1} convergence {2} val {3}'.format(
            N, str(rule),
            '{:.5e}'.format(np.abs(area_new - area_old)),
            area_new))

    # Return final area and number of segments
    return (area_new, N)
```

../fe621/numerical_integration/convergence.py

	Estimated Area	Segments
Trapezoidal Rule	3.14162154e+00	1.00000000e+05
Simpson's Rule	3.14159078e+00	1.00000000e+07

Table 5: Analysis of segments required for convergence under the Trapezoidal and Simpson's quadrature rules.

Analyzing the results in Table 5, it is evident that the number of segments required for convergence under the Trapezoidal quadrature rule is significantly less than that required under Simpson's quadrature rule. This difference is significant, with the Trapezoidal quadrature rule requiring segments two orders of magnitude less than Simpson's quadrature rule for convergence. These behavior is in agreement with the previous analysis of convergence with respect to varying values of N and a, explored in Section 3.2.

3.3.1 Arbitrary Function

Additionally, we also evaluate each quadrature rule with respect to the number of segments required for convergence with an additional arbitrary integral:

$$g(x) = 1 + e^{-x^2} \cos\left(8x^{\frac{2}{3}}\right)$$
$$\int_0^2 g(x)\, dx$$

	Estimated Area	Segments
Trapezoidal Rule	1.95879798e+00	1.00000000e+04
Simpson's Rule	1.95879793e+00	1.00000000e+03

Table 6: Analysis of segments required for convergence of an arbitrary integral under the Trapezoidal and Simpson's quadrature rules.

The estimates and segments required for convergence for the integral $\int_0^2 g(x)\, dx$ are presented in Table 6. The source code for this analysis is reproduced in Appendix D.3.3.

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A Computed Implied Volatility

A.1 SPY Option Chain

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190215P00216000	2019-02-15	P	216.0	0.4180494415790529
SPY190215P00217000	2019-02-15	P	217.0	0.411174376602368
SPY190215P00218000	2019-02-15	P	218.0	0.4024843791561663
SPY190215C00220000	2019-02-15	C	220.0	0.6748545896195169
SPY190215P00220000	2019-02-15	P	220.0	0.39316105415754
SPY190215P00221000	2019-02-15	P	221.0	0.3793626795034579
SPY190215P00222000	2019-02-15	P	222.0	0.3923273574360801
SPY190215P00225000	2019-02-15	P	225.0	0.36844059634391607
SPY190215P00226000	2019-02-15	P	226.0	0.36053417283860617
SPY190215P00227000	2019-02-15	P	227.0	0.45633983124247596
SPY190215P00228000	2019-02-15	P	228.0	0.3263965835961539
SPY190215P00229000	2019-02-15	P	229.0	0.33697590498668156
SPY190215C00230000	2019-02-15	C	230.0	0.6229763478040695
SPY190215P00230000	2019-02-15	P	230.0	0.32519168561072
SPY190215P00231000	2019-02-15	P	231.0	0.30408811691167104
SPY190215P00232000	2019-02-15	P	232.0	0.3023834789500517
SPY190215P00233000	2019-02-15	P	233.0	0.31018948615969294
SPY190215P00234000	2019-02-15	P	234.0	0.2982148002175724
SPY190215C00235000	2019-02-15	C	235.0	0.4914560983347338
SPY190215P00235000	2019-02-15	P	235.0	0.28873788731177447
SPY190215P00236000	2019-02-15	P	236.0	0.28140371717760326
SPY190215P00237000	2019-02-15	P	237.0	0.2792122662829621
SPY190215C00238000	2019-02-15	C	238.0	0.46540345577214726
SPY190215P00238000	2019-02-15	P	238.0	0.2676158548925844
SPY190215P00239000	2019-02-15	P	239.0	0.26702857688259896
SPY190215C00240000	2019-02-15	C	240.0	0.3908137921933775
SPY190215P00240000	2019-02-15	P	240.0	0.25351754844646013
SPY190215C00241000	2019-02-15	C	241.0	0.42809450707468044
SPY190215P00241000	2019-02-15	P	241.0	0.24700246503590928
SPY190215C00242000	2019-02-15	C	242.0	0.3554098606109619
SPY190215P00242000	2019-02-15	P	242.0	0.24634772249499856
SPY190215C00243000	2019-02-15	C	243.0	0.413379016376677
SPY190215P00243000	2019-02-15	P	243.0	0.24172608504819748
SPY190215C00244000	2019-02-15	C	244.0	0.4249940169484992
SPY190215P00244000	2019-02-15	P	244.0	0.23729632882510915
SPY190215C00245000	2019-02-15	C	245.0	0.42333401900071366
SPY190215P00245000	2019-02-15	P	245.0	0.22461953370467477
SPY190215C00246000	2019-02-15	C	246.0	0.3807940775034379
SPY190215P00246000	2019-02-15	P	246.0	0.22283757739054882
SPY190215C00247000	2019-02-15	C	247.0	0.328372902011279
SPY190215P00247000	2019-02-15	P	247.0	0.21837750359264482
SPY190215C00248000	2019-02-15	C	248.0	0.32188292082191117

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190215P00248000	2019-02-15	P	248.0	0.2079607214769134
SPY190215P00249000	2019-02-15	P	249.0	0.20192688993175925
SPY190215C00250000	2019-02-15	C	250.0	0.30153508697237286
SPY190215P00250000	2019-02-15	P	250.0	0.1962907966750357
SPY190215C00251000	2019-02-15	C	251.0	0.3511462409115402
SPY190215P00251000	2019-02-15	P	251.0	0.19062775175284852
SPY190215C00252000	2019-02-15	C	252.0	0.2855393611808692
SPY190215P00252000	2019-02-15	P	252.0	0.18416079108977257
SPY190215C00253000	2019-02-15	C	253.0	0.2862711355719768
SPY190215P00253000	2019-02-15	P	253.0	0.17791476091155617
SPY190215C00254000	2019-02-15	C	254.0	0.29861538103838875
SPY190215P00254000	2019-02-15	P	254.0	0.17023541433427036
SPY190215C00255000	2019-02-15	C	255.0	0.2397400568546861
SPY190215P00255000	2019-02-15	P	255.0	0.16614805402048408
SPY190215C00256000	2019-02-15	C	256.0	0.24582996895785705
SPY190215P00256000	2019-02-15	P	256.0	0.16078886778458304
SPY190215C00257000	2019-02-15	C	257.0	0.2594671356544066
SPY190215P00257000	2019-02-15	P	257.0	0.15432878528409602
SPY190215C00258000	2019-02-15	C	258.0	0.2148171833583287
SPY190215P00258000	2019-02-15	P	258.0	0.14846573705258576
SPY190215C00259000	2019-02-15	C	259.0	0.21678821713316673
SPY190215P00259000	2019-02-15	P	259.0	0.14327652923896184
SPY190215C00260000	2019-02-15	C	260.0	0.20072617344350122
SPY190215P00260000	2019-02-15	P	260.0	0.1371335251556943
SPY190215P00261000	2019-02-15	P	261.0	0.13070634563865563
SPY190215C00262000	2019-02-15	C	262.0	0.18904398203591216
SPY190215P00262000	2019-02-15	P	262.0	0.12552745506891508
SPY190215C00263000	2019-02-15	C	263.0	0.17502479961523587
SPY190215P00263000	2019-02-15	P	263.0	0.11941925643959923
SPY190215C00264000	2019-02-15	C	264.0	0.16773176974937565
SPY190215P00264000	2019-02-15	P	264.0	0.11333376550308578
SPY190215C00265000	2019-02-15	C	265.0	0.1595995284482182
SPY190215P00265000	2019-02-15	P	265.0	0.10747603443272584
SPY190215C00266000	2019-02-15	C	266.0	0.15568796054337375
SPY190215P00266000	2019-02-15	P	266.0	0.10189264936520316
SPY190215C00267000	2019-02-15	C	267.0	0.1475665514426463
SPY190215P00267000	2019-02-15	P	267.0	0.09729181714070118
SPY190215C00268000	2019-02-15	C	268.0	0.13919694924060208
SPY190215P00268000	2019-02-15	P	268.0	0.09244671258170281
SPY190215C00269000	2019-02-15	C	269.0	0.1365114173011097
SPY190215P00269000	2019-02-15	P	269.0	0.08925885495627323
SPY190215C00270000	2019-02-15	C	270.0	0.13298826144479425
SPY190215P00270000	2019-02-15	P	270.0	0.08841481660028248
SPY190215C00271000	2019-02-15	C	271.0	0.1292855172510952
SPY190215P00271000	2019-02-15	P	271.0	0.09264454512340028
SPY190215C00272000	2019-02-15	C	272.0	0.12483576069707455

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190215P00272000	2019-02-15	P	272.0	0.10526036362513862
SPY190215C00273000	2019-02-15	C	273.0	0.12088632949477876
SPY190215P00273000	2019-02-15	P	273.0	0.1284050392677717
SPY190215C00274000	2019-02-15	C	274.0	0.11812486916856693
SPY190215P00274000	2019-02-15	P	274.0	0.16080216068745878
SPY190215C00275000	2019-02-15	C	275.0	0.11595551620054123
SPY190215P00275000	2019-02-15	P	275.0	0.19981958677091866
SPY190215C00276000	2019-02-15	C	276.0	0.11406975329074713
SPY190215P00276000	2019-02-15	P	276.0	0.24198950404096442
SPY190215C00277000	2019-02-15	C	277.0	0.11332081406927474
SPY190215P00277000	2019-02-15	P	277.0	0.2910963165790529
SPY190215C00278000	2019-02-15	C	278.0	0.11290690478156595
SPY190215P00278000	2019-02-15	P	278.0	0.32880091606198675
SPY190215C00279000	2019-02-15	C	279.0	0.11354740318434928
SPY190215P00279000	2019-02-15	P	279.0	0.40279262815899863
SPY190215C00280000	2019-02-15	C	280.0	0.11533855477257458
SPY190215P00280000	2019-02-15	P	280.0	0.4625415192235766
SPY190215C00281000	2019-02-15	C	281.0	0.11863212146417564
SPY190215C00282000	2019-02-15	C	282.0	0.11963784542230084
SPY190215P00282000	2019-02-15	P	282.0	0.5605288234818012
SPY190215C00283000	2019-02-15	C	283.0	0.12342878619728186
SPY190215P00283000	2019-02-15	P	283.0	0.6304874200650188
SPY190215C00284000	2019-02-15	C	284.0	0.12756590343192412
SPY190215C00285000	2019-02-15	C	285.0	0.13167348359246997
SPY190215P00285000	2019-02-15	P	285.0	0.7465776823975546
SPY190215C00286000	2019-02-15	C	286.0	0.13660356516728317
SPY190215P00286000	2019-02-15	P	286.0	0.8140328960955295
SPY190215C00287000	2019-02-15	C	287.0	0.1377358278045264
SPY190215C00288000	2019-02-15	C	288.0	0.15191401362114246
SPY190215C00289000	2019-02-15	C	289.0	0.14361290065833673
SPY190215P00290000	2019-02-15	P	290.0	1.0384364140308118
SPY190215C00292000	2019-02-15	C	292.0	0.17808676375757398
SPY190215C00295000	2019-02-15	C	295.0	0.18663971015559438
SPY190215P00295000	2019-02-15	P	295.0	1.2997933238973398
SPY190215C00315000	2019-02-15	C	315.0	0.3224352985391836
SPY190215C00320000	2019-02-15	C	320.0	0.3540768708719317
SPY190215P00320000	2019-02-15	P	320.0	2.56294915133425
SPY190315P00216000	2019-03-15	P	216.0	0.2567688461459811
SPY190315P00219000	2019-03-15	P	219.0	0.24956958068301305
SPY190315P00220000	2019-03-15	P	220.0	0.24319184100841318
SPY190315P00222000	2019-03-15	P	222.0	0.23931106947876912
SPY190315P00223000	2019-03-15	P	223.0	0.23296903771207766
SPY190315P00225000	2019-03-15	P	225.0	0.2271421608107779
SPY190315P00226000	2019-03-15	P	226.0	0.22420842934142599
SPY190315P00227000	2019-03-15	P	227.0	0.22198551451153767
SPY190315P00228000	2019-03-15	P	228.0	0.2168821373863903

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190315P00229000	2019-03-15	P	229.0	0.21612434435988326
SPY190315C00230000	2019-03-15	C	230.0	0.32262427990253156
SPY190315P00230000	2019-03-15	P	230.0	0.2106313510319156
SPY190315P00231000	2019-03-15	P	231.0	0.2082483847732739
SPY190315P00232000	2019-03-15	P	232.0	0.203957728412755
SPY190315P00233000	2019-03-15	P	233.0	0.2010753880376401
SPY190315P00234000	2019-03-15	P	234.0	0.1978809205467439
SPY190315C00235000	2019-03-15	C	235.0	0.22735218901734253
SPY190315P00235000	2019-03-15	P	235.0	0.19455630456090278
SPY190315P00236000	2019-03-15	P	236.0	0.19152244948365194
SPY190315P00237000	2019-03-15	P	237.0	0.18878969694952222
SPY190315P00238000	2019-03-15	P	238.0	0.18524983349968405
SPY190315C00239000	2019-03-15	C	239.0	0.25748373306903644
SPY190315P00239000	2019-03-15	P	239.0	0.18218483156560328
SPY190315C00240000	2019-03-15	C	240.0	0.23636243382438285
SPY190315P00240000	2019-03-15	P	240.0	0.1786011381222464
SPY190315P00241000	2019-03-15	P	241.0	0.17551201383780946
SPY190315C00242000	2019-03-15	C	242.0	0.2227803819691787
SPY190315P00242000	2019-03-15	P	242.0	0.17209278653039956
SPY190315C00243000	2019-03-15	C	243.0	0.2493386256420399
SPY190315P00243000	2019-03-15	P	243.0	0.1690099062517171
SPY190315P00244000	2019-03-15	P	244.0	0.165990466047126
SPY190315C00245000	2019-03-15	C	245.0	0.22824278877824164
SPY190315P00245000	2019-03-15	P	245.0	0.16262214202100359
SPY190315C00246000	2019-03-15	C	246.0	0.21666697180975986
SPY190315P00246000	2019-03-15	P	246.0	0.15953987150850807
SPY190315C00247000	2019-03-15	C	247.0	0.20501292431292756
SPY190315P00247000	2019-03-15	P	247.0	0.15616198634857412
SPY190315C00248000	2019-03-15	C	248.0	0.20221117845515615
SPY190315P00248000	2019-03-15	P	248.0	0.15337223287128732
SPY190315P00249000	2019-03-15	P	249.0	0.14996359110488305
SPY190315C00250000	2019-03-15	C	250.0	0.19929795387463692
SPY190315P00250000	2019-03-15	P	250.0	0.14705290879739824
SPY190315C00251000	2019-03-15	C	251.0	0.19538619634135604
SPY190315P00251000	2019-03-15	P	251.0	0.14393273521872127
SPY190315C00252000	2019-03-15	C	252.0	0.21840955290343145
SPY190315P00252000	2019-03-15	P	252.0	0.14090295337959932
SPY190315P00253000	2019-03-15	P	253.0	0.13802253986563523
SPY190315C00254000	2019-03-15	C	254.0	0.19070778530483687
SPY190315P00254000	2019-03-15	P	254.0	0.13492058610062466
SPY190315C00255000	2019-03-15	C	255.0	0.18263030235114916
SPY190315P00255000	2019-03-15	P	255.0	0.13203112365644606
SPY190315C00256000	2019-03-15	C	256.0	0.1841575959149529
SPY190315P00256000	2019-03-15	P	256.0	0.12959581506831566
SPY190315C00257000	2019-03-15	C	257.0	0.17389593831718425
SPY190315P00257000	2019-03-15	P	257.0	0.12613319679904167

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190315C00258000	2019-03-15	C	258.0	0.1777208796547502
SPY190315P00258000	2019-03-15	P	258.0	0.12373132778860418
SPY190315C00259000	2019-03-15	C	259.0	0.1795022993746316
SPY190315P00259000	2019-03-15	P	259.0	0.12089371986096473
SPY190315C00260000	2019-03-15	C	260.0	0.1702547805083682
SPY190315P00260000	2019-03-15	P	260.0	0.11850736330232352
SPY190315C00261000	2019-03-15	C	261.0	0.15413590404383667
SPY190315P00261000	2019-03-15	P	261.0	0.11638052323285271
SPY190315C00262000	2019-03-15	C	262.0	0.1599738542990916
SPY190315P00262000	2019-03-15	P	262.0	0.11395265379220323
SPY190315C00263000	2019-03-15	C	263.0	0.1580963842094402
SPY190315P00263000	2019-03-15	P	263.0	0.1122342107241111
SPY190315C00264000	2019-03-15	C	264.0	0.1505485763940055
SPY190315P00264000	2019-03-15	P	264.0	0.11094075029768298
SPY190315C00265000	2019-03-15	C	265.0	0.15280866257065093
SPY190315P00265000	2019-03-15	P	265.0	0.11004800381867783
SPY190315C00266000	2019-03-15	C	266.0	0.14605613620689764
SPY190315P00266000	2019-03-15	P	266.0	0.11004668672371398
SPY190315C00267000	2019-03-15	C	267.0	0.14434518106758137
SPY190315P00267000	2019-03-15	P	267.0	0.11089874960272514
SPY190315C00268000	2019-03-15	C	268.0	0.14659765736221353
SPY190315P00268000	2019-03-15	P	268.0	0.11290280715278957
SPY190315C00269000	2019-03-15	C	269.0	0.1395301501769239
SPY190315P00269000	2019-03-15	P	269.0	0.11642764596378102
SPY190315C00270000	2019-03-15	C	270.0	0.13776912103833444
SPY190315P00270000	2019-03-15	P	270.0	0.12220596108595123
SPY190315C00271000	2019-03-15	C	271.0	0.1356402809357704
SPY190315P00271000	2019-03-15	P	271.0	0.1299093080603558
SPY190315C00272000	2019-03-15	C	272.0	0.13226115185281503
SPY190315P00272000	2019-03-15	P	272.0	0.13956624833519196
SPY190315C00273000	2019-03-15	C	273.0	0.1300222367581809
SPY190315P00273000	2019-03-15	P	273.0	0.15124846602339878
SPY190315C00274000	2019-03-15	C	274.0	0.12726097155714888
SPY190315P00274000	2019-03-15	P	274.0	0.16528594219471182
SPY190315C00275000	2019-03-15	C	275.0	0.12531296371498987
SPY190315P00275000	2019-03-15	P	275.0	0.18053468231045072
SPY190315C00276000	2019-03-15	C	276.0	0.12283753251175747
SPY190315P00276000	2019-03-15	P	276.0	0.20004307827376344
SPY190315C00277000	2019-03-15	C	277.0	0.12082013327752233
SPY190315P00277000	2019-03-15	P	277.0	0.21241694155251584
SPY190315C00278000	2019-03-15	C	278.0	0.11911110499935687
SPY190315P00278000	2019-03-15	P	278.0	0.23596284334616893
SPY190315C00279000	2019-03-15	C	279.0	0.11721956150611039
SPY190315P00279000	2019-03-15	P	279.0	0.2595404285908965
SPY190315C00280000	2019-03-15	C	280.0	0.11594249159478776
SPY190315P00280000	2019-03-15	P	280.0	0.2773567965573362

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190315C00281000	2019-03-15	C	281.0	0.11454295624247597
SPY190315P00281000	2019-03-15	P	281.0	0.29590371319704956
SPY190315C00282000	2019-03-15	C	282.0	0.11325154463043603
SPY190315C00283000	2019-03-15	C	283.0	0.1128300986326564
SPY190315P00283000	2019-03-15	P	283.0	0.35343907983101847
SPY190315C00284000	2019-03-15	C	284.0	0.11202337796730763
SPY190315P00284000	2019-03-15	P	284.0	0.3692597318488314
SPY190315C00285000	2019-03-15	C	285.0	0.11120614493289567
SPY190315P00285000	2019-03-15	P	285.0	0.3994443776357509
SPY190315C00286000	2019-03-15	C	286.0	0.11112863145520925
SPY190315C00287000	2019-03-15	C	287.0	0.11094904311782564
SPY190315C00288000	2019-03-15	C	288.0	0.11073977136246078
SPY190315C00289000	2019-03-15	C	289.0	0.1109923852983948
SPY190315P00289000	2019-03-15	P	289.0	0.5001608246122785
SPY190315C00290000	2019-03-15	C	290.0	0.11165327428247007
SPY190315P00290000	2019-03-15	P	290.0	0.5464020531500697
SPY190315C00291000	2019-03-15	C	291.0	0.11259924115427315
SPY190315C00292000	2019-03-15	C	292.0	0.1129561251081774
SPY190315P00292000	2019-03-15	P	292.0	0.5878950323899994
SPY190315C00293000	2019-03-15	C	293.0	0.11505908673376683
SPY190315C00295000	2019-03-15	C	295.0	0.11651194004146644
SPY190315P00295000	2019-03-15	P	295.0	0.6672172229308302
SPY190315C00300000	2019-03-15	C	300.0	0.12795420253978057
SPY190315C00305000	2019-03-15	C	305.0	0.13398430231587052
SPY190315C00310000	2019-03-15	C	310.0	0.15140132221114605
SPY190418P00216000	2019-04-18	P	216.0	0.21253096782947745
SPY190418P00217000	2019-04-18	P	217.0	0.2116135867965191
SPY190418P00219000	2019-04-18	P	219.0	0.20655662507352318
SPY190418P00220000	2019-04-18	P	220.0	0.2043260759709741
SPY190418P00221000	2019-04-18	P	221.0	0.20202164759721292
SPY190418P00222000	2019-04-18	P	222.0	0.19964860833209494
SPY190418P00223000	2019-04-18	P	223.0	0.19721261680583516
SPY190418P00224000	2019-04-18	P	224.0	0.19533768334352147
SPY190418P00225000	2019-04-18	P	225.0	0.19226094950800357
SPY190418P00226000	2019-04-18	P	226.0	0.18986734892706128
SPY190418P00227000	2019-04-18	P	227.0	0.1877715581518305
SPY190418P00228000	2019-04-18	P	228.0	0.1852854926262975
SPY190418P00229000	2019-04-18	P	229.0	0.18352070732799639
SPY190418C00230000	2019-04-18	C	230.0	0.19810199737548828
SPY190418P00230000	2019-04-18	P	230.0	0.18077536921976778
SPY190418P00231000	2019-04-18	P	231.0	0.17872167670208475
SPY190418P00232000	2019-04-18	P	232.0	0.1756786751320295
SPY190418P00233000	2019-04-18	P	233.0	0.17451294852644586
SPY190418P00234000	2019-04-18	P	234.0	0.17104472040825183
SPY190418P00235000	2019-04-18	P	235.0	0.16934771671929322
SPY190418P00236000	2019-04-18	P	236.0	0.16679486960096432

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190418P00237000	2019-04-18	P	237.0	0.16446012365238746
SPY190418P00238000	2019-04-18	P	238.0	0.1612498510219252
SPY190418P00239000	2019-04-18	P	239.0	0.1610995314615157
SPY190418P00240000	2019-04-18	P	240.0	0.15743783672752282
SPY190418P00241000	2019-04-18	P	241.0	0.1544271527653765
SPY190418P00242000	2019-04-18	P	242.0	0.1544917879811943
SPY190418P00243000	2019-04-18	P	243.0	0.15101746220113066
SPY190418P00244000	2019-04-18	P	244.0	0.14963917110277258
SPY190418P00245000	2019-04-18	P	245.0	0.14623704163924509
SPY190418P00246000	2019-04-18	P	246.0	0.1442417403316254
SPY190418P00247000	2019-04-18	P	247.0	0.14176691889457996
SPY190418P00248000	2019-04-18	P	248.0	0.14114315247596682
SPY190418P00249000	2019-04-18	P	249.0	0.13769204659230264
SPY190418C00250000	2019-04-18	C	250.0	0.168319407021603
SPY190418P00250000	2019-04-18	P	250.0	0.13563111005231854
SPY190418P00251000	2019-04-18	P	251.0	0.1347533638215126
SPY190418P00252000	2019-04-18	P	252.0	0.13139852782344574
SPY190418P00253000	2019-04-18	P	253.0	0.12961079092586741
SPY190418P00254000	2019-04-18	P	254.0	0.1275735864858798
SPY190418P00255000	2019-04-18	P	255.0	0.12591265656454179
SPY190418P00256000	2019-04-18	P	256.0	0.12425845846190782
SPY190418C00257000	2019-04-18	C	257.0	0.14762643048220583
SPY190418P00257000	2019-04-18	P	257.0	0.12276099465997017
SPY190418P00258000	2019-04-18	P	258.0	0.12175334384069418
SPY190418P00259000	2019-04-18	P	259.0	0.119044835610158
SPY190418C00260000	2019-04-18	C	260.0	0.1486221542748649
SPY190418P00260000	2019-04-18	P	260.0	0.11871556186919932
SPY190418C00261000	2019-04-18	C	261.0	0.13939978216615173
SPY190418P00261000	2019-04-18	P	261.0	0.11858248649655706
SPY190418C00262000	2019-04-18	C	262.0	0.14051489817821766
SPY190418P00262000	2019-04-18	P	262.0	0.11752034697081427
SPY190418C00263000	2019-04-18	C	263.0	0.14476057818478635
SPY190418P00263000	2019-04-18	P	263.0	0.1180703072901577
SPY190418P00264000	2019-04-18	P	264.0	0.11810942988871309
SPY190418C00265000	2019-04-18	C	265.0	0.13930426839062626
SPY190418P00265000	2019-04-18	P	265.0	0.11821970000596302
SPY190418C00266000	2019-04-18	C	266.0	0.13701354756074793
SPY190418P00266000	2019-04-18	P	266.0	0.11957399070720233
SPY190418C00267000	2019-04-18	C	267.0	0.13641568400975687
SPY190418P00267000	2019-04-18	P	267.0	0.12203147039389062
SPY190418C00268000	2019-04-18	C	268.0	0.13520112732792144
SPY190418P00268000	2019-04-18	P	268.0	0.1251350095509873
SPY190418C00269000	2019-04-18	C	269.0	0.13374824962957435
SPY190418P00269000	2019-04-18	P	269.0	0.12876836235261024
SPY190418C00270000	2019-04-18	C	270.0	0.1311254013529824
SPY190418P00270000	2019-04-18	P	270.0	0.13355500252960284

Option Name	Expiration Date	Type	Strike	Implied Volatility
SPY190418C00271000	2019-04-18	C	271.0	0.12932551791295982
SPY190418P00271000	2019-04-18	P	271.0	0.13969507973517298
SPY190418C00272000	2019-04-18	C	272.0	0.12772451581247626
SPY190418P00272000	2019-04-18	P	272.0	0.14613784487595033
SPY190418C00273000	2019-04-18	C	273.0	0.12595034011489595
SPY190418P00273000	2019-04-18	P	273.0	0.15478764653510754
SPY190418C00274000	2019-04-18	C	274.0	0.12422406764896325
SPY190418P00274000	2019-04-18	P	274.0	0.1638658699172232
SPY190418C00275000	2019-04-18	C	275.0	0.12282794698729844
SPY190418P00275000	2019-04-18	P	275.0	0.17227954571814183
SPY190418C00276000	2019-04-18	C	276.0	0.12080398666889161
SPY190418P00276000	2019-04-18	P	276.0	0.18565303529314983
SPY190418C00277000	2019-04-18	C	277.0	0.11929305922954589
SPY190418P00277000	2019-04-18	P	277.0	0.19241970823244062
SPY190418C00278000	2019-04-18	C	278.0	0.11827662777717766
SPY190418P00278000	2019-04-18	P	278.0	0.2101958804118359
SPY190418C00279000	2019-04-18	C	279.0	0.11649886665441801
SPY190418P00279000	2019-04-18	P	279.0	0.21964454894785382
SPY190418C00280000	2019-04-18	C	280.0	0.11576322033582136
SPY190418P00280000	2019-04-18	P	280.0	0.23431608439101587
SPY190418C00281000	2019-04-18	C	281.0	0.11447732101011154
SPY190418P00281000	2019-04-18	P	281.0	0.24426300507372298
SPY190418C00282000	2019-04-18	C	282.0	0.11246448282695487
SPY190418C00283000	2019-04-18	C	283.0	0.112595460603914
SPY190418C00284000	2019-04-18	C	284.0	0.11145753933645575
SPY190418P00284000	2019-04-18	P	284.0	0.2944151519814416
SPY190418C00285000	2019-04-18	C	285.0	0.10988344012014091
SPY190418P00285000	2019-04-18	P	285.0	0.3120133212155393
SPY190418C00286000	2019-04-18	C	286.0	0.11003834512227637
SPY190418P00286000	2019-04-18	P	286.0	0.32431142714322375
SPY190418C00287000	2019-04-18	C	287.0	0.10888913098503561
SPY190418C00288000	2019-04-18	C	288.0	0.1072702505399504
SPY190418C00289000	2019-04-18	C	289.0	0.10612576818832047
SPY190418C00290000	2019-04-18	C	290.0	0.10689775657165995
SPY190418P00290000	2019-04-18	P	290.0	0.3956008994061014
SPY190418C00291000	2019-04-18	C	291.0	0.10624911169261883
SPY190418C00292000	2019-04-18	C	292.0	0.10639650437533094
SPY190418C00293000	2019-04-18	C	293.0	0.1059596922696399
SPY190418C00295000	2019-04-18	C	295.0	0.10678382785728825
SPY190418C00296000	2019-04-18	C	296.0	0.10731100731188684
SPY190418C00297000	2019-04-18	C	297.0	0.10846461176567371
SPY190418C00298000	2019-04-18	C	298.0	0.1078549431413031
SPY190418C00300000	2019-04-18	C	300.0	0.11016710335031495
SPY190418P00300000	2019-04-18	P	300.0	0.5862353220010352
SPY190418C00305000	2019-04-18	C	305.0	0.116671235360148
SPY190418C00315000	2019-04-18	C	315.0	0.1295311310712029

A.2 AMZN Option Chain

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190215P01340000	2019-02-15	P	1340.0	0.3931917375920679
AMZN190215C01350000	2019-02-15	C	1350.0	0.6713369118906286
AMZN190215P01350000	2019-02-15	P	1350.0	0.3767203430995307
AMZN190215P01355000	2019-02-15	P	1355.0	0.3629656033137875
AMZN190215P01360000	2019-02-15	P	1360.0	0.3618261698261856
AMZN190215P01375000	2019-02-15	P	1375.0	0.37503629084438317
AMZN190215P01390000	2019-02-15	P	1390.0	0.3419472799276757
AMZN190215C01400000	2019-02-15	C	1400.0	0.9025380237043397
AMZN190215P01400000	2019-02-15	P	1400.0	0.33274951798226826
AMZN190215P01405000	2019-02-15	P	1405.0	0.32782165595637563
AMZN190215P01410000	2019-02-15	P	1410.0	0.32916716602452273
AMZN190215P01425000	2019-02-15	P	1425.0	0.31495315034676086
AMZN190215C01430000	2019-02-15	C	1430.0	0.5020366713058116
AMZN190215P01430000	2019-02-15	P	1430.0	0.3108822049387276
AMZN190215P01440000	2019-02-15	P	1440.0	0.30271163072122637
AMZN190215P01445000	2019-02-15	P	1445.0	0.29856204986572266
AMZN190215C01450000	2019-02-15	C	1450.0	0.5183580259209899
AMZN190215P01450000	2019-02-15	P	1450.0	0.2887842783232784
AMZN190215P01455000	2019-02-15	P	1455.0	0.29096826567979117
AMZN190215P01460000	2019-02-15	P	1460.0	0.28318047828381626
AMZN190215C01465000	2019-02-15	C	1465.0	0.5257828351927967
AMZN190215P01465000	2019-02-15	P	1465.0	0.27593145589999224
AMZN190215P01470000	2019-02-15	P	1470.0	0.2714030395078537
AMZN190215P01475000	2019-02-15	P	1475.0	0.2666375460222249
AMZN190215C01480000	2019-02-15	C	1480.0	0.6397621650395431
AMZN190215P01480000	2019-02-15	P	1480.0	0.2617296355459696
AMZN190215P01485000	2019-02-15	P	1485.0	0.254672274870031
AMZN190215C01490000	2019-02-15	C	1490.0	0.6323267527988978
AMZN190215P01490000	2019-02-15	P	1490.0	0.2567543824920264
AMZN190215P01495000	2019-02-15	P	1495.0	0.24963465493048548
AMZN190215C01500000	2019-02-15	C	1500.0	0.4113160892721795
AMZN190215P01500000	2019-02-15	P	1500.0	0.24507775026209214
AMZN190215P01505000	2019-02-15	P	1505.0	0.243131571718494
AMZN190215C01510000	2019-02-15	C	1510.0	0.4423285819388725
AMZN190215P01510000	2019-02-15	P	1510.0	0.23675475888849828
AMZN190215P01515000	2019-02-15	P	1515.0	0.23299896503653367
AMZN190215C01520000	2019-02-15	C	1520.0	0.362400016528648
AMZN190215P01520000	2019-02-15	P	1520.0	0.2285838432019324
AMZN190215C01525000	2019-02-15	C	1525.0	0.5512893068921435
AMZN190215P01525000	2019-02-15	P	1525.0	0.2221765176719412
AMZN190215C01530000	2019-02-15	C	1530.0	0.510822873719981
AMZN190215P01530000	2019-02-15	P	1530.0	0.22120623332460213
AMZN190215C01535000	2019-02-15	C	1535.0	0.29214825428707497
AMZN190215P01535000	2019-02-15	P	1535.0	0.21658032751449233

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190215C01540000	2019-02-15	C	1540.0	0.6803448791699032
AMZN190215P01540000	2019-02-15	P	1540.0	0.21259450546615874
AMZN190215P01545000	2019-02-15	P	1545.0	0.20798960000352787
AMZN190215C01550000	2019-02-15	C	1550.0	0.3248352650791178
AMZN190215P01550000	2019-02-15	P	1550.0	0.20665857797998297
AMZN190215P01555000	2019-02-15	P	1555.0	0.2039353377983698
AMZN190215C01560000	2019-02-15	C	1560.0	0.2852750468898464
AMZN190215P01560000	2019-02-15	P	1560.0	0.1983834166660943
AMZN190215C01565000	2019-02-15	C	1565.0	0.29585641811491775
AMZN190215P01565000	2019-02-15	P	1565.0	0.19599330394774142
AMZN190215C01570000	2019-02-15	C	1570.0	0.28880543825103017
AMZN190215P01570000	2019-02-15	P	1570.0	0.19241865943459904
AMZN190215C01575000	2019-02-15	C	1575.0	0.45143418434338695
AMZN190215P01575000	2019-02-15	P	1575.0	0.19041726046510973
AMZN190215C01580000	2019-02-15	C	1580.0	0.2855514121179136
AMZN190215P01580000	2019-02-15	P	1580.0	0.18820983369637023
AMZN190215C01585000	2019-02-15	C	1585.0	0.33621239860302193
AMZN190215P01585000	2019-02-15	P	1585.0	0.18601443151683758
AMZN190215C01590000	2019-02-15	C	1590.0	0.3245344796144139
AMZN190215P01590000	2019-02-15	P	1590.0	0.18603491966071947
AMZN190215P01595000	2019-02-15	P	1595.0	0.18335650948917165
AMZN190215C01600000	2019-02-15	C	1600.0	0.28551710231224897
AMZN190215P01600000	2019-02-15	P	1600.0	0.18337636347621908
AMZN190215C01605000	2019-02-15	C	1605.0	0.3114741171717339
AMZN190215P01605000	2019-02-15	P	1605.0	0.18405035023799027
AMZN190215C01610000	2019-02-15	C	1610.0	0.31359220099875995
AMZN190215P01610000	2019-02-15	P	1610.0	0.1883249331618209
AMZN190215C01615000	2019-02-15	C	1615.0	0.2718814039883548
AMZN190215P01615000	2019-02-15	P	1615.0	0.19189350440374117
AMZN190215C01620000	2019-02-15	C	1620.0	0.2825334187968613
AMZN190215P01620000	2019-02-15	P	1620.0	0.19983961149249846
AMZN190215C01625000	2019-02-15	C	1625.0	0.27873999017583745
AMZN190215P01625000	2019-02-15	P	1625.0	0.20896817717100957
AMZN190215C01630000	2019-02-15	C	1630.0	0.2751511015245677
AMZN190215P01630000	2019-02-15	P	1630.0	0.22236998428774002
AMZN190215C01635000	2019-02-15	C	1635.0	0.2770198885437168
AMZN190215P01635000	2019-02-15	P	1635.0	0.23765113957397774
AMZN190215C01640000	2019-02-15	C	1640.0	0.2743386978383564
AMZN190215P01640000	2019-02-15	P	1640.0	0.2588814420773245
AMZN190215C01645000	2019-02-15	C	1645.0	0.2744878222570395
AMZN190215P01645000	2019-02-15	P	1645.0	0.2802935768576229
AMZN190215C01650000	2019-02-15	C	1650.0	0.27077783404103933
AMZN190215P01650000	2019-02-15	P	1650.0	0.3069831648141222
AMZN190215C01655000	2019-02-15	C	1655.0	0.26853240664352845
AMZN190215P01655000	2019-02-15	P	1655.0	0.3282023451822188
AMZN190215C01660000	2019-02-15	C	1660.0	0.2693468347534804

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190215P01660000	2019-02-15	P	1660.0	0.36156299473989345
AMZN190215C01665000	2019-02-15	C	1665.0	0.26807598445726477
AMZN190215P01665000	2019-02-15	P	1665.0	0.4026970168208832
AMZN190215C01670000	2019-02-15	C	1670.0	0.26815718092272045
AMZN190215P01670000	2019-02-15	P	1670.0	0.42779945656466667
AMZN190215C01675000	2019-02-15	C	1675.0	0.26953486225489154
AMZN190215P01675000	2019-02-15	P	1675.0	0.45730353011499586
AMZN190215C01680000	2019-02-15	C	1680.0	0.2670118936797237
AMZN190215P01680000	2019-02-15	P	1680.0	0.4927836537666028
AMZN190215C01685000	2019-02-15	C	1685.0	0.2687679471262276
AMZN190215P01685000	2019-02-15	P	1685.0	0.5071174763047787
AMZN190215C01690000	2019-02-15	C	1690.0	0.26820091335364926
AMZN190215P01690000	2019-02-15	P	1690.0	0.513898808023204
AMZN190215C01695000	2019-02-15	C	1695.0	0.2687012387053741
AMZN190215P01695000	2019-02-15	P	1695.0	0.4995315946886302
AMZN190215C01700000	2019-02-15	C	1700.0	0.2682603045802592
AMZN190215P01700000	2019-02-15	P	1700.0	0.6548640429211394
AMZN190215C01705000	2019-02-15	C	1705.0	0.2690154146355436
AMZN190215P01705000	2019-02-15	P	1705.0	0.6873675075638325
AMZN190215C01710000	2019-02-15	C	1710.0	0.2704009253655553
AMZN190215P01710000	2019-02-15	P	1710.0	0.7601725658797243
AMZN190215C01715000	2019-02-15	C	1715.0	0.27242951990698305
AMZN190215P01715000	2019-02-15	P	1715.0	0.7592116231503694
AMZN190215C01720000	2019-02-15	C	1720.0	0.27154499307617813
AMZN190215P01720000	2019-02-15	P	1720.0	0.8250626639636887
AMZN190215C01725000	2019-02-15	C	1725.0	0.2732527530406747
AMZN190215P01725000	2019-02-15	P	1725.0	0.8546877395161583
AMZN190215C01730000	2019-02-15	C	1730.0	0.2754301061410733
AMZN190215P01730000	2019-02-15	P	1730.0	0.8922332144149429
AMZN190215C01735000	2019-02-15	C	1735.0	0.2776727773954191
AMZN190215C01740000	2019-02-15	C	1740.0	0.2789619694585386
AMZN190215P01740000	2019-02-15	P	1740.0	0.9754144444185144
AMZN190215C01745000	2019-02-15	C	1745.0	0.2848521400900448
AMZN190215C01750000	2019-02-15	C	1750.0	0.28289659553781493
AMZN190215P01750000	2019-02-15	P	1750.0	1.0836620891795439
AMZN190215C01755000	2019-02-15	C	1755.0	0.2882826846578847
AMZN190215P01755000	2019-02-15	P	1755.0	1.1780660170728288
AMZN190215C01760000	2019-02-15	C	1760.0	0.2900647141439531
AMZN190215P01760000	2019-02-15	P	1760.0	1.1937874967179944
AMZN190215C01765000	2019-02-15	C	1765.0	0.2930499830514269
AMZN190215P01765000	2019-02-15	P	1765.0	1.0765382639892267
AMZN190215C01770000	2019-02-15	C	1770.0	0.29585017572583444
AMZN190215P01770000	2019-02-15	P	1770.0	1.285964217027435
AMZN190215C01775000	2019-02-15	C	1775.0	0.29757525305004073
AMZN190215P01775000	2019-02-15	P	1775.0	1.169216334057586
AMZN190215C01780000	2019-02-15	C	1780.0	0.30265258096368114

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190215P01780000	2019-02-15	P	1780.0	1.3553036082431178
AMZN190215C01785000	2019-02-15	C	1785.0	0.304396707383568
AMZN190215P01785000	2019-02-15	P	1785.0	1.4110525550744724
AMZN190215C01790000	2019-02-15	C	1790.0	0.30950957247058447
AMZN190215P01790000	2019-02-15	P	1790.0	1.3081268764212919
AMZN190215C01795000	2019-02-15	C	1795.0	0.3150627619165289
AMZN190215C01800000	2019-02-15	C	1800.0	0.31483673378634636
AMZN190215P01800000	2019-02-15	P	1800.0	1.51802215429828
AMZN190215C01805000	2019-02-15	C	1805.0	0.3214980635191778
AMZN190215C01810000	2019-02-15	C	1810.0	0.3238233215058856
AMZN190215P01810000	2019-02-15	P	1810.0	1.620296724617024
AMZN190215C01815000	2019-02-15	C	1815.0	0.326063626867426
AMZN190215C01820000	2019-02-15	C	1820.0	0.3328457878678656
AMZN190215C01825000	2019-02-15	C	1825.0	0.3367603648349147
AMZN190215C01830000	2019-02-15	C	1830.0	0.338779300679941
AMZN190215P01830000	2019-02-15	P	1830.0	1.8159792795205665
AMZN190215C01835000	2019-02-15	C	1835.0	0.3568941613902216
AMZN190215C01840000	2019-02-15	C	1840.0	0.3477374245138729
AMZN190215P01840000	2019-02-15	P	1840.0	1.90409766438672
AMZN190215C01850000	2019-02-15	C	1850.0	0.35497720284230266
AMZN190215P01850000	2019-02-15	P	1850.0	1.9144002494909573
AMZN190215C01860000	2019-02-15	C	1860.0	0.36597536043132967
AMZN190215P01860000	2019-02-15	P	1860.0	2.0410497841017934
AMZN190215C01865000	2019-02-15	C	1865.0	0.36400847422802235
AMZN190215C01870000	2019-02-15	C	1870.0	0.3726689346001276
AMZN190215P01870000	2019-02-15	P	1870.0	2.1543815861577573
AMZN190215C01880000	2019-02-15	C	1880.0	0.3800619837573117
AMZN190215P01880000	2019-02-15	P	1880.0	2.1759273816862374
AMZN190215C01890000	2019-02-15	C	1890.0	0.3924184808950595
AMZN190215P01890000	2019-02-15	P	1890.0	2.2982461861027477
AMZN190215C01895000	2019-02-15	C	1895.0	0.3835412120575185
AMZN190215C01900000	2019-02-15	C	1900.0	0.3955075076169065
AMZN190215P01900000	2019-02-15	P	1900.0	2.3690766751613763
AMZN190215C01910000	2019-02-15	C	1910.0	0.40474061161051017
AMZN190215P01910000	2019-02-15	P	1910.0	2.450083186254477
AMZN190215C01920000	2019-02-15	C	1920.0	0.4159797731872715
AMZN190215P01920000	2019-02-15	P	1920.0	2.5196788012219207
AMZN190215P01930000	2019-02-15	P	1930.0	2.6225728452053216
AMZN190215C01940000	2019-02-15	C	1940.0	0.41939751266518516
AMZN190215P01940000	2019-02-15	P	1940.0	2.6837522233538613
AMZN190215C01945000	2019-02-15	C	1945.0	0.4330110549926758
AMZN190215P01945000	2019-02-15	P	1945.0	2.769272492059966
AMZN190215C01950000	2019-02-15	C	1950.0	0.4279592640869453
AMZN190215P01950000	2019-02-15	P	1950.0	2.731454476066258
AMZN190215C01955000	2019-02-15	C	1955.0	0.4383469969415299
AMZN190215C01960000	2019-02-15	C	1960.0	0.44400045633925805

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190215P01960000	2019-02-15	P	1960.0	2.877180765351981
AMZN190215P01965000	2019-02-15	P	1965.0	2.9229611813869623
AMZN190315P01315000	2019-03-15	P	1315.0	0.275224175904413
AMZN190315P01320000	2019-03-15	P	1320.0	0.2795055882095376
AMZN190315P01325000	2019-03-15	P	1325.0	0.2748864629994268
AMZN190315C01330000	2019-03-15	C	1330.0	0.5001741631209355
AMZN190315P01330000	2019-03-15	P	1330.0	0.2715393588366106
AMZN190315C01340000	2019-03-15	C	1340.0	0.5088601284079934
AMZN190315P01340000	2019-03-15	P	1340.0	0.2672618246444351
AMZN190315P01345000	2019-03-15	P	1345.0	0.26661354562510614
AMZN190315P01350000	2019-03-15	P	1350.0	0.26483976017788546
AMZN190315P01355000	2019-03-15	P	1355.0	0.26168053717259554
AMZN190315P01360000	2019-03-15	P	1360.0	0.2599025321433611
AMZN190315P01365000	2019-03-15	P	1365.0	0.25642537704818996
AMZN190315P01370000	2019-03-15	P	1370.0	0.2538691396298616
AMZN190315P01375000	2019-03-15	P	1375.0	0.25244038428187066
AMZN190315P01380000	2019-03-15	P	1380.0	0.24847053810763542
AMZN190315C01385000	2019-03-15	C	1385.0	0.4434741302870647
AMZN190315P01385000	2019-03-15	P	1385.0	0.24392824343708167
AMZN190315P01390000	2019-03-15	P	1390.0	0.2416315468985711
AMZN190315P01395000	2019-03-15	P	1395.0	0.2414120554619128
AMZN190315C01400000	2019-03-15	C	1400.0	0.44259760663917025
AMZN190315P01400000	2019-03-15	P	1400.0	0.23820700243001094
AMZN190315P01405000	2019-03-15	P	1405.0	0.2375652844948537
AMZN190315P01410000	2019-03-15	P	1410.0	0.23583969496705037
AMZN190315P01415000	2019-03-15	P	1415.0	0.23317506551132788
AMZN190315P01420000	2019-03-15	P	1420.0	0.23233983217907683
AMZN190315P01425000	2019-03-15	P	1425.0	0.2255889399887046
AMZN190315P01430000	2019-03-15	P	1430.0	0.22774385369342307
AMZN190315P01435000	2019-03-15	P	1435.0	0.22479360975572826
AMZN190315C01440000	2019-03-15	C	1440.0	0.32381322648790145
AMZN190315P01440000	2019-03-15	P	1440.0	0.21562248239736728
AMZN190315P01445000	2019-03-15	P	1445.0	0.21367320624153938
AMZN190315C01450000	2019-03-15	C	1450.0	0.31531679126578316
AMZN190315P01450000	2019-03-15	P	1450.0	0.21721619169425477
AMZN190315P01455000	2019-03-15	P	1455.0	0.21082991529303743
AMZN190315P01460000	2019-03-15	P	1460.0	0.2118801034015158
AMZN190315P01465000	2019-03-15	P	1465.0	0.21485664045719235
AMZN190315P01470000	2019-03-15	P	1470.0	0.21043271359885135
AMZN190315P01475000	2019-03-15	P	1475.0	0.20909830127530696
AMZN190315P01480000	2019-03-15	P	1480.0	0.20565997609092146
AMZN190315P01485000	2019-03-15	P	1485.0	0.20209425855475618
AMZN190315C01490000	2019-03-15	C	1490.0	0.32616853057188766
AMZN190315P01490000	2019-03-15	P	1490.0	0.20302781058699274
AMZN190315C01495000	2019-03-15	C	1495.0	0.3014516830444336
AMZN190315P01495000	2019-03-15	P	1495.0	0.19849197943802074

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190315C01500000	2019-03-15	C	1500.0	0.31362295760523023
AMZN190315P01500000	2019-03-15	P	1500.0	0.20040793796939313
AMZN190315P01505000	2019-03-15	P	1505.0	0.1972504223094267
AMZN190315P01510000	2019-03-15	P	1510.0	0.1971746893490062
AMZN190315C01515000	2019-03-15	C	1515.0	0.3206087206746196
AMZN190315P01515000	2019-03-15	P	1515.0	0.1910213436312078
AMZN190315P01520000	2019-03-15	P	1520.0	0.1946028663069391
AMZN190315C01525000	2019-03-15	C	1525.0	0.33041690826416015
AMZN190315P01525000	2019-03-15	P	1525.0	0.1949395548047312
AMZN190315C01530000	2019-03-15	C	1530.0	0.324357375896797
AMZN190315P01530000	2019-03-15	P	1530.0	0.18860335240278708
AMZN190315P01535000	2019-03-15	P	1535.0	0.17653334781032085
AMZN190315C01540000	2019-03-15	C	1540.0	0.2720038096110026
AMZN190315P01540000	2019-03-15	P	1540.0	0.19332045484381868
AMZN190315C01545000	2019-03-15	C	1545.0	0.39905302969695966
AMZN190315P01545000	2019-03-15	P	1545.0	0.1900670351579671
AMZN190315C01550000	2019-03-15	C	1550.0	0.3073633662270158
AMZN190315P01550000	2019-03-15	P	1550.0	0.19013740217594235
AMZN190315C01555000	2019-03-15	C	1555.0	0.27919923146565756
AMZN190315P01555000	2019-03-15	P	1555.0	0.18802344036834015
AMZN190315C01560000	2019-03-15	C	1560.0	0.30713655759611397
AMZN190315P01560000	2019-03-15	P	1560.0	0.18919677685593705
AMZN190315P01565000	2019-03-15	P	1565.0	0.18985415358677546
AMZN190315C01570000	2019-03-15	C	1570.0	0.2674809843301773
AMZN190315P01570000	2019-03-15	P	1570.0	0.19375597424519336
AMZN190315P01575000	2019-03-15	P	1575.0	0.19115927274269826
AMZN190315P01580000	2019-03-15	P	1580.0	0.19399605138832346
AMZN190315C01585000	2019-03-15	C	1585.0	0.2819899463897471
AMZN190315P01585000	2019-03-15	P	1585.0	0.19265476090219014
AMZN190315C01590000	2019-03-15	C	1590.0	0.28512797392237826
AMZN190315P01590000	2019-03-15	P	1590.0	0.20057133091685106
AMZN190315C01595000	2019-03-15	C	1595.0	0.27487124323540024
AMZN190315P01595000	2019-03-15	P	1595.0	0.20540667921685807
AMZN190315C01600000	2019-03-15	C	1600.0	0.2816049888005952
AMZN190315P01600000	2019-03-15	P	1600.0	0.2071735133295474
AMZN190315C01605000	2019-03-15	C	1605.0	0.28084661039854864
AMZN190315P01605000	2019-03-15	P	1605.0	0.19807775307189474
AMZN190315C01610000	2019-03-15	C	1610.0	0.277194306063835
AMZN190315P01610000	2019-03-15	P	1610.0	0.1861878002391142
AMZN190315C01615000	2019-03-15	C	1615.0	0.33755210964271176
AMZN190315P01615000	2019-03-15	P	1615.0	0.21666023127563164
AMZN190315C01620000	2019-03-15	C	1620.0	0.2871799590947378
AMZN190315P01620000	2019-03-15	P	1620.0	0.2304673621721585
AMZN190315C01625000	2019-03-15	C	1625.0	0.2755928405410493
AMZN190315P01625000	2019-03-15	P	1625.0	0.23554615352464758
AMZN190315C01630000	2019-03-15	C	1630.0	0.2802031851180679

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190315P01630000	2019-03-15	P	1630.0	0.24576168840803453
AMZN190315C01635000	2019-03-15	C	1635.0	0.27782526772345423
AMZN190315P01635000	2019-03-15	P	1635.0	0.25762883598542274
AMZN190315C01640000	2019-03-15	C	1640.0	0.2775746050393185
AMZN190315P01640000	2019-03-15	P	1640.0	0.26502051926634806
AMZN190315C01645000	2019-03-15	C	1645.0	0.27459401913616055
AMZN190315P01645000	2019-03-15	P	1645.0	0.27630729138698723
AMZN190315C01650000	2019-03-15	C	1650.0	0.2719840003401422
AMZN190315P01650000	2019-03-15	P	1650.0	0.2869187108695964
AMZN190315C01655000	2019-03-15	C	1655.0	0.2723185424609562
AMZN190315P01655000	2019-03-15	P	1655.0	0.2981777752147001
AMZN190315C01660000	2019-03-15	C	1660.0	0.2713887953697263
AMZN190315P01660000	2019-03-15	P	1660.0	0.30779644656364263
AMZN190315C01665000	2019-03-15	C	1665.0	0.2735249038852389
AMZN190315P01665000	2019-03-15	P	1665.0	0.31851874592968876
AMZN190315C01670000	2019-03-15	C	1670.0	0.2732999489435454
AMZN190315P01670000	2019-03-15	P	1670.0	0.33132720176521163
AMZN190315C01675000	2019-03-15	C	1675.0	0.269605034147687
AMZN190315P01675000	2019-03-15	P	1675.0	0.3389485473827938
AMZN190315C01680000	2019-03-15	C	1680.0	0.27195602426748444
AMZN190315P01680000	2019-03-15	P	1680.0	0.3661725344255452
AMZN190315C01685000	2019-03-15	C	1685.0	0.27697383900127753
AMZN190315P01685000	2019-03-15	P	1685.0	0.3667534464765388
AMZN190315C01690000	2019-03-15	C	1690.0	0.27180729009916105
AMZN190315P01690000	2019-03-15	P	1690.0	0.3727492042209791
AMZN190315C01695000	2019-03-15	C	1695.0	0.27083115199642716
AMZN190315P01695000	2019-03-15	P	1695.0	0.4090758540746196
AMZN190315C01700000	2019-03-15	C	1700.0	0.2653441831583867
AMZN190315P01700000	2019-03-15	P	1700.0	0.42151413305336255
AMZN190315C01705000	2019-03-15	C	1705.0	0.2667290841222114
AMZN190315P01705000	2019-03-15	P	1705.0	0.4525746469912322
AMZN190315C01710000	2019-03-15	C	1710.0	0.2662659545078912
AMZN190315C01715000	2019-03-15	C	1715.0	0.26260445489907813
AMZN190315P01715000	2019-03-15	P	1715.0	0.4832153734953507
AMZN190315C01720000	2019-03-15	C	1720.0	0.2651385456094961
AMZN190315C01725000	2019-03-15	C	1725.0	0.2645186085225371
AMZN190315P01725000	2019-03-15	P	1725.0	0.5177042368427872
AMZN190315C01730000	2019-03-15	C	1730.0	0.2634405419039909
AMZN190315C01735000	2019-03-15	C	1735.0	0.26506732491885915
AMZN190315C01740000	2019-03-15	C	1740.0	0.25980074997143365
AMZN190315C01745000	2019-03-15	C	1745.0	0.26308368234073415
AMZN190315C01750000	2019-03-15	C	1750.0	0.2613908982337893
AMZN190315P01750000	2019-03-15	P	1750.0	0.6017277734663785
AMZN190315C01755000	2019-03-15	C	1755.0	0.2611676750280668
AMZN190315C01760000	2019-03-15	C	1760.0	0.26180261236322505
AMZN190315P01760000	2019-03-15	P	1760.0	0.5595346850812283

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190315C01765000	2019-03-15	C	1765.0	0.2625203071652776
AMZN190315P01765000	2019-03-15	P	1765.0	0.5918460489843812
AMZN190315C01770000	2019-03-15	C	1770.0	0.26088088979501556
AMZN190315C01775000	2019-03-15	C	1775.0	0.2621715940782786
AMZN190315P01775000	2019-03-15	P	1775.0	0.6279300850675539
AMZN190315C01780000	2019-03-15	C	1780.0	0.2615111685165054
AMZN190315P01780000	2019-03-15	P	1780.0	0.7145935190303246
AMZN190315C01785000	2019-03-15	C	1785.0	0.26196285891715826
AMZN190315C01790000	2019-03-15	C	1790.0	0.2682559142637131
AMZN190315C01800000	2019-03-15	C	1800.0	0.26034839317926667
AMZN190315P01800000	2019-03-15	P	1800.0	0.7312000133192448
AMZN190315C01810000	2019-03-15	C	1810.0	0.26126650593164935
AMZN190315C01820000	2019-03-15	C	1820.0	0.26299182716233044
AMZN190315P01820000	2019-03-15	P	1820.0	0.8711790855583328
AMZN190315C01830000	2019-03-15	C	1830.0	0.2625738934177877
AMZN190315C01840000	2019-03-15	C	1840.0	0.26236552411637953
AMZN190315C01850000	2019-03-15	C	1850.0	0.2637331321111421
AMZN190315C01860000	2019-03-15	C	1860.0	0.26330285669897524
AMZN190315P01860000	2019-03-15	P	1860.0	1.0238794414588557
AMZN190315C01870000	2019-03-15	C	1870.0	0.26702530853583684
AMZN190315P01870000	2019-03-15	P	1870.0	1.0788714733270124
AMZN190315C01880000	2019-03-15	C	1880.0	0.26920495435709846
AMZN190315C01890000	2019-03-15	C	1890.0	0.2711801577711959
AMZN190315C01900000	2019-03-15	C	1900.0	0.2712737446855706
AMZN190315P01900000	2019-03-15	P	1900.0	1.1596539319323762
AMZN190315C01910000	2019-03-15	C	1910.0	0.27913772846426804
AMZN190315C01920000	2019-03-15	C	1920.0	0.2769566679854527
AMZN190315P01920000	2019-03-15	P	1920.0	1.2707021840088202
AMZN190315C01930000	2019-03-15	C	1930.0	0.2773995533623659
AMZN190315P01930000	2019-03-15	P	1930.0	1.3085726155039599
AMZN190315C01940000	2019-03-15	C	1940.0	0.2824651249839217
AMZN190315C01950000	2019-03-15	C	1950.0	0.2829251813766597
AMZN190315P01950000	2019-03-15	P	1950.0	1.3855940606587989
AMZN190315C01960000	2019-03-15	C	1960.0	0.2841904462146027
AMZN190315C01970000	2019-03-15	C	1970.0	0.2887721561714816
AMZN190315P01970000	2019-03-15	P	1970.0	1.4383751291143314
AMZN190418P01320000	2019-04-18	P	1320.0	0.2420685297387945
AMZN190418P01340000	2019-04-18	P	1340.0	0.23732728055675925
AMZN190418P01355000	2019-04-18	P	1355.0	0.23165869895759447
AMZN190418P01360000	2019-04-18	P	1360.0	0.23214690215752254
AMZN190418P01365000	2019-04-18	P	1365.0	0.22818625125738665
AMZN190418P01375000	2019-04-18	P	1375.0	0.22645636897562715
AMZN190418P01380000	2019-04-18	P	1380.0	0.22601484947497277
AMZN190418P01385000	2019-04-18	P	1385.0	0.22358227263935995
AMZN190418P01395000	2019-04-18	P	1395.0	0.22222988440862398
AMZN190418C01400000	2019-04-18	C	1400.0	0.43687075300289846

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190418P01400000	2019-04-18	P	1400.0	0.21727963176834614
AMZN190418P01405000	2019-04-18	P	1405.0	0.2168200875792052
AMZN190418P01415000	2019-04-18	P	1415.0	0.21600285454479326
AMZN190418P01420000	2019-04-18	P	1420.0	0.2056448051081899
AMZN190418C01425000	2019-04-18	C	1425.0	0.2887456502940846
AMZN190418P01425000	2019-04-18	P	1425.0	0.21179732154397404
AMZN190418C01435000	2019-04-18	C	1435.0	0.31537605566289834
AMZN190418P01435000	2019-04-18	P	1435.0	0.21063879017939652
AMZN190418C01440000	2019-04-18	C	1440.0	0.3013491042672771
AMZN190418P01440000	2019-04-18	P	1440.0	0.20912432609616644
AMZN190418P01445000	2019-04-18	P	1445.0	0.20714748850868792
AMZN190418C01460000	2019-04-18	C	1460.0	0.38930867334156083
AMZN190418P01460000	2019-04-18	P	1460.0	0.2031648372445265
AMZN190418P01480000	2019-04-18	P	1480.0	0.19710905411664179
AMZN190418C01500000	2019-04-18	C	1500.0	0.32955405047482544
AMZN190418P01500000	2019-04-18	P	1500.0	0.19711615179505798
AMZN190418C01520000	2019-04-18	C	1520.0	0.2821439733285733
AMZN190418P01520000	2019-04-18	P	1520.0	0.19704356522816222
AMZN190418C01540000	2019-04-18	C	1540.0	0.3028642917837938
AMZN190418P01540000	2019-04-18	P	1540.0	0.19247266032811625
AMZN190418C01555000	2019-04-18	C	1555.0	0.28673592735739317
AMZN190418P01555000	2019-04-18	P	1555.0	0.20284449048054493
AMZN190418P01560000	2019-04-18	P	1560.0	0.20630676728075423
AMZN190418C01565000	2019-04-18	C	1565.0	0.285433442391398
AMZN190418P01565000	2019-04-18	P	1565.0	0.20039896221112108
AMZN190418C01575000	2019-04-18	C	1575.0	0.28621138209272223
AMZN190418P01575000	2019-04-18	P	1575.0	0.20974699493564303
AMZN190418C01585000	2019-04-18	C	1585.0	0.276497440874729
AMZN190418P01595000	2019-04-18	P	1595.0	0.2216804362928776
AMZN190418C01600000	2019-04-18	C	1600.0	0.2862739441035044
AMZN190418P01600000	2019-04-18	P	1600.0	0.22347612454153387
AMZN190418C01605000	2019-04-18	C	1605.0	0.3455523883595186
AMZN190418P01605000	2019-04-18	P	1605.0	0.23465326070175757
AMZN190418P01615000	2019-04-18	P	1615.0	0.24473523239955267
AMZN190418C01620000	2019-04-18	C	1620.0	0.2774217488515712
AMZN190418P01620000	2019-04-18	P	1620.0	0.24735444646967036
AMZN190418C01625000	2019-04-18	C	1625.0	0.2814957186998919
AMZN190418P01625000	2019-04-18	P	1625.0	0.24744296012936956
AMZN190418C01635000	2019-04-18	C	1635.0	0.280478848215869
AMZN190418P01635000	2019-04-18	P	1635.0	0.26573601891012755
AMZN190418C01640000	2019-04-18	C	1640.0	0.28482543233105595
AMZN190418P01640000	2019-04-18	P	1640.0	0.26743829097894145
AMZN190418C01645000	2019-04-18	C	1645.0	0.2810819313654204
AMZN190418P01645000	2019-04-18	P	1645.0	0.28212924137749634
AMZN190418C01655000	2019-04-18	C	1655.0	0.281185859914326
AMZN190418P01655000	2019-04-18	P	1655.0	0.2938338008987934

Option Name	Expiration Date	Type	Strike	Implied Volatility
AMZN190418C01660000	2019-04-18	C	1660.0	0.2811121269869987
AMZN190418P01660000	2019-04-18	P	1660.0	0.2945779351627125
AMZN190418C01665000	2019-04-18	C	1665.0	0.27539276405978386
AMZN190418P01665000	2019-04-18	P	1665.0	0.29625671903800477
AMZN190418C01675000	2019-04-18	C	1675.0	0.2774616275601985
AMZN190418P01675000	2019-04-18	P	1675.0	0.31055361413589827
AMZN190418C01680000	2019-04-18	C	1680.0	0.2812185433819471
AMZN190418P01680000	2019-04-18	P	1680.0	0.343132006847645
AMZN190418C01685000	2019-04-18	C	1685.0	0.2799948889886022
AMZN190418P01685000	2019-04-18	P	1685.0	0.3128225053362834
AMZN190418C01700000	2019-04-18	C	1700.0	0.27353653822408613
AMZN190418P01700000	2019-04-18	P	1700.0	0.3772860597771452
AMZN190418C01720000	2019-04-18	C	1720.0	0.27736047954510545
AMZN190418P01720000	2019-04-18	P	1720.0	0.428796570624232
AMZN190418C01725000	2019-04-18	C	1725.0	0.2677525644717009
AMZN190418P01725000	2019-04-18	P	1725.0	0.4001241937622695
AMZN190418C01740000	2019-04-18	C	1740.0	0.2716831417034959
AMZN190418C01760000	2019-04-18	C	1760.0	0.2699733085339636
AMZN190418P01760000	2019-04-18	P	1760.0	0.5230233980261761
AMZN190418C01780000	2019-04-18	C	1780.0	0.2691921736578197
AMZN190418C01785000	2019-04-18	C	1785.0	0.2705791966079751
AMZN190418P01785000	2019-04-18	P	1785.0	0.5784737057698047
AMZN190418C01790000	2019-04-18	C	1790.0	0.26068703292885703
AMZN190418P01790000	2019-04-18	P	1790.0	0.5206362365761681
AMZN190418C01795000	2019-04-18	C	1795.0	0.26029629475625277
AMZN190418P01795000	2019-04-18	P	1795.0	0.602663106015881
AMZN190418C01800000	2019-04-18	C	1800.0	0.2656045777108663
AMZN190418P01800000	2019-04-18	P	1800.0	0.5920504913915454
AMZN190418C01805000	2019-04-18	C	1805.0	0.2617792705135882
AMZN190418C01810000	2019-04-18	C	1810.0	0.26277601871344136
AMZN190418C01820000	2019-04-18	C	1820.0	0.2613774833776762
AMZN190418C01825000	2019-04-18	C	1825.0	0.2598911173203412
AMZN190418C01830000	2019-04-18	C	1830.0	0.26095847644464437
AMZN190418C01840000	2019-04-18	C	1840.0	0.2635895199787891
AMZN190418C01850000	2019-04-18	C	1850.0	0.26487732177500223
AMZN190418C01855000	2019-04-18	C	1855.0	0.2600204853145668
AMZN190418C01860000	2019-04-18	C	1860.0	0.27086795748347214
AMZN190418C01865000	2019-04-18	C	1865.0	0.27064900264105834
AMZN190418C01870000	2019-04-18	C	1870.0	0.2620139573236256
AMZN190418C01875000	2019-04-18	C	1875.0	0.2612693108561094
AMZN190418C01880000	2019-04-18	C	1880.0	0.2659066314892391
AMZN190418C01885000	2019-04-18	C	1885.0	0.2650399342217409
AMZN190418C01890000	2019-04-18	C	1890.0	0.26456307267289025
AMZN190418C01895000	2019-04-18	C	1895.0	0.25696136152652826
AMZN190418C01900000	2019-04-18	C	1900.0	0.26440775302974767
AMZN190418P01900000	2019-04-18	P	1900.0	0.854484004437771

Option Name	**Expiration Date**	**Type**	**Strike**	**Implied Volatility**
AMZN190418C01905000	2019-04-18	C	1905.0	0.2654209405260013
AMZN190418C01910000	2019-04-18	C	1910.0	0.2644220703398175
AMZN190418C01915000	2019-04-18	C	1915.0	0.26904395169309336
AMZN190418C01920000	2019-04-18	C	1920.0	0.2681516930270378
AMZN190418C01925000	2019-04-18	C	1925.0	0.2657634096072458
AMZN190418C01930000	2019-04-18	C	1930.0	0.2630787554299435
AMZN190418C01935000	2019-04-18	C	1935.0	0.28071777899856765
AMZN190418C01940000	2019-04-18	C	1940.0	0.2775561901004723
AMZN190418C01950000	2019-04-18	C	1950.0	0.2625715763062772
AMZN190418C01960000	2019-04-18	C	1960.0	0.2628836790314111
AMZN190418C01965000	2019-04-18	C	1965.0	0.2610769174287996
AMZN190418P01965000	2019-04-18	P	1965.0	1.0254512479543076

B Analytically Computed and Estimated Greeks

B.1 SPY Option Chain Greeks

Option Name	Analytical			Estimated		
	Δ	Γ	ν	Δ	Γ	ν
SPY190215C00256000	0.9539155	0.0091741	4.1393352	0.9539155	0.0090949	46.6598299
SPY190215C00257000	0.9334452	0.0116174	5.5325278	0.9334452	0.0116529	46.7960534
SPY190215C00258000	0.9546070	0.0103717	4.0893400	0.9546070	0.0105160	53.3652495
SPY190215C00259000	0.9409217	0.0126714	5.0418890	0.9409217	0.0122213	55.5473864
SPY190215C00260000	0.9409396	0.0136821	5.0406894	0.9409396	0.0144951	60.0887959
SPY190215C00262000	0.9191308	0.0185067	6.4213282	0.9191308	0.0187583	65.0937635
SPY190215C00263000	0.9147231	0.0208030	6.6828194	0.9147231	0.0216005	70.2705305
SPY190215C00264000	0.9005609	0.0243224	7.4878185	0.9005609	0.0250111	72.1745461
SPY190215C00265000	0.8845512	0.0284666	8.3387404	0.8845512	0.0281375	73.4239080
SPY190215C00266000	0.8584682	0.0336174	9.6062420	0.8584682	0.0329692	69.4811793
SPY190215C00267000	0.8338012	0.0394492	10.6846356	0.8338012	0.0397904	66.4012291
SPY190215C00268000	0.8037780	0.0464089	11.8567324	0.8037780	0.0457590	60.7043663
SPY190215C00269000	0.7573635	0.0534719	13.3976455	0.7573635	0.0537170	46.3265509
SPY190215C00270000	0.7047417	0.0605839	14.7878354	0.7047417	0.0602540	30.9461509
SPY190215C00271000	0.6446102	0.0672412	15.9558296	0.6446102	0.0673595	16.0404919
SPY190215C00272000	0.5774678	0.0731832	16.7681212	0.5774678	0.0730438	4.6150349
SPY190215C00273000	0.5031323	0.0770287	17.0908485	0.5031323	0.0770228	-0.0123559
SPY190215C00274000	0.4248419	0.0774287	16.7871666	0.4248419	0.0774492	5.6041044
SPY190215C00275000	0.3469803	0.0743243	15.8181558	0.3469803	0.0744649	22.0987790
SPY190215C00276000	0.2732765	0.0680665	14.2507515	0.2732765	0.0684963	46.7649072
SPY190215C00277000	0.2088892	0.0591785	12.3085707	0.2088892	0.0591882	72.8792935
SPY190215C00278000	0.1546784	0.0491947	10.1946600	0.1546784	0.0493827	94.9375514
SPY190215C00279000	0.1127330	0.0393391	8.1985314	0.1127330	0.0394351	107.6447395
SPY190215C00280000	0.0821787	0.0307028	6.4995860	0.0821787	0.0310152	110.3859187
SPY190215C00281000	0.0614365	0.0238760	5.1987466	0.0614365	0.0239808	105.5705443
SPY190215C00282000	0.0428260	0.0177692	3.9018390	0.0428260	0.0177636	97.4023359
SPY190215C00283000	0.0322972	0.0136774	3.0985061	0.0322972	0.0136957	86.6356686
SPY190215C00284000	0.0247898	0.0106187	2.4862242	0.0247898	0.0105960	75.9115083
SPY190315C00256000	0.8763100	0.0127739	17.7502685	0.8763100	0.0125056	122.4314862
SPY190315C00257000	0.8751785	0.0136141	17.8636939	0.8751785	0.0130740	129.5956650
SPY190315C00258000	0.8552029	0.0147500	19.7799150	0.8552029	0.0153477	118.1518154
SPY190315C00259000	0.8367969	0.0158073	21.4102329	0.8367969	0.0162004	108.1848771
SPY190315C00260000	0.8316153	0.0170061	21.8472491	0.8316153	0.0162004	111.7190013
SPY190315C00261000	0.8358137	0.0184807	21.4938803	0.8358137	0.0184741	126.5262695
SPY190315C00262000	0.8074323	0.0196894	23.7669924	0.8074323	0.0193268	105.4849613
SPY190315C00263000	0.7888690	0.0210521	25.1137383	0.7888690	0.0210321	95.8849874
SPY190315C00264000	0.7767008	0.0228349	25.9399765	0.7767008	0.0221689	93.5242431
SPY190315C00265000	0.7494775	0.0239667	27.6343453	0.7494775	0.0244427	75.9516272
SPY190315C00266000	0.7326208	0.0259339	28.5812246	0.7326208	0.0261480	69.7572825
SPY190315C00267000	0.7074397	0.0274123	29.8566049	0.7074397	0.0272848	56.4553676
SPY190315C00268000	0.6765780	0.0282070	31.2016049	0.6765780	0.0281375	40.1238962

Option Name	Analytical			Estimated		
	Δ	Γ	ν	Δ	Γ	ν
SPY190315C00269000	0.6536573	0.0304424	32.0508537	0.6536573	0.0306954	31.8455727
SPY190315C00270000	0.6236601	0.0317213	32.9758077	0.6236601	0.0318323	20.4579763
SPY190315C00271000	0.5924170	0.0329464	33.7202027	0.5924170	0.0321165	11.0755180
SPY190315C00272000	0.5600843	0.0343295	34.2604800	0.5600843	0.0338218	4.2714960
SPY190315C00273000	0.5256823	0.0352488	34.5824258	0.5256823	0.0358114	0.3944854
SPY190315C00274000	0.4899042	0.0360769	34.6431620	0.4899042	0.0358114	0.4535290
SPY190315C00275000	0.4531783	0.0363968	34.4153126	0.4531783	0.0369482	5.0893920
SPY190315C00276000	0.4154249	0.0365447	33.8725737	0.4154249	0.0365219	14.8862204
SPY190315C00277000	0.3775794	0.0362082	33.0095593	0.3775794	0.0362377	29.8464742
SPY190315C00278000	0.3402099	0.0354219	31.8358771	0.3402099	0.0353850	49.5197658
SPY190315C00279000	0.3031528	0.0343077	30.3448181	0.3031528	0.0346745	73.7326776
SPY190315C00280000	0.2681687	0.0327183	28.6237100	0.2681687	0.0324007	100.0344294
SPY190315C00281000	0.2344957	0.0308485	26.6621489	0.2344957	0.0306244	128.2001839
SPY190315C00282000	0.2028988	0.0287047	24.5296139	0.2028988	0.0292033	156.1760845
SPY190315C00283000	0.1753870	0.0263377	22.4231328	0.1753870	0.0265032	179.6899371
SPY190315C00284000	0.1494109	0.0238977	20.2003122	0.1494109	0.0238742	201.3301222
SPY190418C00257000	0.8466773	0.0133179	28.4676048	0.8466773	0.0130740	188.6911486
SPY190418C00260000	0.7992159	0.0156914	33.7671664	0.7992159	0.0156319	147.3722344
SPY190418C00261000	0.7960737	0.0168856	34.0822145	0.7960737	0.0176215	155.0486259
SPY190418C00262000	0.7763412	0.0176776	35.9662759	0.7763412	0.0179057	135.7478207
SPY190418C00263000	0.7516798	0.0181777	38.1012718	0.7516798	0.0176215	110.2043601
SPY190418C00265000	0.7192627	0.0201075	40.5575900	0.7192627	0.0204636	87.7747669
SPY190418C00266000	0.7009076	0.0210601	41.7805288	0.7009076	0.0216005	74.9826881
SPY190418C00267000	0.6796395	0.0217960	43.0516855	0.6796395	0.0213163	59.8746670
SPY190418C00268000	0.6582358	0.0225668	44.1773002	0.6582358	0.0227374	46.3572888
SPY190418C00269000	0.6361925	0.0233294	45.1795798	0.6361925	0.0233058	34.0386442
SPY190418C00270000	0.6140679	0.0242434	46.0289159	0.6140679	0.0241585	23.6228702
SPY190418C00271000	0.5903166	0.0249762	46.7693273	0.5903166	0.0244427	14.1483884
SPY190418C00272000	0.5655780	0.0256059	47.3547979	0.5655780	0.0258638	6.6606457
SPY190418C00273000	0.5400672	0.0261902	47.7625151	0.5400672	0.0264322	1.7188029
SPY190418C00274000	0.5137605	0.0266729	47.9762702	0.5137605	0.0267164	-0.2703295
SPY190418C00275000	0.4868470	0.0269775	47.9787365	0.4868470	0.0268585	1.1225444
SPY190418C00276000	0.4591583	0.0273005	47.7530482	0.4591583	0.0274269	6.3174017
SPY190418C00277000	0.4312221	0.0273780	47.2896500	0.4312221	0.0275691	15.5142687
SPY190418C00278000	0.4035056	0.0272068	46.5936565	0.4035056	0.0272848	28.5280232
SPY190418C00279000	0.3749830	0.0270496	45.6280116	0.3749830	0.0270006	46.1900927
SPY190418C00280000	0.3478579	0.0265307	44.4701436	0.3478579	0.0261480	66.4329178
SPY190418C00281000	0.3203631	0.0259730	43.0518334	0.3203631	0.0262901	90.7675930
SPY190418C00282000	0.2920004	0.0253756	41.3220421	0.2920004	0.0251532	120.1365418
SPY190418C00283000	0.2682425	0.0243245	39.6564987	0.2682425	0.0245848	145.3860380
SPY190418C00284000	0.2429928	0.0233358	37.6600793	0.2429928	0.0233058	175.5833008

B.2 AMZN Option Chain Greeks

Option Name	Analytical			Estimated		
	Δ	Γ	ν	Δ	Γ	ν
AMZN190215C01560000	0.8760679	0.0027848	52.7036186	0.8760679	0.0022737	237.1293785
AMZN190215C01565000	0.8524446	0.0030263	59.3977324	0.8524446	0.0022737	210.3054265
AMZN190215C01570000	0.8416058	0.0032494	62.2560160	0.8416058	0.0068212	206.2430219
AMZN190215C01575000	0.7312256	0.0028372	84.9688184	0.7312255	0.0068212	63.3166951
AMZN190215C01580000	0.8079519	0.0037140	70.3554181	0.8079519	0.0022737	177.0330359
AMZN190215C01585000	0.7538748	0.0036392	81.1689931	0.7538748	0.0045475	105.1024489
AMZN190215C01590000	0.7414344	0.0038694	83.3067572	0.7414344	0.0045475	99.2387102
AMZN190215C01600000	0.7223743	0.0045585	86.3439432	0.7223742	0.0090949	97.2392208
AMZN190215C01605000	0.6846925	0.0044299	91.5349917	0.6846925	0.0090949	61.0407581
AMZN190215C01610000	0.6608483	0.0045321	94.2842448	0.6608484	0.0045475	45.5850609
AMZN190215C01615000	0.6549543	0.0052616	94.9017322	0.6549542	0.0022737	49.5531241
AMZN190215C01620000	0.6238710	0.0052157	97.7597970	0.6238710	0.0045475	29.6320284
AMZN190215C01625000	0.5983016	0.0053872	99.6183356	0.5983016	0.0045475	18.2557884
AMZN190215C01630000	0.5716309	0.0055383	101.0929088	0.5716309	0.0022737	9.1081946
AMZN190215C01635000	0.5434752	0.0055581	102.1429784	0.5434752	0.0045475	2.6449197
AMZN190215C01640000	0.5155374	0.0056416	102.6757867	0.5155374	0.0068212	-0.0601022
AMZN190215C01645000	0.4873685	0.0056400	102.7022167	0.4873685	0.0068212	0.8859310
AMZN190215C01650000	0.4585589	0.0056893	102.1988438	0.4585589	0.0068212	5.7573856
AMZN190215C01655000	0.4297365	0.0056783	101.1558752	0.4297365	0.0045475	14.6198347
AMZN190215C01660000	0.4021813	0.0055768	99.6492191	0.4021813	0.0045475	26.5763198
AMZN190215C01665000	0.3742852	0.0054885	97.6087316	0.3742852	0.0068212	42.3199307
AMZN190215C01670000	0.3476676	0.0053497	95.1685931	0.3476676	0.0068212	60.2833582
AMZN190215C01675000	0.3227431	0.0051695	92.4359438	0.3227431	0.0011369	79.2580734
AMZN190215C01680000	0.2960222	0.0050250	89.0104463	0.2960222	0.0045475	103.2178616
AMZN190215C01685000	0.2734159	0.0048064	85.6974800	0.2734159	0.0051159	123.8590547
AMZN190215C01690000	0.2500820	0.0046010	81.8624964	0.2500820	0.0045475	147.4202004
AMZN190215C01695000	0.2287670	0.0043743	77.9739454	0.2287670	0.0022737	169.2571862
AMZN190215C01700000	0.2077856	0.0041451	73.7685460	0.2077856	0.0022737	191.6945679
AMZN190215C01705000	0.1890917	0.0039049	69.6885297	0.1890917	0.0045475	210.8216574
AMZN190215C01710000	0.1722068	0.0036635	65.7177009	0.1722068	0.0045475	227.0166982
AMZN190215C01715000	0.1571178	0.0034264	61.9260297	0.1571178	0.0036948	240.0034104
AMZN190215C01720000	0.1405022	0.0031901	57.4676551	0.1405022	0.0014211	255.6927918
AMZN190215C01725000	0.1274118	0.0029641	53.7317100	0.1274118	0.0017053	264.5802459
AMZN190315C01555000	0.7493043	0.0021826	166.1991450	0.7493043	0.0068212	233.4812777
AMZN190315C01560000	0.7213345	0.0020938	175.3893753	0.7213345	0.0068212	163.8696128
AMZN190315C01570000	0.7207987	0.0024065	175.5533422	0.7207987	0.0000000	192.0653148
AMZN190315C01585000	0.6749762	0.0024441	187.9661284	0.6749761	-0.0045475	110.0550322
AMZN190315C01590000	0.6609297	0.0024581	191.1519357	0.6609297	0.0068212	90.2048063
AMZN190315C01595000	0.6522065	0.0025744	192.9883817	0.6522065	0.0045475	83.4523285
AMZN190315C01600000	0.6365070	0.0025523	196.0232424	0.6365070	0.0011369	63.0616436
AMZN190315C01605000	0.6235982	0.0025884	198.2614603	0.6235982	0.0045475	50.1420821
AMZN190315C01610000	0.6112891	0.0026479	200.1821467	0.6112891	0.0011369	39.6908155
AMZN190315C01615000	0.5873686	0.0022086	203.3257981	0.5873686	-0.0011369	15.0688230

Option Name	Analytical			Estimated		
	Δ	Γ	ν	Δ	Γ	ν
AMZN190315C01620000	0.5824850	0.0026030	203.8729039	0.5824849	0.0045475	17.2710753
AMZN190315C01625000	0.5706869	0.0027283	205.0631243	0.5706869	0.0045475	11.9785372
AMZN190315C01630000	0.5565435	0.0026988	206.2460216	0.5565435	0.0045475	5.5476862
AMZN190315C01635000	0.5430193	0.0027336	207.1296898	0.5430193	0.0045475	1.5777823
AMZN190315C01640000	0.5293000	0.0027447	207.7800147	0.5293000	0.0022737	-0.8179998
AMZN190315C01645000	0.5153694	0.0027799	208.1875422	0.5153694	0.0011369	-1.4284224
AMZN190315C01650000	0.5012060	0.0028086	208.3412211	0.5012060	0.0022737	-0.1935301
AMZN190315C01655000	0.4873270	0.0028038	208.2370437	0.4873270	0.0011369	2.8783835
AMZN190315C01660000	0.4732555	0.0028085	207.8738340	0.4732554	0.0011369	7.8877705
AMZN190315C01665000	0.4600045	0.0027788	207.2942877	0.4600045	0.0000000	14.2706273
AMZN190315C01670000	0.4462916	0.0027698	206.4512666	0.4462916	0.0011369	22.6507134
AMZN190315C01675000	0.4313654	0.0027914	205.2512370	0.4313654	0.0045475	34.0572333
AMZN190315C01680000	0.4187539	0.0027505	204.0066090	0.4187539	0.0034106	44.8709264
AMZN190315C01685000	0.4076931	0.0026839	202.7400397	0.4076931	0.0034106	54.9748985
AMZN190315C01690000	0.3921114	0.0027071	200.6762767	0.3921114	0.0011369	72.8512237
AMZN190315C01695000	0.3785168	0.0026888	198.6062560	0.3785168	0.0056843	89.7530012
AMZN190315C01700000	0.3622667	0.0027056	195.7985964	0.3622667	0.0045475	113.6095564
AMZN190315C01705000	0.3502225	0.0026597	193.4805778	0.3502225	0.0022737	131.0626924
AMZN190315C01710000	0.3372143	0.0026267	190.7472152	0.3372143	0.0011369	151.9279349
AMZN190315C01715000	0.3220300	0.0026145	187.2502732	0.3220300	0.0017053	179.7611841
AMZN190315C01720000	0.3116082	0.0025536	184.6561374	0.3116082	0.0039790	196.9888001
AMZN190315C01725000	0.2990576	0.0025133	181.3189797	0.2990576	0.0000000	220.8857933
AMZN190418C01555000	0.6992483	0.0016781	251.8152333	0.6992483	0.0022737	181.5179462
AMZN190418C01565000	0.6819387	0.0017274	258.0460960	0.6819387	0.0022737	148.5242254
AMZN190418C01575000	0.6633872	0.0017628	264.0495175	0.6633872	0.0000000	114.9676721
AMZN190418C01585000	0.6481318	0.0018553	268.4744530	0.6481318	0.0034106	95.3889738
AMZN190418C01600000	0.6167403	0.0018433	276.1595615	0.6167403	0.0022737	48.8870333
AMZN190418C01605000	0.5984830	0.0015470	279.7669648	0.5984830	0.0045475	19.5916999
AMZN190418C01620000	0.5796159	0.0019481	282.8396659	0.5796159	0.0011369	16.0908683
AMZN190418C01625000	0.5694465	0.0019293	284.2222057	0.5694465	0.0000000	8.9766327
AMZN190418C01635000	0.5499584	0.0019507	286.3401968	0.5499583	0.0045475	0.2374377
AMZN190418C01640000	0.5403183	0.0019262	287.1309662	0.5403183	-0.0011369	-2.4886239
AMZN190418C01645000	0.5304362	0.0019562	287.7657727	0.5304362	0.0011369	-3.7217389
AMZN190418C01655000	0.5109752	0.0019605	288.4968766	0.5109752	0.0034106	-2.7241893
AMZN190418C01660000	0.5012599	0.0019617	288.6046581	0.5012599	0.0034106	-0.3917518
AMZN190418C01665000	0.4903919	0.0020019	288.5223923	0.4903919	0.0011369	3.6728159
AMZN190418C01675000	0.4713287	0.0019824	287.8604428	0.4713287	0.0045475	14.5001064
AMZN190418C01680000	0.4628108	0.0019524	287.3512107	0.4628108	0.0011369	20.7365263
AMZN190418C01685000	0.4528878	0.0019558	286.5913138	0.4528878	0.0011369	29.3029721
AMZN190418C01700000	0.4217149	0.0019771	283.0314516	0.4217149	0.0000000	65.0184386
AMZN190418C01720000	0.3863970	0.0019071	276.8241740	0.3863970	-0.0022737	118.4498728
AMZN190418C01725000	0.3714646	0.0019517	273.4936503	0.3714646	0.0011369	149.4367243

C DATA2 Computed Prices

C.1 SPY Option Chain

Option Name	Expiration Date	Type	Strike	Computed Price
SPY190215C00256000	2019-02-15	C	256.00	14.68
SPY190215P00256000	2019-02-15	P	256.00	0.25
SPY190215C00257000	2019-02-15	C	257.00	13.88
SPY190215P00257000	2019-02-15	P	257.00	0.28
SPY190215C00258000	2019-02-15	C	258.00	12.66
SPY190215P00258000	2019-02-15	P	258.00	0.32
SPY190215C00259000	2019-02-15	C	259.00	11.77
SPY190215P00259000	2019-02-15	P	259.00	0.38
SPY190215C00260000	2019-02-15	C	260.00	10.76
SPY190215P00260000	2019-02-15	P	260.00	0.43
SPY190215P00261000	2019-02-15	P	261.00	0.49
SPY190215C00262000	2019-02-15	C	262.00	8.92
SPY190215P00262000	2019-02-15	P	262.00	0.58
SPY190215C00263000	2019-02-15	C	263.00	7.94
SPY190215P00263000	2019-02-15	P	263.00	0.66
SPY190215C00264000	2019-02-15	C	264.00	7.03
SPY190215P00264000	2019-02-15	P	264.00	0.76
SPY190215C00265000	2019-02-15	C	265.00	6.14
SPY190215P00265000	2019-02-15	P	265.00	0.87
SPY190215C00266000	2019-02-15	C	266.00	5.33
SPY190215P00266000	2019-02-15	P	266.00	1.00
SPY190215C00267000	2019-02-15	C	267.00	4.49
SPY190215P00267000	2019-02-15	P	267.00	1.14
SPY190215C00268000	2019-02-15	C	268.00	3.69
SPY190215P00268000	2019-02-15	P	268.00	1.26
SPY190215C00269000	2019-02-15	C	269.00	3.03
SPY190215P00269000	2019-02-15	P	269.00	1.36
SPY190215C00270000	2019-02-15	C	270.00	2.42
SPY190215P00270000	2019-02-15	P	270.00	1.41
SPY190215C00271000	2019-02-15	C	271.00	1.87
SPY190215P00271000	2019-02-15	P	271.00	1.46
SPY190215C00272000	2019-02-15	C	272.00	1.39
SPY190215P00272000	2019-02-15	P	272.00	1.57
SPY190215C00273000	2019-02-15	C	273.00	0.99
SPY190215P00273000	2019-02-15	P	273.00	1.84
SPY190215C00274000	2019-02-15	C	274.00	0.69
SPY190215P00274000	2019-02-15	P	274.00	2.28
SPY190215C00275000	2019-02-15	C	275.00	0.46
SPY190215P00275000	2019-02-15	P	275.00	2.85
SPY190215C00276000	2019-02-15	C	276.00	0.30
SPY190215P00276000	2019-02-15	P	276.00	3.49
SPY190215C00277000	2019-02-15	C	277.00	0.19

Option Name	Expiration Date	Type	Strike	Computed Price
SPY190215P00277000	2019-02-15	P	277.00	4.27
SPY190215C00278000	2019-02-15	C	278.00	0.12
SPY190215P00278000	2019-02-15	P	278.00	4.83
SPY190215C00279000	2019-02-15	C	279.00	0.08
SPY190215P00279000	2019-02-15	P	279.00	6.10
SPY190215C00280000	2019-02-15	C	280.00	0.05
SPY190215P00280000	2019-02-15	P	280.00	7.10
SPY190215C00281000	2019-02-15	C	281.00	0.04
SPY190215C00282000	2019-02-15	C	282.00	0.02
SPY190215P00282000	2019-02-15	P	282.00	8.71
SPY190215C00283000	2019-02-15	C	283.00	0.02
SPY190215P00283000	2019-02-15	P	283.00	9.92
SPY190215C00284000	2019-02-15	C	284.00	0.01
SPY190315C00256000	2019-03-15	C	256.00	16.17
SPY190315P00256000	2019-03-15	P	256.00	1.61
SPY190315C00257000	2019-03-15	C	257.00	15.14
SPY190315P00257000	2019-03-15	P	257.00	1.70
SPY190315C00258000	2019-03-15	C	258.00	14.42
SPY190315P00258000	2019-03-15	P	258.00	1.83
SPY190315C00259000	2019-03-15	C	259.00	13.67
SPY190315P00259000	2019-03-15	P	259.00	1.95
SPY190315C00260000	2019-03-15	C	260.00	12.67
SPY190315P00260000	2019-03-15	P	260.00	2.09
SPY190315C00261000	2019-03-15	C	261.00	11.50
SPY190315P00261000	2019-03-15	P	261.00	2.24
SPY190315C00262000	2019-03-15	C	262.00	10.91
SPY190315P00262000	2019-03-15	P	262.00	2.38
SPY190315C00263000	2019-03-15	C	263.00	10.13
SPY190315P00263000	2019-03-15	P	263.00	2.54
SPY190315C00264000	2019-03-15	C	264.00	9.21
SPY190315P00264000	2019-03-15	P	264.00	2.71
SPY190315C00265000	2019-03-15	C	265.00	8.60
SPY190315P00265000	2019-03-15	P	265.00	2.88
SPY190315C00266000	2019-03-15	C	266.00	7.74
SPY190315P00266000	2019-03-15	P	266.00	3.06
SPY190315C00267000	2019-03-15	C	267.00	7.06
SPY190315P00267000	2019-03-15	P	267.00	3.24
SPY190315C00268000	2019-03-15	C	268.00	6.54
SPY190315P00268000	2019-03-15	P	268.00	3.43
SPY190315C00269000	2019-03-15	C	269.00	5.74
SPY190315P00269000	2019-03-15	P	269.00	3.63
SPY190315C00270000	2019-03-15	C	270.00	5.14
SPY190315P00270000	2019-03-15	P	270.00	3.87
SPY190315C00271000	2019-03-15	C	271.00	4.57
SPY190315P00271000	2019-03-15	P	271.00	4.13
SPY190315C00272000	2019-03-15	C	272.00	3.99

Option Name	Expiration Date	Type	Strike	Computed Price
SPY190315P00272000	2019-03-15	P	272.00	4.44
SPY190315C00273000	2019-03-15	C	273.00	3.48
SPY190315P00273000	2019-03-15	P	273.00	4.79
SPY190315C00274000	2019-03-15	C	274.00	2.99
SPY190315P00274000	2019-03-15	P	274.00	5.21
SPY190315C00275000	2019-03-15	C	275.00	2.57
SPY190315P00275000	2019-03-15	P	275.00	5.67
SPY190315C00276000	2019-03-15	C	276.00	2.16
SPY190315P00276000	2019-03-15	P	276.00	6.29
SPY190315C00277000	2019-03-15	C	277.00	1.81
SPY190315P00277000	2019-03-15	P	277.00	6.64
SPY190315C00278000	2019-03-15	C	278.00	1.51
SPY190315P00278000	2019-03-15	P	278.00	7.40
SPY190315C00279000	2019-03-15	C	279.00	1.24
SPY190315P00279000	2019-03-15	P	279.00	8.17
SPY190315C00280000	2019-03-15	C	280.00	1.01
SPY190315P00280000	2019-03-15	P	280.00	8.73
SPY190315C00281000	2019-03-15	C	281.00	0.82
SPY190315P00281000	2019-03-15	P	281.00	9.32
SPY190315C00282000	2019-03-15	C	282.00	0.65
SPY190315C00283000	2019-03-15	C	283.00	0.53
SPY190315P00283000	2019-03-15	P	283.00	11.30
SPY190315C00284000	2019-03-15	C	284.00	0.42
SPY190315P00284000	2019-03-15	P	284.00	11.80
SPY190418P00256000	2019-04-18	P	256.00	2.99
SPY190418C00257000	2019-04-18	C	257.00	16.29
SPY190418P00257000	2019-04-18	P	257.00	3.14
SPY190418P00258000	2019-04-18	P	258.00	3.32
SPY190418P00259000	2019-04-18	P	259.00	3.41
SPY190418C00260000	2019-04-18	C	260.00	14.04
SPY190418P00260000	2019-04-18	P	260.00	3.62
SPY190418C00261000	2019-04-18	C	261.00	12.97
SPY190418P00261000	2019-04-18	P	261.00	3.83
SPY190418C00262000	2019-04-18	C	262.00	12.30
SPY190418P00262000	2019-04-18	P	262.00	3.98
SPY190418C00263000	2019-04-18	C	263.00	11.78
SPY190418P00263000	2019-04-18	P	263.00	4.20
SPY190418P00264000	2019-04-18	P	264.00	4.38
SPY190418C00265000	2019-04-18	C	265.00	10.24
SPY190418P00265000	2019-04-18	P	265.00	4.55
SPY190418C00266000	2019-04-18	C	266.00	9.51
SPY190418P00266000	2019-04-18	P	266.00	4.75
SPY190418C00267000	2019-04-18	C	267.00	8.88
SPY190418P00267000	2019-04-18	P	267.00	4.98
SPY190418C00268000	2019-04-18	C	268.00	8.24
SPY190418P00268000	2019-04-18	P	268.00	5.22

Option Name	Expiration Date	Type	Strike	Computed Price
SPY190418C00269000	2019-04-18	C	269.00	7.61
SPY190418P00269000	2019-04-18	P	269.00	5.46
SPY190418C00270000	2019-04-18	C	270.00	6.96
SPY190418P00270000	2019-04-18	P	270.00	5.72
SPY190418C00271000	2019-04-18	C	271.00	6.36
SPY190418P00271000	2019-04-18	P	271.00	6.02
SPY190418C00272000	2019-04-18	C	272.00	5.80
SPY190418P00272000	2019-04-18	P	272.00	6.32
SPY190418C00273000	2019-04-18	C	273.00	5.26
SPY190418P00273000	2019-04-18	P	273.00	6.71
SPY190418C00274000	2019-04-18	C	274.00	4.74
SPY190418P00274000	2019-04-18	P	274.00	7.11
SPY190418C00275000	2019-04-18	C	275.00	4.27
SPY190418P00275000	2019-04-18	P	275.00	7.47
SPY190418C00276000	2019-04-18	C	276.00	3.80
SPY190418P00276000	2019-04-18	P	276.00	8.06
SPY190418C00277000	2019-04-18	C	277.00	3.38
SPY190418P00277000	2019-04-18	P	277.00	8.32
SPY190418C00278000	2019-04-18	C	278.00	3.01
SPY190418P00278000	2019-04-18	P	278.00	9.14
SPY190418C00279000	2019-04-18	C	279.00	2.63
SPY190418P00279000	2019-04-18	P	279.00	9.53
SPY190418C00280000	2019-04-18	C	280.00	2.33
SPY190418P00280000	2019-04-18	P	280.00	10.20
SPY190418C00281000	2019-04-18	C	281.00	2.03
SPY190418P00281000	2019-04-18	P	281.00	10.62
SPY190418C00282000	2019-04-18	C	282.00	1.73
SPY190418C00283000	2019-04-18	C	283.00	1.53
SPY190418C00284000	2019-04-18	C	284.00	1.31
SPY190418P00284000	2019-04-18	P	284.00	12.96

C.2 AMZN Option Chain

Option Name	Expiration Date	Type	Strike	Computed Price
AMZN190215P01555000	2019-02-15	P	1555.00	9.69
AMZN190215C01560000	2019-02-15	C	1560.00	64.17
AMZN190215P01560000	2019-02-15	P	1560.00	10.24
AMZN190215C01565000	2019-02-15	C	1565.00	61.18
AMZN190215P01565000	2019-02-15	P	1565.00	11.18
AMZN190215C01570000	2019-02-15	C	1570.00	56.93
AMZN190215P01570000	2019-02-15	P	1570.00	11.98
AMZN190215C01575000	2019-02-15	C	1575.00	68.06
AMZN190215P01575000	2019-02-15	P	1575.00	12.95
AMZN190215C01580000	2019-02-15	C	1580.00	49.62
AMZN190215P01580000	2019-02-15	P	1580.00	13.86
AMZN190215C01585000	2019-02-15	C	1585.00	50.98
AMZN190215P01585000	2019-02-15	P	1585.00	14.72
AMZN190215C01590000	2019-02-15	C	1590.00	46.77
AMZN190215P01590000	2019-02-15	P	1590.00	15.73
AMZN190215P01595000	2019-02-15	P	1595.00	16.31
AMZN190215C01600000	2019-02-15	C	1600.00	37.04
AMZN190215P01600000	2019-02-15	P	1600.00	17.04
AMZN190215C01605000	2019-02-15	C	1605.00	36.83
AMZN190215P01605000	2019-02-15	P	1605.00	17.66
AMZN190215C01610000	2019-02-15	C	1610.00	34.39
AMZN190215P01610000	2019-02-15	P	1610.00	18.44
AMZN190215C01615000	2019-02-15	C	1615.00	27.65
AMZN190215P01615000	2019-02-15	P	1615.00	18.93
AMZN190215C01620000	2019-02-15	C	1620.00	26.34
AMZN190215P01620000	2019-02-15	P	1620.00	19.67
AMZN190215C01625000	2019-02-15	C	1625.00	23.70
AMZN190215P01625000	2019-02-15	P	1625.00	20.37
AMZN190215C01630000	2019-02-15	C	1630.00	21.23
AMZN190215P01630000	2019-02-15	P	1630.00	21.37
AMZN190215C01635000	2019-02-15	C	1635.00	19.44
AMZN190215P01635000	2019-02-15	P	1635.00	22.48
AMZN190215C01640000	2019-02-15	C	1640.00	17.34
AMZN190215P01640000	2019-02-15	P	1640.00	24.18
AMZN190215C01645000	2019-02-15	C	1645.00	15.65
AMZN190215P01645000	2019-02-15	P	1645.00	25.86
AMZN190215C01650000	2019-02-15	C	1650.00	13.73
AMZN190215P01650000	2019-02-15	P	1650.00	28.13
AMZN190215C01655000	2019-02-15	C	1655.00	12.09
AMZN190215P01655000	2019-02-15	P	1655.00	29.78
AMZN190215C01660000	2019-02-15	C	1660.00	10.84
AMZN190215P01660000	2019-02-15	P	1660.00	32.81
AMZN190215C01665000	2019-02-15	C	1665.00	9.53
AMZN190215P01665000	2019-02-15	P	1665.00	36.76

Option Name	**Expiration Date**	**Type**	**Strike**	**Computed Price**
AMZN190215C01670000	2019-02-15	C	1670.00	8.45
AMZN190215P01670000	2019-02-15	P	1670.00	38.89
AMZN190215C01675000	2019-02-15	C	1675.00	7.55
AMZN190215P01675000	2019-02-15	P	1675.00	41.53
AMZN190215C01680000	2019-02-15	C	1680.00	6.48
AMZN190215P01680000	2019-02-15	P	1680.00	44.89
AMZN190215C01685000	2019-02-15	C	1685.00	5.79
AMZN190215P01685000	2019-02-15	P	1685.00	45.79
AMZN190215C01690000	2019-02-15	C	1690.00	5.03
AMZN190215P01690000	2019-02-15	P	1690.00	45.78
AMZN190215C01695000	2019-02-15	C	1695.00	4.41
AMZN190215P01695000	2019-02-15	P	1695.00	43.18
AMZN190215C01700000	2019-02-15	C	1700.00	3.81
AMZN190215P01700000	2019-02-15	P	1700.00	60.91
AMZN190215C01705000	2019-02-15	C	1705.00	3.33
AMZN190215P01705000	2019-02-15	P	1705.00	64.03
AMZN190215C01710000	2019-02-15	C	1710.00	2.94
AMZN190215P01710000	2019-02-15	P	1710.00	71.92
AMZN190215C01715000	2019-02-15	C	1715.00	2.61
AMZN190215P01715000	2019-02-15	P	1715.00	71.15
AMZN190215C01720000	2019-02-15	C	1720.00	2.22
AMZN190215P01720000	2019-02-15	P	1720.00	78.29
AMZN190215C01725000	2019-02-15	C	1725.00	1.95
AMZN190215P01725000	2019-02-15	P	1725.00	81.16
AMZN190315C01555000	2019-03-15	C	1555.00	93.24
AMZN190315P01555000	2019-03-15	P	1555.00	28.75
AMZN190315C01560000	2019-03-15	C	1560.00	95.20
AMZN190315P01560000	2019-03-15	P	1560.00	30.07
AMZN190315P01565000	2019-03-15	P	1565.00	31.22
AMZN190315C01570000	2019-03-15	C	1570.00	81.49
AMZN190315P01570000	2019-03-15	P	1570.00	33.01
AMZN190315P01575000	2019-03-15	P	1575.00	33.33
AMZN190315P01580000	2019-03-15	P	1580.00	34.72
AMZN190315C01585000	2019-03-15	C	1585.00	75.35
AMZN190315P01585000	2019-03-15	P	1585.00	35.16
AMZN190315C01590000	2019-03-15	C	1590.00	73.14
AMZN190315P01590000	2019-03-15	P	1590.00	37.43
AMZN190315C01595000	2019-03-15	C	1595.00	68.33
AMZN190315P01595000	2019-03-15	P	1595.00	38.95
AMZN190315C01600000	2019-03-15	C	1600.00	66.99
AMZN190315P01600000	2019-03-15	P	1600.00	39.75
AMZN190315C01605000	2019-03-15	C	1605.00	64.21
AMZN190315P01605000	2019-03-15	P	1605.00	38.23
AMZN190315C01610000	2019-03-15	C	1610.00	60.91
AMZN190315P01610000	2019-03-15	P	1610.00	36.06
AMZN190315C01615000	2019-03-15	C	1615.00	70.77

Option Name	**Expiration Date**	**Type**	**Strike**	**Computed Price**
AMZN190315P01615000	2019-03-15	P	1615.00	42.45
AMZN190315C01620000	2019-03-15	C	1620.00	58.05
AMZN190315P01620000	2019-03-15	P	1620.00	45.36
AMZN190315C01625000	2019-03-15	C	1625.00	53.33
AMZN190315P01625000	2019-03-15	P	1625.00	46.41
AMZN190315C01630000	2019-03-15	C	1630.00	52.00
AMZN190315P01630000	2019-03-15	P	1630.00	48.47
AMZN190315C01635000	2019-03-15	C	1635.00	49.30
AMZN190315P01635000	2019-03-15	P	1635.00	50.82
AMZN190315C01640000	2019-03-15	C	1640.00	47.11
AMZN190315P01640000	2019-03-15	P	1640.00	52.20
AMZN190315C01645000	2019-03-15	C	1645.00	44.43
AMZN190315P01645000	2019-03-15	P	1645.00	54.36
AMZN190315C01650000	2019-03-15	C	1650.00	41.90
AMZN190315P01650000	2019-03-15	P	1650.00	56.36
AMZN190315C01655000	2019-03-15	C	1655.00	40.03
AMZN190315P01655000	2019-03-15	P	1655.00	58.48
AMZN190315C01660000	2019-03-15	C	1660.00	37.97
AMZN190315P01660000	2019-03-15	P	1660.00	60.22
AMZN190315C01665000	2019-03-15	C	1665.00	36.59
AMZN190315P01665000	2019-03-15	P	1665.00	62.19
AMZN190315C01670000	2019-03-15	C	1670.00	34.81
AMZN190315P01670000	2019-03-15	P	1670.00	64.61
AMZN190315C01675000	2019-03-15	C	1675.00	32.42
AMZN190315P01675000	2019-03-15	P	1675.00	65.88
AMZN190315C01680000	2019-03-15	C	1680.00	31.26
AMZN190315P01680000	2019-03-15	P	1680.00	71.53
AMZN190315C01685000	2019-03-15	C	1685.00	30.65
AMZN190315P01685000	2019-03-15	P	1685.00	71.21
AMZN190315C01690000	2019-03-15	C	1690.00	28.18
AMZN190315P01690000	2019-03-15	P	1690.00	72.08
AMZN190315C01695000	2019-03-15	C	1695.00	26.57
AMZN190315P01695000	2019-03-15	P	1695.00	79.90
AMZN190315C01700000	2019-03-15	C	1700.00	24.23
AMZN190315P01700000	2019-03-15	P	1700.00	82.28
AMZN190315C01705000	2019-03-15	C	1705.00	23.17
AMZN190315P01705000	2019-03-15	P	1705.00	88.95
AMZN190315C01710000	2019-03-15	C	1710.00	21.84
AMZN190315C01715000	2019-03-15	C	1715.00	20.04
AMZN190315P01715000	2019-03-15	P	1715.00	95.14
AMZN190315C01720000	2019-03-15	C	1720.00	19.33
AMZN190315C01725000	2019-03-15	C	1725.00	18.15
AMZN190315P01725000	2019-03-15	P	1725.00	102.28
AMZN190418C01555000	2019-04-18	C	1555.00	117.40
AMZN190418P01555000	2019-04-18	P	1555.00	47.23
AMZN190418P01560000	2019-04-18	P	1560.00	49.11

Option Name	**Expiration Date**	**Type**	**Strike**	**Computed Price**
AMZN190418C01565000	2019-04-18	C	1565.00	111.11
AMZN190418P01565000	2019-04-18	P	1565.00	48.22
AMZN190418C01575000	2019-04-18	C	1575.00	105.57
AMZN190418P01575000	2019-04-18	P	1575.00	52.37
AMZN190418C01585000	2019-04-18	C	1585.00	97.35
AMZN190418P01595000	2019-04-18	P	1595.00	57.93
AMZN190418C01600000	2019-04-18	C	1600.00	92.09
AMZN190418P01600000	2019-04-18	P	1600.00	58.83
AMZN190418C01605000	2019-04-18	C	1605.00	106.20
AMZN190418P01605000	2019-04-18	P	1605.00	62.31
AMZN190418P01615000	2019-04-18	P	1615.00	65.65
AMZN190418C01620000	2019-04-18	C	1620.00	79.67
AMZN190418P01620000	2019-04-18	P	1620.00	66.56
AMZN190418C01625000	2019-04-18	C	1625.00	78.46
AMZN190418P01625000	2019-04-18	P	1625.00	66.70
AMZN190418C01635000	2019-04-18	C	1635.00	73.60
AMZN190418P01635000	2019-04-18	P	1635.00	72.03
AMZN190418C01640000	2019-04-18	C	1640.00	72.62
AMZN190418P01640000	2019-04-18	P	1640.00	72.50
AMZN190418C01645000	2019-04-18	C	1645.00	69.39
AMZN190418P01645000	2019-04-18	P	1645.00	76.72
AMZN190418C01655000	2019-04-18	C	1655.00	65.24
AMZN190418P01655000	2019-04-18	P	1655.00	79.95
AMZN190418C01660000	2019-04-18	C	1660.00	63.19
AMZN190418P01660000	2019-04-18	P	1660.00	80.01
AMZN190418C01665000	2019-04-18	C	1665.00	59.61
AMZN190418P01665000	2019-04-18	P	1665.00	80.31
AMZN190418C01675000	2019-04-18	C	1675.00	56.39
AMZN190418P01675000	2019-04-18	P	1675.00	84.13
AMZN190418C01680000	2019-04-18	C	1680.00	55.60
AMZN190418P01680000	2019-04-18	P	1680.00	93.78
AMZN190418C01685000	2019-04-18	C	1685.00	53.47
AMZN190418P01685000	2019-04-18	P	1685.00	84.22
AMZN190418C01700000	2019-04-18	C	1700.00	46.62
AMZN190418P01700000	2019-04-18	P	1700.00	103.23
AMZN190418C01720000	2019-04-18	C	1720.00	41.46
AMZN190418P01720000	2019-04-18	P	1720.00	118.23
AMZN190418C01725000	2019-04-18	C	1725.00	37.55
AMZN190418P01725000	2019-04-18	P	1725.00	108.76

D Solution Source Code

D.1 Question 1 Implementation

D.1.1 Bloomberg Terminal Data Download

```
library("Rblpapi")

# Connect to Bloomberg Terminal backend service
blpConnect(host = "localhost", port = 8194)


#----------------------------------
# Data Download Functionality
#----------------------------------


getPrice <- function(security, startTime, endTime, timeZone) {
  # Downloads and returns the closing price of a given security
  # for each minute in the trading day.
  #
  # Args:
  #   security: Name of the security to be downloaded.
  #   startTime: Datetime object with the start time.
  #   endTime: Datetime object with the end time.
  #   timeZone: Time zone of the target start and end times.
  #
  # Returns:
  #   DataFrame with the closing price for each minute in the
  #   trading day.

  # Getting price data
  data <- getBars(security = security, barInterval = 1,
                  startTime = startTime, endTime = endTime,
                  tz = timeZone)

  # Isolate time and closing price
  data <- data[c("times", "close")]

  # Rename columns
  colnames(data) <- c("Dates", "Close")

  # Return
  data
}


createOptionName <- function(security, dates, prices, type, suffix) {
  # Creates the Bloomberg-standard option name, given a security, date, price,
  # option type and suffix.
  #
  # Args:
  #   security: Name of the security to be included in the option price.
  #   dates: Dates to be included in option name.
  #   prices: Prices to be included in the option name.
  #   type: Type of the option ("C" or "P").
  #   suffix: Suffix for option name (typically "Index" or "Equity").
  #
  # Returns:
  #   Vector of Bloomberg-compatible option names.
```

```

  # Empty vector to store names
  names <- c()

  # Iterate over each date and price
  for (date in dates) {
    for (price in prices) {
      # Building option name
      name <- paste(security, date, paste(type, price, sep = ""), suffix)

      # Appending to list of option names
      names <- c(names, name)
    }
  }

  # Returning names
  names
}


#----------------------------------
# DATA1
#----------------------------------


# Define Start and End times (DATA1)
data1Start <- ISOdatetime(year = 2019, month = 2, day = 6,
                          hour = 9, min = 30, sec = 0)
data1End <- ISOdatetime(year = 2019, month = 2, day = 6,
                        hour = 16, min = 0, sec = 0)

# Defining time zone
timeZone = "America/New_York"

# Defining top-level securities
securities <- c("SPY US Equity", "AMZN US Equity", "VIX Index")

# Getting prices for each of the top-level securities
for (security in securities) {
  data <- getPrice(security, data1Start, data1End, timeZone)
  write.csv(data, file = paste(security, "DATA1", "csv", sep = "."),
            row.names = FALSE)
}

# Expiration dates
expDates <- c("2/15/19", "3/15/19", "4/18/19")

# Defining put and call prices for SPY and AMZN options
# Grabbing prices for 15% +/- current price

# Defining bounds
lowerBoundPct <- 0.85
upperBoundPct <- 1.15

# Current SPY price
spyCurrent <- 270
spyPrices <- c(floor(
  lowerBoundPct * spyCurrent):ceiling(upperBoundPct * spyCurrent))

# Function to round to the nearest 'base', given an input 'x'. This is to
# compute strike prices for AMZN options, which are in intervals of 5.
```

```r
# Source: http://r.789695.n4.nabble.com/Rounding-to-the-nearest-5-td863189.html
mround <- function(x, base) {
  base * round(x / base)
}

# Current AMZN price (need to do this manually because of option strikes)
amznCurrent <- 1640
roundingLevel <- 5
amznPrices <- seq(mround(amznCurrent * lowerBoundPct, roundingLevel),
                  mround(amznCurrent * upperBoundPct, roundingLevel), by=5)

# Creating option names for SPY and AMZN
spyOptions <- createOptionName("SPY", expDates, spyPrices, "C", "Equity")
spyOptions <- c(spyOptions, createOptionName("SPY", expDates, spyPrices,
                                             "P", "Equity"))

amznOptions <- createOptionName("AMZN", expDates, amznPrices, "C", "Equity")
amznOptions <- c(amznOptions, createOptionName("AMZN", expDates, amznPrices,
                                               "P", "Equity"))

# Getting prices for each of the options
for (option in c(amznOptions, spyOptions)) {
  data <- getPrice(option, data1Start, data1End, timeZone)
  # Only print to file if option exists
  if (all(dim(data) > 0)) {
    optionFileName <- gsub("/", "-", option) # Need to do this for Windows
    write.csv(data, file = paste(optionFileName, "csv", sep = "."),
              row.names = FALSE)
  }
}


#----------------------------------
# DATA2
#----------------------------------

# Define Start and End times (DATA2)
data2Start <- ISOdatetime(year = 2019, month = 2, day = 7,
                          hour = 9, min = 30, sec = 0)
data2End <- ISOdatetime(year = 2019, month = 2, day = 7,
                        hour = 16, min = 0, sec = 0)

# Getting prices for each of the top-level securities
for (security in securities) {
  data <- getPrice(security, data2Start, data2End, timeZone)
  write.csv(data, file = paste(security, "DATA2", "csv", sep = "."),
            row.names = FALSE)
}
```

question_solutions/question_1.R

D.2 Question 2 Implementation

D.2.1 Optimization Method Convergence Comparison

```
from context import fe621

from datetime import datetime
import numpy as np
import pandas as pd


# Defining dates
data1_date = '2019-02-06'

# Loading DATA1
spy_data1 = fe621.util.loadData(folder_path='Homework 1/data/DATA1/SPY',
                                date=data1_date)
amzn_data1 = fe621.util.loadData(folder_path='Homework 1/data/DATA1/AMZN',
                                 date=data1_date)

# Loading Risk-free rate (effective federal funds rate)
rf = pd.read_csv('Homework 1/data/ffr.csv')

# Setting comparison tolerance level
tol = 1e-3

# Number of input options
input_count = len(spy_data1.columns) - 1

def compareConvergenceTime():
    """Function to compare the convergence times of the Newton and Bisection
    method solvers, on the SPY option chain.
    """

    # Newton's Method
    start = datetime.now().timestamp()
    spy_vol_newton = fe621.util.computeAvgImpliedVolNewton(
        data=spy_data1,
        name='SPY',
        rf=rf[data1_date][0],
        current_date=data1_date,
        tol=tol
    )
    end = datetime.now().timestamp()

    # Computing time  and number of options for Newton
    newton_time = end - start
    newton_count = spy_vol_newton.count(axis=0)[0]

    # Bisection Method
    start = datetime.now().timestamp()
    spy_vol_bisection = fe621.util.computeAvgImpliedVolNewton(
        data=spy_data1,
        name='SPY',
        rf=rf[data1_date][0],
        current_date=data1_date,
        tol=tol
    )
    end = datetime.now().timestamp()

    # Computing time and number of options for Bisection
```

```
    bisection_time = end - start
    bisection_count = spy_vol_bisection.count(axis=0)[0]

    # Building DataFrame, and saving to CSV
    convergence_table = pd.DataFrame({
        'Number of Input Options': [input_count, input_count],
        'Number of Output Options': [newton_count, bisection_count],
        'Number of Dropped Options': [input_count - newton_count,
                                      input_count - bisection_count],
        'Time Elapsed for Computation (s)': [newton_time, bisection_time],
        'Average Time per Option (s)': [newton_time / input_count,
                                        bisection_time / input_count]
    })
    convergence_table = convergence_table.T  # Transposing so cols are methods
    convergence_table.columns = ['Newton Method', 'Bisection Method']
    convergence_table.to_csv('Homework 1/bin/imp_vol_convergence.csv')

if __name__ == '__main__':
    compareConvergenceTime()
```

question_solutions/question_2_convergence.py

D.2.2 Implied Volatility Computation

```
from context import fe621

import numpy as np
import pandas as pd


# Defining dates
data1_date = '2019-02-06'
data2_date = '2019-02-07'

# Loading DATA1
spy_data1 = fe621.util.loadData(folder_path='Homework 1/data/DATA1/SPY',
                                date=data1_date)
amzn_data1 = fe621.util.loadData(folder_path='Homework 1/data/DATA1/AMZN',
                                 date=data1_date)
vix_data1 = fe621.util.loadData(folder_path='Homework 1/data/DATA1/VIX',
                                date=data1_date)

# Loading DATA2
spy_data2 = fe621.util.loadData(folder_path='Homework 1/data/DATA2/SPY',
                                date=data2_date)
amzn_data2 = fe621.util.loadData(folder_path='Homework 1/data/DATA2/AMZN',
                                 date=data2_date)
vix_data2 = fe621.util.loadData(folder_path='Homework 1/data/DATA2/VIX',
                                date=data2_date)

# Loading Risk-free rate (effective federal funds rate)
rf = pd.read_csv('Homework 1/data/ffr.csv')

# Tolerance level for optimization
tol = 1e-6

def computeImpVolatilities():
    """Function to compute the implied volatilities for the SPY and AMZN option
    chains, for all maturities. Computed implied volatilities are output to
```

```
    CSV files.
    """

    # SP 500
    spy_data1_vol = fe621.util.computeAvgImpliedVolBisection(
                                                data=spy_data1,
                                                name='SPY',
                                                rf=rf[data1_date][0],
                                                current_date=data1_date,
                                                tol=tol)
    # Saving to CSV
    spy_data1_vol.to_csv('Homework 1/bin/spy_data1_vol.csv', index=False)

    # AMZN
    amzn_data1_vol = fe621.util.computeAvgImpliedVolBisection(
                                                data=amzn_data1,
                                                name='AMZN',
                                                rf=rf[data1_date][0],
                                                current_date=data1_date,
                                                tol=tol)
    # Saving to CSV
    amzn_data1_vol.to_csv('Homework 1/bin/amzn_data1_vol.csv', index=False)


if __name__ == "__main__":
    # Part 1 - Implied Volatility Computation
    computeImpVolatilities()
```

question_solutions/question_2_imp_vol.py

D.2.3 Implied Volatility Analysis

```
from context import fe621

import numpy as np
import pandas as pd


# Defining date
data1_date = '2019-02-06'

# Loading computed average daily implied volatilities
spy_imp_vol = pd.read_csv('Homework 1/bin/spy_data1_vol.csv',
                          index_col=False, header=0)
amzn_imp_vol = pd.read_csv('Homework 1/bin/amzn_data1_vol.csv',
                           index_col=False, header=0)

# Loading price information (for daily close)
spy_prices = pd.read_csv('Homework 1/data/DATA1/SPY/SPY.csv',
                         index_col=False, header=0)
amzn_prices = pd.read_csv('Homework 1/data/DATA1/AMZN/AMZN.csv',
                          index_col=False, header=0)

# Isolating daily close prices
spy_close = spy_prices.iloc[-1][1]
amzn_close = amzn_prices.iloc[-1][1]
print(amzn_close)
# Defining 'money-ness' ratio
# NOTE: This needs to be changed when more data is available
```

```
lower_bound_pct = 0.975
upper_bound_pct = 1.025

def analyzeVolAvg(data: pd.DataFrame, underlying_close: float) -> list:
    """Function to compute the average daily implied volatility of in-the-money
    and out-of-the-money options.

    Arguments:
        data {pd.DataFrame} -- Input data containing implied volatilities.
        underlying_close {float} -- Daily closing price of the underlying asset.

    Returns:
        list -- List containing [itm_avg_vol, otm_avg_vol].
    """

    # Computing upper and lower bounds for 'moneyness'
    lower_bound = underlying_close * lower_bound_pct
    upper_bound = underlying_close * upper_bound_pct

    # Isolating in-the-money and out-of-the-money options
    out_money_options = data[(data['strike'] < lower_bound) | \
        (data['strike'] > upper_bound)]
    in_money_options = data[(data['strike'] >= lower_bound) | \
        (data['strike'] <= upper_bound)]

    # Computing average daily implied volatility of in and out the money options
    otm_vol_avg = np.mean(out_money_options['implied_vol'])
    itm_vol_avg = np.mean(in_money_options['implied_vol'])

    return [itm_vol_avg, otm_vol_avg]

if __name__ == '__main__':
    # Computing average daily implied volatility for itm and otm options
    spy_avg = analyzeVolAvg(data=spy_imp_vol, underlying_close=spy_close)
    amzn_avg = analyzeVolAvg(data=amzn_imp_vol, underlying_close=amzn_close)

    # Building output DataFrame
    output = pd.DataFrame({
        'SPY': spy_avg,
        'AMZN': amzn_avg
    })
    # Renaming index
    output.index = ['In-the-money Options Average Daily Implied Vol',
                    'Out-of-the-money Options Average Daily Implied Vol']
    # Write to CSV
    output.to_csv('Homework 1/bin/itm_otm_vol_analysis.csv')
```

question_solutions/question_2_vol_analysis.py

D.2.4 Volatility Plots

```
from context import fe621

from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd


```

```python
# Loading implied volatility data from CSV files
spy_imp_vol = pd.read_csv('Homework 1/bin/spy_data1_vol.csv',
                          index_col=False, header=0)
amzn_imp_vol = pd.read_csv('Homework 1/bin/amzn_data1_vol.csv',
                           index_col=False, header=0)

# Defining date of DATA1
data1_date = '2019-02-06'


def plot2DVolSmile(data: pd.DataFrame, name: str, save_loc: str):
    """Function to plot a 2D Volatility Smile for a given option chain.

    Arguments:
        data {pd.DataFrame} -- Input data containing implied volatilities.
        name {str} -- Name of the underlying asset.
        save_loc {str} -- Location (folder) to save the output image.
    """

    # Iterating through types of options for 2 separate put/call imp vol plots
    for option_type_group in data.groupby('type'):
        # Isolating current option type
        option_type = option_type_group[0]

        # Iterating through expiration dates for individual lines for each
        for exp_date_group in option_type_group[1].groupby('expiration'):
            # Isolating current expiration date
            exp_date = exp_date_group[0]

            # Sorting data to be ascending on 'strike'
            plt_data = exp_date_group[1].sort_values(by='strike')

            # Plotting strike vs implied vol
            plt.plot(plt_data['strike'], plt_data['implied_vol'],
                     label=('Maturity on ' + exp_date))

        # Formatting plot
        #================

        ax = plt.gca()  # Get current axes

        # Setting y ticks and label
        ax.set_yticklabels(['{:,.1%}'.format(i) for i in ax.get_yticks()])
        ax.set_ylabel('Implied Volatility')
        # Setting x ticks and label
        ax.set_xticklabels(['$%i' % i for i in ax.get_xticks()])
        ax.set_xlabel('Strike Price')

        # Setting legend and setting plot dimensions to tight
        plt.legend()
        plt.tight_layout()

        # Saving to file
        full_option_type = 'Call' if (option_type == 'C') else 'Put'
        fname = '_'.join([name, full_option_type, '2DVolSmile.png'])
        plt.savefig(fname=(save_loc + '/' + fname))

        # Closing plot for next one
        plt.close()


```

```python
def plot3DVolatilitySurface(data: pd.DataFrame, name: str, save_loc: str):
    """Fuction to plot a 3D Volatility Surface for a given option chain.

    Arguments:
        data {pd.DataFrame} -- Input data containing implied volatilities
        name {str} -- Name of the underlying asset.
        save_loc {str} -- Location (folder) to save the output image.
    """

    # Iterating through types of options for 2 separate put/call imp vol plots
    for option_type_group in data.groupby('type'):
        # Isolating current option type
        option_type = option_type_group[0]

        # Isolating plot data
        plot_data = option_type_group[1]

        # Creating new column with time to maturity information for each option
        ttm = plot_data.apply(lambda row: fe621.util.getTTM(
                                name=row.loc['name'],
                                current_date=data1_date),
                              axis=1)
        # Converting TTM to days
        ttm_days = ttm * 365

        # Isolating data for each axis
        x = np.array(ttm_days)
        y = np.array(plot_data['strike'])
        z = np.array(plot_data['implied_vol'])

        # Plotting surface
        fig = plt.figure()
        ax = fig.gca(projection='3d')
        ax.plot_trisurf(x, y, z, cmap='plasma')

        # Formatting plot
        #----------------

        # Setting x label
        ax.set_xlabel('TTM (Days)')
        # Setting y label
        ax.set_ylabel('Strike Price ($)')
        # Setting z label
        ax.set_zlabel('Implied Volatility')

        # Modifying z ticks to be percentages
        ax.set_zticklabels(['{:,.0%}'.format(i) for i in ax.get_zticks()])

        # Setting plot dimensions to tight
        plt.tight_layout()

        # Saving to file
        full_option_type = 'Call' if (option_type == 'C') else 'Put'
        fname = '_'.join([name, full_option_type, '3DVolSurface.png'])
        plt.savefig(fname=(save_loc + '/' + fname))

        # Closing plot for next one
        plt.close()

if __name__ == '__main__':
    # Plotting 2D Volatility Smile for AMZN and SPY option chains
```

```
    plot2DVolSmile(data=amzn_imp_vol, name='AMZN',
                   save_loc='Homework 1/bin/vol_smile/')
    plot2DVolSmile(data=spy_imp_vol, name='SPY',
                   save_loc='Homework 1/bin/vol_smile/')

    # Plotting 3D Volatility Surface for AMZN and SPY option chains
    plot3DVolatilitySurface(data=spy_imp_vol, name='SPY',
                            save_loc='Homework 1/bin/vol_surface/')
    plot3DVolatilitySurface(data=amzn_imp_vol, name='AMZN',
                            save_loc='Homework 1/bin/vol_surface/')
```

question_solutions/question_2_vol_plots.py

D.2.5 The Greeks

```
from context import fe621

import pandas as pd


# Defining date
data1_date = '2019-02-06'

# Loading computed average daily implied volatilities
spy_options = pd.read_csv('Homework 1/bin/spy_data1_vol.csv',
                          index_col=False, header=0)
amzn_options = pd.read_csv('Homework 1/bin/amzn_data1_vol.csv',
                           index_col=False, header=0)

# Isolating call options
spy_call_options = spy_options[spy_options['type'] == 'C']
amzn_call_options = amzn_options[amzn_options['type'] == 'C']

# Loading price information (for daily close)
spy_prices = pd.read_csv('Homework 1/data/DATA1/SPY/SPY.csv',
                         index_col=False, header=0)
amzn_prices = pd.read_csv('Homework 1/data/DATA1/AMZN/AMZN.csv',
                          index_col=False, header=0)

# Isolating daily close prices
spy_close = spy_prices.iloc[-1][1]
amzn_close = amzn_prices.iloc[-1][1]

# Loading Risk-free rate (effective federal funds rate) for DATA1
rf = pd.read_csv('Homework 1/data/ffr.csv')[data1_date][0]

# Step size for computation
h = 1e-5

def computeAnalyticalAndEstimatedGreeks(data: pd.DataFrame, close: float) \
    -> pd.DataFrame:
    """Function to compute the Greeks for a given set of call options. It does
    this both using the analytical formulas and by numerical approximation. It
    uses the central finite difference method. It computes the Delta
    (first derivative w.r.t. underlying price), Gamma (second derivative w.r.t.
    underlying price), and the Vega (first derivative w.r.t. volatility).

    Arguments:
        data {pd.DataFrame} -- Option DataFrame with implied volatilities.
```

```
        close {float} -- Closing price of the underlying asset.

    Returns:
        pd.DataFrame -- Formatted DataFrame with computed results.
    """

    # Creating DataFrame for results
    results = pd.DataFrame()

    for _, option_data in data.iterrows():
        # Isolating required arguments
        volatility = option_data['implied_vol']
        ttm = fe621.util.getTTM(name=option_data['name'],
                                current_date=data1_date)
        strike = fe621.util.getStrikePrice(name=option_data['name'])

        # Computing analytical (prefix: a_*) and estimated (prefix: e_*) greeks

        # Delta (first derivative w.r.t. underlying price, S)
        a_delta = fe621.black_scholes.greeks.callDelta(current=close,
                                                       volatility=volatility,
                                                       ttm=ttm,
                                                       strike=strike,
                                                       rf=rf)
        e_delta = fe621.numerical_differentiation.firstDerivative(
            f=lambda x: fe621.black_scholes.call(
                x, volatility, ttm, strike, rf),
            x=close,
            h=h
        )

        # Gamma (second derivative w.r.t. underlying price, S)
        a_gamma = fe621.black_scholes.greeks.callGamma(current=close,
                                                       volatility=volatility,
                                                       ttm=ttm,
                                                       strike=strike,
                                                       rf=rf)
        e_gamma = fe621.numerical_differentiation.secondDerivative(
            f=lambda x: fe621.black_scholes.call(
                x, volatility, ttm, strike, rf),
            x=close,
            h=h
        )

        # Vega (first derivative w.r.t. volatility, $\sigma$)
        a_vega = fe621.black_scholes.greeks.vega(current=close,
                                                 volatility=volatility,
                                                 ttm=ttm,
                                                 strike=strike,
                                                 rf=rf)
        e_vega = fe621.numerical_differentiation.firstDerivative(
            f=lambda x: fe621.black_scholes.greeks.vega(
                close, x, ttm, strike, rf),
            x=volatility,
            h=h
        )

        # Adding to output DataFrame
        results = results.append(pd.Series([option_data['name'],
                                            a_delta, a_gamma,
                                            a_vega, e_delta,
```

```
                                             e_gamma, e_vega]),
                                  ignore_index=True)

    # Setting column names
    results.columns = ['name',
                       'delta_analytical', 'gamma_analytical',
                       'vega_analytical', 'delta_estimated',
                       'gamma_estimated', 'vega_estimated']

    return results


if __name__ == '__main__':
    # Computing Greeks for SPY
    spy_greeks = computeAnalyticalAndEstimatedGreeks(data=spy_call_options,
                                                     close=spy_close)
    # Saving to CSV
    spy_greeks.to_csv('Homework 1/bin/greeks/spy_greeks.csv', index=False,
                      float_format='%.7f')

    # Computing Greeks for AMZN
    amzn_greeks = computeAnalyticalAndEstimatedGreeks(data=amzn_call_options,
                                                      close=amzn_close)
    # Saving to CSV
    amzn_greeks.to_csv('Homework 1/bin/greeks/amzn_greeks.csv', index=False,
                       float_format='%.7f')
```

question_solutions/question_2_greeks.py

D.2.6 DATA2 Price Computation

```
from context import fe621

import pandas as pd


# Defining date
data2_date = '2019-02-06'

# Loading computed average daily implied volatilities
spy_options = pd.read_csv('Homework 1/bin/spy_data1_vol.csv',
                          index_col=False, header=0)
amzn_options = pd.read_csv('Homework 1/bin/amzn_data1_vol.csv',
                           index_col=False, header=0)

# Loading daily closing price information (for daily close)
spy_data2_close = pd.read_csv('Homework 1/data/DATA2/SPY/SPY.csv',
                              index_col=False, header=0).iloc[-1][1]
amzn_data2_close = pd.read_csv('Homework 1/data/DATA2/AMZN/AMZN.csv',
                               index_col=False, header=0).iloc[-1][1]

# Getting risk-free date (effective federal funds rate) for DATA2
rf = pd.read_csv('Homework 1/data/ffr.csv')[data2_date][0]


def computeData2Prices(data: pd.DataFrame, close: float) -> pd.DataFrame:
    """Function to compute the prices for a given set of options with implied
    volatilities, and a closing price.

```

```
    Arguments:
        data {pd.DataFrame} -- Input option data with implied volatility.
        close {float} -- Closing price of underlying asset.

    Returns:
        pd.DataFrame -- Formatted results DataFrame with DATA2 prices.
    """

    # Creating Series for results
    computed_prices = pd.Series()

    for idx, option_data in data.iterrows():
        # Isolating required arguments
        volatility = option_data['implied_vol']
        ttm = fe621.util.getTTM(name=option_data['name'],
                                current_date=data2_date)
        strike = fe621.util.getStrikePrice(name=option_data['name'])

        # Deciding price computation function based on type
        if option_data['type'] == 'C':
            computePrice = fe621.black_scholes.call
        else:
            computePrice = fe621.black_scholes.put

        # Computing price
        price = computePrice(current=close, volatility=volatility, ttm=ttm,
                             strike=strike, rf=rf)

        # Adding to output Series
        computed_prices.at[idx] = price

    # Copying `data` DataFrame for output
    results = data.copy(deep=True)

    # Dropping implied volatility column
    results.drop(labels=['implied_vol'], axis=1, inplace=True)

    # Adding computed prices
    results = results.assign(computed_prices=computed_prices)

    return results


if __name__ == '__main__':
    # Computing DATA2 prices for SPY
    spy_data2 = computeData2Prices(data=spy_options, close=spy_data2_close)

    # Saving to CSV
    spy_data2.to_csv('Homework 1/bin/data2/spy_prices.csv', index=False,
                     float_format='%.2f')

    # Computing DATA2 prices for AMZN
    amzn_data2 = computeData2Prices(data=amzn_options, close=amzn_data2_close)

    # Saving to CSV
    amzn_data2.to_csv('Homework 1/bin/data2/amzn_prices.csv', index=False,
                      float_format='%.2f')
```

question_solutions/question_2_data2.py

D.3 Question 3 Implementation

D.3.1 Truncation Error Analysis

```python
from context import fe621

import numpy as np
import pandas as pd


def truncationErrorAnalysis():
    """Function to analyze the truncation error of the Trapezoidal and Simpson's
    quadature rules.
    """

    # Objective function
    def f(x: float) -> float:
        return np.where(x == 0.0, 1.0, np.sin(x) / x)

    # Setting values for N
    N = np.power(10, np.arange(3, 8))

    # Setting values for a
    a = np.power(10, np.arange(2, 7))

    trapezoidal_vals = np.ndarray((N.size, a.size))
    simpsons_vals = np.ndarray((N.size, a.size))

    # Building function approximation table, varying N and A
    for i in range(0, N.size):
        for j in range(0, a.size):
            # Trapezoidal rule approximation
            trapezoidal_vals[i, j] = fe621.numerical_integration \
                .trapezoidalRule(f=f, N=N[i], start=-a[j], stop=a[j])
            # Simpsons rule trunc approximation
            simpsons_vals[i, j] = fe621.numerical_integration \
                .simpsonsRule(f=f, N=N[i], start=-a[j], stop=a[j])

    # Computing the absolute difference from Pi (i.e. trunc error)
    # and casting to DataFrame
    trapezoidal_df = pd.DataFrame(np.abs(trapezoidal_vals - np.pi))
    simpsons_df = pd.DataFrame(np.abs(simpsons_vals - np.pi))

    # Setting row and column names
    trapezoidal_df.columns = ['N = ' + str(i) for i in N]
    trapezoidal_df.index = ['a = ' + str(i) for i in a]
    simpsons_df.columns = ['N = ' + str(i) for i in N]
    simpsons_df.index = ['a = ' + str(i) for i in a]

    # Saving to CSV
    trapezoidal_df.to_csv(
        'Homework 1/bin/numerical_integration/trapezoidal_trunc_error.csv',
        header=True, index=True, float_format='%.8e'
    )
    simpsons_df.to_csv(
        'Homework 1/bin/numerical_integration/simpsons_trunc_error.csv',
        header=True, index=True, float_format='%.8e'
    )


if __name__ == '__main__':
```

```python
    # Part 2 - Truncation Error Analysis
    truncationErrorAnalysis()
```

question_solutions/question_3_trunc_error.py

D.3.2 Convergence Segment Analysis

```python
from context import fe621

import numpy as np
import pandas as pd


def convergenceSegmentLimit():
    """Function to compute the number of segments required for convergence of
    various quadrature methods.
    """

    # Objective function
    def f(x: float) -> float:
        return np.where(x == 0.0, 1.0, np.sin(x) / x)

    # Setting target tolerance level for termination
    epsilon = 1e-3

    # Using Trapezoidal rule
    trapezoidal_result = fe621.numerical_integration.convergenceApproximation(
        f=f,
        rule=fe621.numerical_integration.trapezoidalRule,
        epsilon=epsilon
    )

    # Using Simpson's rule
    simpsons_result = fe621.numerical_integration.convergenceApproximation(
        f=f,
        rule=fe621.numerical_integration.simpsonsRule,
        epsilon=epsilon
    )

    # Building DataFrame of results for output
    results = pd.DataFrame(np.abs(np.array([trapezoidal_result,
                                            simpsons_result])))

    # Setting row and column names
    results.columns = ['Estimated Area', 'Segments']
    results.index = ['Trapezoidal Rule', 'Simpson\'s Rule']

    # Saving to CSV
    results.to_csv('Homework 1/bin/numerical_integration/convergence.csv',
                   header=True, index=True, float_format='%.8e')


if __name__ == '__main__':
    # Part 3 - Convergence Analysis
    convergenceSegmentLimit()
```

question_solutions/question_3_convergence.py

D.3.3 Arbitrary Function Convergence Segment Analysis

```python
from context import fe621

import numpy as np
import pandas as pd


def arbitraryFunctionSegmentAnalysis():
    """Function to analyze number of segments required for an arbitrary function
    to converge under the Trapezoidal and Simpson's quadrature rules.
    """

    # Defining objective function
    def g(x: float) -> float:
        return 1 + np.exp(-1 * np.power(x, 2)) * np.cos(8 * np.power(x, 2/3))

    # Setting target tolerance level for termination
    epsilon = 1e-4

    # Setting start and stop limits
    start = 0
    stop = 2

    # Trapezoidal rule
    trapezoidal_result = fe621.numerical_integration.convergenceApproximation(
        f=g,
        rule=fe621.numerical_integration.trapezoidalRule,
        start=start,
        stop=stop,
        epsilon=epsilon
    )

    # Simpson's rule
    simpsons_result = fe621.numerical_integration.convergenceApproximation(
        f=g,
        rule=fe621.numerical_integration.simpsonsRule,
        start=start,
        stop=stop,
        epsilon=epsilon
    )

    # Building DataFrame of results for output
    results = pd.DataFrame(np.abs(np.array([trapezoidal_result,
                                            simpsons_result])))

    # Setting row and column names
    results.columns = ['Estimated Area', 'Segments']
    results.index = ['Trapezoidal Rule', 'Simpson\'s Rule']

    # Saving to CSV
    results.to_csv('Homework 1/bin/numerical_integration/arb_convergence.csv',
                   header=True, index=True, float_format='%.8e')


if __name__ == '__main__':
    # Part 4 - Arbitrary Function
    arbitraryFunctionSegmentAnalysis()
```

question_solutions/question_3_arbitrary_area.py