Logarithms

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1 Problem

Evaluate: $\log_a\left(\frac{\sqrt{140}}{2\sqrt{30}}\right)+\log_a\left(\frac{3\sqrt{12}}{2\sqrt{27}}\right)+\log_a\left(\frac{a^3\sqrt{b^2}}{b\sqrt{a^2}}\right)$
$= \log_a\left(\frac{\sqrt{2^2\times5\times7}}{2\sqrt{2\times3\times5}}\right)+\log_a\left(\frac{3\sqrt{2^2\times3}}{2\sqrt{3^3}}\right)+\log_a\left(\frac{a^3\times b}{b\times a}\right)$
$= \log_a\left(\frac{\sqrt{2^2\times5\times7}}{\sqrt{2^3\times3\times5}}\right)+\log_a\left(\frac{\sqrt{2^2\times3^3}}{\sqrt{2^2\times3^3}}\right)+\log_a\left(a^2\right)$
$= \log_a\left(\frac{2^2\times5\times7}{2^3\times3\times5}\right)^{\frac{1}{2}}+\log_a(1)+2\log_a(a)$
$= \log_a\left(\frac{7}{6}\right)^{\frac{1}{2}}+0+2\times1$
$= \frac{1}{2}\left(\log_a 7-\log_a 6\right)+2$

2 Problem

Evaluate: $2\log_{10}3+3\log_{10}4+2\log_{10}5$
$= \log_{10}3^2+\log_{10}4^3+\log_{10}5^2$
$= \log_{10}\left(3^2\times4^3\times5^2\right)$
$= \log_{10}14400$
$= \log_{10}(120)^2$
$= 2\log_{10}120$

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3 Problem

let,
Initial principle $= p$
$\therefore$ Compound principle (A) $= p+p\times40\%$
Interest rate (r) $= 12\%$
time (n) $=$?

We Know that,

$$A = p\,(1+r)^n$$

$$or,\ p+p\times40\% = p\,(1+r)^n$$

$$or,\ p+p\times\frac{2}{5} = p\,(1+12\%)^n$$

$$or,\ \frac{5p+2p}{5} = p\,(1+12\%)^n$$

$$or,\ \frac{7p}{5} = p\,(1+0.12)^n$$

$$or,\ \frac{7}{5} = (1.12)^n$$

$$or,\ \log\frac{7}{5} = \log{(1.12)}^n$$
$$or,\ \log\frac{7}{5} = n\log{(1.12)}$$
$$or,\ n = \frac{\log\frac{7}{5}}{log(1.12)}$$
$$\therefore\ n \approx 3$$

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4 Problem

let,

Initial price $=P$

$\therefore$ After 5 years, decreased price $(P_T) = P - P \times 60\%$

We know that,

$$P_T = P\,(1-R)^T$$
$$or,\ \frac{P}{2} = P(1-R)^5$$
$$or,\ \frac{1}{2} = (1-R)^5$$
$$or,\ \left(\frac{1}{2}\right)^{\frac{1}{5}} = (1-R)$$
$$or,\ R = 1 - \left(\frac{1}{2}\right)^{\frac{1}{5}}$$
$$\therefore R = 0.129$$

As initial price $=P$

$\therefore$ After n years, the decreased price will be $= P - P \times 60\%$

According to the question,

$$P - P \times 60\% = P(1-0.129)^n$$
$$or,\ P - \frac{3P}{5} = P(0.871)^n$$
$$or,\ \frac{2P}{5} = P(0.871)^n$$
$$or,\ \frac{2}{5} = (0.871)^n$$
$$or,\ (0.871)^n = \frac{2}{5}$$
$$or,\ n = \log_{0.871}\frac{2}{5}$$
$$\therefore n \approx 6.63$$

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5 Problem

let,

Intensity of the first earthquake $= I_5$
Intensity of the second earthquake $= I_7$
Intensity of an ideal earthquake $= S$

$\therefore$ The Richter magnitude of the first earthquake is $\log_{10}\left(\frac{I_5}{S}\right) = 5$ ... (i)
$\therefore$ The Richter magnitude of the first earthquake is $\log_{10}\left(\frac{I_7}{S}\right) = 7$ ... (ii)

(ii)-(i)

$$\log_{10}\left(\frac{I_7}{S}\right) - \log_{10}\left(\frac{I_5}{S}\right) = 7 - 5$$

$$or,\ \log_{10}\frac{\left(\frac{I_7}{S}\right)}{\left(\frac{I_5}{S}\right)} = 2$$

$$or,\ \log_{10}\left(\frac{I_7}{S} \times \frac{S}{I_5}\right) = 2$$

$$or,\ \log_{10}\left(\frac{I_7}{I_5}\right) = 2$$

$$or,\ \frac{I_7}{I_5} = 10^2$$

$$or,\ \frac{I_7}{I_5} = 100$$

$$\therefore I_7 = 100 \times I_5$$

Again,
Intensity of the first earthquake $= I_5$
Intensity of the second earthquake $= I_8$
Intensity of an ideal earthquake $= S$

$\therefore$ The Richter magnitude of the first earthquake is $\log_{10}\left(\frac{I_5}{S}\right) = 5$ ... (i)
$\therefore$ The Richter magnitude of the first earthquake is $\log_{10}\left(\frac{I_8}{S}\right) = 8$ ... (ii)

(ii)-(i)

$$\log_{10}\left(\frac{I_8}{S}\right) - \log_{10}\left(\frac{I_5}{S}\right) = 8 - 5$$

$$or,\ \log_{10}\frac{\left(\frac{I_8}{S}\right)}{\left(\frac{I_5}{S}\right)} = 3$$

$$or,\ \log_{10}\left(\frac{I_8}{S} \times \frac{S}{I_5}\right) = 3$$

$$or,\ \log_{10}\left(\frac{I_8}{I_5}\right) = 3$$

$$or,\ \frac{I_8}{I_5} = 10^3$$

$$or,\ \frac{I_8}{I_5} = 1000$$

$$\therefore I_8 = 1000 \times I_5$$

(Showed)

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6 Problem

let,

Intensity of the earthquake measured in Manikganj = I_1
Intensity of the earthquake measured in Rangamati = I_2
Intensity of an ideal earthquake = S

$\therefore$ The Richter magnitude of the measured in Manikganj is $\log_{10}\left(\frac{I_1}{S}\right) = 7.0$ ... (i)
$\therefore$ The Richter magnitude of the earthquake measured in Rangamati is $\log_{10}\left(\frac{I_2}{S}\right) = 5.1$ ... (ii)

(i)-(ii)

$$\log_{10}\left(\frac{I_1}{S}\right) - \log_{10}\left(\frac{I_2}{S}\right) = 7.0 - 5.1$$
$$or,\ \log_{10}\frac{\left(\frac{I_1}{S}\right)}{\left(\frac{I_2}{S}\right)} = 1.9$$
$$or,\ \log_{10}\left(\frac{I_1}{S} \times \frac{S}{I_2}\right) = 1.9$$
$$or,\ \log_{10}\left(\frac{I_1}{I_2}\right) = 1.9$$
$$or,\ \frac{I_7}{I_5} = 10^{1.9}$$
$$or,\ \frac{I_1}{I_2} = 79.43$$
$$or,\ \frac{I_1}{I_2} \approx 80$$
$$\therefore I_1 \approx 80 \times I_2$$

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7 Problem

We know that

Sound level is $= d\log_{10})\left(\frac{I}{S}\right)$
here,
$I = 2.35 \times 10^{-6} w/m^2$
$S = 10^{-12} w/m^2$

$\therefore d = 10\log_{10}\left(\frac{2.35\times10^{-6} w/m^2}{10^{-12} w/m^2}\right)$
$= 10\log_{10}\left(\frac{2.35\times10^{-6}}{10^{-12}}\right)$
$= 10\log_{10}\left(2.35 \times 10^6\right)$
$= 10 \times 6.371$
$= 63.71$
≈ 64
$\therefore$ The sound level is approximately 64 decibel

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8 Problem

8.1

Evaluate: $2\sqrt[3]{343} + 2\sqrt[5]{243} - 12\sqrt[6]{64}$
$= 2\sqrt[3]{7^3} + 2\sqrt[5]{3^5} - 12\sqrt[6]{2^6}$
$= 2 \times 7 + 2 \times 3 + 2 \times 2$
$= 24$

8.2

Evaluate: $\frac{y^{a+b}}{y^{2c}} \times \frac{y^{b+c}}{y^{2a}} \times \frac{y^{c+a}}{y^{2b}}$
$= y^{a+b-2c} \times y^{b+c-2a} \times y^{c+a-2b}$
$= y^{a+b-2c+b+c-2a+c+a-2b}$
$= y^0$
$= 1$

9 Problem

Evaluate: $\left(\frac{z^a}{z^b}\right)^{a+b-c} \times \left(\frac{z^b}{z^c}\right)^{b+c-a} \times \left(\frac{z^a}{z^a}\right)^{c+a-b}$
$= z^{(a-b)(a+b-c)} \times z^{(b-c)(b+c-a)} \times z^{(c-a)(c+a-b)}$
$= z^{a^2-b^2-ac+bc} \times z^{b^2-c^2-ab+ac} \times z^{c^2-a^2-bc+ab}$
$= z^{a^2-b^2-ac+bc+b^2-c^2-ab+ac+c^2-a^2-bc+ab}$
$= z^0$
$= 1$

10 Problem

10.1

$2^x = 64$
or, $\log 2^x = \log 64$
or, $x \log 2 = \log 2^6$
or, $x = \frac{6 \log 2}{\log 2}$
$\therefore x = 6$

10.2

$(1.2)^x = 100$
or, $\log (1.2)^x = \log 100$
or, $x \log (1.2) = \log 10^2$
or, $x = \frac{2 \log 10}{\log (1.2)}$
$\therefore x = 25.259$

10.3

$7^x = 5$
or, $\log 7^x = \log 5$
or, $x \log 7 = \log 5$
or, $x = \frac{log5}{\log 7}$
$\therefore x = 0.827$

10.4

$\left(\frac{2}{3}\right)^x = 7$
or, $\log \left(\frac{2}{3}\right)^x = \log 7$
or, $x \log \left(\frac{2}{3}\right) = \log 7$
or, $x = \frac{\log 7}{\log \left(\frac{2}{3}\right)}$
$\therefore x = -4.8$

11 Problem

let,
Initial principle = p
$\therefore$ Compound principle (A) = $3p$
Interest rate (r) = 10%
time (n) = ?

We Know that,

$$A = p\,(1+r)^n$$
$$or,\ 3p = p\,(1+10\%)^n$$
$$or,\ 3 = (1+0.1)^n$$
$$or,\ \log 3 = \log\,(1.1)^n$$
$$or,\ \log 3 = n\log\,(1.1)$$
$$or,\ n = \frac{\log 3}{log(1.1)}$$
$$\therefore\ n \approx 11.53$$

12 Problem

After 1 day the number of affected people will be $= 3^1$
After 2 days the number of affected people will be $= 3^2$
After 3 days the number of affected people will be $= 3^3$
$\therefore$ After 30 days the number of affected people will be $= 3^{30}$
$= 2.0589 \times 10^1 4$

After 1 day the number of affected people will be $= 3^1$
$\therefore$ After n days the number of affected people will be $= 3^n$

According to the question,

$$3^n = 10^7$$
$$or,\ \log 3^n = \log 10^7$$
$$or,\ n\log 3 = 7\log 10$$
$$or,\ n = \frac{7\log 10}{log 3}$$
$$\therefore\ n = 14.67$$

13 Problem

We know that ,
1 Bigha = 20 Katha
$\therefore$ 3 Bigha = 20×3 Katha
= 60 Katha

1 kg fertilizer increase the fertility by = 3%
$\therefore$ 30 kg fertilizer increase the fertility by = $30 \times 3\%$
= 90%
Given that,
The amount of fertile land (P) = 60 Katha
The fertility reduction rate (R) = 90%
time $(n) = 1$
We know that,

$$Depriciation \quad (P_T) = P(1-R)^n$$
$$or,\ (P_T) = 60(1-90\%)^1$$
$$= 60 \times (1 - \frac{90}{100})^1$$
$$= 60 \times \frac{1}{10}$$
$$= 6$$

$\therefore$ The amount of time it would take the land to lose it's fertility is $= \frac{60}{6}$ years $= 10$ years

14 Problem

let,

Intensity of the earthquake measured in Sreemangal $= I_1$
Intensity of the earthquake measured in Chattogram $= I_2$
Intensity of an ideal earthquake $= S$

$\therefore$ The Richter magnitude of the earthquake measured in Sreemangal is $\log_{10}\left(\frac{I_1}{s}\right) = 7.6$ ... (i)
$\therefore$ The Richter magnitude of the earthquake measured in Chattogram is $\log_{10}\left(\frac{I_2}{s}\right) = 6.0$ ... (ii)

(i)-(ii)

$$\log_{10}\left(\frac{I_1}{s}\right) - \log_{10}\left(\frac{I_2}{s}\right) = 7.6 - 6.0$$

$$or,\ \log_{10}\frac{\left(\frac{I_1}{S}\right)}{\left(\frac{I_2}{S}\right)} = 1.6$$

$$or,\ \log_{10}\left(\frac{I_1}{S} \times \frac{S}{I_2}\right) = 1.6$$

$$or,\ \log_{10}\left(\frac{I_1}{I_2}\right) = 1.6$$

$$or,\ \frac{I_7}{I_5} = 10^{1.6}$$

$$or,\ \frac{I_1}{I_2} = 39.81$$

$$or,\ \frac{I_1}{I_2} \approx 40$$

$$\therefore I_1 \approx 40 \times I_2$$

15 Problem

let,

Intensity of the first earthquake $= I_1$
$\therefore$ Intensity of the second earthquake $= 6I_1$
Intensity of an ideal earthquake $= S$

$\therefore$ The Richter magnitude of the first earthquake is $= \log_{10}\left(\frac{I_1}{S}\right)$
$\therefore$ The Richter magnitude of the second earthquake is $= \log_{10}\left(\frac{6I_1}{S}\right)$
According to the question,

$$\log_{10}\left(\frac{I_1}{S}\right) = 8$$

$$or,\ \frac{I_1}{S} = 10^8$$

$$or,\ \frac{6I_1}{S} = 6 \times 10^8$$

$$or,\ \log\left(\frac{6I_1}{S}\right) = \log(6 \times 10^8)$$

$$or,\ \log\left(\frac{6I_1}{S}\right) = 8.78$$

$\therefore$ The Richter magnitude of the second earthquake is 8.78

16 Problem

let,

Intensity of the earthquake measured in Cox's Bazar = I_1
Intensity of the earthquake measured in Turkey = $398I_1$
Intensity of an ideal earthquake = S

$\therefore$ The Richter magnitude of earthquake measured in Cox's Bazar is = $\log_{10}\left(\frac{I_1}{S}\right)$
$\therefore$ The Richter magnitude of earthquake measured in Turkey is = $\log_{10}\left(\frac{398I_1}{S}\right)$
According to the question,

$$\log_{10}\left(\frac{I_1}{S}\right) = 5.2$$

$$or,\ \frac{I_1}{S} = 10^{5.2}$$

$$or,\ \frac{398I_1}{S} = 398 \times 10^{5.2}$$

$$or,\ \log\left(\frac{398I_1}{S}\right) = \log(398 \times 10^{5.2})$$

$$or,\ \log\left(\frac{6I_1}{S}\right) = 7.8$$

$\therefore$ The Richter magnitude of earthquake measured in Turkey is 7.8