X

M

K

X

$K \cap$

$M =$

$\emptyset$

$N \in$

X

$K \cap$

$N =$

$\emptyset.$

C

C

x_1

x_2

C

$x =$

$\alpha x_1 +$

$1 - \alpha x_2$

$0 \leq$

$\alpha \leq$

1

X

$f :$

$X \to$

X

$f :$

$X \to$

$\sup_{x \in X} x, x - f(x)$

$f :$

$X \to$

$f :$

$X \to$

x, y

X

$t \in$

$[0, 1]$

$\sup_{x \in X} tx + (1-t)y, x - f(x)$

$= \sup_{x \in X} tx + (1-t)y, x - tf(x) - (1-t)f(x)$

$= \sup_{x \in X} tx, x + (1-t)y, x - tf(x) - (1-t)f(x)$

$\leq \sup_{x,y \in X} tx, x + (1-t)y, y - tf(x) - (1-t)f(y)$

$= t \sup_{x \in X} x, x - f(x) +$

$(1-$

$t)t \sup_{y \in X} y, y - f(y)$

$= tf(x) +$

$(1-$

$t)f(y).$

f

f

X

f^*

f

ϱ

$\varrho(x) \leq$

$f(x)$

$\forall x \in$

X

$f^{**}(x) \leq$

$f(x)$

$\varrho(x) \leq$

$f^{**}(u)$

$\forall x \in$

U

$f^{**} =$

f

$f :^d \to$

$f^{**} =$

f

$A \subseteq^d$

$\times^d$

$N \in$

σ

$(x_1, p_1), \dots, (x_N, p_N) \in$

A

N

$$\sum_{i=1}^{N} x_i, p_i \geq \sum_{i=1}^{N} x_i, p_{\sigma_i}.$$

(1)

$f :$

$X \to$

$(-\infty, \infty]$

$\partial f :$

$X^* \to$

$\partial f(x) =$

$x^* \in X^*; f(x) - f(y) \leq x^*, x - y, \forall y \in X$

$d \times$

Here
we